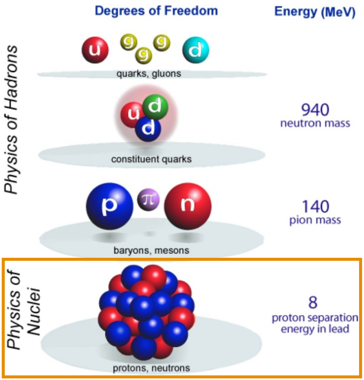
Olivier VASSEUR
IPN Orsay, Theory Group
Second year PhD student

A microscopic treatment of correlated nucleons: Collective properties in stable and exotic nuclei

PhD supervised by **Marcella GRASSO (IPN Orsay)**
Co-supervised by **Danilo GAMBACURTA (ELI-NP, Romania)**

Contents

General assumptions

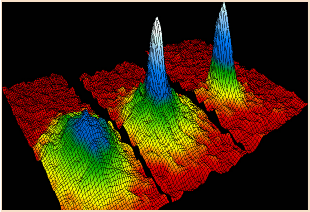


Framework

- Low-energy scales
  - $\longrightarrow$ nucleons are point-like, structureless particles
    relevant degrees of freedom = nucleons
- Solve the nuclear many-body problem
  - $\longrightarrow$ Use of **effective interactions**
    $\longrightarrow$ Energy-Density Functionals (EDF): functionals derived in most cases from effective interactions

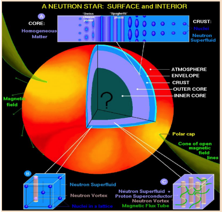
Interdisciplinarity of many-body techniques

Atomic physics



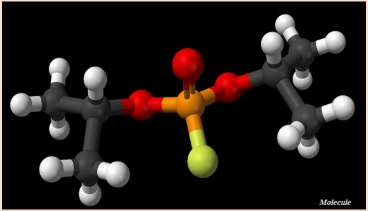
Bose-Einstein condensate of ultra-cold trapped atoms

Astrophysics (neutron stars)



Nuclei and other nuclear systems in star crusts

Chemistry & Condensed matter physics



Strong analogy between Energy-Density Functionals (EDF) and Density Functional Theory (DFT)

General motivation: What we are interested in

Describe nuclear excitation spectra: low-lying states and giant resonances.

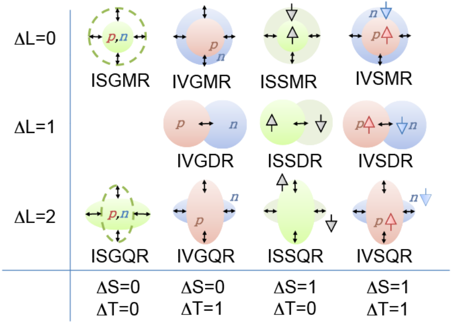
Methodology

Going **beyond the mean-field approximation** (single-particle degrees of freedom): complex configurations and correlations, within the Energy-Density Functional theory (EDF).

Objective

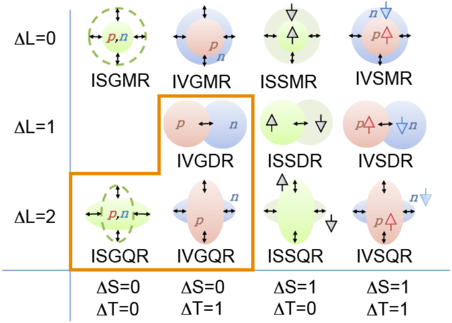
Owing to the coupling of single-particle degrees of freedom with more complex configurations: physical description of **fragmentation** and **spreading width** of excitations.

Phenomenology



Schematic view of giant resonances

- Low-lying states
- Giant resonances:
  - → higher in energy
  - → more collective states



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Schematic view of giant resonances

SRPA formalism

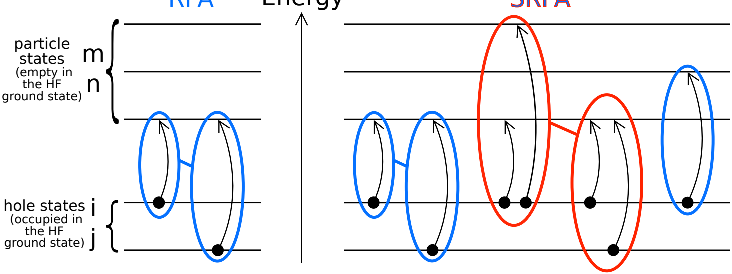
SRPA model: Include complex configurations (2 particle-2 hole) (2p2h)

Da Providencia, *Nucl. Phys.* 61, 87 (1965) Yannouleas, *Phys. Rev. C* 35, 1159 (1987)

1. Choose the form of the excitation operator $Q_\nu^\dagger$ that creates the excited state $|\nu\rangle$ on top of the ground state $|0\rangle$:

$$Q_\nu^\dagger := \sum_{m,i} \left(X_{mi}(\nu)\, a_m^\dagger a_i - Y_{mi}(\nu)\, a_i^\dagger a_m \right) + \sum_{\substack{m,\, n>m \\ i,\, j>i}} \left(X_{mnij}(\nu)\, a_m^\dagger a_n^\dagger a_j a_i - Y_{mnij}(\nu)\, a_i^\dagger a_j^\dagger a_n a_m \right)$$

where $\begin{cases} Q_\nu^\dagger \,|0\rangle = |\nu\rangle \\ Q_\nu \,|0\rangle = 0 \end{cases}$



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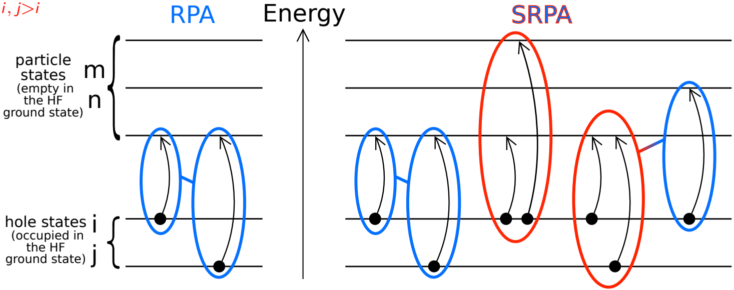


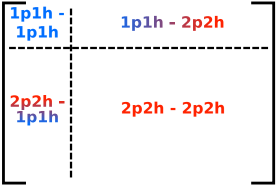
SRPA formalism

❷ Get the **RPA-type equations** = matrix form of equations of motion

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix}$$

- Same equations for RPA and SRPA, **much larger dimension** in SRPA

$$A = \begin{pmatrix} (A_{mi,nj}) & (A_{mi,pqkl}) \\ (A_{pqkl,mi}) & (A_{mnij,pqkl}) \end{pmatrix} =$$



- Well known method, but very strong truncations and approximations in early times due to **important computational effort** in SRPA
- Calculations without truncations in matrices and large cutoffs: only recently **Papakonstantinou, Roth, *Phys. Lett. B* 671, 356 (2009)** **Gambacurta, Grasso, Catara, *Phys. Rev. C* 81, 054312 (2010)**

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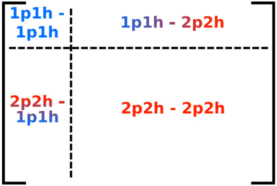

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  $\rightarrow$ **problems highlighted**

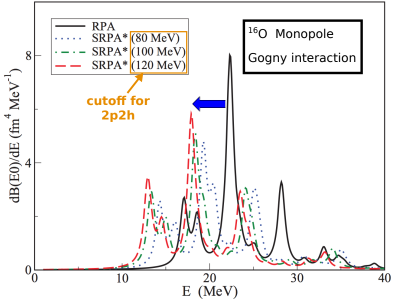
- Strong abnormal shift to low energies
- Instabilities (non-real eigenstates)

} The Thouless theorem cannot be extended from RPA to SRPA

- Double-counting of correlations
- Cutoff dependence with zero-range interactions

} Use of an effective interaction "EDF problems"

Tselyaev, *Phys. Rev. C* 88, 054301 (2013)
Papakonstantinou, *Phys. Rev. C* 90, 024305 (2014)



Gambacurta, Grasso, *et al.*, *Phys. Rev. C* 86, 021304 (R) (2012)

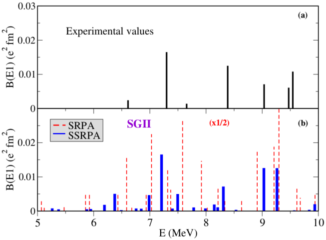
We consider **two methods to address these problems**:

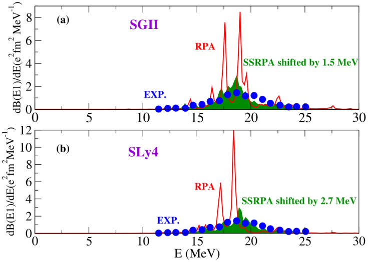
- Subtraction method
- Include correlations in the ground state

In the EDF framework, it allows to cure all the problems of SRPA
Gambacurta, Grasso, Engel, *Phys. Rev. C* 92, 034303 (2015)

One example of application: dipole response in ^{48}Ca

Gambacurta, Grasso, Vasseur, *Phys. Lett. B*, 777, 163 (2018)





→ Better description of low-lying states with respect to SRPA (and to RPA!)

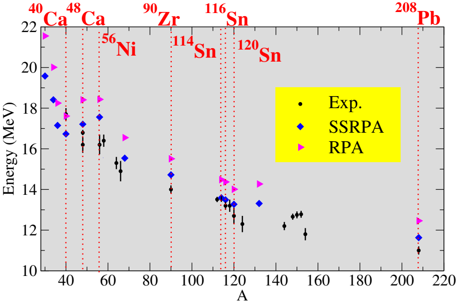
→ Better description of fragmentation and width with respect to RPA

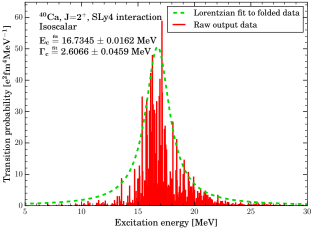
Exp: { **Hartmann *et al.*, *Phys. Rev. C* 65, 034301 (2002)**
Birkhan *et al.*, *Phys. Rev. Lett.* 118, 252501 (2017)

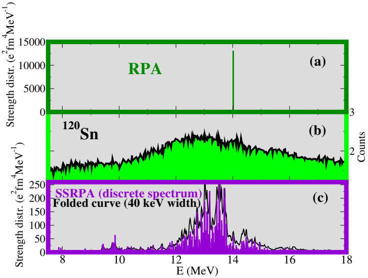
Results (preliminary): Quadrupole response

Work in progress on **isoscalar giant quadrupole resonances** for a range of nuclei from ^{30}Si to to ^{208}Pb.

Centroids: always lower in subtracted SRPA than in RPA







Widths: larger in subtracted SRPA than in RPA

Vasseur, Gambacurta, Grasso, in progress

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix}$$

In RPA: $G_{mi,nj} = \delta_{mn}\boldsymbol{\rho_{ji}} - \delta_{ji}\boldsymbol{\rho_{mn}}$ $\longleftarrow$ **One-body density matrix**

$\rho_{\alpha\alpha'} \simeq \delta_{\alpha\alpha'}\boldsymbol{n_\alpha}$ $\longleftarrow$ **occupation number**

In standard RPA: $n_\alpha \in \{0,1\}$. In renormalized RPA: $n_\alpha \in [0,1]$.

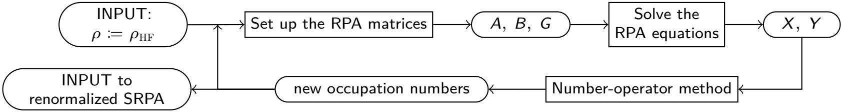
Number operator method: **Rowe, *Phys. Rev.* 175, 1283 (1968)**

$$n_m = \frac{1}{2(2j_m+1)} \sum_\nu (2j_\nu+1) \sum_i |Y_{mi}(\nu)|^2 \cancel{+O(|Y|^4)}$$

$$n_i = 1 - \frac{1}{2(2j_i+1)} \sum_\nu (2j_\nu+1) \sum_m |Y_{mi}(\nu)|^2 \cancel{+O(|Y|^4)}$$

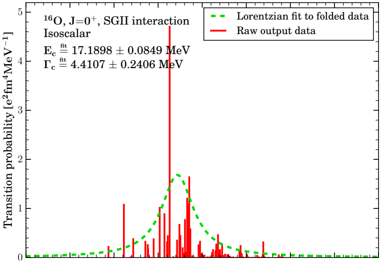
Used as input to SRPA calculation

Gambacurta, Catara, *Phys. Rev. B* 81, 085418 (2010)

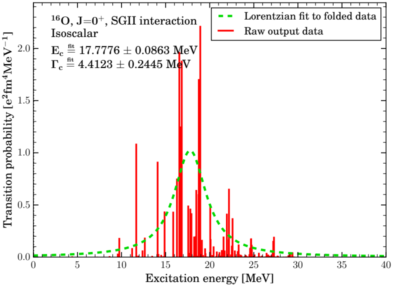


Renormalized SRPA: Example (preliminary)

Isoscalar monopole response in ^{16}O
SGII interaction



Standard SRPA
Centroid $\simeq$ **17.2 MeV**



Renormalized SRPA
Centroid $\simeq$ **17.8 MeV**

$\rightarrow$ Consistent shift to higher energies
$\rightarrow$ But rather small correction

Include pairing and non-zero temperature

- Correlations produced by the RPA amplitudes are (too) weak. This may be due to the used diagonal approximation for the density (analysis in progress)
- Additional correlations could be included via occupation numbers $\rightarrow$ next step in the PhD

Next step

Renormalize SRPA with occupation numbers with pairing correlations (Hartree-Fock-Bogoliubov/Bogoliubov-de Gennes occupation numbers), at non-zero temperature:

- pairing or superfluid effects at $T = 0$
- $T \neq 0$ $\rightarrow$ Study of hot resonances (experimental results exist *e.g.* from an Italian-Polish collaboration. Maj *et al.* and Bracco *et al.*)

Summary

- It is necessary to take more complex configurations (*2p2h*) into account to describe the spreading width and the fragmentation of spectra
- Standard SRPA — straightforward transposition of standard RPA to *2p2h* configurations — presents important drawbacks, due to the formalism itself or due to the effective interaction
- To address these drawbacks, we have considered two methods:
  1. Subtraction method $\longrightarrow$ cures all the problems, gives a better description of spectra
  2. Inclusion of correlations in the ground state with RPA occupation numbers $\longrightarrow$ preliminary results are as expected, but weaker correction
- Occupation-number method easily generalized to include pairing and non-zero temperature (in progress)

Summary

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Thank you for your attention!

Backup slide: What is the subtraction method

- In the EDF framework, it allows to cure all the problems of SRPA
- It was developed prior to this thesis Gambacurta, Grasso, Engel, *Phys. Rev. C* 92, 034303 (2015)

How it works:

1. Response functions: $\rho^{(1)}_{kl} = \sum_{p,q} R_{klpq} f^{pq}$ — $\rho^{(1)}$: transition density; f: external field

$$R^{\mathrm{RPA}}(E) = \begin{pmatrix} E - A & -B \\ -B & -E - A \end{pmatrix}^{-1}$$

$$R^{\mathrm{SRPA}}(E) = \left[\begin{pmatrix} E - A & -B \\ -B & -E - A \end{pmatrix} - \begin{pmatrix} \Sigma(E) & 0 \\ 0 & \Sigma(E) \end{pmatrix} \right]^{-1}$$

$\Sigma(E)$: energy-dendenpent 2nd-order self-energy

2. Double-counting is canceled if one requires $R^{\mathrm{SRPA}}(0) = R^{\mathrm{RPA}}(0)$
This may be guaranteed by a subtraction method
$\longrightarrow$ It was demonstrated that the Thouless theorem is satisfied under this condition Tselyaev, *Phys. Rev. C* 88, 054301 (2013)
$\longrightarrow$ Cutoff dependence is also eliminated

Renormalized SRPA

The occupation numbers calculated iteratively in a renormalized RPA calculation are used in SRPA, to renormalize all the matrix elements

For example, the **1p1h-1p1h** part of A reads:

$$A_{mi,nj} = \delta_{ij}\delta_{mn}(t_m - t_i)\boldsymbol{(n_i - n_m)} + \overline{v}_{mjni}\boldsymbol{(n_i - n_m)(n_j - n_n)}$$

$\longrightarrow$ **renormalizing factors**

$\longrightarrow$ Do the same for all the other terms in the A, B and G matrices

$\Rightarrow$ Include correlations in the ground state