A microscopic treatment of correlated nucleons: Collective properties in stable and exotic nuclei

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1. Introduction: Phenomenology & context

2. Starting point: Formalism of the standard Second Random-Phase Approximation (SRPA). Limitations and drawbacks

3. 1st correction method: Subtraction

4. 2nd correction method: Renormalized SRPA

5. Summary
1 Introduction: Phenomenology & context

2 Starting point: Formalism of the standard Second Random-Phase Approximation (SRPA). Limitations and drawbacks

3 1st correction method: Subtraction

4 2nd correction method: Renormalized SRPA

5 Summary
General assumptions

Framework

- Low-energy scales
  → nucleons are point-like, structureless particles
  relevant degrees of freedom
  = nucleons

- Solve the nuclear many-body problem
  → Use of effective interactions
  → Energy-Density Functionals (EDF): functionals derived in most cases from effective interactions
Interdisciplinarity of many-body techniques

Atomic physics

Bose-Einstein condensate of ultra-cold trapped atoms

Chemistry & Condensed matter physics

Strong analogy between Energy-Density Functionals (EDF) and Density Functional Theory (DFT)

Astrophysics (neutron stars)

Nuclei and other nuclear systems in star crusts
Aim and method

General motivation: What we are interested in
Describe nuclear excitation spectra: low-lying states and giant resonances.

Methodology
Going beyond the mean-field approximation (single-particle degrees of freedom): complex configurations and correlations, within the Energy-Density Functional theory (EDF).

Objective
Owing to the coupling of single-particle degrees of freedom with more complex configurations: physical description of fragmentation and spreading width of excitations.
### Phenomenology

**Schematic View of Giant Resonances**

<table>
<thead>
<tr>
<th>ΔL=0</th>
<th>ΔL=1</th>
<th>ΔL=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISGMR</td>
<td>IVGMR</td>
<td>ISSMR</td>
</tr>
<tr>
<td>ISGDR</td>
<td>IVGDR</td>
<td>ISSDR</td>
</tr>
<tr>
<td>ISGQR</td>
<td>IVGQR</td>
<td>ISSQR</td>
</tr>
</tbody>
</table>

- **Low-lying states**
- **Giant resonances:**
  - Higher in energy
  - More collective states

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**Olivier VASSEUR**

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Phenomenology

Schematic view of giant resonances

- Low-lying states
- Giant resonances:
  - higher in energy
  - more collective states

ΔL=0
- ISGMR
- IVGMR
- ISSMR
- IVSMR

ΔL=1
- IVGDR
- ISSDR
- IVSDR

ΔL=2
- ISGQR
- IVGQR
- ISSQR
- IVSQR

ΔS=0
ΔT=0
ΔS=0
ΔT=1
ΔS=1
ΔT=0
ΔS=1
ΔT=1

Schematic view of giant resonances
1 Introduction: Phenomenology & context

2 Starting point: Formalism of the standard Second Random-Phase Approximation (SRPA). Limitations and drawbacks
   SRPA formalism
   Problems of standard SRPA

3 1st correction method: Subtraction

4 2nd correction method: Renormalized SRPA

5 Summary
Choose the form of the excitation operator \( Q^\dagger_\nu \) that creates the excited state \( |\nu\rangle \) on top of the ground state \( |0\rangle \):

\[
Q^\dagger_\nu := \sum_{m,i} (X_{mi}(\nu) \ a^\dagger_m a_i - Y_{mi}(\nu) \ a^\dagger_i a_m ) \\
+ \sum_{m,n>m \ i,j>i} (X_{mnij}(\nu) \ a^\dagger_m a^\dagger_n a_j a_i - Y_{mnij}(\nu) \ a^\dagger_i a^\dagger_j a_n a_m )
\]

where

\[
\begin{align*}
Q^\dagger_\nu \ |0\rangle &= |\nu\rangle \\
Q_\nu \ |0\rangle &= 0
\end{align*}
\]
SRPA formalism

Choose the form of the excitation operator $Q_{\nu}^\dagger$ that creates the excited state $|\nu\rangle$ on top of the ground state $|0\rangle$:

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Q_{\nu}^\dagger := \sum_{m,i} \left( X_{mi}(\nu) a_m^\dagger a_i - Y_{mi}(\nu) a_i^\dagger a_m \right) + \sum_{m,n>m, i,j>i} \left( X_{mnij}(\nu) a_m^\dagger a_n^\dagger a_j a_i - Y_{mnij}(\nu) a_i^\dagger a_j^\dagger a_n a_m \right)
$$

where

$$
\begin{cases}
Q_{\nu}^\dagger |0\rangle = |\nu\rangle \\
Q_{\nu} |0\rangle = 0
\end{cases}
$$
SRPA formalism

2 Get the **RPA-type equations** = matrix form of equations of motion

\[
\begin{pmatrix}
A & B \\
B^* & A^*
\end{pmatrix}
\begin{pmatrix}
X(\nu) \\
Y(\nu)
\end{pmatrix}
= \hbar \omega_\nu
\begin{pmatrix}
G & 0 \\
0 & -G^*
\end{pmatrix}
\begin{pmatrix}
X(\nu) \\
Y(\nu)
\end{pmatrix}
\]

- Same equations for RPA and SRPA, **much larger dimension** in SRPA

\[
A = \begin{pmatrix}
(A_{mi,nj}) & (A_{mi,pqkl}) \\
(A_{pqkl,mi}) & (A_{mnij,pqkl})
\end{pmatrix}
= \begin{pmatrix}
\begin{array}{ll}
1p1h - 1p1h & 1p1h - 2p2h \\
2p2h - 1p1h & 2p2h - 2p2h
\end{array}
\end{pmatrix}
\]

- Well known method, but very strong truncations and approximations in early times due to **important computational effort** in SRPA

- Calculations without truncations in matrices and large cutoffs: only recently


SRPA formalism

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\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix} = \hbar \omega_\nu \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix}
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  \[\rightarrow\] problems highlighted
Problems of standard SRPA

- Strong abnormal shift to low energies
- Instabilities (non-real eigenstates)
- Double-counting of correlations
- Cutoff dependence with zero-range interactions

The Thouless theorem cannot be extended from RPA to SRPA

Use of an effective interaction “EDF problems”


We consider two methods to address these problems:
- Subtraction method
- Include correlations in the ground state

Introduction: Phenomenology & context

Starting point: Formalism of the standard Second Random-Phase Approximation (SRPA). Limitations and drawbacks

1st correction method: Subtraction

What we get with the subtraction method
Results (preliminary): Quadrupole response

2nd correction method: Renormalized SRPA

Summary
What we get with the subtraction method

In the EDF framework, it allows to cure all the problems of SRPA

One example of application: dipole response in $^{48}$Ca

→ Better description of low-lying states with respect to SRPA (and to RPA!)

→ Better description of fragmentation and width with respect to RPA

*Exp*: 
Results (preliminary): Quadrupole response

Work in progress on **isoscalar giant quadrupole resonances** for a range of nuclei from $^{30}\text{Si}$ to $^{208}\text{Pb}$.

Centroids: always lower in subtracted SRPA than in RPA

Widths: larger in subtracted SRPA than in RPA

Vasseur, Gambacurta, Grasso, in progress
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4 2nd correction method: Renormalized SRPA
   Renormalize SRPA in an iterative way
   Renormalized SRPA: Example (preliminary)
   Include pairing and non-zero temperature

5 Summary
Renormalize SRPA in an iterative way

\[
\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix} = \hbar \omega_\nu \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix} \begin{pmatrix} X(\nu) \\ Y(\nu) \end{pmatrix}
\]

In RPA: \( G_{mi,nj} = \delta_{mn}\rho_{ji} - \delta_{ji}\rho_{mn} \) ← One-body density matrix

\( \rho_{\alpha\alpha'} \simeq \delta_{\alpha\alpha'} n_\alpha \) ← occupation number

In standard RPA: \( n_\alpha \in \{0,1\} \). In renormalized RPA: \( n_\alpha \in [0,1] \).

Number operator method: \( n_m = \frac{1}{2(2j_m + 1)} \sum_{\nu} (2j_\nu + 1) \sum_i |Y_{mi}(\nu)|^2 + O(|Y|^4) \)

\( n_i = 1 - \frac{1}{2(2j_i + 1)} \sum_{\nu} (2j_\nu + 1) \sum_m |Y_{mi}(\nu)|^2 + O(|Y|^4) \)

Used as input to SRPA calculation


INPUT: \( \rho \equiv \rho_{HF} \)

Set up the RPA matrices \( A, B, G \)

Solve the RPA equations \( X, Y \)

INPUT to renormalized SRPA

new occupation numbers

Number-operator method
Renormalized SRPA: Example (preliminary)

Isoscalar monopole response in $^{16}\text{O}$ SGII interaction

Standard SRPA
Centroid $\simeq 17.2$ MeV

Renormalized SRPA
Centroid $\simeq 17.8$ MeV

→ Consistent shift to higher energies
→ But rather small correction
Include pairing and non-zero temperature

- Correlations produced by the RPA amplitudes are (too) weak. This may be due to the used diagonal approximation for the density (analysis in progress)
- Additional correlations could be included via occupation numbers → next step in the PhD

Next step

Renormalize SRPA with occupation numbers with pairing correlations (Hartree-Fock-Bogoliubov/Bogoliubov-de Gennes occupation numbers), at non-zero temperature:

- pairing or superfluid effects at $T = 0$
- $T \neq 0$ → Study of hot resonances (experimental results exist e.g. from an Italian-Polish collaboration. Maj et al. and Bracco et al.)
Summary

- It is necessary to take more complex configurations ($2p2h$) into account to describe the spreading width and the fragmentation of spectra.
- Standard SRPA — straightforward transposition of standard RPA to $2p2h$ configurations — presents important drawbacks, due to the formalism itself or due to the effective interaction.
- To address these drawbacks, we have considered two methods:
  1. Subtraction method → cures all the problems, gives a better description of spectra.
  2. Inclusion of correlations in the ground state with RPA occupation numbers → preliminary results are as expected, but weaker correction.
- Occupation-number method easily generalized to include pairing and non-zero temperature (in progress).
Summary

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Thank you for your attention!
In the EDF framework, it allows to cure all the problems of SRPA

It was developed prior to this thesis


How it works:

1. Response functions:
   \[ \rho_{kl}^{(1)} = \sum_{p,q} R_{klpq} f^{pq} \]
   \[ \rho^{(1)}: \text{transition density} \]
   \[ f: \text{external field} \]

   \[ R_{RPA}(E) = \left( \begin{array}{cc} E - A & -B \\ -B & -E - A \end{array} \right)^{-1} \]

   \[ R_{SRPA}(E) = \left[ \left( \begin{array}{cc} E - A & -B \\ -B & -E - A \end{array} \right) - \left( \begin{array}{cc} \Sigma(E) & 0 \\ 0 & \Sigma(E) \end{array} \right) \right]^{-1} \]

   \[ \Sigma(E): \text{energy-dependent} \]
   \[ 2\text{nd-order self-energy} \]

2. Double-counting is canceled if one requires \( R_{SRPA}(0) = R_{RPA}(0) \)
   This may be guaranteed by a subtraction method
   \[ \Rightarrow \text{It was demonstrated that the Thouless theorem is satisfied under this condition} \]
   \[ \Rightarrow \text{Cutoff dependence is also eliminated} \]

Renormalize SRPA matrix elements

Renormalized SRPA

The occupation numbers calculated iteratively in a renormalized RPA calculation are used in SRPA, to renormalize all the matrix elements.

For example, the $1p1h-1p1h$ part of $A$ reads:

$$A_{mi,nj} = \delta_{ij}\delta_{mn}(t_m - t_i)(n_i - n_m) + \bar{v}_{mjni}(n_i - n_m)(n_j - n_n)$$

$\rightarrow$ renormalizing factors

$\rightarrow$ Do the same for all the other terms in the $A$, $B$ and $G$ matrices

$\Rightarrow$ Include correlations in the ground state