Neutrino Oscillation & Other Quantum Oscillations

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Flow of this Seminar

Part-I

* Neutrino Oscillation
* Oscillation parameter measurements
  - Status
  - Future

Part-II

* Collection of Quantum Oscillations
  - Cabbibo Angle: θ_C,
  - Chirality Oscillation (why μ^-_R can decay)
  - Weinberg angle: θ_W
  - K^0 oscillation and CP violation

Part-I: Neutrino Oscillation

Fermion(spin=1/2) — **Flavor**

Name	Charge	1st generation	2nd generation	3rd generation
Lepton	-1	e	μ	τ
	0	ν_e	ν_μ	ν_τ
Quark	+2/3	u	c	t
	-1/3	d	s	b

← neutrino

Gauge boson(spin=1)

charge	EM	W	S
0	γ	Z^0	G
±1		$W^\pm$	

Higgs boson(spin=0)

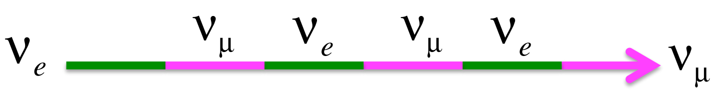
charge	
0	H^0

What is Neutrino Oscillation?

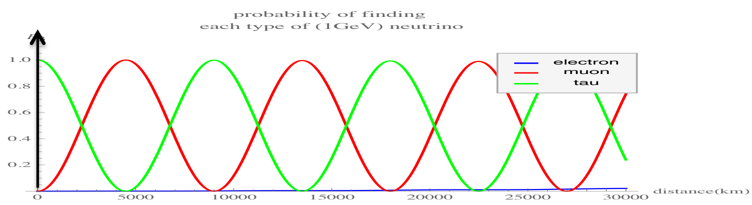
Electron stays as electron while it travels in space.

$e^- \longrightarrow e^-$

However, neutrinos change their flavors periodically.



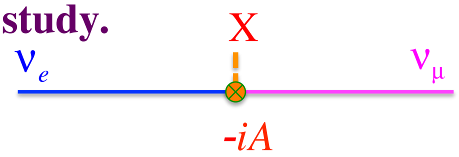
$P(\nu_e)$



This phenomenon is called neutrino oscillation

What causes the neutrino to oscillate?

We do not know yet.
In order for N.O. to happen, something(X) has to change the neutrino flavor. To know what is X is the important purpose of N.O. study.



"A" indicates the strength of the transition (amplitude).

In this case

State equation

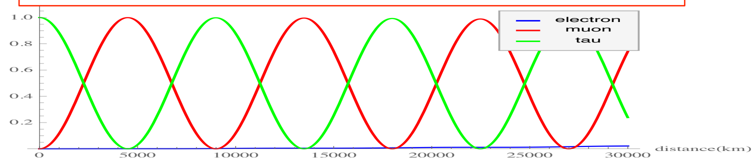
$$\frac{d}{dt}\nu_e = -iA\nu_\mu, \quad \frac{d}{dt}\nu_\mu = -iA\nu_e$$

Initial condition

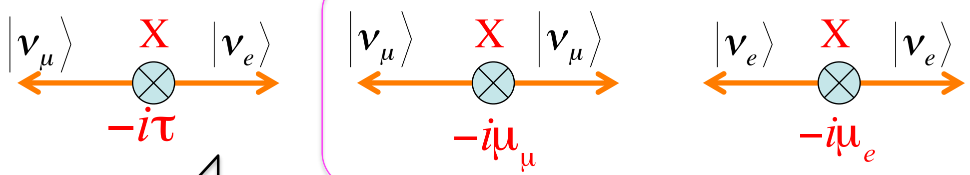
$$\nu_e[t=0]=1, \quad \nu_\mu[t=0]=0$$

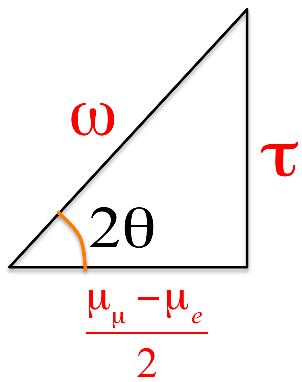
→ Oscillation

$$|\nu_e[t]|^2 = \cos^2 At, \quad |\nu_\mu[t]|^2 = \sin^2 At$$



General transition amplitudes & mass eigenstates





$$\tan 2\theta = \frac{2\tau}{\mu_\mu - \mu_e}$$

If there are self-transitions (original mass), the mass eigenstates become the superposition of flavor eigenstates;

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

The neutrino masses are

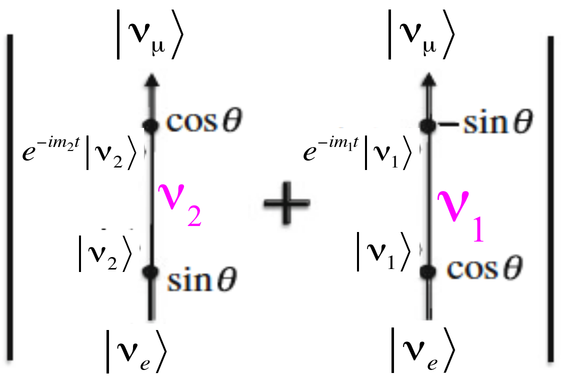
$$\begin{cases} m_1 = \bar{\mu} - \omega \\ m_2 = \bar{\mu} + \omega \end{cases}, \quad \bar{\mu} = \frac{\mu_\mu + \mu_e}{2}, \quad \omega = \frac{A}{\sin 2\theta}$$

ν oscillation (non relativistic case)

A Q.M. principle: Probability for something to happen is the absolute square of the sum of amplitudes of all possible diagrams.

There are 2 amplitudes for $\nu_e \rightarrow \nu_\mu$

$P[\nu_e \rightarrow \nu_\mu] = \Big|$



$\Big|^2$

$$= \left|\sin\theta\cos\theta e^{-im_2 t} - \sin\theta\cos\theta e^{-im_1 t}\right|^2$$

$$= \sin^2 2\theta \sin^2 \frac{m_2 - m_1}{2} t$$

Oscillation in time

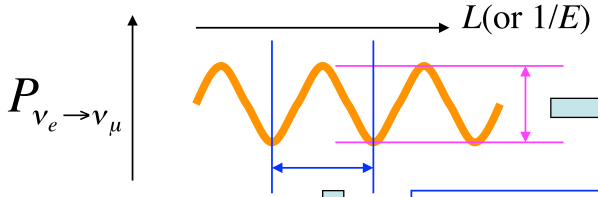
Relativistic Neutrino Oscillation

In experimental condition, neutrino is traveling relativistic

Lorentz Boost $(\gamma = E/m)$

$$mt \rightarrow \frac{mt}{\gamma} = \frac{m^2}{E} t = \frac{m^2 L}{E} \quad \Rightarrow \quad P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} L$$

What we can measure,



Amplitude ==> $\sin^2 2\theta = (\tau/\omega)^2$

Frequency => $\Delta m^2 = |m_2^2 - m_1^2| = 4\bar{\mu}\omega$

Why we measure ν oscillations?

There are many oscillations (irrespective to it is observable or not).

* $K^0 \Leftrightarrow \overline{K^0}$ oscillation. → CP violation
* $|u\bar{u}\rangle \Leftrightarrow |d\bar{d}\rangle$ oscillation in π^0, ρ, etc. → Hadron mass pattern
* Cabbibo angle, quark mass ← $d \Leftrightarrow s$ oscillation
* Weinberg angle, W, Z^0 mass ← $B \Leftrightarrow W_3$ oscillation

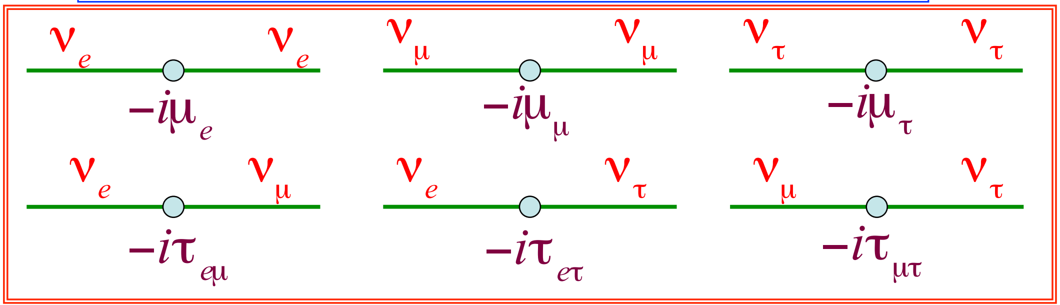
←We have learned a lot from these "Oscillations"

We can expect to learn more from ν oscillations;

$$\nu_\alpha \Leftrightarrow \nu_\beta$$

What is X??

3 Flavor Neutrino Oscillation



→ **The mixing matrix becomes 3x3.**

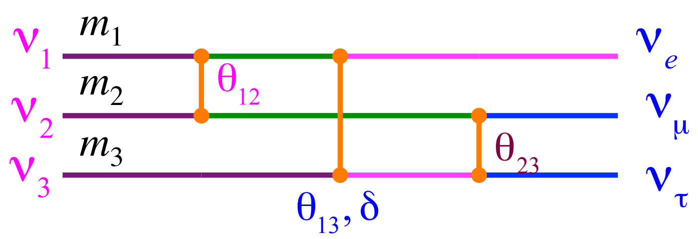
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad m_1, m_2, m_3 = \ldots$$

$\tau_{\alpha\beta}$ can be complex number and $U_{\alpha i}$ can also be complex number
→ CPV possible.

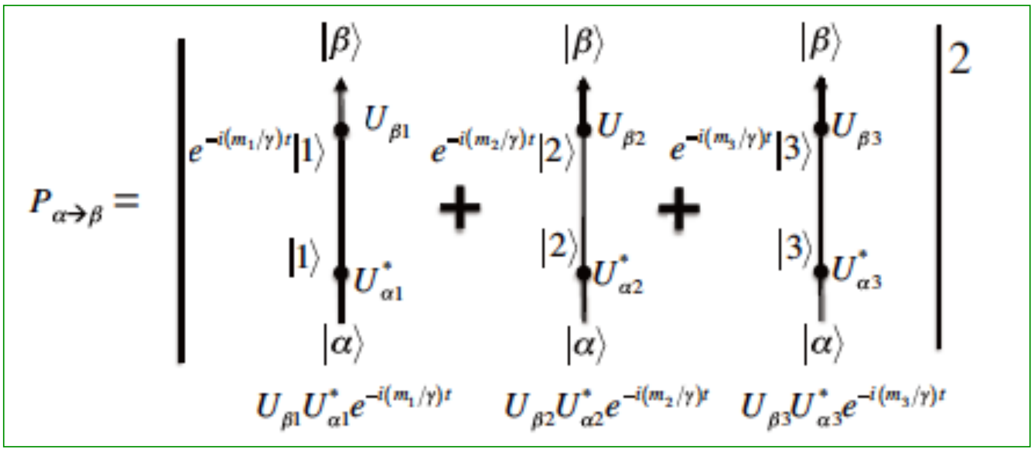
A useful parametrization of the mixing matrix

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$$



3 flavor oscillation probabilities



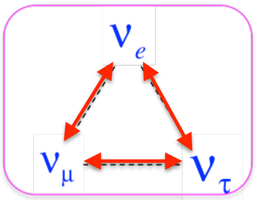
$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left[\Omega_{ij}^{\alpha\beta}\right]\sin^2\Phi_{ij} \mp 2\sum_{i>j}\mathrm{Im}\left[\Omega_{ij}^{\alpha\beta}\right]\sin 2\Phi_{ij}$$

$$\Omega_{ij}^{\alpha\beta} \equiv U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} \qquad \Phi_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E_\nu}, \quad \Delta m_{ij}^2 \equiv m_j^2 - m_i^2$$

There are 6 independent oscillation parameters;

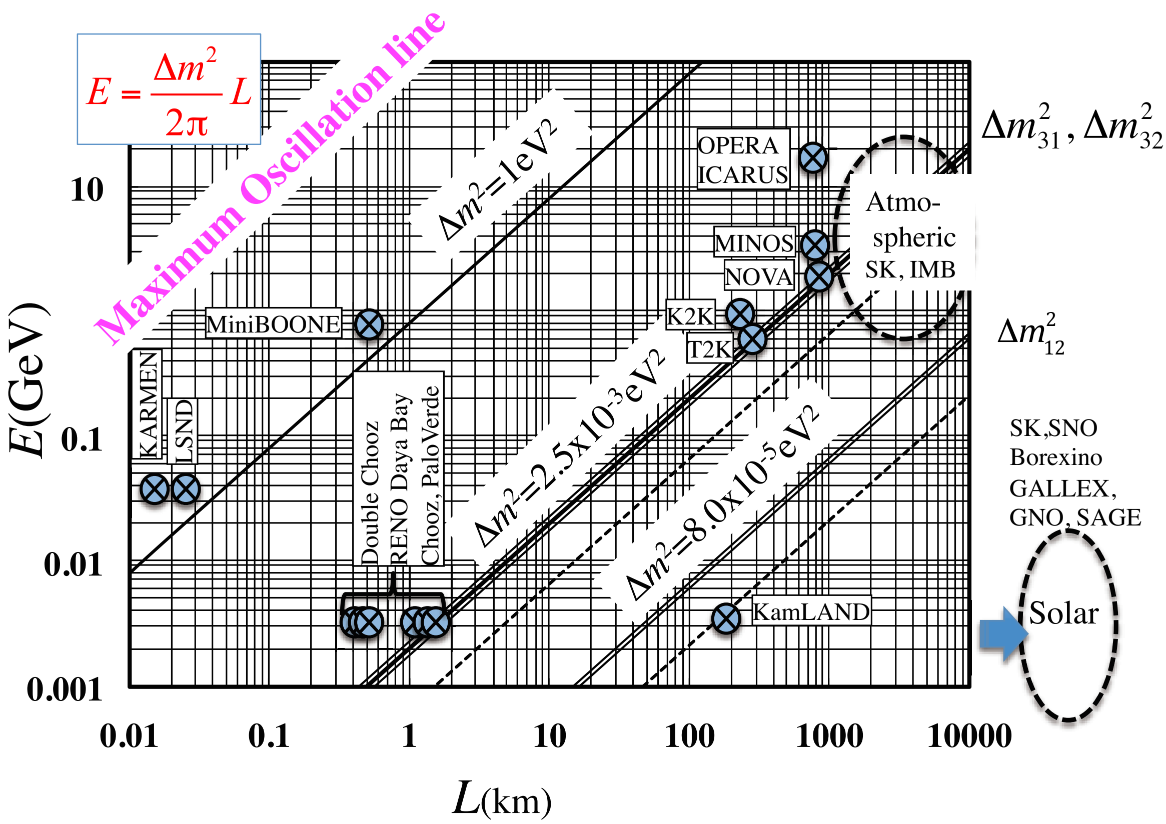
Neutrino Oscillation Experiments

Parameters to measure
- **Mixing angles:** $\theta_{12}, \theta_{23}, \theta_{13}$
- **Square mass differences:** $\Delta m^2_{12}, \Delta m^2_{23} \left(, \Delta m^2_{13}\right)$
- **CP violation phase:** δ_{CP}



It has been a long story with labors and efforts, brilliant ideas, severe competitions and sometimes errors.
But allow me to make it short .....

E-L relation of N.O. experiments



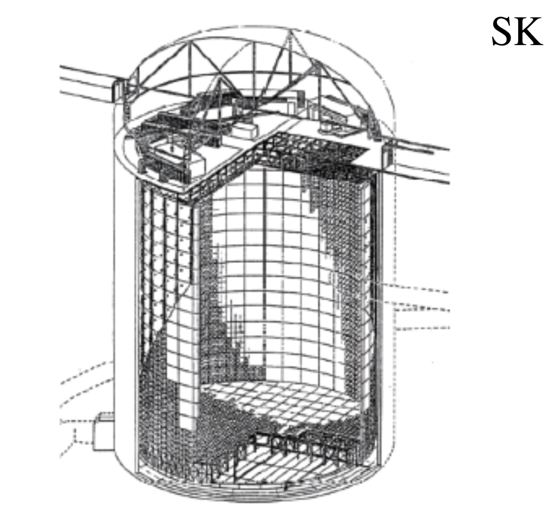
$\theta_{23}, \quad \Delta m^2_{32}$

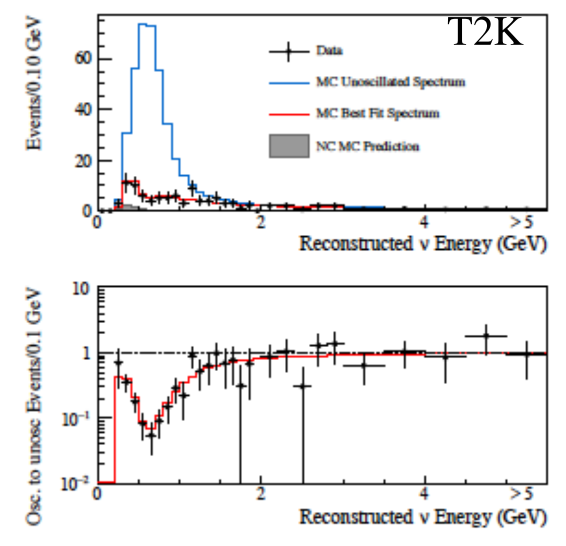
Atmospheric (SK. etc.), T2K, MINOS, NOVA ....

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{32}}{4E} L$$

arXiv:1502.01550v1

SK

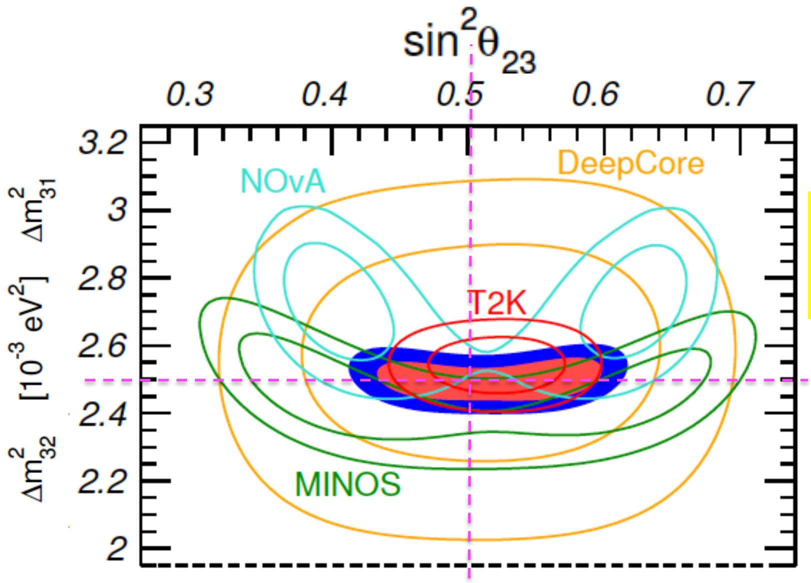




➔ Discovery of N.O. : Nobel prize in 2015 (T.Kajita)

$\theta_{23}, \quad \Delta m^2_{32}$

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{32}}{4E} L$$



Normal Hierarchy ($m_3 > m_2$) case

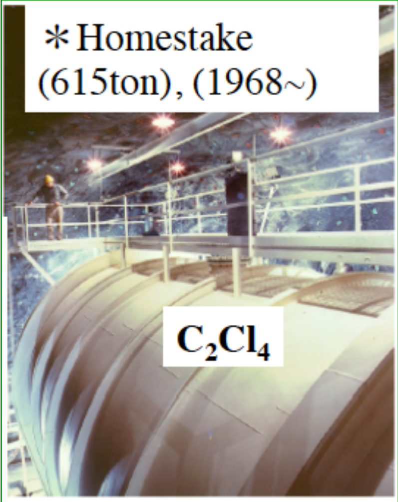
NuFIT 3.1(2017), www.nu-fit.org, JHEP01(2017)087,

$$\sin^2 2\theta_{23} \sim 1, \quad \Delta m^2_{32} \sim 2.5 \times 10^{-3} [eV^2]$$

$\theta_{12},\quad \Delta m_{12}^2$

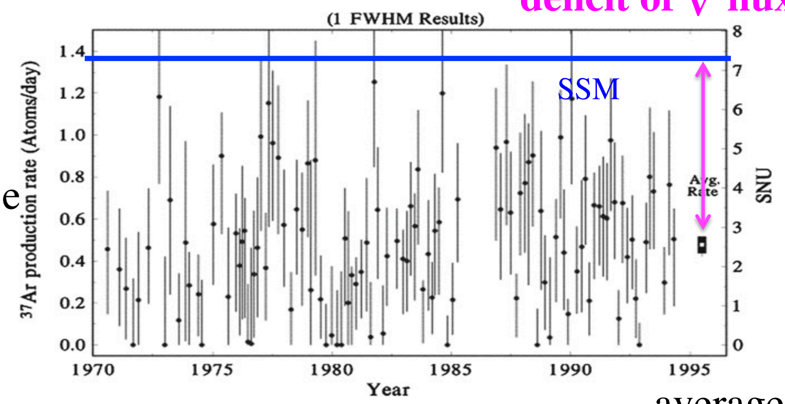
Solar Neutrino Experiments
(Homestake, SuperK, SAGE, GALLEX, BNO, Borexino, SNO, etc.)

$$P(\nu_e \to \nu_e; @\Delta m_{21}^2) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2}{4E} L$$



$^{37}Cl + \nu_e \to {}^{37}Ar + e^-$

deficit of ν flux

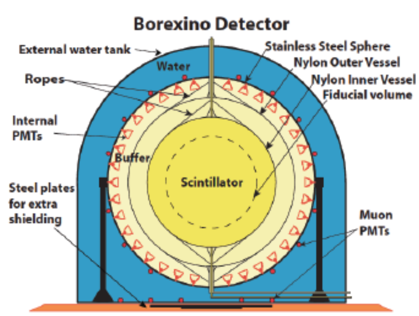


average

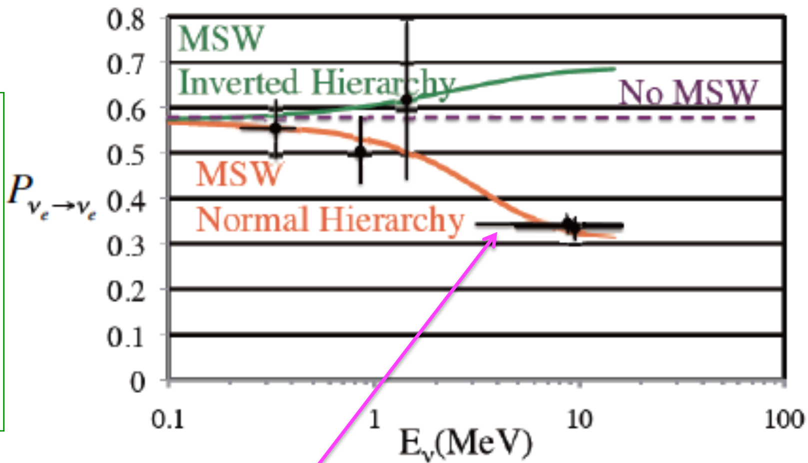
➔ **Nobel prize in 2002 (R.Davis)**

Δm_{21}^2 **mass hierarchy (m_2>m_1 or m_2<m_1)**

Genuine N.O. can not resolve it but
Matter Effect depends on M.H. and it can be used.



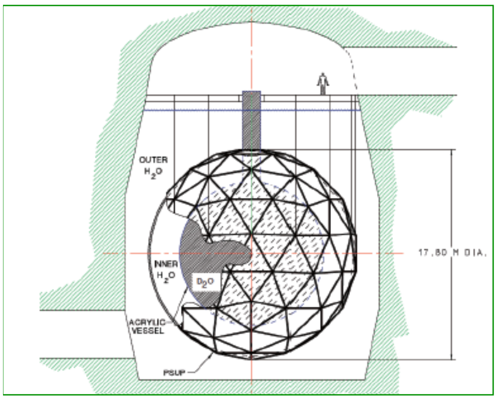
etc.

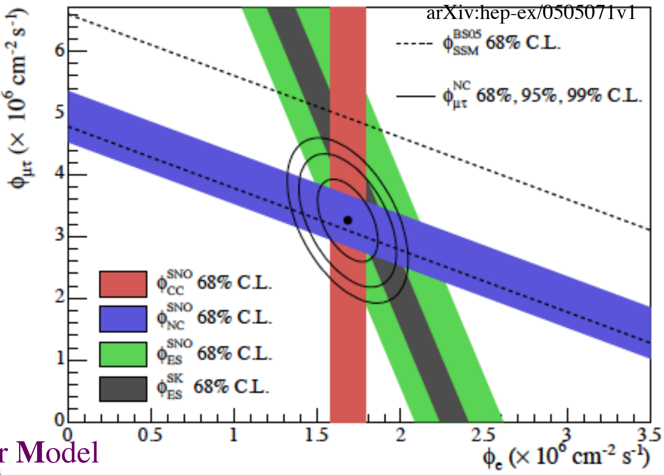


m_2>m_1 determined

Flavor Transmutation: SNO experiment

$\nu_x + D \rightarrow \nu_x + p + n$ **NC interaction: possible to count all flavors**





Standard Solar Model

Although $\Phi(\nu_e) < \Phi(SSM)$, $\Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau) = \Phi(SSM)$

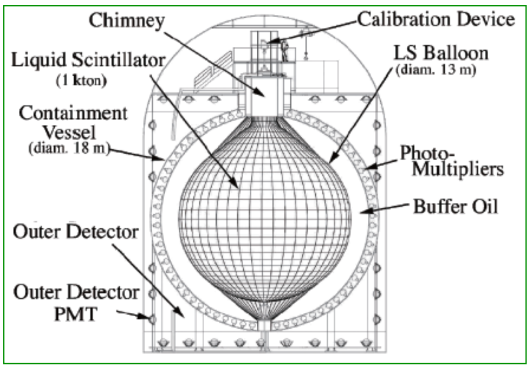
➔ Total # of ν does not change. ν_e changed to other neutrinos.

➔ Nobel prize in 2015 (A.McDonald)

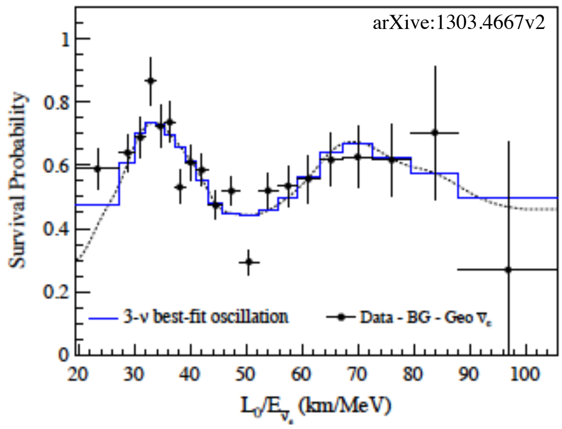
$\theta_{12}, \quad \Delta m^2_{12}$

KamLAND Reactor Neutrino Oscillation

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; @\Delta m^2_{21}) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{12}}{4E} L$$



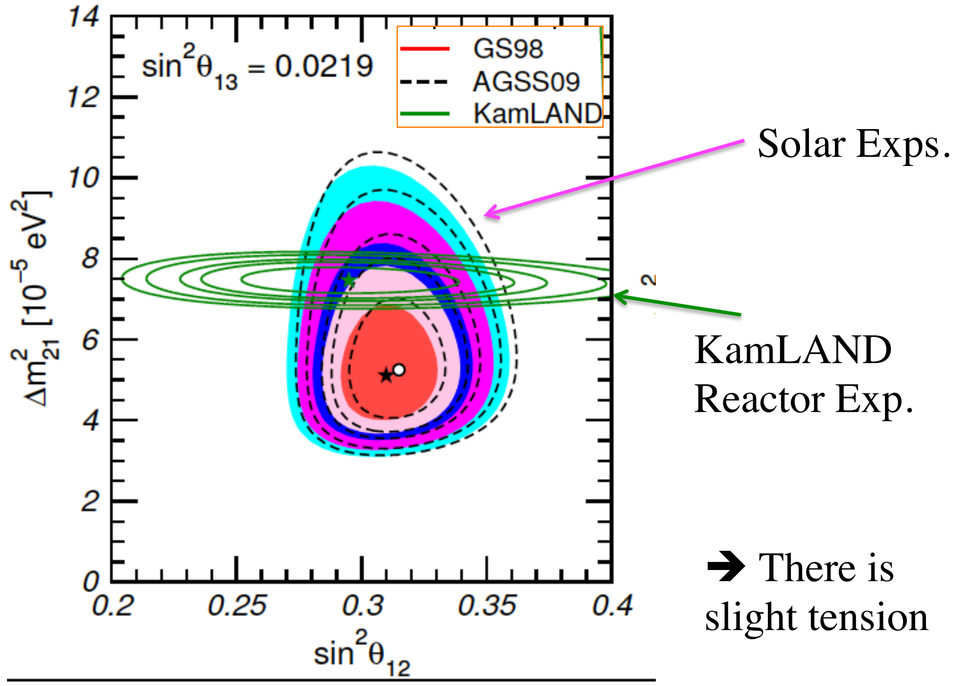
L~180km



KamLAND : $\tan^2\theta_{12} = 0.436^{+0.029}_{-0.025}$, $|\Delta m^2_{21}| = 7.53^{+0.18}_{-0.18} \times 10^{-5}\, eV^2$

$\theta_{12}, \quad \Delta m^2_{12}$

NuFIT 3.1(2017), www.nu-fit.org, JHEP01(2017)087,



➔ There is slight tension

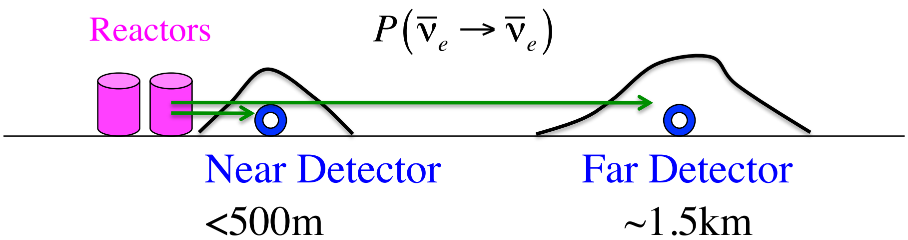
$\theta_{13}, \quad \Delta m^2_{31}$

Reactor-θ_{13} Experiment

θ_{13} was a key parameter to proceed to CPV measurement.
But it was known small ($\sin^2 2\theta_{13}$)

➔ Reactor measurement of θ_{13}

$$P\left(\bar{\nu}_e \to \bar{\nu}_e; L \sim 1.5km\right) \sim 1 - \sin^2 2\theta_{13}$$

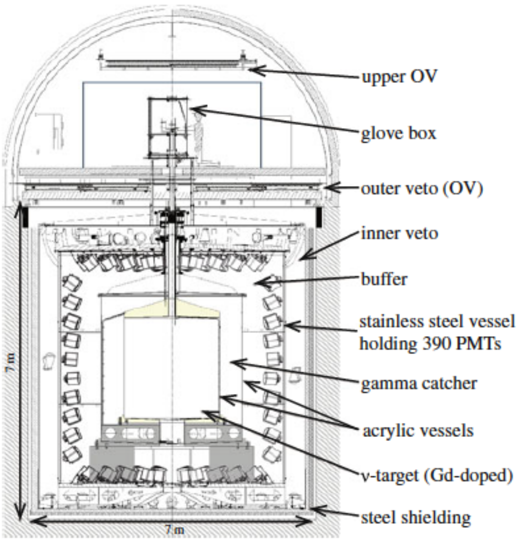


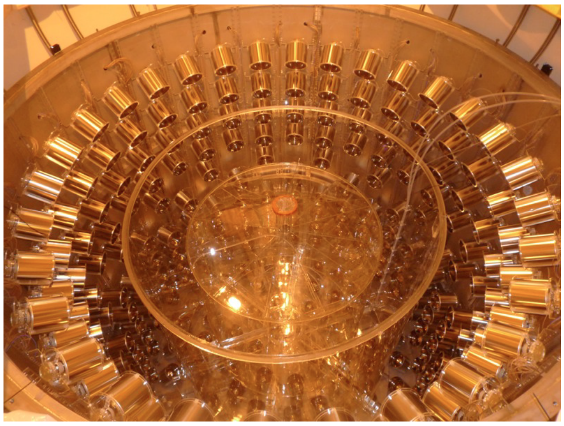
Two detector concept: Cancel uncertainty of neutrino flux and detection efficiency by comparing near & far detector

$\theta_{13}, \quad \Delta m^2_{31}$:

Our experiment: Double Chooz

Most of the ideas of the reactor θ_{13} experiment/detector were proposed first by the DC group members.

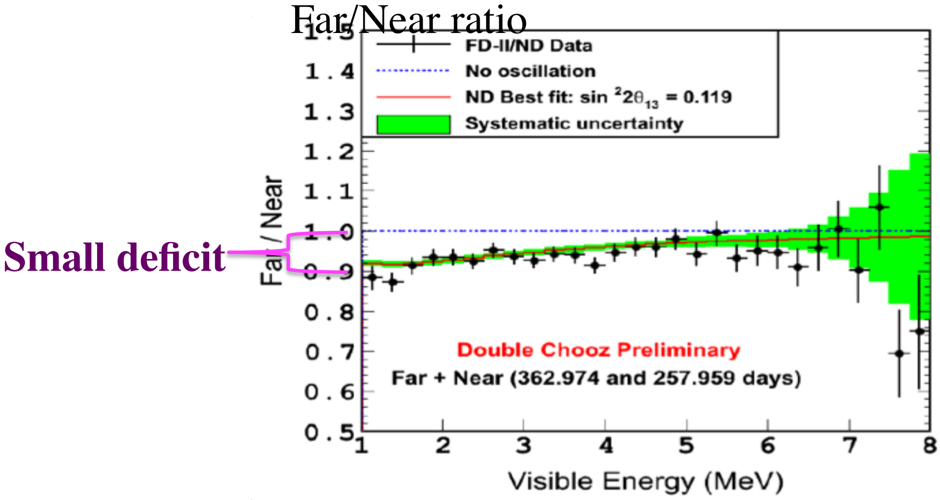




Double Chooz Oscillation fit result

Far detector/Near detector concept to cancel most of the systematics.

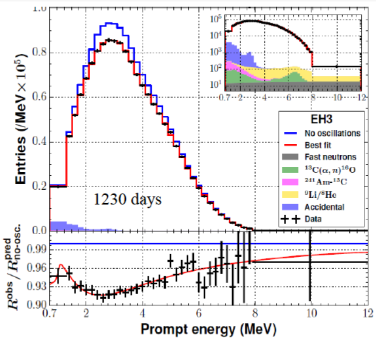
Far/Near ratio



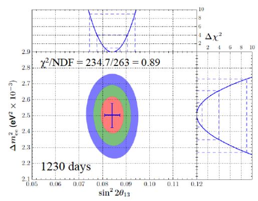
$\sin^2 2\theta_{13} = 0.119 \pm 0.016$ with $\chi^2/\mathrm{ndf} = 236.2/114$ (preliminary)

Daya Bay Result

Logan Lebanowski @ 2016.11 NNN16



Oscillation analysis result



$\sin^2 2\theta_{13} = 0.0841 \pm 0.0027(stat.) \pm 0.0019(syst.)$

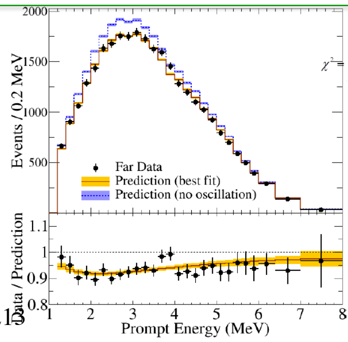
$|\Delta m^2_{ee}| = [2.50 \pm 0.06(stat.) \pm 0.06(syst.)] \times 10^{-3} eV^2$

Multiple analyses yield consistent results.

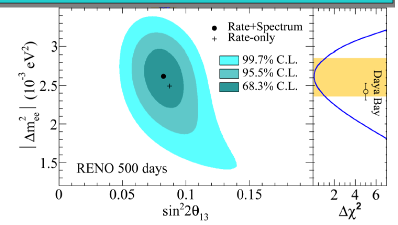
[arXiv:1610.04802]

RENO result

Hyunkwan Seo @ 2016.11 NNN16



Results from Spectral Fit



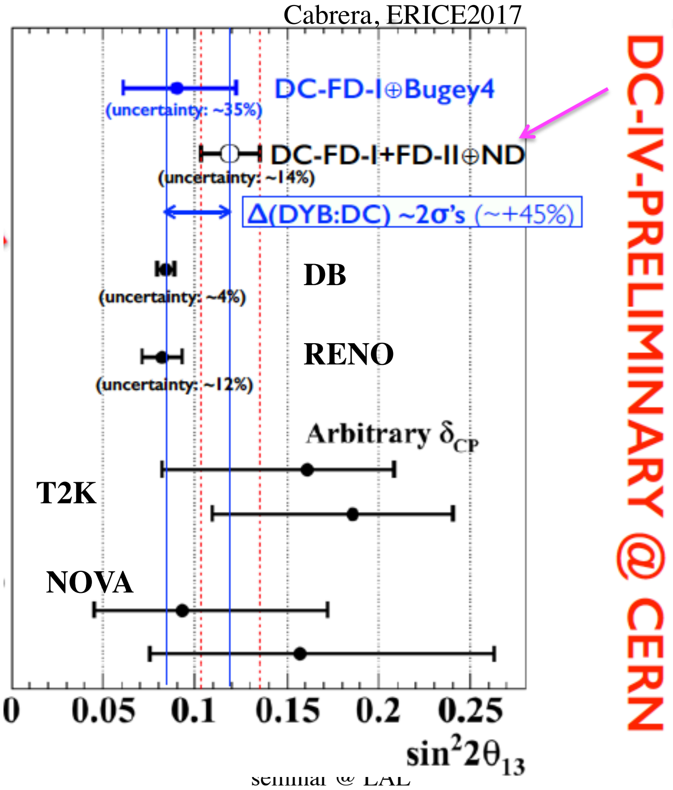
Rate+shape new results

$\sin^2 2\theta_{13} = 0.082 \pm 0.009(\text{stat.}) \pm 0.006(\text{syst.})$

$|\Delta m^2_{ee}| = 2.62^{+0.21}_{-0.23}(\text{stat.})^{+0.12}_{-0.13}(\text{syst.})\ (10^{-3} eV^2)$

There may be a tension between DC and DB, RENO

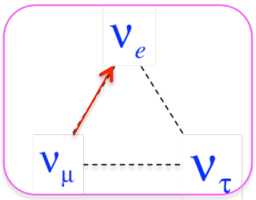
Cabrera, ERICE2017



CP violation δ

T2K and NOVA measure

$$\left(\nu_\mu \to \nu_e\right),\ \left(\bar{\nu}_\mu \to \bar{\nu}_e\right)$$

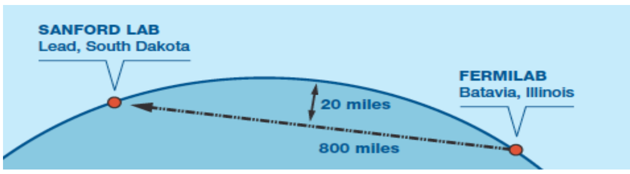


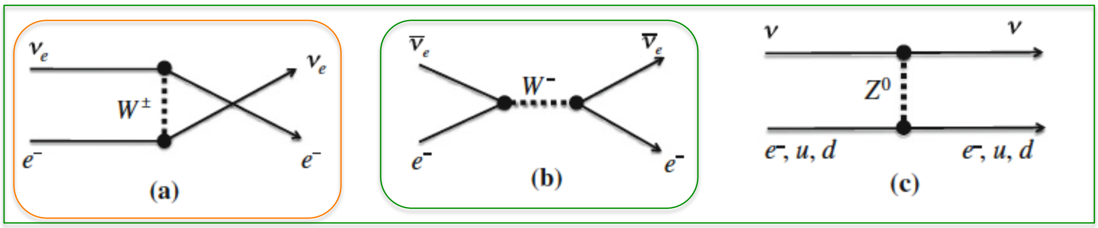
$$\nu_\mu \to \nu_e \qquad P_A \sim 0.5\sin^2 2\theta_{13} - 0.043\sin 2\theta_{13}\,\underline{\sin\delta}$$

$$\bar{\nu}_\mu \to \bar{\nu}_e \qquad \bar{P}_A \sim 0.5\sin^2 2\theta_{13} + 0.043\sin 2\theta_{13}\,\underline{\sin\delta}$$

$$A_{CP} = \frac{P_A - \bar{P}_A}{P_A + \bar{P}_A} \sim 0.3\sin\delta$$

Earth Matter Effect





ν_e and $\bar{\nu}_e$ feel different weak potential

Effective Weak Potential

$$V_W = 2\sqrt{2}E_\nu \frac{n_e G_F}{m_3^2 - m_1^2}$$

changes the coupling sign depending on ν or $\bar{\nu}$

changes sign depending on the mass hierarchy

Energy dependent

Weak Potential & Oscillation Probability

After lengthy calculation, main effect of the weak potential on the oscillation:

$$\sin\Phi_{31} \rightarrow \frac{\sin\left((1-V_W)\Phi_{31}\right)}{1-V_W}; \qquad \Phi_{31} = \frac{\Delta m^2_{31}}{4E}L$$

Then, the appearance probability with the matter effect is,

$$P\left(\nu_\mu \rightarrow \nu_e; @\Phi_{31}\right) \sim \frac{0.5\sin^2 2\theta_{13}}{\left(1-V_W\right)^2} \pm \frac{0.043\sin 2\theta_{13}}{\left(1-V_W\right)}\sin\delta$$

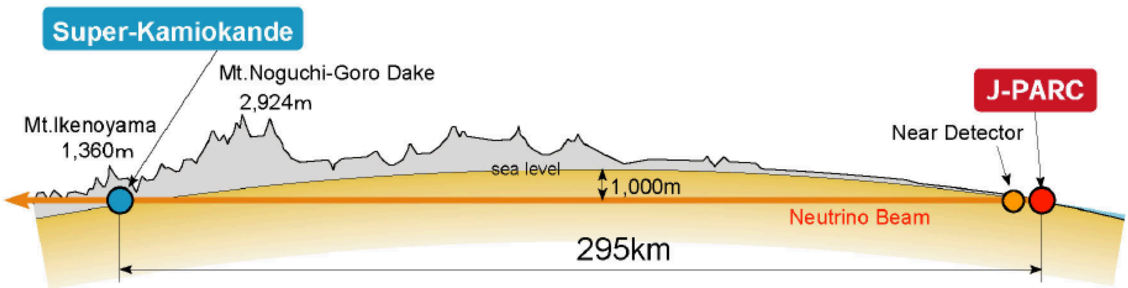
	L[km]	V_W (=L/L_0)
T2K/HK	295	**±0.055**
NOVA	810	**±0.15**
DUNE	1,300	**±0.24**

The sign of the potential depends on Mass Hierarchy:
* Normal H.: $m_3 > m_1$
* Inverted H.: $m_3 < m_1$

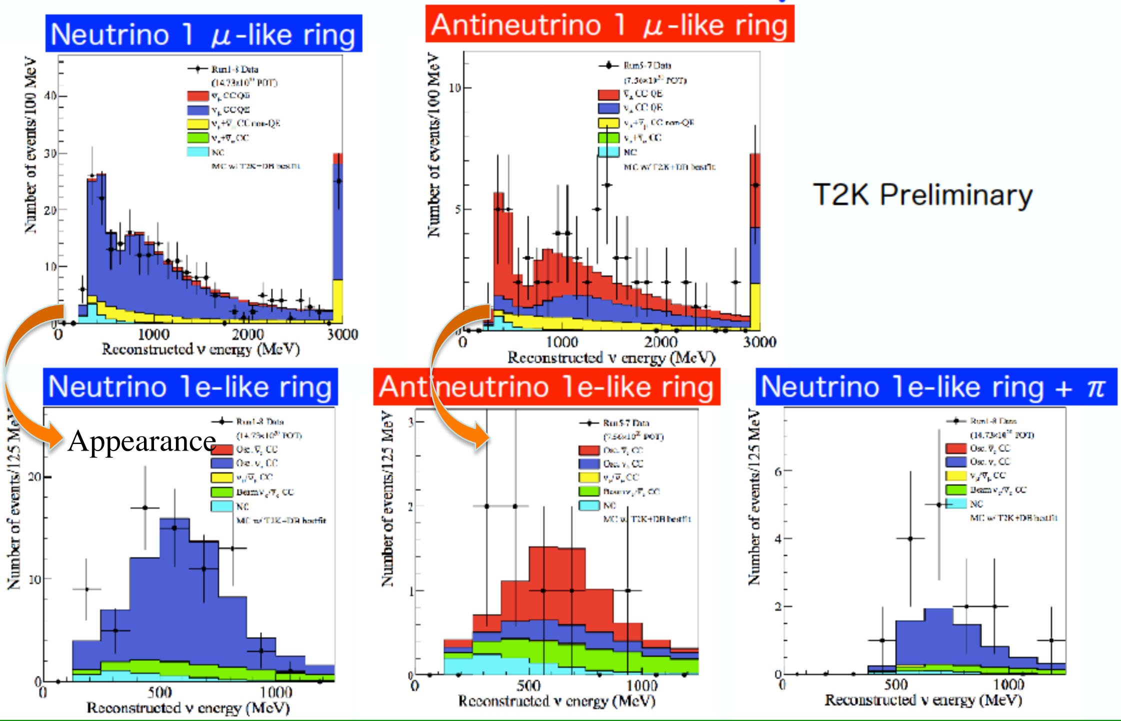
➔ Interpretation of data depends on MH.

T2K Experiment

K.Iwamoto@ICHEP16



Observation at Super-K



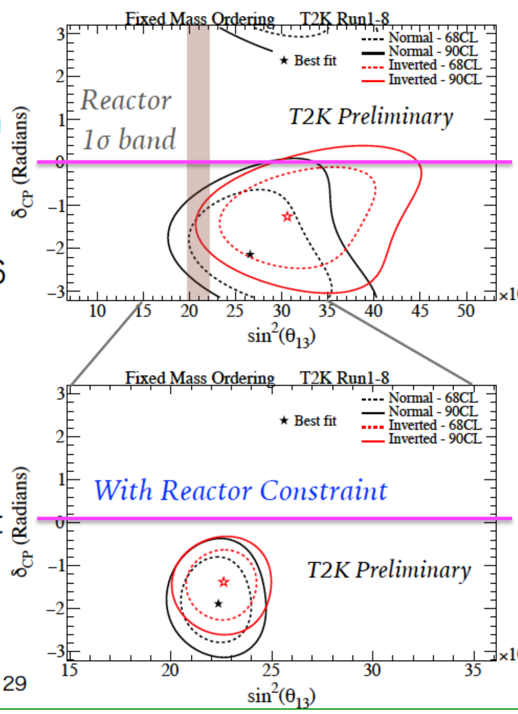
θ_{13} and δ_{CP}

- Fit without the reactor constraint: closed contours in δ_{cp} at 90% CL
- The T2K value for $\sin^2\theta_{13}$ is consistent with the PDG 2016

T2K Best Fit:
$\sin^2\theta_{13} = 0.0277^{+0.0054}_{-0.0047}$ (NH)

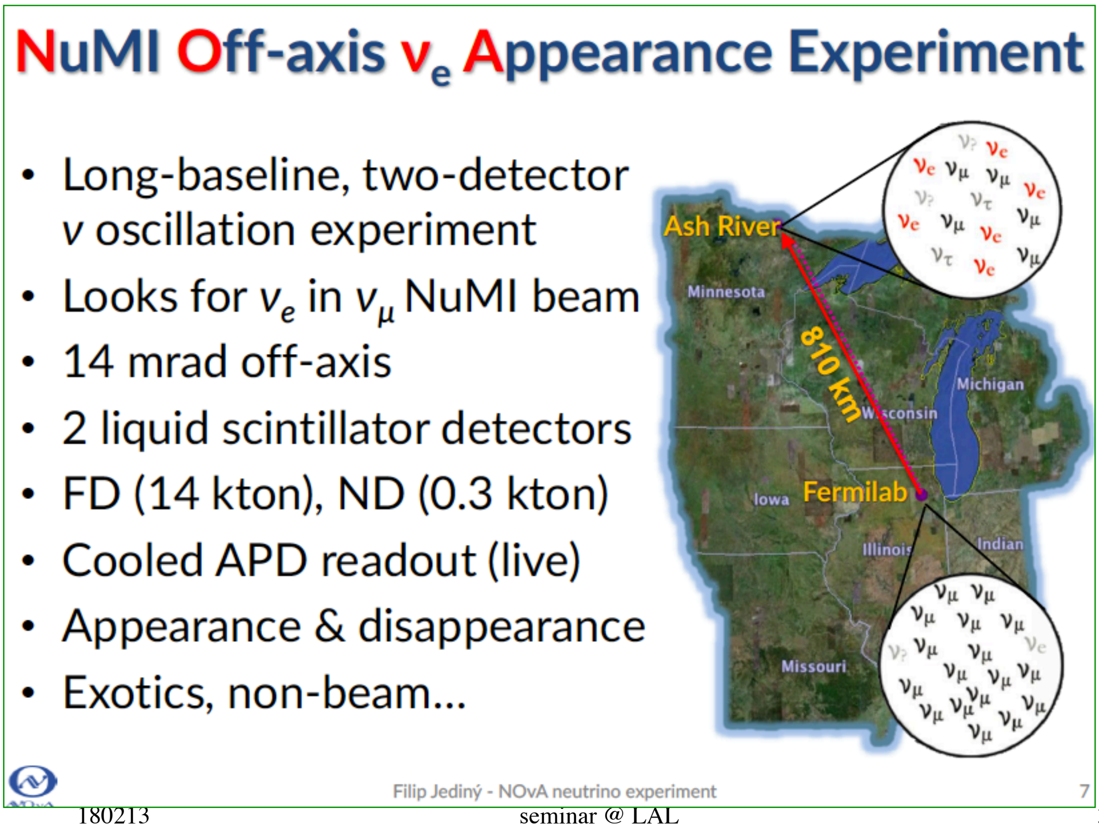
PDG 2016:
$\sin^2\theta_{13} = 0.0210 \pm 0.0011$

- Adding the reactor constraint improves the constraint on δ_{cp} average:

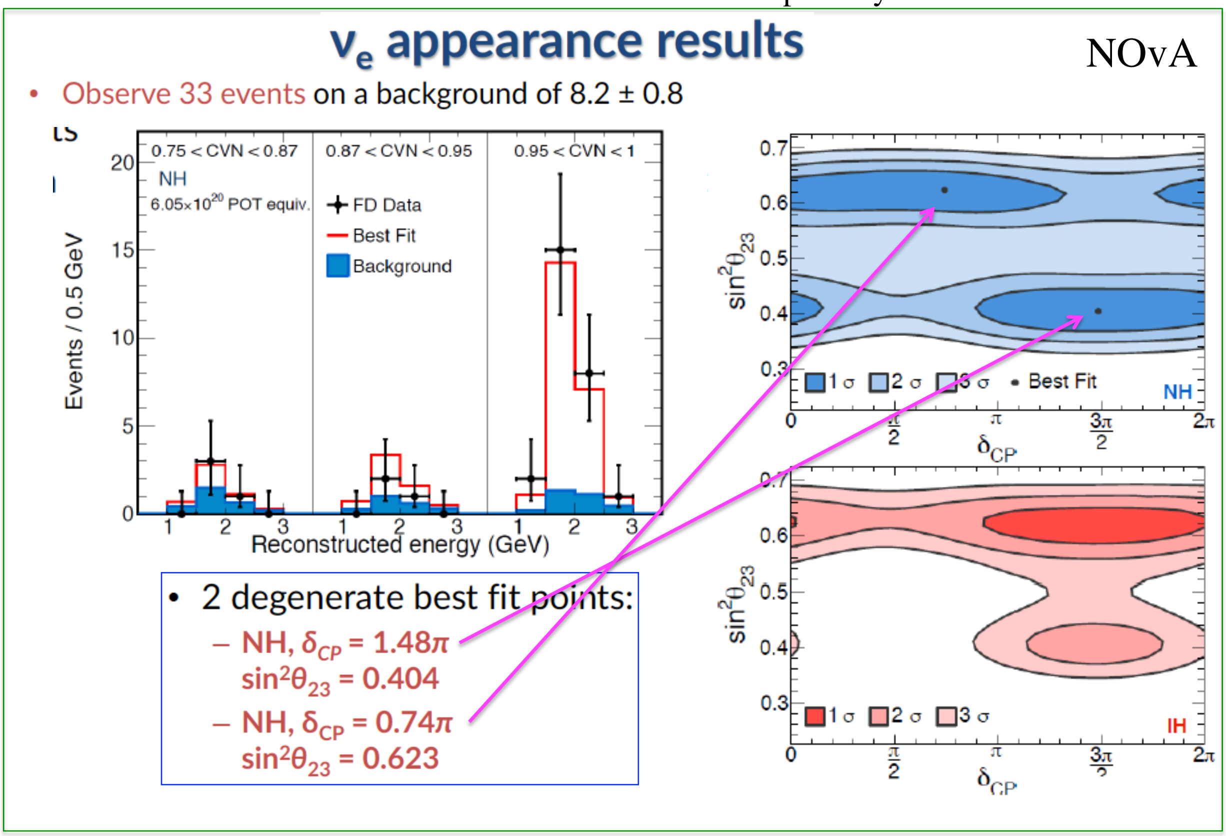


NOVA

Filip Jediny @ 2017.3 Moriond



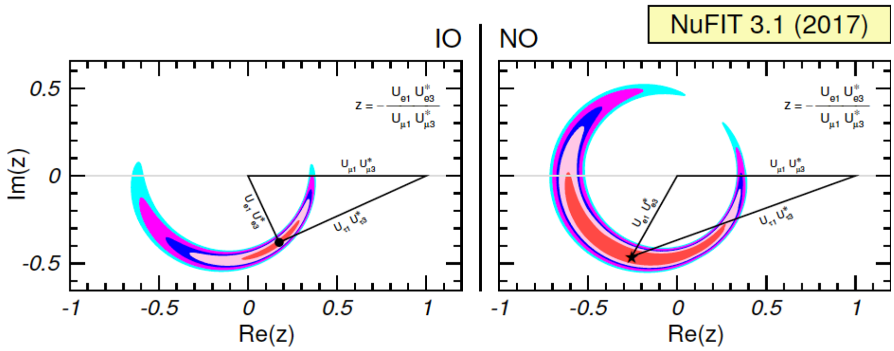
Filip Jediny @ 2017.3 Moriond



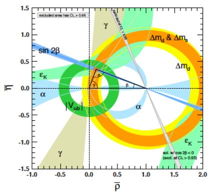
What we know now

NuFIT 3.1(2017), www.nu-fit.org, JHEP01(2017)087,

	θ_{12}	θ_{23}	θ_{13}	δ_{CP}	Δm^2_{12}	Δm^2_{31}
N.H.	33.6°	48.7°	8.52°	228°,	7.40×10^{-5} eV^2	2.515×10^{-3} eV^2
I.H.	33.6°	49.1°,	8.55°	281°,	7.40×10^{-5} eV^2	-2.483×10^{-3} eV^2



c.f. CKM



Our Current Knowledge of Neutrino Transition Amplitude

For Example: If NH and $\delta_{CP} = -\pi/2$

$$U_{NH} \sim \begin{pmatrix} 0.82 & 0.55 & -0.09+0.13i \\ -0.36+0.07i & 0.65+0.05i & 0.67 \\ 0.43+0.08i & -0.53+0.05i & 0.73 \end{pmatrix}$$

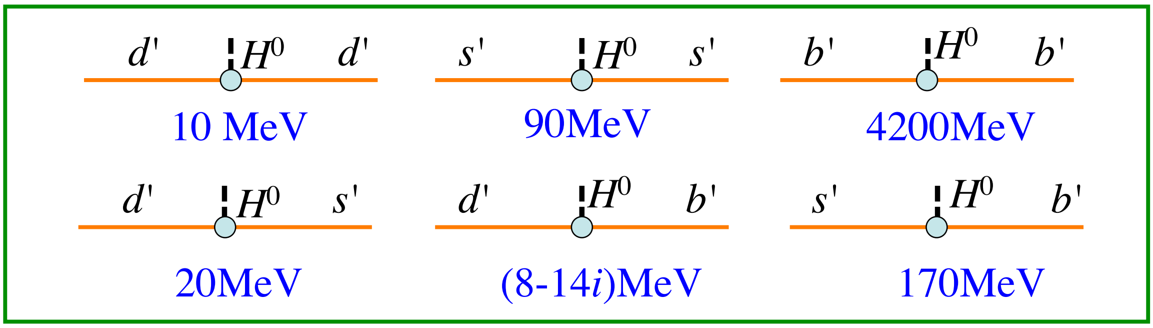
Assumption: m_1~0, ➔ m_2 =8.7meV, m_3=50meV

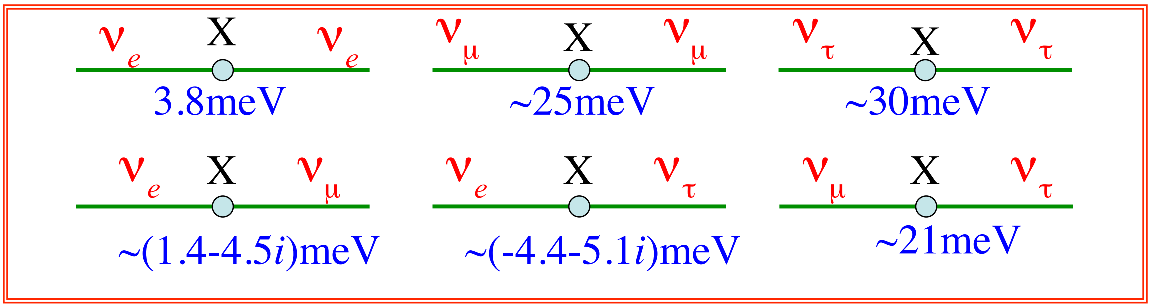
ν_e X ν_e: 3.8meV	ν_μ X ν_μ: ~25meV	ν_τ X ν_τ: ~30meV
ν_e X ν_μ: ~(1.4-4.5i)meV	ν_e X ν_τ: ~(-4.4-5.1i)meV	ν_μ X ν_τ: ~21meV

 What is X and how this pattern can be explained??

Comparison with quark transition amplitudes

Quark Mass + CKM Mixing Matrix

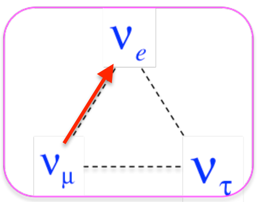


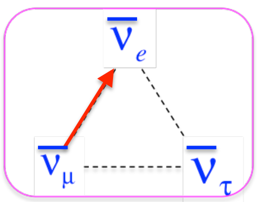


 What is X and how this pattern can be explained??

Future

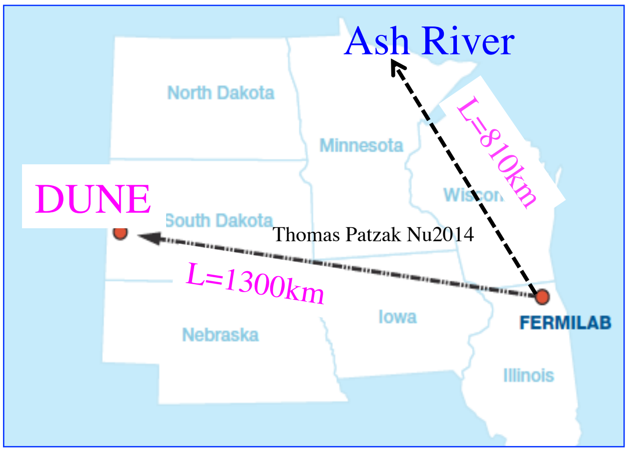
More precise CP Asymmetry
Mass Hierarchy determination
:





CP asymmetry by Future Long baseline experiments

NOVA

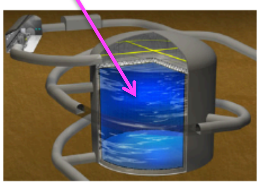


E. O'Sullivan, NuFACT 2017

Hyper Kamiokande:

516kt W.C.

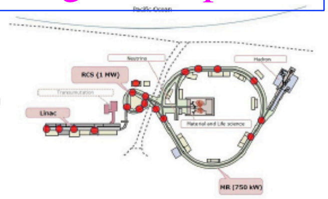
T2HK



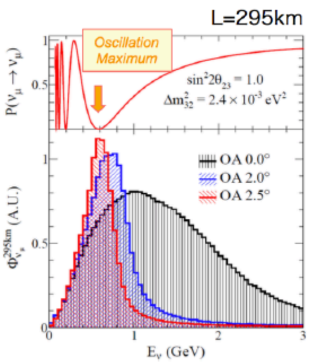
Hyper-K detector (Kamioka)

T2K→
*Larger target
*Larger acc. power

295 km



J-PARC neutrino Beam (Tokai)

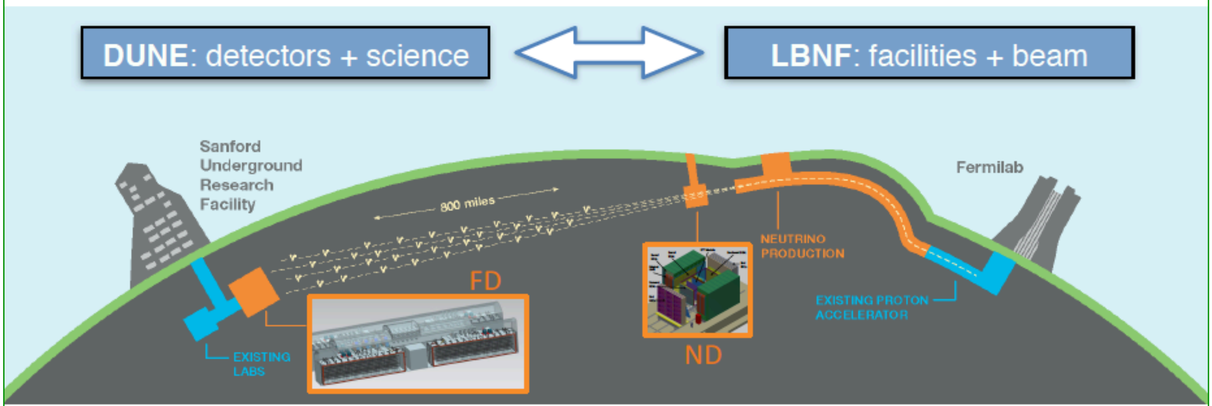


Building off the successful T2K program, Hyper-K also plans to measure neutrinos from the J-PARC neutrino beam

Data taking expected in 2026

DUNE at LBNF

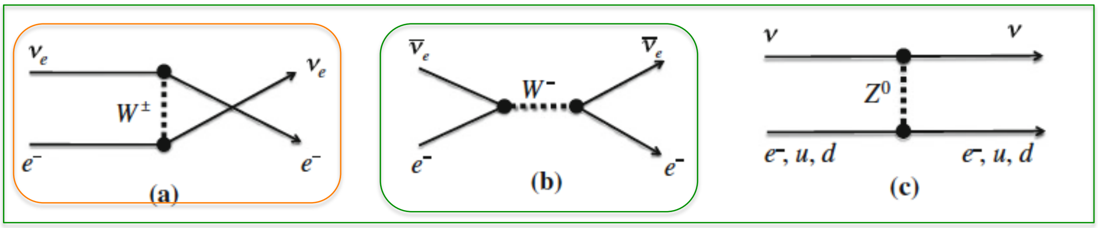
Deep Underground Neutrino Experiment at the Long Baseline Neutrino Facility



- High intensity, wide-band, neutrino beam from Fermilab
- Highly capable neutrino near detector at Fermilab
- 40-kt fiducial mass far detector at SURF based on LAr-TPCs

3 **2026, Neutrino Beam available**

CP asymmetry with the matter effect



Difference of ν_e and $\bar{\nu}_e$ interactions with Earth matter

$$A_{CP}(@\Phi_{13}) \sim -0.3\sin\delta_{CP} \pm 2(L/L_0)$$

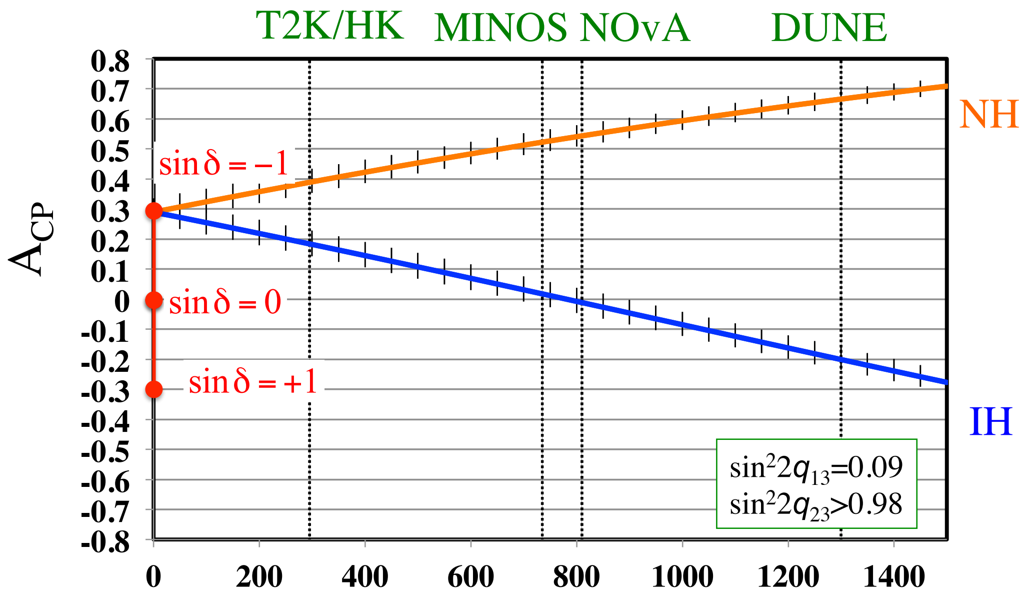
fake CP asymmetry

	L[km]	$A_{FK}=2(L/L_0)$
T2K/HK	295	**±0.11**
NOVA	810	**±0.30**
DUNE	1,300	**±0.48**

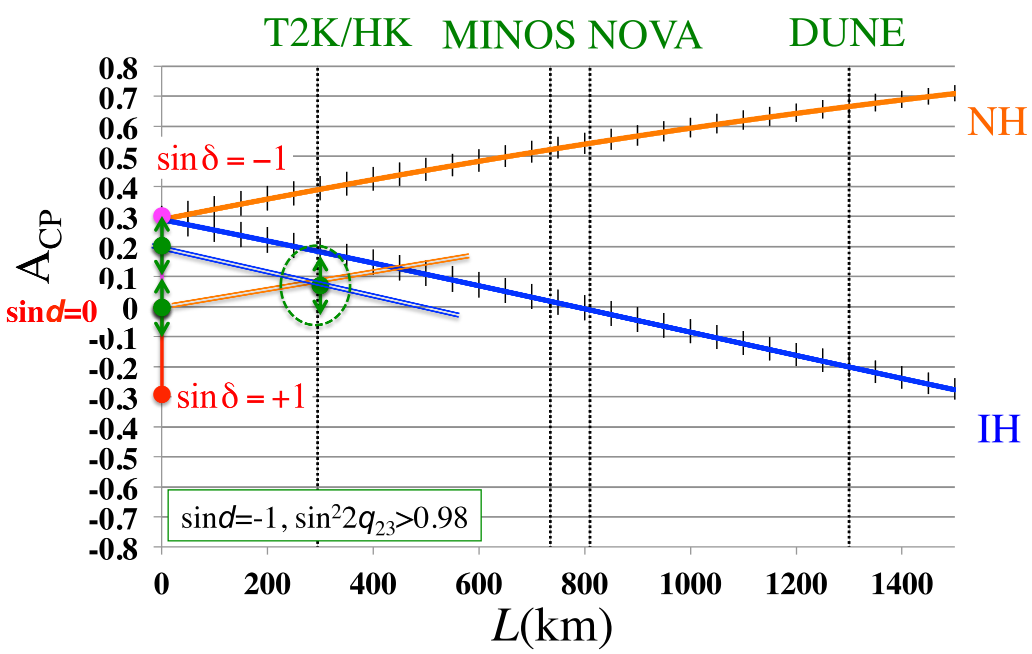
The sign depends on mass hierarchy

Baseline Dependence of Matter effect

$$A_{CP} \sim -0.29\sin\delta \pm 2\left(\frac{L}{L_0}\right)$$

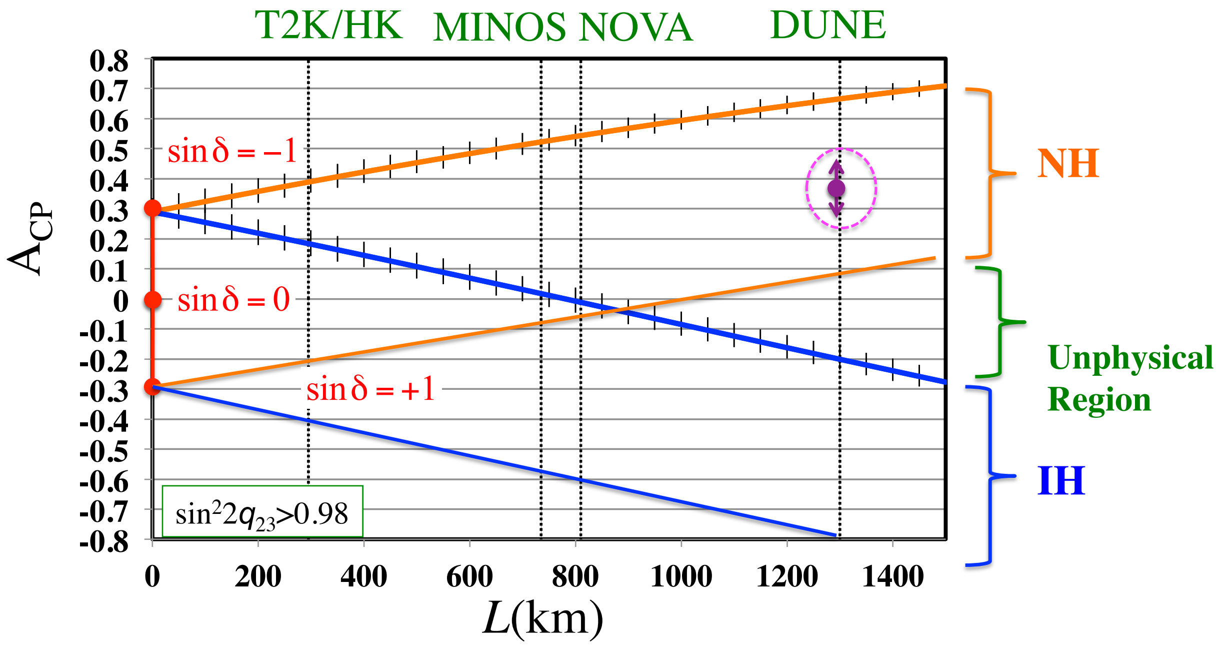


HK only case



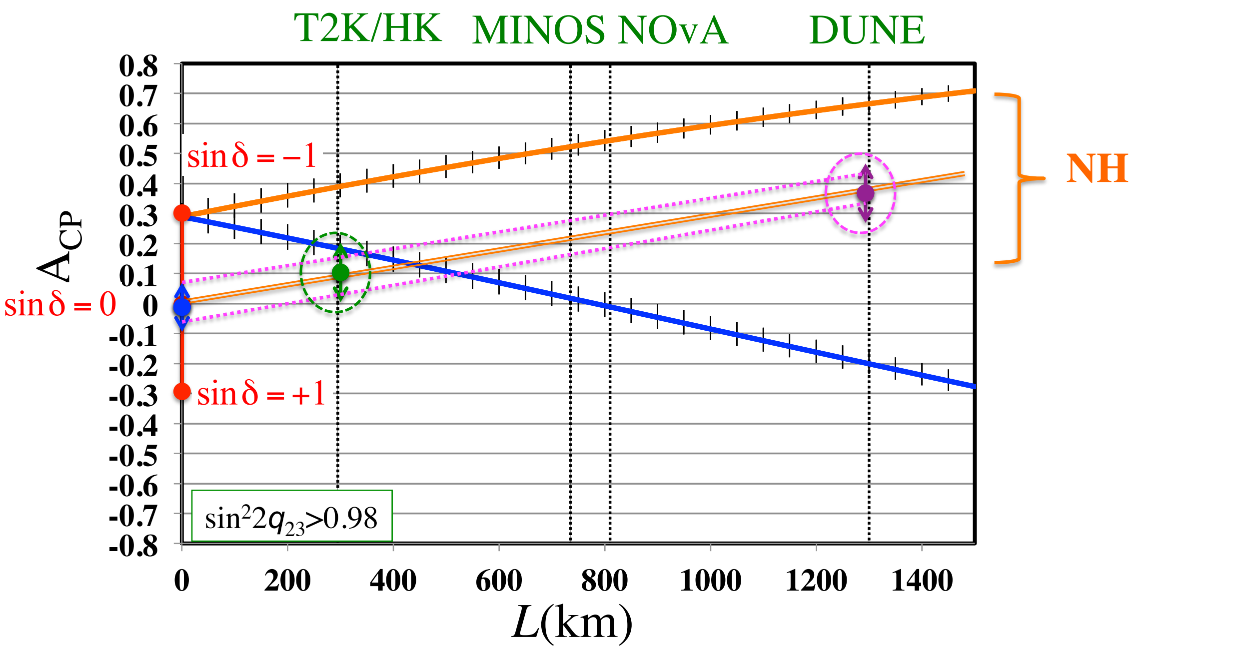
If M.H. is not known, there are two solutions.
sind=0 can not be confirmed. ➔ M.H. is necessary.

DUNE only case



This case, M.H. is determined to be N.H.
But measurement is somewhat model dependent.

HK+DUNE



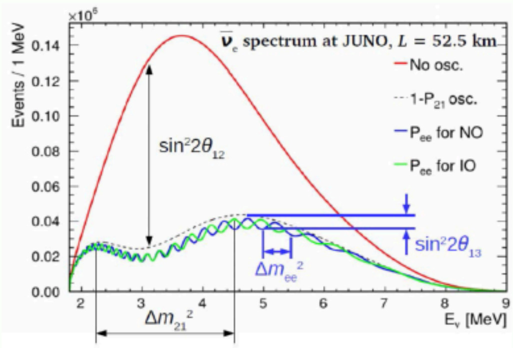
* **Matter effect independent measurement is possible.**
➔ **good to have both experiments.**

J.P.O.Ricoux, NuFact2017

Δm^2_{23} mass hierarchy:

JUNO

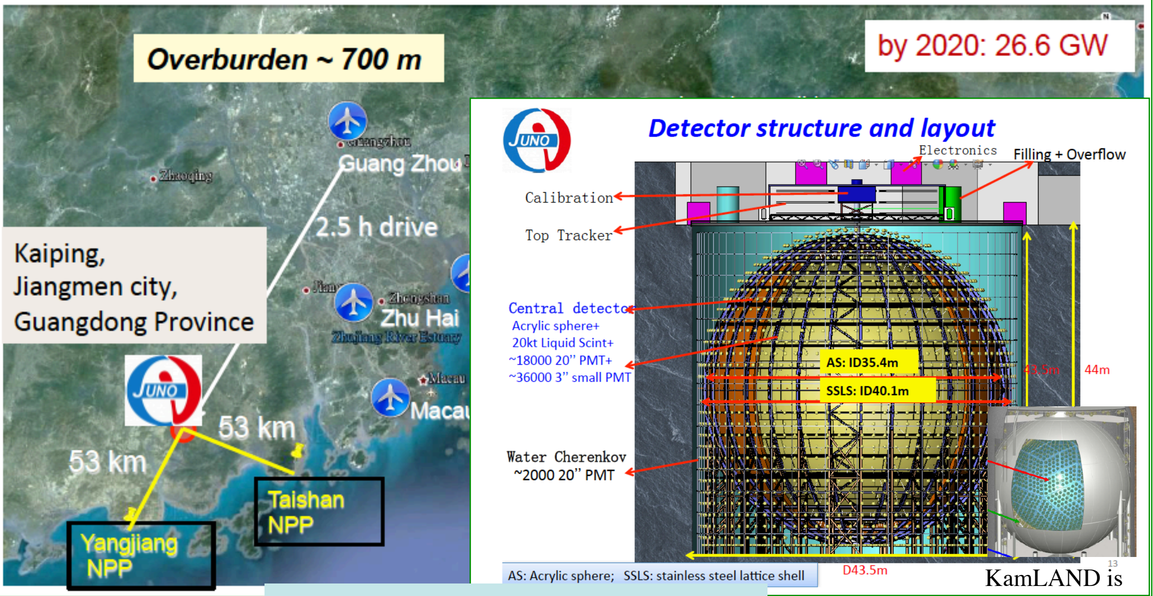
- The **J**iangmen **U**nderground **N**eutrino **O**bservatory (JUNO) is a multipurpose experiment under construction in China:
  - Rich physics program: neutrino mass hierarchy, sub-% measurement of oscillation parameters, astrophysical neutrinos, geo-neutrinos, atmospheric neutrinos, search for exotic physics… etc.
- Main keys to accomplishing the physics goals:
  - **Optimal baseline**
  - **High statistics**
  - **Superb energy resolution (3% @ 1 MeV)**
  - **Excellent control of energy response systematics**
  - **Background reduction**



Yee Bob Hsiung @ 2016.11 NNN16

Location of JUNO

NPP	Daya Bay	Huizhou	Lufeng	Yangjiang	Taishan
Status	Operational	Planned	Planned	Under construction	Under construction
Power	17.4 GW	17.4 GW	17.4 GW	17.4 GW	18.4 GW



KamLAND is this size

Filling and data taking 2020

Summary: Part-I

* Thanks to the huge experimental efforts, $\theta_{12}, \theta_{23}, \theta_{13}$
  Δm^2_{12}, $|\Delta\tilde{m}^2_{32}|$, $|\Delta\tilde{m}^2_{31}|$ have been measured.
* Decisive measurements of δ, M.H. are planned
* There are several tensions.
  ➔ Redundant experiments to check each other are important.
  ➔ New physics might be behind them.

Part-II

Other Quantum Oscillations:
= A collection of various oscillations & mixings =

The origin of the neutrino oscillation is transitions between different flavor neutrinos, such as,

$$\nu_e \Leftrightarrow \nu_\mu$$

In fact, many kinds of transitions take place in various physics phenomena; many of them bear important physics effects. Such important physics can be understood as the same way as neutrino oscillation mechanism.

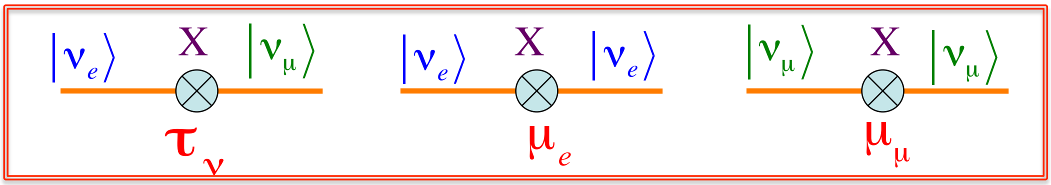
In some cases, abstract concepts, such as Parity, can be understood by a concrete idea of oscillation and mixing.

It should be useful to teach various physics using such unified and concrete point of view.

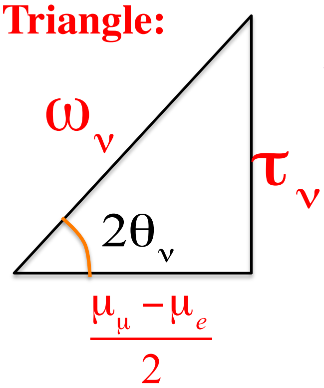
Cabbibo angle θ_C

Neutrino Oscillation case

Something (X) changes ν_μ to ν_e and gives self-transition



Mixing Triangle:



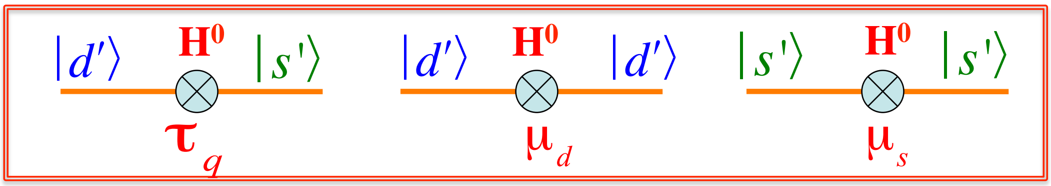
Mass eigenstate:
$$\begin{cases} \nu_1 = \left(\cos\theta_\nu \left|\nu_e\right\rangle - \sin\theta_\nu \left|\nu_\mu\right\rangle\right)\exp\left[-im_1 t\right] \\ \nu_2 = \left(\sin\theta_\nu \left|\nu_e\right\rangle + \cos\theta_\nu \left|\nu_\mu\right\rangle\right)\exp\left[-im_2 t\right] \end{cases}$$

Masses:
$$m_{1,2} = \bar{\mu}_\nu \mp \omega_\nu$$

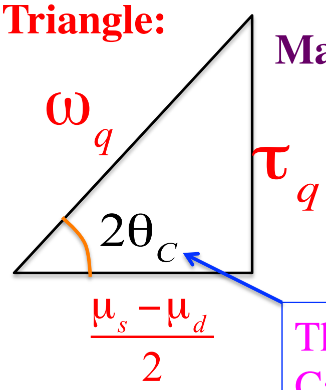
$$\bar{\mu}_\nu = \frac{\mu_e + \mu_\mu}{2}, \quad \omega_\nu = \frac{1}{2}\sqrt{\left(\mu_\mu - \mu_e\right)^2 + 4\tau_\nu^2}$$

Quark case $(\nu_e,\nu_\mu)\rightarrow(d',s')$

Higgs potential (H^0) changes *d'* to *s'* and gives self-transition



Mixing Triangle:



This is the Cabbibo angle

Mass eigenstate:

$$\begin{cases} d=\left(\cos\theta_C|d'\rangle-\sin\theta_C|s'\rangle\right)\exp\left[-im_d t\right] \\ s=\left(\sin\theta_C|d'\rangle+\cos\theta_C|s'\rangle\right)\exp\left[-im_s t\right] \end{cases}$$

Masses: $m_{d,s}=\bar{\mu}_q\mp\omega_q$

$$\bar{\mu}_q=\frac{\mu_d+\mu_s}{2},\quad \omega_q=\frac{1}{2}\sqrt{\left(\mu_s-\mu_d\right)^2+4\tau_q^2}$$

Important Difference

neutrino and quark oscillations are two extreme cases of the uncertainty principle.

$$P[t]=\sin^2 2\theta\sin^2\frac{\Delta m^2}{4E}t$$

Oscillation length for 1GeV particle

$$\lambda_\nu\sim10^6 m$$
$$\lambda_q\sim10^{-14} m$$

ν: Measurement of the flavor evolution pattern possible but it is impossible to distinguish ν_1 and ν_2 by measuring masses

q*: It is possible to distinguish *d* and *s* by their masses but the oscillation is too quick to observe the evolution of *d'* and *s'

Chirality Oscillation

A problem:

μ_R^- can be produced by EM interaction.

$\sim 2\mu s$ after, it decays weakly.

$$e^+ + e^- \to \gamma^* \to \mu_L^+ + \mu_R^-$$

EM

$$\hookrightarrow e^- + \nu_\mu + \overline{\nu}_e$$

Weak

But why this μ_R^- can decay weakly?

Chirality Oscillation & Muon Decay

Definition of Chirality State:

$$\begin{cases} \psi_R = \dfrac{1+\gamma^5}{2}\begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{u+v}{\sqrt{2}}\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \equiv \dfrac{u+v}{\sqrt{2}}|R\rangle, \\ \psi_L = \dfrac{1-\gamma^5}{2}\begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{u-v}{\sqrt{2}}\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \dfrac{u-v}{\sqrt{2}}|L\rangle \end{cases}$$

Muon satisfies the Dirac equation

$$\frac{d}{dt}\psi_\mu = -im_\mu\gamma_0\psi_\mu$$

Wave function can be expressed in chirality basis

$$\frac{d}{dt}\psi_\mu = \dot{C}_R|\mu_R\rangle + \dot{C}_L|\mu_L\rangle$$

The right-hand side of the Dirac equation is expressed as

$$\gamma_0\psi_\mu = \begin{pmatrix} u \\ -v \end{pmatrix} = C_L|\mu_R\rangle + C_R|\mu_L\rangle$$

Therefore, the Dirac equation can be expressed as

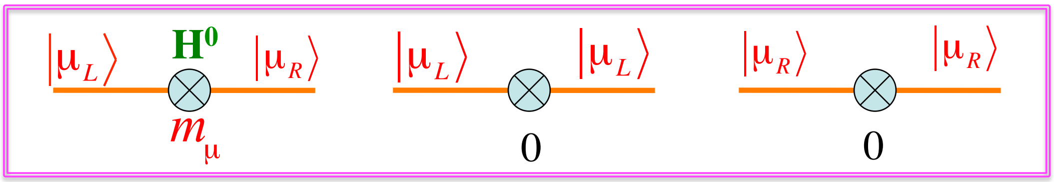
$$\Rightarrow \quad i\left(\dot{C}_L|\mu_L\rangle + \dot{C}_R|\mu_R\rangle\right) = m_\mu\left(C_R|\mu_L\rangle + C_L|\mu_R\rangle\right)$$

or,

$$i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$$

← The Dirac equation is actually chirality swapping equation

$$i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$$



mass eigenstates:

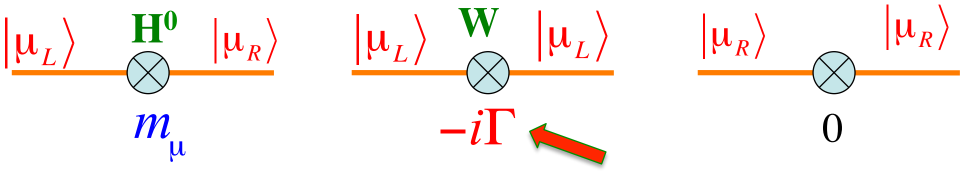
$$\begin{cases} \mu[E>0] = \frac{1}{\sqrt{2}}\left(|\mu_L\rangle + |\mu_R\rangle\right)\exp\left[-im_\mu t\right] \\ \mu[E<0] = \frac{1}{\sqrt{2}}\left(|\mu_L\rangle - |\mu_R\rangle\right)\exp\left[+im_\mu t\right] \end{cases}$$

$\mu_R \Leftrightarrow \mu_L$ **oscillation is taking place**

$$P\left[\mu_R \Leftrightarrow \mu_L\right] = \sin^2 m_\mu t$$

Muon decays weakly while it is in μ_L state

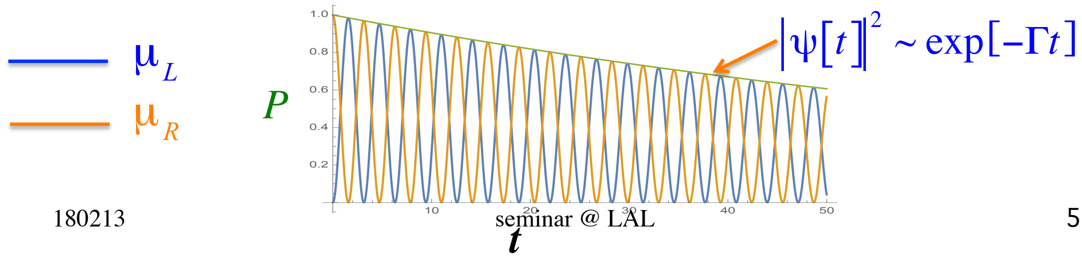
The weak decay effect can be included by putting imaginary amplitude to the μ_L self transition



State equation: $i\begin{pmatrix}\dot{C}_L\\ \dot{C}_R\end{pmatrix}=\begin{pmatrix}-i\Gamma & m_\mu\\ m_\mu & 0\end{pmatrix}\begin{pmatrix}C_L\\ C_R\end{pmatrix}$ $\quad\Gamma=\frac{1}{\tau_\mu}=3\times10^{-10}eV\ll m_\mu$

The solution for the condition $\psi[0]=|\mu_R\rangle$ **is,**

$$\psi[t]\sim\left(\cos[m_\mu t]|\mu_R\rangle-i\sin[m_\mu t]|\mu_L\rangle\right)\exp\left[-\frac{\Gamma}{2}t\right]$$



Weinberg Angle θ_W

The Lagrangian for the interaction of the gauge boson and Higgs fields is written as,

U(1) G SU(3) G Higgs

$$\mathcal{L}_{\Phi G_{\smile}} = \frac{1}{4}\left|(g' B^{\mu} + g(\vec{W}^{\mu}\cdot\vec{\sigma}))\Phi\right|^2 \xrightarrow{\text{SSB}}$$

Neutral component

$$\frac{(v_0+h)^2}{8}(g^2(|W^+|^2+|W^-|^2) + (g^2 W_3^2 + g'^2 B^2 - gg'(W_3 B + B W_3)))$$

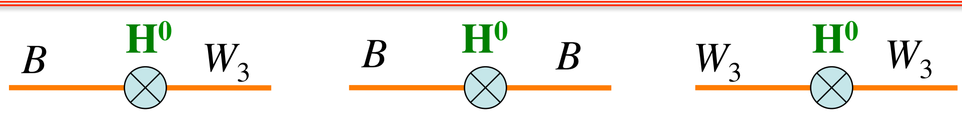
Euler-Lagrange equation
➔State equation,

$$\frac{d^2}{dt^2}\begin{pmatrix} B \\ W_3 \end{pmatrix} = \frac{v_0^2}{4}\begin{pmatrix} -g'^2 & gg' \\ gg' & -g^2 \end{pmatrix}\begin{pmatrix} B \\ W_3 \end{pmatrix}$$

This 2nd differential equation can be rewritten by 2 times of the 1st differential equation:

$$i\frac{d}{dt}\begin{pmatrix} B \\ W_3 \end{pmatrix} = \frac{v_0}{2\sqrt{g^2+g'^2}}\begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix}\begin{pmatrix} B \\ W_3 \end{pmatrix}$$

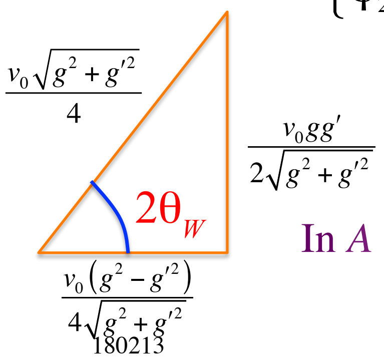
Oscillation View of Gauge Bosons



$-gg' \times \frac{v_0}{2\sqrt{g^2+g'^2}}$ $\quad g'^2 \times \frac{v_0}{2\sqrt{g^2+g'^2}}$ $\quad g^2 \times \frac{v_0}{2\sqrt{g^2+g'^2}}$

Mass eigenstate

$$\begin{cases} \psi_1 = (B\cos\theta_W - W_3\sin\theta_W)\exp[-i\times 0\times t] & \Rightarrow A \\ \psi_2 = (B\sin\theta_W + W_3\cos\theta_W)\exp[-iM_Z t] & \Rightarrow Z^0 \end{cases}$$



$$M_Z = \frac{v_0}{2}\sqrt{g^2+g'^2}, \quad M_A = 0$$

$$\tan 2\theta_W = \frac{2gg'}{g^2-g'^2} \text{ or } \tan\theta_W = \frac{g'}{g}$$

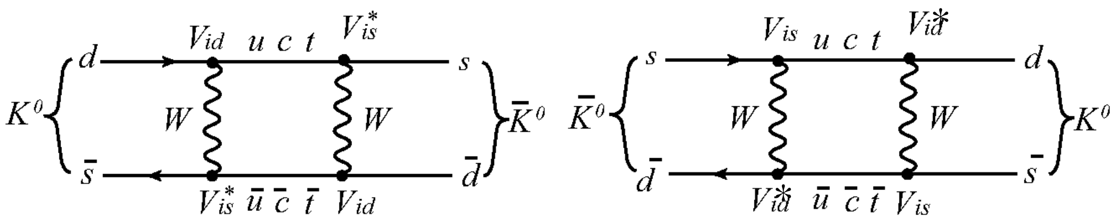
In A and Z^0, B and W_3 are oscillating very quickly.

$$P[B \Leftrightarrow W_3] = \sin^2 2\theta_W \sin^2\left[\frac{1}{2}M_Z t\right]$$

$K^0 \Leftrightarrow \bar{K}^0$ Oscillation and CP non-conservation:

A case of imaginary cross transition amplitude

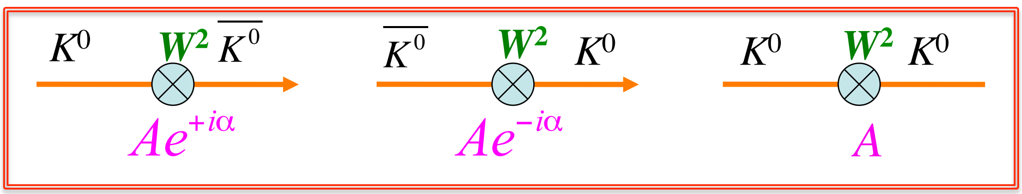
$K^0,\ \bar{K}^0$ system



(a) $K^0 \to \bar{K}^0$ (b) $\bar{K}^0 \to K^0$

Transition amplitudes includes imaginary number

$$\begin{cases} M\left(K^0 \to \overline{K^0}\right) = g_W^4 \left(\sum_{i=u,c,t} V_{id} V_{is}^* \Pi_K\left(m_i\right) \right)^2 = A \exp\left[+i\alpha\right] \\ M\left(\overline{K^0} \to K^0\right) = g_W^4 \left(\sum_{i=u,c,t} V_{id}^* V_{is} \Pi_K\left(m_i\right) \right)^2 = A \exp\left[-i\alpha\right] \\ M\left(K^0 \to K^0\right) = g_W^4 \left| \sum_{i=u,c,t} V_{id} V_{is}^* \Pi_K\left(m_i\right) \right|^2 = A \end{cases}$$



State equation: $i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} = A\begin{pmatrix} 1 & e^{-i\alpha} \\ e^{i\alpha} & 1 \end{pmatrix}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}$

On the other hand, CP eigenstates are

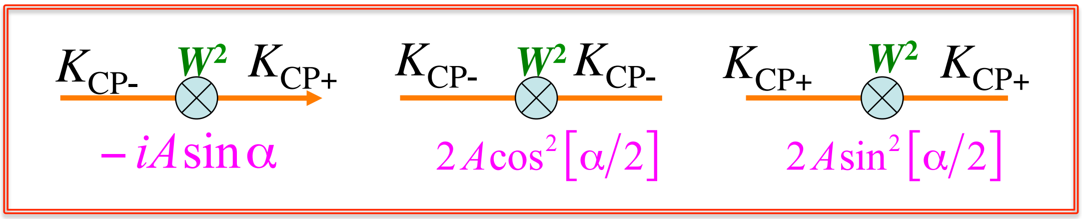
$$\begin{pmatrix} K_{CP+} \\ K_{CP-} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}$$

$$\left(\mathrm{CP}K_{CP+} = -\frac{\overline{K^0} - K^0}{\sqrt{2}} = K_{CP+}, \quad \mathrm{CP}K_{CP-} = -\frac{\overline{K^0} + K^0}{\sqrt{2}} = -K_{CP-} \right)$$

The state equation in CP basis:

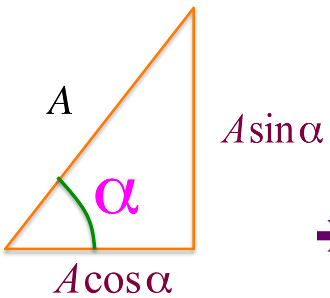
$$i\frac{d}{dt}\begin{pmatrix} K_{CP+} \\ K_{CP-} \end{pmatrix} = A\begin{pmatrix} 2\sin^2[\alpha/2] & -i\sin\alpha \\ i\sin\alpha & 2\cos^2[\alpha/2] \end{pmatrix}\begin{pmatrix} K_{CP+} \\ K_{CP-} \end{pmatrix}$$

Transitions between CP eigenstates



Mass eigenstate

$$\begin{cases} \psi_1 = \left(\cos[\alpha/2]K_{CP+} - i\sin[\alpha/2]K_{CP-}\right)\exp[-iM_K t] \\ \psi_2 = \left(-i\sin[\alpha/2]K_{CP+} + \cos[\alpha/2]K_{CP-}\right)\exp[-i(M_K + 2A)t] \end{cases}$$



$K_{CP+} \Leftrightarrow K_{CP-}$ **oscillation takes place**

$$P\left[K_{CP+} \Leftrightarrow K_{CP-}\right] = \sin^2\alpha \sin^2 At$$
$$\omega = 2A \sim 1/10ns$$

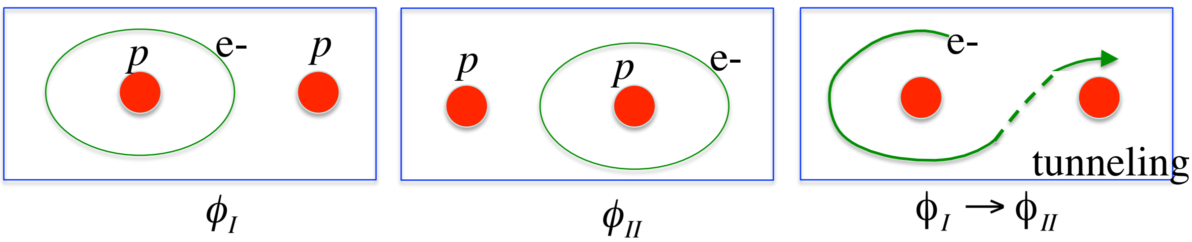
➔ **CPV effect can be explained by oscillation of CP eigenstate.**

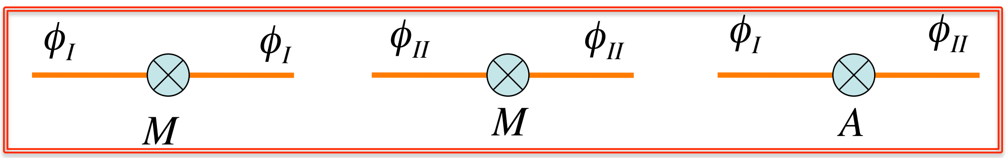
The oscillation and mixing view is useful to understand abstract properties concretely

* Parity
* C-Parity
* Isospin

What is parity? (Feynman's explanation)

An Hydrogen ion H_2^+ has two basis states: ϕ_I and ϕ_{II}

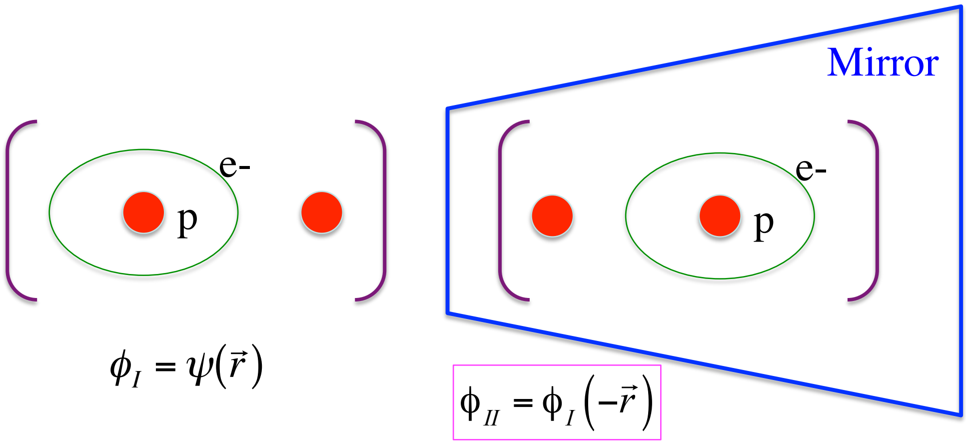




➔ The energy eigenstates

$$\begin{cases} \Phi_+ = \frac{1}{\sqrt{2}}(\phi_I + \phi_{II})e^{-i(M+A)t} \\ \Phi_- = \frac{1}{\sqrt{2}}(\phi_I - \phi_{II})e^{-i(M-A)t} \end{cases}$$

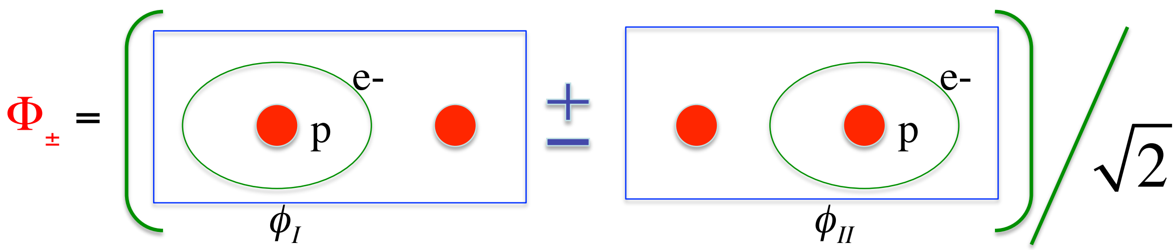
ϕ_I and ϕ_{II} are oscillating



Actually, ϕ_{II} is a mirror image of ϕ_I

Therefore, the energy eigenstates are

$$\begin{cases} \Phi_+(\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(\vec{r}) + \phi_I(-\vec{r})\right)\exp\left[-i(M+A)t\right] \\ \Phi_-(\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(\vec{r}) - \phi_I(-\vec{r})\right)\exp\left[-i(M-A)t\right] \end{cases}$$



If parity of the mass eigenstates is reversed

Parity = + structure

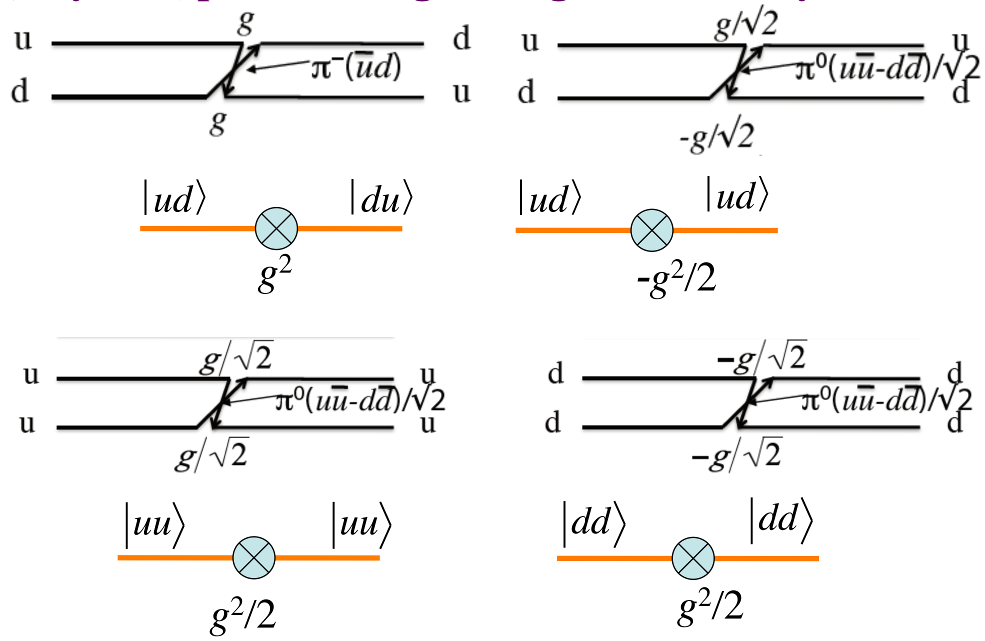
$$\begin{cases} \Phi_+(-\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(-\vec{r}) + \phi_I(\vec{r})\right)\exp\left[-i(M+A)t\right] = +\Phi_+(\vec{r}) \\ \Phi_-(-\vec{r}) = \frac{1}{\sqrt{2}}\left(\phi_I(-\vec{r}) - \phi_I(\vec{r})\right)\exp\left[-i(M-A)t\right] = -\Phi_-(\vec{r}) \end{cases}$$

Parity =- structure

=> Energy eigenstates have fixed parities.

Isospin

For *u,d* system, pion exchange changes the basis system



State equation

$$i\frac{d}{dt}\begin{pmatrix} C_{uu} \\ C_{ud} \\ C_{du} \\ C_{dd} \end{pmatrix} = \begin{pmatrix} g^2/2 & 0 & 0 & 0 \\ 0 & -g^2/2 & g^2 & 0 \\ 0 & g^2 & -g^2/2 & 0 \\ 0 & 0 & 0 & g^2/2 \end{pmatrix}\begin{pmatrix} C_{uu} \\ C_{ud} \\ C_{du} \\ C_{dd} \end{pmatrix}$$

←This is the same form as spin dipole moment interaction (cf. 21cm line)

Mass eigenstates

$$\begin{cases} \psi_U = |uu\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_D = |dd\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_+ = \frac{|ud\rangle + |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_- = \frac{|ud\rangle - |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 - \frac{3}{2}g^2\right)t\right] \end{cases}$$

Analogy to the Spin combination

3 same mass state: I=1 state

$$|\Sigma^+\rangle = |uu\rangle|s\rangle$$

$$|\Sigma^-\rangle = |dd\rangle|s\rangle$$

$$|\Sigma^0\rangle = \frac{|ud\rangle + |du\rangle}{2}|s\rangle$$

singlet state: I=0 → $|\Lambda\rangle = \frac{|ud\rangle - |du\rangle}{2}|s\rangle$

There are a lot more interesting oscillations and mixings . . .

Name	Origin	Transition	Energy eigenstate
Neutrino Oscillation	X	$\vert\nu_e\rangle \Leftrightarrow \vert\nu_\mu\rangle \Leftrightarrow \vert\nu_\tau\rangle$	$\nu_1,\ \nu_2,\ \nu_3$
Cabbibo Angle	Higgs	$\vert d'\rangle \Leftrightarrow \vert s'\rangle$	$d,\ s$
Chirality Osc.	Higgs	$\vert L\rangle \Leftrightarrow \vert R\rangle$	$\vert R\rangle \pm \vert L\rangle$
Majorana Neutrino	X	$\vert\nu_L\rangle \Leftrightarrow \vert\bar{\nu}_R\rangle$	$\vert\nu_L\rangle \pm \vert\bar{\nu}_R\rangle$
Seesaw Mechanism	X	$\vert\nu_R\rangle \Leftrightarrow \vert\bar{\nu}_L\rangle,\ \vert\nu_L\rangle \Leftrightarrow \vert\nu_R\rangle$	$\nu = \vert\nu_L\rangle - \vert\bar{\nu}_R\rangle,\ N = \vert\nu_R\rangle + \vert\bar{\nu}_L\rangle$
Weinberg angle	Higgs	$W_3 \Leftrightarrow B$	$\gamma,\ Z^0$
Hydrogen 21 cm line	$\vec{\mu}_p \cdot \vec{\mu}_e$	$\vert p(\Uparrow)e(\Downarrow)\rangle \Leftrightarrow \vert p(\Downarrow)e(\Uparrow)\rangle$	$\vert\Uparrow\Downarrow\rangle \pm \vert\Downarrow\Uparrow\rangle$
$\pi^+ - \rho^+$ mass difference	Strong	$\vert\Uparrow\Downarrow\rangle_S \Leftrightarrow \vert\Downarrow\Uparrow\rangle_S$	$\pi^+,\ \rho^+$
CPV	Weak	$K_{CP+} \Leftrightarrow K_{CP-}$	$K_1,\ K_2$
Hydrogen Ion (H_2^+)	tunneling	$\vert(pe^-)p\rangle \Leftrightarrow \vert p(e^-p)\rangle$	$\vert(pe^-)p\rangle \pm \vert p(e^-p)\rangle$
Positronium	EM	$\vert e^+e^-\rangle \Leftrightarrow \vert e^-e^+\rangle$	o-Ps, p-Ps
Isospin	S	$\vert ud\rangle \Leftrightarrow \vert du\rangle,\ \vert u\bar{u}\rangle \Leftrightarrow \vert d\bar{d}\rangle$	$(\Lambda,\ \Sigma),\ (\rho,\omega)$
Baryon Color	Strong	$\vert RGB\rangle \Leftrightarrow \vert GRB\rangle$	$\vert RGB\rangle - \vert RBG\rangle + \vert BRG\rangle - \cdots$
ρ^0, ω, ϕ structure	Strong	$\vert u\bar{u}\rangle \Leftrightarrow \vert d\bar{d}\rangle \Leftrightarrow \vert s\bar{s}\rangle$	$\rho^0,\ \omega,\ \phi$
Spin precession in $\vec{B}$	$\vec{\mu}\vec{B}$	$\vert\Uparrow\rangle \Leftrightarrow \vert\Downarrow\rangle$	$\vert\Uparrow\rangle_\theta$
Deuteron	S	$\vert pn\rangle \Leftrightarrow \vert np\rangle$	$(\vert pn\rangle - \vert np\rangle)\vert\Uparrow\Uparrow\rangle$
sp^3 hybrid orbit	EM	$\psi_{2S} \Leftrightarrow \psi_{2P_i}$	$\psi_{2S} \pm \psi_{2P_x} \pm \psi_{2P_y} \pm \psi_{2P_z}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Summary: Part-II

*** The same mechanism as neutrino oscillation is working in various other places and is playing important physics roles.**

*** Many important physics can be understood by analogy of neutrino oscillation mechanism (or vice versa).**

*** Abstract properties, such as parity, etc. can be attributed to the structure of the mixings.**

*** It should be educative to teach such ideas to students.**