Theoretical models and simulations of high energy electron radiation in oriented crystals

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March 30, 2018 Orsay LAL

Plan

Intoduction. *Crystal field strength*

Diversity of orientation effects in crystals

- *synchrotron type radiation and pair production, intensity growth and saturation. Shift to zero incidence angle*
- *polarization and spin effects, charm and beauty hyperon magnetic and electric dipole moments*
- *gamma-telescopes, gamma-background suppression (CLEVER)*
- *crystal undulators (PEARL), Ferrara experiments in u-short bent crystals*

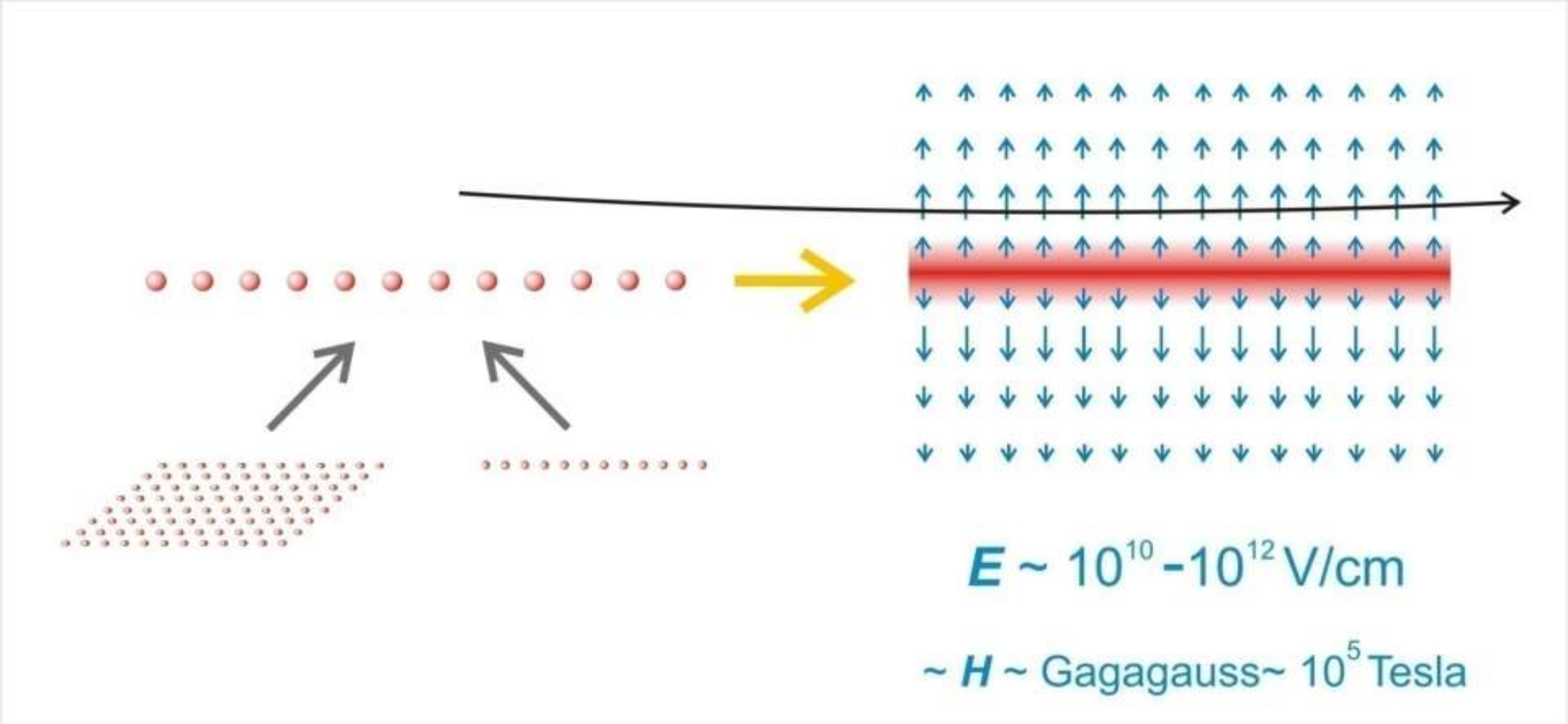
Crystal assisted collimation *Crustal cut. Multiple volume reflection. Miscut problem. Scattering by atomic strings.*

Simulation of high-energy electron radiation in crystals.

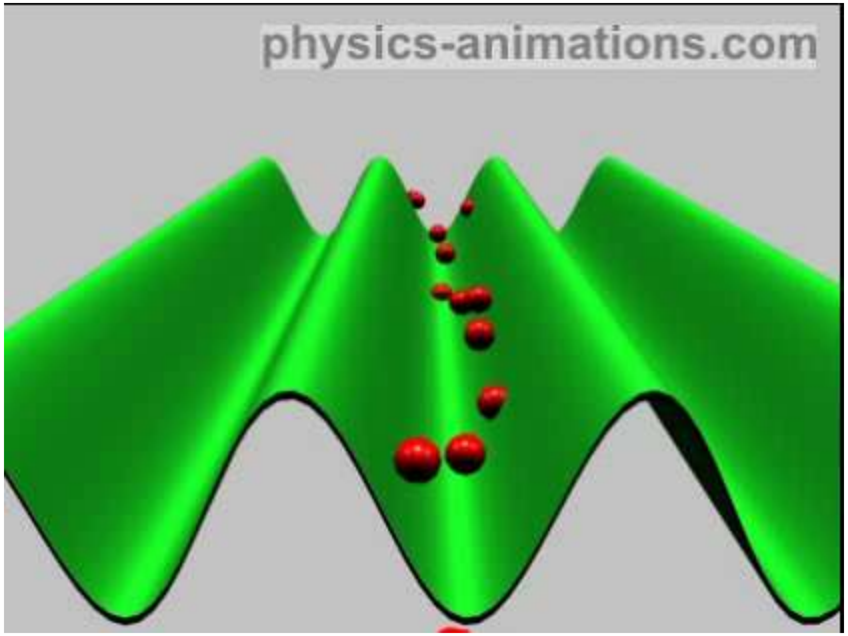
− *the need of detailed simulations (CFA insufficiency)*
− *incoherent processes in crystals*
− *radiation under multiple volume reflection*
− *crystalline scintillator performance (PRIN)*
− *medium energy electron radiation in W <111>*

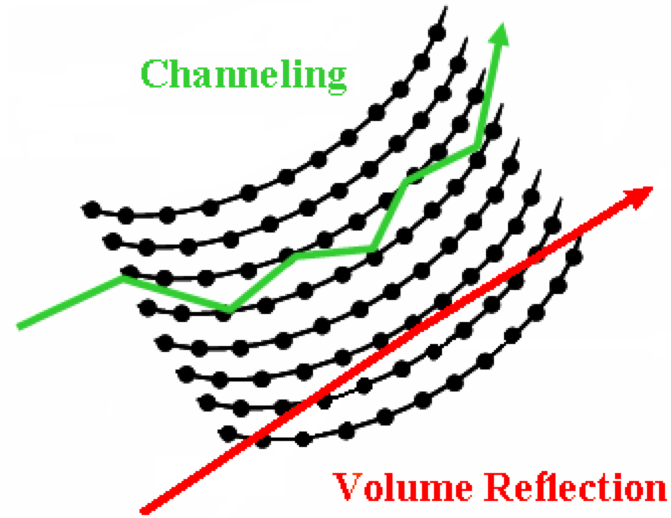
The uniqueness of crystal field

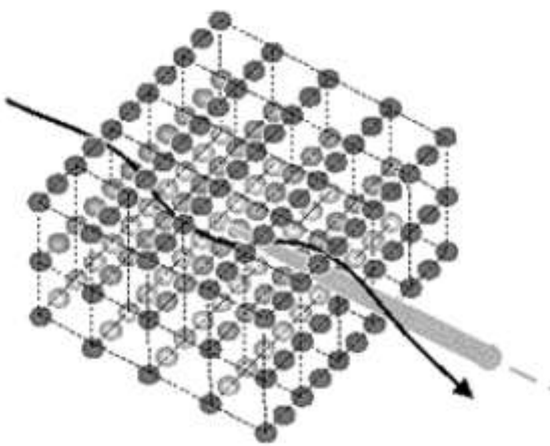
Moving in oriented crystals, particles come under the action of the practically inter-atomic-scale ***effective crystal field***

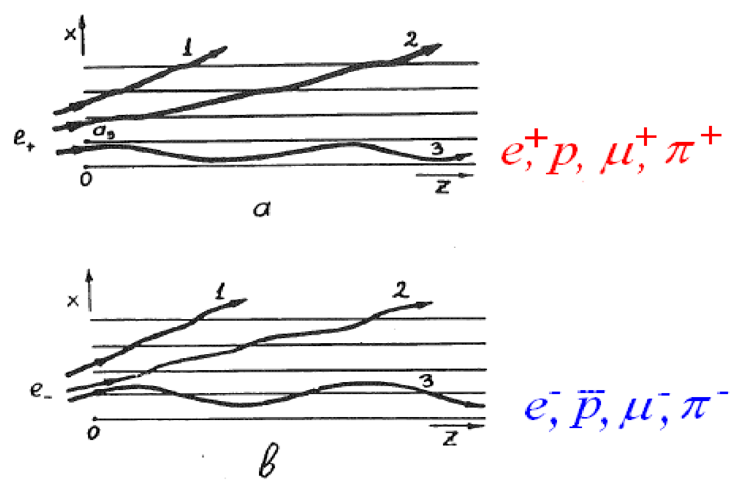


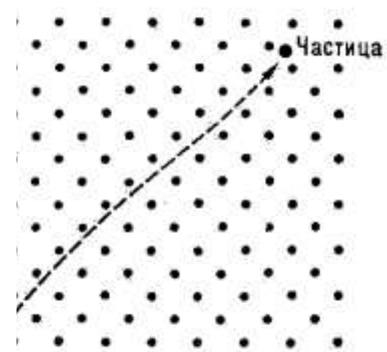
Planar channeling



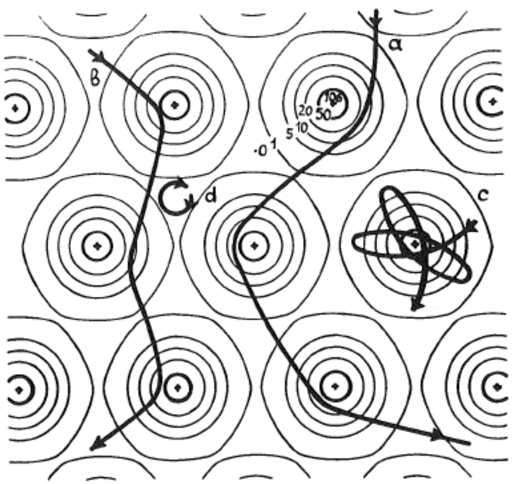


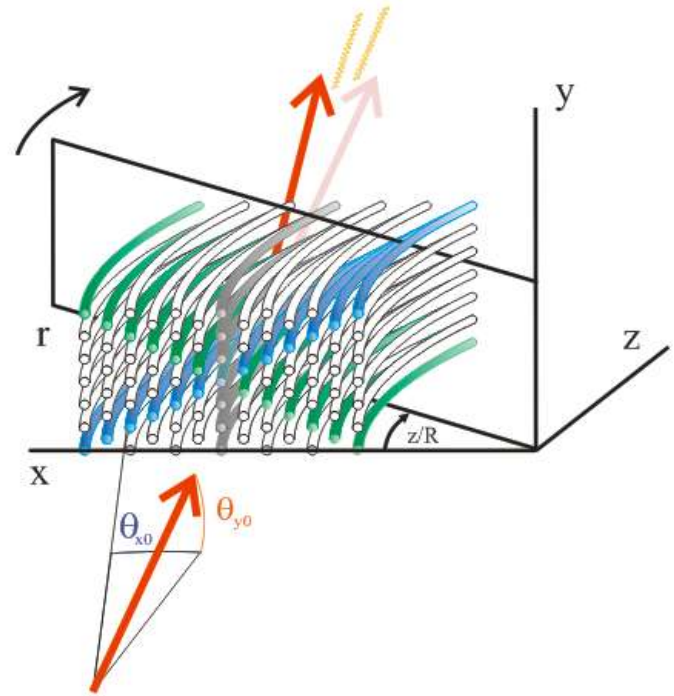


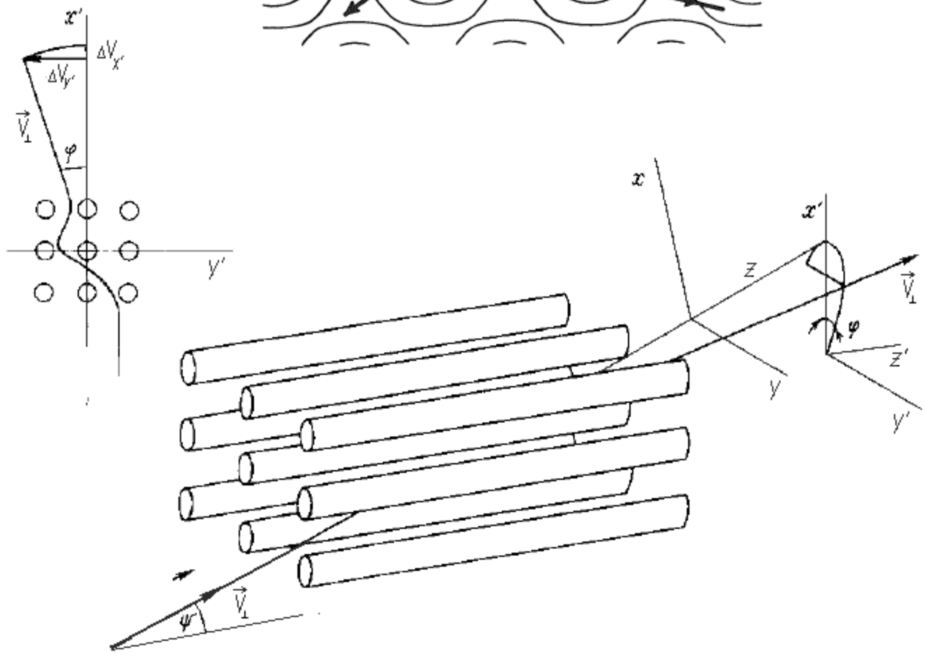


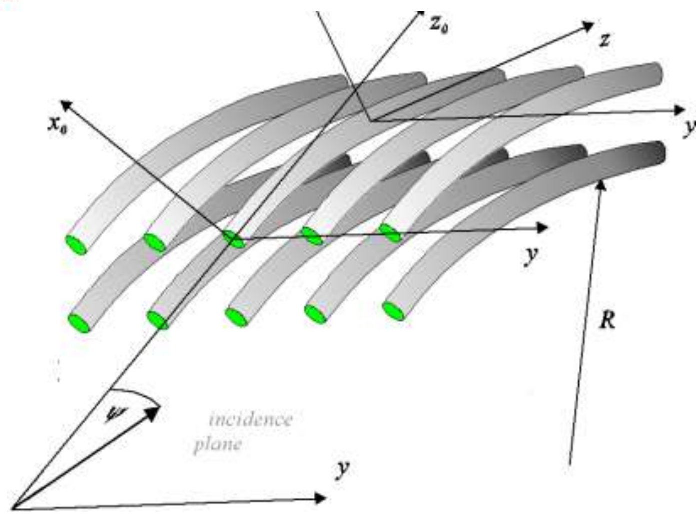


Scattering by atomic strings





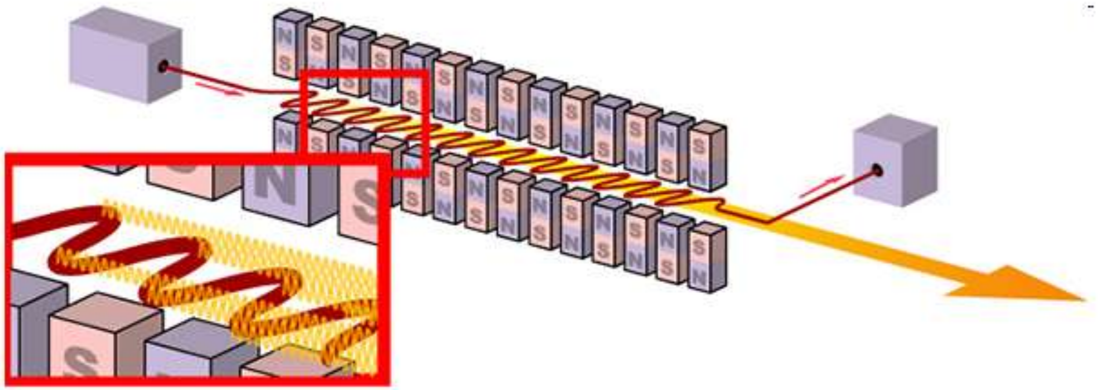


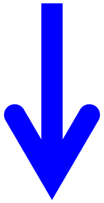


When channeling radiation becomes synchrotron-like radiation:

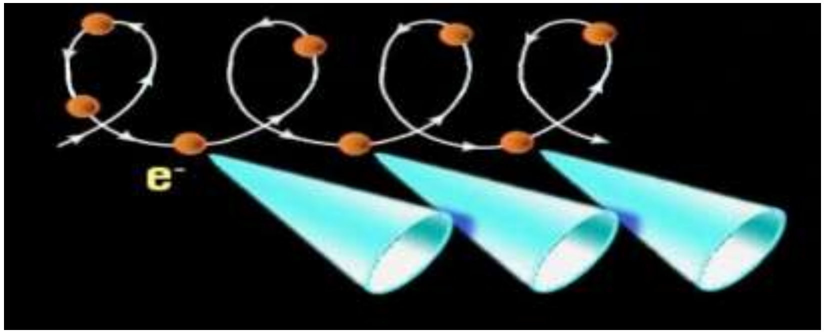
$$\vartheta \approx \sqrt{\frac{2V_0}{\varepsilon}} > \frac{m}{\varepsilon} \Rightarrow \varepsilon \gg \frac{m^2}{2V_0} \sim 1 \div 10 GeV$$

Dipole radiation

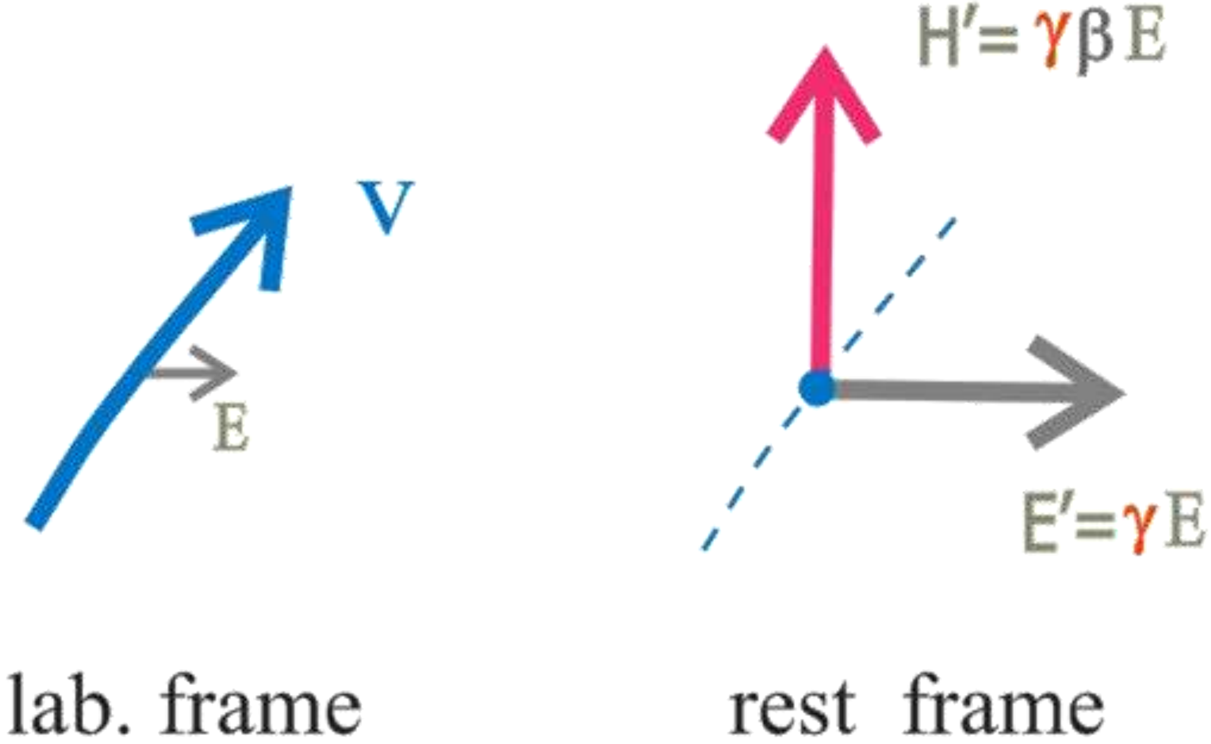




Synchrotron-like radiation



Field amplification in the particle rest frame



Lorentz amplification of field can exceed γ **~ 10^5** times

Critical energy for hard synchrotron-like radiation and pair production by gamma-quanta

$$E_0 = \frac{m^2 c^3}{e\hbar} \approx 1.32 \times 10^{16} eV$$

$$E_{cryst} \sim 10^{10} \div 10^{12} \frac{Volt}{cm} \equiv 3 \times 10^3 \div 3 \times 10^5 tesla$$

$$\varepsilon_{\chi=1} = \hbar \omega_{\kappa=1} = \frac{E_0}{E_{cryst}} mc^2 \sim 10 \div 1000 GeV$$

13.6 GeV for ⟨111⟩ W 293 K

Synchrotron-like radiation quantum parameter χ

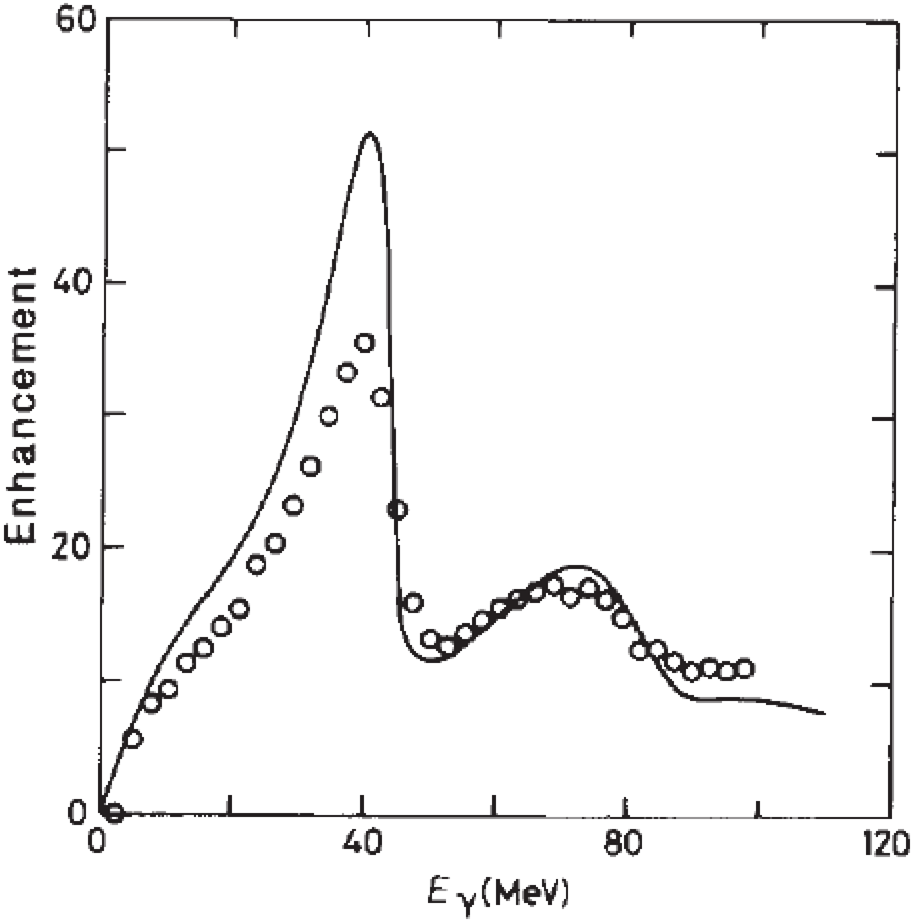
Invariant parameter: $\chi = \frac{\sqrt{\left(F_{\mu\nu}k^{\nu}\right)^{2}}}{m^{3}} \rightarrow \frac{E}{(m^{2}c^{3}/e\hbar)}\frac{\varepsilon_{e^{\pm}}}{mc^{2}} = \frac{E\gamma}{E_{0}} = \frac{E_{com}}{E_{0}}.$

Critical field: $E_{0} = \frac{m^{2}c^{3}}{e\hbar} \approx 1.32\times 10^{16}\, eV$

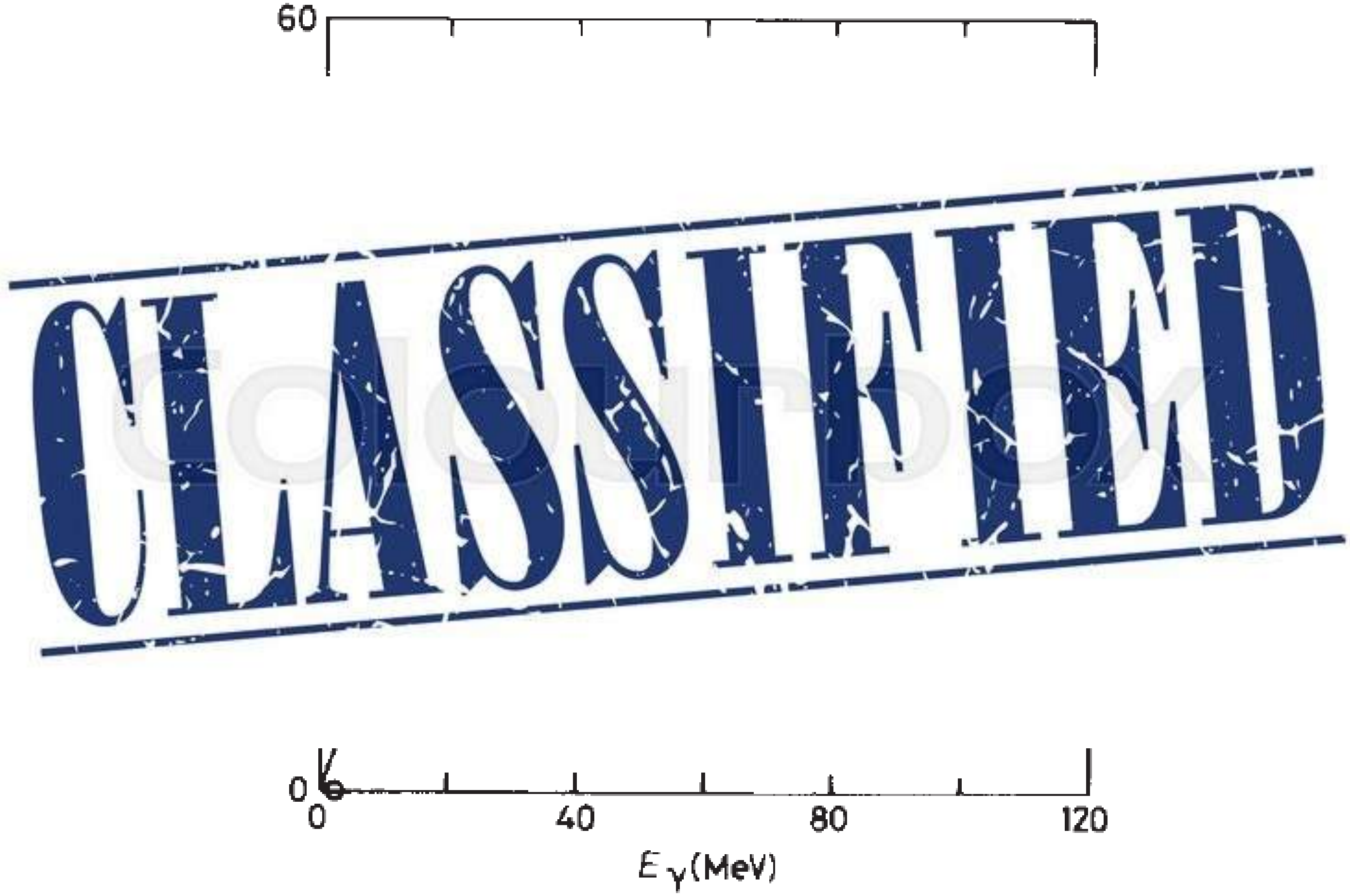
$$\varepsilon_{\gamma} \equiv \hbar\omega \approx \frac{\chi}{\chi + 2/3}\,\varepsilon_{e^{\pm}}$$

“Quantum” synchrotron-like radiation is observable in crystals:

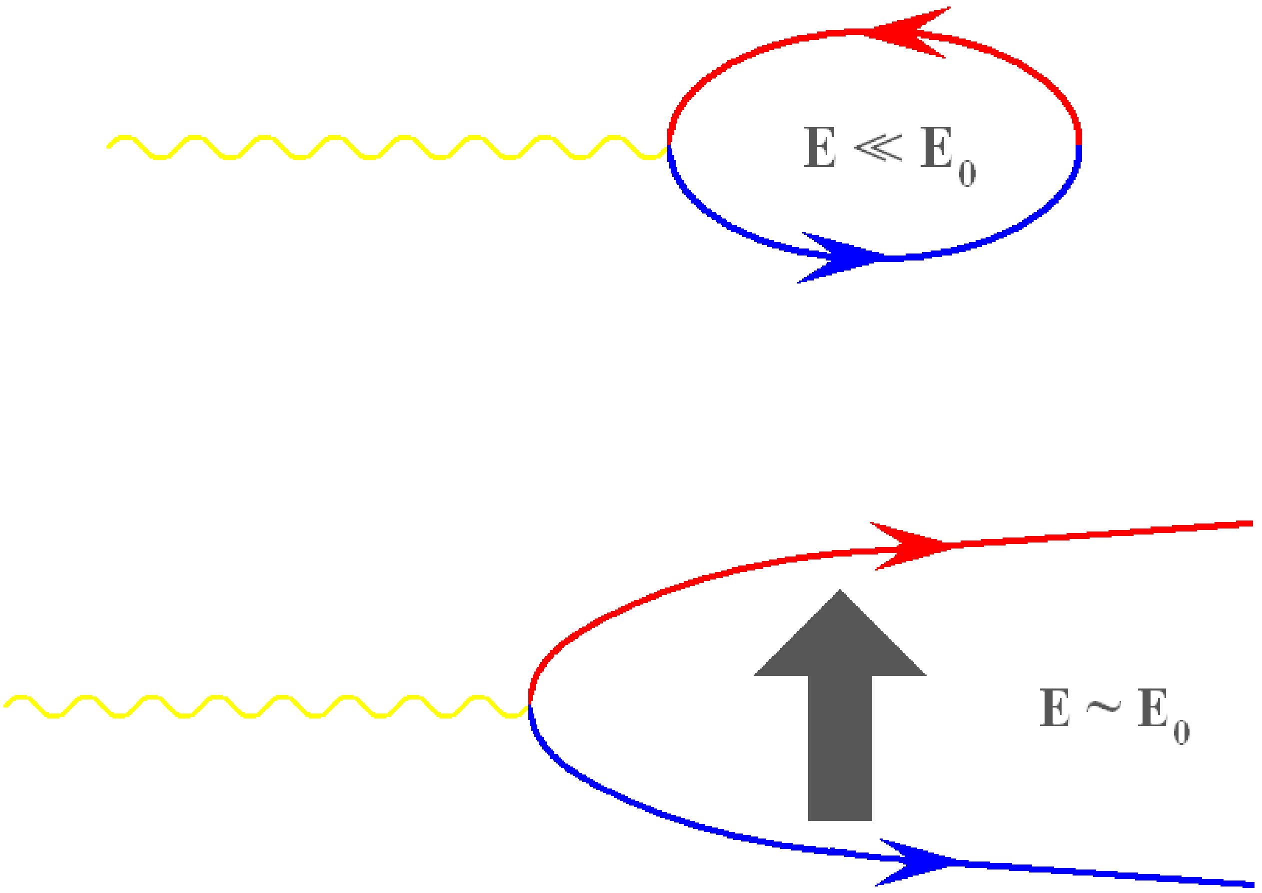
Channeling radiation



Channeling radiation



A virtual pair conversion to a real one in electric field $E \sim E_0$



Electron-positron pair production by gamma-quanta in the uniform field

Invariant parameter: $\kappa = \frac{\sqrt{\left(F_{\mu\nu}k^{\nu}\right)^2}}{m^3} \to \frac{\mathrm{E}}{(m^2c^3/e\hbar)}\frac{\hbar\omega}{mc^2} = \frac{\mathrm{E}\gamma}{\mathrm{E}_0} = \frac{\mathrm{E}_{com}}{\mathrm{E}_0}.$

Critical electric field: $\mathrm{E}_0 = \frac{m^2c^3}{e\hbar} \approx 1.32\times 10^{16}\, eV$

PP in vacuum (Schwinger) $\frac{dP_{e^+e^-}}{d\vec{r}} = \frac{\alpha \mathrm{E}^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{\mathrm{E}_0}{\mathrm{E}_{com}}\right)$

this process is also observable in crystals:

Critical electron and photon energies

Baryshevsky, Tikhomirov UFN, 1989

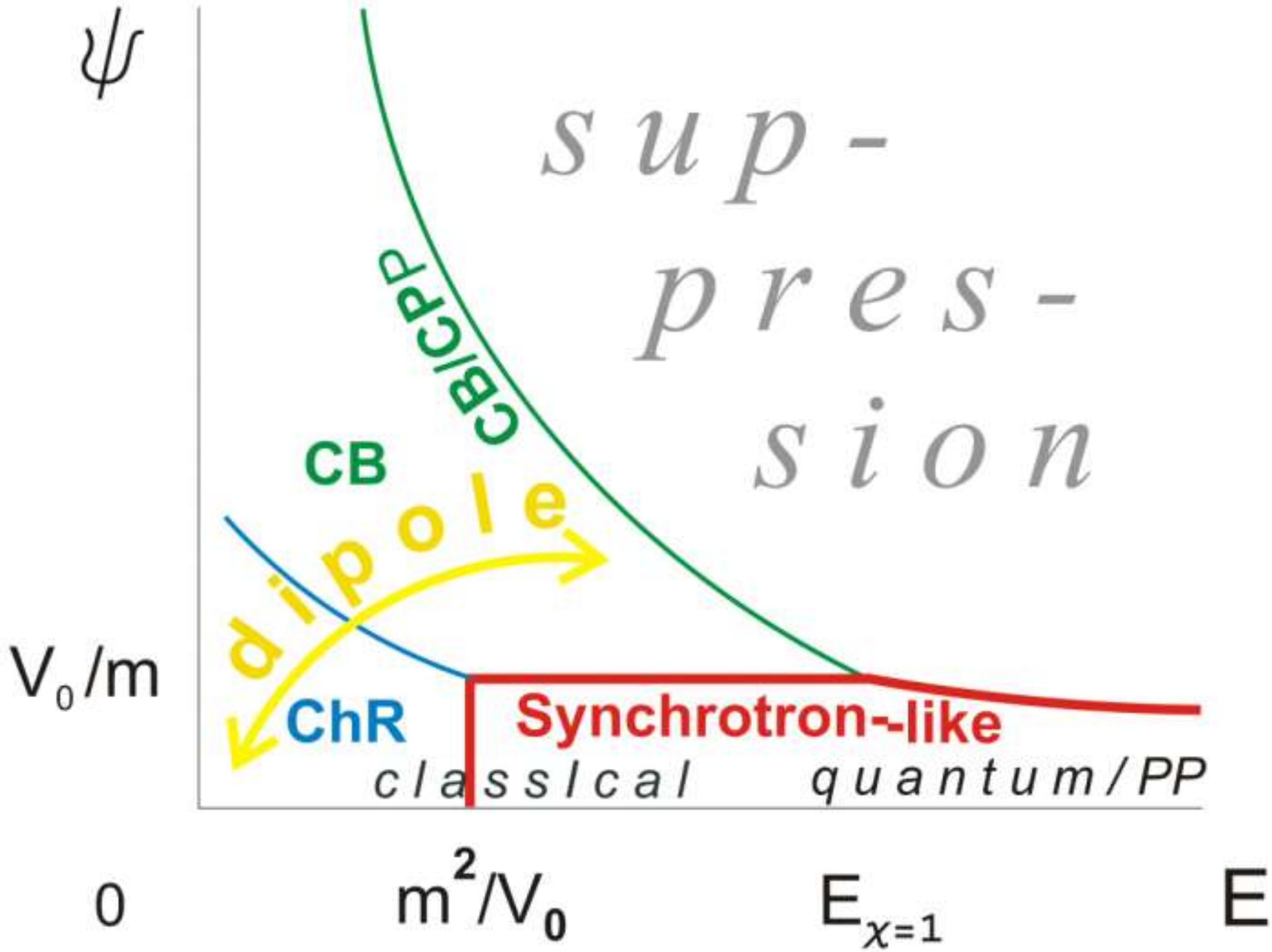
element	Z	*plane / axis*	$E_{\mathbf{max}}$ (GV/cm)	**H_{eff} (kilotesla)**	**$\hbar\omega_{cr} = \varepsilon_{cr}$ (GeV)**
Si	14	*plane* (110)	5.7	**1.9**	**1200**
Ge	32	*axis* ‹110›100K	144	**48**	**47**
W	74	*axis* ‹111›	500	**167**	**13.6**

Typical angle V_0/m for synchrotron-like radiation and PP processes in crystals

$$\vartheta \sim \frac{\left|\delta \vec{p}_\perp\right|}{\varepsilon} = \frac{1}{\varepsilon}\left|\int e\vec{\mathrm{E}}(z)dz\right| = \frac{1}{\varepsilon}\left|\int e\vec{\mathrm{E}}(z)\frac{d\vec{\rho}}{\psi}\right| \approx \frac{V_0}{\varepsilon\psi} \approx \frac{m}{\varepsilon} \Rightarrow$$

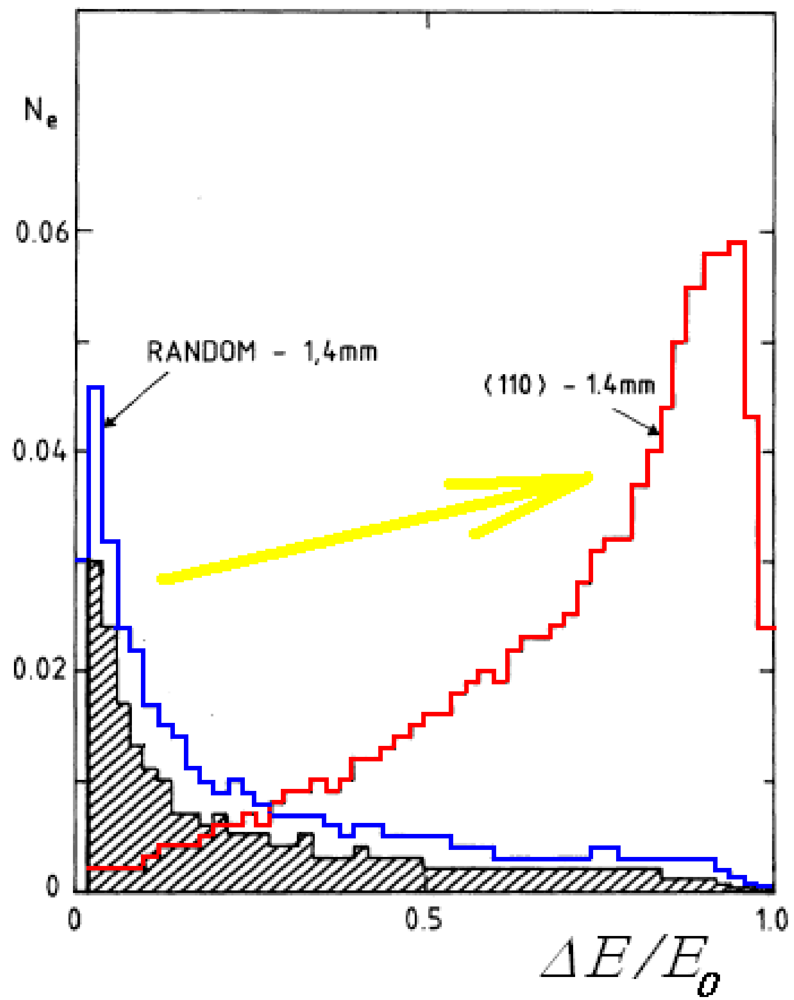
$$\psi \approx \frac{V_0}{m} \gg \sqrt{\frac{2V_0}{\varepsilon}}$$

At high energies both radiation and PP processes acquire synchrotron nature



25/14-time increase of radiative energy losses of 150 GeV electrons in 0.4/1.4 mm Ge<110> 100K

A. Belkacem, PRL 1985



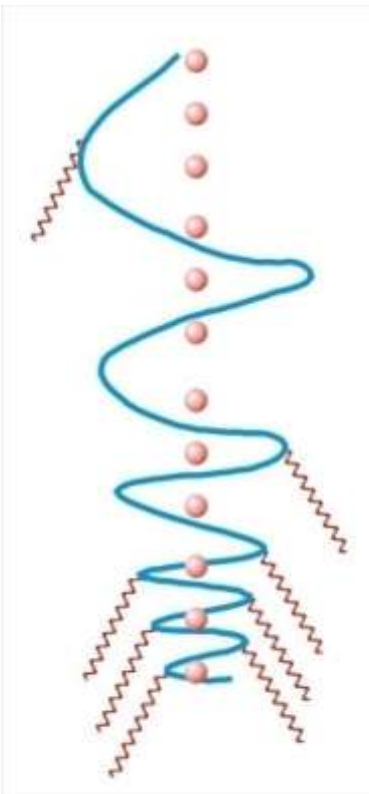
Electron radiative cooling

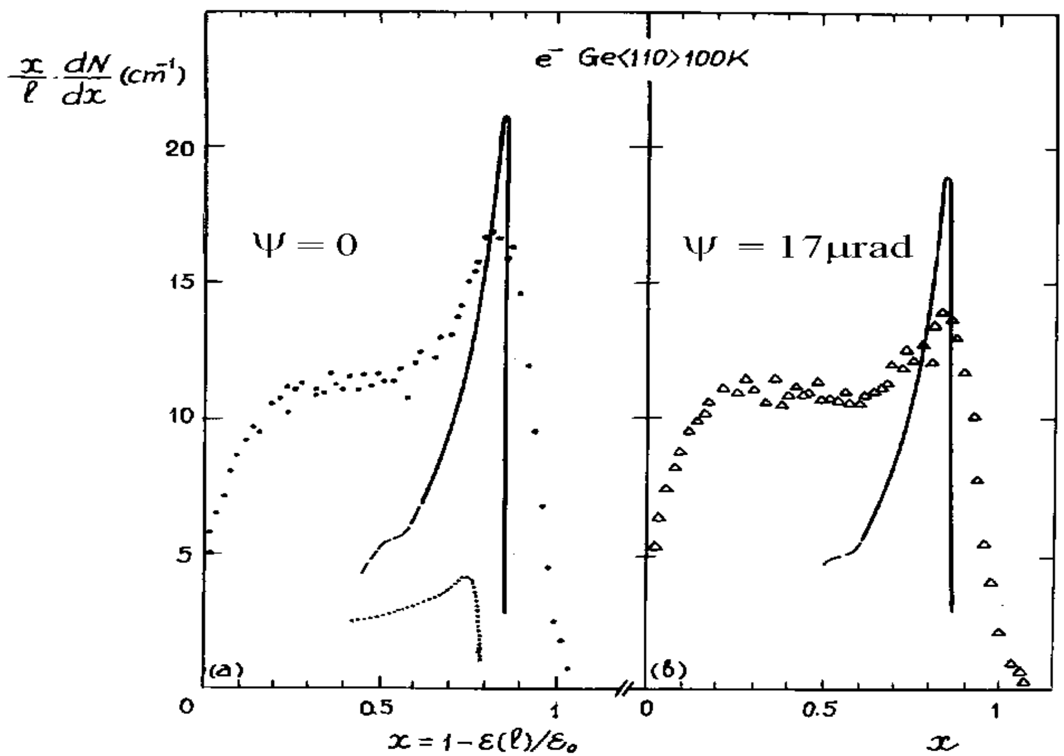
Baryshevskii V. G., Dubovskaya I. Ya. // Phys. Lett. 1977. Vol. A62. P. 45.

Belkacem A. et al. // Phys. Lett. 1986. Vol. B177. P. 211.

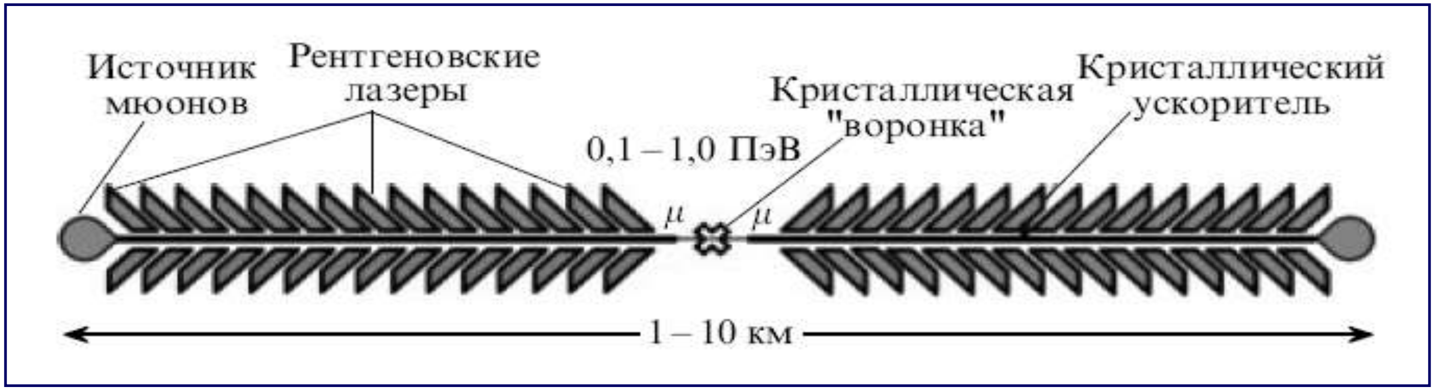
Tikhomirov V. V. // Phys. Lett. 1987. Vol. A125. P. 411.

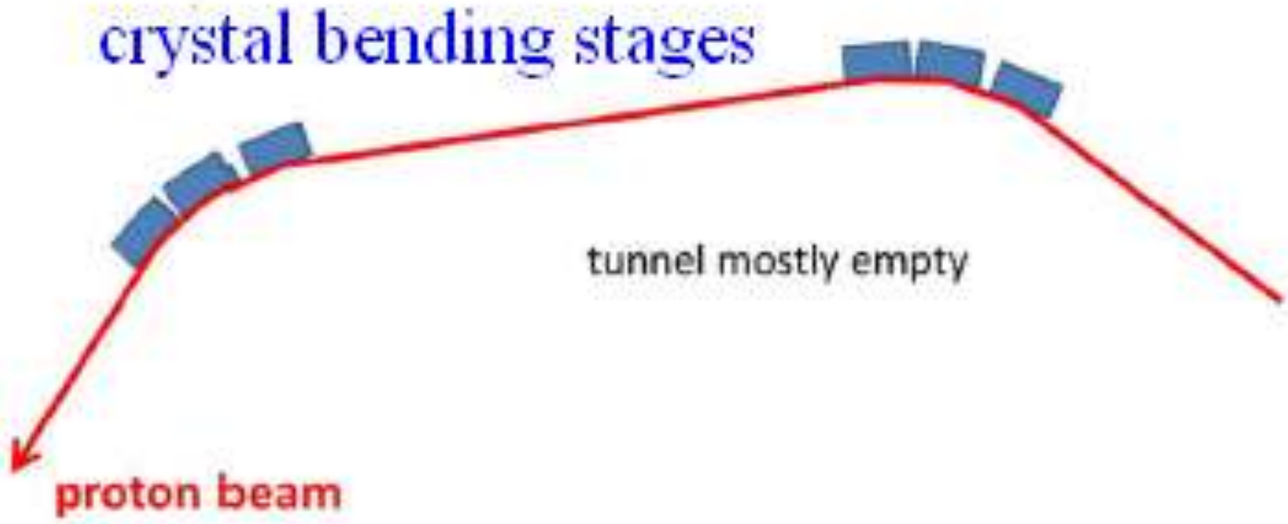
Tikhomirov V. V. // Nucl. Instr. Meth. 1989. Vol. B36. P. 282.





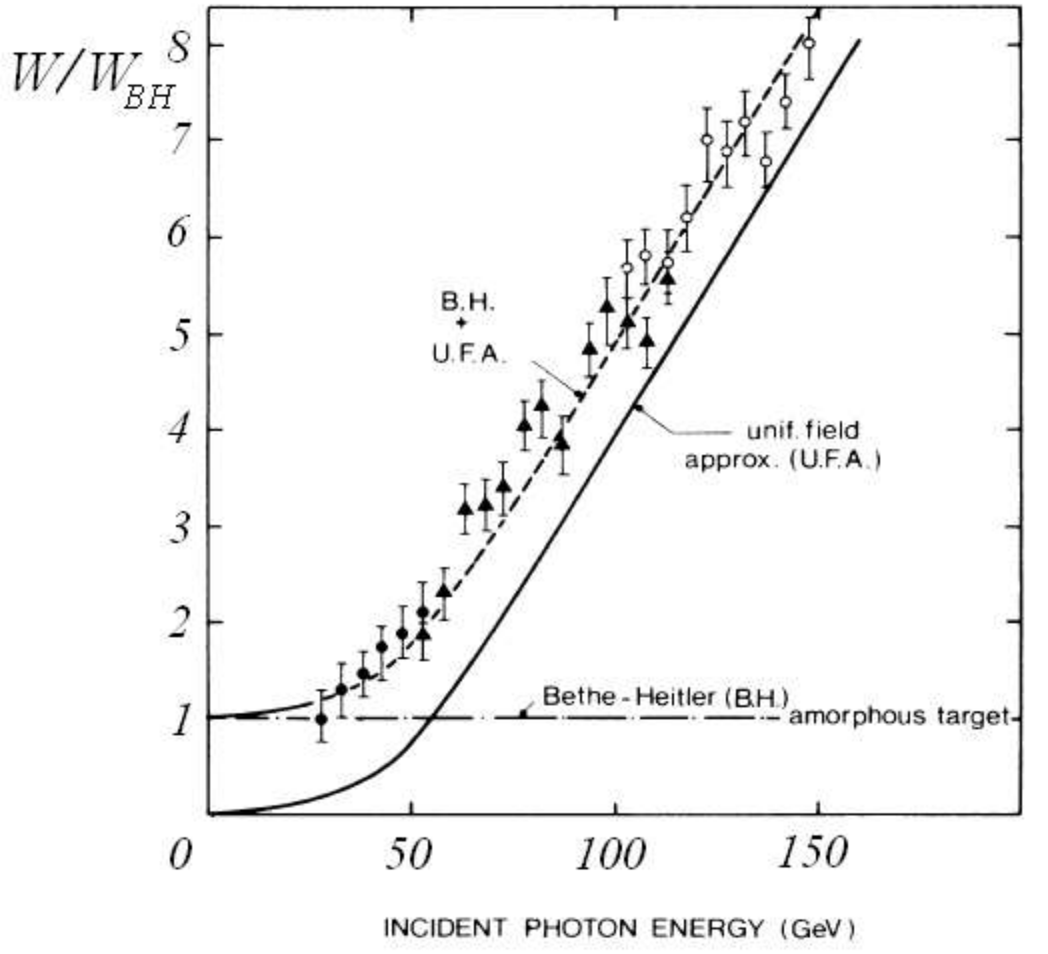
All plans for **PeV energies** and higher are based on **radiative cooling** in crystlas





8-time increase of PP probability in Ge<110> 100K at 150 GeV

A. Belkacem, PRL 1987



Crystal dichroism and birefringence

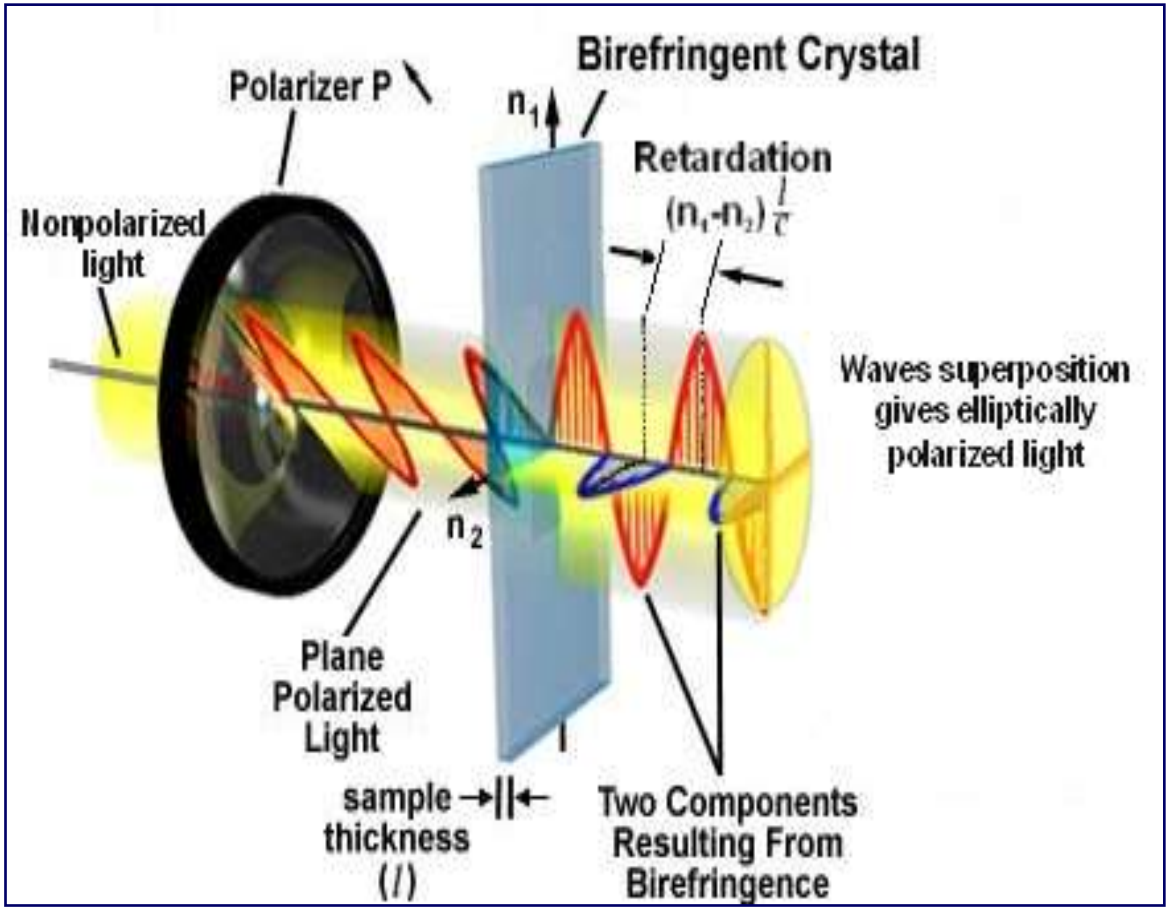
$$L_{\lambda/4}\frac{\omega}{c}(n_{\parallel}-n_{\perp})=\frac{\pi}{2}$$

For visible light

$$\Delta t = 10^{-15}\,\mathrm{s}$$

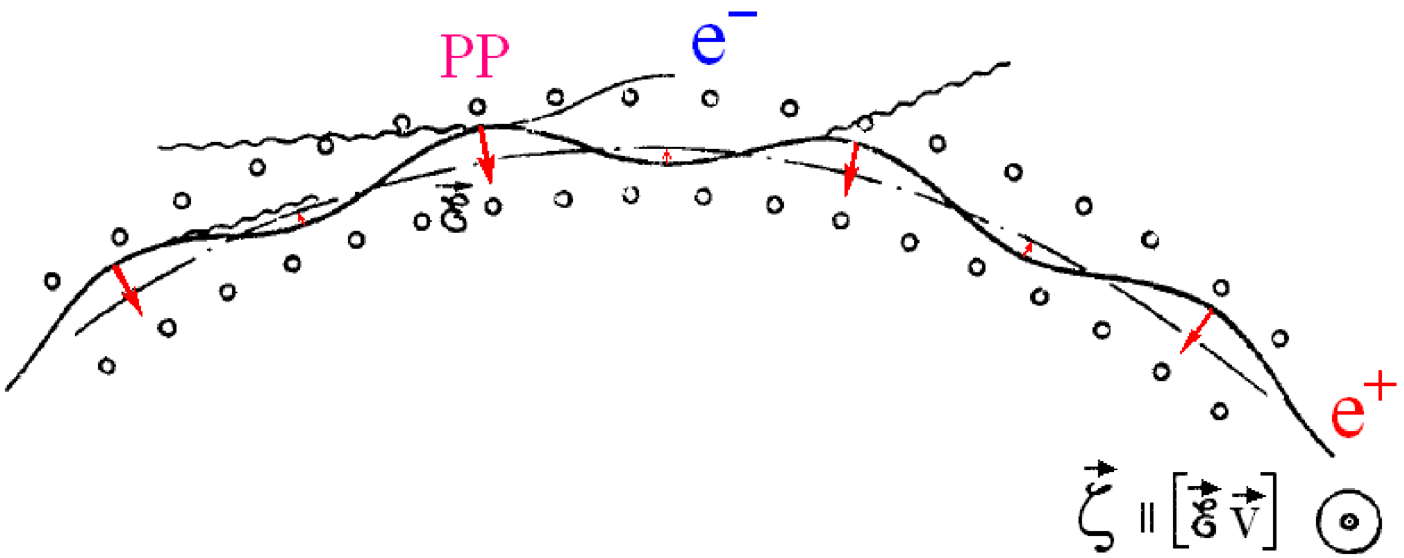
For γ - quanta

$$\Delta t = 10^{-28}\,\mathrm{s}$$



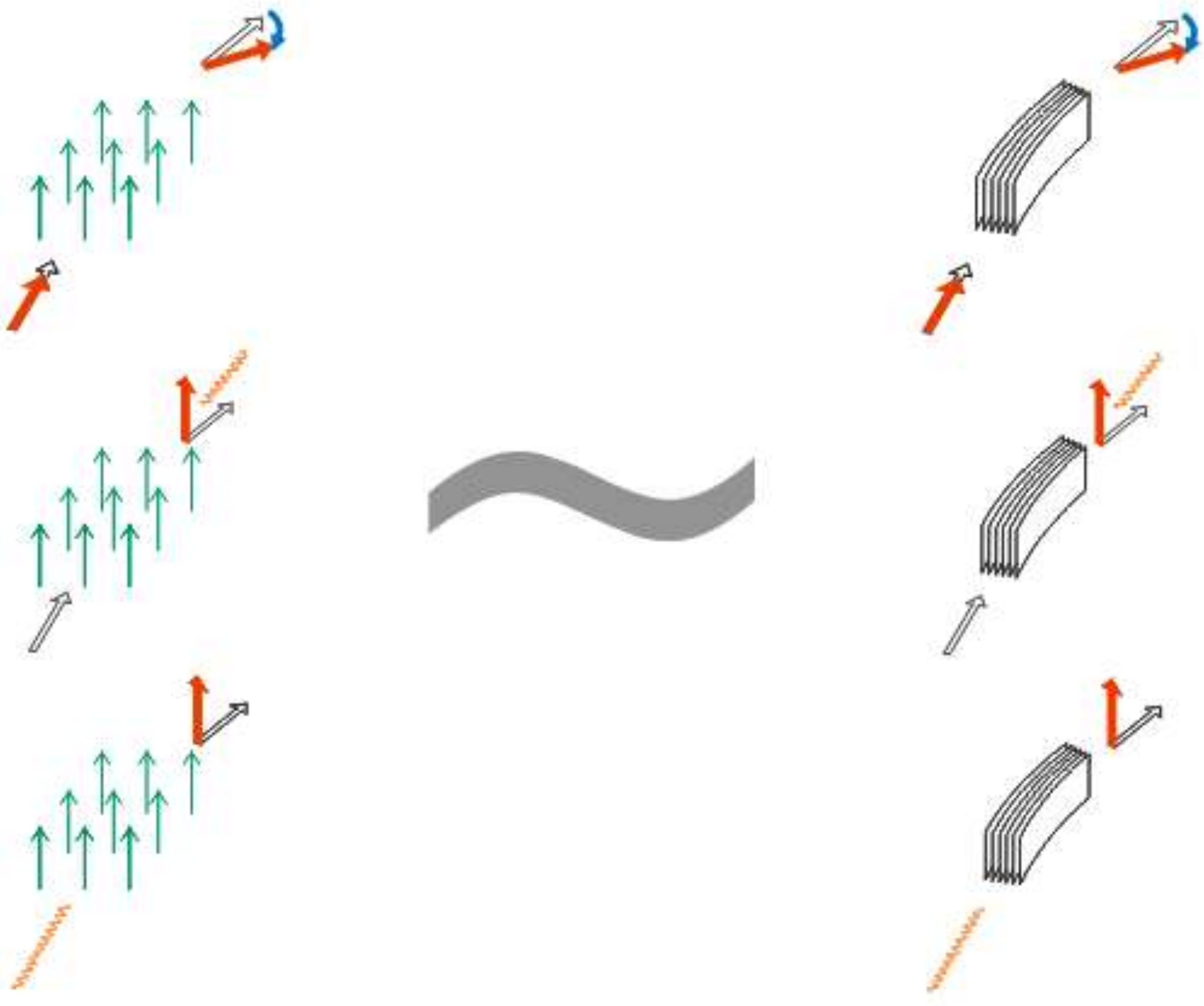
Spin effects in bent crystal

V. G. Baryshevsky. Pis'ma Zh. Tekh. Fiz. 5(1979)182; 5(1979)1529.



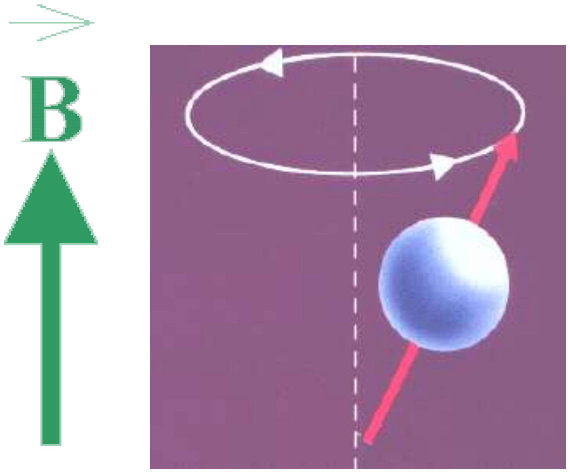
Channeled e^+ and e^- move or are produced by gamma-quanta in bent crystals in the regions with **dominating direction** of the planar electric field. which represents itself an origin of a numberof **spin effects**.

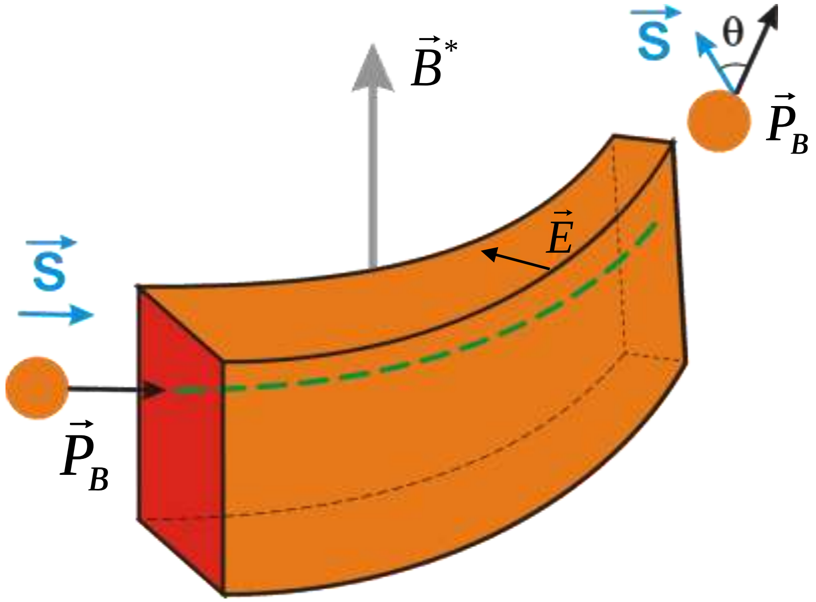
Spin effects in bent crystal



Spin rotation in bent crystals and magnetic and electric moment measurement

$$\Omega = \gamma_S B = \frac{g \mu_B B}{\hbar}$$



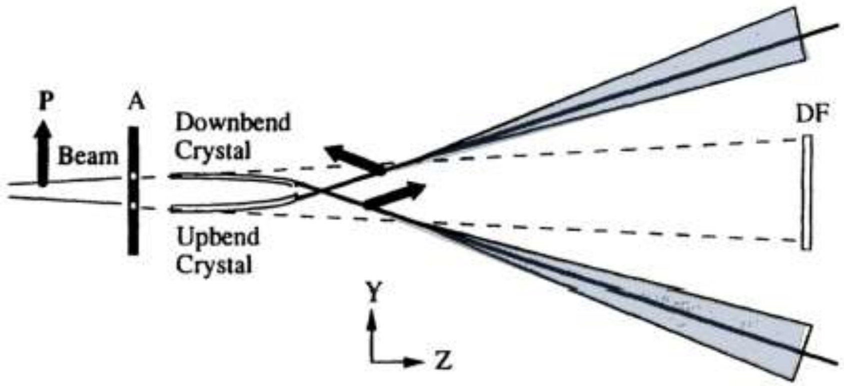


E761 Collaboration, **FERMILAB**

”First observation of spin precession of polarized hyperons channeled in bent crystals”, LNPI Research Reports (1990-1991) 129.

Energy of : 200 – 300 GeV

D. Chen ”First Observation of Magnetic Moment Precession of Channeled Particles in Bent Crystals”, Phys. Rev. Lett. 69 (1992) 3286.



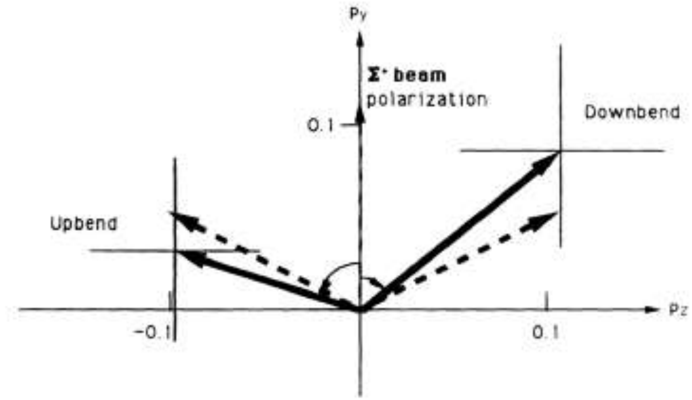
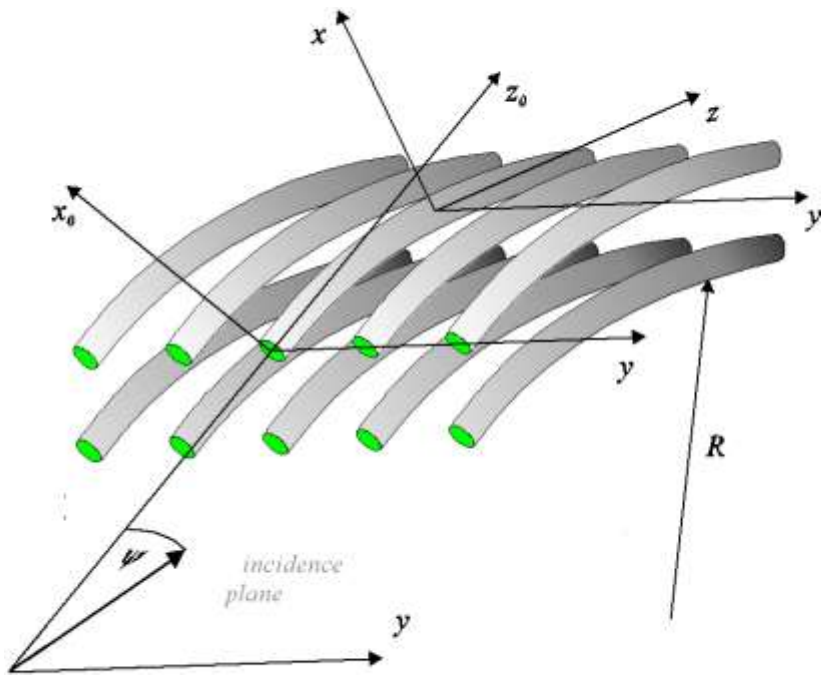
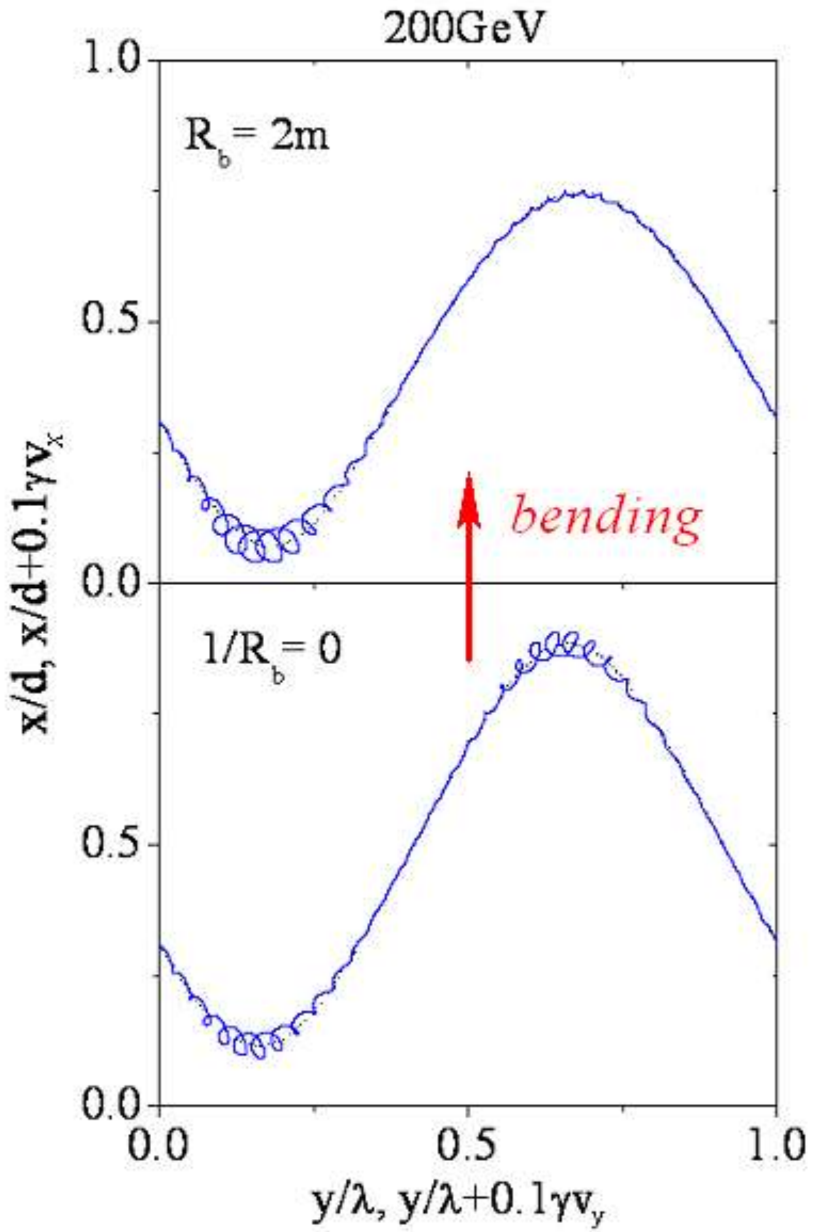


FIG. 3. Measured polarizations and uncertainties (1σ statistical errors) after spins have been precessed by the two crystals. The dashed arrows show the expected precessions.

Baryshevsky V.G., The possibility to measure the magnetic moments of short-lived particles (**charm and beauty baryons**) at LHC and FCC energies using the phenomenon of spin rotation in crystals, Physics Letters B, V. 757, 2016, pp 426–429.

Circularly polarized SOS radiation of channeling positrons





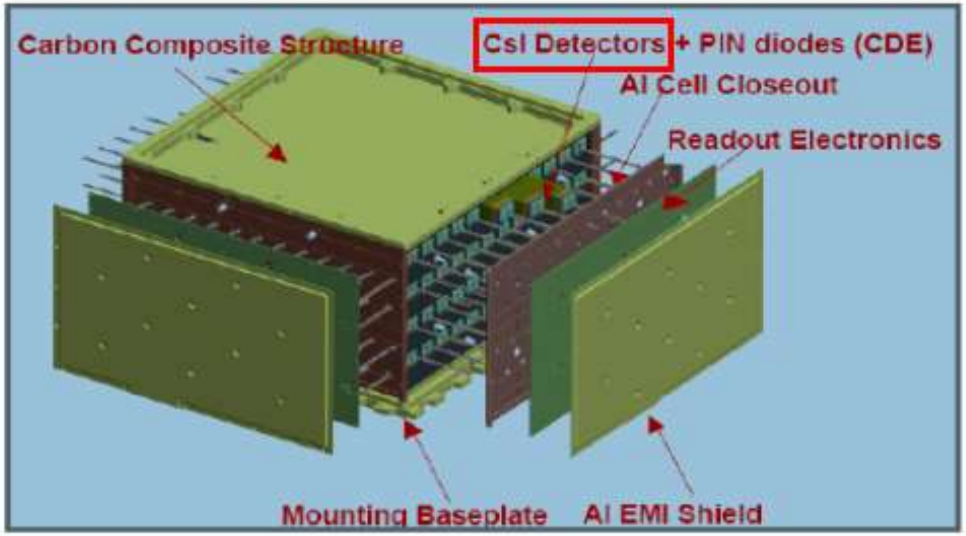
Crystal-based angular-sensitive gamma-telescope

V.A. Baskov, V.A. Khablo, V.V. Kim et al., NIM B 122(1997)194.

V.N. Baier, V.M. Katkov, V.M. Strakhovenko, *Electromagnetic Processes at High Energies in Oriented Single Crystals*, World Scientific, Singapore, 1998.

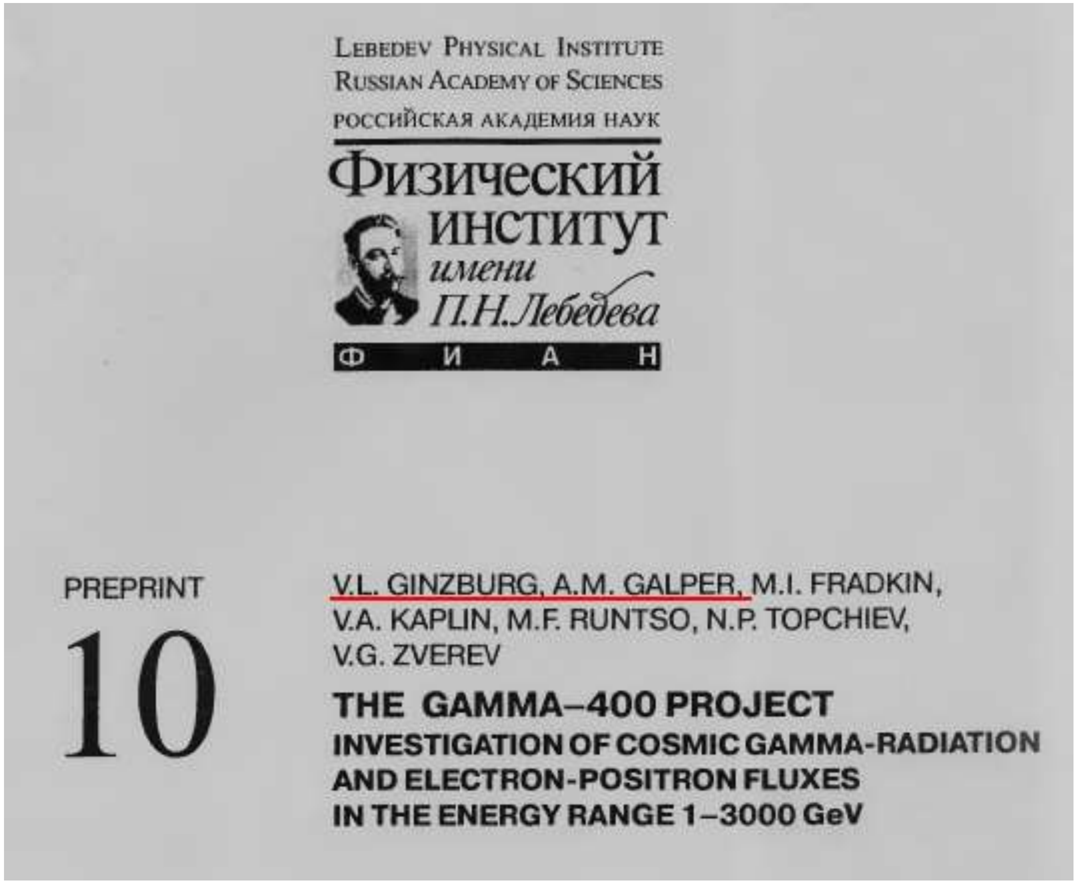
CsI scintillators in the **Fermi** Large Aperture Telescope

GLAST Observatory after the integration of the Large Area Space Telescope. Picture taken at the General Dynamic on December 2006.



Schematic view of the GLAST imaging calorimeter.

The GAMMA-400 project



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10

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V.A. KAPLIN, M.F. RUNTSO, N.P. TOPCHIEV,
V.G. ZVEREV

THE GAMMA–400 PROJECT

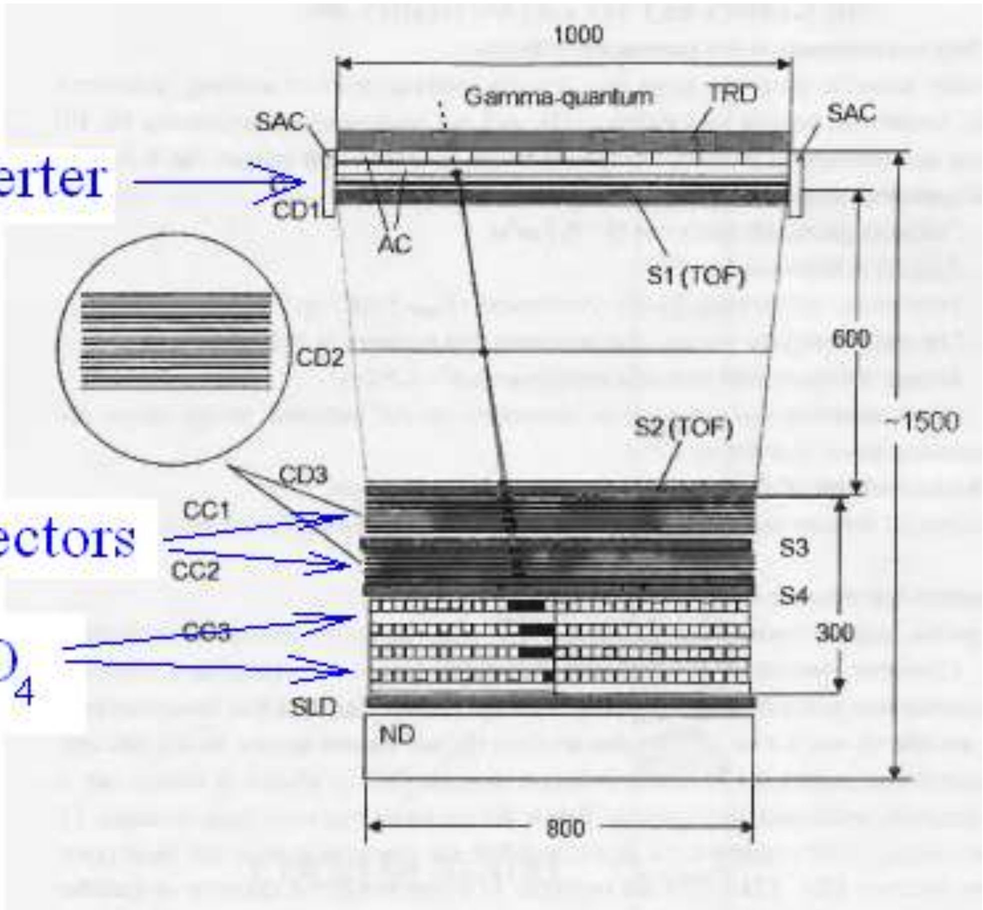
INVESTIGATION OF COSMIC GAMMA-RADIATION AND ELECTRON-POSITRON FLUXES IN THE ENERGY RANGE 1–3000 GeV

Crystals in the GAMMA-400 telescope

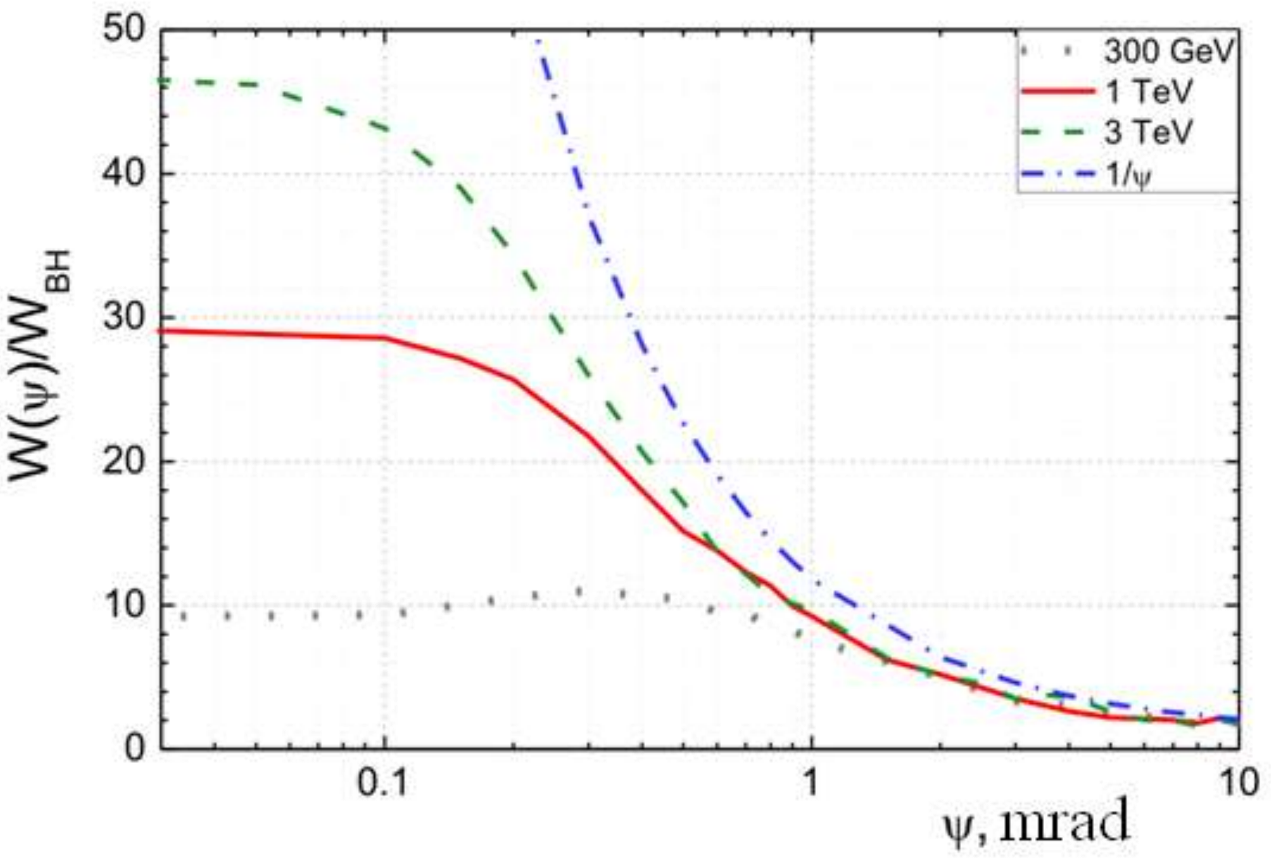
tunsten converter

W + silicon strip detectors

W + PWO_4



To the crystal gamma-telescope development



Pair production probability by 300Gev. 1 Tev and 3 TeV gamma-quanta vs the angles of incidence w.r.t. <110> Si axis.

The probability is measured in units of Beth-Heitler PP probability WBH ≈ 0.083/cm.

Any ($E_\gamma \geq 10$ TeV) energy

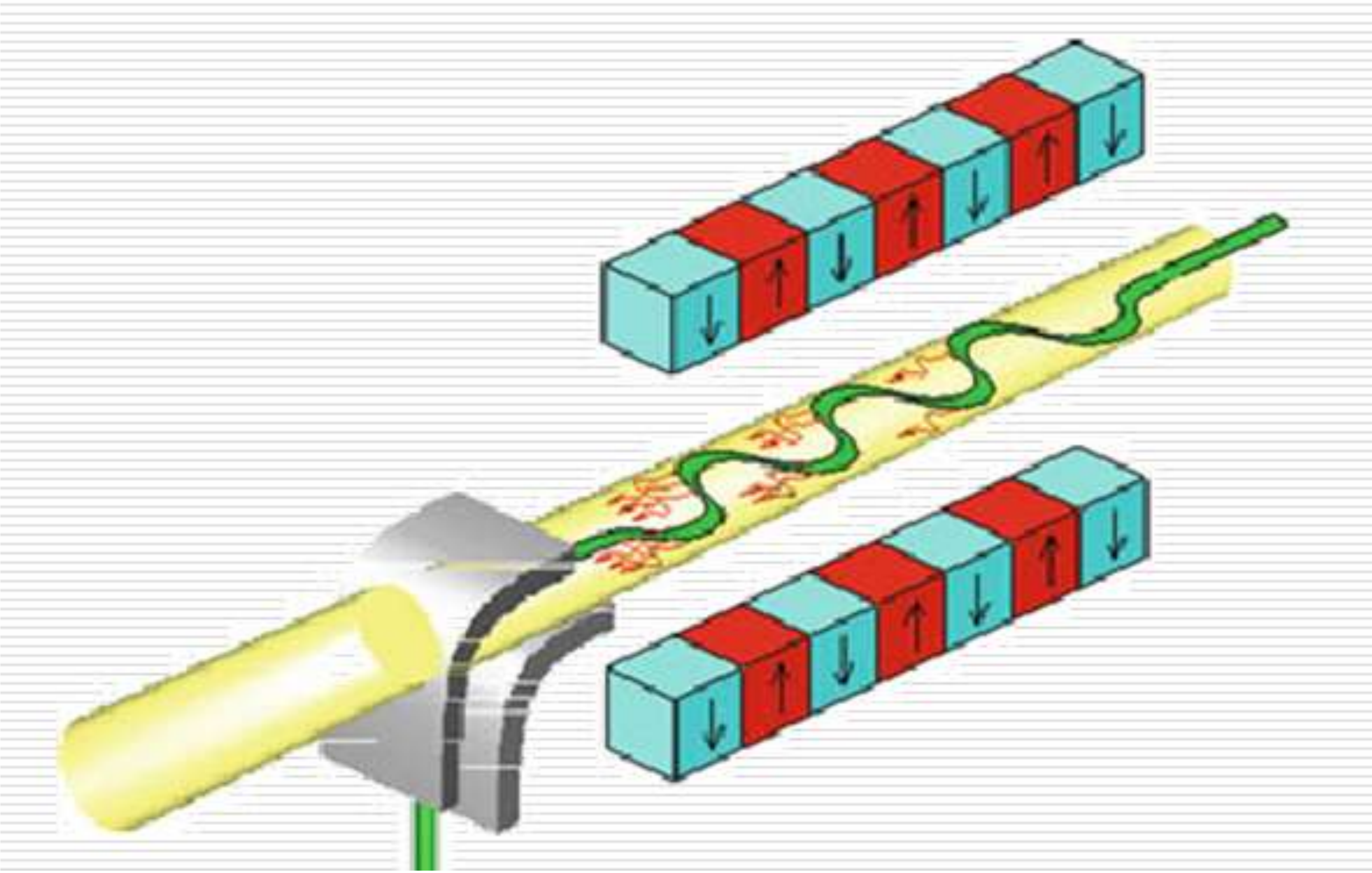
can be measured

with the same

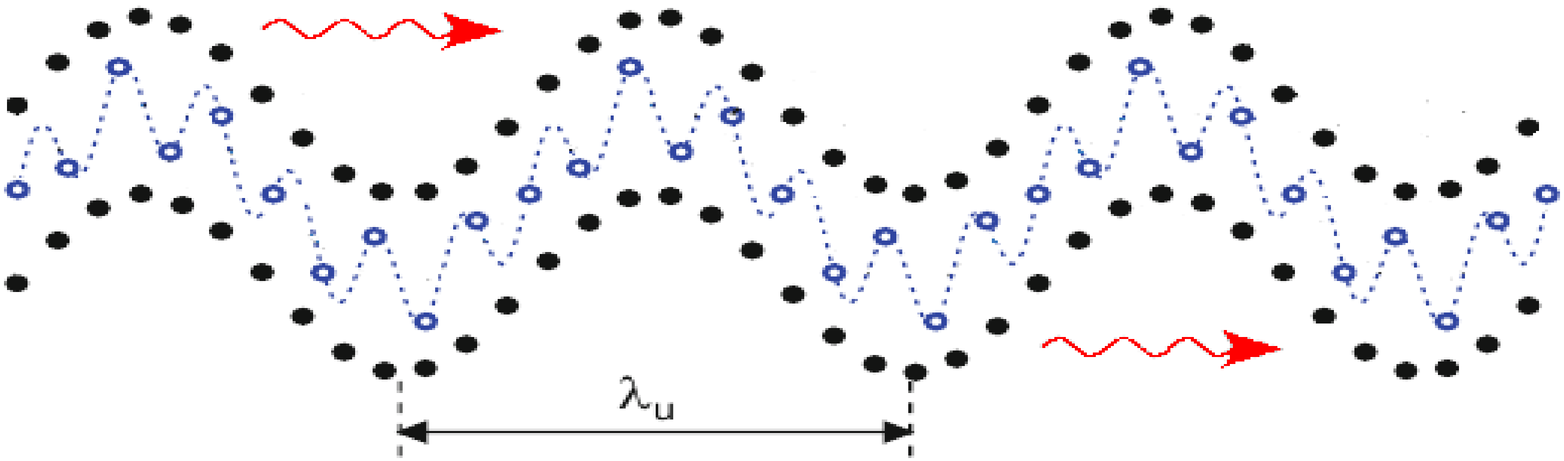
detector thickness

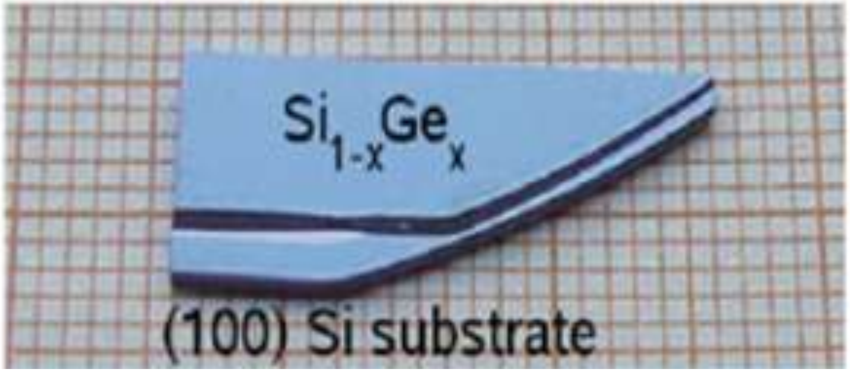

γ
γ
γ
γ
γ
γ
γ

Magnetic undulator (Ginzburg, 1947; Motz, 1953)

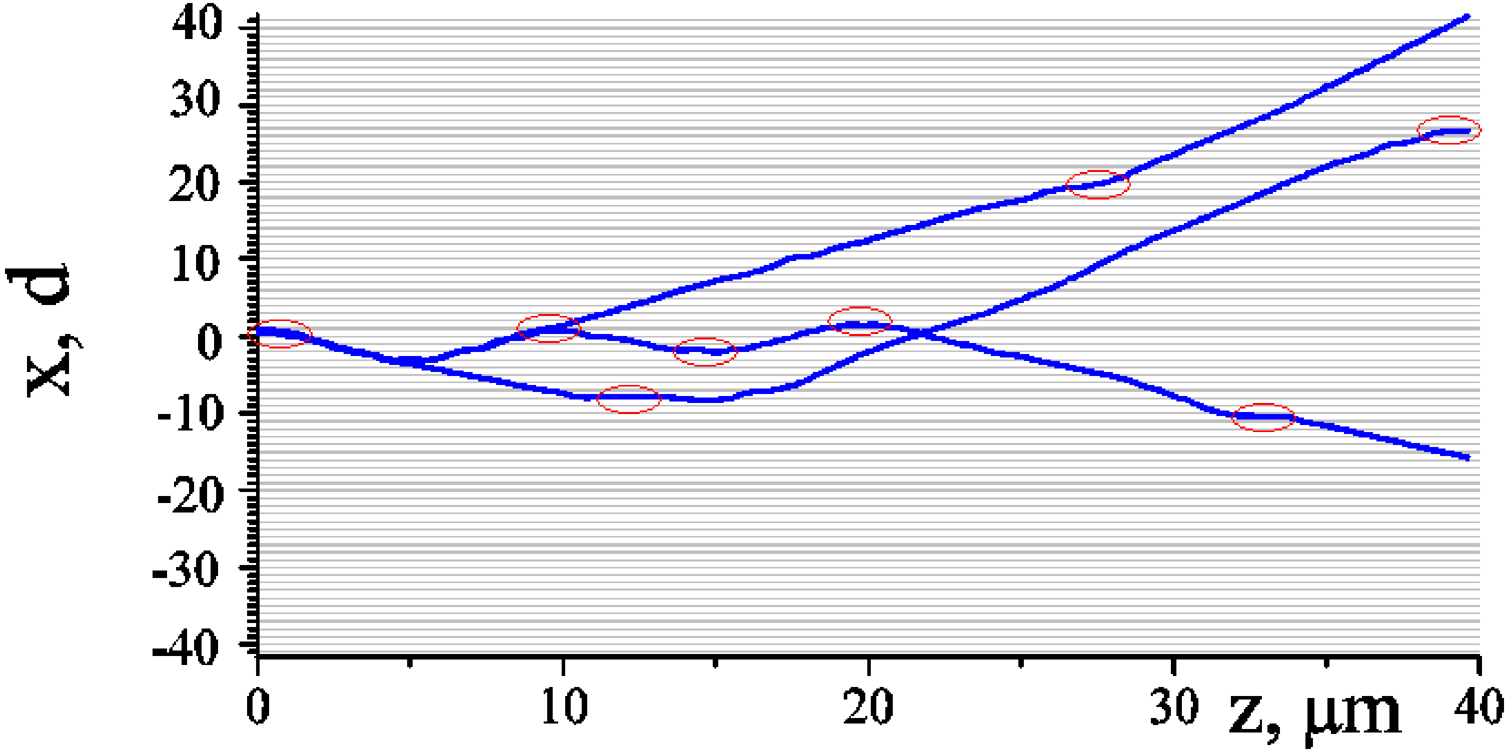


Crystal Undulators

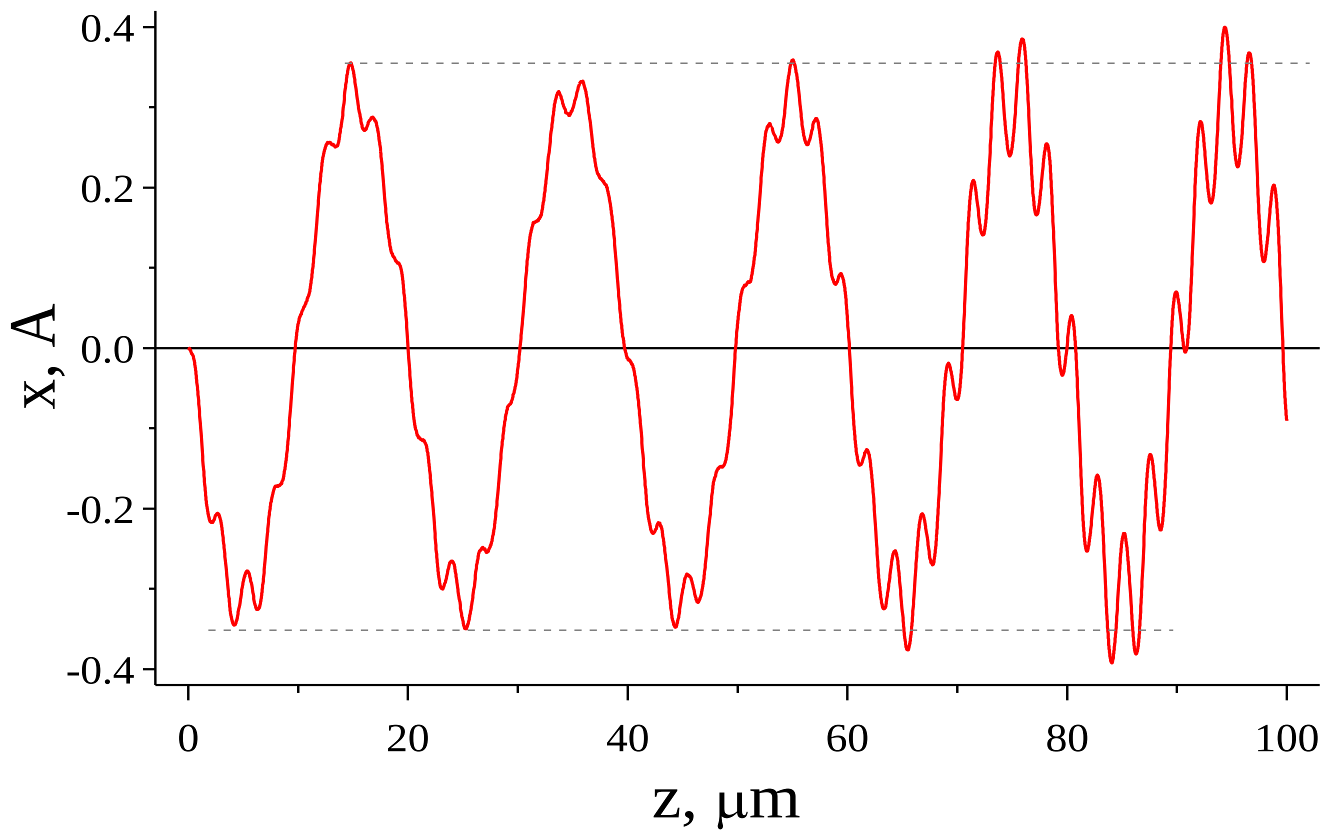




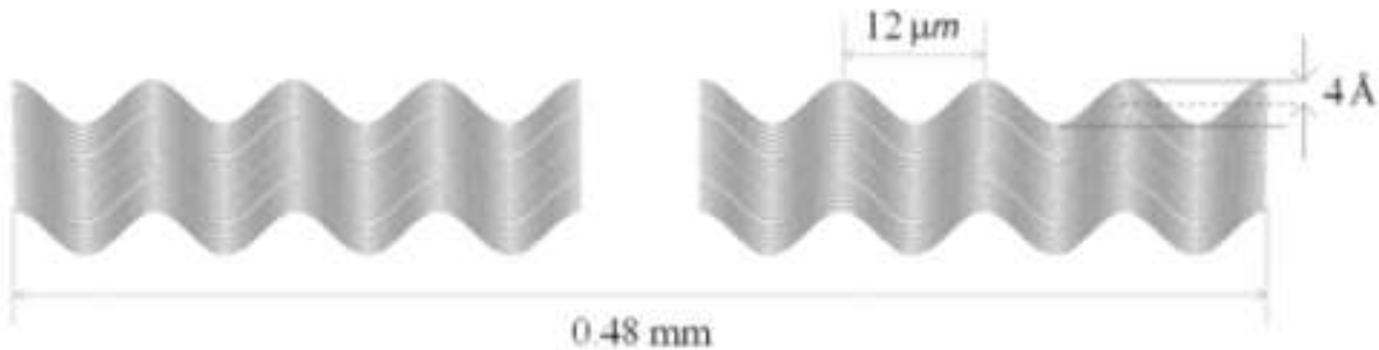
855 MeV electron rechanneling in “Backe CU”



A "*fortunate*" 500 MeV positron trajectory



Optimal undulator for 1.5 GeV



“Principles” of construction of the “optimal” positron CU

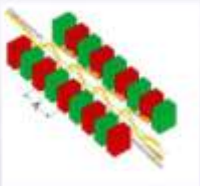
- Maximal undulator radiation **energy**
- Minimal CU radiation spectral **width**
- $\mathbf{L_{CU}} \approx L_{dech}$
- $\mathbf{K} = eF_0 \lambda_U / 2\pi m \approx \mathbf{1}$

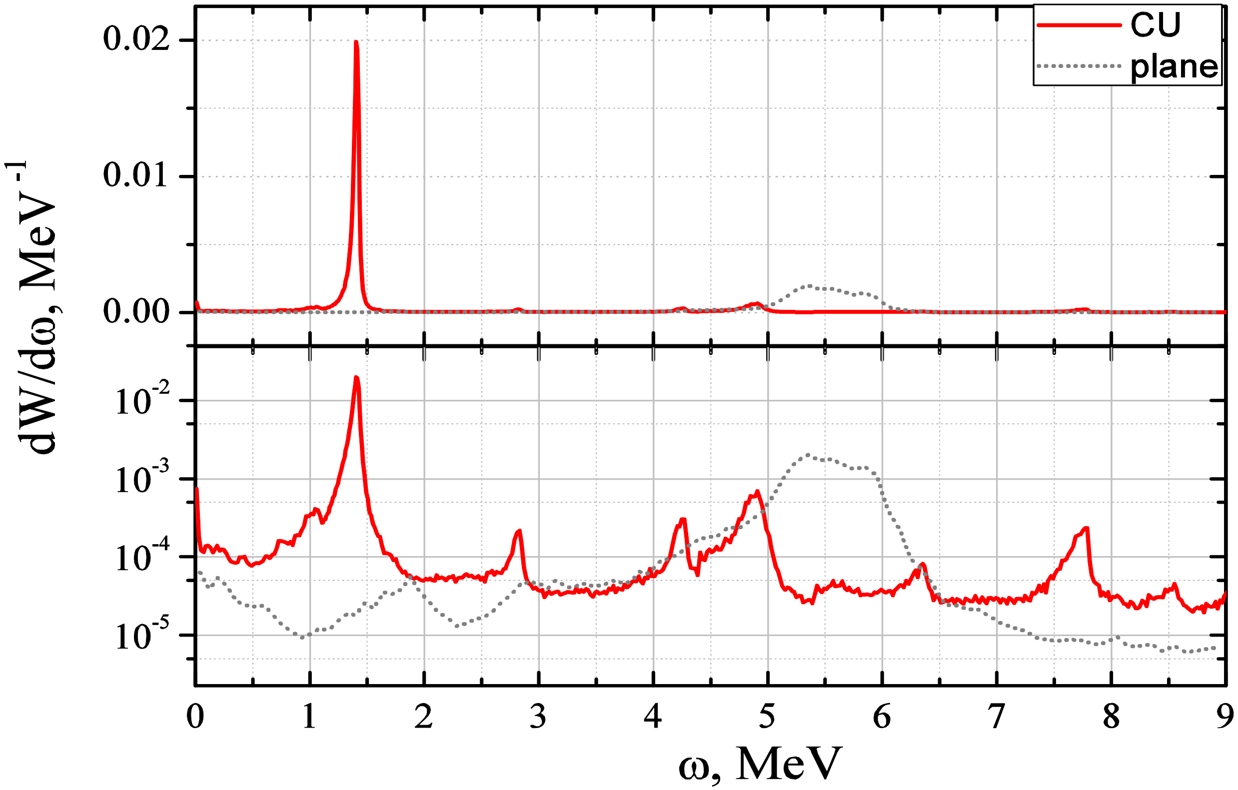
Optimal undulator for 1.5 GeV

$$\Delta\omega/\omega \approx 3\%,$$

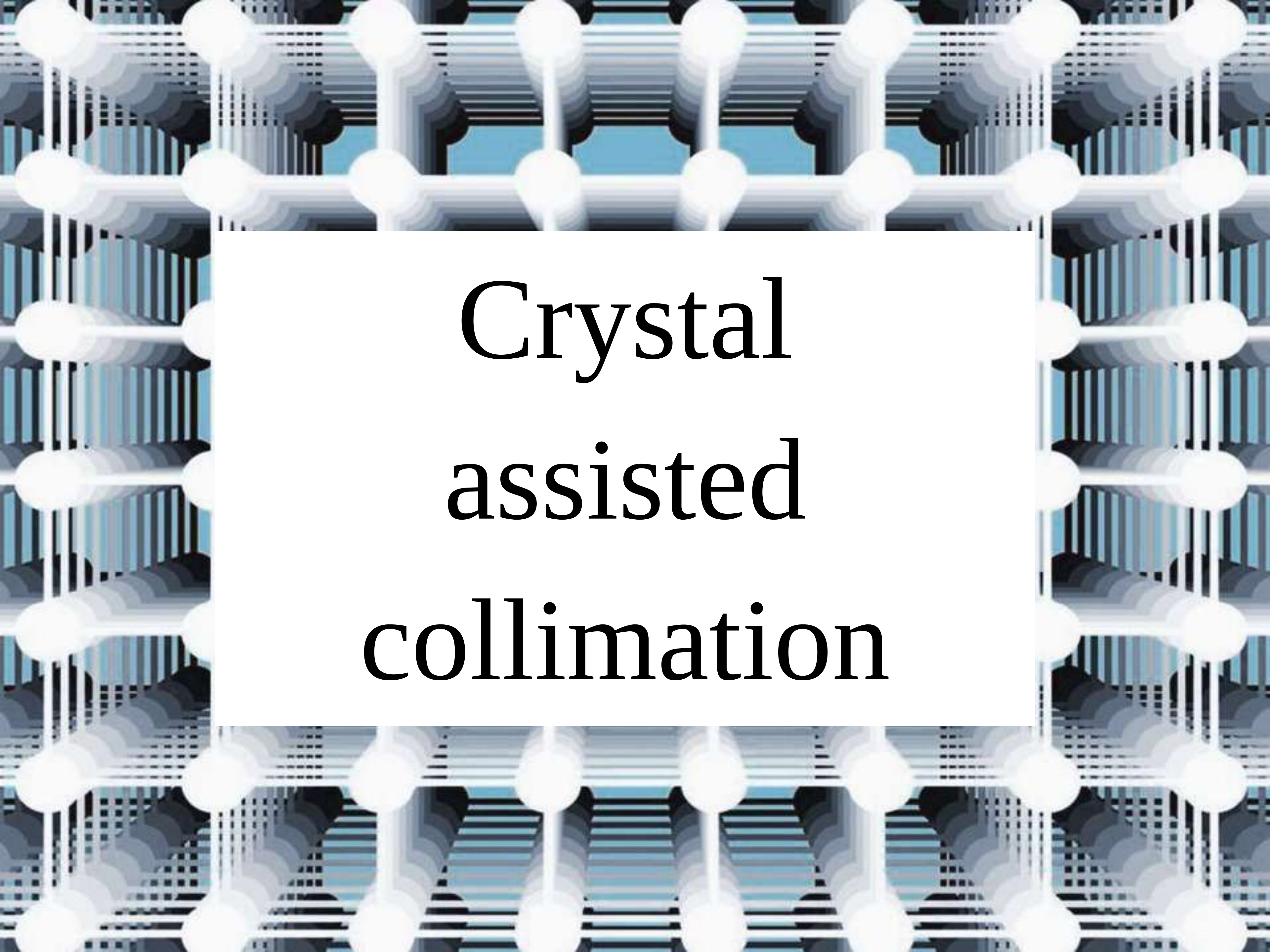
$$\Delta N_\gamma/N_e \sim 10^{-3}\gamma/e^+,$$

$$\Delta\theta_e \le 20 - 30\ \mu\text{rad}$$



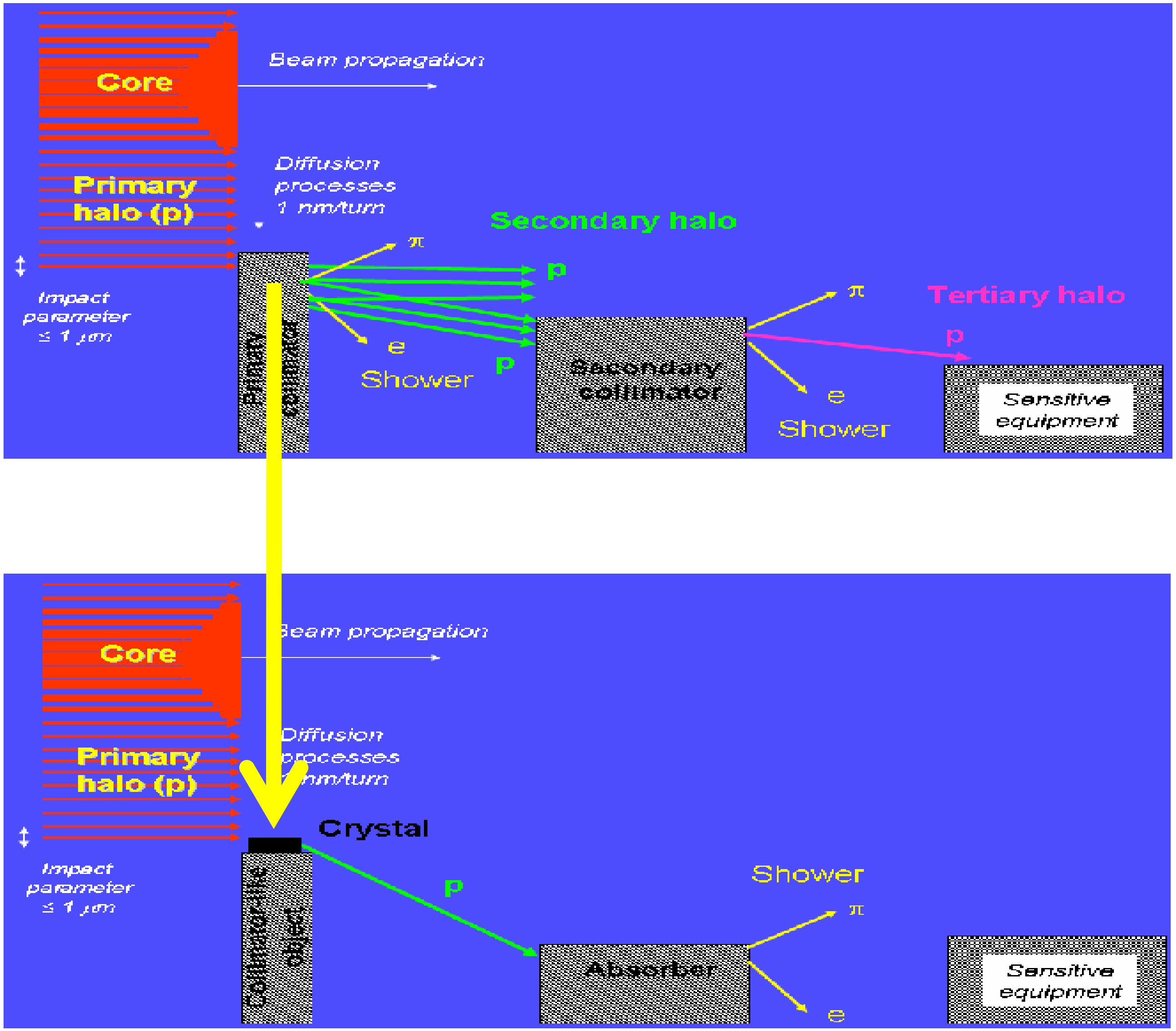


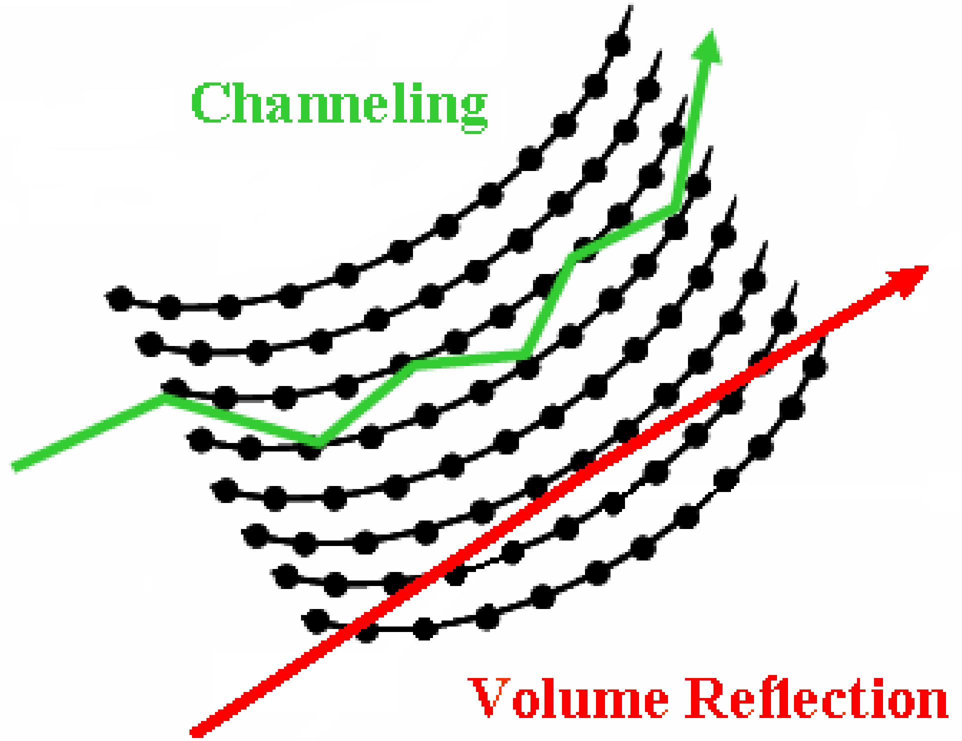
Spectral distribution of the radiation emitted by a 1.5 GeV positron:
***Solid line* − in the "optimal" Si (110) CU**
***Dotted line* − in a plane 0.48 mm Si (110) crystal**
Positron beam incidence angle equals zero,
incident beam angular divergence is $\Delta\theta_e$ = 10 μrad
and collimation semi-apex angle is $\theta_\gamma = 1/8\gamma$ = 42.6 μrad.



Crystal assisted collimation

Orientational effects in crystals allow to facilitate collimation





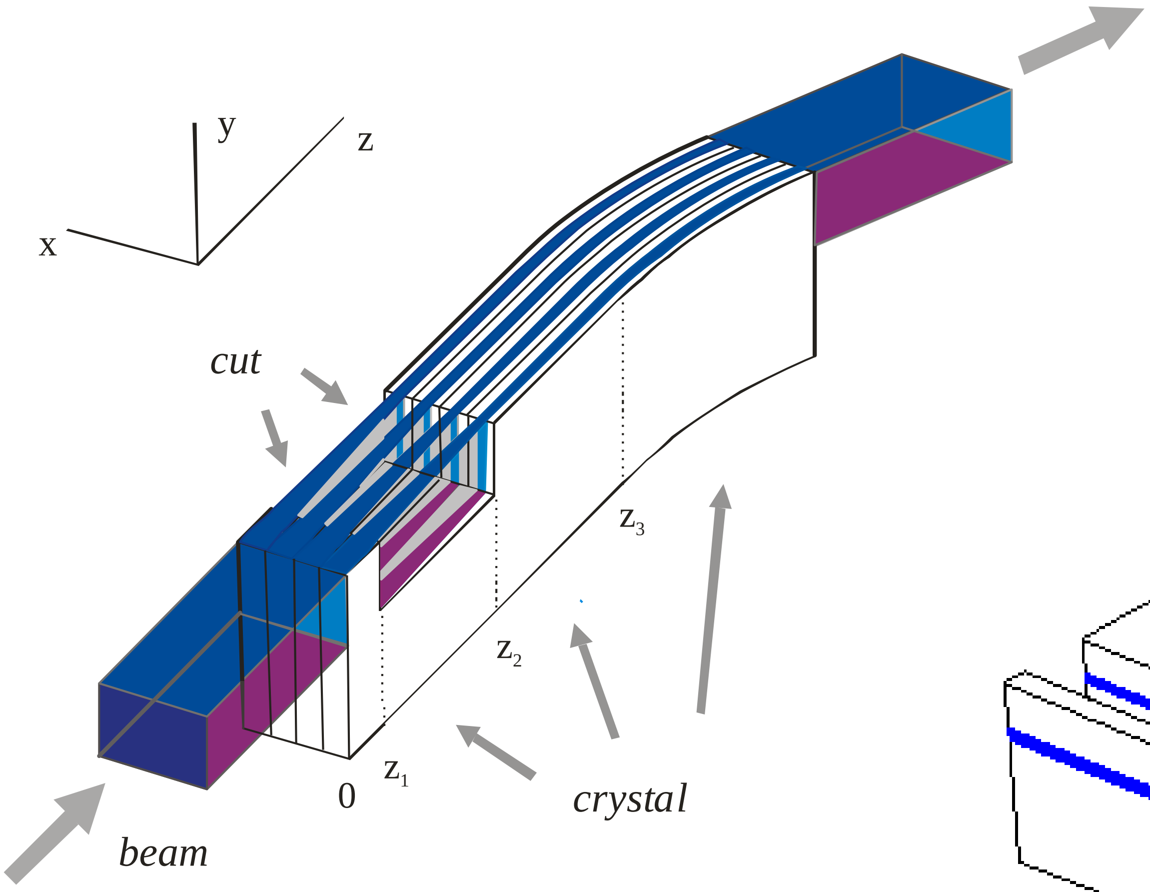
channeling – limited efficiency

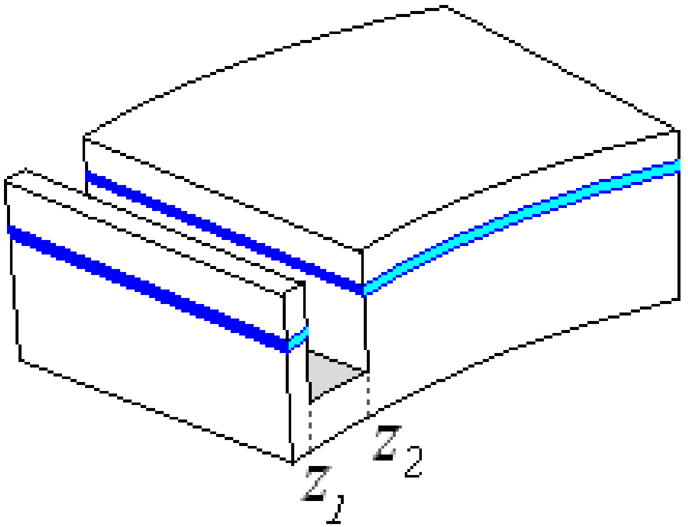
volume reflection – small deflection angle

Channeling **efficiency** can be increased by **crystal cut**

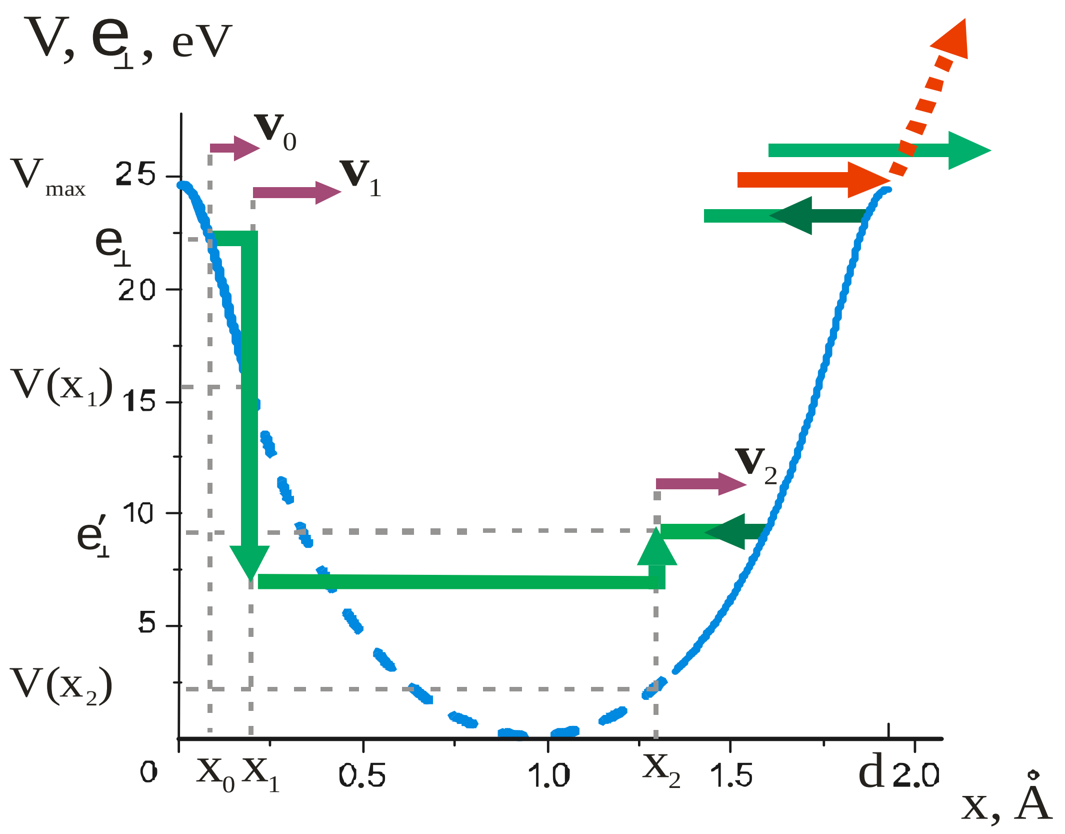
The capture probability **increase by crystal cut**

V.V.*Tikhomirov, JINST, 2(2007)P08006*



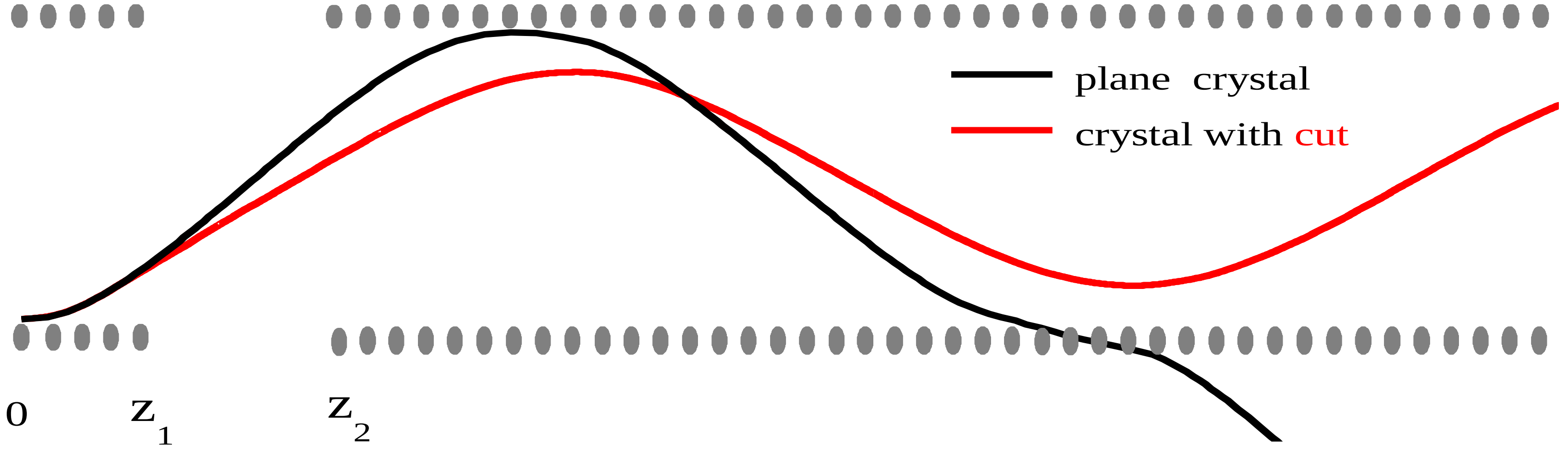


Transverse energy reduction by the cut

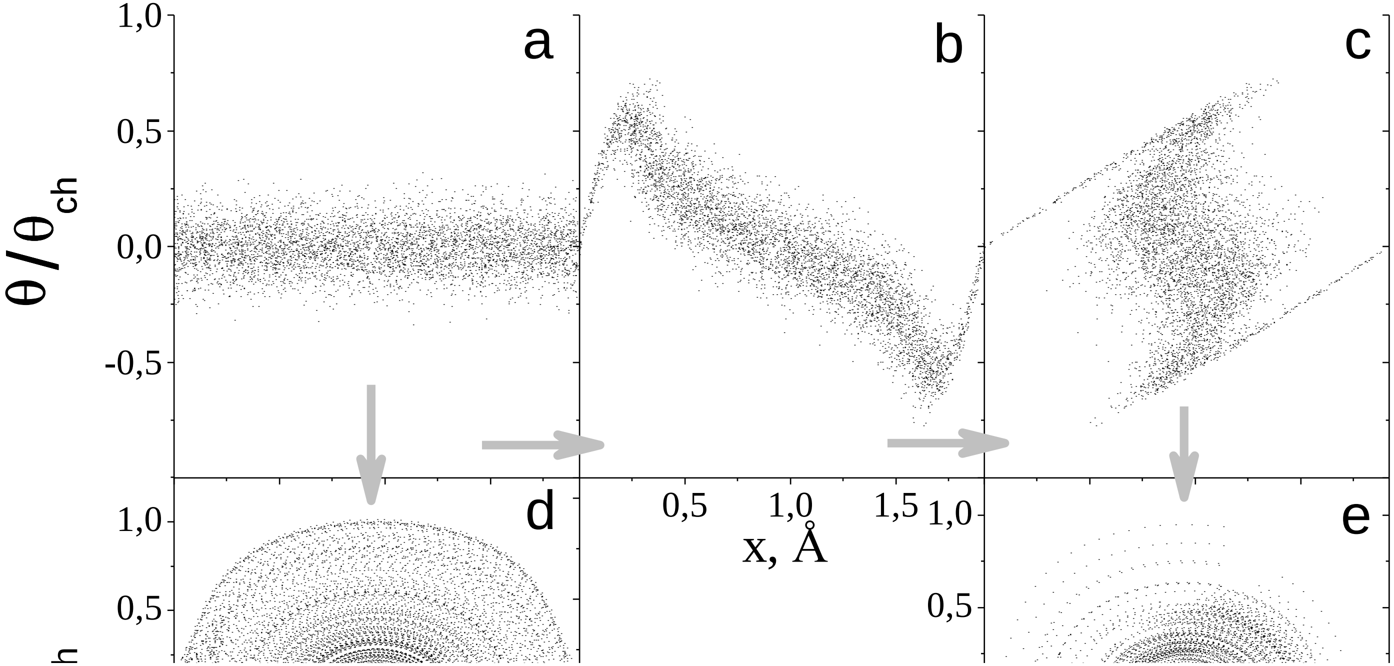


The cut diminishes the potential energy preserving the transverse kinetic one

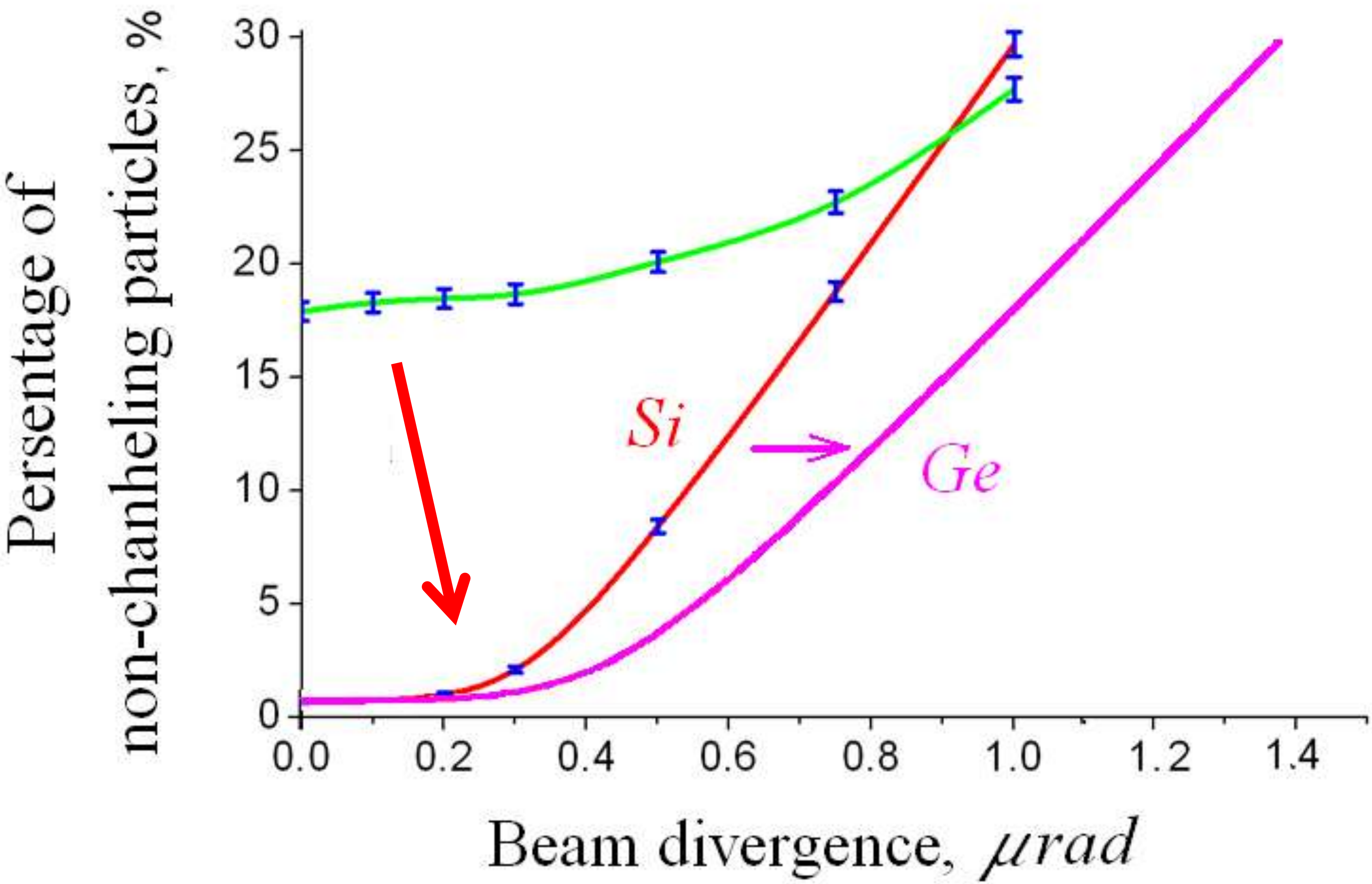
Protons cease to reach the high nuclear density regions



Phase space transformation by the cut



Channeling efficiency increase by crystal cut

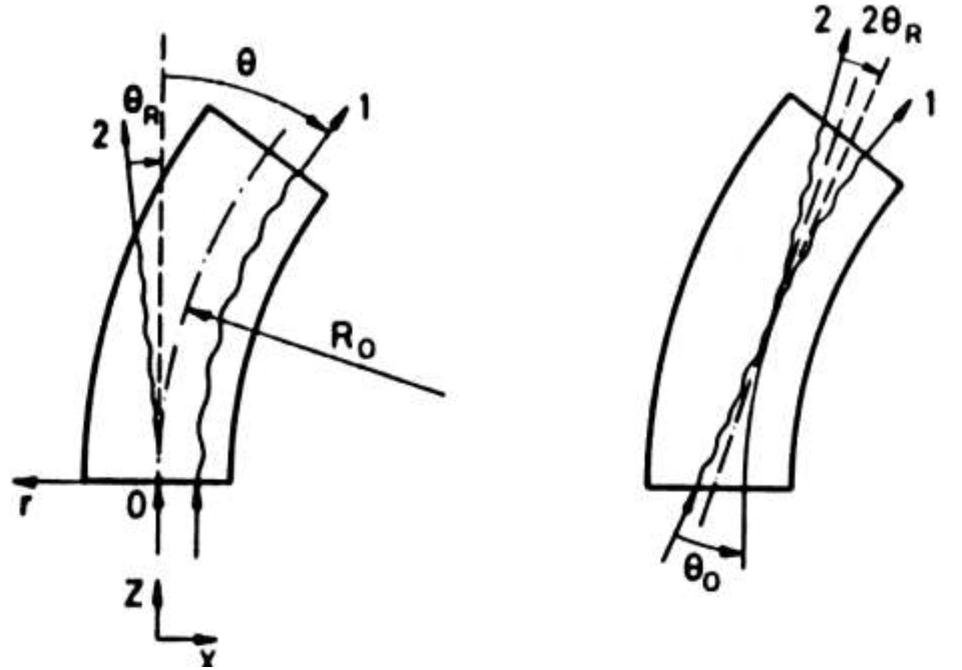


Deflection of nonchanneled particles can be increased by **multiple volume reflection** in one crystal

Volume Reflection prediction

A.M.Taratin and .A.Vorobiev

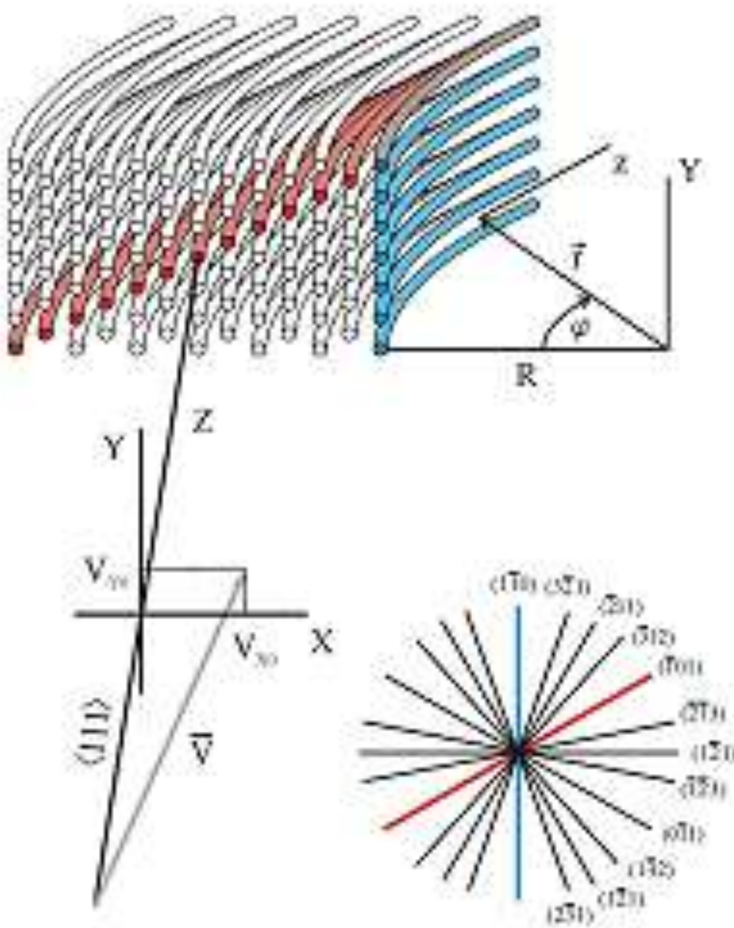
Phys. Lett. A119 (1987) 425, NIM B26 (1987) 512

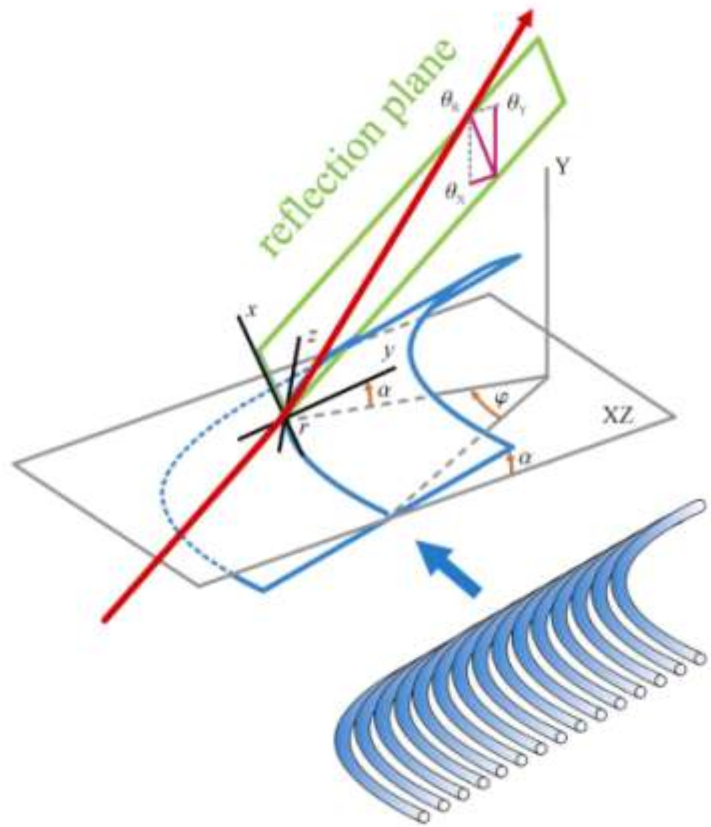


Large acceptance, however small deflection angles

Multiple Volume Reflection in One Crystal (MVROC)

V.V. Tikhomirov, PLB 655(2007)217



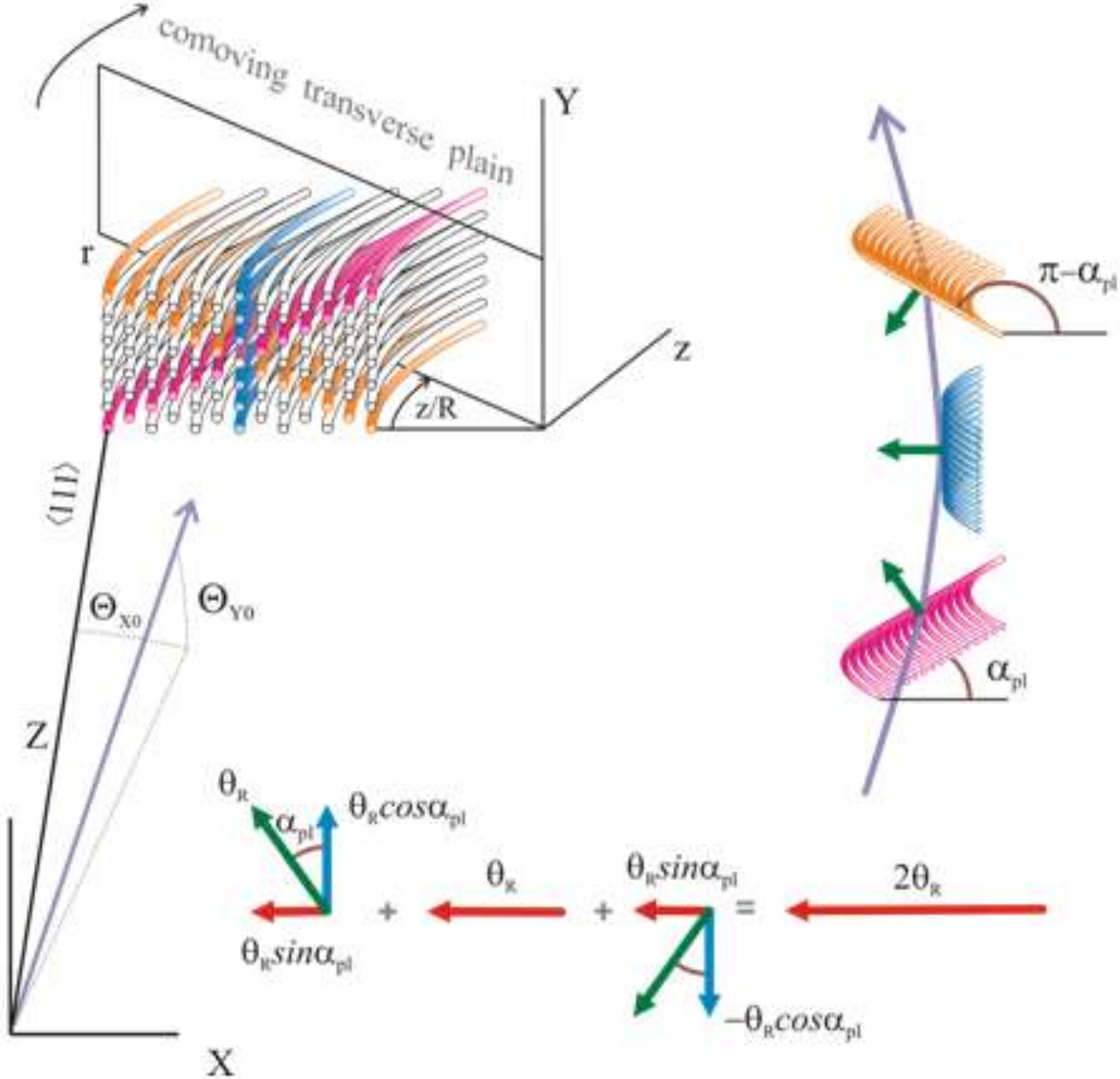


Axes form ***many*** inclined reflecting planes

In this talk only and just for short

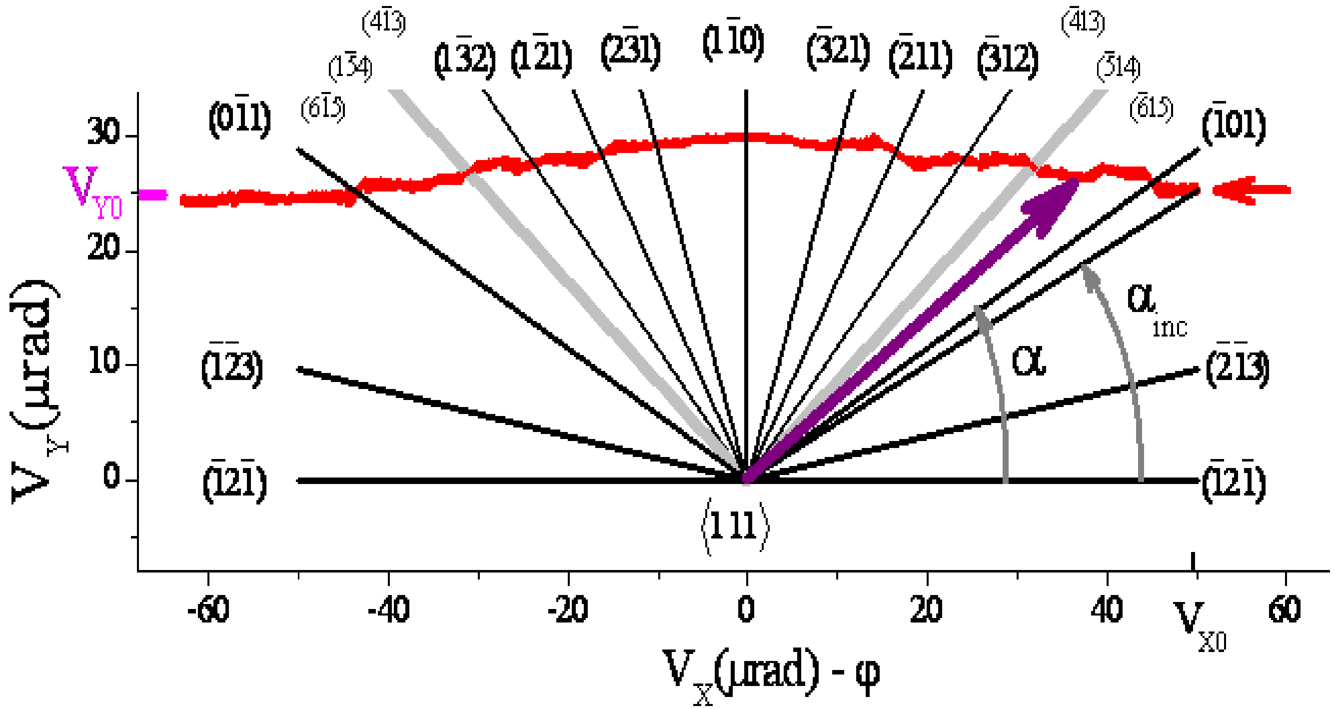
MVR = multiple volume reflection
in one crystal/”axial VR”

VR = “one plane VR”/”planar VR”

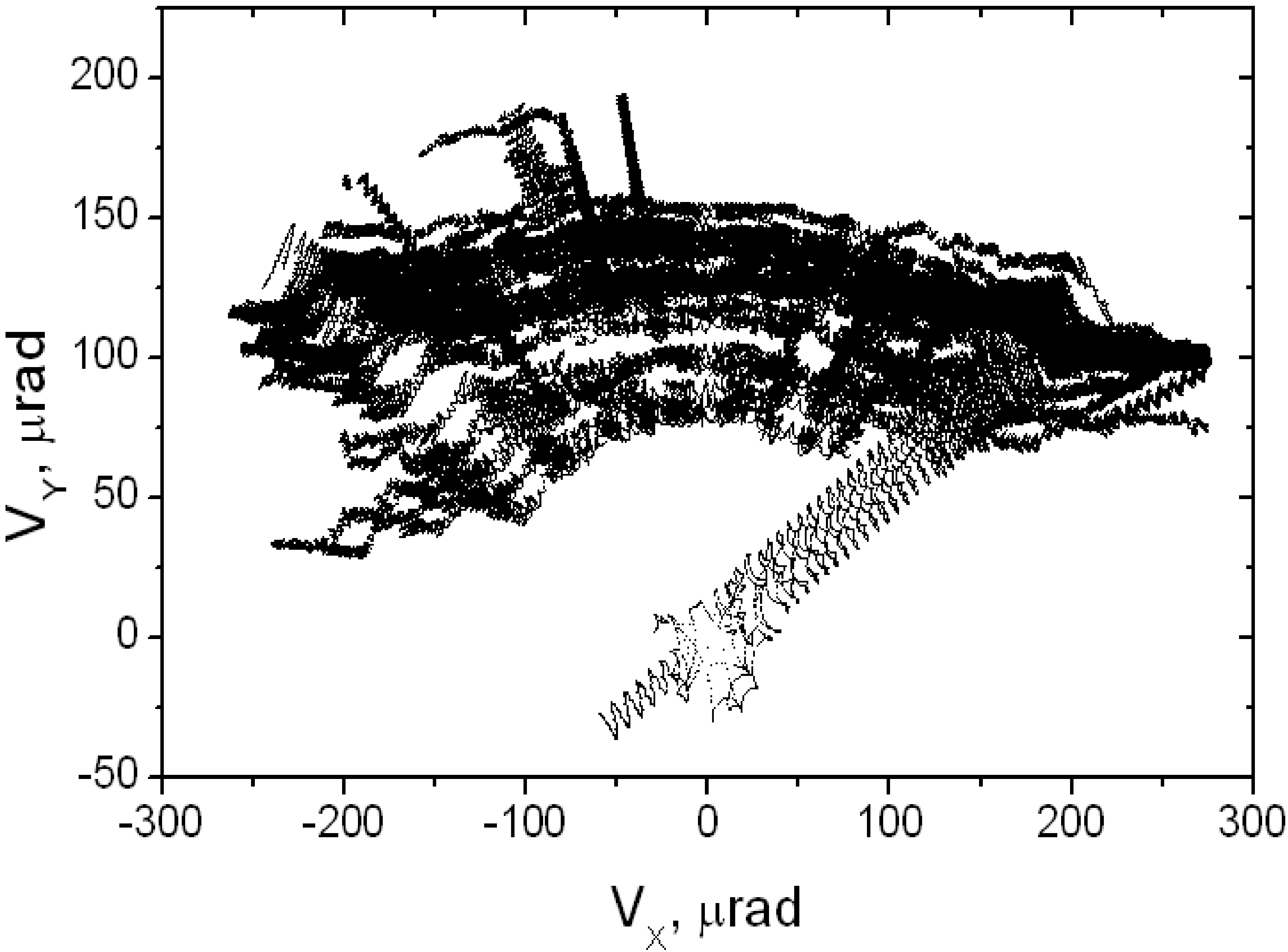


comoving transverse plain
Y
r
z
z/R
⟨111⟩
Θ_{X0}
Θ_{Y0}
Z
Y
X
$\pi-\alpha_{pl}$
α_{pl}
θ_R
α_{pl}
$\theta_R cos\alpha_{pl}$
$\theta_R sin\alpha_{pl}$
+
θ_R
+
$\theta_R sin\alpha_{pl}$
=
$2\theta_R$
$-\theta_R cos\alpha_{pl}$

Proton motion in comoving reference plane

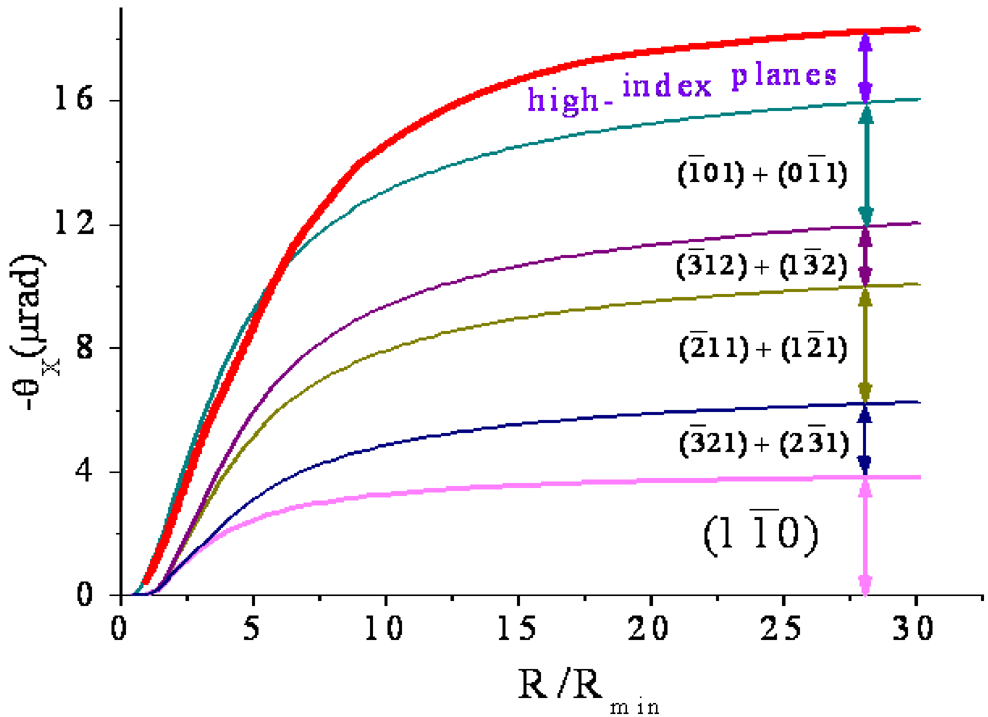


Protons are reflected from ***many*** different crystal plane sets in ***one*** crystal



200
150
100
50
0
-50
-300
-200
-100
0
100
200
300
V_Y, μrad
V_X, μrad

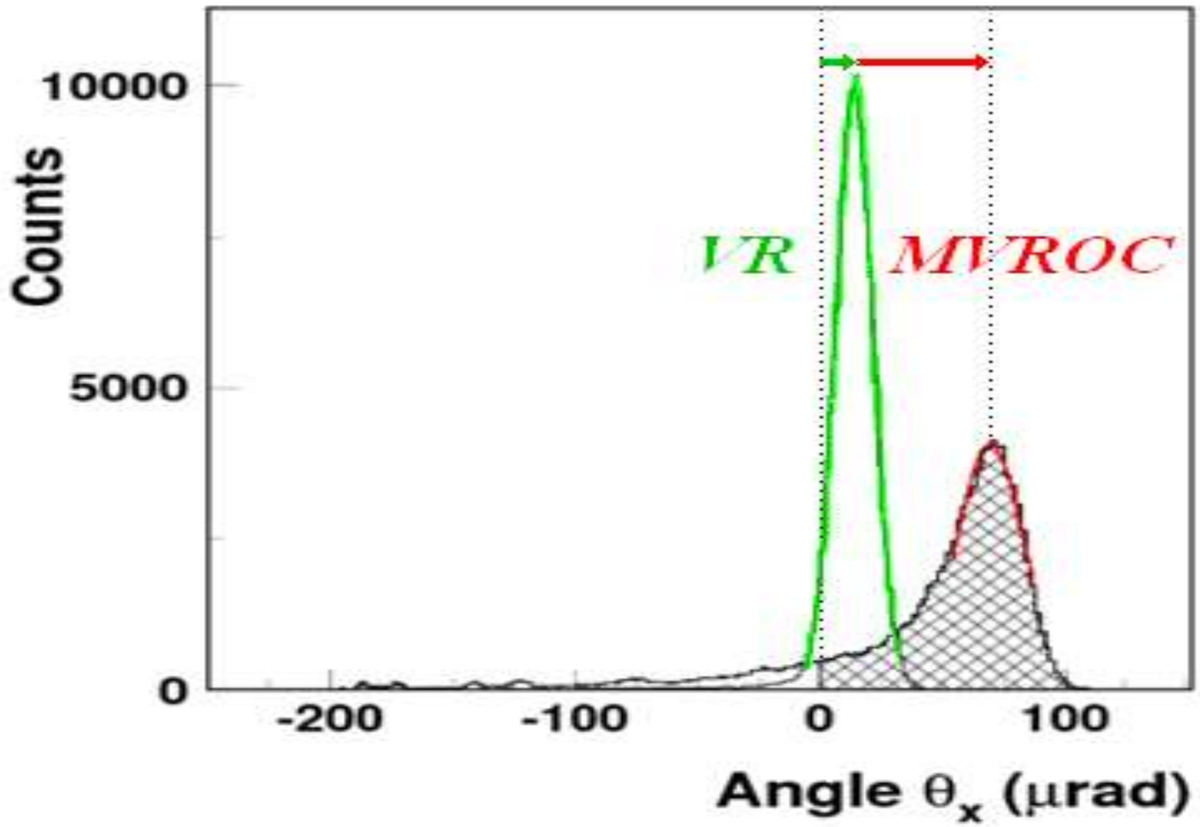
Reflection angles from planes of one crystal *vs* bending radius



Reflection from different crystal planes increases VR angle about ***5 times***

First MVROC observation

W. Scandale et al, PLB 682(2009)274



MVROC indeed increases reflection angle **5 times**

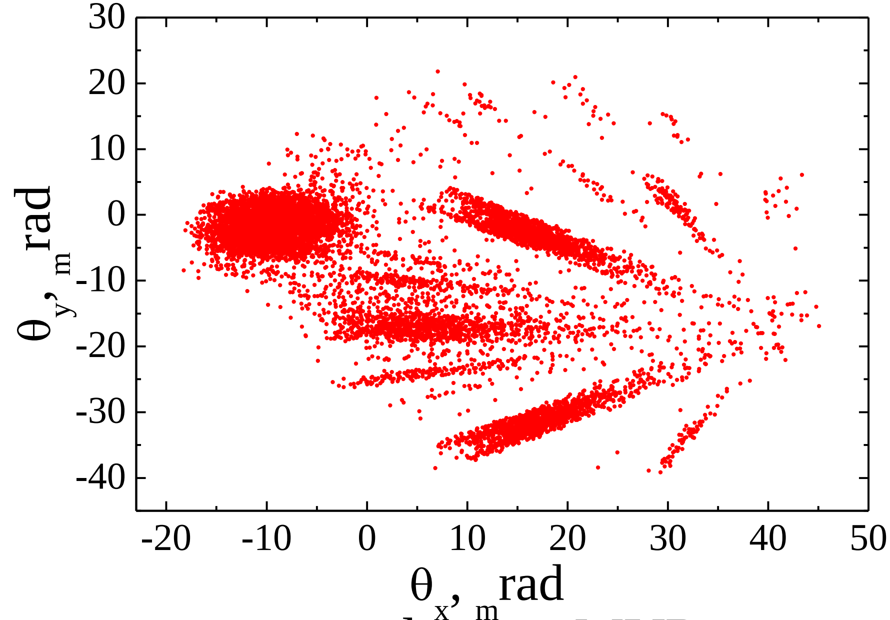
multiple volume reflection

in one crystal can be

combined with **planar**

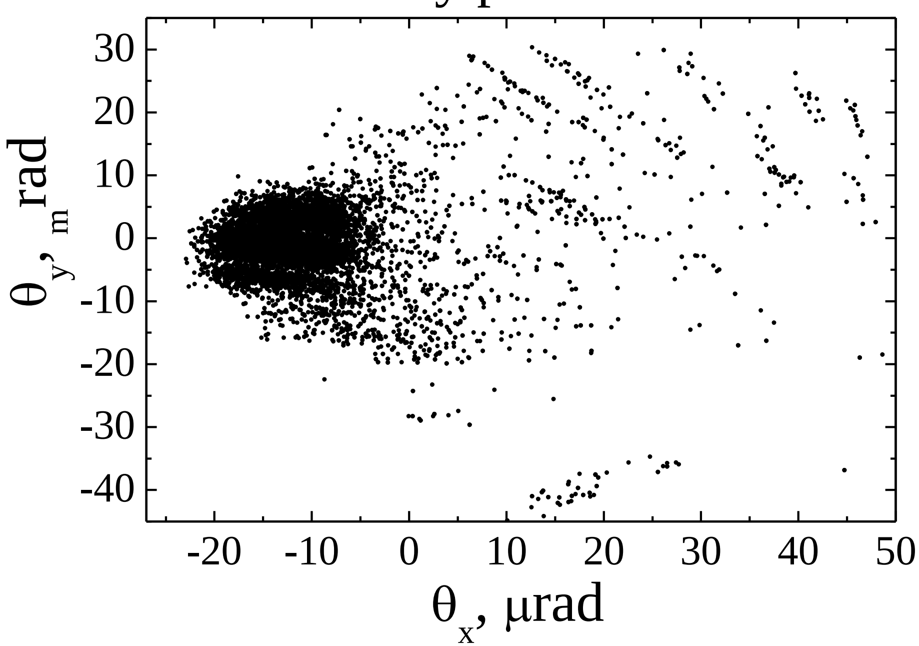
channeling

MVR + Channeling

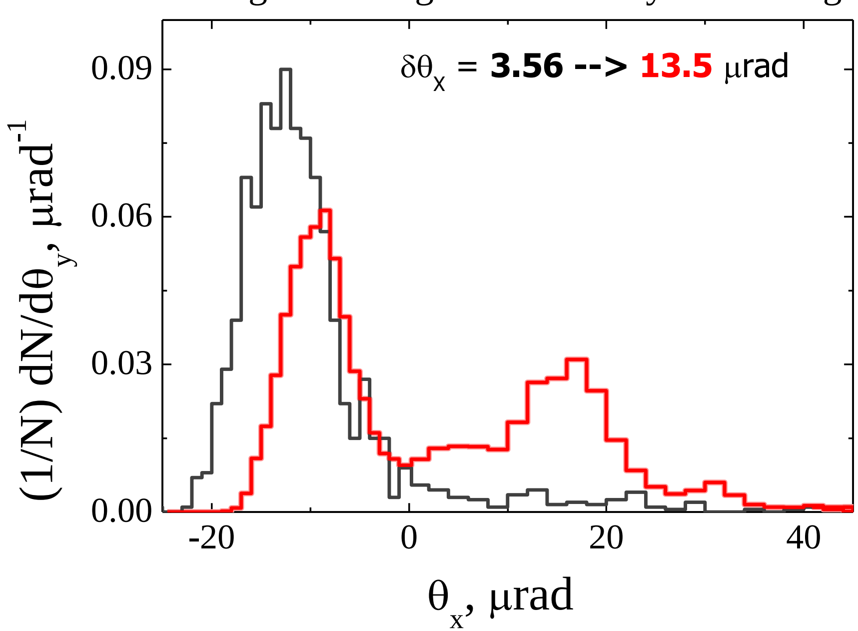


Enhanced beam angular dispersion by MVR can be combined with channeling

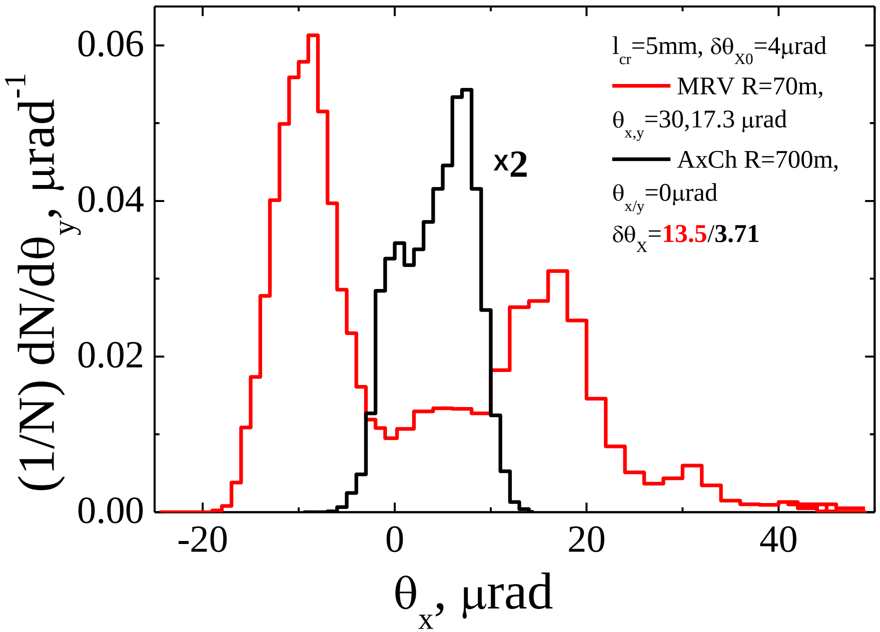
nearly pure MVR



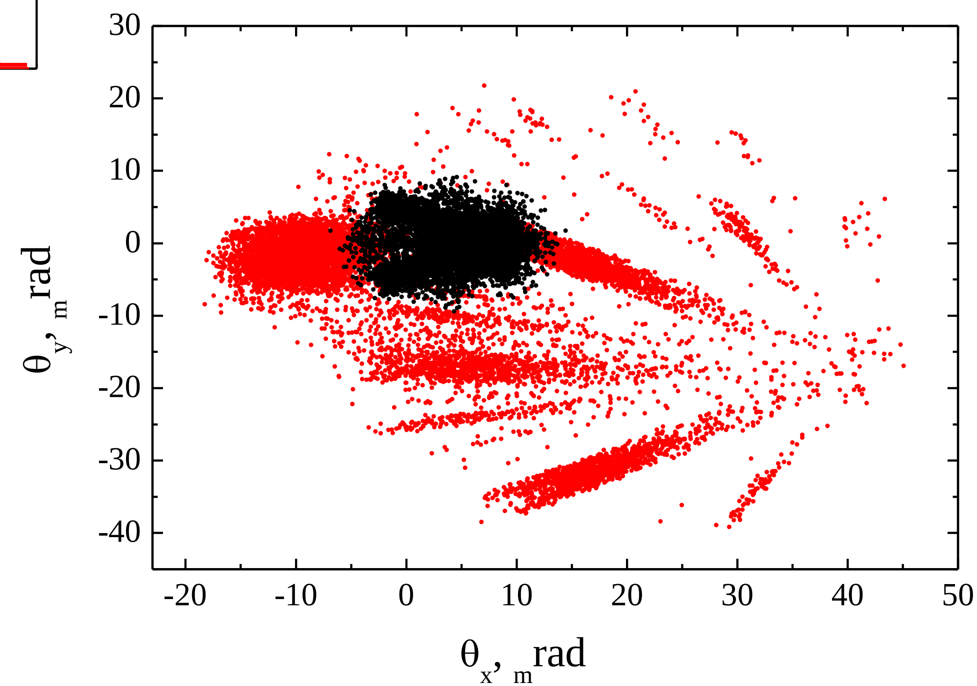
drastic MVR angular divergion increase by channeling



MVR+channeling in comparison with axial channeling of 7 TeV protons



MVR + Planar Channeling vs Axial Channeling



Uncorrelated scattering

by atomic strings

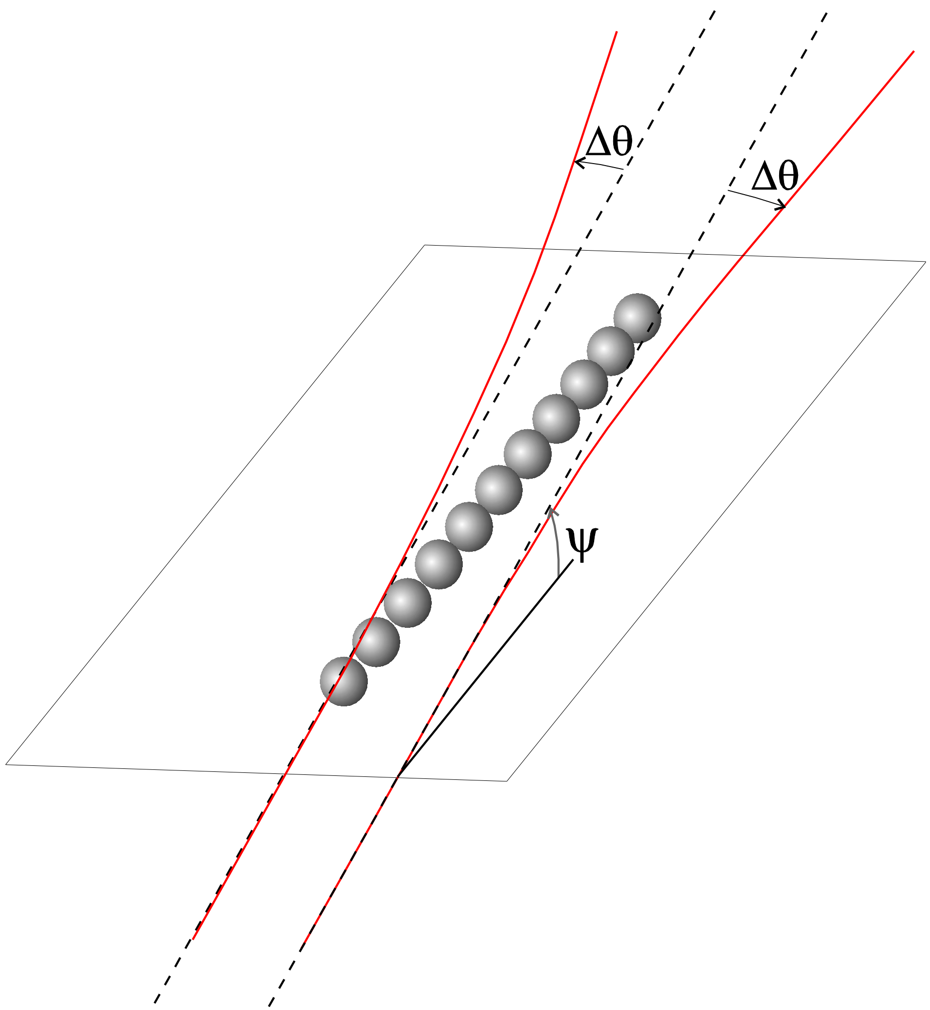
will enhance random

scattering practically

at any alignment

requirements

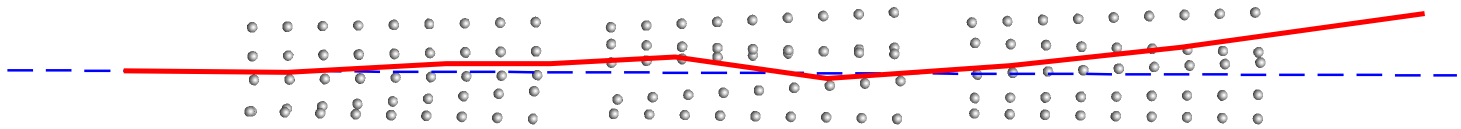
Particle scattering by atoms constituting a string



Particle scattering
by the atoms
constituting a string
are correlated (coherent)

coherent effects
in particle scattering
by the atomic string
will increase
the “scattering power”
by nearly 1000 times

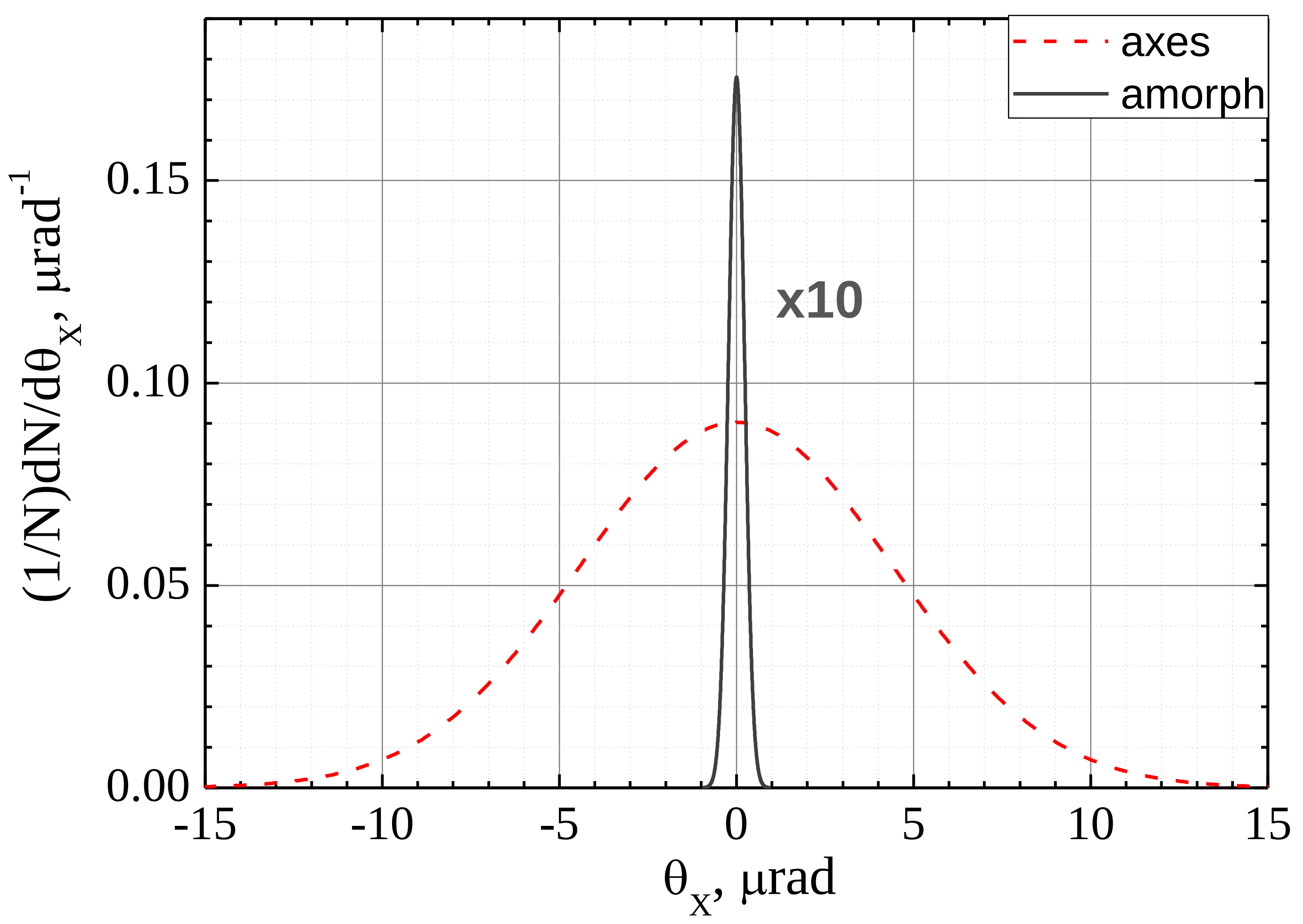
Multiple uncorrelated scattering by atomic strings



$$\psi\, l_{string} \sim 10^{-7}\, cm, \quad l_{string} \sim 0.01 cm$$

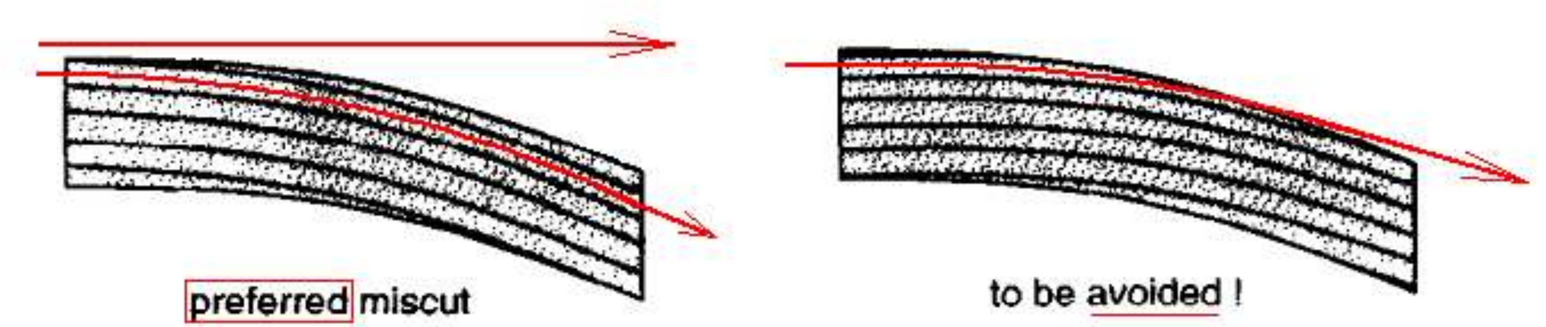
$$\frac{d\left\langle \vartheta_{ax}^2 \right\rangle}{dx} \approx \frac{4\pi^2 Z^2 \alpha^2 n}{\varepsilon^2 \psi} \frac{R-\tilde{u}}{d_{inat}} \approx \frac{\pi}{4\ln(190/Z^{1/3})} \frac{R-\tilde{u}}{d_{inat}\,\psi} \frac{d\left\langle \vartheta_{am}^2 \right\rangle}{dx}$$

Uncorrelated scattering amplification by atomic strings in 1 cm W target



The “miscut problem”

V.V. Tikhomirov, A.I. Sytov, *The miscut angle influence on the future LHC crystal based collimation system*. Problems of Atomic Science and Technology № 1 2012. 88-92. arXiv:1109.5051.



No big difference in fact!

Need of comprehensive simulations of high-energy electron radiation in crystals

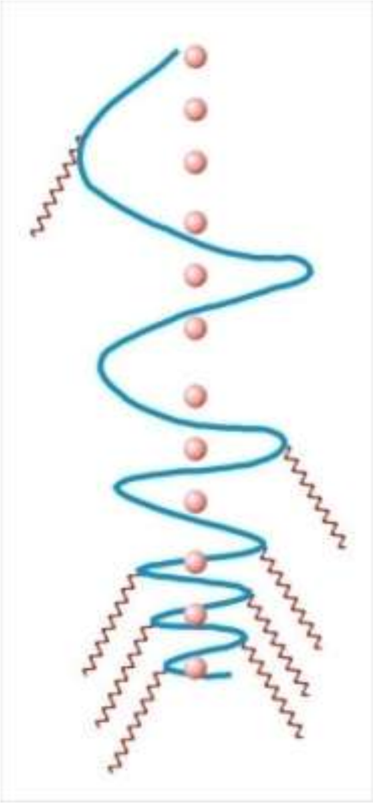
Electron radiative cooling

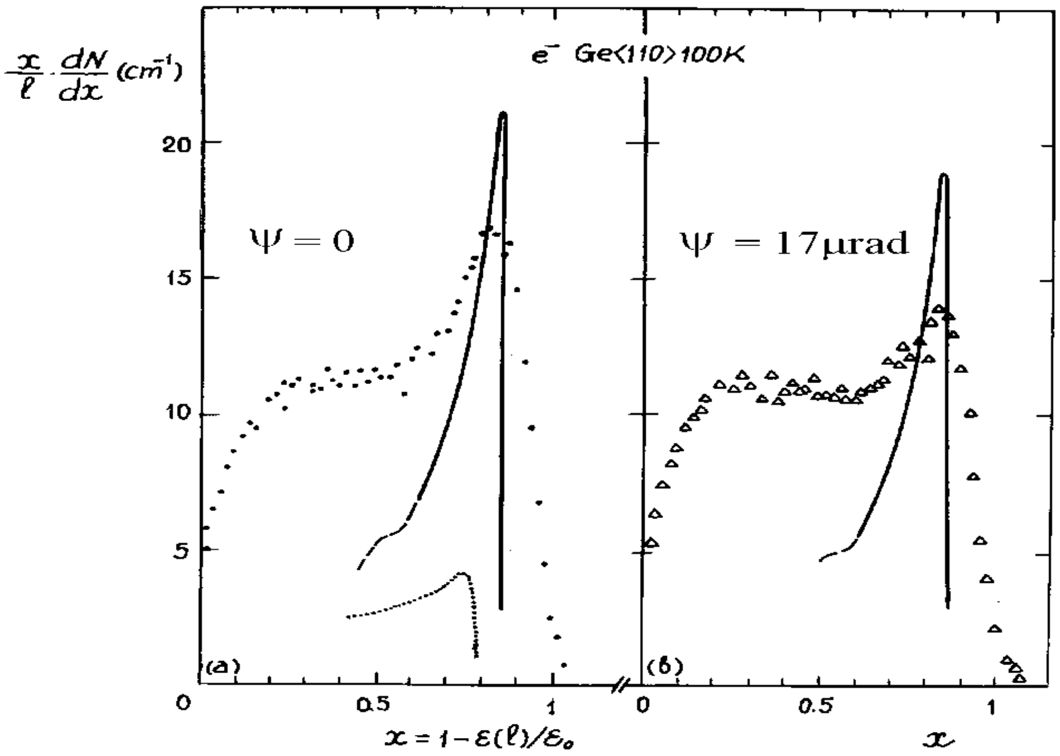
Baryshevskii V. G., Dubovskaya I. Ya. // Phys. Lett. 1977. Vol. A62. P. 45.

Belkacem A. et al. // Phys. Lett. 1986. Vol. B177. P. 211.

Tikhomirov V. V. // Phys. Lett. 1987. Vol. A125. P. 411.

Tikhomirov V. V. // Nucl. Instr. Meth. 1989. Vol. B36. P. 282.





A SIMULATION CODE FOR CHANNELING RADIATION BY ULTRARELATIVISTIC ELECTRONS OR POSITRONS

Xavier ARTRU *

Nuclear Instruments and Methods in Physics Research B48 (1990) 278–282

Laboratoire de Physique Théorique et Hautes Energies **, *Université de Paris-XI, 91405 Orsay, France*

2.2. *Multiple scattering*

After instruction (2a), we insert

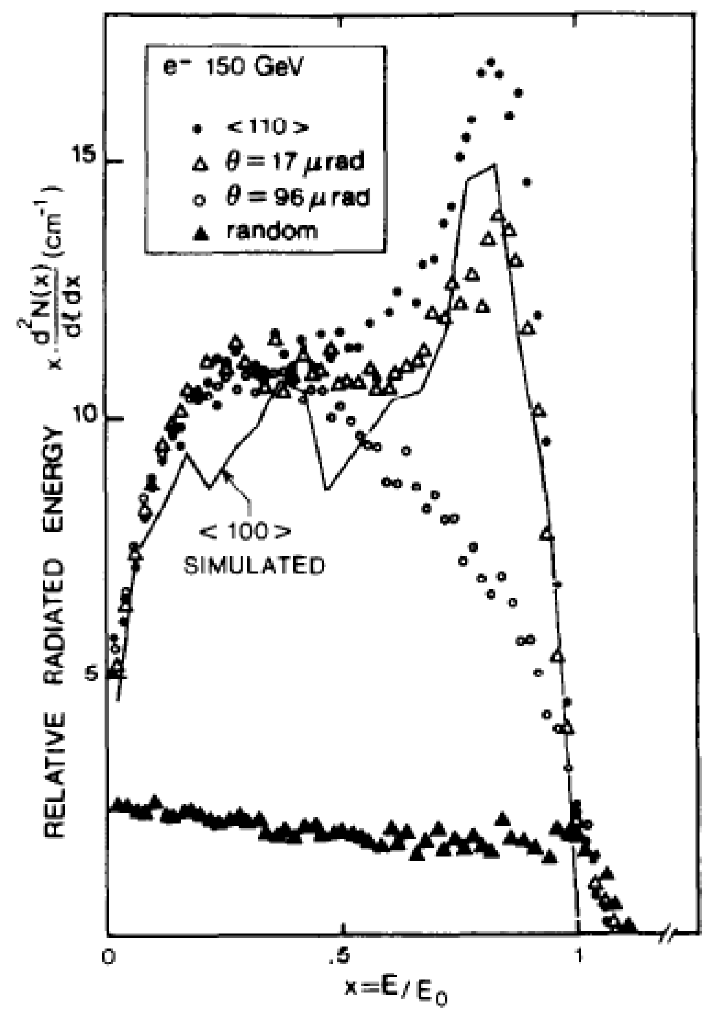
$$\boldsymbol{p}_l = \boldsymbol{p}_l + \boldsymbol{Q}, \tag{3}$$

where $\boldsymbol{Q} = \Sigma_1^n \boldsymbol{q}_j$ is the cumulated momentum transfer of n incoherent collisions during $\mathrm{d}t_l$. n is chosen at random about $\langle n \rangle = \mathrm{d}t_1 \rho_N(\boldsymbol{x}_l)\sigma_{\text{inco}}$, where $\rho_N = (2\pi u_1^2)^{-1} \exp(-r^2/2u_1^2)$ is the local density of nuclei and

$$\sigma_{\text{inco}} = 4(Z/137)^2 \int q^{-4}\, \mathrm{d}^2\boldsymbol{q} f_A(q^2) f_{DW}(q^2) \tag{4}$$

is the incoherent cross section. $f_A \simeq [1 + (a_{TF}q)^{-2}]^{-2}$ represents the atomic screening and $f_{DW} = 1 - \exp(-u_1^2 q^2)$ is the Debye–Waller factor. For a fast calculation of $\boldsymbol{Q}$ we take

$$f_A(q^2) f_{DW}(q^2) = (q/q_0)^4 \int_0^1 h \exp(-hq^2/q_0^2)\, \mathrm{d}h,$$



General simulation method

Key simulation points:

Trajectory simulations in most **realistic potentials**

Simulation of **incoherent scattering** on both nuclei and electrons

Separate simulation of **single** and **multiple** scattering

Direct integration of **Baier-Katkov formula**

Infinite trajectories, **density** effect…

Radiation process simulations from the *"**First Principles**"*

The general expression for radiation intensity

$$\frac{d^2 I}{d\omega d^2\theta} = \frac{\alpha\omega^2 d\omega}{8\pi^2\varepsilon'^2} \times \int\int dt_1 dt_2 \left[(\varepsilon^2 + \varepsilon'^2)(\mathbf{v}_\perp(t_1) - \boldsymbol{\theta})(\mathbf{v}_\perp(t_2) - \boldsymbol{\theta}) + \omega^2/\gamma^2\right]$$

$$\exp\left\{i\frac{\omega\varepsilon}{2\varepsilon'}\left[\int_{-\infty}^{t_1}\left(\gamma^{-2} + (\mathbf{v}_\perp(t') - \boldsymbol{\theta})^2\right)dt' + \int_{-\infty}^{t_2}\left(\gamma^{-2} + (\mathbf{v}_\perp(t'') - \boldsymbol{\theta})^2\right)dt''\right]\right\}$$

contains two integrals

$$A = \int \exp\left\{i\frac{\omega\varepsilon}{2\varepsilon'}\int_{-\infty}^{t}\left[\gamma^{-2} + (\mathbf{v}_\perp(t') - \boldsymbol{\theta})^2\right]dt'\right\}dt,$$

$$\mathbf{B} = \int (\mathbf{v}_\perp(t) - \boldsymbol{\theta})\exp\left\{i\frac{\omega\varepsilon}{2\varepsilon'}\int_{-\infty}^{t}\left[\gamma^{-2} + (\mathbf{v}_\perp(t') - \boldsymbol{\theta})^2\right]dt'\right\}dt$$

and slowly decreases with radiation angle $\boldsymbol{\theta}$, complicating its numerical integration.

Radiation at sharp change of particle trajectory

$$\frac{d\mathcal{E}}{d\omega do} = \frac{e^2}{4\pi^2 c}[\boldsymbol{k}, \boldsymbol{I}]^2,$$

$$\boldsymbol{I} = \frac{ic}{\omega}\int_{-\infty}^{\infty} dt\, e^{i(\omega/c)[ct-\boldsymbol{n}\cdot\boldsymbol{r}(t)]}\frac{d}{dt}\frac{\boldsymbol{v}(t)}{c-\boldsymbol{n}\cdot\boldsymbol{v}(t)}.$$

$$\boldsymbol{I} \approx \frac{ic}{\omega}\left(\frac{\boldsymbol{v}'}{c-\boldsymbol{n}\cdot\boldsymbol{v}'} - \frac{\boldsymbol{v}}{c-\boldsymbol{n}\cdot\boldsymbol{v}}\right),$$

$$\frac{d\mathcal{E}}{d\omega} = \frac{2e^2}{\pi c}\left(\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln\left(\xi+\sqrt{\xi^2+1}\right) - 1\right).$$

Single scattering effects are treated separately

$$A = \int_{-\infty}^{\infty} \exp\{i\varphi(t)\}dt = \frac{i}{\dot\varphi(+0)} - \frac{i}{\dot\varphi(-0)} +$$

$$i\sum_{i=1}^{N}\left\{\left[\frac{1}{\dot\varphi(t_i+0)} - \frac{1}{\dot\varphi(t_i-0)}\right]\exp i\varphi(t_i) - \frac{2\ddot\varphi(\bar t_i)}{\dot\varphi^3(\bar t_i)}\sin\left[\frac{\varphi(t_i-0)-\varphi(t_{i-1}+0)}{2}\right]\exp i\varphi(\bar t_i)\right\},$$

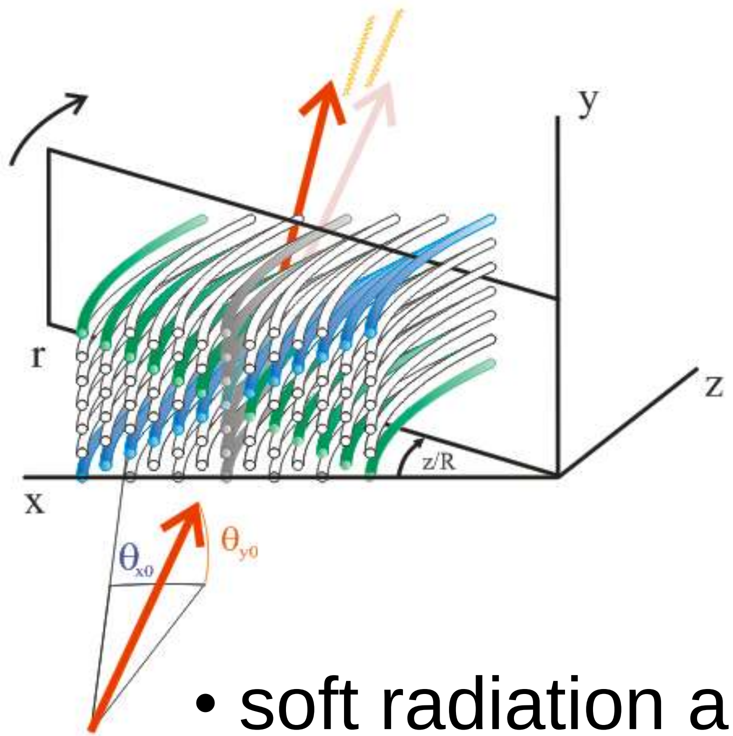
$$\vec B = \int_{-\infty}^{\infty}\left[\vec v_\perp(t) - \vec\theta\right]\exp\{i\varphi(t)\}dt = \left[\frac{i}{\dot\varphi(+0)} - \frac{i}{\dot\varphi(-0)}\right]\left(\vec v_\perp(0) - \vec\theta\right) +$$

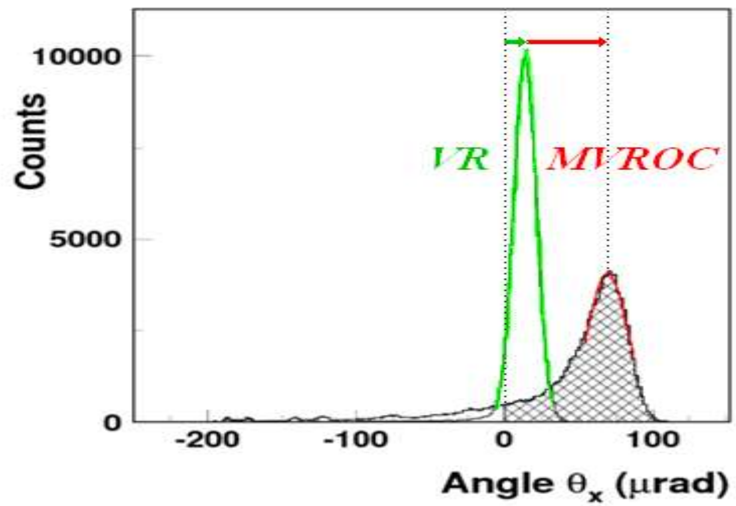
$$i\sum_{i=1}^{N}\left\{\begin{array}{l}\left[\underline{\frac{\vec v_\perp(t_i)+\vec\vartheta_i-\vec\theta}{\dot\varphi(t_i+0)} - \frac{\vec v_\perp(t_i)-\vec\theta}{\dot\varphi(t_i-0)}}\right]\exp i\varphi(t_i) - \\ \frac{2}{\dot\varphi^2(\bar t_i)}\left[\dot{\vec v}_\perp(\bar t_i) - \left(\vec v_\perp(\bar t_i) - \vec\theta\right)\frac{\ddot\varphi(\bar t_i)}{\dot\varphi(\bar t_i)}\right]\sin\left[\frac{\varphi(t_i-0)-\varphi(t_{i-1}+0)}{2}\right]\exp i\varphi(\bar t_i)\end{array}\right\},$$

where $\omega' = \varepsilon/(\varepsilon-\omega)$, $\ddot\varphi(t) = \omega'\left(\vec v_\perp(t_i) - \vec\theta\right)\dot{\vec v}_\perp(t)$ and $\bar t_i = (t_i + t_{i-1})/2$.

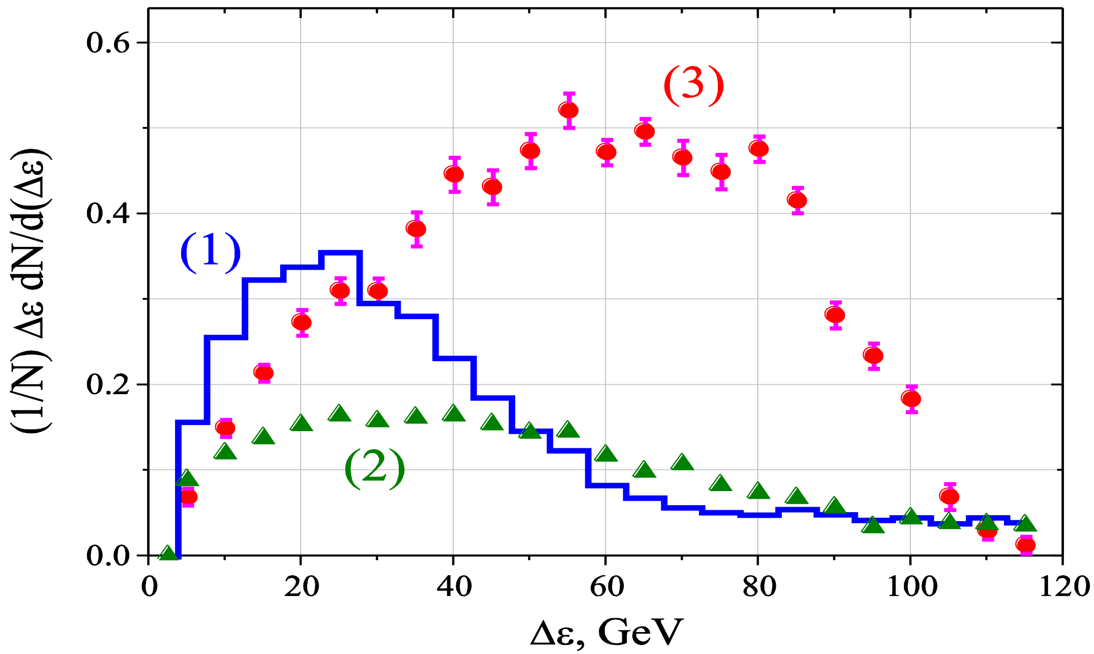
Simulation of radiation accompanying multiple volume reflection

Radiation amplification under Multiple Volume Reflection in One Crystal (120 GeV e^-, 2 mm Si ‹111›)

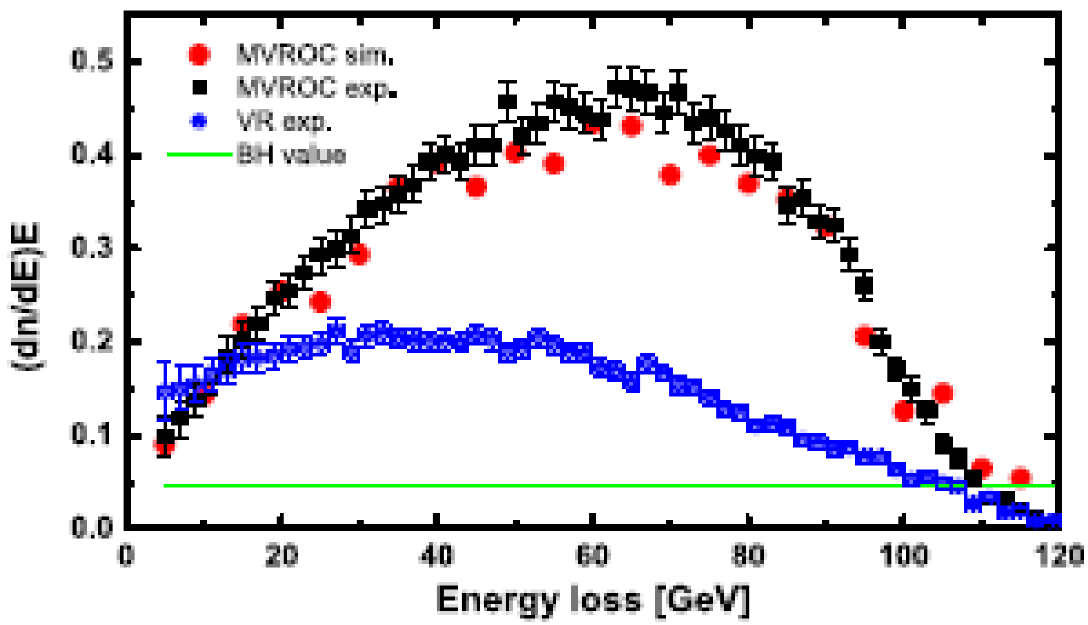




- soft radiation amplification by reflection
  by different planes
- hard radiation amplification
  by axial field



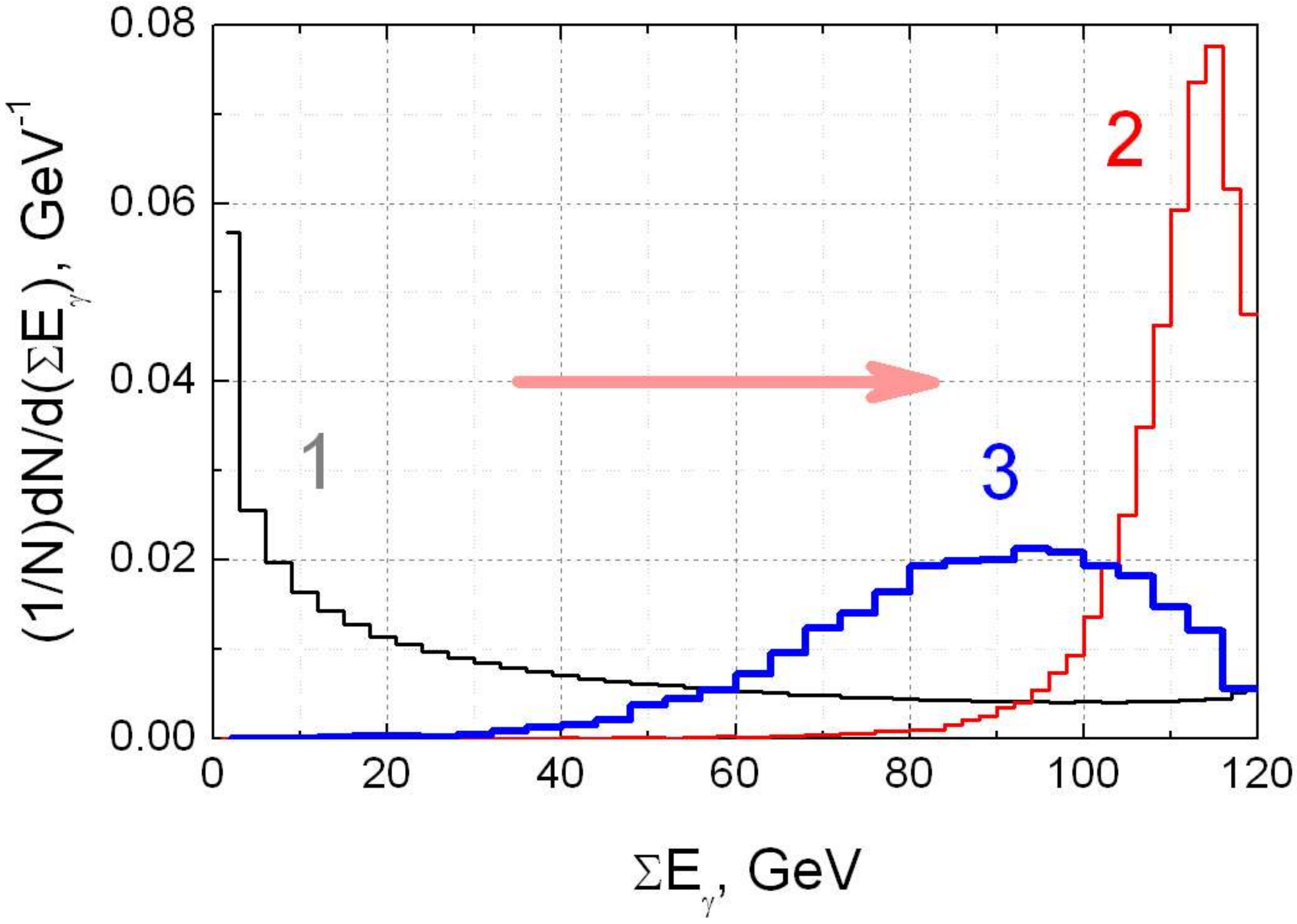
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
(1/N) Δε dN/d(Δε)
Δε, GeV
(1)
(2)
(3)



MVROC sim.
MVROC exp.
VR exp.
BH value
0.5
0.4
0.3
0.2
0.1
0.0
0
20
40
60
80
100
120
(dn/dE)E
Energy loss [GeV]

120 GeV electron radiation in <001> PWO

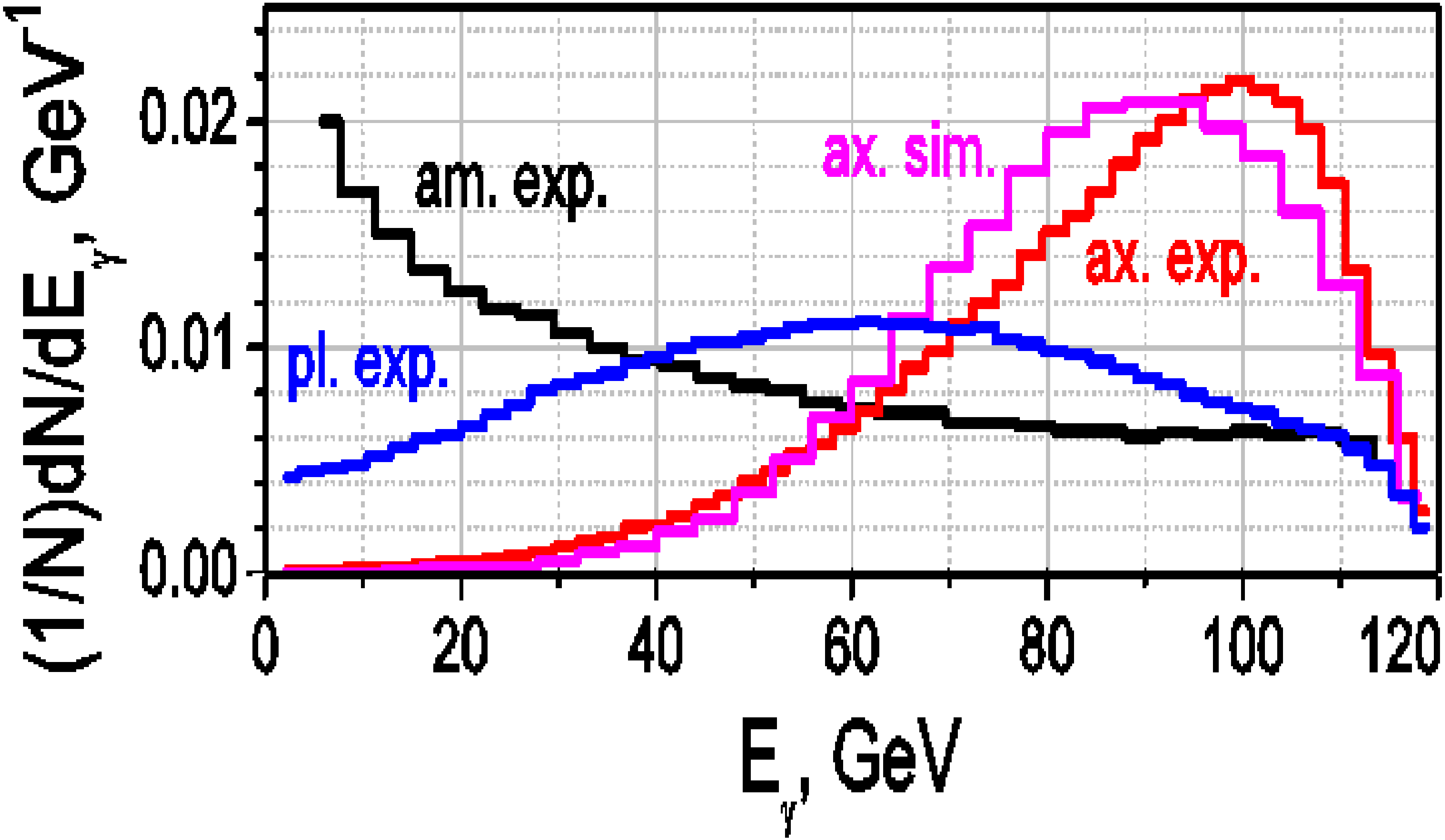
(Dr. L. Bandiera PRIN project)



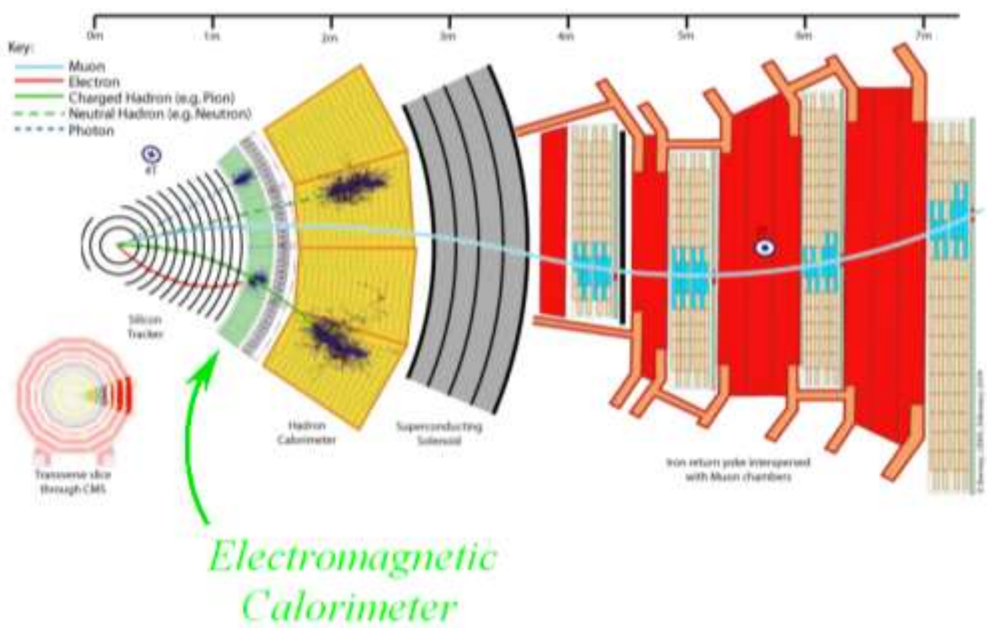
120 GeV electron energy losses on 4 mm of **amorphous** $PbWO_4$ (1),

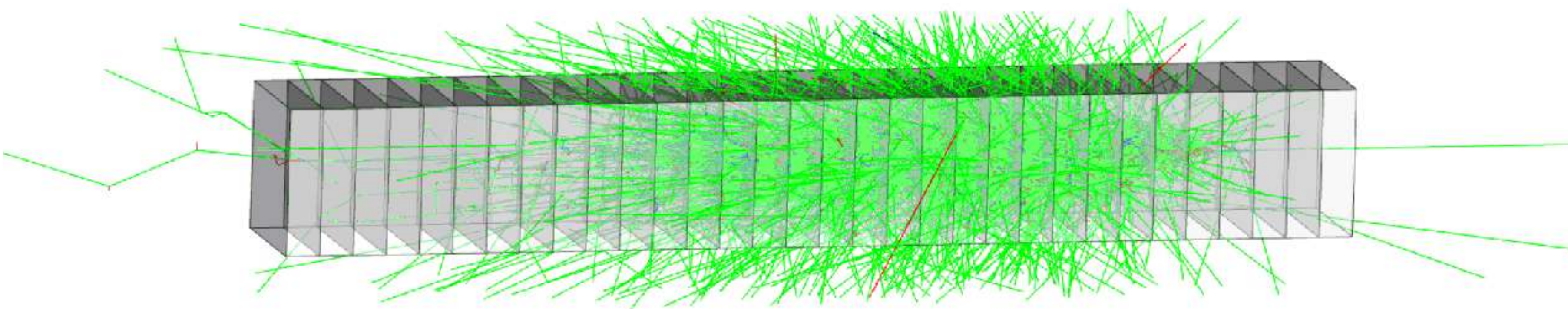
crystalline $PbWO_4$ with (**3**) and **without** (**2**) PP by radiated photons

Electromagnetic shower development acceleration in PWO at E_e =120 GeV

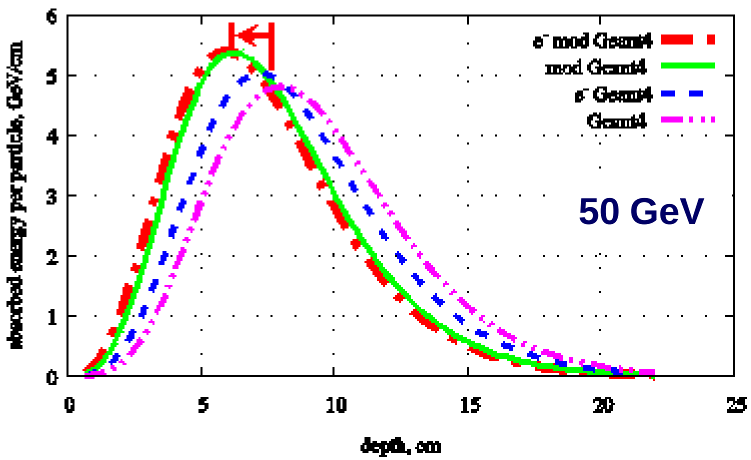


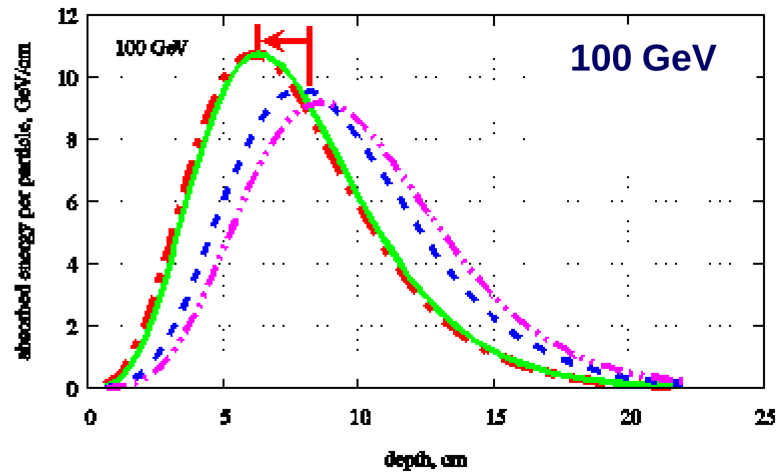
First simulations of shower development in ECAL CMS

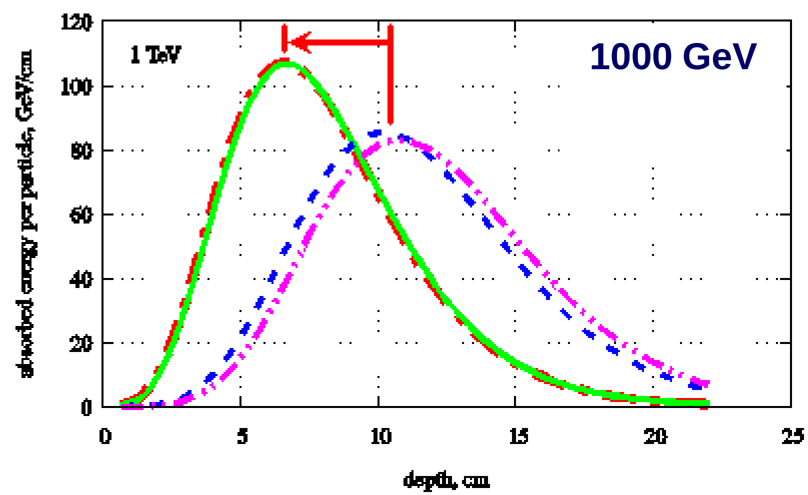




Electromagnetic shower acceleration in PWO at 50, 100 and 1000 GeV







em shower maximum shifts

by 2 ÷ 5

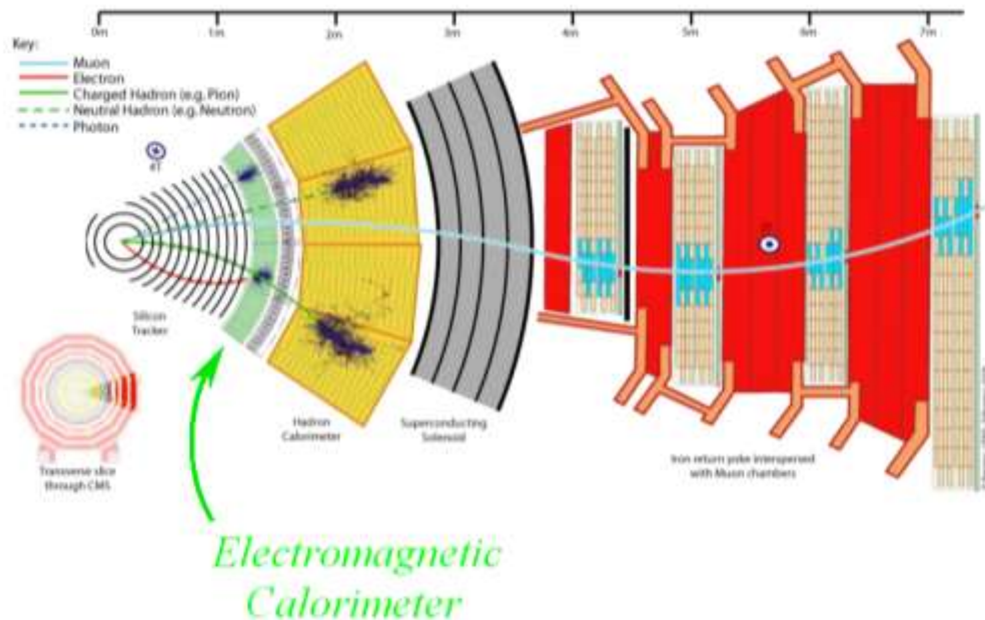
radiation lengths

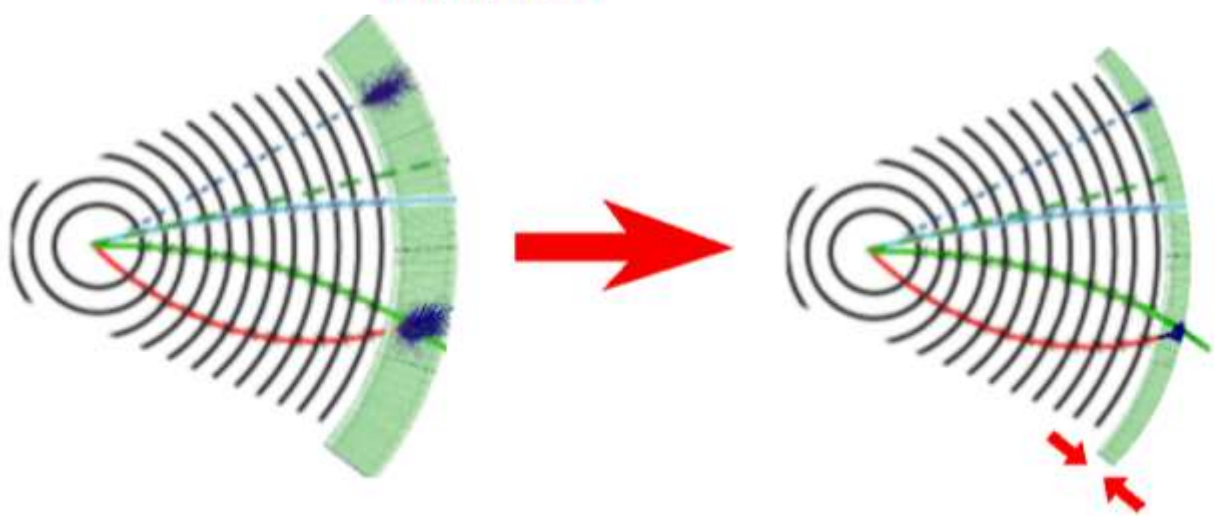
Electromagnetic shower acceleration in PWO can influence H boson mass measurements

ATLAS $m_{H\to\gamma\gamma} = 125.98 \pm 0.50$ GeV

CMS $m_{H\to\gamma\gamma} = 124.70 \pm 0.34$ GeV

Thickness of the CMS ECAL calorimeters can be made smaller

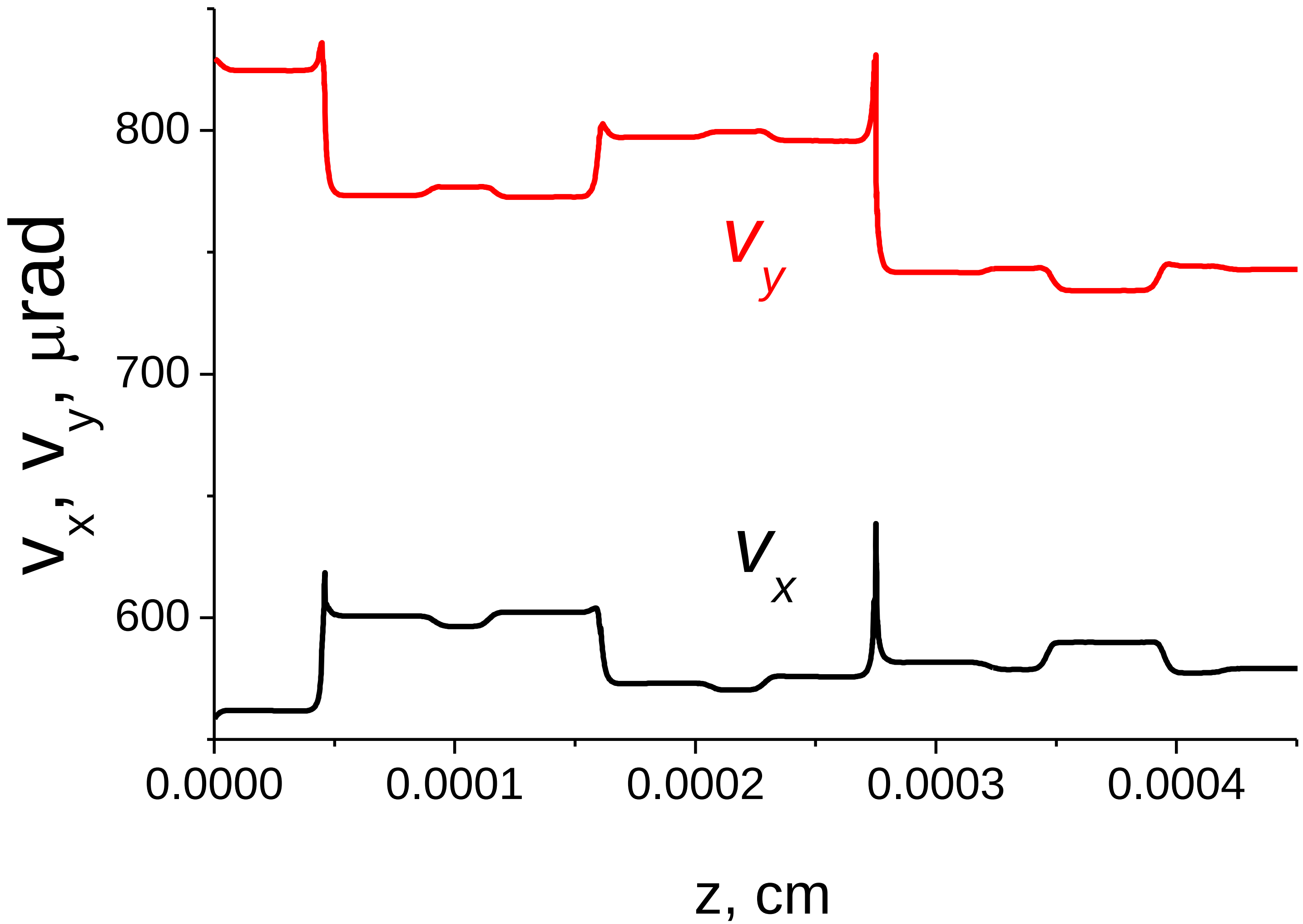


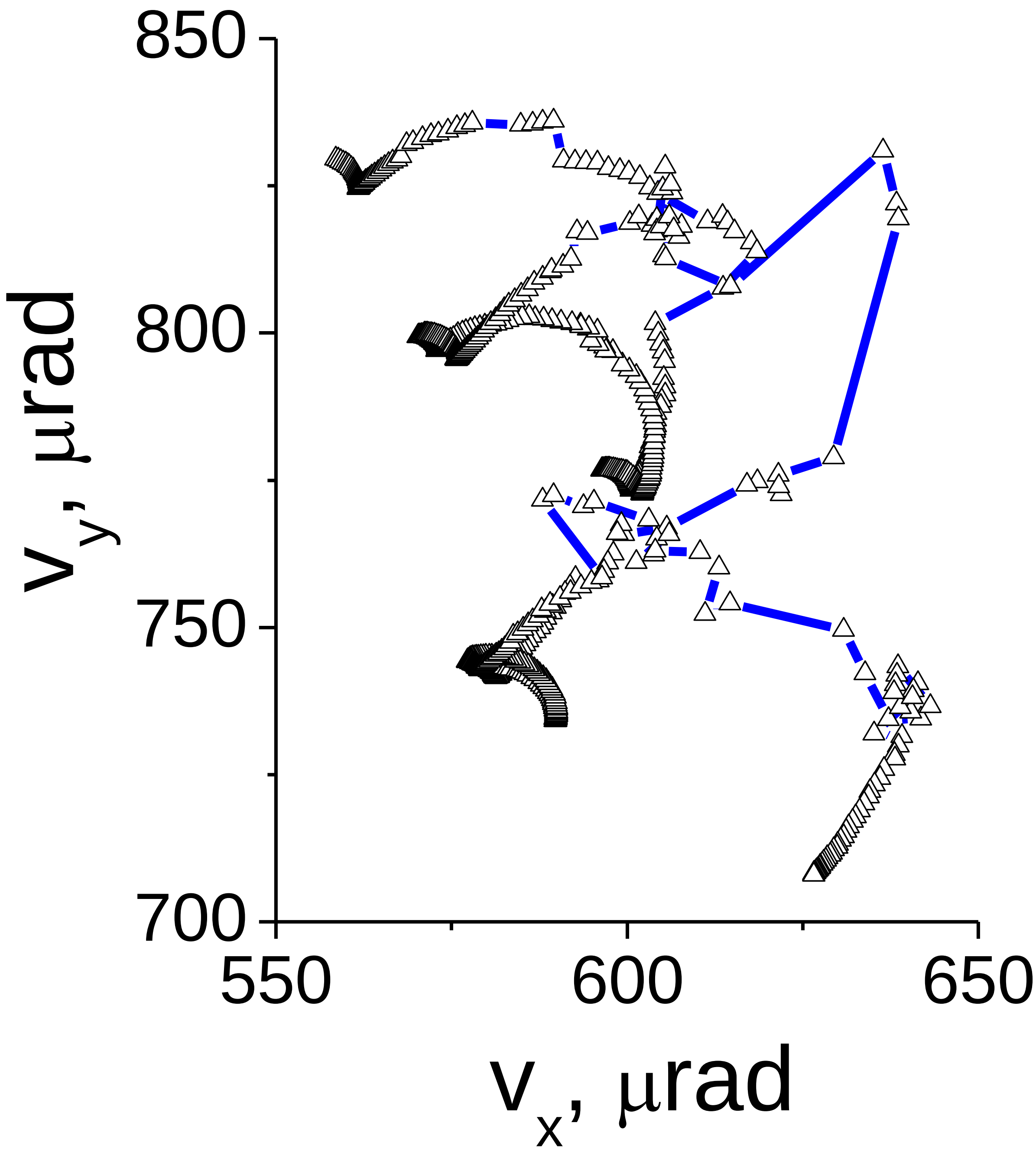


A first step to positron source simulations:

electron radiation in 1 mm W <111>

A 20 GeV in <110> W field trajectory



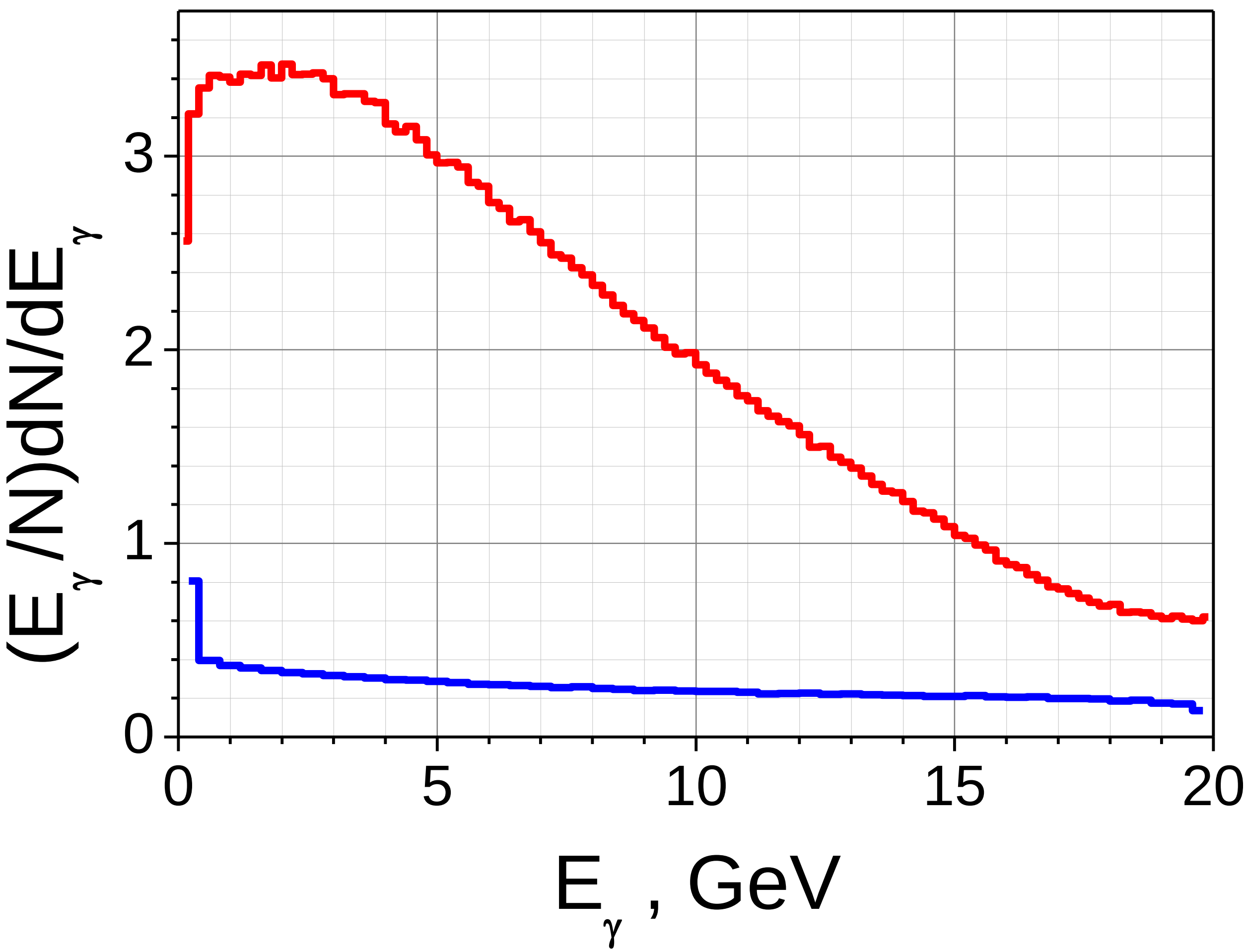


Transverse velocity component evolution

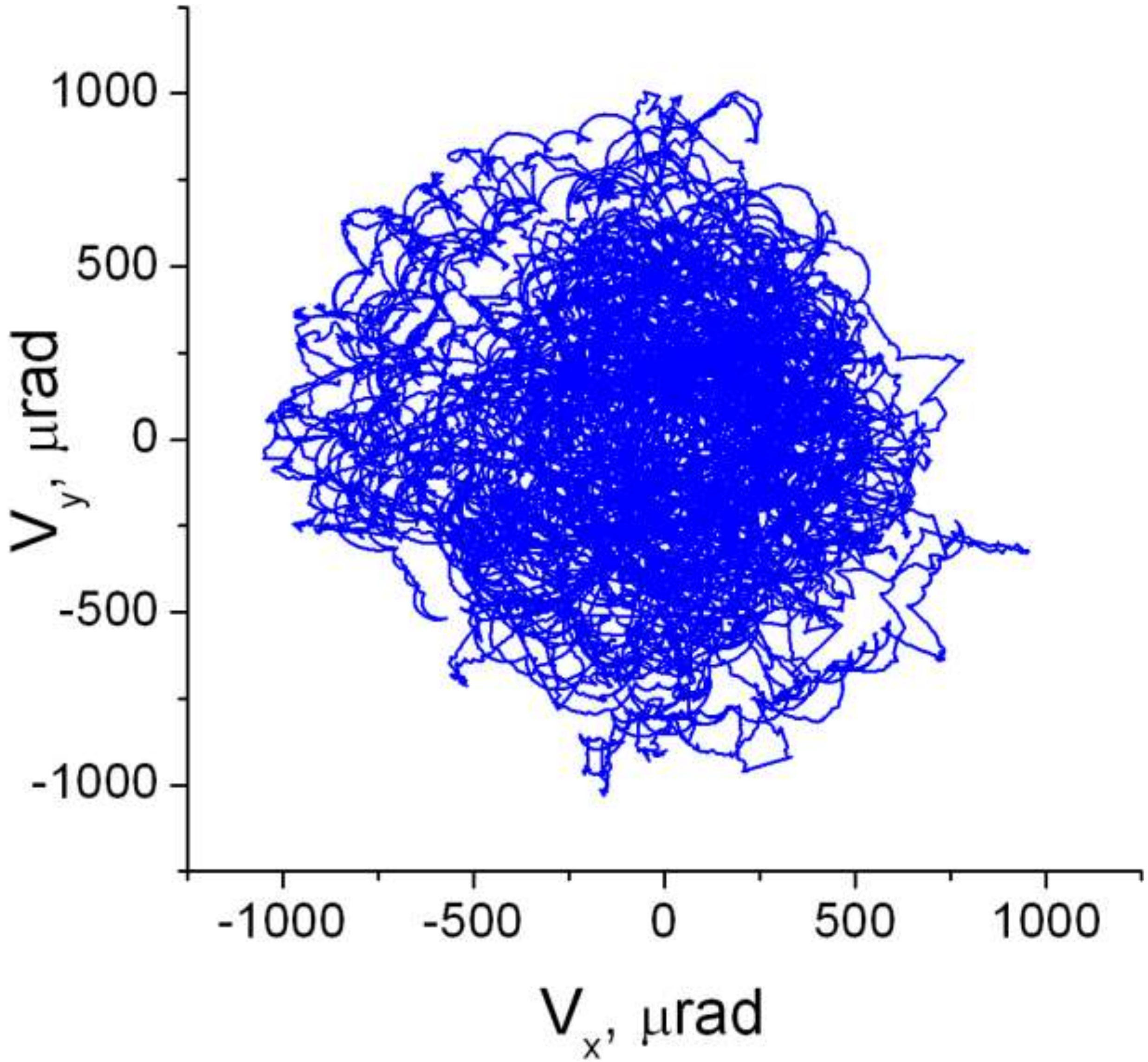
Mind single scattering!

Radiation amplification in <110> W at 20 GeV

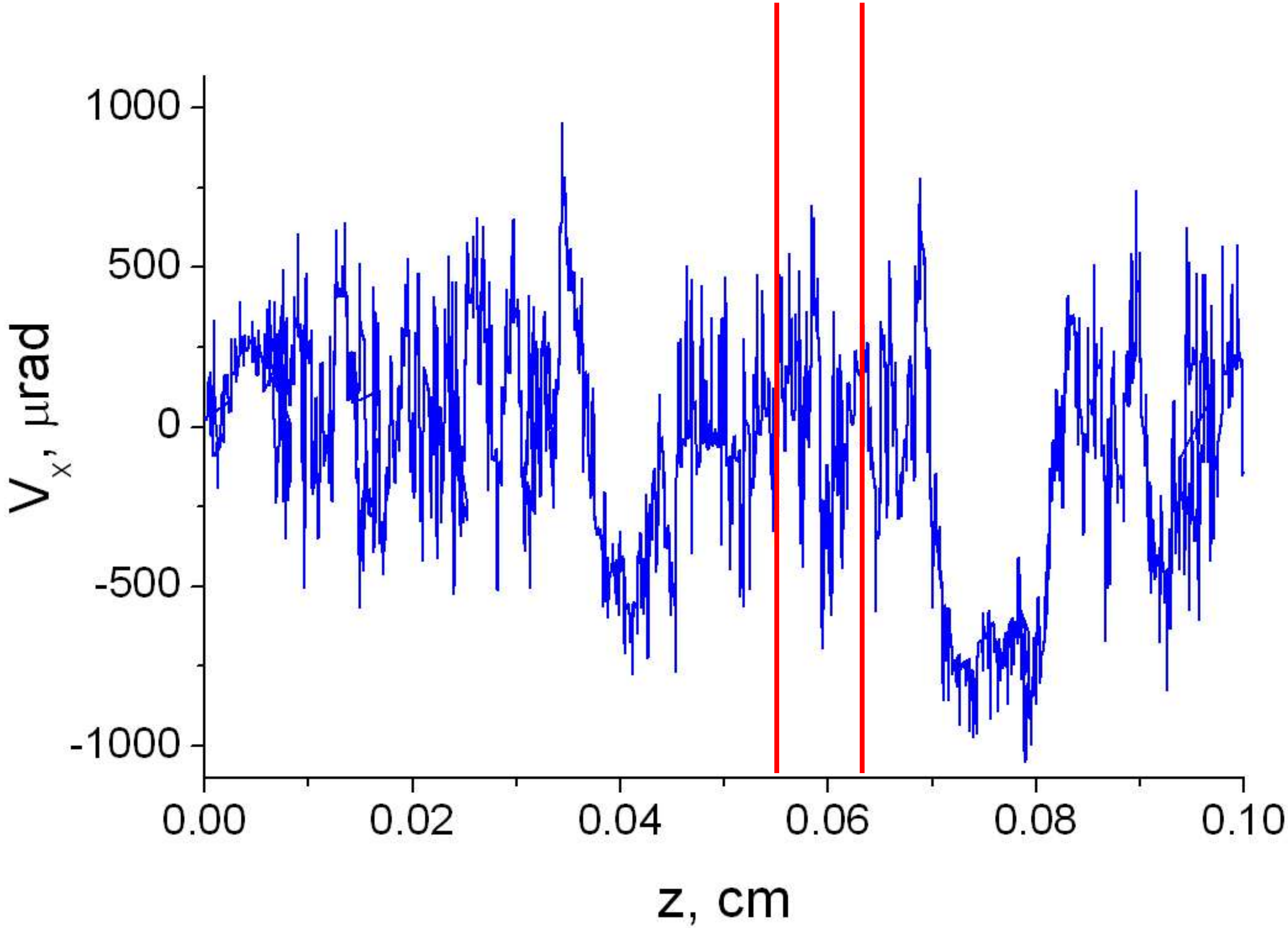
*(**photon** spectra thin crystal limit)*



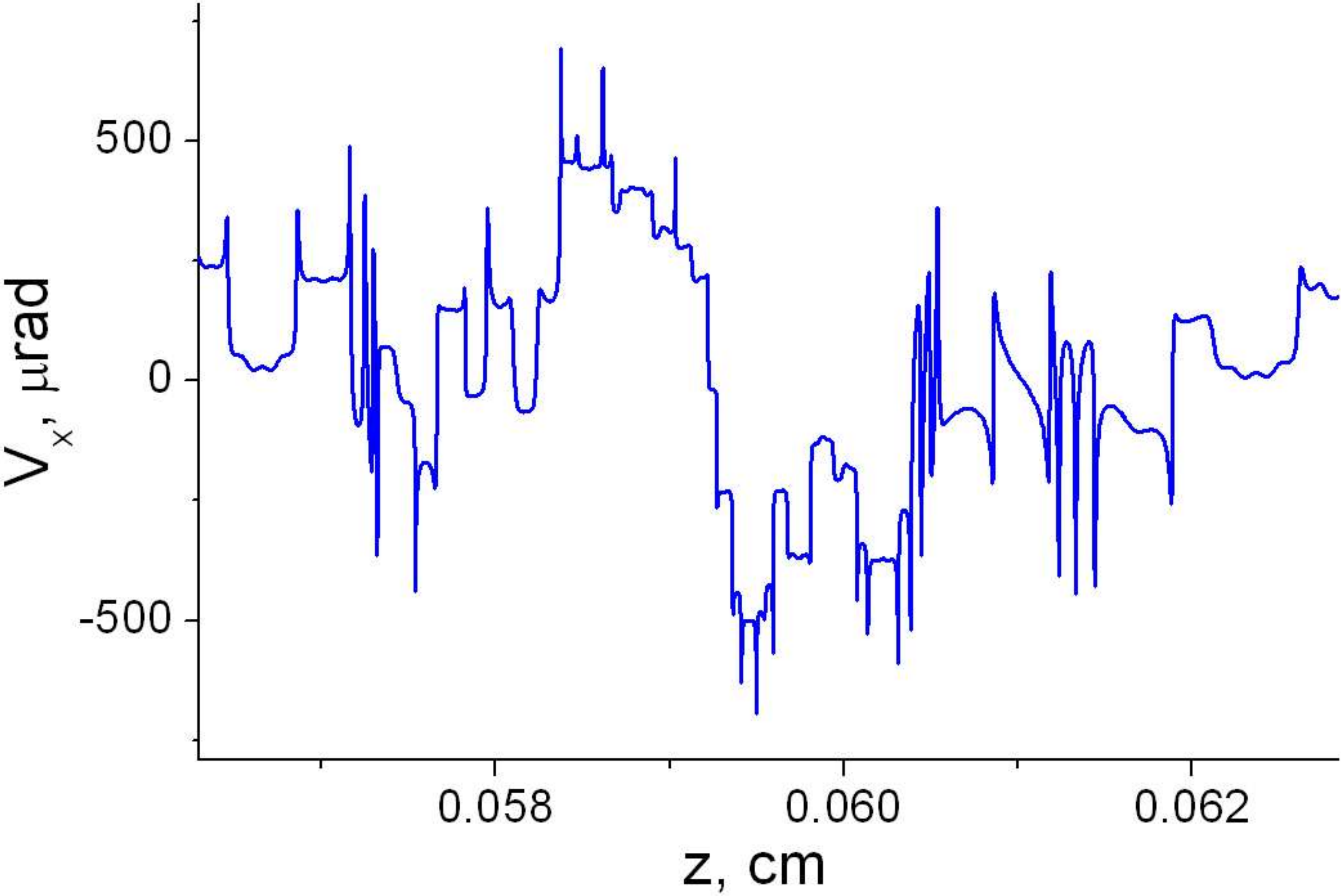
20 GeV electron trajectory in W<111> 1 mm



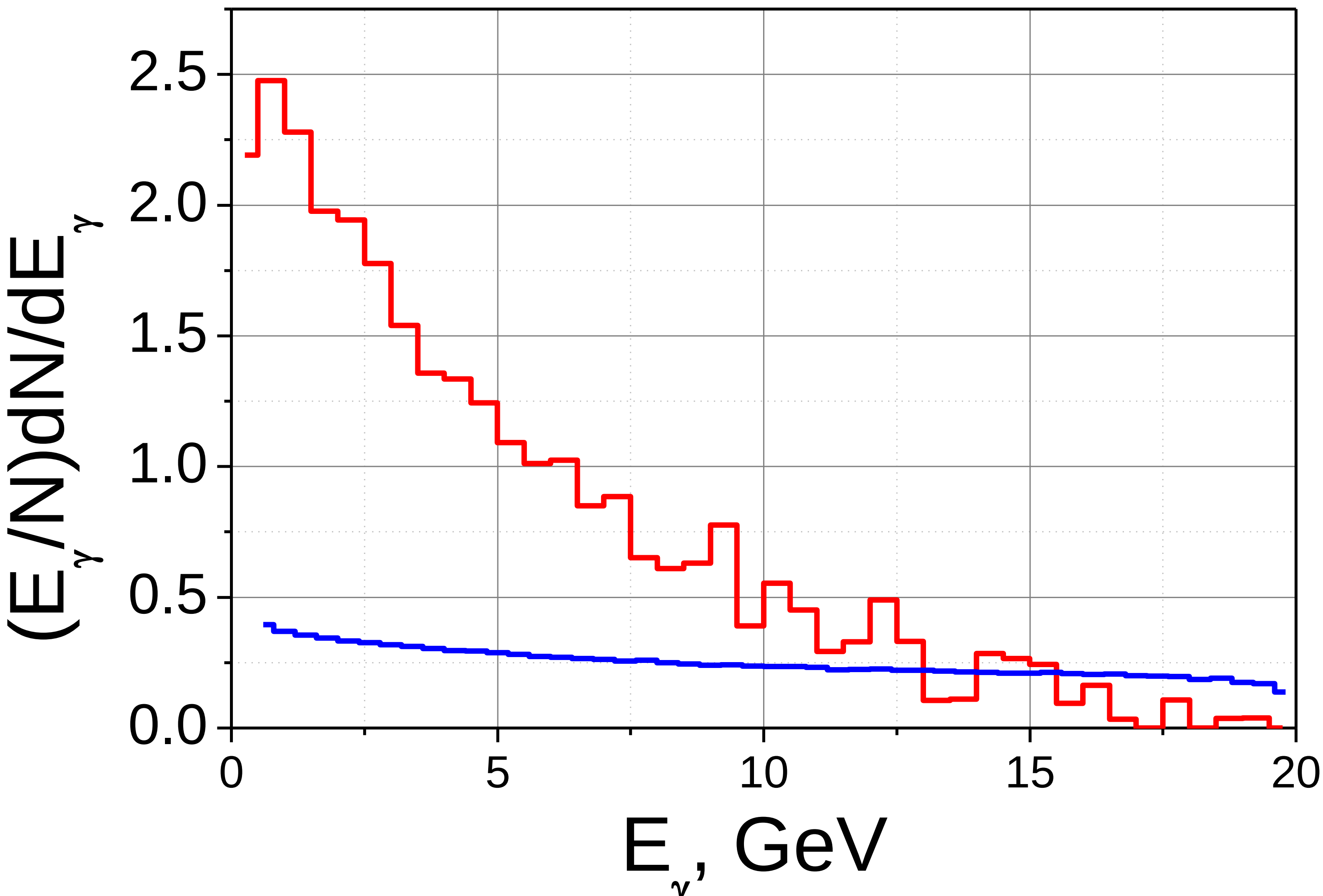
20 GeV electron trajectory in W<111> 1 mm



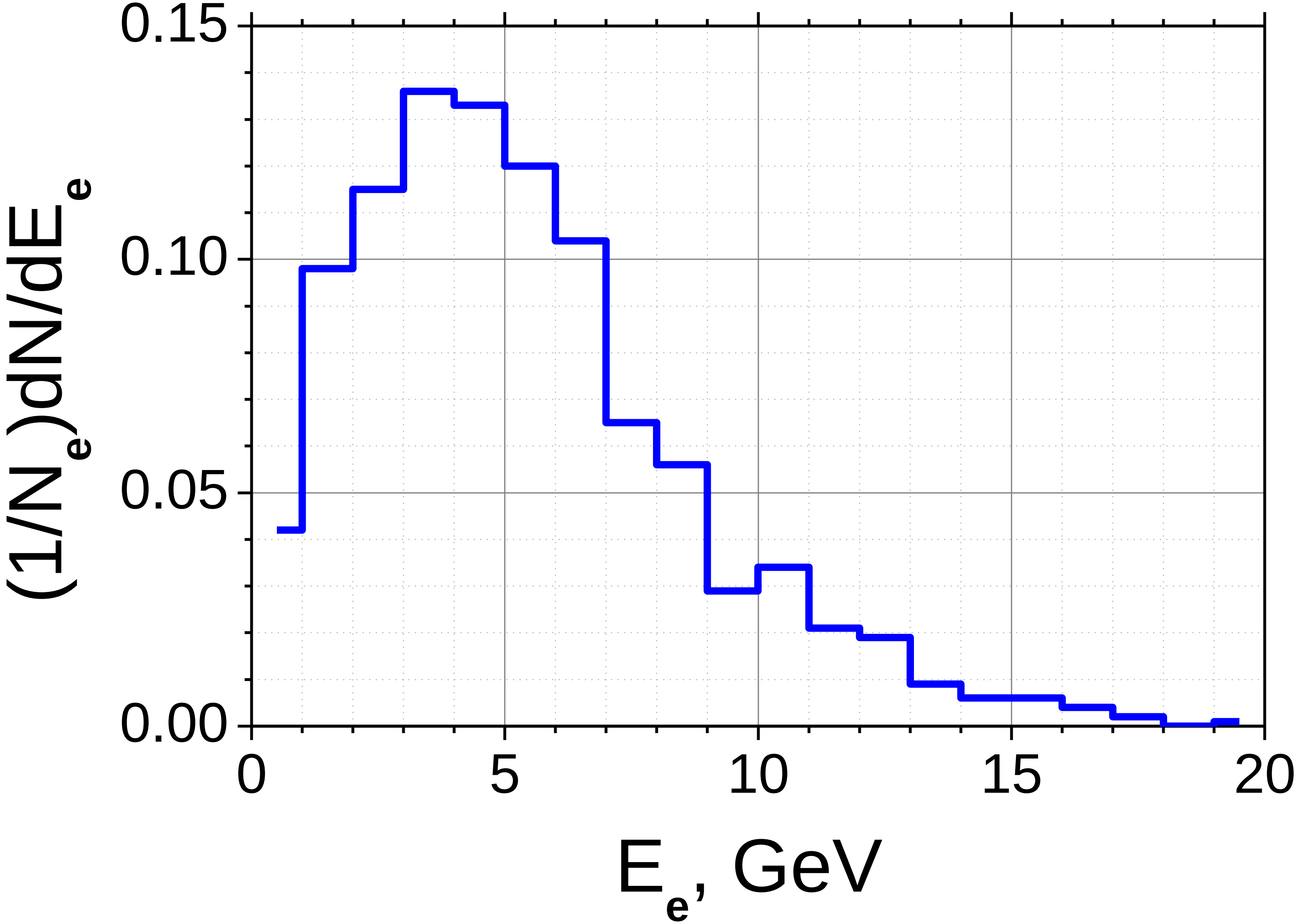
A part of electron trajectory in W<111>



20 GeV e⁻ radiation amplification in <110> W **1 mm**



20 GeV **electron** spectrum behind 1 mm <110> W



as a conclusion

Possible cooperation directions

– positron source

– crystal assisted collimation

– crystal undulators

– crystal scintillators

– gamma-telescopes

– radiation pair production at high energies,
polarization and spin effects

– charm and beauty hyperon magnetic and
electric dipole moments

Thank you for attention!