Simulations in High-Energy Physics

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PART I: Introduction

PART II: Monte Carlo for Perturbative QCD

PART III: Precision Simulations

PART IV: Monte Carlo for Non–Perturbative QCD

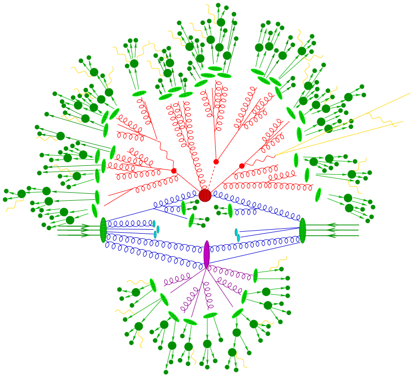
PART I: INTRODUCTION

IMPROVING EVENT GENERATORS

Strategy of event generators

principle: divide et impera

- hard process:
  fixed order perturbation theory
  traditionally: Born-approximation
- bremsstrahlung:
  resummed perturbation theory
- hadronisation:
  phenomenological models
- hadron decays:
  effective theories, data
- "underlying event":
  phenomenological models

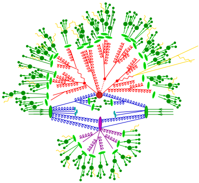


. . . and possible improvements

possible strategies:

- improving the phenomenological models:
  - "tuning" (fitting parameters to data)
  - replacing by better models, based on more physics

    (my hot candidate: "minimum bias" and "underlying event" simulation)



- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

    "NLO-Matching" & "Multijet-Merging"
  - systematic improvement of the parton shower: next-to leading (or higher) logs & colours

Motivation – precision edge of particcle physics

- after Higgs discovery: time for precision studies
  is it the SM Higgs boson or something else?
  relevant: spin/parity (✓), couplings to other particles
- Higgs signal suffers from different backgrounds, depending on production and decay channel considered in the analysis
- decomposing in bins of different jet multiplicities yields
  - different signal composition (e.g. WBF vs. ggF)
  - different backgrounds (most notably: $t\bar{t}$ in WW final states)
- to this end: must understand jet production in big detail
  name of the game: uncertainties and their control

  despite far-reaching claims: analytic resummation and fixed-order calculations will not be sufficient
- same reasoning also true for new resonances/phenomena

Motivation – BSM edge of particle physics

- to date no survivors in searches for new physics & phenomena

  (a pity, but that's what Nature hands to us)

- push into precision tests of the Standard Model

  (find it or constrain it!)

- statistical uncertainties approach zero

  (because of the fantastic work of accelerator, DAQ, etc.)

- systematic experimental uncertainties decrease

  (because of ingenious experimental work)

- theoretical uncertainties are or become dominant

  (it would be good to change this to fully exploit LHC's potential)

$\Longrightarrow$ more accurate tools for more precise physics needed!

Aim of the lectures

- review the state of the art in precision simulations

  (celebrate success)

- highlight missing or ambiguous theoretical ingredients

  (acknowledge failure)

- (maybe) suggest some further studies – experiment and theory

  (. . .)

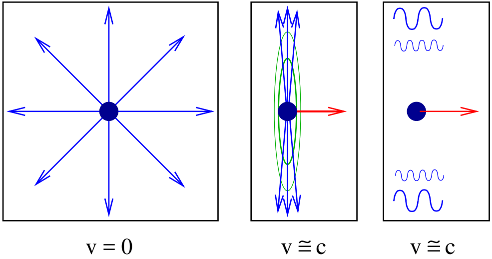
QCD BASICS

SCALES & KINEMATICS

Contents

An electromagnetic analogy

- consider a charge Z moving at constant velocity v



- at $v = 0$: radial E field only
- at $v = c$: B field emerges: $\vec{E} \perp \vec{B}$, $\vec{B} \perp \vec{v}$, $\vec{E} \perp \vec{v}$,
  energy flow $\sim$ Poynting vector $\vec{S} \sim \vec{E} \times \vec{B}$, $\parallel \vec{v}$
- approximate classical fields by "equivalent quanta": photons

- spectrum of photons:

(in dependence on energy ω and transverse distance $b_\perp$)

$$\mathrm{d}n_\gamma = \frac{Z^2\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2} \xrightarrow{\text{electron}(Z=1)} \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2}$$

- Fourier transform to transverse momenta $k_\perp$:

$$\mathrm{d}n_\gamma = \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

note: divergences for $k_\perp \to 0$ (collinear) and $\omega \to 0$ (soft)

- therefore: Fock state for lepton = superposition (coherent):

$$|e\rangle_{\text{phys}} = |e\rangle + |e\gamma\rangle + |e\gamma\gamma\rangle + |e\gamma\gamma\gamma\rangle + \ldots$$

photon fluctuations will “recombine”

QED Initial and Final State Radiation

- consider final state radiation in $\gamma^* \to \ell\bar{\ell}$
  (electron velocities/momenta labelled as v and v'/p and p')
- classical electromagnetic spectrum from radiation function:

(this is from Jackson or any other reasonable book on ED)

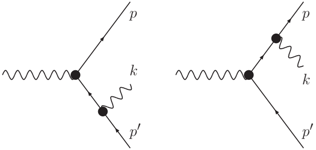
$$\frac{\mathrm{d}^2 I}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{e^2}{4\pi^2}\left|\vec{\epsilon}^{\,*}\cdot\left(\frac{\vec{v}}{1-\vec{v}\cdot\vec{n}} - \frac{\vec{v}'}{1-\vec{v}'\cdot\vec{n}}\right)\right|^2,$$

with ϵ the polarisation vector and $\vec{n}(\Omega)$ the direction of the radiation
- recast with four–momenta, equivalent photon spectrum:

$$\mathrm{d}N = \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}\,\frac{\alpha}{\pi}\left|\epsilon^*_\mu\left(\frac{p^\mu}{p\cdot k} - \frac{p'^\mu}{p'\cdot k}\right)\right|^2 = \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}\,\frac{\alpha}{\pi}\left|\mathrm{W}_{pp';k}\right|^2$$

with the eikonal $\mathrm{W}_{pp';k}$

- repeat exercise in QFT, Feynman diagrams:



$$\mathcal{M}_{X\to e^+e^-\gamma} = e\bar{u}(p)\left[\Gamma\,\frac{\not{p}'-\not{k}}{(p'-k)^2}\,\gamma^\mu - \gamma^\mu\,\frac{\not{p}+\not{k}}{(p+k)^2}\,\Gamma\right]u(p')\epsilon^*_\mu(k)$$

$$\stackrel{\text{soft}}{\longrightarrow} \quad e\epsilon^*_\mu(k)\left[\frac{p^\mu}{p\cdot k}-\frac{p'^\mu}{p'\cdot k}\right]\bar{u}(p')\Gamma u(p) = e\mathcal{M}_{X\to e^+e^-\gamma}\cdot \mathrm{W}_{pp';k}$$

- manifestation of Low's theorem:
  soft radiation independent of spin ($\to$ classical)

(radiation decomposes into soft, classical part with logs – i.e. dominant – and hard collinear part)

DGLAP equations for QED

(Dokshitser–Gribov–Lipatov–Altarelli–Parisi Equations)

- define probability to find electron or photon in electron:

$$\text{at LO in } \alpha(\text{noemission}): \ \ell(x,\, k_\perp^2) = \delta(1-x)$$
$$\text{and} \quad \gamma(x,\, k_\perp^2) = 0$$

(introduced $x =$ energy fraction w.r.t. physical state)

- including emissions:
  - probabilities change
  - energy fraction ξ of lepton parton w.r.t. the physical lepton object reduced by some fraction $z = x/\xi$
  - reminder: differential of photon number w.r.t. $k_\perp^2$:

$$\mathrm{d}n_\gamma = \frac{\alpha}{\pi}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}\omega}{\omega} \leftrightarrow \frac{\mathrm{d}n_\gamma}{\mathrm{d}\log k_\perp^2} = \frac{\alpha}{\pi}\frac{\mathrm{d}x}{x}$$

- evolution equations (trivialised)

$$\frac{d\ell(x,\,k_\perp^2)}{d\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int\limits_x^1 \frac{d\xi}{\xi}\mathcal{P}_{\ell\ell}\left(\frac{x}{\xi},\,\alpha(k_\perp^2)\right)\ell(\xi,\,k_\perp^2)$$

$$\frac{d\gamma(x,\,k_\perp^2)}{d\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int\limits_x^1 \frac{d\xi}{\xi}\mathcal{P}_{\gamma\ell}\left(\frac{x}{\xi},\,\alpha(k_\perp^2)\right)\ell(\xi,\,k_\perp^2)\,.$$

- $k_\perp^2$ plays the role of "resolution parameter"
- the $\mathcal{P}_{ab}(z)$ are the splitting functions, encoding quantum mechanics of the "splitting cross section", for example (at LO)

$$\mathcal{P}_{\ell\ell}(z) = \left(\frac{1+z^2}{1-z}\right)_+ + \frac{3}{2}\delta(1-z)$$

- if $\gamma \to \ell\bar{\ell}$ splittings included, have to add entries/splitting functions into evolution equations above

Running of α_s and bound states

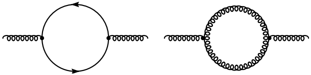
- quantum effect due to loops: couplings change with scale
- running driven by β–function

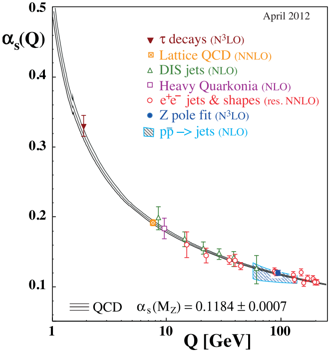
$$\beta(\alpha_s) = \mu_R^2 \frac{\partial \alpha_s(\mu_R^2)}{\partial \mu_R^2} = \frac{\beta_0}{4\pi}\,\alpha_s^2 + \frac{\beta_1}{(4\pi)^2}\alpha_s^3 + \dots$$

with

$$\beta_0 = \frac{11}{3}\,C_A - \frac{4}{3}\,T_R n_f$$

$$\beta_1 = \frac{34}{3}\,C_A^2 - \frac{20}{3}\,C_A T_R n_f - 4\,C_F T_R n_f$$





- Casimir operators in the fundamental and adjoint representation:

$$C_F = \frac{N_c^2 - 1}{2N_c} \quad \text{and} \quad C_A = N_c$$

with $N_c = 3$ colours and $T_R = 1/2$.

- n_f = the number of (quark) flavours
- the Casimirs correspond to quark and gluon colour charges
- explicit expression for strong coupling

$$\alpha_s(\mu_R^2) \equiv \frac{g_s^2(\mu_R^2)}{4\pi} = \frac{1}{\frac{\beta_0}{4\pi} \log \frac{\mu_R^2}{\Lambda_{\rm QCD}{}^2}}$$

with $\Lambda_{\rm QCD}$ the Landau pole of QCD, $\Lambda_{\rm QCD} \approx 250{\rm MeV}$.

Picture of Hard QCD Interactions

- borrowed from QED: lifetime of electron–photon fluctuations: $e(P) \to e(p) + \gamma(k)$
- estimate: use uncertainty relation and Lorentz time dilation
  - $P^2 = (p+k)^2 = M^2_{\rm virt}$ the virtual mass of the incident electron
  - life time = life time in rest frame · time dilation

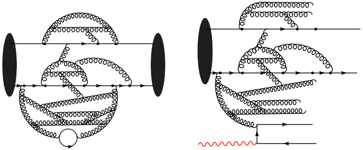
$$\tau \sim \frac{1}{M_{\rm virt}} \cdot \frac{E}{M_{\rm virt}} = \frac{E}{(p+k)^2} \sim \frac{E}{2Ek(1-\cos\theta)} \approx \frac{k}{k^2 \sin^2\theta/2} \approx \frac{\omega}{k_\perp^2}$$

- lifetime larger with smaller transverse momentum

(i.e. with larger transverse distance)

- same pattern also in QCD

- physical interpretation:
  equivalent quanta = quantum manifestation of accompanying fields
- in absence of interaction: recombination enforced by coherence
- but: hard interaction possibly "kicks out" quantum
  $\longrightarrow$ coherence broken
  $\longrightarrow$ equivalent (virtual) quanta become real
  $\longrightarrow$ emission pattern unravels



- alternative idea:
  initial state radiation of photons off incident electron

Hadrons in initial state: DGLAP equations of QCD

- define probabilities (at LO) to find a parton q – quark or gluon – in hadron h at energy fraction x and resolution parameter/scale Q:
  parton distribution function (PDF) $f_{q/h}(x,\, Q^2)$
- scale-evolution of PDFs: DGLAP equations

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} f_{q/h}(x,\, Q^2) \\ f_{g/h}(x,\, Q^2) \end{pmatrix}$$
$$= \frac{\alpha_s(Q^2)}{2\pi}\int\limits_x^1 \frac{\mathrm{d}z}{z}\begin{pmatrix} \mathcal{P}_{qq}\left(\frac{x}{z}\right) & \mathcal{P}_{qg}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gq}\left(\frac{x}{z}\right) & \mathcal{P}_{gg}\left(\frac{x}{z}\right) \end{pmatrix}\begin{pmatrix} f_{q/h}(z,\, Q^2) \\ f_{g/h}(z,\, Q^2) \end{pmatrix},$$

- QCD splitting functions:

$$\mathcal{P}_{qq}^{(1)}(x) \;=\; C_F\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right] \;=\; \left[P_{qq}^{(1)}(x)\right]_+ + \gamma_q^{(1)}\delta(1-x)$$

$$\mathcal{P}_{qg}^{(1)}(x) \;=\; T_R\left[x^2+(1-x)^2\right] \;=\; P_{qg}^{(1)}(x)$$

$$\mathcal{P}_{gq}^{(1)}(x) \;=\; C_F\left[\frac{1+(1-x)^2}{x}\right] \;=\; P_{gq}^{(1)}(x)$$

$$\mathcal{P}_{gg}^{(1)}(x) \;=\; 2C_A\left[\frac{x}{(1-x)_+}+\frac{1-x}{x}+x(1-x)\right] + \frac{11C_A-4n_fT_R}{6}\delta(1-x) \;=\; \left[P_{gg}^{(1)}(x)\right]_+ + \gamma_g^{(1)}\delta(1-x)\,.$$

- remark: IR regularisation by $+$–prescription & terms $\sim\delta(1-x)$ from physical conditions on splitting functions

(flavour conservation for $q\to qg$ and momentum conservation for $g\to gg,\,q\bar{q}$)

Hadron production: Scales

- consider QCD final state radiation
- pattern for $q \to qg$ similar to $\ell \to \ell\gamma$ in QED:

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi}\,C_F\,\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\frac{\mathrm{d}\omega}{\omega}\,\left[1+\left(1-\frac{\omega}{E}\right)^2\right]$$

$$\stackrel{\omega=E(1-z)}{=} \frac{\alpha_s(k_\perp^2)}{2\pi}\,C_F\,\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\mathrm{d}z\,\frac{1+z^2}{1-z} = \frac{\alpha_s(k_\perp^2)}{2\pi}\,C_F\,\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\mathrm{d}z\,P_{qg}^{(1)}(z)\,.$$

- divergent structures for:

$z \to 1$ (soft divergence)	$\longleftrightarrow$	infrared/soft logarithms
$k_\perp^2 \to 0$ (collinear/mass divergence)	$\longleftrightarrow$	collinear logarithms

- cut regularise with cut-off $k_{\perp,\mathrm{min}} \sim 1\mathrm{GeV} > \Lambda_{\mathrm{QCD}}$

- find two perturbative regimes:
  - a regime of jet production, where $k_\perp \sim k_\parallel \sim \omega \gg k_{\perp,\rm min}$ and emission probabilities scale like $w \sim \alpha_s(k_\perp) \ll 1$; and
  - a regime of jet evolution, where $k_{\perp,\rm min} \leq k_\perp \ll k_\parallel \leq \omega$ and therefore emission probabilities scale like $w \sim \alpha_s(k_\perp)\log^2 k_\perp^2 \gtrsim 1$.
- in jet production:

  standard fixed–order perturbation theory
- in jet evolution regime,

  perturbative parameter not α_s any more
  but rather towers of $\exp\left[\alpha_s \log k_\perp^2 \log k_\parallel\right]$
- induces counting of leading logarithms (LL), $\alpha_s L^{2n}$,
  next-to leading logarithms (NLL), $\alpha_s L^{2n-1}$, etc.

PART II: MONTE CARLO

FOR PERTURBATIVE QCD

MONTE CARLO FOR

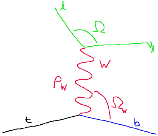
PARTON LEVEL

Contents

Simulating hard processes (signals & backgrounds)

- Simple example: $t \to bW^+ \to b\bar{l}\nu_l$:

$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_W}\right)^2 \frac{p_t\cdot p_\nu \; p_b\cdot p_l}{(p_W^2-M_W^2)^2+\Gamma_W^2 M_W^2}$$



- Phase space integration (5-dim):

$$\Gamma = \frac{1}{2m_t}\frac{1}{128\pi^3}\int \mathrm{d}p_W^2 \frac{\mathrm{d}^2\Omega_W}{4\pi}\frac{\mathrm{d}^2\Omega}{4\pi}\left(1-\frac{p_W^2}{m_t^2}\right)|\mathcal{M}|^2$$

- 5 random numbers $\Longrightarrow$ four-momenta $\Longrightarrow$ "events".
- Apply smearing and/or arbitrary cuts.
- Simply histogram any quantity of interest - no new calculation for each observable

Calculating matrix elements efficiently

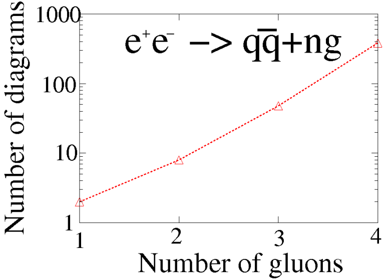
- stating the problem(s):
  - multi-particle final states for signals & backgrounds.
  - need to evaluate $\mathrm{d}\sigma_N$:

$$\int\limits_{\mathrm{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i q_i\right) \left|\mathcal{M}_{p_1 p_2 \to N}\right|^2 .$$

  - problem 1: factorial growth of number of amplitudes.
  - problem 2: complicated phase-space structure.
  - solutions: numerical methods.

- example for factorial growth: $e^+e^- \to q\bar{q} + ng$

n	$\#_{\text{diags}}$
0	1
1	2
2	8
3	48
4	384



- obvious: traditional textbook methods (squaring, completeness relations, traces) fail
  $\Longrightarrow$ result in proliferation of terms $(\mathcal{M}_i\mathcal{M}_j^*)$
- better ideas of efficient ME calculation:
  $\Longrightarrow$ realise: amplitudes just are complex numbers,
  $\Longrightarrow$ add them before squaring!
- remember: spinors, gamma matrices have explicit form
  could be evaluated numerically (brute force)
  but: Rough method, lack of elegance, CPU-expensive
- can do better with smart basis for spinors (see detour)
- this is still on the base of traditional Feynman diagrams!

Phase spacing for professionals

("Amateurs study strategy, professionals study logistics")

- democratic, process-blind integration methods:
  - Rambo/Mambo: Flat & isotropic

    R.Kleiss, W.J.Stirling & S.D.Ellis, Comput. Phys. Commun. **40** (1986) 359;
  - HAAG/Sarge: Follows QCD antenna pattern

    A.van Hameren & C.G.Papadopoulos, Eur. Phys. J. C **25** (2002) 563.
- multi-channelling: each Feynman diagram related to a phase space mapping (= "channel"), optimise their relative weights

  R.Kleiss & R.Pittau, Comput. Phys. Commun. **83** (1994) 141.
- main problem: practical only up to $\mathcal{O}(10\mathrm{k})$ channels.
- some improvement by building phase space mappings recursively: more channels feasible, efficiency drops a bit.

basic idea of multichannel sampling (again):
use a sum of functions $g_i(\vec{x})$ as Jacobean $g(\vec{x})$.

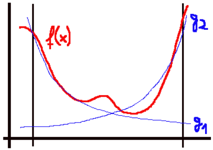
$\Longrightarrow \quad g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x})$;

$\Longrightarrow$ condition on weights like stratified sampling;

("combination" of importance & stratified sampling).

algorithm for one iteration:

- select g_i with probability $\alpha_i \to \vec{x}_j$.
- calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$
- add $f(\vec{x}_j)/g(\vec{x}_j)$ to total result and $f(\vec{x}_j)/g_i(\vec{x}_j)$ to partial (channel-) results.
- after N sampling steps, update a-priori weights.

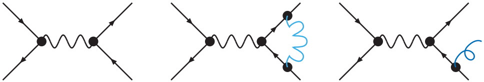


this is the method of choice for parton level event generation!

- quality measure for integration performance: unweighting efficiency
- want to generate events "as in nature".
- basic idea: use hit-or-miss method;
  - generate $\vec{x}$ with integration method,
  - compare actual $f(\vec{x})$ with maximal value during sampling $\Longrightarrow$ "Unweighted events".
- comments:
  - unweighting efficiency, $w_{\rm eff} = \langle f(\vec{x}_j)/f_{\rm max}\rangle$ = number of trials for each event.
  - expect $\log_{10} w_{\rm eff} \approx 3 - 5$ for good integration of multi-particle final states at tree-level.
  - maybe acceptable to use $f_{\rm max,eff} = Kf_{\rm max}$ with $K < 1$.
    problem: what to do with events where $f(\vec{x}_j)/f_{\rm max,eff} > 1$?
    answer: Add $\mathrm{int}[f(\vec{x}_j)/f_{\rm max,eff}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\rm max,eff} - k$.

Including higher order corrections

- obtained from adding diagrams with additional:
  loops (virtual corrections) or
  legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F
  NLO allows for meaningful estimate of uncertainties
- additional difficulties when going NLO:
  ultraviolet divergences in virtual correction
  infrared divergences in real and virtual correction

  enforce
  UV regularisation & renormalisation
  IR regularisation & cancellation

(Kinoshita–Lee–Nauenberg–Theorem)

Structure of NLO calculations

$$\mathrm{d}\sigma = \overbrace{\mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})}^{\text{Born term}} + \overbrace{\mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_N(\Phi_{\mathcal{B}})}^{\text{virtual correction (renormalized)}} + \overbrace{\mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_N(\Phi_{\mathcal{R}})}^{\text{real correction}}$$

$$= \underbrace{\mathrm{d}\Phi_{\mathcal{B}}\left(\mathcal{B}_N + \mathcal{V}_N + \mathcal{I}_N^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\left(\mathcal{R}_N - \mathcal{S}_N\right)}_{\text{after adding a zero: } \mathcal{I}_N^{(\mathcal{S})} = \int \mathrm{d}\Phi_{\mathcal{R}/\mathcal{B}}\mathcal{S}_N}$$

- phase space factorisation assumed here ($\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1$)

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) = \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

- process independent, universal subtraction kernels

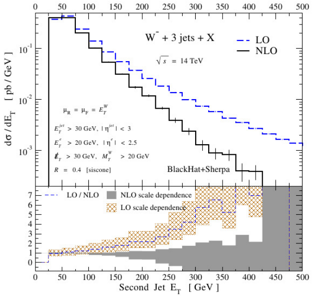
$$\begin{aligned} \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1) \\ \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}}), \end{aligned}$$

  and invertible phase space mapping (e.g. Catani-Seymour)

$$\Phi_{\mathcal{R}} \longleftrightarrow \Phi_{\mathcal{B}} \otimes \Phi_1$$

Aside: choices . . .

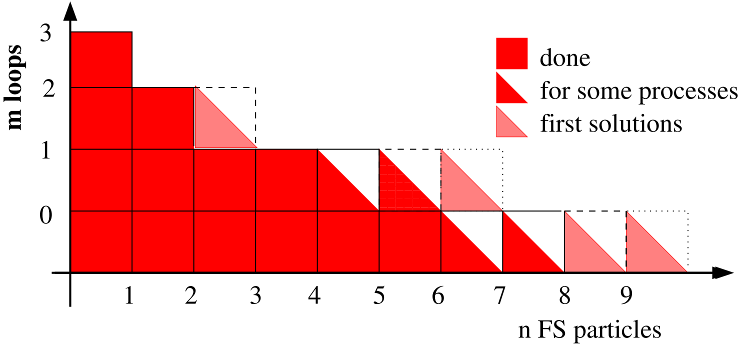
- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:
  unphysical scale choices will yield unphysical results



- more ways of botching it at higher orders

Availability of exact calculations (hadron colliders)

- fixed order matrix elements ("parton level") are exact to a given perturbative order. (and often quite a pain!)
- important to understand limitations:
  only tree-level and one-loop level fully automated, beyond: prototyping



Survey of existing parton-level tools @ tree–level

	Models	$2 \longrightarrow n$	Ampl.	Integ.	public?	lang.
ALPGEN	SM	$n = 8$	rec.	Multi	yes	Fortran
AMEGIC++	SM, UFO	$n = 6$	hel.	Multi	yes	C++
COMIX	SM, UFO	$n = 8$	rec.	Multi	yes	C++
COMPHEP	SM, LANHEP	$n = 4$	trace	1Channel	yes	C
HELAC	SM	$n = 8$	rec.	Multi	yes	Fortran
MADEVENT	SM, UFO	$n = 6$	hel.	Multi	yes	Python/Fortran
WHIZARD	SM, UFO	$n = 8$	rec.	Multi	yes	O'Caml

Survey of existing parton-level tools @ NLO

	type	technology dependencies on other codes
LoopTools	integrals	
OneLoop	integrals	
QCDLoop	integrals	
Collier	reduction	
CutTools	reduction	OPP
FormCalc	reduction	PV
Ninja	reduction	Laurent expansion
Samurai	reduction	
BlackHat	library (amplitudes)	OPP (unitarity)
McFm	library (full calculation)	PV & OPP
MJet	library (amplitudes)	OPP
GoSam	generator (amplitudes)	OPP Samurai +Ninja +
MadLoop	generator (full calculation)	OL+OPP CutTools +
OpenLoops	generator (amplitudes)	OL+OPP Collier +CutTools +
Recola	generator (amplitudes)	TR Collier +CutTools +
Helac-NLO	generator (full calculation)	OPP CutTools +

GOING MONTE CARLO

PARTON SHOWERS – THE BASICS

Contents

An analogy: Radioactive decays

- consider radioactive decay of an unstable isotope with half-life τ.

(and ignore factors of ln 2.)

- "survival" probability after time t is given by

$$\mathcal{S}(t) = \mathcal{P}_{\rm nodec}(t) = \exp[-t/\tau]$$

(note "unitarity relation": $\mathcal{P}_{\rm dec}(t) = 1 - \mathcal{P}_{\rm nodec}(t)$.)

- probability for an isotope to decay at time t:

$$\frac{{\rm d}\mathcal{P}_{\rm dec}(t)}{{\rm d}t} = -\frac{{\rm d}\mathcal{P}_{\rm nodec}(t)}{{\rm d}t} = \frac{1}{\tau}\exp(-t/\tau)$$

- now: connect half-life with width $\Gamma = 1/\tau$.
- probability for isotope decay at any fixed time t determined by Γ.

- spice things up now: add time–dependence, $\Gamma = \Gamma(t')$
- rewrite

$$\Gamma t \longrightarrow \int_0^t \mathrm{d}t' \Gamma$$

- decay-probability at a given time t is given by

$$\frac{\mathrm{d}\mathcal{P}_{\rm dec}(t)}{\mathrm{d}t} = \Gamma(t) \exp\left[-\int_0^t \mathrm{d}t' \Gamma(t')\right] = \Gamma(t)\,\mathcal{P}_{\rm nodec}(t)$$

(unitarity strikes again: $\mathrm{d}\mathcal{P}_{\rm dec}(t)/\mathrm{d}t = -\mathrm{d}\mathcal{P}_{\rm nodec}(t)/\mathrm{d}t$.)

- interpretation of l.h.s.:
  - first term is for the actual decay to happen.
  - second term is to ensure that no decay before t
    $\Longrightarrow$ conservation of probabilities.
    the exponential is - of course - called the Sudakov form factor.

The pattern of QCD radiation

- a detour: Altarelli-Parisi equation, once more
- AP describes the scaling behaviour of the parton distribution function

(which depends on Bjorken-parameter and scale Q^2)

$$\frac{\mathrm{d}q(x,\,Q^2)}{\mathrm{d}\ln Q^2} = \int\limits_x^1 \frac{\mathrm{d}y}{y}\left[\alpha_s(Q^2)P_q(x/y)\right]q(y,Q^2)$$

- term in square brackets determines the probability that the parton emits another parton at scale Q^2 and Bjorken-parameter y

(after the splitting, $x \to yx + (1-y)x$.)

- driving term: Splitting function $P_q(x)$
  important property: universal, process independent

Rederiving the splitting functions

- differential cross section for gluon emission in $e^+e^- \to$ jets

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

singular for $x_{1,2} \to 1$.

- rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\mathrm{c.m.}}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

singular for $x_3 \to 0$ ("soft"), $\sin\theta_{qg} \to 0$ ("collinear").

- re-express collinear singularities

$$\begin{aligned}\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} &= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}}\\ &= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}\end{aligned}$$

- independent evolution of two jets (q and $\bar{q}$)

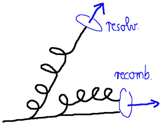
$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z)\,,$$

- note: same form for any $t \propto \theta^2$:
- transverse momentum $k_\perp^2 \approx z^2(1-z)^2E^2\theta^2$
- invariant mass $q^2 \approx z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

- parametrisation-independent observation:
  (logarithmically) divergent expression for $t \to 0$.
- practical solution: cut-off Q_0^2.
  $\Longrightarrow$ divergence will manifest itself as $\log Q_0^2$.
- similar for $P(z)$: divergence for $z \to 0$ cured by cut-off.

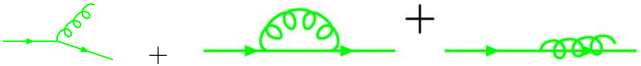
- what is a parton?
  collinear pair/soft parton recombine!
- introduce resolution criterion $k_\perp > Q_0$.



- combine virtual contributions with unresolvable emissions:
  cancels infrared divergences $\Longrightarrow$ finite at $\mathcal{O}(\alpha_s)$

  (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- unitarity: probabilities add up to one
  $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1.$



$=1.$

- the Sudakov form factor, once more
- differential probability for emission between q^2 and $q^2 + \mathrm{d}q^2$:

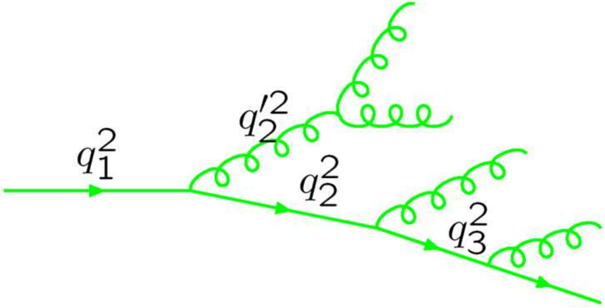
$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int\limits_{z_{\min}}^{z_{\max}}\mathrm{d}zP(z) =: \mathrm{d}q^2\,\Gamma(q^2)$$

- from radioactive example: evolution equation for Δ

$$-\frac{\mathrm{d}\Delta(Q^2,\, q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\, q^2)\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}q^2} = \Delta(Q^2,\, q^2)\Gamma(q^2)$$

$$\Longrightarrow \Delta(Q^2,\, q^2) = \exp\left[-\int\limits_{q^2}^{Q^2}\mathrm{d}k^2\Gamma(k^2)\right]$$

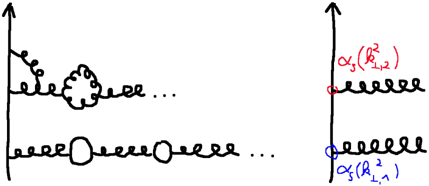
- maximal logs if emissions ordered
- impacts on radiation pattern: in each emission t becomes smaller



$$q_1^2 > q_2^2 > q_3^2,\; q_1^2 > {q_2'}^2$$

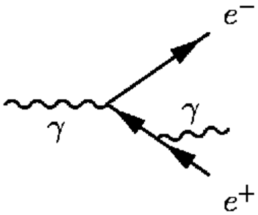
Quantum improvements

- improvement: inclusion of various quantum effects
- trivial: effect of summing up higher orders (loops) $\alpha_s \to \alpha_s(k_\perp^2)$



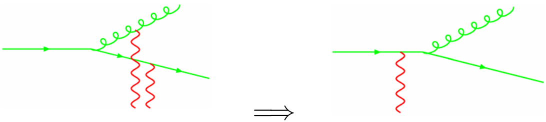
- much faster parton proliferation, especially for small $k_\perp^2$.
- avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda^2_{\rm QCD} \Longrightarrow Q_0^2 =$ physical parameter.

- soft limit for single emission also universal
- problem: soft gluons come from all over (not collinear!) quantum interference? still independent evolution?
- answer: not quite independent.
- consider case in QED



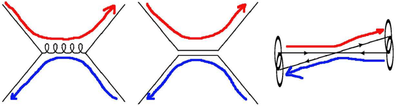
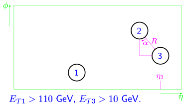
- assume photon into e^+e^- at θ_{ee} and photon off electron at θ photon momentum denoted as k
- energy imbalance at vertex: $k_\perp^\gamma \sim k_\parallel\theta$, hence $\Delta E \sim k_\perp^2/k_\parallel \sim k_\parallel\theta^2$.
- formation time for photon emission: $\Delta t \sim 1/\Delta E \sim k_\parallel/k_\perp^2 \sim 1/(k_\parallel\theta^2)$.
- ee-separation: $\Delta b \sim \theta_{ee}\Delta t$
- must be larger than transverse wavelength of photon: $\theta_{ee}/(k_\parallel\theta^2) > 1/k_\perp = 1/(k_\parallel\theta)$
- thus: $\theta_{ee} > \theta$ must be satisfied for photon to form
- angular ordering as manifestation of quantum coherence

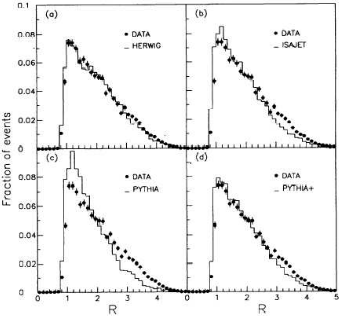
- pictorially:

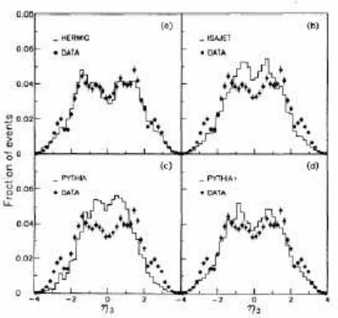


gluons at large angle from combined colour charge!

- experimental manifestation:
  ΔR of $2^{\rm nd}$ & $3^{\rm rd}$ jet in multi-jet events in pp-collisions





Parton showers, compact notation

- Sudakov form factor (**no-decay** probability)

$$\Delta_{ij,k}^{(\mathcal{K})}(t,t_0) = \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\frac{\alpha_s}{2\pi}\int \mathrm{d}z\frac{\mathrm{d}\phi}{2\pi}\underbrace{\mathcal{K}_{ij,k}(t,z,\phi)}_{\text{splitting kernel for } (ij)\to ij \text{ (spectator } k)}\right]$$

- evolution parameter t defined by kinematics

  generalised angle (HERWIG ++) or transverse momentum (PYTHIA, SHERPA)

- will replace $\frac{\mathrm{d}t}{t}\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi} \longrightarrow \mathrm{d}\Phi_1$
- scale choice for strong coupling: $\alpha_s(k_\perp^2)$ — resums classes of higher logarithms
- regularisation through cut-off t_0

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_s}{2\pi} \sum_{\text{all }\{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}), \quad \Delta_n^{(\mathcal{K})}(t,t_0) = \exp\left[-\int\limits_{t_0}^{t} \mathrm{d}\Phi_1\, \mathcal{K}_n(\Phi_1)\right]$$

- consider first emission only off Born configuration

$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\, \mathcal{B}_N(\Phi_N) \cdot \underbrace{\left\{\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1))\right]\right\}}_{\text{integrates to unity} \longrightarrow \text{"unitarity" of parton shower}}$$

- further emissions by recursion with $Q^2 = t$ of previous emission

Connection to resummation

- consider standard Collins-Soper-Sterman Q_T-formalism (CSS):

$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}y\mathrm{d}Q_\perp^2} = \mathrm{d}\Phi_X\,\mathcal{B}_{ij}(\Phi_X)\cdot\underbrace{\int\frac{\mathrm{d}^2b_\perp}{(2\pi)^2}\exp(i\vec{b}_\perp\cdot\vec{Q}_\perp)}_{\text{guarantee 4-mom conservation}}\underbrace{\tilde{W}_{ij}(b;\,\Phi_X)}_{\text{higher orders}}$$

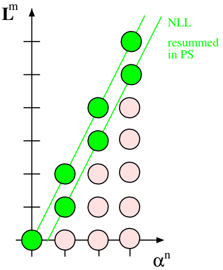
with

$$\tilde{W}_{ij}(b;\,\Phi_X) = \overbrace{C_i(b;\,\Phi_X,\,\alpha_s)C_j(b;\,\Phi_X,\,\alpha_s)}^{\text{collinear bits}}\overbrace{H_{ij}(\alpha_s)}^{\text{loops}}$$

$$\underbrace{\exp\left[-\int\limits_{1/b_\perp^2}^{Q_X^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A(\alpha_s(k_\perp^2))\log\frac{Q_X^2}{k_\perp^2}+B(\alpha_s(k_\perp^2))\right)\right]}_{\text{Sudakov form factor, }A,\,B\text{ expanded in powers of }\alpha_s}$$

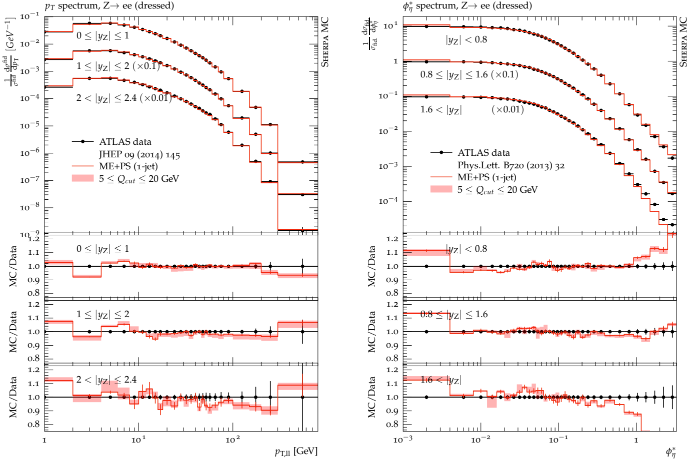
- analyse structure of emissions above
- logarithmic accuracy in $\log \frac{\mu_N}{k_\perp}$ (a la CSS) possibly up to next-to leading log,
  - if evolution parameter $\sim$ transverse momentum,
  - if argument in α_s is $\propto k_\perp$ of splitting,
  - if $K_{ij,k} \to$ terms $A_{1,2}$ and B_1 upon integration

  (OK, if soft gluon correction is included, and if $K_{ij,k} \to$ AP splitting kernels)



- in CSS $k_\perp$ typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions
- resummation scale $\mu_N \approx \mu_F$ given by (Born) kinematics – simple for cases like $q\bar{q}' \to V$, $gg \to H$, ... tricky for more complicated cases

Example: achievable precision of shower alone in DY



Another systematic uncertainty

- parton showers are approximations, based on leading colour, leading logarithmic accuracy, spin-averaged
- parametric accuracy by comparing Sudakov form factors:

$$\Delta = \exp\left\{-\int \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\left[A\log\frac{k_\perp^2}{Q^2}+B\right]\right\}\,,$$

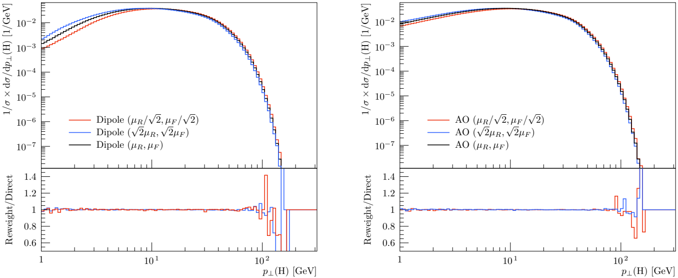
  where A and B can be expanded in $\alpha_s(k_\perp^2)$
- showers usually include terms $A_{1,2}$ and B_1 (NLL)
- A_2 realised by pre-factor multiplying scale $\mu_R \simeq k_\perp$

  (CMW rescaling: Catani, Marchesini, Webber, Nucl Phys B,349 635)
- fixed-order precision necessitates to consistently assess uncertainties from parton showers (quite often just used as black box)
- maybe improve by including higher orders?

Event generation (on-the-fly scale variations)

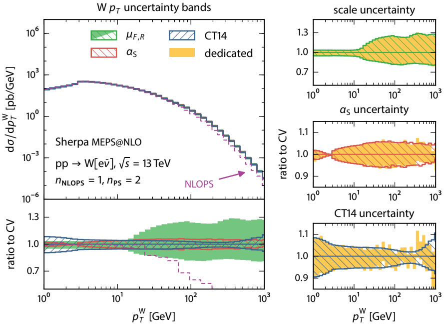
- basic idea: want to vary scales to assess uncertainties
- simple reweighting in matrix elements straightforward
- reweighting in parton shower more cumbersome
    - shower is probabilistic, concept of weight somewhat alien
    - introduce relative weight
    - evaluate (trial-)emission by (trial-)emission

Implementation in HERWIG7

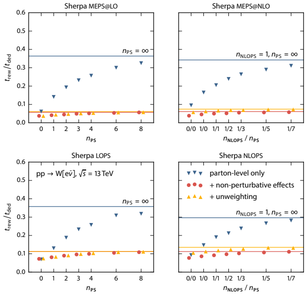


Weight variation for W+jets with MEPS@NLO

- uncertainties in $p_\perp^W$



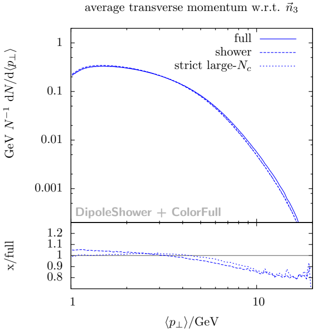
- CPU budget



Going beyond leading colour

- start including next-to leading colour

(first attempts by Platzer & Sjodahl; Nagy & Soper)



- also included in 1st emission in SHERPA's MC@NLO

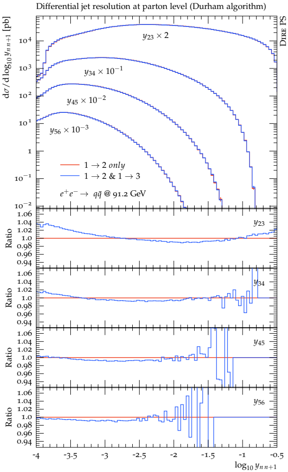
Towards higher logarithmic accuracy

- reproduce DGLAP evolution at NLO include all NLO splitting kernels
- corrections to standard $1 \to 2$ trivial
  - 2-loop cusp term subtracted & combined with LO soft contribution
  - use weighting algorithms

(Hoeche, Schumann, Siegert, 0912.3501)

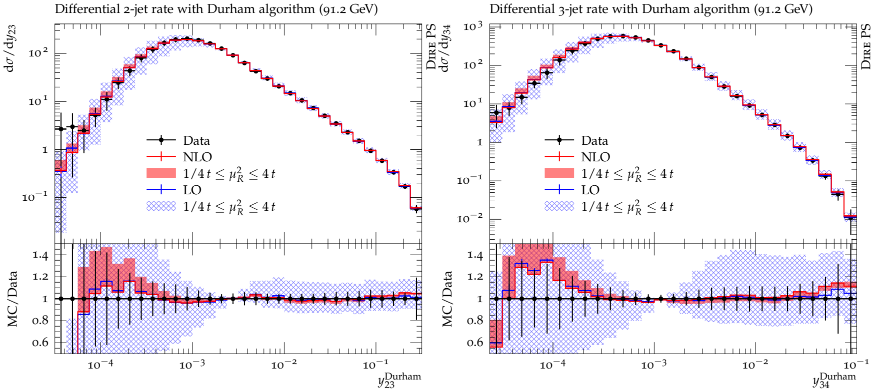
- new topology at NLO from $q \to \bar{q}$ and $q \to q'$ splittings
- generic $1 \to 3$ process in parton shower
- implementation complete and cross-checked (PYTHIA vs. SHERPA)

(Hoeche, FK, Prestel)



Comparison with data: $e^-e^+ \to$ hadrons

(Hoeche, FK & Prestel, 1705.00982)



Comparison with data: $e^- e^+ \to$ hadrons

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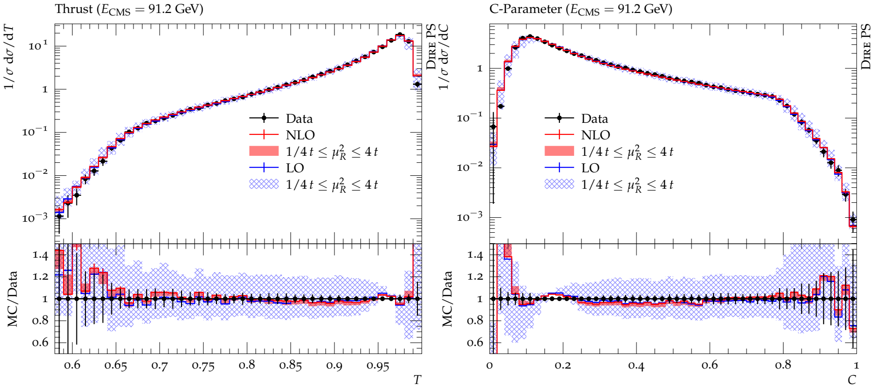
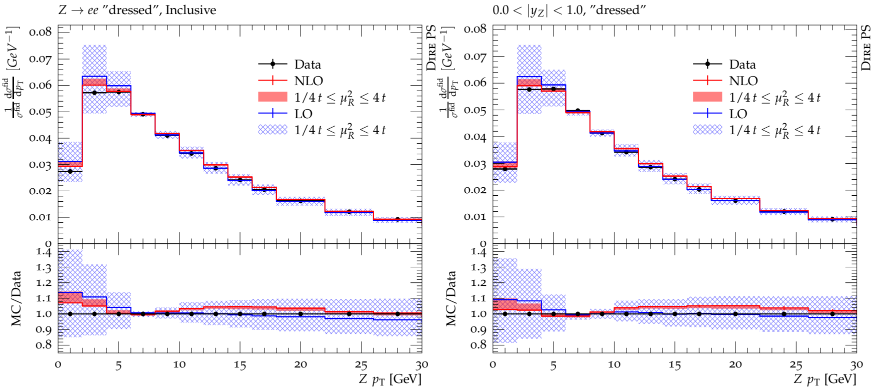


Comparison with data: DY at LHC

(Hoeche, FK & Prestel, 1705.00982)



ROUND III: PRECISION MONTE CARLO

FIRST IMPROVEMENTS:

ME CORRECTIONS

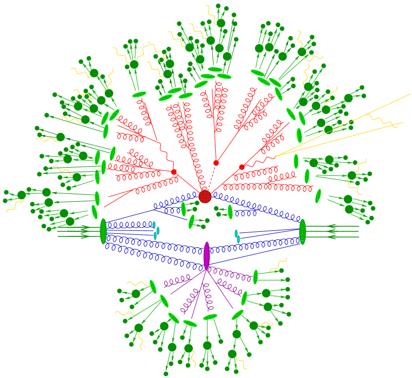
Contents

Improving event generators

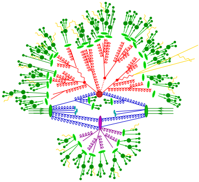
The inner working of event generators

. . . simulation: divide et impera

- hard process:
  fixed order perturbation theory
  traditionally: Born-approximation
- bremsstrahlung:
  resummed perturbation theory
- hadronisation:
  phenomenological models
- hadron decays:
  effective theories, data
- "underlying event":
  phenomenological models



... and possible improvements
possible strategies:



- improving the phenomenological models:
  - “tuning” (fitting parameters to data)
  - replacing by better models, based on more physics

    (my hot candidate: “minimum bias” and “underlying event” simulation)
- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

    “NLO-Matching” & “Multijet-Merging”
  - systematic improvement of the parton shower: next-to leading (or higher) logs & colours

- remember structure of NLO calculation for N-body production

$$\begin{aligned} \mathrm{d}\sigma &= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_N(\Phi_{\mathcal{R}}) \\ &= \mathrm{d}\Phi_{\mathcal{B}}\left(\mathcal{B}_N + \mathcal{V}_N + \mathcal{I}_N^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\left(\mathcal{R}_N - \mathcal{S}_N\right) \end{aligned}$$

- phase space factorisation assumed here ($\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1$)

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) = \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

- process independent subtraction kernels

$$\begin{aligned} \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1) \\ \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}}) \end{aligned}$$

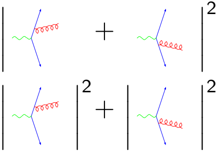
with universal $\mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1)$ and $\mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})$

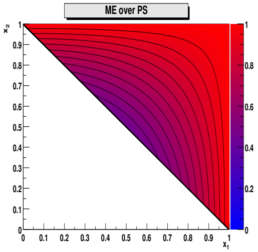
Matrix element corrections

- parton shower ignores interferences typically present in matrix elements
- pictorially

ME :

PS :



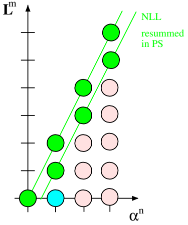


- form many processes $\mathcal{R}_N < \mathcal{B}_N \times \mathcal{K}_N$
- typical processes: $q\bar{q}' \to V$, $e^-e^+ \to q\bar{q}$, $t \to bW$
- practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $\mathcal{P} = \mathcal{R}_N/(\mathcal{B}_N \times \mathcal{K}_N)$

- analyse **first** emission, given by

$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\,\mathcal{B}_N(\Phi_N) \cdot \underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\frac{\mathcal{R}_N(\Phi_N\times\Phi_1)}{\mathcal{B}_N(\Phi_N)}\,\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t(\Phi_1))\right]\right\}}_{\text{once more: integrates to unity} \longrightarrow \text{“unitarity” of parton shower}}$$

- radiation given by $\mathcal{R}_N$ (correct at $\mathcal{O}(\alpha_s)$)
  (but modified by logs of higher order in α_s from $\Delta_N^{(\mathcal{R}/\mathcal{B})}$)
- emission phase space constrained by μ_N
- also known as “soft ME correction”
  hard ME correction fills missing phase space
- used for “power shower”:
  $\mu_N \to E_{pp}$ and apply ME correction



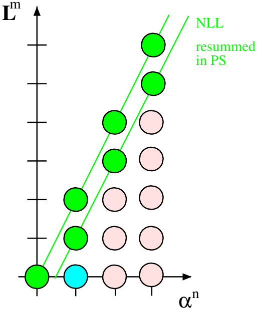
PRECISION MONTE CARLO

(N)NLO MATCHING

Contents

NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- adjust ("match") terms:
  - cross section at NLO accuracy &
    correct hardest emission in PS to exactly
    reproduce ME at order α_s
    ($\mathcal{R}$-part of the NLO calculation)
    (this is relatively trivial)
  - maintain (N)LL-accuracy of parton shower
    (this is not so simple to see)



PowHeg

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\;\xrightarrow{\text{IR}}\;\mathrm{d}\Phi_1\,\frac{\alpha_s}{2\pi}\,\mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) = \exp\left[-\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\right],$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_s to parton shower scale

- define local K-factors
- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$\begin{aligned} d\sigma_N^{(\mathrm{NLO})} &= d\Phi_N\, \bar{\mathcal{B}}(\Phi_N) \\ &= d\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N) \otimes \mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)} \right. \\ &\qquad \left. + \int d\Phi_1 \left[\mathcal{R}_N(\Phi_N \otimes \Phi_1) - \mathcal{B}_N(\Phi_N) \otimes d\mathcal{S}(\Phi_1) \right] \right\} \end{aligned}$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes d\mathcal{S}$

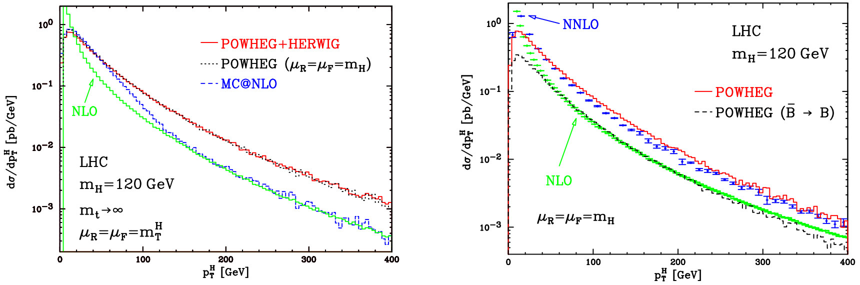
(relevant for MC@NLO)

- analyse accuracy of radiation pattern
- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0)$:

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\, \bar{\mathcal{B}}(\Phi_N)$$
$$\times \underbrace{\left\{ \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_N \otimes \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, k_\perp^2(\Phi_1)) \right\}}_{\text{integrating to yield 1 - ``unitarity of parton shower''}}$$

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2 (this is vanilla POWHEG!)
- apart from logs: which configurations enhanced by local K-factor

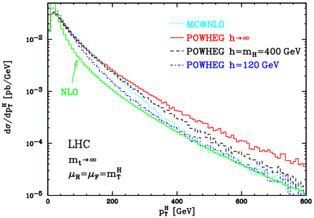
(K-factor for inclusive production of X adequate for $X+$ jet at large $p_\perp$?)



- large enhancement at high $p_{T,h}$
- can be traced back to large NLO correction
- fortunately, NNLO correction is also large $\to\sim$ agreement

- improving PowHeg
- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(F)}}\right)$$



- can "tune" h to mimick NNLO - or other (resummation) result
  - differential event rate up to first emission

$$\begin{aligned}\mathrm{d}\sigma \quad &= \quad \mathrm{d}\Phi_B \bar{\mathcal{B}}^{(\mathrm{R}^{(\mathrm{S})})}\left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, t_0) + \int\limits_{t_0}^{s} \mathrm{d}\Phi_1 \frac{\mathcal{R}^{(S)}}{\mathcal{B}}\, \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, k_\perp^2)\right] \\ &\quad + \mathrm{d}\Phi_R\, \mathcal{R}^{(F)}(\Phi_R)\end{aligned}$$

MC@NLO

- MC@NLO paradigm: divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}_1 + \mathcal{H}_N$$

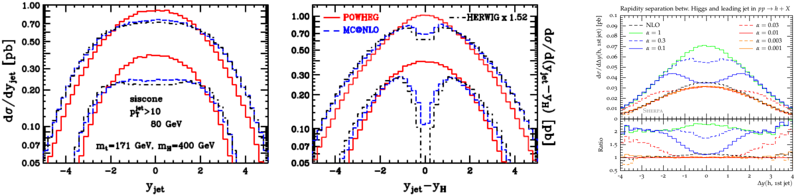
- identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum\limits_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1st emission to account for colour)

$$\begin{aligned}\mathrm{d}\sigma_N \;=\;& \mathrm{d}\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}}\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1\, \mathcal{K}_{ij,k}(\Phi_1)\, \Delta_N^{(\mathcal{K})}(\mu_N^2,\, k_\perp^2)\right] \\ &+ \mathrm{d}\Phi_{N+1}\, \mathcal{H}_N\end{aligned}$$

- effect: only resummed parts modified with local K-factor

- phase space effects: shower vs. fixed order



- problem: impact of subtraction terms on local K-factor (filling of phase space by parton shower)
- studied in case of $gg \to H$ above
- proper filling of available phase space by parton shower paramount

NNLOPS in the MINLO approach: merging without Q_J

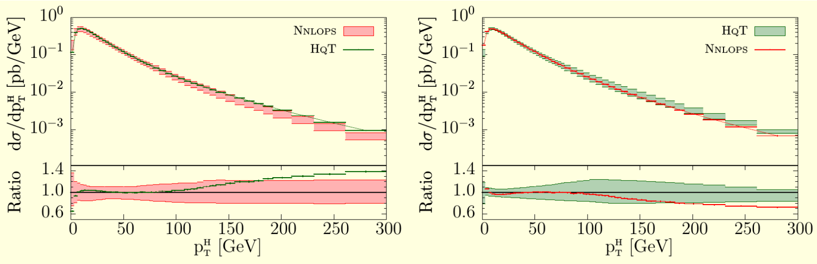
(K.Hamilton, P.Nason, C.Oleari & G.Zanderighi, JHEP 1305 (2013) 082)

- based on POWHEG + shower from PYTHIA or HERWIG
- up to today only for singlet S production, gives NNLO + PS
- basic idea:
  - use S+jet in POWHEG
  - push jet cut to parton shower IR cutoff
  - apply analytical NNLL Sudakov rejection weight for intrinsic line in Born configuration

    (kills divergent behaviour at order α_S)
  - don't forget double-counted terms
  - reweight to NNLO fixed order

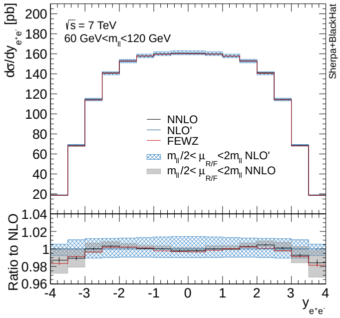
NNLOPS for H production

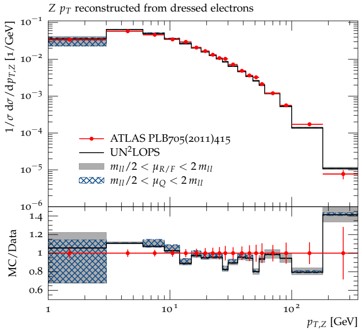
(K.Hamilton, P.Nason, E.Re & G.Zanderighi, JHEP 1310 (2013) 222)



NNLOPS for Z production: UNNLOPS

S. Hoche, Y. Li, & S. Prestel, Phys.Rev.D90 & D91





- also available for H production

NNLOPS: shortcomings/limitations

- MINLO relies on knowledge of B_2 terms from analytic resummation
  $\longrightarrow$ to date only known for colour singlet production
- MINLO relies on reweighting with full NNLO result
  $\longrightarrow$ one parameter for H (y_H), more complicated for Z, ...

- UNNLOPS relies on integrating single- and double emission to low scales and combination of unresolved with virtual emissions
  $\longrightarrow$ potential efficiency issues, need NNLO subtraction
- UNNLOPS puts unresolved & virtuals in "zero-emission" bin
  $\longrightarrow$ no parton showering for virtuals (?)

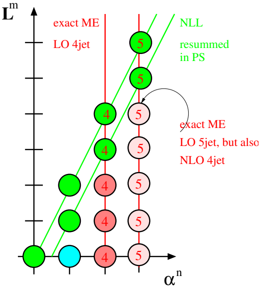
PRECISION MONTE CARLO

MULTIJET MERGING

Contents

Multijet merging: basic idea

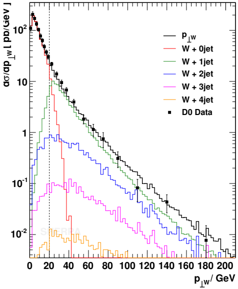
- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- combine ("merge") both:
  result: "towers" of MEs with increasing number of jets evolved with PS
  - multijet cross sections at Born accuracy
  - maintain (N)LL accuracy of parton shower



- separate regions of jet production and jet evolution with jet measure Q_J

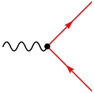
  ("truncated showering" if not identical with evolution parameter)

- matrix elements populate hard regime
- parton showers populate soft domain

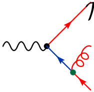


Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \to$ hadrons
  Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor α_S and up to $\log^2 \frac{E_{\rm c.m.}}{Q_J}$ per jet
- use Sudakov form factor for resummation &
  replace approximate fixed order by exact expression:



$$\mathcal{R}_2(Q_J) = \left[\Delta_q(E^2_{\rm c.m.},\, Q_J^2)\right]^2$$



$$\mathcal{R}_3(Q_J) = 2\Delta_q(E^2_{\rm c.m.},\, Q_J^2) \int\limits_{Q_J^2}^{E^2_{\rm c.m.}} \frac{{\rm d}k_\perp^2}{k_\perp^2} \left[\frac{\alpha_s(k_\perp^2)}{2\pi}{\rm d}z\mathcal{K}_q(k_\perp^2, z) \right.$$
$$\left. \times\Delta_q(E^2_{\rm c.m.},\, k_\perp^2)\Delta_q(k_\perp^2,\, Q_J^2)\Delta_g(k_\perp^2,\, Q_J^2)\right]$$

Multijet merging at LO

- expression for first emission

$$\begin{aligned} d\sigma \;=\; & d\Phi_N\,\mathcal{B}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_0)\right. \\ & \left. + \int\limits_{t_0}^{\mu_N^2} d\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_{N+1})\Theta(Q_J - Q_{N+1})\right] \\ & + d\Phi_{N+1}\,\mathcal{B}_{N+1}\,\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2,\,t_{N+1})\Theta(Q_{N+1} - Q_J) \end{aligned}$$

- note: $N+1$-contribution includes also $N+2$, $N+3$, ...

(no Sudakov suppression below t_{n+1}, see further slides for iterated expression)

- potential occurence of different shower start scales: $\mu_{N,N+1,\dots}$
- "unitarity violation" in square bracket: $\mathcal{B}_N\mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &

S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])

$$\mathrm{d}\sigma = \sum_{n=N}^{n_{\max}-1}\left\{\mathrm{d}\Phi_n\,\mathcal{B}_n\,\overbrace{\left[\prod_{j=N}^{n-1}\Theta(Q_{j+1}-Q_J)\right]}^{(n-N)\text{ extra jets}}\overbrace{\left[\prod_{j=N}^{n-1}\Delta_j^{(\mathcal{K})}(t_j,\,t_{j+1})\right]}^{\text{no emissions off internal lines}}\right.$$

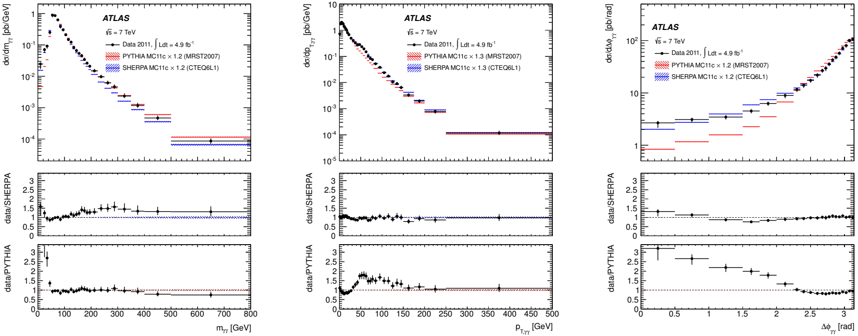
$$\times\left[\underbrace{\Delta_n^{(\mathcal{K})}(t_n,t_0)}_{\text{no emission}}+\underbrace{\int\limits_{t_0}^{t_n}\mathrm{d}\Phi_1\,\mathcal{K}_n\Delta_n^{(\mathcal{K})}(t_n,t_{n+1})\Theta(Q_J-Q_{n+1})}_{\text{next emission no jet \& below last ME emission}}\right]$$

$$+\mathrm{d}\Phi_{n_{\max}}\,\mathcal{B}_{n_{\max}}\left[\prod_{j=N}^{n_{\max}-1}\Theta(Q_{j+1}-Q_J)\right]\left[\prod_{j=N}^{n_{\max}-1}\Delta_j^{(\mathcal{K})}(t_j,\,t_{j+1})\right]$$

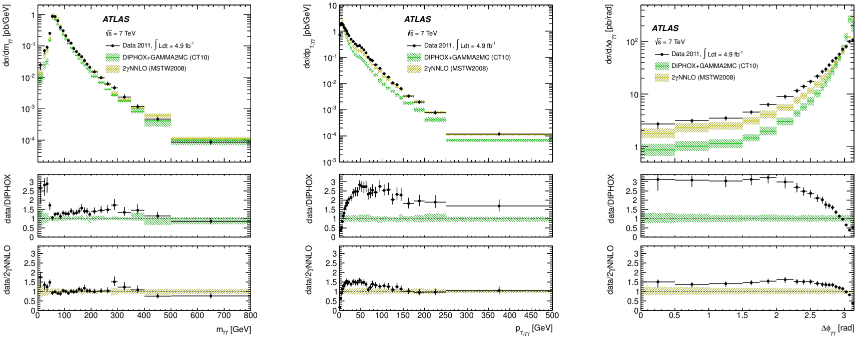
$$\times\left[\Delta_{n_{\max}}^{(\mathcal{K})}(t_{n_{\max}},t_0)+\int\limits_{t_0}^{t_{n_{\max}}}\mathrm{d}\Phi_1\,\mathcal{K}_{n_{\max}}\Delta_{n_{\max}}^{(\mathcal{K})}(t_{n_{\max}},t_{n_{\max}+1})\right]$$

Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])



Aside: Comparison with higher order calculations



Multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

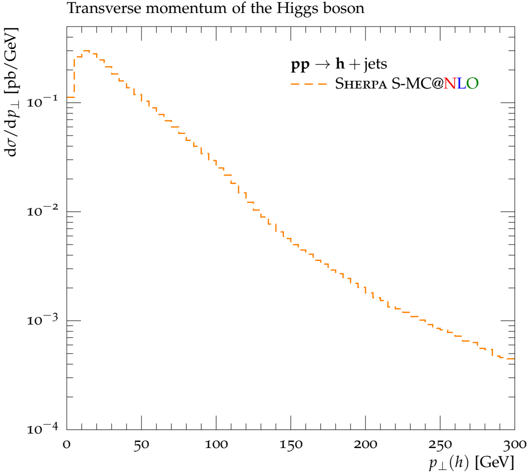
maintain NLO and (N)LL accuracy of ME and PS

- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities

First emission(s), once more

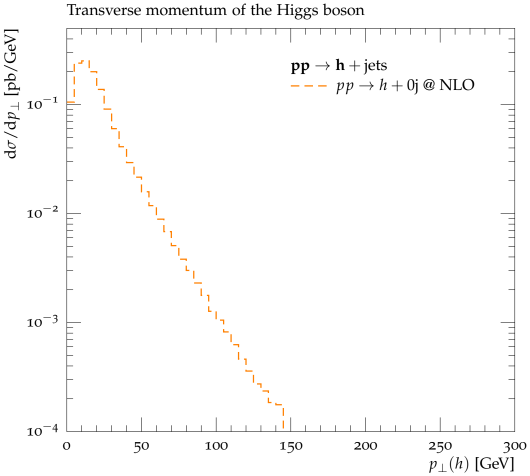
$$\begin{aligned}
\mathrm{d}\sigma \;=\; & \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right]\\
& +\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\\
& +\mathrm{d}\Phi_{N+1}\,\tilde{\mathcal{B}}_{N+1}\left(1+\frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int_{t_{N+1}}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\right)\Theta(Q_{N+1}-Q_J)\\
& \quad\cdot\left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_0)+\int_{t_0}^{t_{N+1}}\mathrm{d}\Phi_1\,\mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\right]\\
& +\mathrm{d}\Phi_{N+2}\,\mathcal{H}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\Theta(Q_{N+1}-Q_J)+\ldots
\end{aligned}$$

$p_\perp^H$ in MEPS@NLO



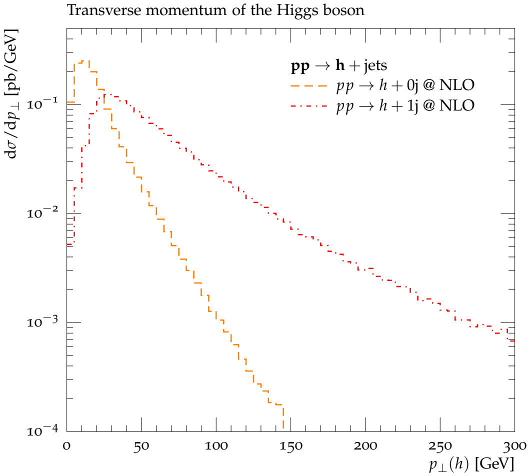
- first emission by MC@NLO

$p_\perp^H$ in MEPS@NLO



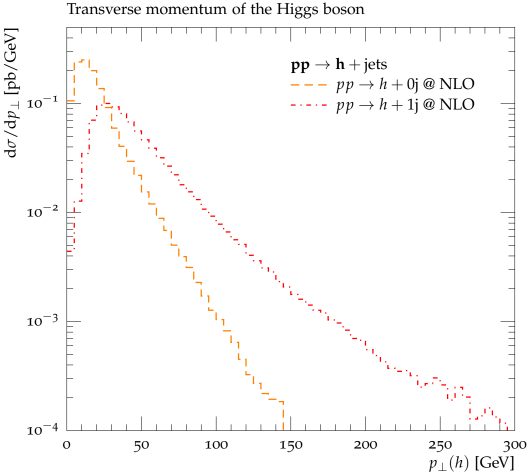
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



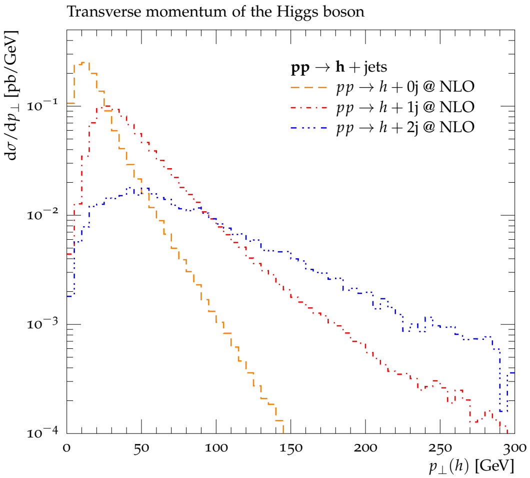
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h + \mathrm{jet}$ for $Q_{n+1} > Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



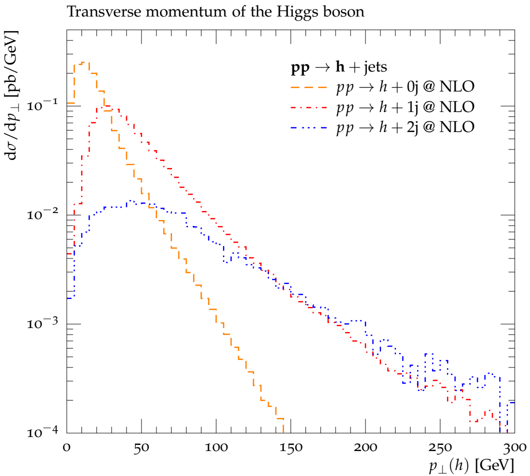
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+$jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+$jet to $Q_{n+2} < Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



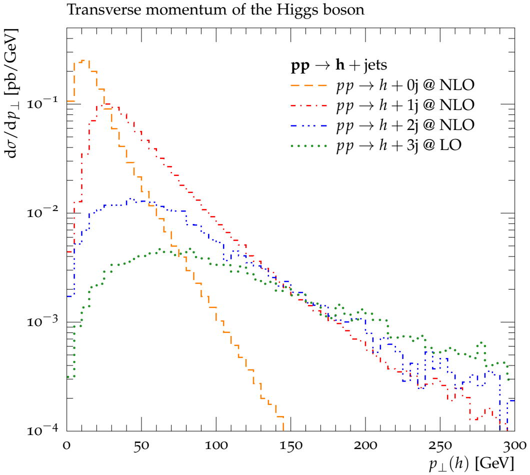
- first emission by MC@NLO, restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+$jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+$jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h+$2jets for $Q_{n+2} > Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



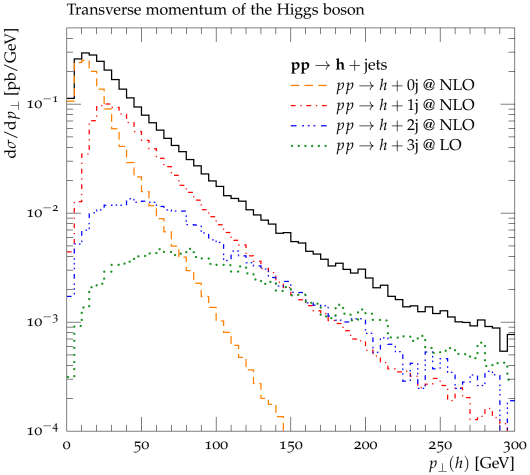
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate

$p_\perp^H$ in MEPs@NLO



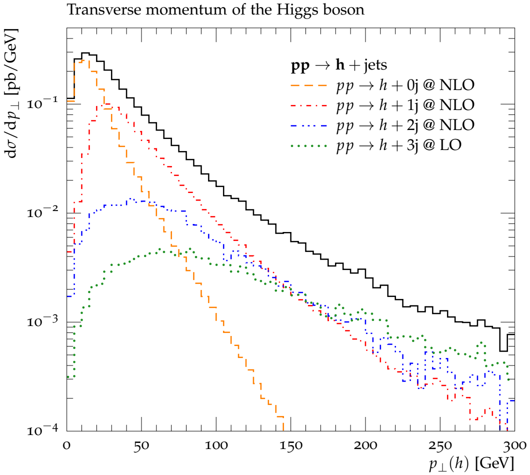
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h +$ jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h +$ 2jets for $Q_{n+2} > Q_{\rm cut}$
- iterate

$p_\perp^H$ in MEPS@NLO



- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h+$jet for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h+$jet to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h+$2jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

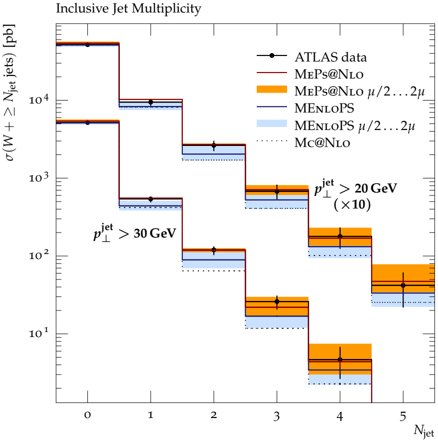
$p_\perp^H$ in MEPS@NLO

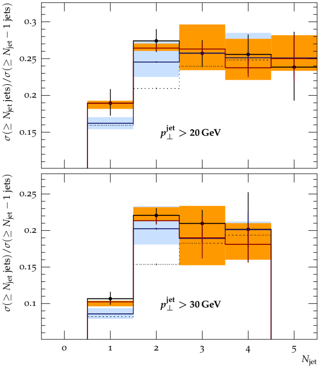


- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{2jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions
- eg. $p_\perp(h) > 200$ GeV has contributions fr. multiple topologies

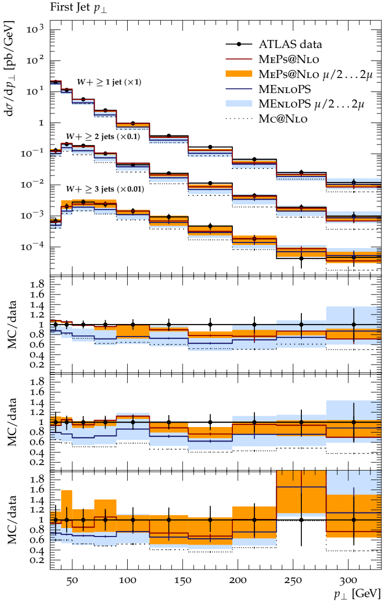
Example: MEPS@NLO for $W+$jets

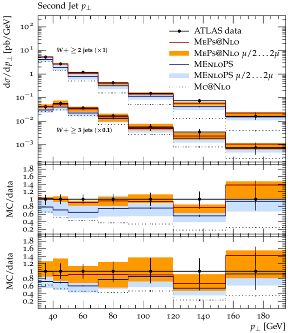
(up to two jets @ NLO, from BLACKHAT, see arXiv: 1207.5031 [hep-ex])

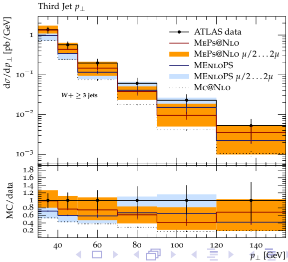


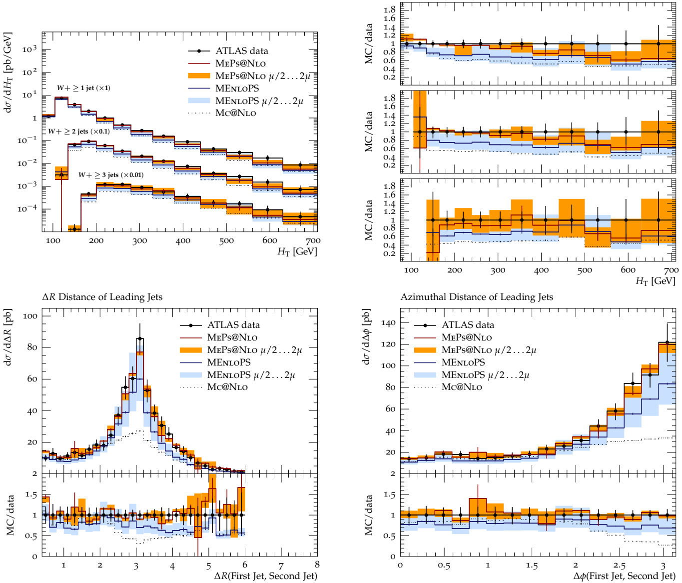


Multijet merging at NLO





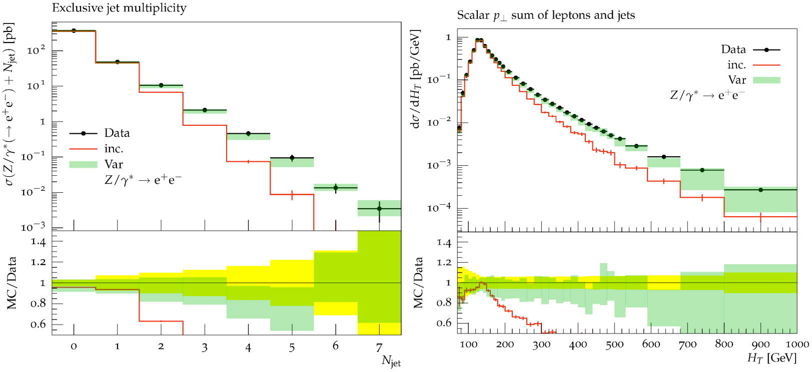



ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2 \ldots 2\mu$
MENLOPS
MENLOPS $\mu/2 \ldots 2\mu$
MC@NLO
$W + \geq 1$ jet ($\times 1$)
$W + \geq 2$ jets ($\times 0.1$)
$W + \geq 3$ jets ($\times 0.01$)
$d\sigma/dH_T$ [pb/GeV]
H_T [GeV]
MC/data
ΔR Distance of Leading Jets
$d\sigma/d\Delta R$ [pb]
ΔR(First Jet, Second Jet)
Azimuthal Distance of Leading Jets
$d\sigma/d\Delta\phi$ [pb]
$\Delta\phi$(First Jet, Second Jet)

FxFx: validation in Z+jets

(Data from ATLAS, 1304.7098, aMC@NLO _MADGRAPH with HERWIG++)

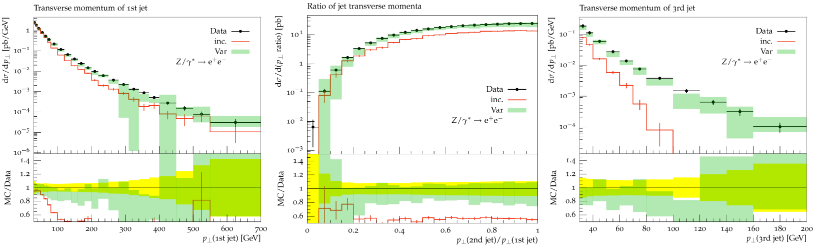
(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



FxFx: validation in Z+jets

(Data from ATLAS, 1304.7098, aMC@NLO _MADGRAPH with HERWIG++)

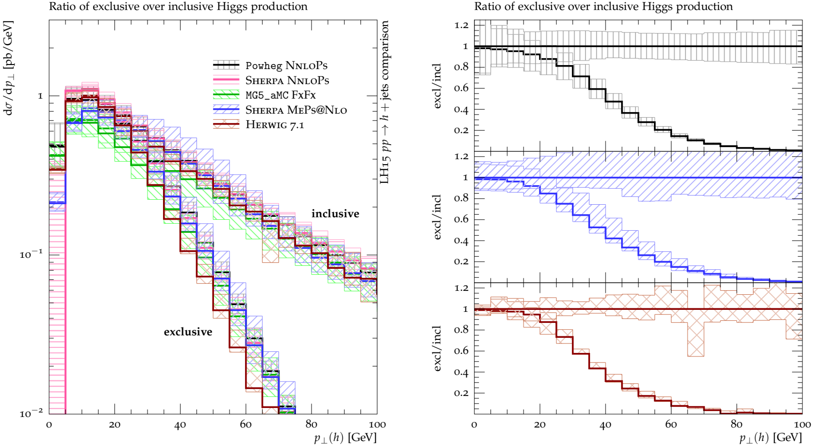
(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



Differences between MEPS@NLO, UNLOPS & FxFx

	FxFx	MEPS@NLO	UNLOPS
ME	all internal aMC@NLO_MADGRAPH	$\mathcal{V}$ external COMIX or AMEGIC++ $\mathcal{V}$ from OPENLOOPS, BLACKHAT, MJET, ...	all external
shower	external HERWIG or PYTHIA	intrinsic	intrinsic PYTHIA
Δ_N $\Theta(Q_J)$	analytical a-posteriori	from PS per emission	from PS per emission
Q_J-range	relatively high (but changed)	$>$ Sudakov regime $\approx$ 10%	$\approx$ Sudakov regime $\approx$ 10%

Higgs-$p_\perp$: exclusive over inclusive rate



- $\approx 20\%$ of Higgs with $p_\perp = 60\,$GeV are not accompanied by a jet

PRECISION MONTE CARLO

ELECTROWEAK CORRECTIONS

Contents

Motivation: the size of EW corrections

- EW corrections sizeable $\mathcal{O}(10\%)$ at large scales: must include them!
- but: more painful to calculate
- need EW showering & possibly corresponding PDFs

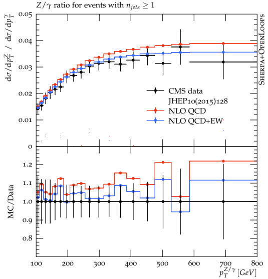
(somewhat in its infancy: chiral couplings)

- example: $Z\gamma$ vs. p_T (right plot)

(handle on $p_\perp^Z$ in $Z \to \nu\bar{\nu}$)

(Kallweit, Lindert, Pozzorini, Schoenherr for LH'15)

- difference due to EW charge of Z
- no real correction (real V emission)
- improved description of $Z \to \ell\ell$



Inclusion of electroweak corrections in simulation

- incorporate approximate electroweak corrections in MEPS@NLO
  1. using electroweak Sudakov factors

$$\tilde{\mathrm{B}}_n(\Phi_n) \;\approx\; \tilde{\mathrm{B}}_n(\Phi_n)\,\Delta_{\mathrm{EW}}(\Phi_n)$$

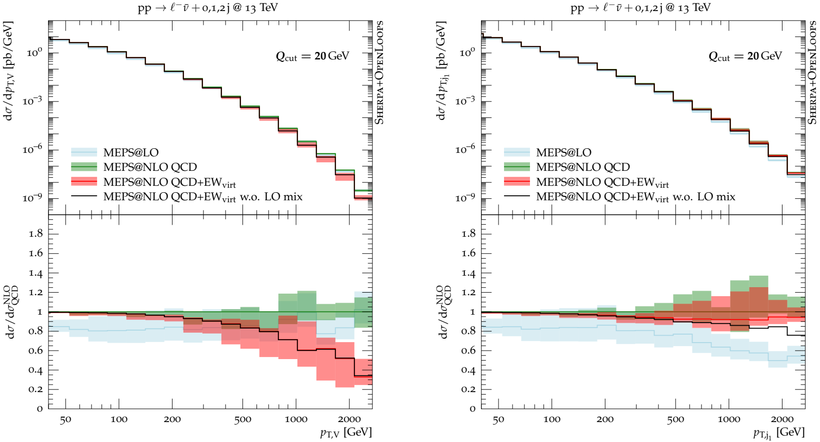
  2. using virtual corrections and approx. integrated real corrections

$$\tilde{\mathrm{B}}_n(\Phi_n) \;\approx\; \tilde{\mathrm{B}}_n(\Phi_n) + \mathrm{V}_{n,\mathrm{EW}}(\Phi_n) + \mathrm{I}_{n,\mathrm{EW}}(\Phi_n) + \mathrm{B}_{n,\mathrm{mix}}(\Phi_n)$$

- real QED radiation can be recovered through standard tools (parton shower, YFS resummation)
- simple stand-in for proper QCD$\oplus$EW matching and merging
  $\rightarrow$ validated at fixed order, found to be reliable,
  difference $\lesssim 5\%$ for observables not driven by real radiation

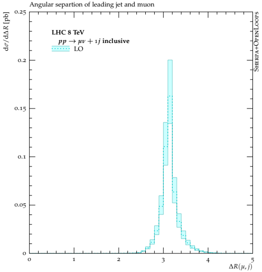
Results: $pp \to \ell^- \bar{\nu} + \text{jets}$

(Kallweit, Lindert, Maierhöfer, Pozzorini, Schoenherr JHEP04(2016)021)



$\Rightarrow$ particle level events including dominant EW corrections

NLO EW predictions for $\Delta R(\mu, j_1)$



measure collinear W emission?

LHC@8TeV, $p_\perp^{j_1} > 500\,\mathrm{GeV}$, central μ and jet

- LO $pp \to Wj$ with $\Delta\phi(\mu, j) \approx \pi$

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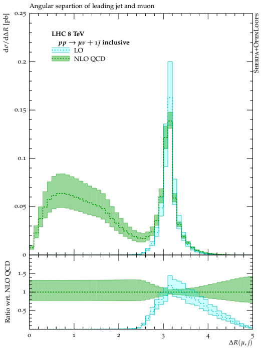

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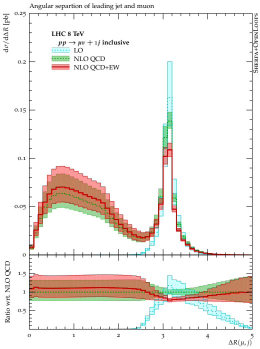

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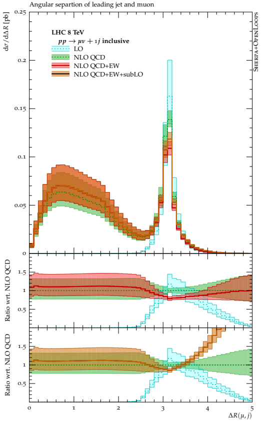

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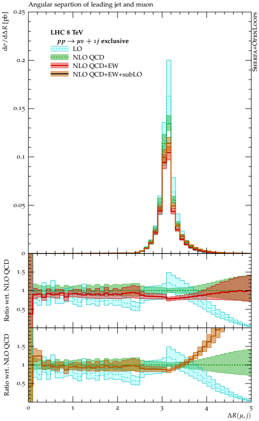

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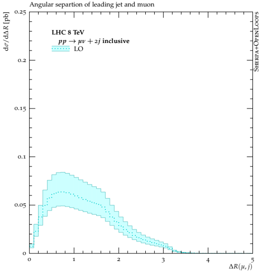

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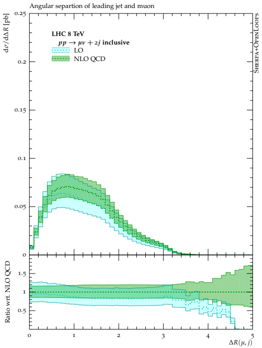

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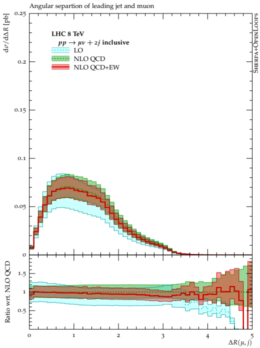

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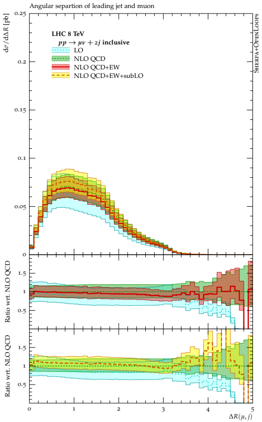

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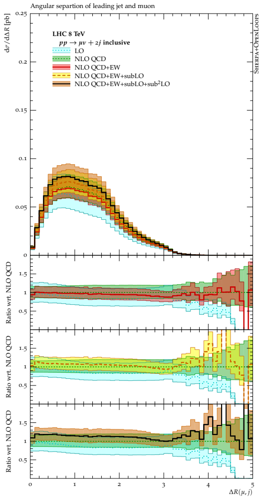

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- sub^{2}leading Born (diboson etc) conts. pos.
  $\to$ possible double counting with BG

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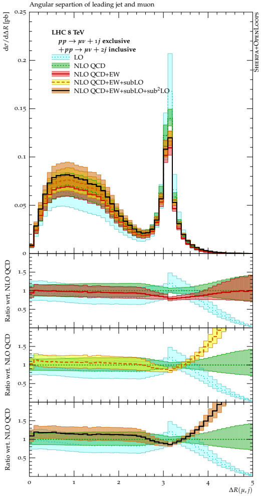

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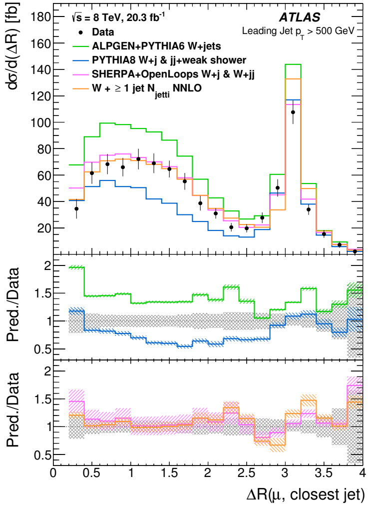
  $\to$ possible double counting with BG
- merge using exclusive sums

NLO EW predictions for $\Delta R(\mu, j_1)$



Data comparison

(M. Wu ICHEP'16, ATLAS arXiv:1609.07045)

- ALPGEN+PYTHIA
  $pp \to W +$ jets MLM merged
  (Mangano et.al., JHEP07(2003)001)
- PYTHIA 8
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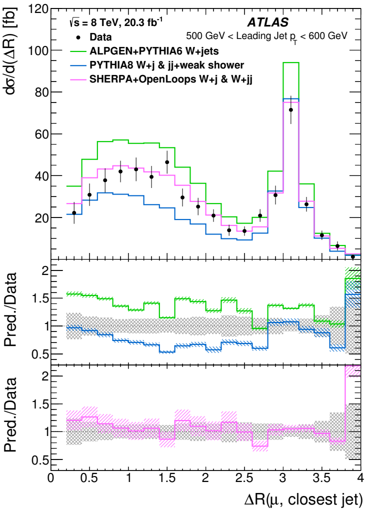

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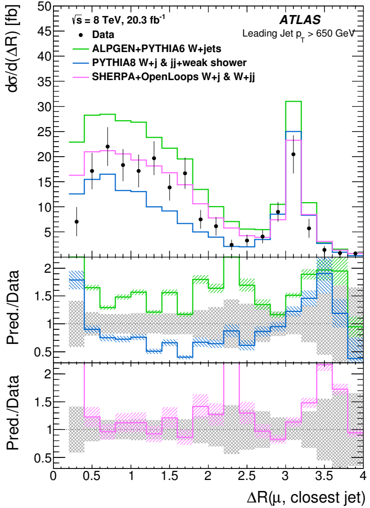

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SIMULATING SOFT QCD

SIMULATING SOFT QCD

HADRONISATION

Contents

QCD radiation, once more

- remember QCD emission pattern

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi}\,C_F\,\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\frac{\mathrm{d}\omega}{\omega}\,\left[1+\left(1-\frac{\omega}{E}\right)\right].$$

- spectrum cut-off at small transverse momenta and energies by onset of hadronization, at scales $R \approx 1\,\mathrm{fm}/\Lambda_{\mathrm{QCD}}$
- two (extreme) classes of emissions: gluons and gluers determined by relation of formation and hadronization times

- gluers formed at times R, with momenta $k_\parallel \sim k_\perp \sim \omega \gtrsim 1/R$
- assuming that hadrons follow partons,

$$\mathrm{d}N_{\rm (hadrons)} \sim \int\limits_{k_\perp > 1/R}^{Q} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{C_F\,\alpha_s(k_\perp^2)}{2\pi} \left[1 + \left(1 - \frac{\omega}{E}\right)\right] \frac{\mathrm{d}\omega}{\omega}$$

$$\sim \frac{C_F \alpha_s(1/R^2)}{\pi} \log(Q^2 R^2) \frac{\mathrm{d}\omega}{\omega}$$

or - relating their energyn with that of the gluers -

$$\mathrm{d}N_{\rm (hadrons)}/\mathrm{d}\log\epsilon \;=\; \mathrm{const.}\,,$$

a plateau in log of energy (or in rapidity)

- impact of additional radiation
- new partons must separate before they can hadronize independently
- therefore, one more time

$$
\begin{aligned}
t^{\rm form} &\sim \frac{k_\parallel}{k_\perp^2} \\
t^{\rm sep} &\sim R\theta \sim t^{\rm form}\,(Rk_\perp) \\
t^{\rm had} &\sim k_\parallel R^2 \sim t^{\rm form}\,(Rk_\perp)^2\,.
\end{aligned}
$$

- for gluers $Rk_\perp \approx 1$: all times the same
- naively; new & more hadrons following new partons
- but: colour coherence
  primary and secondary partons not separated enough in

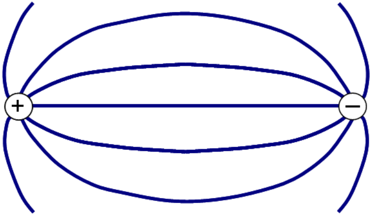
$$1/R \lesssim \omega_{\rm (hadron)} \lesssim 1/(R\theta)$$

and therefore no independent radiation

Hadronisation: General thoughts

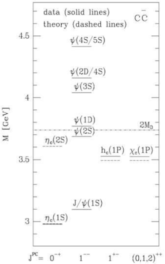
- confinement the striking feature of low–scale sotrng interactions
- transition from partons to their bound states, the hadrons
- the Meissner effect in QCD

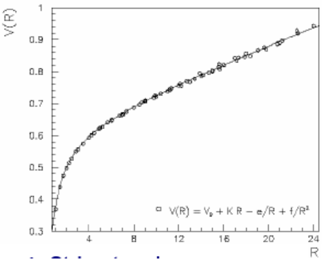
QED:



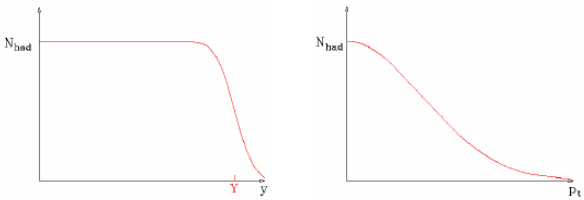
QCD:

- linear QCD potential in Quarkonia – like a string





- combine some experimental facts into a naive parameterisation
- in $e^+e^- \to$ hadrons: exponentially decreasing $p_\perp$, flat plateau in y for hadrons



- try "smearing": $\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$

- use parameterisation to "guesstimate" hadronisation effects:

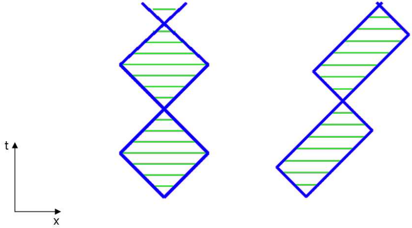
$$\begin{aligned} E &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2 \rho(p_\perp^2) p_\perp \cosh y = \lambda \sinh Y \\ P &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2 \rho(p_\perp^2) p_\perp \sinh y = \lambda(\cosh Y - 1) \approx E - \lambda \\ \lambda &= \int \mathrm{d}p_\perp^2 \rho(p_\perp^2) p_\perp = \langle p_\perp \rangle \,. \end{aligned}$$

- estimate $\lambda \sim 1/R_{\rm had} \approx m_{\rm had}$, with $m_{\rm had}$ 0.1-1 GeV.
- effect: jet acquire non-perturbative mass $\sim 2\lambda E$ ($\mathcal{O}(10{\rm GeV})$ for jets with energy $\mathcal{O}(100{\rm GeV})$).

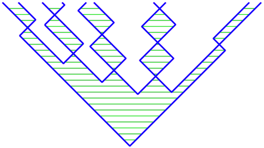
- similar parametrization underlying Feynman-Field model for independent fragmentation
- recursively fragment $q \to q' +$ had, where
  - transverse momentum from (fitted) Gaussian;
  - longitudinal momentum arbitrary (hence from measurements);
  - flavour from symmetry arguments + measurements.
- problems: frame dependent, "last quark", infrared safety, no direct link to perturbation theory, ...

The string model

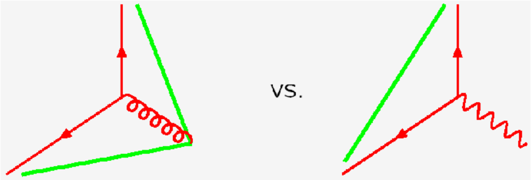
- a simple model of mesons: yoyo strings
  - light quarks ($m_q = 0$) connected by string, form a meson
  - area law: $m^2_{\rm had} \propto$ area of string motion
  - L=0 mesons only have 'yo-yo' modes:



- turn this into hadronisation model $e^+e^- \to q\bar{q}$ as test case
- ignore gluon radiation: $q\bar{q}$ move away from each other, act as point-like source of string
- intense chromomagnetic field within string:
  more $q\bar{q}$ pairs created by tunnelling and string break-up
- analogy with QED (Schwinger mechanism):
  $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t \exp\left(-\pi m_q^2/\kappa\right)$, $\kappa =$ "string tension".



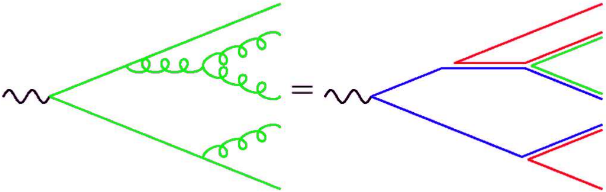
- string model = well motivated model, constraints on fragmentation (Lorentz-invariance, left-right symmetry, ...)
- how to deal with gluons?
- interpret them as kinks on the string $\Longrightarrow$ the string effect



- infrared-safe, advantage: smooth matching with PS.

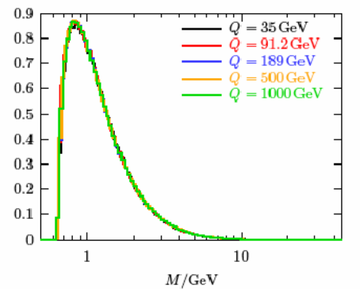
The cluster model

- underlying idea: preconfinement/LPHD
  - typically, neighbouring colours will end in same hadron
  - hadron flows follow parton flows $\longrightarrow$ don't produce any hadrons at places where you don't have partons
  - works well in large–N_c limit with planar graphs
- follow evolution of colour in parton showers



- paradigm of cluster model: clusters as continuum of hadron resonances
- trace colour through shower in $N_c \to \infty$ limit
- force decay of gluons into $q\bar{q}$ or $\bar{d}d$ pairs, form colour singlets from neighbouring colours, usually close in phase space
- mass of singlets: peaked at low scales $\approx Q_0^2$
- decay heavy clusters into lighter ones or into hadrons (here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like)
- if light enough, clusters will decay into hadrons
- naively: spin information washed out, decay determined through phase space only $\to$ heavy hadrons suppressed (baryon/strangeness suppression)

- self–similarity of parton shower will end with roughly the same **local** distribution of partons, with roughly the same invairant mass for colour singlets
- adjacent pairs colour connected, form colourless (white) clusters.
- clusters ("≈ excited hadrons) decay into hadrons



Practicalities

- practicalities of hadronisation models: parameters
  - kinematics of string or cluster decay: 2-5 parameters
  - must "pop" quark or diquark flavours in string or cluster decay — cannot be completely democratic or driven by masses alone
    $\longrightarrow$ suppression factors for strangeness, diquarks 2-10 parameters
  - transition to hadrons, cannot be democratic over multiplets
    $\longrightarrow$ adjustment factors for vectors/tensors etc. 2-6 parameters
- tuned to LEP data, overall agreement satisfying
- validity for hadron data not quite clear

  (beam remnant fragmentation not in LEP.)
- there are some issues with inclusive strangeness/baryon production

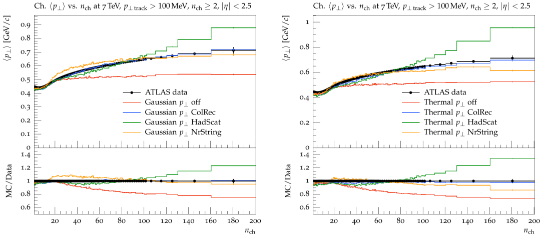
Colour reconnections and friends

(Fischer, Sjostrand, 1610:09818)

Collective flow observed in pp at LHC. Partly unexpected.
New mechanisms required; could also (partly) replace CR.

Active field, e.g. N. Fischer & TS, arXiv:1610:09818 [hep-ph]:

- Thermal $exp(-p_\perp/T) \to exp(-m_\perp/T)$ hadronic spectrum.
- Close-packed strings $\Rightarrow$ increased string κ or T.
- Dense hadronic gas $\Rightarrow$ hadronic rescattering.

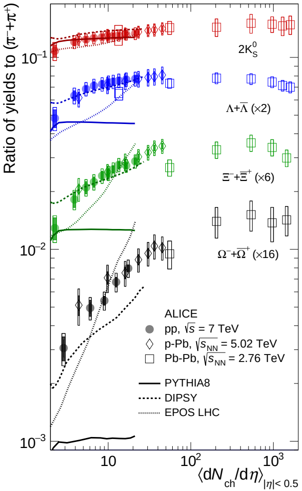


(slide stolen from Torbjorn Sjostrand)

Strange strangeness

- universality of hadronisation assumed
- parameters tuned to LEP data
  in particular: strangeness suppression
- for strangeness: flat ratios
  but data do not reproduce this
- looks like $SU(3)$ restoration
  not observed for protons
- needs to be investigated

(ALICE collaboration)



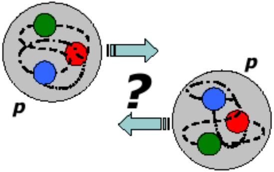
SIMULATING SOFT QCD

UNDERLYING EVENT

Contents

Multiple parton scattering

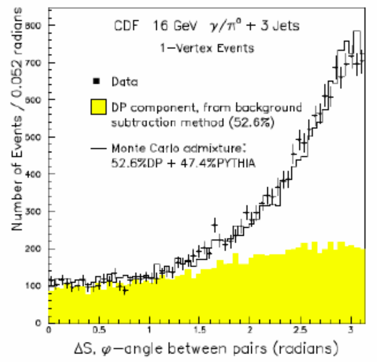
- hadrons = extended objects!
- no guarantee for one scattering only.
- running of α_S
  $\Longrightarrow$ preference for soft scattering.



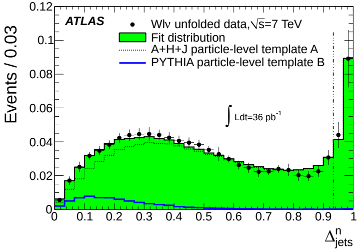
- first experimental evidence for double–parton scattering: events with $\gamma + 3$ jets:
  - cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| <1.3$;
  - "clean sample": two softest jets with $E_T < 7$ GeV;
- cross section for DPS

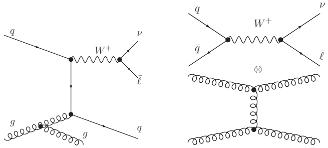
$$\sigma_{\rm DPS} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\rm eff}}$$

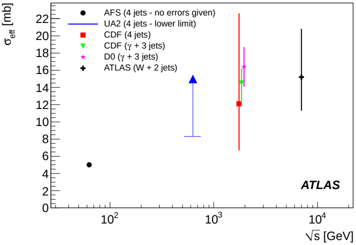
$\sigma_{\rm eff} \approx 14 \pm 4$ mb.

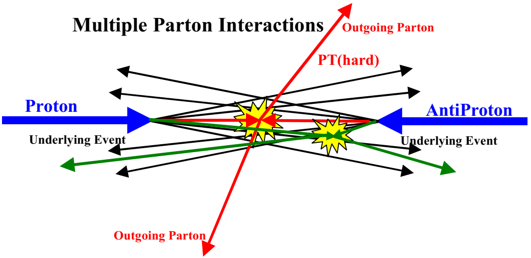


- more measurements, also at LHC
- ATLAS results from $W + 2$ jets









but: how to define the underlying event?

1. everything apart from the hard interaction, but including IS showers, FS showers, remnant hadronisation.
2. remnant-remnant interactions, soft and/or hard.
3. lesson: hard to define

- origin of MPS: parton–parton scattering cross section exceeds hadron–hadron total cross section

$$\sigma_{\rm hard}(p_{\perp,\rm min}) = \int\limits_{p^2_{\perp,\rm min}}^{s/4} {\rm d}p_\perp^2 \, \frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} > \sigma_{pp,\rm total}$$

  for low $p_{\perp,\rm min}$
- remember

$$\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} = \int\limits_0^1 {\rm d}x_1 {\rm d}x_2 f(x_1,\, q^2) f(x_2,\, q^2)\, \frac{{\rm d}\hat\sigma_{2\to 2}}{{\rm d}p_\perp^2}$$

- $\langle \sigma_{\rm hard}(p_{\perp,\rm min})/\sigma_{pp,\rm total}\rangle \geq 1$
- depends strongly on cut-off $p_{\perp,\rm min}$ (energy-dependent)!

Modelling the underlying event

- take the old PYTHIA model as example:
  - start with hard interaction, at scale $Q^2_{\rm hard}$.
  - select a new scale $p_\perp^2$ from

$$\exp\left[-\frac{1}{\sigma_{\rm norm}}\int\limits_{p_\perp^2}^{Q^2_{\rm hard}} {\rm d}p_\perp{}'^2 \frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp'^2}\right]$$

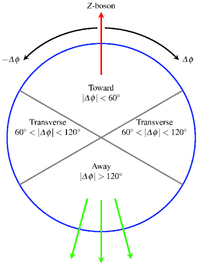
    with constraint $p_\perp^2 > p^2_{\perp,\rm min}$
  - rescale proton momentum (“proton-parton = proton with reduced energy”).
  - repeat until no more allowed $2 \to 2$ scatter

Modelling the underlying event

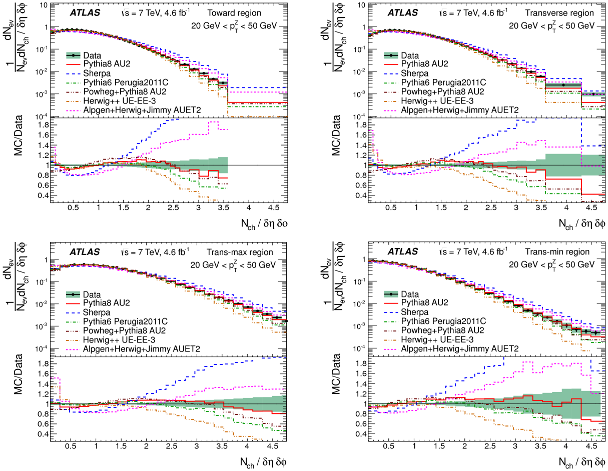
- possible refinements:
  - may add impact-parameter dependence $\longrightarrow$ more fluctuations
  - add parton showers to UE
  - "regularisation" to dampen sharp dependence on $p_{\perp,\rm min}$: replace $1/\hat{t}$ in MEs by $1/(t+t_0)$, also in α_s.
  - treat intrinsic $k_\perp$ of partons ($\to$ parameter)
  - model proton remnants ($\to$ parameter)

Some results for MPS in Z production

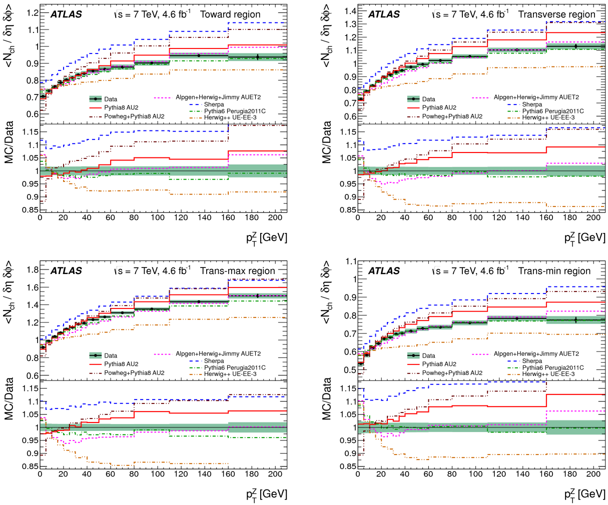
- observables sensitive to MPS
- classical analysis: transverse regions in QCD/jet events
- idea: find the hardest system, orient event into regions:
    - toward region along system
    - away region back-to-back
    - transverse regions
- typically each in 120°



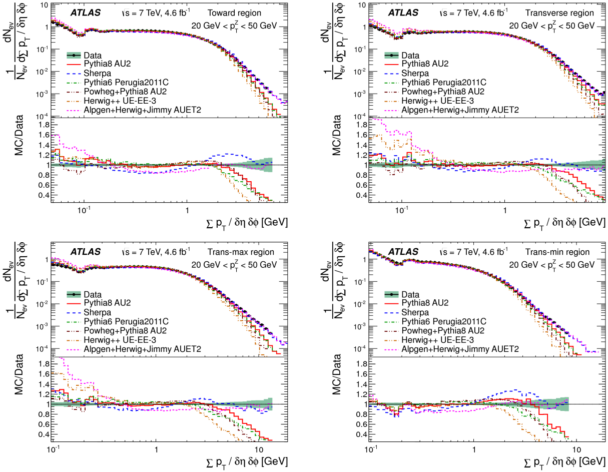
Some results in Z production



Some results in Z production



Some results in Z production



- see some data comparison in Minimum Bias
- practicalities of underlying event models: parameters
  - profile in impact parameter space — 2-3 parameters
  - IR cut-off at reference energy, its energy evolution, dampening paramter and normalisation cross section — 4 parameters
  - treating colour connections to rest of event — 2-5 parameters
- tuned to LHC data, overall agreement satisfying
- energy extrapolation not exactly perfect, plus other process categories such as diffraction etc..

SUMMARY

Summary of fixed order

- NLO (QCD) "revolution" consolidated:
  - lots of routinely used tools for large FS multis (4 and more)
  - incorporation in MC tools done, need comparisons, critical appraisals and a learning curve in their phenomenological use
  - to improve: description of loop–induced processes
- amazing success in NNLO (QCD) calculations:
  - emergence of first round of $2 \to 2$ calculations
  - next revolution imminent (with question marks)
  - first MC tools for simple processes ($gg \to H$, DY), more to be learnt by comparison etc. (see above)
- first N^{3}LO calculation in $gg \to H$, more to come (?)
- attention turning to NLO (EW)
  - first benchmarks with new methods (V+3j)
  - calculational setup tricky
  - need maybe faster approximation for high-scales (EW Sudakovs)

Limitations of fixed order

- practical limitations/questions to be overcome:
  - dealing with IR divergences at NNLO: slicing vs. subtracting

    (I'm not sure we have THE solution yet)
  - how far can we push NNLO? are NLO automated results stable enough for NNLO at higher multiplicity?
  - users of codes: higher orders tricky $\rightarrow$ training needed

    (MC = black box attitude problematic - a new brand of pheno/experimenters needed?)
- limitations of perturbative expansion:
  - breakdown of factorisation at HO (Seymour et al.)
  - higher-twist: compare $(\alpha_s/\pi)^n$ with $\Lambda_{\rm QCD}/M_Z$
- limitations in analytic resummation: process- and observable-dependent
  - first attempts at automation (CAESAR and some others) – checks/cross-comparison necessary
- showering needs to be improved

  (for NNLO the "natural" accuracy is NNLL)

Summary for event generation

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "POWHEG")
  - MENLOPS NLO matching & merging
  - MEPS@NLO ("SHERPA", "UNLOPS", "MINLO", "FxFx")

- multijet merging an important tool for many relevant signals and backgrounds - pioneering phase at LO & NLO over
- complete automation of NLO calculations done
  $\longrightarrow$ must benefit from it!

(it's the precision and trustworthy & systematic uncertainty estimates!)

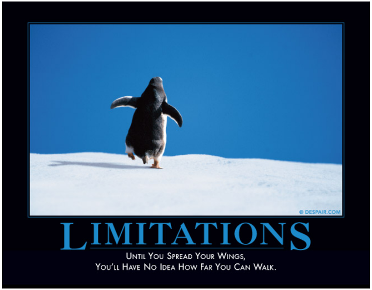
Vision

- we have constructed lots of tools for precision physics at LHC
  but we did not cross-validate them careful enough (yet)
  but we did not compare their theoretical foundations (yet)
- we also need unglamorous improvements:
  - systematically check advanced scale-setting schemes (MINLO)
  - automatic (re-)weighting for PDFs & scales (ME: ✓, PS: -)
  - scale compensation in PS is simple (implement and check)
  - PDFs: to date based on FO vs. data — will we have to move to resummed/parton showered?

(reminder: LO* was not a big hit, though)

- . . . and maybe we will have to go to the "dirty" corners:
  higher-twist, underlying event, hadronization, . . .

(many of those driven by experiment)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LIMITATIONS
Until You Spread Your Wings,
You'll Have No Idea How Far You Can Walk.