Measurements of the magnetic form factor at low Q^2

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RBRC
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Cross section for elastic scattering

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

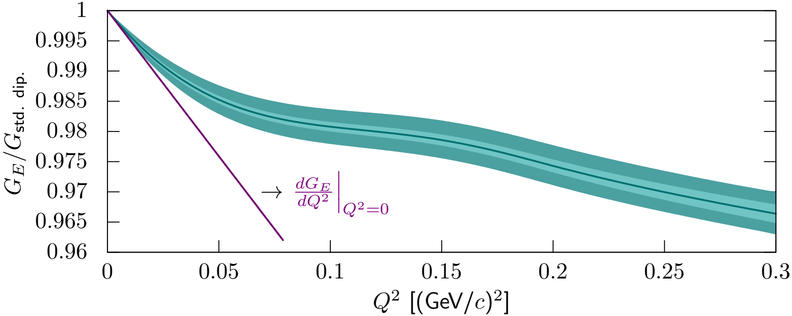
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

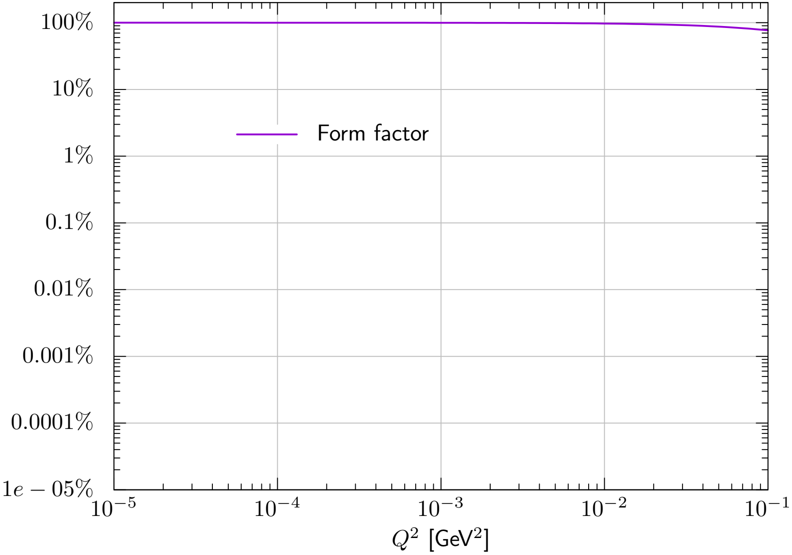
- Rosenbluth formula
- Electric and magnetic form factor encode the shape of the proton
- Fourier transform (almost) gives the spatial distribution, in the Breit frame

Radius

$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}\left(G_M/\mu_p\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$



Why is getting radii out so hard?



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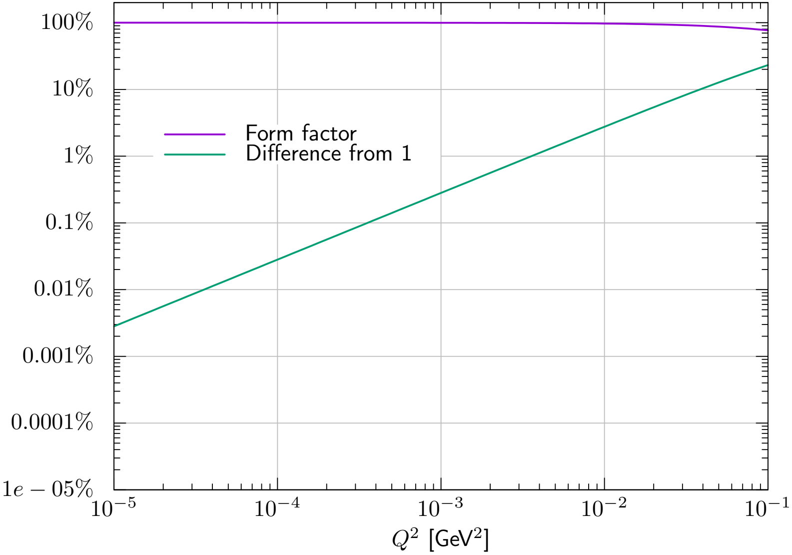

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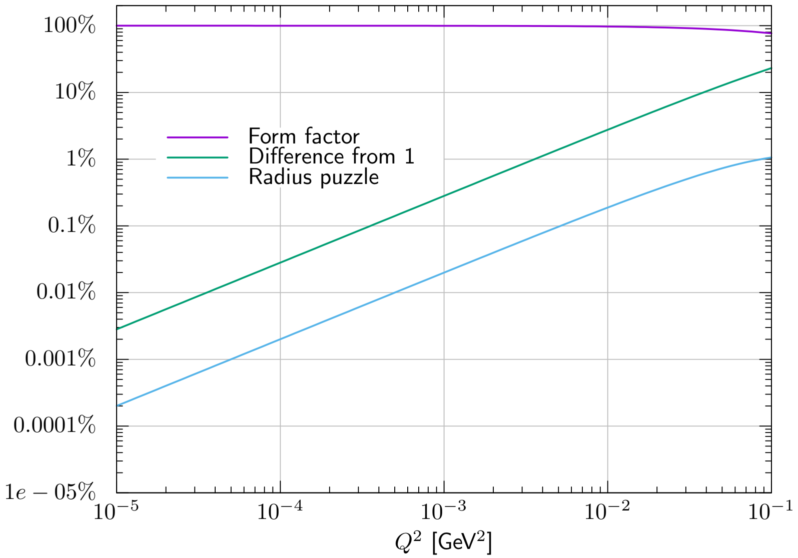

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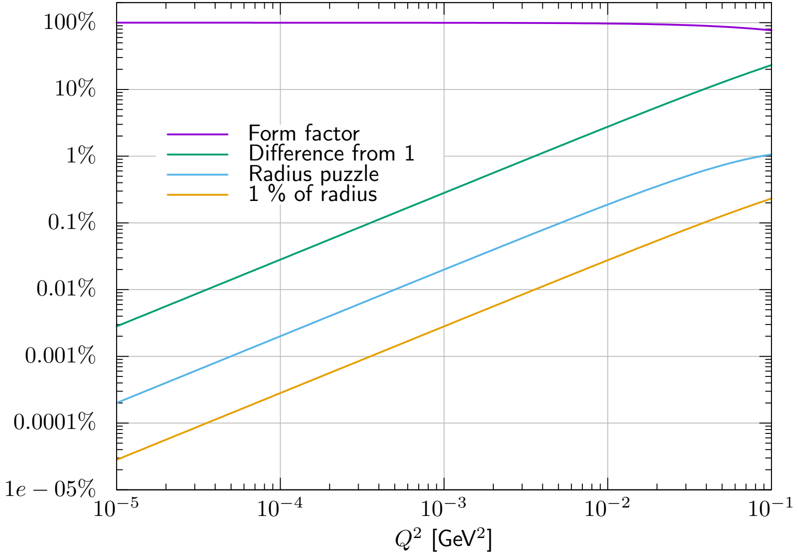

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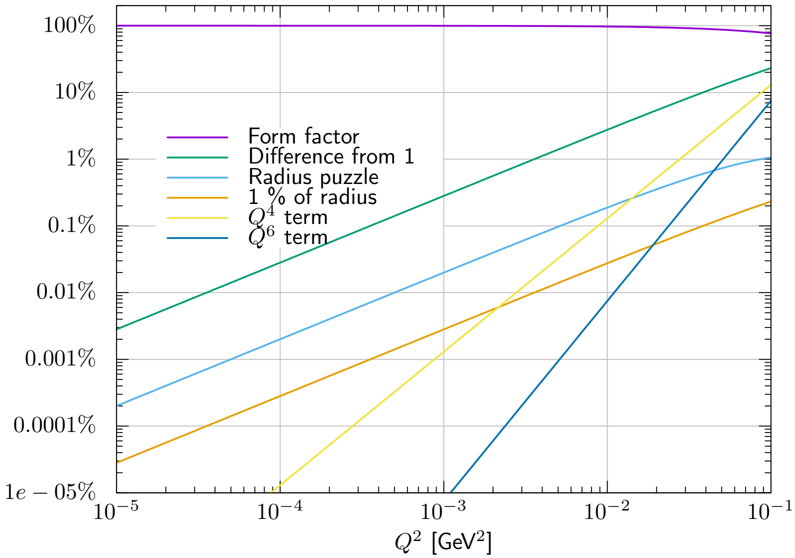


Why is G_M at low Q^2 important?

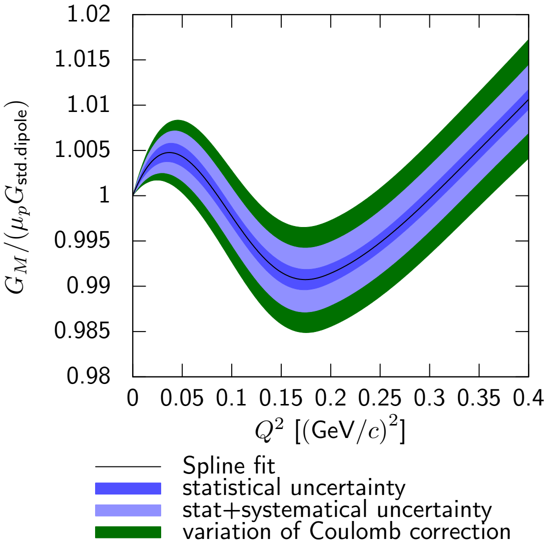
- Long range behavior of magnetisation in the nucleus!
- Gives the magnetic radius
- Zemach radius
- Structure seen in Mainz data

Zemach radius

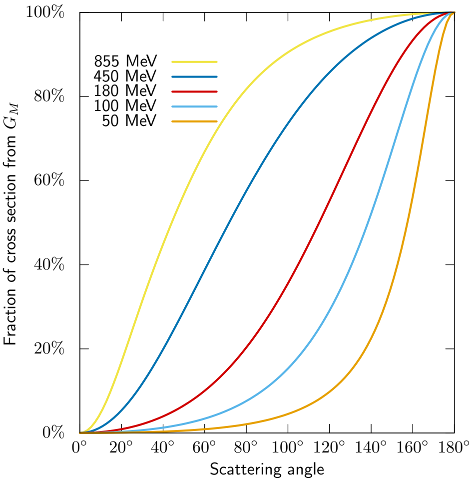
$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(\frac{1}{\mu_p}G_E(Q^2)G_M(Q^2)-1\right)$$

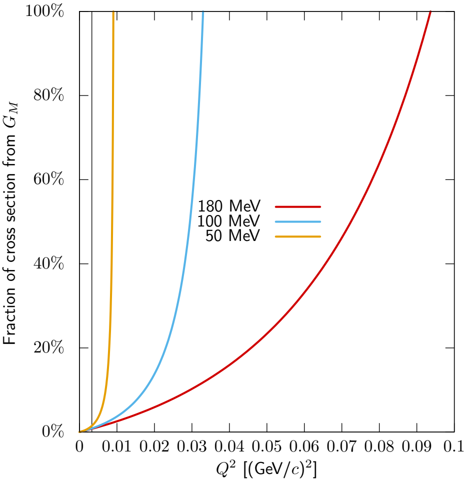
- Another connection point to spectroscopy!
- Dominated by FF. difference from 1 at low-Q^2
- I.e. similar problems as charge/magnetic radii

Mainz data structure in G_M



Low-Q G_M is hard





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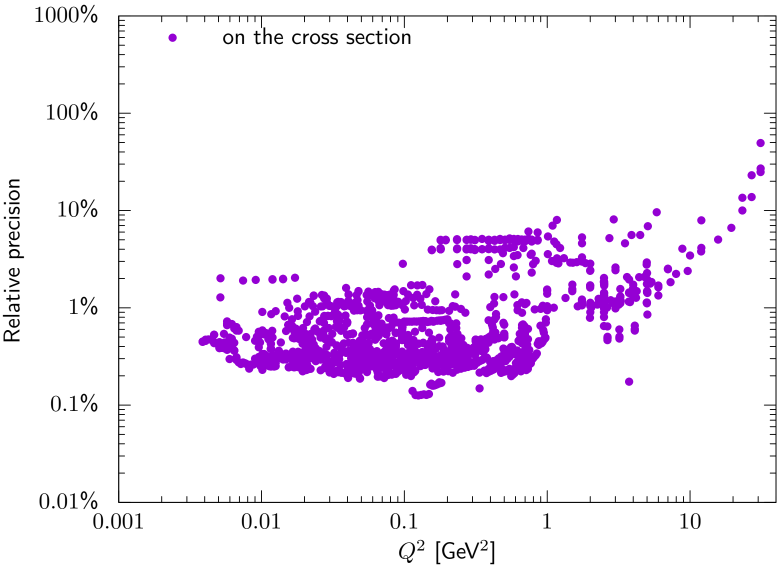

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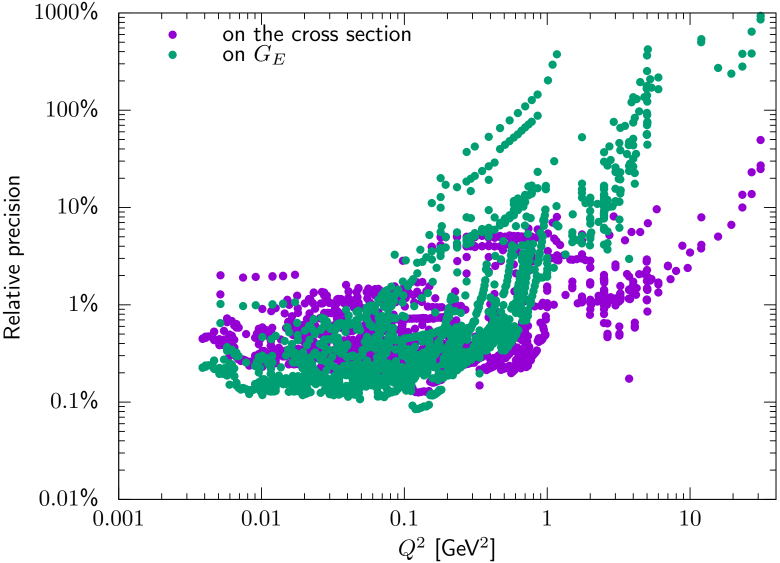

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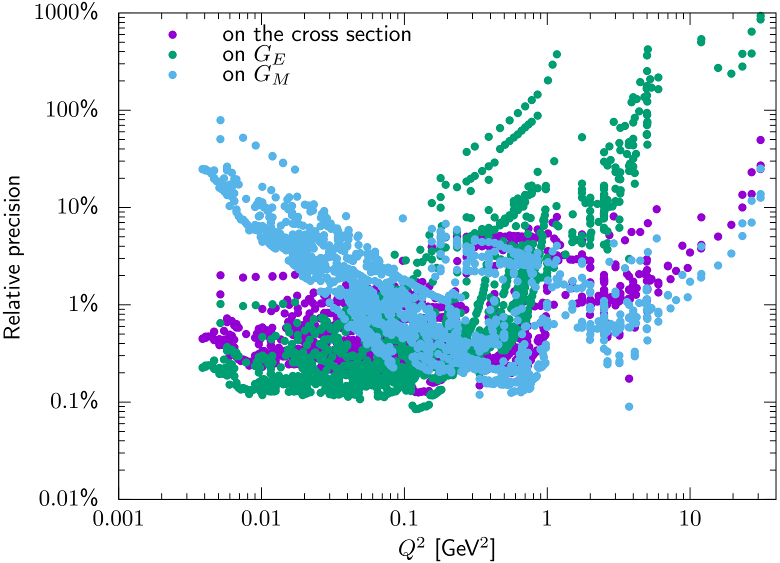
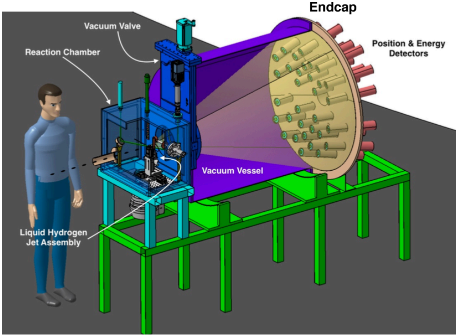


What could PRAE do?

We need measurements at backward angle, and small beam energy so that Q^2 is smallish.

For a start:

ProRad



What could PRAE do?

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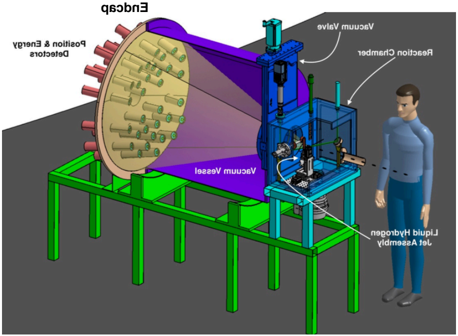
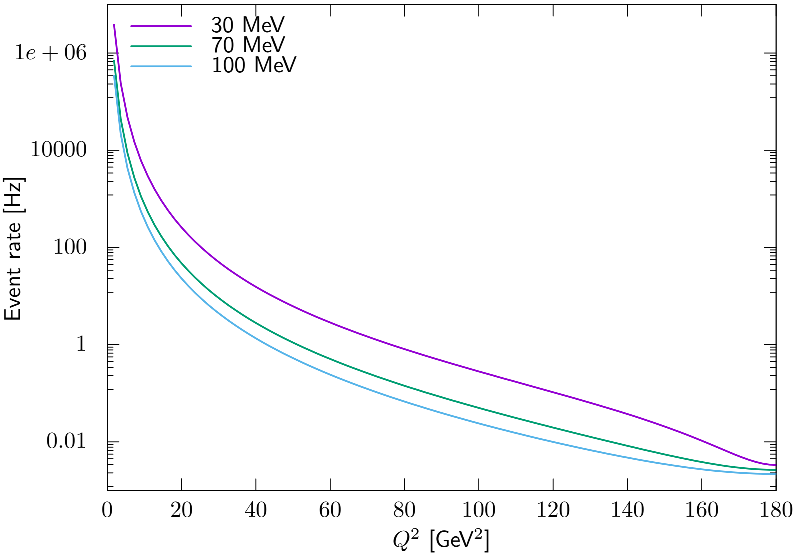
For a start:
Flip over ProRad

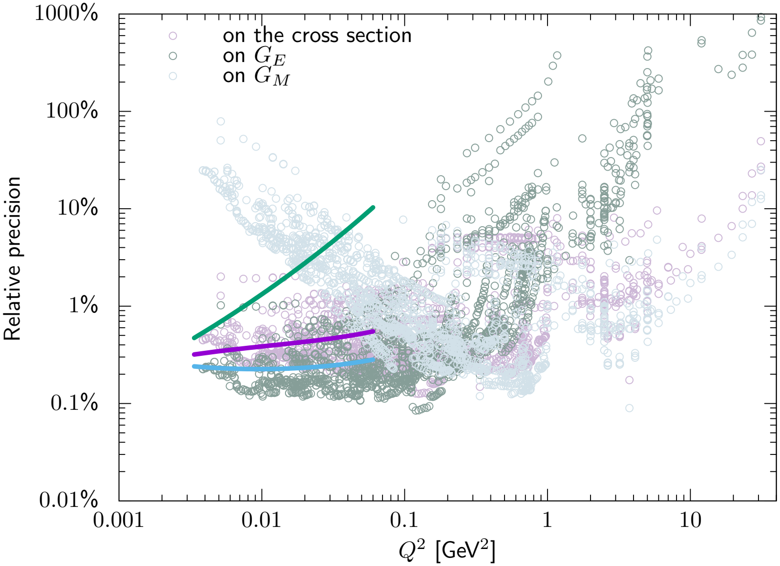
- $15 \mu m$ solid hydrogen target
- 32×0.87 msr detectors at $\approx 170^\circ$



reverse ProRad rates (single detector)



Reach with one week of beamtime

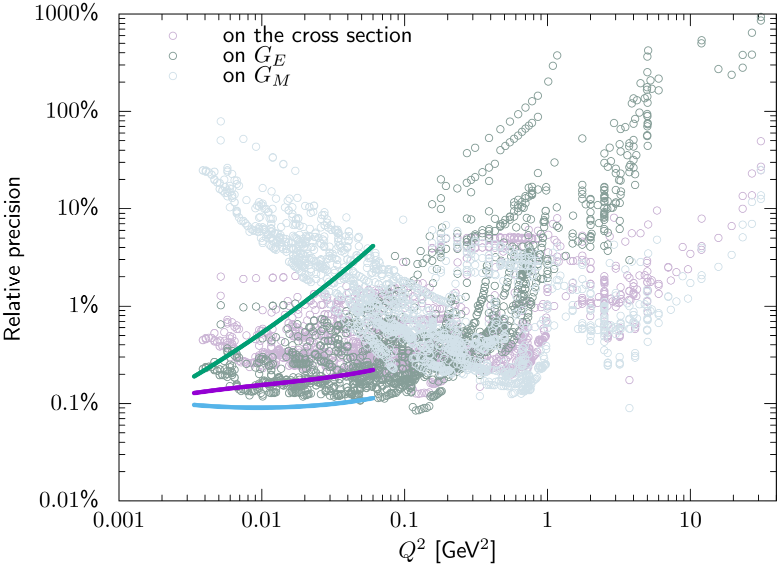


Can we do better?

Rate is small. Thicker target?
E.g. 2 cm liquid hydrogen target (as in Mainz)

- 1000 times more rate
- Background from scattering of target wall!
  - Empty cell
  - Cut elastics via momentum resolution

Reach with 1h beamtime, liquid target

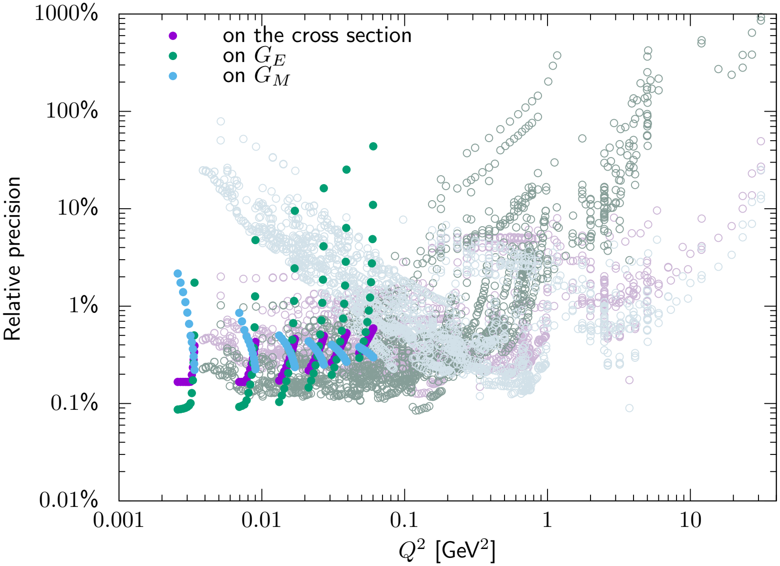


Alternative detector, proposed program

Instead of ProRad, assume 1msr detector
Movable from 120° to 175° in 5° steps
Energies: 30, 50, 70, 90, 110, 140 MeV
4h each measurement

Total of 288 hours!

Reach with 4h, alt. detector



Things to worry about

- Need good normalization, at least relative over all points
  - Møller detector for relative normalization
  - G_E dominated part can give absolute normalization
- Background for liquid target cell
  - empty cell measurement and/or magnetic spectrometer
- Radiative corrections larger, especially two photon exchange
  - build a positron source and measure it!

Conclusion

A measurement of G_M at low Q^2 is important:

- Connection to spectroscopy
- long range structure of the proton

PRAE could provide a crucial dataset. Measurements are possible

- with a flipped-around ProRad (many weeks / few month)
- plus a different target (few weeks)
- alternative detector (fewer weeks, more points)

Extrapolation to $Q^2 = 0$

Have to extrapolate form factor to $Q^2 = 0$.
Mainz lowest $Q^2 = 0.0033\,(\mathrm{GeV/c})^2$.
We use a 10th order polynomial to fit data up to $1\,(\mathrm{GeV/c})^2$. This gets people scared.

Can we fit just a linear term?

Can a linear fit work?

$$\frac{d\sigma}{d\Omega} \propto 1 - \underbrace{A}_{\mathcal{O}(6)} \cdot Q^2 + \underbrace{B}_{\mathcal{O}(30)} \cdot Q^4 + \ldots$$

(Q in units of GeV/c)

We want to measure the radius (~$\sqrt{A}$) to within 0.5%, without knowing B. So:

$$B/A \cdot Q^2 \ll 0.01 \longrightarrow Q^2 \ll 0.002\,(\mathrm{GeV}/c)^2$$

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But: Need to measure A to 1%, so measure $\frac{d\sigma}{d\Omega}$ to $6 \cdot 0.002 \cdot 0.01 = 0.012\%$. Now I'm feeling depressed.