Updated determinations of $|V_{us}|$ with tau data

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Precision Tau Physics measurements

- LEP experiments were very successful in providing tau precision measurements
- *B*-factories and charm-tau factories have significantly different experimental conditions and factories luminosities cannot be directly compared to LEP luminosities, however
  - factors 10-100-1000 increase w.r.t. previous factories of same kind do matter
  - *B* and charm-tau factories did provide several precision inputs to tau Physics tests
- a lot of work and time is needed to fully exploit the collected data
  - conclusive ALEPH paper on tau results 2005, for data collected up to 1995
  - *BABAR* and Belle are still producing results, and systematics studies increasing efforts
- encouraging prospects for further good progress from next *B* and charm-tau factories

Experimental progress on some precision measurements

α_s

- $R_{\tau,\mathrm{V+A}} = N_C \, |V_{ud}|^2 \, [1 + \delta_{\mathrm{P}}(\alpha_s) + \delta_{\mathrm{NP}}]$

	α_s	$R_{\tau,\mathrm{V+A}}$
ALEPH 2005	0.340 ± 0.015	3.4820 ± 0.0140
A. Pich 2014 (using HFLAV 2012 global fit)	0.344 ± 0.009	3.4712 ± 0.0079

- major uncertainty theory uncertainty on perturbative term
- significant uncertainty from tau spectral functions uncertainties
- uncertainty from non-perturbative theory also depends on tau spectral functions
- $R_{\tau,\mathrm{V+A}}$ uncertainty contribution minor
  part of experimental improvement due to HFLAV fit

Experimental progress on some precision measurements

g_τ / g_μ from leptonic BRs and tau lifetime

- $$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}{\tau_\tau m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^\tau}}$$

	g_τ / g_μ
A. Pich 2007	$1.00040 \pm 0.00140(B_e) \pm 0.00170(\tau_\tau) \pm 0.00037(m_\tau)$
HFLAV Spring 2017	$1.00103 \pm 0.00115(B_e) \pm 0.00090(\tau_\tau) \pm 0.00017(m_\tau)$

- precision improvements from factories after LEP
  - tau lifetime (Belle, better than former world average)
  - tau mass (KEDR, BES III)
  - $\mathcal{B}(\tau \to e \nu \bar{\nu})$ (BABAR and Belle tau BRs, despite no direct measurement)
    - part of experimental improvement on B_e due to HFLAV fit

Experimental progress on some precision measurements

$|V_{us}|$ from $\tau \to s$ inclusive

- $\dfrac{R(\tau \to X_s \nu)}{|V_{us}|^2} = \dfrac{R(\tau \to X_{\text{non-}s} \nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}}$

	$|V_{us}|$
Gamiz-Jamin-Pich-Prades-Schwab 2005	$0.2208 \pm 0.0033(\text{exp}) \pm 0.0009(\text{th})$
HFLAV Spring 2017	$0.2185 \pm 0.0019(\text{exp}) \pm 0.0011(\text{th})$

- *B*-factories effective on improving statistics-limited Cabibbo-suppressed decay modes

$|V_{us}|$ can be measured

from kaon decays

- $\Gamma(K \to \pi \ell \bar{\nu}_\ell [\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 \, S_{EW}^K \left(|V_{us}| f_+^{K\pi}(0) \right)^2 I_K^\ell \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi} \right)^2$ — $K_{\ell 3}$
- $\frac{\Gamma(K^- \to \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K (1 - m_\ell^2/m_K^2)^2}{m_\pi (1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{EM})$ — $K_{\ell 2}$

from tau decays

- $\frac{R(\tau \to X_s \nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-}s} \nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}}$, — $\tau \to s$ inclusive method
- $\frac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{\left(1 - m_K^2/m_\tau^2\right)^2}{\left(1 - m_\pi^2/m_\tau^2\right)^2} \frac{r_{LD}(\tau^- \to K^- \nu_\tau)}{r_{LD}(\tau^- \to \pi^- \nu_\tau)}$ — $\tau \to K/\tau \to \pi$ method

- $\tau \to s$ inclusive method does not require form factors from lattice QCD (and therefore has theory systematics uncorrelated to lattice QCD form factors)

"$\tau \to s$ inclusive" $|V_{us}|$ determination

Determine $|V_{us}|$ and/or m_s from $\mathcal{B}(\tau \to s)$ inclusive

Gamiz, Jamin, Pich, Prades, Schwab, JHEP 01 (2003) 06, PRL 94 (2005) 011803

$$\frac{R(\tau \to X_s \nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-}s} \nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}}$$

- $\delta R_{\tau,\text{SU3 breaking}}$ computed with OPE techniques

Determine $|V_{us}|$ using the world average for m_s

- E. Gamiz *et al.*, JHEP 01 (2003) 060, PRL 94 (2005) 011803,
  Nucl. Phys .Proc. Suppl. 169 (2007) 85, PoS KAON (2008) 008

Tau BRs best estimated with global fit of all relevant measurements

HFLAV Tau Spring 2017 fit, in HFLAV Summer 2016 report, Eur.Phys.J. C77 (2017)

Tau BRs Measurements

experiment	number of results
ALEPH	39
CLEO	35
BaBar	23
OPAL	19
Belle	15
DELPHI	14
L3	11
CLEO3	6
TPC	3
ARGUS	2
HRS	2
CELLO	1
total	170

- update published results using their systematic dependence on quantities whose measurements have been improved
- take into account correlations induced by common systematic dependencies from external parameters

- 170 measurements, 88 constraint equations
- fit 135 quantities: 47 BRs, 88 derived quantities (ratios of linear combinations of BRs)
- $\chi^2/\text{d.o.f.} = 137/123, \quad \text{CL} = 17.79\%$
- consistent with unitarity within 0.1% uncertainty, residual $= (0.03 \pm 0.10)\%$

Using the global tau BR fit results

$$\frac{R(\tau \to X_s\nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-}s}\nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}}$$

- $R(\tau \to X_s\nu) = \dfrac{\mathcal{B}(\tau \to X_s\nu)}{\mathcal{B}_{\text{ui}}(\tau \to e\bar{\nu}\nu)}$
- $R(\tau \to X_{\text{non-}s}\nu) = \dfrac{\mathcal{B}(\tau \to X_{\text{non-}s}\nu)}{\mathcal{B}_{\text{ui}}(\tau \to e\bar{\nu}\nu)}$

Universality improved $\mathcal{B}_{\text{ui}}(\tau \to e\nu\bar{\nu})$ (M. Davier, 2005)

- assuming the Standard Model, compute $\mathcal{B}_{\text{ui}}(\tau \to e\bar{\nu}_e\nu_\tau)$ averaging:

$$\mathcal{B}_e(e) = \mathcal{B}_e, \qquad \mathcal{B}_e(\mu) = \mathcal{B}_\mu \cdot f_{\tau e}/f_{\tau\mu} \qquad \mathcal{B}_e(\tau_\tau) = \frac{\tau_\tau}{\tau_\mu}\frac{m_\tau^5}{m_\mu^5}\frac{f_{\tau e}R_\gamma^\tau R_W^\tau}{f_{\mu e}R_\gamma^\mu R_W^\mu}$$

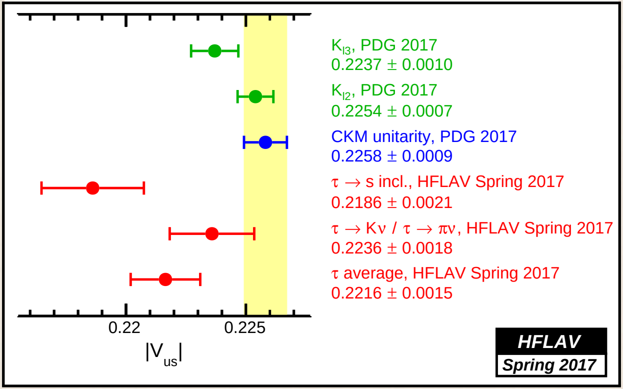
  - $f_{\tau e}$, $f_{\tau\mu}$ denote the phase space factors related to e, μ masses,
  - R's indicates radiative corrections

- using tau global branching fractions fit results all correlations are properly included
- $\mathcal{B}(\tau \to X_s\nu)$ is largest source of uncertainty for $|V_{us}|$

$\mathcal{B}(\tau \to X_s\nu)$ from HFLAV Spring 2017 fit

Tau decay mode	Branching fraction (%)
$K^-\nu_\tau$	0.6960 ± 0.0096
$K^-\pi^0\nu_\tau$	0.4327 ± 0.0149
$K^-2\pi^0\nu_\tau$ (ex. K^0)	0.0640 ± 0.0220
$K^-3\pi^0\nu_\tau$ (ex. K^0, η)	0.0428 ± 0.0216
$\pi^-\bar{K}^0\nu_\tau$	0.8386 ± 0.0141
$\pi^-\bar{K}^0\pi^0\nu_\tau$	0.3812 ± 0.0129
$\pi^-\bar{K}^0\pi^0\pi^0\nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\bar{K}^0h^-h^-h^+\nu_\tau$	0.0222 ± 0.0202
$K^-\eta\nu_\tau$	0.0155 ± 0.0008
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012
$\pi^-\bar{K}^0\eta\nu_\tau$	0.0094 ± 0.0015
$K^-\omega\nu_\tau$	0.0410 ± 0.0092
$K^-\phi\nu_\tau$ ($\phi \to K^+K^-$)	0.0022 ± 0.0008
$K^-\phi\nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0015 ± 0.0006
$K^-\pi^-\pi^+\nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$K^-2\pi^-2\pi^+\nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^-\nu_\tau$	2.9087 ± 0.0482

$|V_{us}|$ from tau BRs using the HFLAV Spring 2017 fit



- $\tau \to s$ inclusive vs. CKM unitarity discrepancy: $-3.0\,\sigma$
  - no significant change since the first HFLAV fit in 2010
- $m_s = 95.00 \pm 6.70\,\text{MeV}$ (PDG 2015), form factors from FLAG 2016
- $\delta R_\tau = 0.242 \pm 0.033$ (E. Gamiz *et al.*, arXiv:hep-ph/0612154v1)
- details in HFLAV Spring 2017 report, Eur. Phys. J. C77 (2017) 895 , arXiv:1612.07233

BABAR preliminary tau BRs measurements, T. Lueck, ICHEP 2018

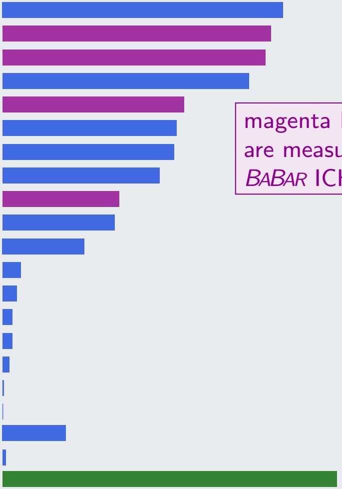
simultaneous measurement of 6 modes:

$$\tau \to K\ n\pi^0\ \nu \quad (\text{ex. } K^0 \to 2\pi^0, \text{ ex. } \eta \to 3\pi^0) \quad n = 0, 1, 2, 3$$
$$\tau \to \pi\ n\pi^0\ \nu \quad (\text{ex. } K^0 \to 2\pi^0, \text{ ex. } \eta \to 3\pi^0) \quad n = 3, 4$$

- simultaneous measurements to deal with cross-feeds between signal modes
- photon efficiency calibrated on data
- correct with data studies poor simulation of hadronic split-off fake photons in EMC

HFLAV $|V_{us}|$ from $\tau \to s$ inclusive uncertainties budget (%) for the HFLAV Spring 2017 determination

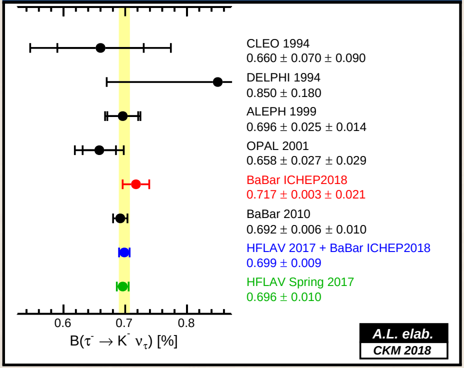
$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.3963
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.3789
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.3715
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3478
$K^- \pi^0 \nu_\tau$	0.2561
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2456
$\pi^- \bar{K}^0 \nu_\tau$	0.2424
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2219
$K^- \nu_\tau$	0.1646
$K^- \omega \nu_\tau$	0.1585
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1157
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0256
$K^- \pi^0 \eta \nu_\tau$	0.0200
$K^- \eta \nu_\tau$	0.0138
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0138
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0096
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0896
$\mathcal{B}_e^{\text{univ}}$	0.0045
theory	0.4722



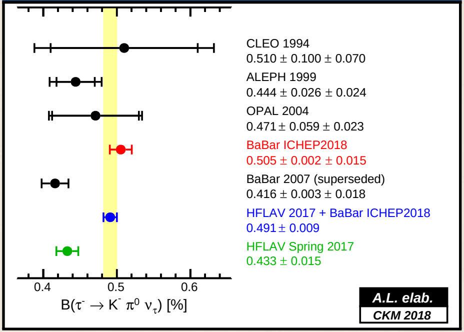
magenta BRs are measured in BABAR ICHEP 2018

Preliminary global HFLAV fit with BABAR ICHEP 2018 results

$\mathcal{B}(\tau \to K\nu)$ measurements and fits



$\mathcal{B}(\tau \to K\pi^0\nu)$ measurements and fits



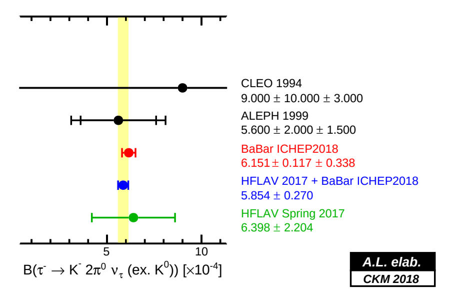
- BABAR 2010 $\mathcal{B}(\tau^- \to K^- \nu_\tau)$ (more precise) statistically independent because measured on 3–1 prongs tau pairs

- new result supersedes BABAR 2007 $\mathcal{B}(\tau^- \to K^- \pi^0 \nu_\tau)$

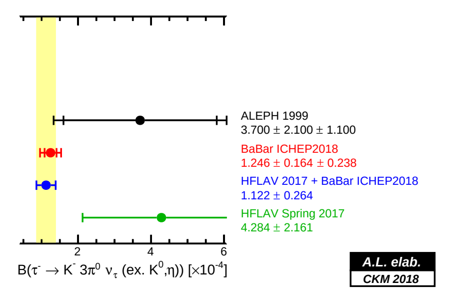
- the plots report a branching fraction and its existing measurements, but note that in the HFLAV global fit also other indirect measurements (ratios of branching fractions, or more inclusive branching fractions) can contribute in a significant way to determine the fit result for the plotted quantities.

Preliminary global HFLAV fit with *BABAR* ICHEP 2018 results

$\mathcal{B}(\tau \to K 2\pi^0 \nu)$ measurements and fits



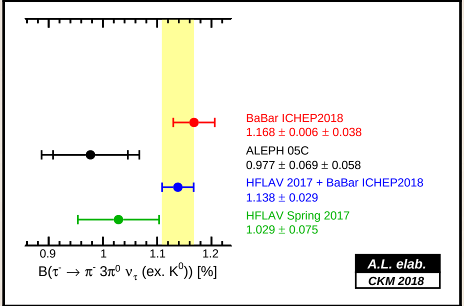
$\mathcal{B}(\tau \to K 3\pi^0 \nu)$ measurements and fits



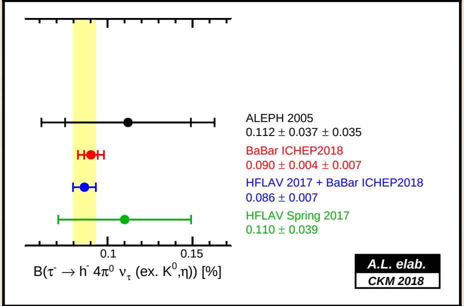
- the plots report a branching fraction and its existing measurements, but note that in the HFLAV global fit also other indirect measurements (ratios of branching fractions, or more inclusive branching fractions) can contribute in a significant way to determine the fit result for the plotted quantities.

Preliminary global HFLAV fit with BABAR ICHEP 2018 results

$\mathcal{B}(\tau\to\pi 3\pi^0\nu)$ measurements and fits



$\mathcal{B}(\tau\to\pi 4\pi^0\nu)$ measurements and fits



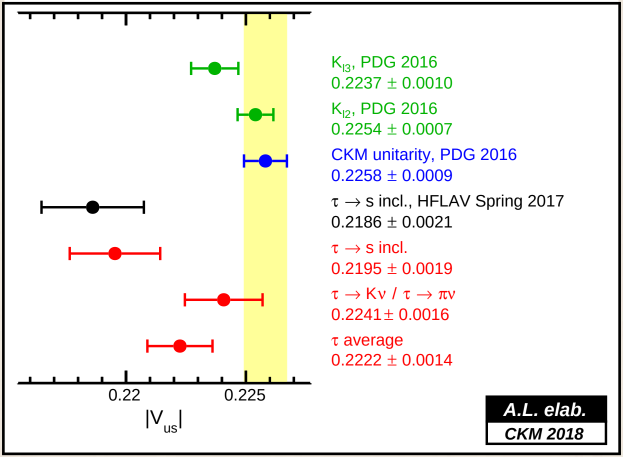
- $\mathcal{B}(\tau^-\to\pi^- 4\pi^0\nu_\tau) = \mathcal{B}(\tau^-\to h^- 4\pi^0\nu_\tau)$, where $h = \pi, K$, since $\mathcal{B}(\tau^-\to K^- 4\pi^0\nu_\tau)$ has not yet been measured and is considered to be negligible

- the plots report a branching fraction and its existing measurements, but note that in the HFLAV global fit also other indirect measurements (ratios of branching fractions, or more inclusive branching fractions) can contribute in a significant way to determine the fit result for the plotted quantities.

HFLAV Spring 2017 + BaBar ICHEP 2018 fit

Tau decay mode	Branching fraction (%)
$K^- \nu_\tau$	0.6986 ± 0.0086
$K^- \pi^0 \nu_\tau$	0.4910 ± 0.0091
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0585 ± 0.0027
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0112 ± 0.0026
$\pi^- \bar{K}^0 \nu_\tau$	0.8388 ± 0.0141
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3811 ± 0.0129
$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0154 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0022 ± 0.0008
$K^- \phi \nu_\tau$ ($\phi \to K_S^0 K_L^0$)	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9327 ± 0.0413

$|V_{us}|$ from tau, HFLAV 2017 + BABAR ICHEP 2018



- $\tau \to s$ inclusive vs. CKM unitarity discrepancy: $-3.0\,\sigma$
  - no significant change, $|V_{us}|$ increased a bit, uncertainty reduced

Use precisely measured kaon BRs to predict tau BRs

M. Antonelli *et al.*, JHEP 10 (2013) 76

$$\mathcal{B}(\tau \to K\nu_\tau) = \frac{m_\tau^3}{2 m_K m_\mu^2} \frac{S_{\mathbf{EW}}^\tau}{S_{\mathbf{EW}}^K} \left(\frac{1 - m_K^2/m_\tau^2}{1 - m_\mu^2/m_K^2} \right)^2 \frac{\tau_\tau}{\tau_K} R_{\mathbf{EM}}^{\tau/K} \mathcal{B}(K_{\mu 2})$$

new:

$$\mathcal{B}(\tau \to \bar{K}\pi\nu_\tau) = \frac{2 m_\tau^5}{m_K^5} \frac{S_{\mathbf{EW}}^\tau}{S_{\mathbf{EW}}^K} \frac{I_K^\tau}{I_K^\ell} \frac{\left(1 + \delta_{\text{EM}}^{K\tau} + \tilde{\delta}_{\text{SU(2)}}^{K\pi}\right)^2}{\left(1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}\right)^2} \frac{\tau_\tau}{\tau_K} \mathcal{B}(K \to \pi e \bar{\nu}_e)$$

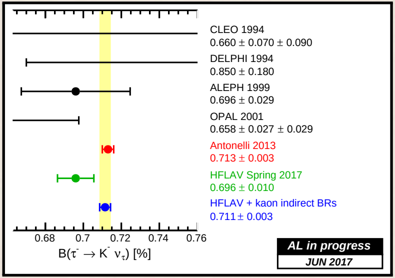
[and similar formula for $\mathcal{B}(\tau \to K\pi^0\nu)$]

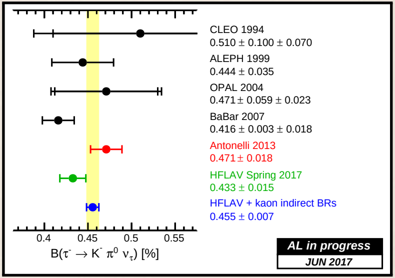
phase space integrals I_K^τ require tau spectral functions

$$I_K^\tau = \frac{1}{m_\tau^2} \int_{s_{K\pi}}^{m_\tau^2} \frac{ds}{s\sqrt{s}} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + \frac{2s}{m_\tau^2}\right) q_{K\pi}^3(s) |\bar{f}_+(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi}(s) \, |\bar{f}_0(s)|^2 \right]$$

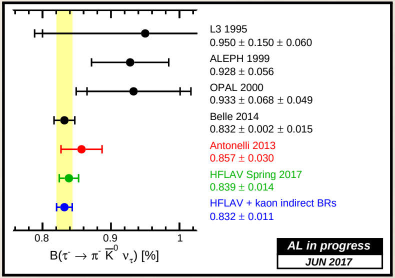
- results:
  - $\mathcal{B}(\tau \to K\nu) = (0.713 \pm 0.003)\%$
  - $\mathcal{B}(\tau \to K\pi^0\nu) = (0.471 \pm 0.018)\%$
  - $\mathcal{B}(\tau \to K^0\pi\nu) = (0.857 \pm 0.030)\%$
- note: the latter two uncertainties are 100% correlated

Tau BRs from kaon BRs, compared to measurements of tau BRs

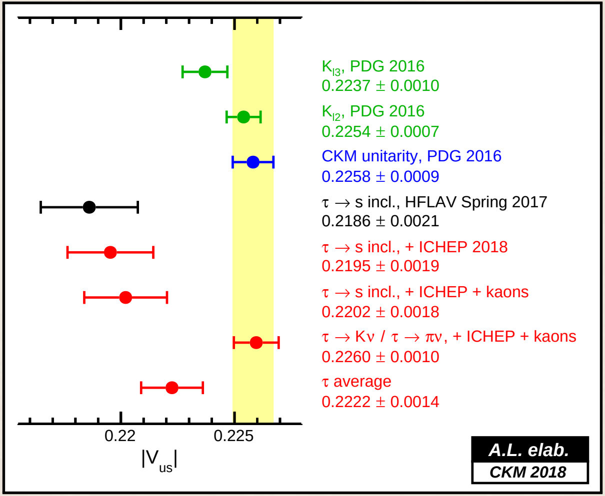




- red: tau BR predicted using kaon BR
- green: HFLAV average using only tau BRs inputs
- blue: HFLAV average combining tau BRs inputs and predictions from kaon BRs



$|V_{us}|$ from tau using HFLAV, *BABAR* ICHEP 2018, kaon predictions



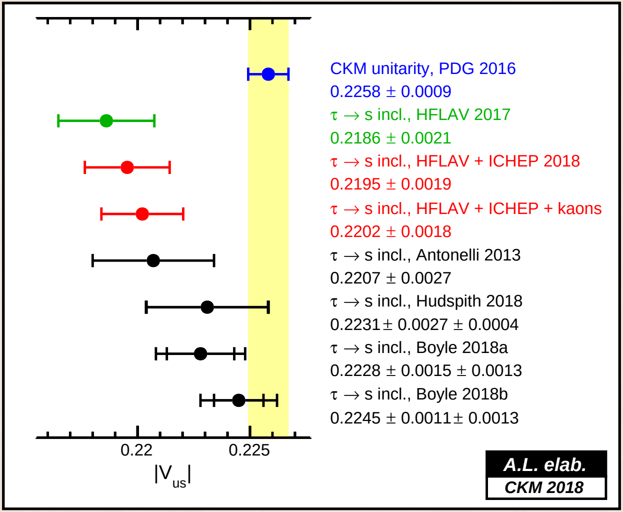
- $\tau \to s$ inclusive vs. CKM unitarity discrepancy: $-2.7\,\sigma$
- most complete unbiased usage of exp. data for $|V_{us}|$ with $\tau \to s$ inclusive

Other $|V_{us}|$ from $\tau \to s$ inclusive determinations (non exhaustive)

- M. Antonelli *et al.*, JHEP 10 (2013) 76
  predict K, $K\pi^0$, $K_s^0\pi$ tau BRs from kaon decays to replace tau results and compute $|V_{us}|$
  - predict K, $K\pi^0$, $K_s^0\pi$ tau BRs from kaon decays to replace tau results
  - compute $|V_{us}|$ with the standard technique
- J. Hudspith *et al.*, PLB 781 (2018) 206
  - compute $|V_{us}|$ from tau inclusive using also the tau spectral functions
- P. Boyle *et al.*, arXiv:1803.07228 [hep-lat]
  compute $|V_{us}|$ from tau inclusive using lattice QCD
  - Boyle 2018a: uses HFLAV tau BRs + tau spectral functions
  - Boyle 2018b: like 2018a but replaces $\mathcal{B}(\tau \to K\nu)$ with prediction from kaon BRs

- there are differences in the set of experimental inputs that are used

Comparison of $|V_{us}|$ from tau inclusive determinations



$|V_{us}|$ inclusive uncertainties budget (%)

after adding both BABAR ICHEP 2018 and kaon indirect results

$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.3908
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3430
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2422
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2178
$K^- \omega \nu_\tau$	0.1563
$\pi^- \bar{K}^0 \nu_\tau$	0.1501
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1140
$K^- \pi^0 \nu_\tau$	0.0881
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0703
$K^- \nu_\tau$	0.0473
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0414
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0253
$K^- \pi^0 \eta \nu_\tau$	0.0197
$K^- \eta \nu_\tau$	0.0136
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0136
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0095
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0020
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0853
$\mathcal{B}_e^{\text{univ}}$	0.0045
theory	0.4724

Conclusions

- B and charm-tau factories have contributed to precision tau measurements
- improved precision $|V_{us}|$ with BABAR 2018 prelim. results and kaon indirect tau BRs
- using today's most complete set of experimental inputs in an unbiased way gives $|V_{us}|$ from tau inclusive determination 2.7 σ away from CKM unitarity
- alternative more complex tau-inclusive $|V_{us}|$ determinations give $|V_{us}|$ values compatible with kaon results and CKM unitarity
- future B and charm-tau factories are useful to improve $|V_{us}|$-related SM tests by measuring small high-multiplicity tau BRs that currently dominate the uncertainty