Higgs theoretical predictions in the precision era

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Higgs measurements: a snapshot

- All major channels observed
- More differential information is now available
- Precise studies of the Higgs sector well ongoing
- By and large, theoretical predictions in very good status… but experiments are catching up quickly
Higgs theoretical predictions

- Thorough investigations of the Higgs sector possible at the (HL-)LHC
- They require accurate predictions for several complex processes → highly non trivial
- Higgs was a key player in pushing forward collider phenomenology → in general, very refined predictions available
Gluon fusion: inclusive results

- SM prediction for ggF cross-section extremely advanced
- $N^3$LO corrections to inclusive cross-section known [Anastasiou et al. (2015), Mistlberger (2018)]
- $N^3$LO residual uncertainty: few percent. At this level, many other effects play a role…

\[
\sigma = 48.58 \text{ pb}^{+2.22 \text{ pb}}_{-3.27 \text{ pb}}(+4.56\%) \ (\text{theory}) \pm 1.56 \text{ pb} (3.20\%) \ (\text{PDF}+\alpha_s).
\]

\[
48.58 \text{ pb} = 16.00 \text{ pb} \ (+32.9\%) \ (\text{LO, rEFT})
\]
\[
\quad \quad + 20.84 \text{ pb} \ (+42.9\%) \ (\text{NLO, rEFT})
\]
\[
\quad \quad - 2.05 \text{ pb} \ (-4.2\%) \ ((t, b, c), \text{exact NLO})
\]
\[
\quad \quad + 9.56 \text{ pb} \ (+19.7\%) \ (\text{NNLO, rEFT})
\]
\[
\quad \quad + 0.34 \text{ pb} \ (+0.7\%) \ (\text{NNLO, } 1/m_t)
\]
\[
\quad \quad + 2.40 \text{ pb} \ (+4.9\%) \ (\text{EW, QCD-EW})
\]
\[
\quad \quad + 1.49 \text{ pb} \ (+3.1\%) \ (\text{N}^3\text{LO, rEFT})
\]

Todo List:
- Full mass dependent NNLO
- Mixed $\mathcal{O}(\alpha\alpha_s)$ corrections
- $N^3$LO PDFs

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progress: Melnikov, Penin (2016); Melnikov, et al. (2016-18); Jones, Kerner, Luisoni (2018)

progress: Bonetti, Melnikov, Tancredi (2017-18); Anastasiou et al. (2018)

SEE E. FURLAN’S TALK THIS AFTERNOON
Gluon fusion: going differential

- Inclusive cross section is an idealised quantity, very far from what we measure
- Reliable prediction: properly model fiducial volume of experiment → fully differential. Only known at NNLO [+PS]
- H is scalar: fully differential = p_t + rapidity

Higgs rapidity (ggF)

- Computed @N^3LO in a soft expansion (~inclusive)
- Expected to work very well (apart from end-points)
- Remarkably flat K-factor (as expected from previous orders)
- Combined with p_t@NNLO, can give access to N^3LO fiducial volume
$p_{t,H}$: a major probe for Higgs physics

- **Light Yukawas…**
- **Highest precision**
- **ggH vs ttH, EFT…**

**Low $p_t$**

**Bulk of the distribution**
$p_{T,H}$: a major probe for Higgs physics

**ATLAS Preliminary**

$H \rightarrow \gamma \gamma$, $\sqrt{s} = 13$ TeV, 79.8 fb$^{-1}$

- Data, tot. unc.
- Syst. unc.
- $gg \rightarrow H$ default MC + XH
- NNLOJET@SCET + XH
- $XH = VBF + VH + ttH + bbH$

**CMS Preliminary**

$137.1$ fb$^{-1}$ (13 TeV)

- Systematic uncertainty
- $gg \rightarrow H$ (NNLOPS) + XH
- $gg \rightarrow H$ (POWHEG) + XH
- $XH = VBF + VH + ttH$ (POWHEG)

(LHC HXSWG YR4, $m_H = 125.09$ GeV)

- Ratio to NNLOPS
- $\sigma_{f/d} = 2.73 \pm 0.23$ fb
- $\sigma_{SM} = 2.76 \pm 0.14$ fb

**Higgs coupling to light quarks**

- $H$ produced dominantly via top-quark loop (largest coupling)
- but interference effects with light quarks are not negligible
- provided theoretical predictions are accurate enough (few%?), constraint on charm (and possible strange) Yukawa can be significantly improved
**Higgs $p_t$: the bulk of the distribution**

- In the region $p_t \ll m_t$: $ggF$ effective vertex, point-like interaction $\rightarrow$ **massive simplification for calculations**

- $m_t$-suppressed terms under good control see e.g. [Neumann et al (2016)]

- In the HEFT approximation: fully differential NNLO $p_t$ distribution known for quite a long time:
  - Boughezal, FC et al. (2015)
  - Boughezal et al., SCET-based (2015)
  - Chen, Gehrmann, Glover, Jaquier (2015)
  - Ellis, Campbell, Seth (2019) $\rightarrow$ **detailed validation of the different methods**

- At small $p_t$: fixed-order non reliable $\rightarrow$ match with resummation
The bulk of the distribution: \( \text{NNLO} + N^3\text{LL} \)

- Sophisticated \( \text{NNLO} + N^3\text{LL} \) results available, done in two different ways
- The two calculation have the same fixed-order and resummation accuracies
- They only differ by subleading effects (matching procedure...) \( \rightarrow \) \text{test for robustness}
- By and large: very stable fixed-order result down to \(~40\ \text{GeV} \) \( \rightarrow \) very good (fully differential) control of the bulk of the distribution
Low $p_t$: light quark effects

- For $m_q \ll p_t \ll m_H$: amplitude develops non-Sudakov double logs
  \[ y_q m_q / m_H \left[ \ln^2 \left( m_H^2 / m_q^2 \right), \ln^2 \left( p_t^2 / m_q^2 \right) \right] \]

- Despite $y_{b,c} \ll y_t$, interference effects may be visible \(\rightarrow\) constrain Yukawas!

- Also: direct $q\bar{q} \rightarrow Hg$ impacts Higgs $p_t \rightarrow$ powerful constraints for light Yukawas

**PROBLEM: control over QCD corrections**

- Resolved quark loop \(\rightarrow\) very difficult loop amplitudes
  \* beyond state-of-the-art for analytic calculations
  \* large logs \(\rightarrow\) numerical approaches difficult

- Low $p_t$, large logs \(\rightarrow\) all-order effects must be considered
t/b interference: not so long ago

- *t/b* interference only known at LO
- non trivial interplay with collinear gluons → “standard” resummation machinery does not work. All-order effects non-trivial, and unknown
- not enough information for a proper fixed-order / resummation matching

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**A pragmatic approach**

- use all the available information
- resum under 2 extreme assumptions:
  - *b/t* contributions on the same footing
  - *no resummation after* $p_T \sim m_b$
- Large residual uncertainties
t/b interference: NLO

- 2-loop amplitude for b contribution computed in the limit $m_b \ll p_t \ll m_h$ [Melnikov, Tancredi, Wever (2016-17)]

- Approximation expected to be very good for all pheno applications

![Graph showing relative top-bottom interference contribution to $pp \rightarrow H + j$ at 13 TeV](image)

- Large $K$-factor…
- … but similar to HEFT
- Large source of unc. from $b$-mass scheme
- Non-trivial logarithmic structure
- Still don’t know how to resum [some work in this direction: Melnikov, Penin (2016); Forte et al (2016); Penin, Liu (2018)]
t/b interference: matching with resummation

NLO result allows for a proper matching → resummation ambiguities much less severe
Major source of uncertainty from $b$-mass scheme → can only be improved with higher order calculation

It will be very hard to improve in this direction

A common feature of processes involving (active) massive virtual quarks...
• Boosted Higgs very sensitive to BSM contributions, internal structure of $ggH$ coupling…

• **Problem**: very difficult (multi)-loop amplitudes. Going beyond LO non trivial
Boosted Higgs very sensitive to BSM contributions, internal structure of ggH coupling…

- Boosted Higgs very sensitive to BSM contributions, internal structure of ggH coupling.
- Problem: very difficult (multi)-loop amplitudes. Going beyond LO non trivial.
- NLO would require complicated 2-loop amplitudes, currently under investigation.

J. Henn

CMS-PAS-HIG-17-010

Higgs @ large $p_T$:...
Boosted Higgs: NLO

NLO is finally known. 2 approaches:

- analytic result under the assumption $m_{t,h} \ll p_t$ [Kudashkin, Melnikov, Wever; Lindert (2017-18)]
- exact numerical result [Jones, Kerner, Luisoni (2018)]

They agree within expectation $\rightarrow$ important validation

- Large $K$-factor
- Very similar to HEFT $K$-factor:
  As expected from
  *merged samples approach* [see e.g. Frederix et al (2016), Greiner et al (2016)]
  *approximate $m_t$ treatment* [see e.g. Neumann and Williams (2016)]
  *resummation analysis* [see e.g. Muselli et al (2016)]

Can combine with NNLO
HEFT $K$-factor
Boosted Higgs: all channels

At large $p_t$, the ggF dominance becomes less pronounced → important to include all channels.

LHCHXSWG (2019)

- Interesting interplay of different channels. Different pattern of radiative corrections
- NLO EW corrections in ggF? $\ln^2(p_t/m_t)$?
Beyond ggF: vector boson fusion

Also in this case, N^3LO predictions are known for quantities inclusive over jet activity (not jet requirement/cut possible) [Dreyer, Karlberg (2016)]

- Tiny corrections ~ few permill
- No kinematic feature on top of NNLO
- Is it the end of it? Not so fast…
VBF beyond the DIS approximation

• Typically, VBF predictions are computed in the DIS/“structure functions”/“factorized” approximation [Han, Valencia, Willenbrock (1992)]

• In this approximation, one consider emission from the two quark legs independently, without considering any cross talk

• Results can be borrowed from DIS → much simpler

• Corrections to this approximation expected to be small after VBF cuts (first appear at NNLO, color/kinematics suppression)

• … but are they small compared to (inclusive) precision (~per mill)?
VBF beyond the DIS approximation

- NNLO exact VBF calculation out of reach \((\text{two-loop } 2 \to 3 \text{ amplitudes well beyond what we can imagine doing in the near future})\)

- However, possible to estimate the leading non-factorizable contributions the VBF region \((\text{two forward/backward tagging jets}) [\text{Liu, Melnikov, Penin (2019)}]\)

- As expected, corrections to inclusive quantities small \((\sim 4 \text{ permill}), \text{ although larger than } N^3\text{LO}\)

- Interestingly, small corrections come as a cancellation between positive and negative corrections to differential distributions \(\rightarrow \text{ can reach percent-level in differential distributions}\)
VBF: fully differential results

• For VBF, crucial to proper model the experimental setup (jet requirements)
• Full NNLO(+NLO EW) results in the DIS approximation known

[Cacciari et al (2015)]

• Corrections in the VBF region much larger than for the inclusive case (most likely due to non-trivial jet dynamics)
• Residual uncertainty ~2-3% → non-factorizable contributions smaller, but barely
• For some distribution, bad disagreement with PS → NNLOPS?

[Cruz-Martinez, Glover, Gehrmann, Huss (2018)]
**VBF: fully differential results**

- For VBF, crucial to proper model the experimental setup (*jet requirements*)
- Large differential corrections: VBF very sensitive to tagging jet cuts and jet radius

![Graph showing differential corrections with jet radius](image)

- NNLO corrections change by ~20% from R=0.1 to R=1.0
- It would be interesting to understand it better
  - NNLO for VBF+j
  - NNLOPS [only major channel where this is missing…]
VH: status

- VH: known at NNLO QCD + NLO EW for quite some time

![Diagram of VH process]

- DY-like: very good control

- gg → ZH
  - delicate SM unitarity cancellations between box/triangle → very good probe for new physics [see e.g. Englert, McCullough, Spannowsky (2013), Harlander et al (2019)]
  - formally, it starts contributing at NNLO, but enhanced by gluon flux (~10/20% of total NNLO cross-section)
  - only known to LO → large residual uncertainties
  - currently, only approximate result [Hasselhuhn, Luthe, Steinhauser (2016)], but simplest yet-to-be-computed gg → XY process mediated by top loop. Within current (numerical) technology → expect results soon?
VH: a faithful description of measurements

- **VH:** important channel for $H \rightarrow b\bar{b}$
- **Ideally:** boosted region. In **practice:** semi-boosted ($p_{t,V} > 150$ GeV)
- In the boosted region, decay should be very collinear → well described by PS
- Interesting to study the interplay fixed-order/PS in the semi-boosted region…
- **Fixed-order:** full NNLO (production⊗decay) [Ferrera, Grazzini Tramontano (2017); FC, Luisoni, Melnikov, Röntsch (2017); Gauld, Gehrmann-de Ridder, Glover, Huss, Majer (2019)]

**WH, Reconstructed Higgs $p_t$, $NLO_{dec}$ vs $NNLO_{dec}$**

Large impact of radiation from final state $bs$
VH: a faithful description of measurements

How well is the radiation pattern described by PS?

- Off-the-shelf PS seems to capture some of the radiation pattern
- Now very sophisticated NNLOPS_{prod}⊗NLOPS_{dec} available [Astill et al (2018)]. Similar pattern
- Delicate issues about HF-identification ($b$-tagging vs flavour $k_t$)
- More apple-to-apple investigations desirable (massive $b$…)

**WH, NNLO vs off-the-shelf PS**

$WH, NNLO vs off-the-shelf PS$

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$WH, NNLO vs off-the-shelf PS$

$WH, NNLO vs off-the-shelf PS$
VH: a faithful description of measurements

How well is the radiation pattern described by PS?

- Massive $b$ calculation available [Berneuther, Chen, Si (2018); Primo, Sasso, Somogyi, Tramontano (2018)] → jet algorithm/full $b$-reconstruction studies

- Furthermore: fully differential $H \rightarrow b\bar{b}$ available [Mondini, Schiavi, Williams (2019)] → more detailed studies on radiation patterns
\( \bar{t}tH \): the devil in the background...

- Direct probe of top Yukawa coupling
- Known to NLOQCD (+NNLL) + NLOEW, including off-shellness and interference
- Fiducial cuts enhance tails \( \rightarrow \) NLOEW
- \( d\sigma \propto y_t^2 \) no longer true @NLOEW

- Proper description of background problematic.
  Most famous example: \( ttbb \)

<table>
<thead>
<tr>
<th>Selection</th>
<th>Tool</th>
<th>( \sigma_{\text{NLO}} ) [fb]</th>
<th>( \sigma_{\text{NLO+PS}} ) [fb]</th>
<th>( \sigma_{\text{NLO+PS}} / \sigma_{\text{NLO}} )</th>
</tr>
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<tbody>
<tr>
<td>( n_b \geq 1 )</td>
<td>SHERPA+OPENLOOPS</td>
<td>( 12820^{+35%}_{-28%} )</td>
<td>( 12939^{+30%}_{-27%} )</td>
<td>1.01</td>
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<td></td>
<td>MadGraph5_AMC@NLO</td>
<td>( 13835^{+37%}_{-29%} )</td>
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<tr>
<td></td>
<td>PowHEl</td>
<td>( 10073^{+45%}_{-29%} )</td>
<td></td>
<td>0.79</td>
</tr>
<tr>
<td>( n_b \geq 2 )</td>
<td>SHERPA+OPENLOOPS</td>
<td>( 2268^{+30%}_{-27%} )</td>
<td>( 2413^{+21%}_{-24%} )</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>MadGraph5_AMC@NLO</td>
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<td>1.41</td>
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<tr>
<td></td>
<td>PowHEl</td>
<td>( 2570^{+35%}_{-28%} )</td>
<td></td>
<td>1.13</td>
</tr>
</tbody>
</table>

- Shower effects enhanced in the Higgs region...
\( \bar{t}tH: \) the devil in the background...


- A lot of complex delicate issues... cannot make justice to it in a few minutes. Just few highlights, see talks by S. Pozzorini at the HXSWG meetings for more details

Most likely cause of bad behavior: \textbf{LARGE K-FACTOR ENHANCED BY SHOWER}

### NLOPS YR4 scales

- Here is a chart showing the distribution of \( p_T \) for the first light-jet (ttbb cuts) for different scales.

### NLOPS 0.5 rescaling

- Here is another chart showing the distribution of \( p_T \) for the first light-jet (ttbb cuts) with 0.5 rescaling.

- **The good news**: a more appropriate scale choice removes part of the issue

- **The bad news**: this does not remove large shower corrections in the \( N_b=2 \) bin
**ttH: the devil in the background...**

**Most likely cause of bad behavior:** LARGE K-FACTOR ENHANCED BY SHOWER

- **The bad news:** clever scale choice does not remove large shower corrections in the $N_b=2$ bin
- **Most likely culprit:** large recoil effect / bin migration
- **To fix it:** need to understand better QCD radiation pattern, find good observables sensitive to it

Once again, it would be crucial to better understand jet dynamics, $g \rightarrow b \bar{b}$ splitting etc...

Very interesting theoretical problem, not limited to $ttH$ (e.g.: $V+HF$ for VH...)

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Azimuthal correlation $\Delta \phi_{\text{rec,1}}$ between recoil and 1st top
HH: good theory, but difficult to improve…

• HH production: direct probe of Higgs self-coupling

Still far from measurements, but still important to have good theoretical control.

• The (usual) problem: LO is loop-induced → NLO is already 2-loop, with massive virtual fermions → cannot do it analytically yet (although a lot of progress…)

• Same problem of boosted Higgs, gg→ZH, gg→VV/off-shell interference…

• In some sense, the "simplest" process in this class → a lot of attention.

• Analytic side: several approximations.

• Numerical techniques developed, we now have full NLO result [Borowka et al (2016), Baglio et al (2018)]
HH@NLO: lesson learned

• Reasonable approximations to extend $1/m_t$ result beyond the top threshold (rescaled Born, exact real radiation) can fail quite significantly

• Exact K-factor much less flat than for $m_t$ approximations

• It would be interesting to study different approximations, to understand better what is going on [see e.g. Xu, Yang (2018)]

• It would be interesting to study other processes, to gain extra information (ZZ, Hj, VH)
HH@NLO: applications

- NLO calculations used as a basis for several applications. For example:
  - NLOPS [Heinrich et al. (2017)]
  - Informing analytic approximations to extend calculation at high invariant mass [Davies et al. (2019)]
  - NLO+NNLO$_{mt\to\infty}$ [de Florian et al. (2016), Grazzini et al. (2018)], +NNLL$_{\text{soft}}$ [de Florian, Mazzitelli (2018)]

A very good control...

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>13 TeV</th>
<th>14 TeV</th>
<th>27 TeV</th>
<th>100 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO [fb]</td>
<td>27.78 $^{+13.8%}_{-12.8%}$</td>
<td>32.88 $^{+13.5%}_{-12.5%}$</td>
<td>127.7 $^{+11.5%}_{-10.4%}$</td>
<td>1147 $^{+10.7%}_{-9.9%}$</td>
</tr>
<tr>
<td>NLO$_{\text{FTapprox}}$ [fb]</td>
<td>28.91 $^{+15.0%}_{-13.4%}$</td>
<td>34.25 $^{+14.7%}_{-13.2%}$</td>
<td>134.1 $^{+12.7%}_{-11.1%}$</td>
<td>1220 $^{+11.9%}_{-10.6%}$</td>
</tr>
<tr>
<td>NNLO$_{\text{NLO-i}}$ [fb]</td>
<td>32.69 $^{+5.3%}_{-7.7%}$</td>
<td>38.66 $^{+5.3%}_{-7.7%}$</td>
<td>149.3 $^{+4.8%}_{-6.7%}$</td>
<td>1337 $^{+4.1%}_{-5.4%}$</td>
</tr>
<tr>
<td>NNLO$_{\text{B-proj}}$ [fb]</td>
<td>33.42 $^{+1.5%}_{-4.8%}$</td>
<td>39.58 $^{+1.4%}_{-4.7%}$</td>
<td>154.2 $^{+0.7%}_{-3.8%}$</td>
<td>1406 $^{+0.5%}_{-2.8%}$</td>
</tr>
<tr>
<td>NNLO$_{\text{FTapprox}}$ [fb]</td>
<td>31.05 $^{+2.2%}_{-5.0%}$</td>
<td>36.69 $^{+2.1%}_{-4.9%}$</td>
<td>139.9 $^{+1.3%}_{-3.9%}$</td>
<td>1224 $^{+0.9%}_{-3.2%}$</td>
</tr>
<tr>
<td>$M_t$ unc. NNLO$_{\text{FTapprox}}$</td>
<td>±2.6%</td>
<td>±2.7%</td>
<td>±3.4%</td>
<td>±4.6%</td>
</tr>
<tr>
<td>NNLO$_{\text{FTapprox}}$/NLO</td>
<td>1.118</td>
<td>1.116</td>
<td>1.096</td>
<td>1.067</td>
</tr>
</tbody>
</table>

... with a very big caveat
HH@NLO: mass-scheme dependence

- Result depends non-trivially on the renormalisation scheme and scale for the top quark mass [Baglio et al (2019)]
- Ambiguities substantially larger than "standard" uncertainties (careful in identify TH uncertainty with "naive" scale variation...)

\[
\begin{align*}
\frac{d\sigma(gg \rightarrow HH)}{dQ} \bigg|_{Q=300 \text{ GeV}} &= 0.0298(7)^{+6\%}_{-34\%} \text{ fb/GeV}, \\
\frac{d\sigma(gg \rightarrow HH)}{dQ} \bigg|_{Q=400 \text{ GeV}} &= 0.1609(4)^{+0\%}_{-13\%} \text{ fb/GeV}, \\
\frac{d\sigma(gg \rightarrow HH)}{dQ} \bigg|_{Q=600 \text{ GeV}} &= 0.03204(9)^{+0\%}_{-30\%} \text{ fb/GeV}, \\
\frac{d\sigma(gg \rightarrow HH)}{dQ} \bigg|_{Q=1200 \text{ GeV}} &= 0.000435(4)^{+0\%}_{-35\%} \text{ fb/GeV},
\end{align*}
\]

- Unfortunately, natural to expect (and also seen in b-contribution to the Higgs \( p_t \) spectrum [Melnikov, Tancredi, Wever (2017)])
- For bulk ggF: top is not active \( \rightarrow \) effect not there (this is the exception!)
- Honest solution: one order higher, i.e. NNLO for a process for which we can only barely compute NLO... quite some fun ahead...
Conclusions

• A 125 GeV Higgs: sweet spot for thorough studies of its properties
• LHC measurements progressing very fast
• Higgs has always been one of the main player in pushing our understanding of QCD and collider phenomenology
• A lot of recent progress → could not make justice to it. Among missing items
  • Off-shell
  • Background issues (ggF contamination to VBF, PS/UE effects…)
  • EW
  • Higgs and complex final states
  • EFT/BSM. Future colliders…
• Apologies if I skipped your favourite topic!

• The general picture: theory in a pretty good shape, but still a lot to be done
• In many cases, this requires some non-trivial improvement in our understanding of QCD/EW/collider pheno, that would have actual implication for real-world Higgs explorations → **EXCITING TIMES AHEAD!**
Thank you very much!