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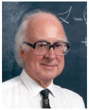
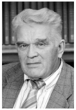
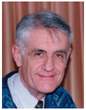
Paris/Orsay
10th Workshop ≪Higgs Hunting≫
July 29, 2019

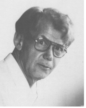
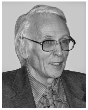
Gerard 't Hooft

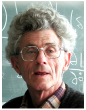
Higgs particle and renormalization





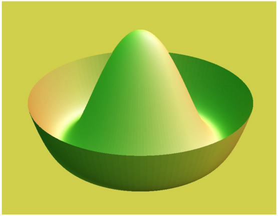






Cargèse June/July 1970

M. Lévy,
B.W. Lee,
J.-L. Gervais
K. Symanzik



about the Lee - Lévy σ model

(pions are the Goldstone bosons of the partially broken, spontaneously broken chiral $SU(2)_L \times SU(2)_R$ symmetry):

The model can be renormalized; the renormalised pions are still Goldstone bosons - but counter terms have to be chosen carefully.

Yet it is all about scalar (spin 0) fields and Dirac (spin ½) fields. Vector (Yang-Mills, spin 1) fields were out of the question.

For renormalisability, a new, unstable hadron, σ, was required.

Even for these scalar theories (now considered to be simple), renormalization of two-loop diagrams, and certainly proving renormalisability at all orders, was considered to be very difficult.

Theories for vector particles seemed to be hopeless.
Infinities generated were not Lorentz invariant (Schwinger terms).

M. Veltman had developed processes to investigate how Feynman diagrams have to be arranged, and how infinities have to be regularised so as to examine
unitarity and causality:
insert "ghost particles" to be removed again later.

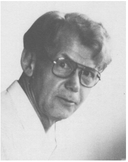
"Bell - Treiman transformation": a transformation that looks like a gauge transformation, involving this ghost field.

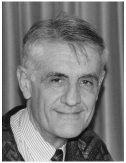
Veltman had noticed that the Yang-Mills Lagrangian could be useful to describe weak interactions. But a mass term for the vector bosons was needed. This would violate local gauge invariance, but only "softly", so he expected that the UV divergences could be kept under control.

He saw no use for the BEH mechanism,

"that's for mathematicians"

Here, the insights obtained by Lee, Gervais, and Symanzik should be helpful.





But only Veltman had the machinery to do what had to be done.

On-shell and off-shell propagators. In momentum space,

$$k = (k^0,\ \vec{k})\ , \qquad g_{\mu\nu} = (-,+,+,+)\ :$$

$$\Delta^F(k) = \frac{-i}{k^2 + m^2 - i\varepsilon}\ , \qquad \Delta^{\pm}(k) = 2\pi\delta(k^2 + m^2)\theta(\pm k^0)\ .$$

In position space:

$$\Delta^F(x) = \int \mathrm{d}^4k\, e^{ikx}\, \Delta^F(k)\ , \qquad \Delta^{\pm}(x) = \int \mathrm{d}^4k\, e^{ikx}\, \Delta^{\pm}(k)\ .$$

By contour integration:

$$\begin{aligned} \Delta^F(x) &= \theta(x^0)\Delta^+(x) + \theta(-x^0)\Delta^-(x)\ , \\ \Delta^{F*}(x) &= \theta(x^0)\Delta^-(x) + \theta(-x^0)\Delta^+ x)\ . \end{aligned} \tag{1}$$

For the vertices $n = 1, \ldots, N$ in a diagram, arrange the coordinates x_n in the order of time, $x_1^0 < x_2^0 < \cdots < x_N^0$, to obtain the *cutting relations*, dispersion equations relating diagrams on-shell and off-shell:

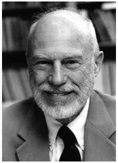
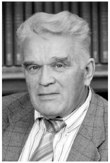
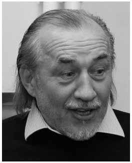
$$\sum S|\ \rangle\langle\ |S^\dagger = \mathbb{I}\ .$$

The sum is over the intermediate states $|\ \rangle$. These are on-shell. In S, we use the propagators $\Delta^F(k)$, in $S^\dagger$, we use $\Delta^{F*}(k)$.

The relation only holds if the Lagrangian $\mathcal{L}$ is real: $\mathcal{L} = \mathcal{L}^*$.

All this should be obvious: the evolution operator is unitary in a theory where the Lagrangian $\mathcal{L}$ and the ensuing Hamiltonian H are real (Hermitian) expressions of the fields.

But here we see how this is related to the Feynman diagrams, so that unitarity is seen to be safeguarded if the relations in question are ensured to hold when the infinities are subtracted.

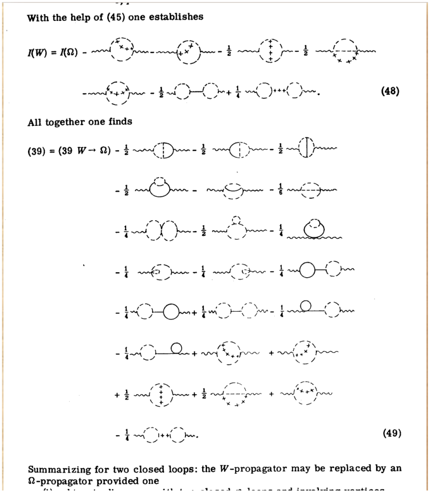

Massive and massless Yang-Mills theories had been studied (inter alios) by Feynman, DeWitt, Faddeev and Popov, and Mandelstam, motivated by gravity theory.

All used *different* Feynman rules. Were most of these wrong?

No.

Fixing the gauge can be done in many ways. But the mass term would break gauge invariance. Even if the mass term is soft, it has to be made gauge-invariant. Here is where the Higgs mechanism comes in.

Veltman had developed a computer program, “Schoonschip”, to handle the complicated algebra for Yang-Mills fields, enriched with a mass term.


With the help of (45) one establishes

$I(W) = I(\Omega)$ - ... (48)

All together one finds

(39) = (39 $W \to \Omega$) - ... (49)

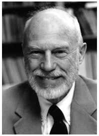
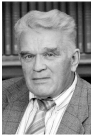
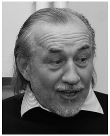
Summarizing for two closed loops: the W-propagator may be replaced by an Ω-propagator provided one

But if you merely add a mass term to the Yang-Mills equations, you add longitudinal degrees of freedom to the vector particles,

where the kinetic terms are missing.

This cannot be unitary.

“Why not? Feyman did it the same way . . . .”

But Feynman had not done it quite right. He had made pioneering attempts in his Polish lecture notes. He found the ghost particles of massive Yang-Mills, at one-loop level. $-1 = 1 - 2$
He wisely stopped there.

B.S. DeWitt had it almost right, but he thought that the infinities should not depend on the gauge-fixing. They do.

Faddeev and Popov had a short paper containing all the essentials needed to understand how it should be done: *path integrals!*

None of them handled the BEH mechanism.

My first priority was to handle Yang-Mills theories at the level of Feynman diagrams, as Veltman was trying to do, but now using *absolute gauge invariance* and *gauge fixing*.

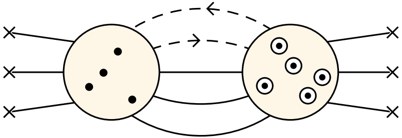
Problem was: *This looked like handling some symmetry for the gauge-fixed theory*, but:

This was *not* a symmetry! The *signs* were all wrong!

But we could handle it. Doing the combinatorics the hard way, I could prove unitarity of such theories, by proving a generalisation of the *Ward Takahashi identities*. Includes the BEH mechanism!

We derived identities for all diagrams with particles and/or ghosts *on mass shell* in the external lines.

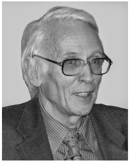
$\sum_{\text{ghosts}}$



$= 0$.

Now it became clear why Veltman had not yet found a unitary subtraction method for the infinities: he had a massive Yang-Mills boson but had not included the Higgs. He had expected a scalar particle, but with the same quantum numbers as his heavy vector bosons. Why should you add a massive scalar particle with the "wrong" quantum numbers?

But this turned out to contain the cure to all problems of non-cancelling infinities. With the Higgs particle present, all unwanted infinities disappeared (the remaining infinities should be handled by renormalization).

The Slavnov-Taylor identities

We decided first to publish this result for theories without Higgs mechanism.
I soon received 2 reactions from the outside world: A.A. Slavnov and J.C. Taylor independently noted that the identities we derived can be generalised for all external lines off mass shell. These identities became known as the *Slavnov-Taylor identities.*

In this form, these identities are easier to prove. They express the condition for the theory to obey causality and unitarity.

Anomalies?

Understanding how infinities in Feynman diagrams cancel out is important, but not the whole story. Physics is about the finite parts after removal of infinities.
In calculating them, ambiguities could arise, such as the

Adler-Bell-Jackiw anomaly:

algebra suggests that the amplitude for $\pi^0 \to \gamma\,\gamma$ should cancel to zero, but boundary terms give a non-vanishing amplitude – and so does experiment!

Can such anomalies destroy renormalisability?

The Adler-Bell-Jackiw anomaly itself could signify danger!

Are there more such anomalies ?

Dimensional Regularisation and Renormalization

And so we had to ask the question: how do we explicitly remove the infinities by some *gauge-invariant regularisation procedure ?*

Pauli-Villars or other procedures known to work for Abelian theories, here fail (diagrams where vector particles interact with one another are not regularised properly).

But we noted something:

If we add a 5^{th} dimension to the theory, while all external lines stay in 4 dimensions, then the 5^{th} dimension acts as a gauge-invariant regulator. This works only for diagrams with one closed loop.

What to do with multi-loop diagrams?

How about 6, 7, or more dimensions? The extra dimensions cause havoc in multi-loop diagrams – cannot serve as regulators. Finally we asked:

> Does the number of dimensions have to be integer?
> If we substitute a non-integer n in functions such as $\Gamma(\frac{1}{2}nd + \ell)$, what happens to the Slavnov-Taylor identities?

Turns out: they remain exactly valid. Infinities, residing in the poles of the Γ functions, become finite when d is not on a pole.

> This means we have a regulator!
> Just let $d \to 4$ sufficiently carefully.

A few more steps towards the Standard Model:

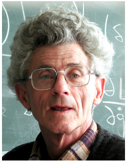
Cornwall, Levin, Tiktopoulos: Gauge theories are the *only* renormalizable theories for vector particles !

Veltman:

The massive vector bosons as well as the Higgs particles are theoretical inventions, and while few people doubt the existence of vector bosons, many are sceptical with respect to the Higgs system.

© Martin J. Veltman 1980





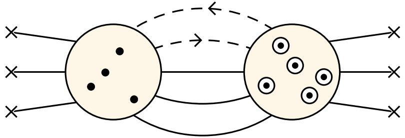
The Slavnov-Taylor identities we produced by manipulating Feynman diagrams can be derived in a much more elegant way, as was discovered by Becchi, Rouet and Stora, an independently by Tyutin.

We knew that the determinant in the functional integral can be written in terms of an anti-commuting field variable, η and $\overline{\eta}$.

If $A(x)$ represents the non-gauge-invariant fields, and the gauge-fixing field operator is $C(A)$, then the gauge fixing Lagrangian is to be written as:

$$\Delta\mathcal{L} = -\lambda C(A,\varphi) + \overline{\eta}\,\frac{\partial C(A,\varphi)}{\partial\Lambda}\eta\ ,$$

This describes the *Faddeev-Popov ghosts* η and $\overline{\eta}$ and λ is a Lagrange multiplier field.

$$\sum_{\text{ghosts}}$$  $$= 0\ .$$

This is not a symmetry, but a *super*-symmetry for the gauge-fixed Lagrangian:

BRST

$$\mathcal{L} = \mathcal{L}^{\rm inv} + \lambda^a(x) C^a(A,x) + \overline{\eta}^a(x) \frac{\partial C^a(x)}{\partial \Lambda^b(x')} \, \eta^b(x') \; + f(\lambda^a) \; .$$

$$\begin{aligned}
\delta A^a(x) &= \; \overline{\varepsilon} \, \frac{\partial A^a(x)}{\partial \Lambda^b(x')} \, \eta^b(x') \; ; \\
\delta \eta^a(x) &= \; \tfrac{1}{2} \overline{\varepsilon} \, f^{abc} \, \eta^b(x) \, \eta^c(x) \; ; \\
\delta \overline{\eta}^a(x) &= \; -\overline{\varepsilon} \, \lambda^a(x) \; ; \\
\delta \lambda^a(x) &= \; 0 \; ,
\end{aligned}$$

$$\rightarrow \quad \delta S = 0 \; .$$





Gell-Mann, Fritzsch, Minkowski: Quantum Chromo Dynamics:

No *direct* BEH mechanism.

Vacuum and particle spectrum are invariant under the local group $SU(3)$.

But the BEH mechanism is essential for understanding the mechanism that removes from the physical spectrum all particles whose fields are not invariant under $SU(3)$ (quarks) from the spectrum: they cannot occur as free particles but are *confined.* Their masses are infinite.

The kinematic properties of the quarks are understood as if the quarks are connected by vortex lines that act like strings (the dual resonance model).

These vortex lines are similar to the Meissner flux tubes.

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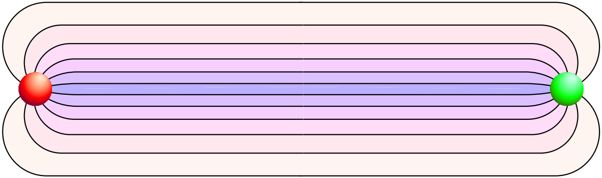

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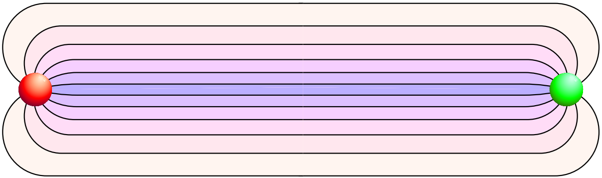

if we make the dual transformation electric $\leftrightarrow$ magnetic.

THANK YOU