Durham University

Higgs & Cosmology

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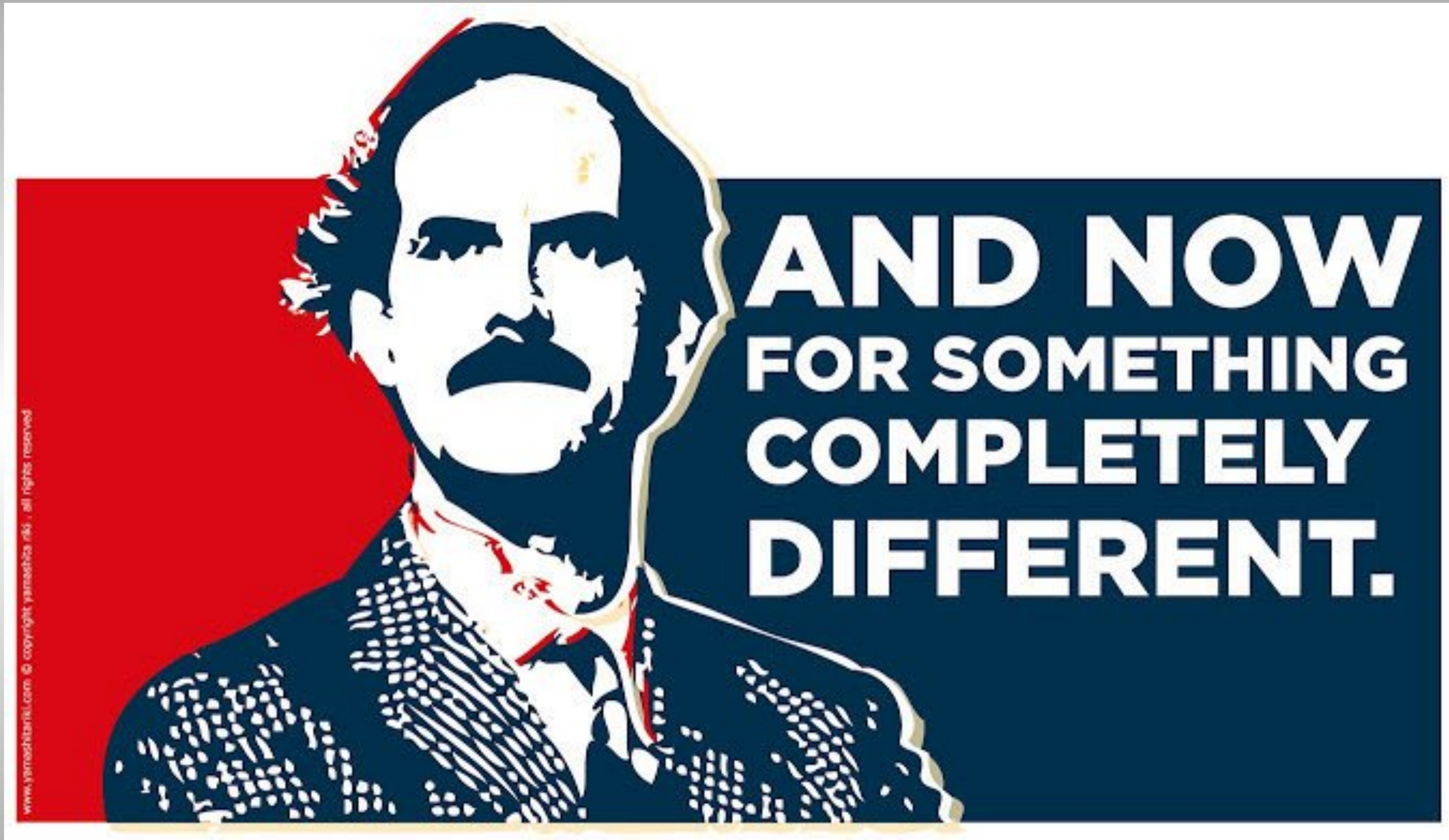
Higgs Hunting, Paris, 31/7/19

Ian Moss and Ben Withers,1401.0017

Philipp Burda, Ian Moss 1501.04937, 1503.07331, 1601.02152

Tom Billam, Florent Michel, Ian Moss: 1811.09169


AND NOW
FOR SOMETHING
COMPLETELY
DIFFERENT.

Implications of Vacuum Metastability

Vacuum Metastability and Nonperturbative Processes

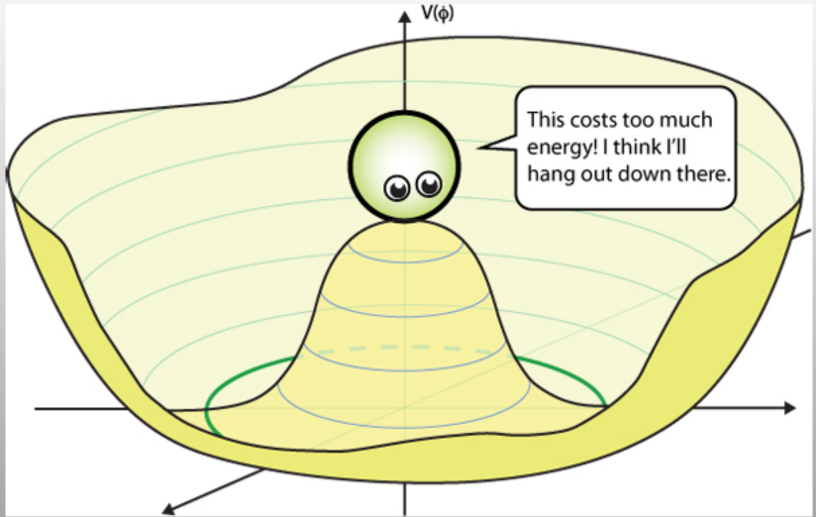
Tunnelling Techniques

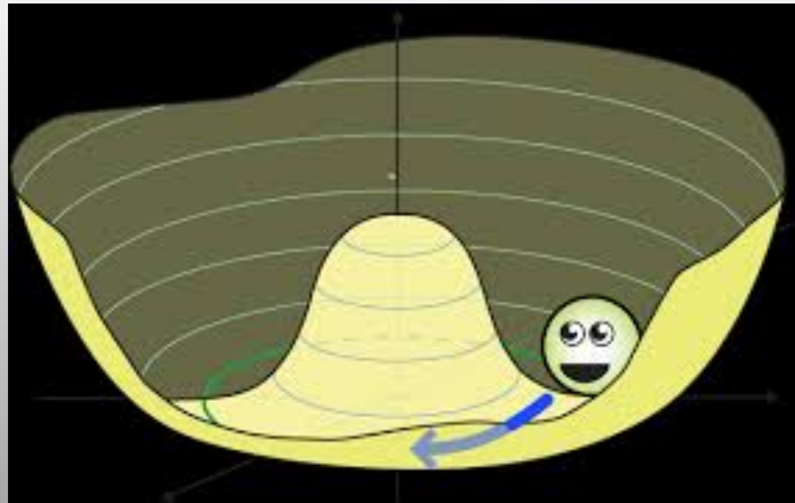
Seeded First Order Phase Transitions

Primordial Black Holes

Vacuum Metastability

We define the vacuum as the lowest energy state, but in practise, how do we know if we are in that state?

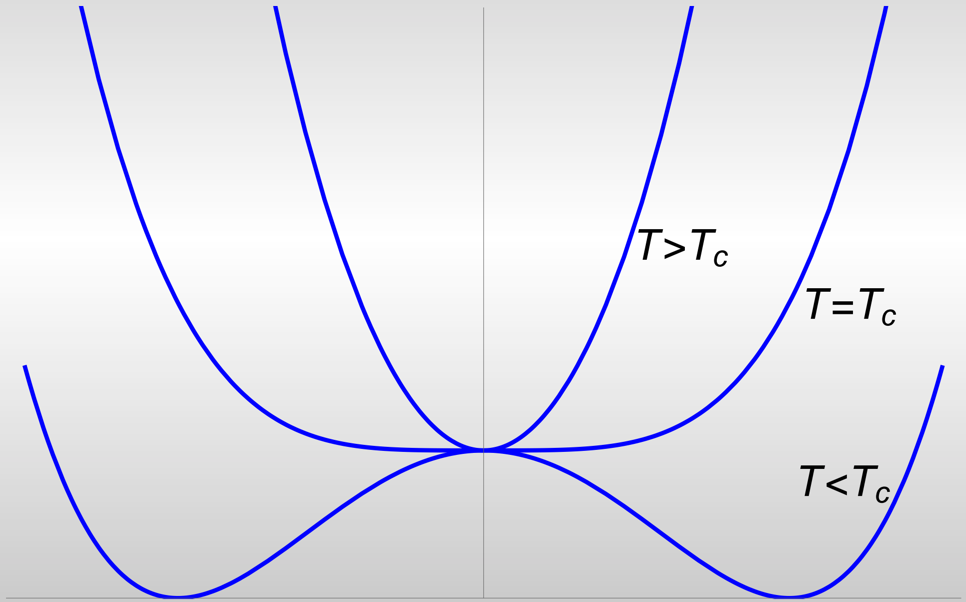


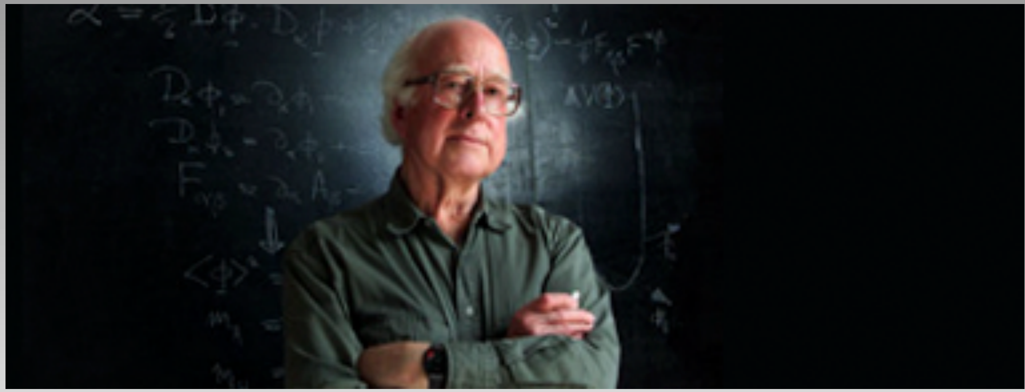


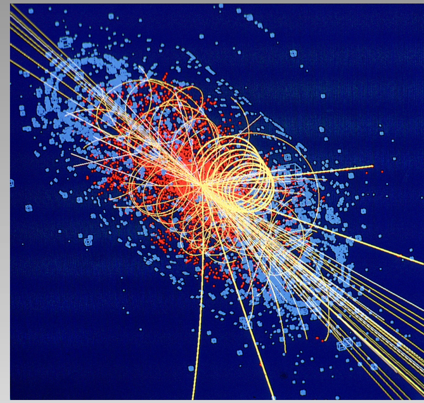
Phase Transitions

The potential depends on scale – in cosmology we expect phase transitions in the early universe as the universe cools.

What is surprising is that the universe might have one more phase transition up its sleeve!

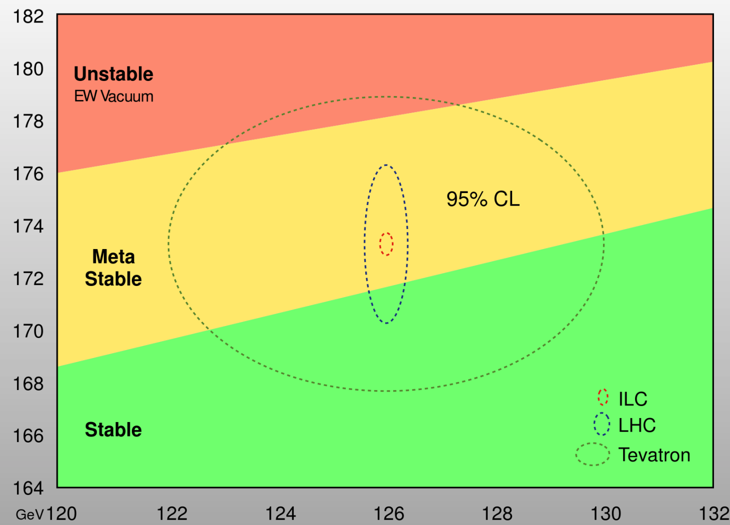






Is Our Vacuum Stable?

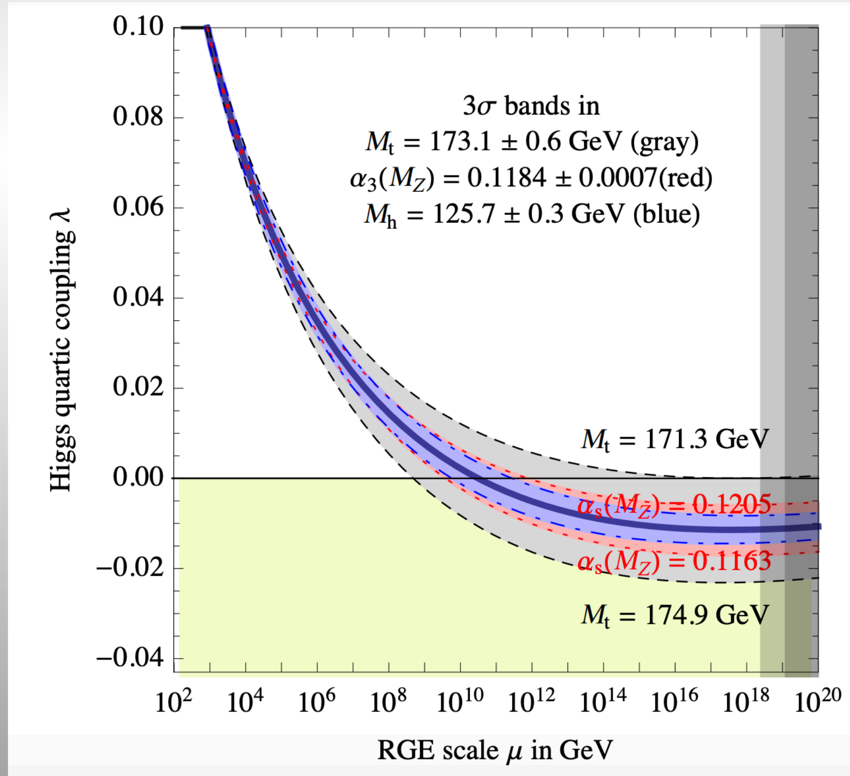
Calculating the running of the Higgs coupling tells us that we seem to be in a sweet spot between stability and instability – metastability.



Higgs Potential

At high energies, the Higgs self-coupling becomes negative, opening the possibility of vacuum tunnelling that could destroy the universe as we know it.

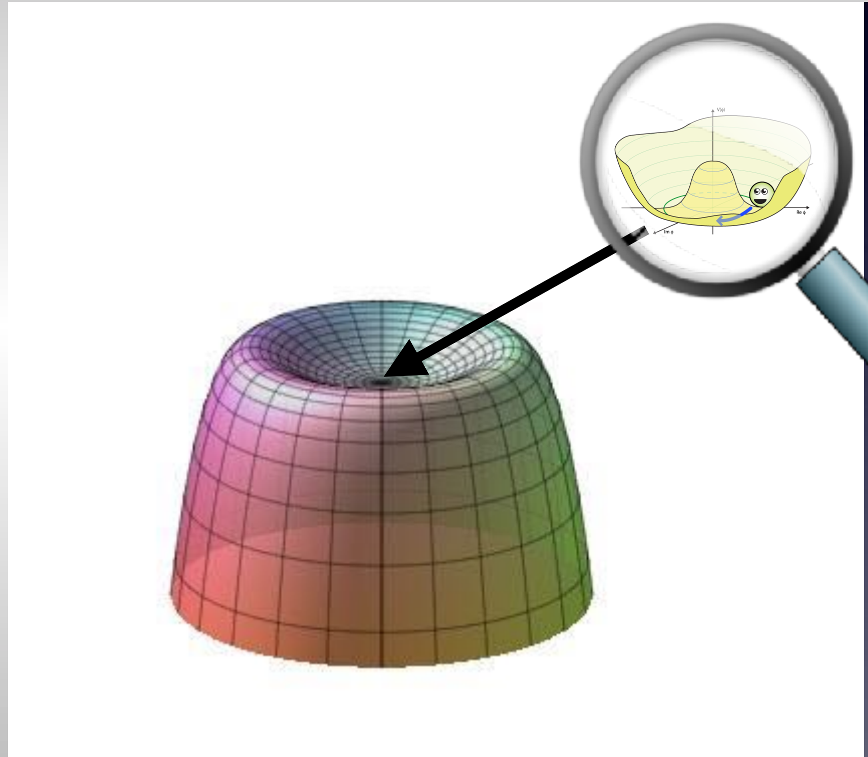
$$V(h) = \lambda(h)\frac{|h|^4}{4}$$



Degrassi et al. 1205.5497

Higgs Vacuum?

The bigger picture from the standard model tells us our universe is…

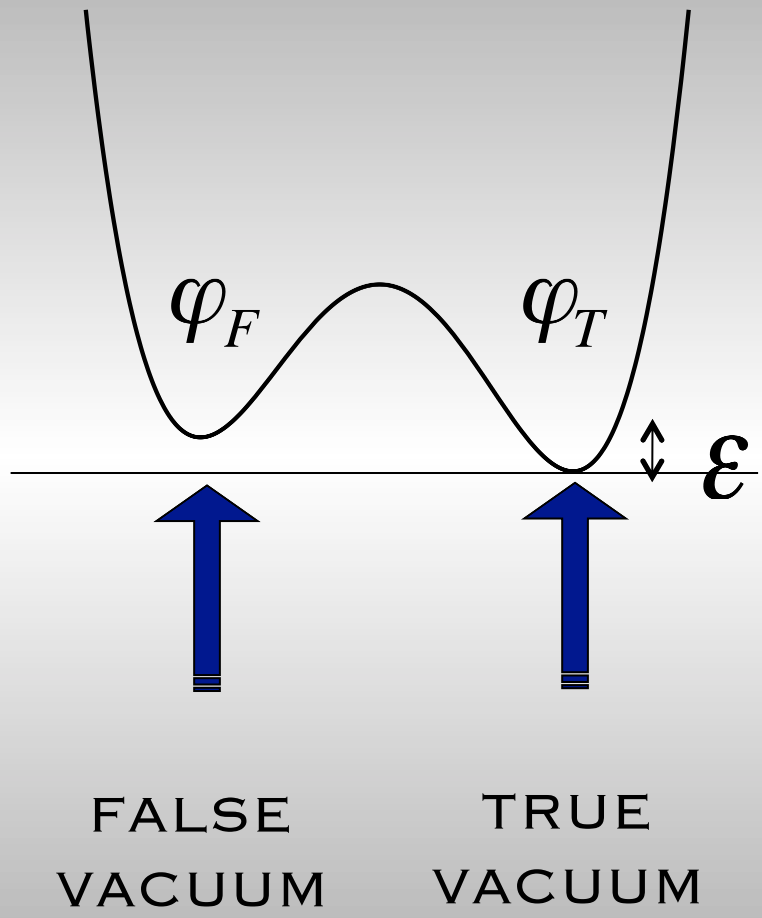


….not entirely stable!

False and True Vacuum

We call this local – not global – minimum a false vacuum, and expect there is a tunneling process to the true minimum / true vacuum.

This will give a first order phase transition, where we tunnel from one local energy minimum to a region with lower overall energy.

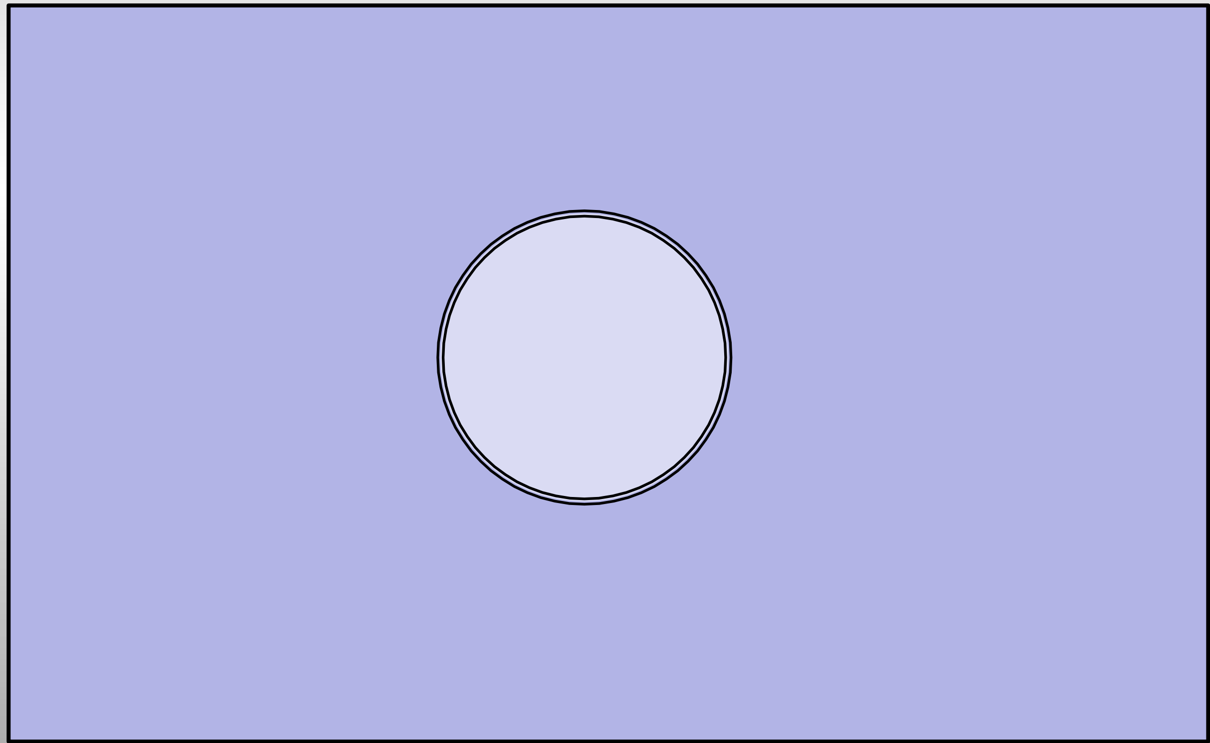


First Order Phase Transition

A first order phase transition proceeds by bubble nucleation – in this case of true vacuum within false. This is described by quantum mechanical tunnelling, and was explored by Coleman and collaborators in the 70's and 80's.

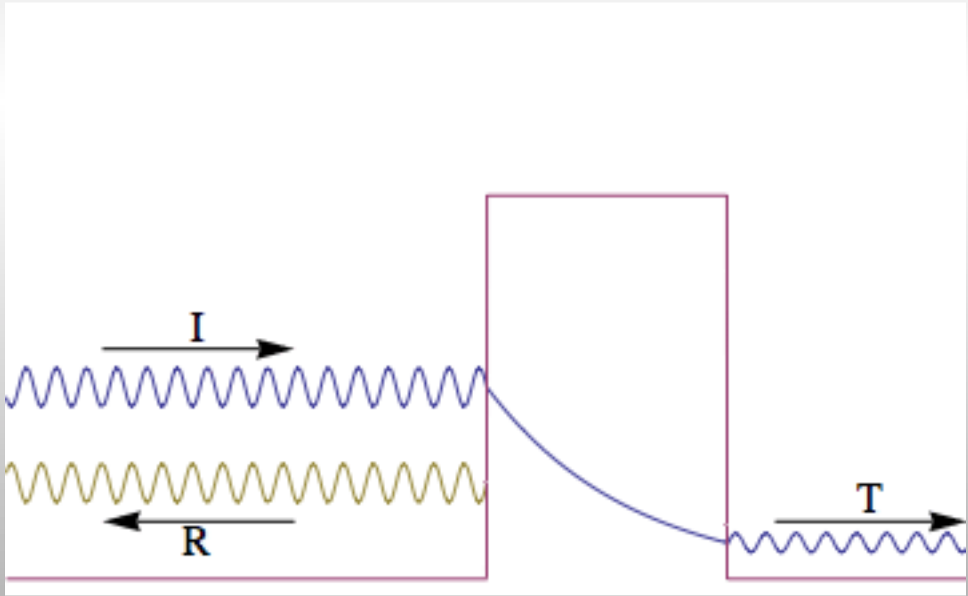
Coleman, 1977

Coleman described the process via bubble nucleation. Due to quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum, then expands.



How to Calculate?

To motivate his calculation, step back to 1st year QM.
First meet tunneling in the Schrodinger equation. Standard 1+1 Schrodinger tunneling exactly soluble. Recall tunnelling probabilities exponentially suppressed.



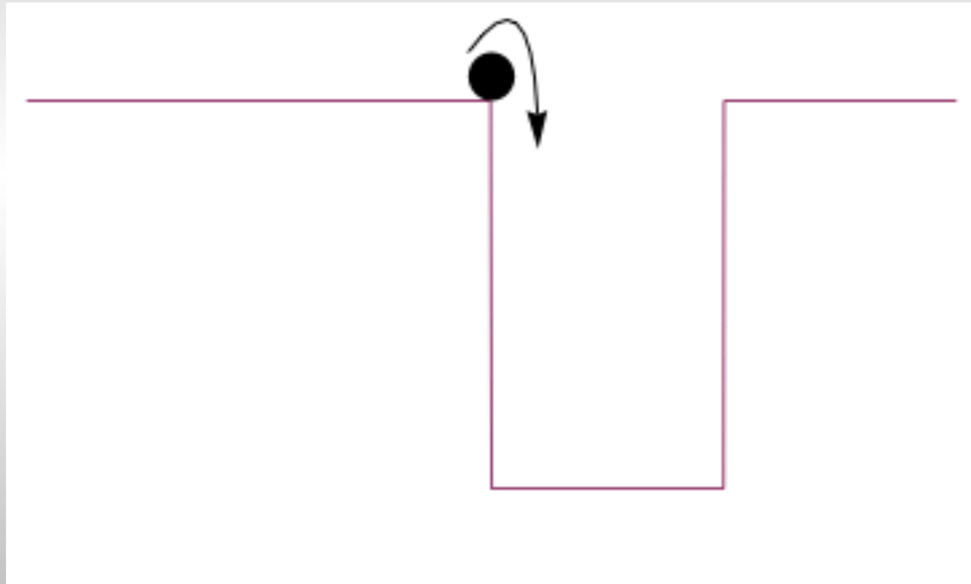
$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}dx$$

Euclidean Perspective

A simple and intuitive way of extracting this leading order behaviour is to take “classical” motion in Euclidean time:



$$\boxed{t \to i\tau}$$

$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 = \Delta V$$

$$\int \sqrt{2\Delta V}\,dx = \int 2\Delta V\,d\tau = \int \left(\Delta V + \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2\right) d\tau = \int L_E\,d\tau$$

Euclidean 'Motion'

For more general potentials, this gives an intuitive visualisation of the tunneling amplitude calculation.
The particle rolls from the (now) unstable point to the "exit" and back again – a "bounce".

In QFT, we construct instantons as solutions to the Euclidean equations of motion. Can think of this by analogy to Schrodinger problem, or via functional Schrodinger approach.

Goldilocks Bubble

Back to Coleman's bubble: If a bubble fluctuates into existence, we gain energy from moving to true vacuum, but the bubble wall costs energy.

Too small and the bubble has too much surface area – recollapses.

Too large and it is too expensive to form.

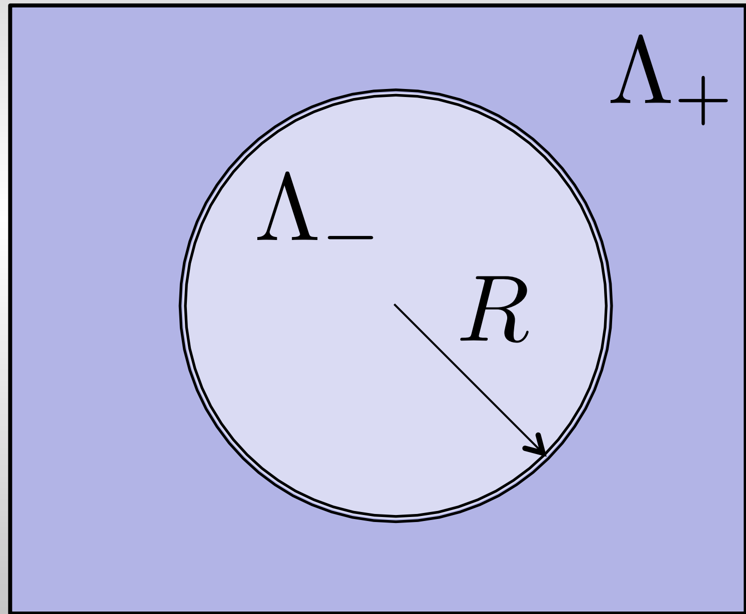
"Just Right" means the bubble will not recollapse, but is still "cheap enough" to form.

Euclidean Action

This corresponds beautifully to the Euclidean calculation of the tunneling solution: “The Bounce”

ENERGY COST $\sigma \times 2\pi^2 R^3$

ENERGY GAIN $\varepsilon \times \pi^2 R^4/2$



Solution stationary wrt R,

$$\Rightarrow R = 3\sigma/\varepsilon$$

Coleman Bounce

This gives us the bubble radius, and the amplitude for the decay – backed up by full field theory calculations.

$$\mathcal{B} = \frac{\pi^2 R^3}{2}\left(-\sigma + \varepsilon R\right) \sim \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$$

Tunneling amplitude, leading order:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

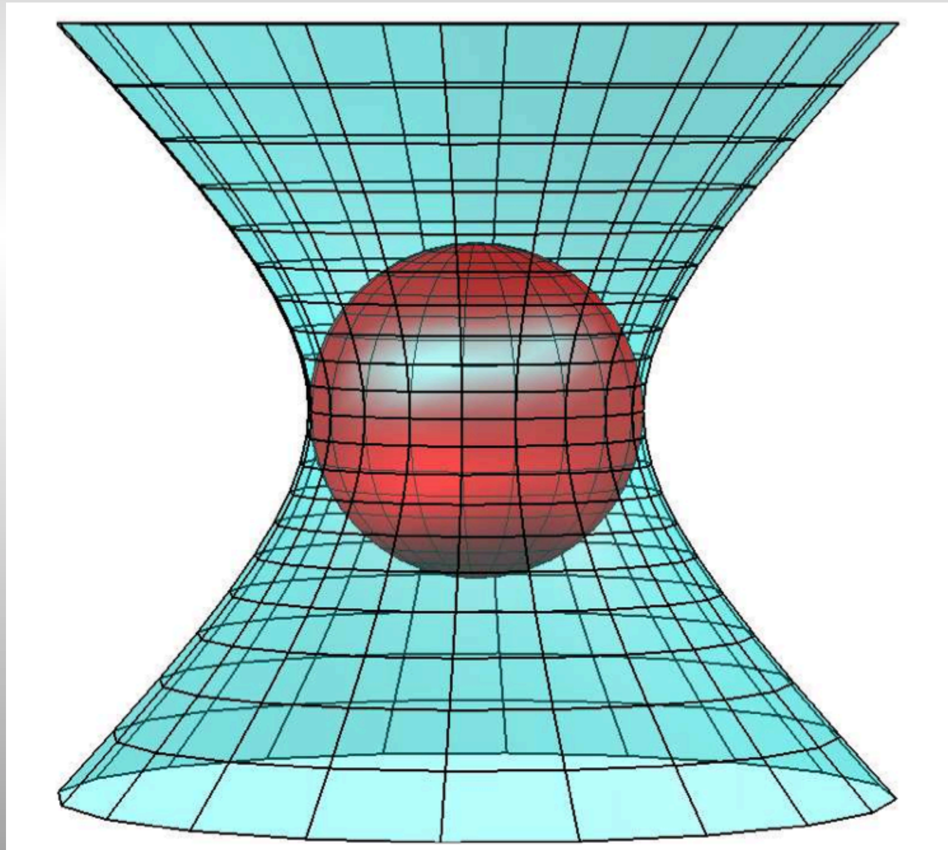
This gives the leading order or saddle point approximation to the amplitude. We must also include fluctuations:

$$\frac{\Gamma}{V} = \left| \frac{\det\, S''[\phi_{FV}]}{\det' S''[\phi_B]} \right|^{1/2} \left(\frac{\mathcal{B}}{2\pi} \right)^{D/2} e^{-\mathcal{B}/\hbar}$$

To get the nett decay rate per unit volume, per unit time.

Does this Euclidean calculation ***mean*** anything real?
Conventional answer is to rotate back to real time: $i\tau \rightarrow t$

$$r^2 + \tau^2 = R^2 \rightarrow r^2 - t^2 = R^2$$

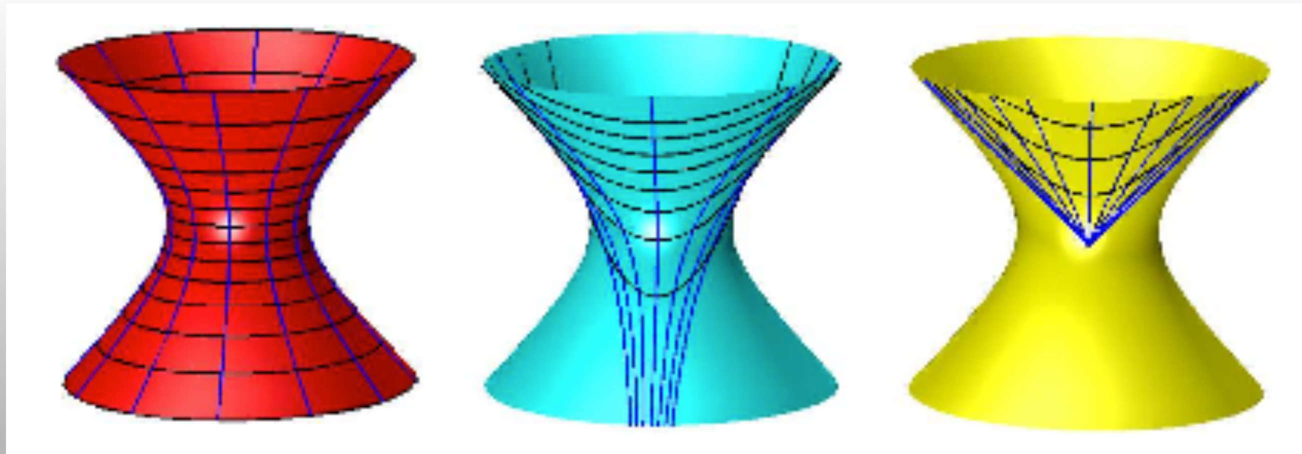


Real time picture is that the bubble expands rapidly.

$$r^2 = R^2 + t^2$$

Gravity and the Vacuum

This is not the full story! Vacuum energy gravitates – e.g. a positive cosmological constant gives us de Sitter spacetime – so we must add gravity to this picture



Quantum Gravity?

Although we do not have an uncontested theory of quantum gravity, we do have ideas on how quantum effects in gravity behave below the Planck scale.

Quantum Effects in Gravity

Below the Planck scale, we expect that spacetime is essentially classical, but that gravity can contribute to quantum effects through the wave functions of fields, and through the back-reaction of quantum fields on the spacetime.

We use this in black hole thermodynamics, cosmological perturbation theory, and for non-perturbative solutions in field theory, this method is particularly unambiguous, but can we test these ideas in a broader sense?

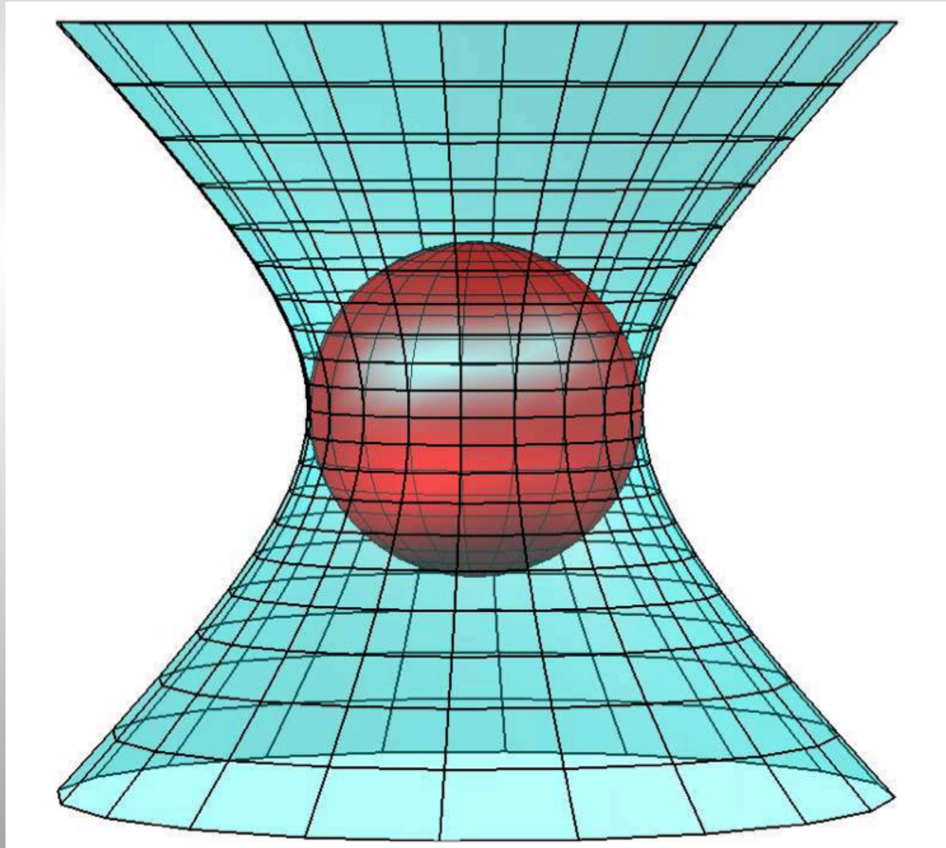
Gibbons-Hawking Euclidean Approach

Extend partition function description to include the Einstein-Hilbert action – at finite temperature we take finite periodicity of Euclidean time.

$$S = -\frac{M_p^2}{2}\int d^4x\sqrt{|g|}R + \int d^4x\mathcal{L}_{SM}$$

Fluctuations treated with caution, but saddle points unambiguous.

De Sitter spacetime has a Lorentzian (real time) and Euclidean (imaginary time) spacetime. The real time expanding universe looks like a hyperboloid and the Euclidean a sphere:

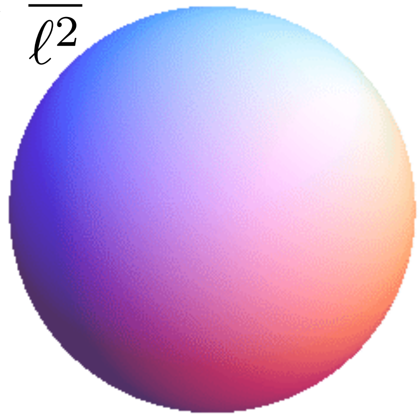


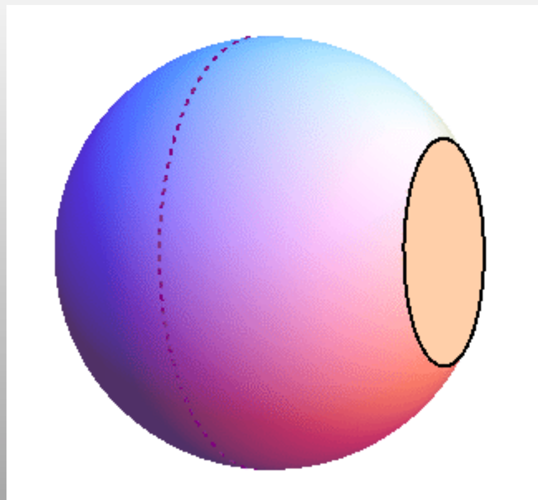
Our instanton must cut the sphere and replace it with flat space (true vacuum).

Coleman de Luccia (CDL)

Coleman and de Luccia showed how to do this with a bubble wall: Euclidean de Sitter space is a sphere, of radius ℓ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$





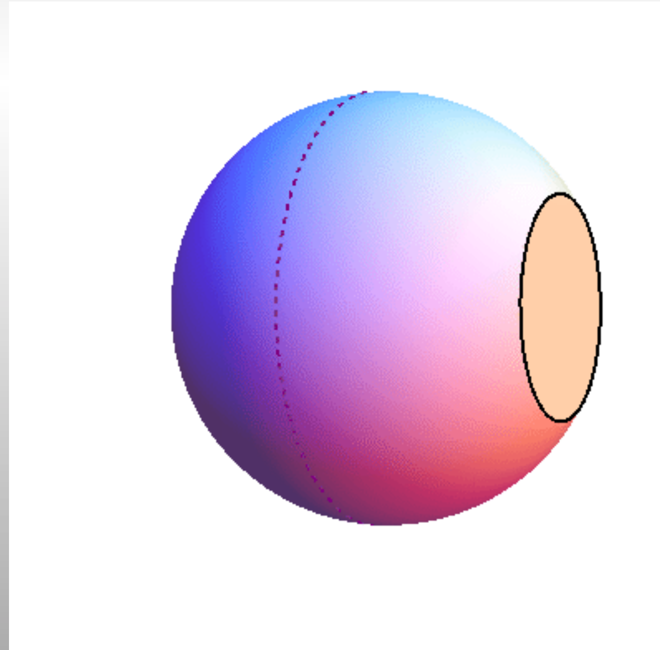
The bounce looks like a truncated sphere.

Coleman and de Luccia, PRD21 3305 (1980)

Goldilocks with Gravity

We can play the same "Goldilocks bubble" game – finding the cost of making this truncated sphere, but adding in the effect of gravity.

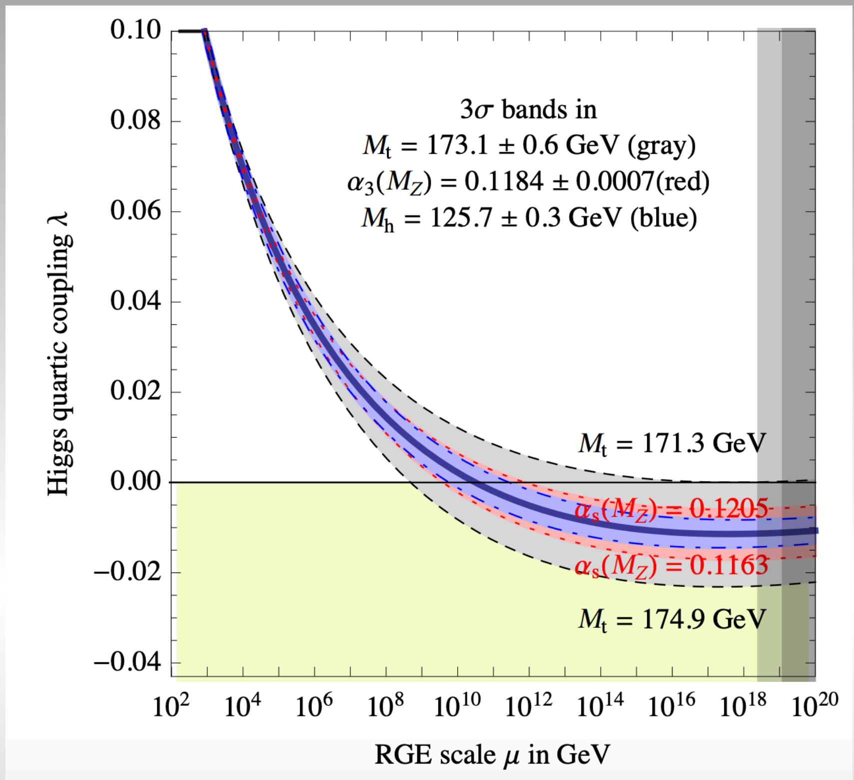
$$\mathcal{B}(R) = \frac{4}{3}\pi^2\varepsilon\ell^4\left[1 - \left(1 - \frac{R^2}{\ell^2}\right)^{\frac{3}{2}}\right] - 2\pi^2\varepsilon\ell^2R^2 + 2\pi^2\sigma R^3$$



CDL Picture

Once again, too small a bubble will recollapse, and large bubbles are harder to make, so there is a “just right” bubble that corresponds to a solution of the Euclidean Einstein equations that we can find either numerically with the full field theory, or analytically if we take our bubble wall to be thin, and we can find our instanton action.

$$
\begin{aligned}
\mathcal{B} &= -\frac{\Lambda}{8\pi G}\int_{\mathrm{int}} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h} \\
&= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{G}\frac{16\bar{\sigma}^4\ell^4}{(1+4\bar{\sigma}^2\ell^2)^2}
\end{aligned}
$$



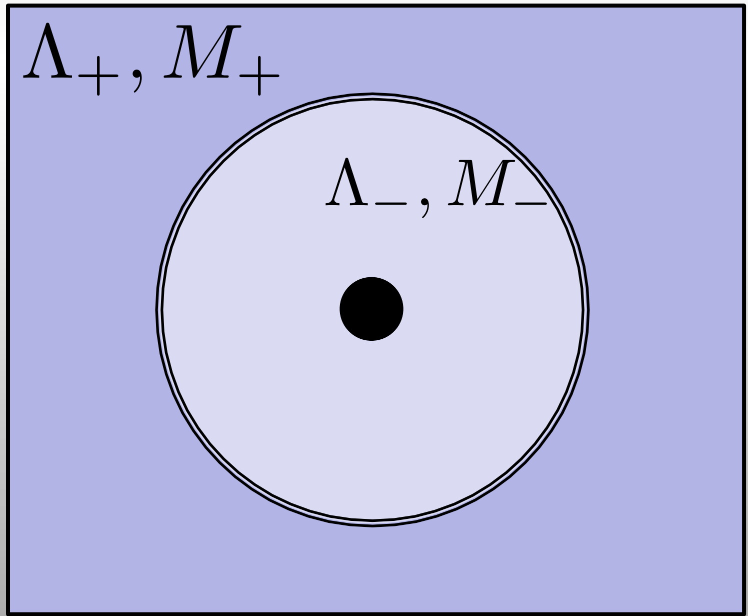
For the Higgs, this gives a half-life of many hundreds of billions of years.

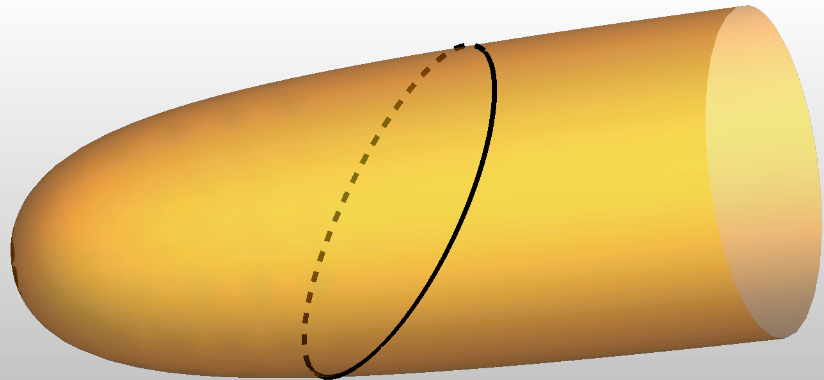
BUT

Most first order phase transitions do not proceed by ideal bubble nucleation, but by seeds.
These calculations are very idealised – an empty and featureless background – what if we throw in a little impurity?

Tweaking CDL

A black hole is an inhomogeneity, and also exactly soluble:





RG, Moss & Withers, 1401.0017

Goldilocks Black Hole Bubbles

- The bubble with a black hole inside, can have a different mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time, but the bubble has no black hole inside it – no remnant black hole.

- For larger seed masses the bubble does not depend on Euclidean time, and has a remnant black hole.

This last case is the relevant one – the action is the difference in entropy (area) between the seed and remnant black holes!

Black Hole Bounces

Balance of action changes because of periodic time:

$$B \sim \sigma \times 4\pi R^2 L - \varepsilon \times \frac{4}{3}\pi R^3 L$$

$$R \sim 2\sigma/\varepsilon$$

$$B \sim \frac{\sigma^3}{\varepsilon^2} L$$

The result is that the action is the difference in entropy of the seed and remnant black hole masses:

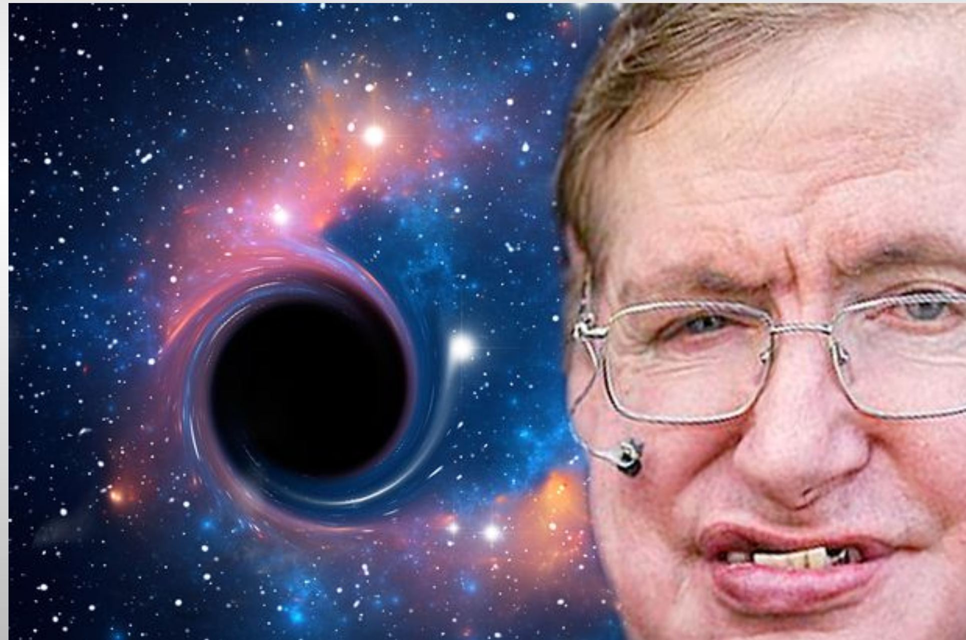
$$B \sim \mathcal{A}_+ - \mathcal{A}_-$$

Seeded tunneling is much more likely than CDL!

The fate of the black hole?

Vacuum decay is not all that can happen! Hawking tells us that black holes are black bodies, and radiate:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$



So we must compare evaporation rate to tunneling half-life.

Tunneling v Evaporation

Although we have computed bubble actions in full, we can estimate the dependence of the action on mass using input from our solutions which show that the seed and remnant masses are very close:

$$\begin{aligned}\mathcal{B} &= \pi(r_s^2 - r_r^2) \\ &\sim 4\pi(M_s + M_r)(M_s - M_r) \\ &\sim 8\pi M_s \delta M \quad \Rightarrow \Gamma_D \propto e^{-8\pi M_s \delta M}\end{aligned}$$

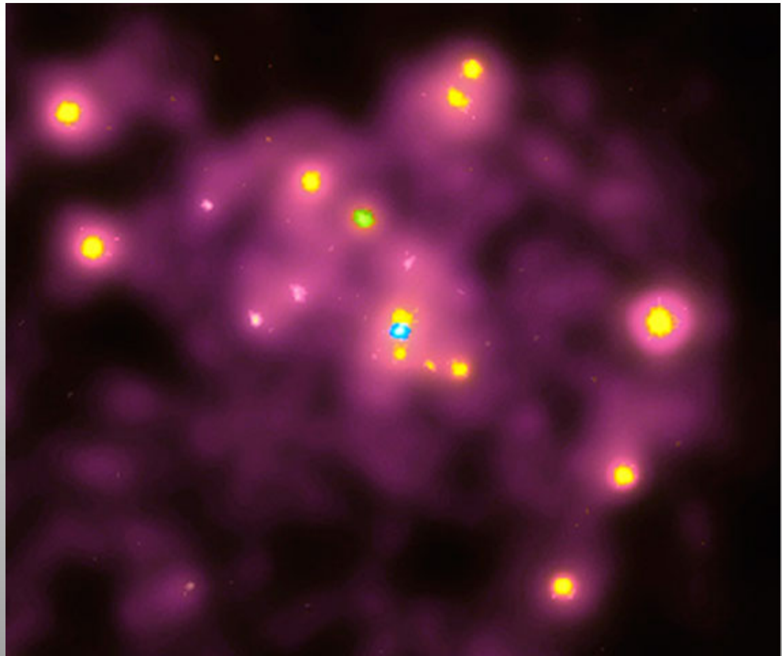
So our decay rate depends on an exponential of M_s, whereas evaporation depends on an inverse power of M – tunneling becomes important for smaller M

$$\Gamma_H \approx 3.6 \times 10^{-4} M^{-3}$$

Page, PRD-76

Primordial Black Holes

Comparing power law with prefactor to exponential shows that decay can only dominate for small black holes. Primordial Black Holes are tiny black holes with masses of order a ton, conjectured to form in the very early universe.

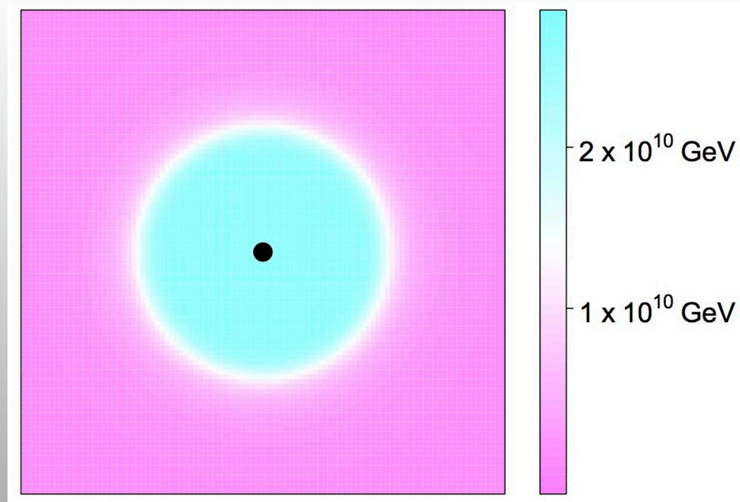


Some think they may explain the dark matter in the universe.

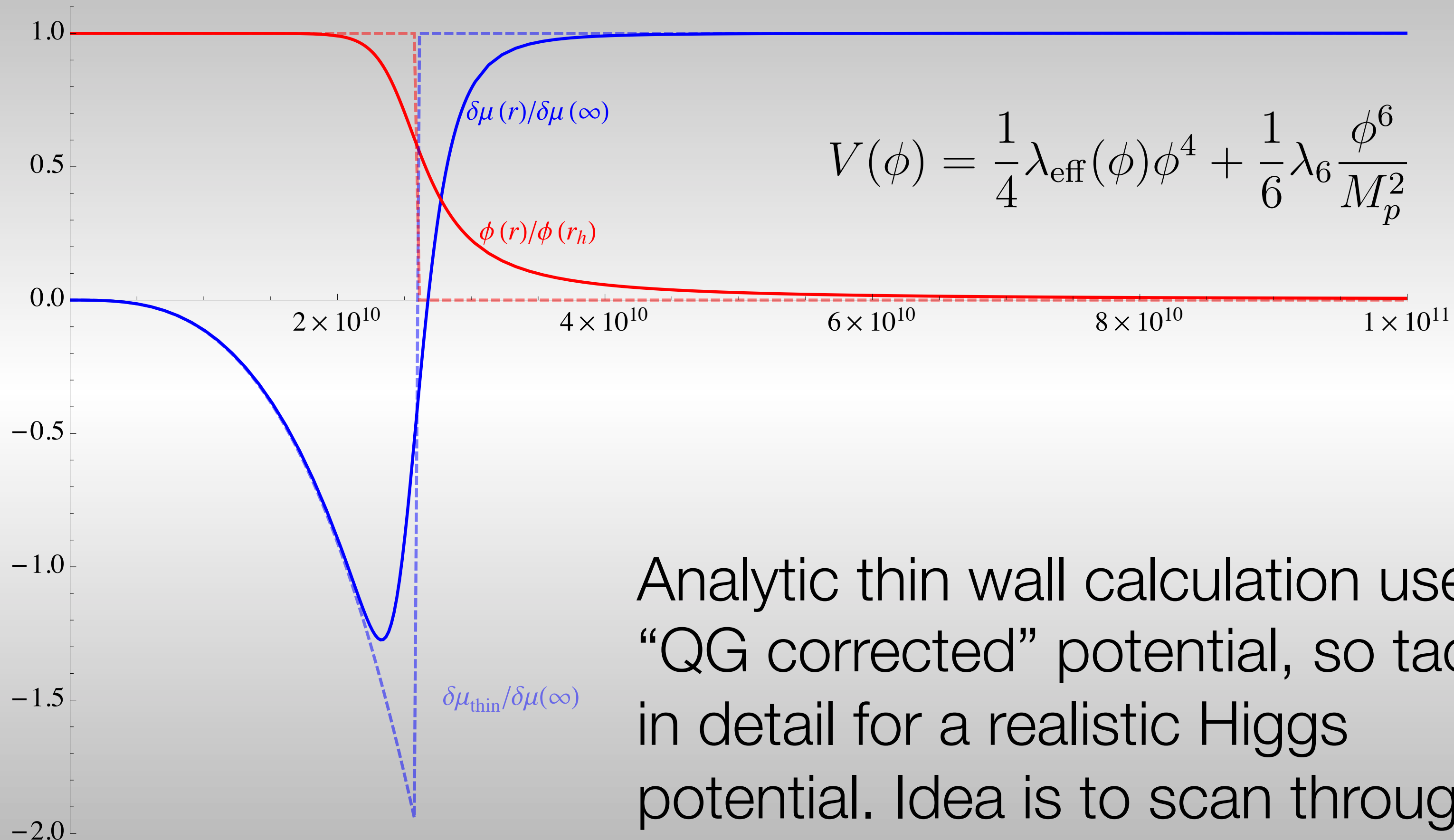
Primordial Black Holes

Primordial black holes have a temperature above the CMB, so these do evaporate over time. Eventually, they become light enough that they hit the "danger range" for vacuum decay and WILL catalyse it.

For the Goldilocks bubble argument, we used Coleman's "thin wall" picture – this does not correspond to the SM Higgs potential! However, can add ad hoc term to potential to tune between thin wall and SM & integrate numerically.



Numerical Results

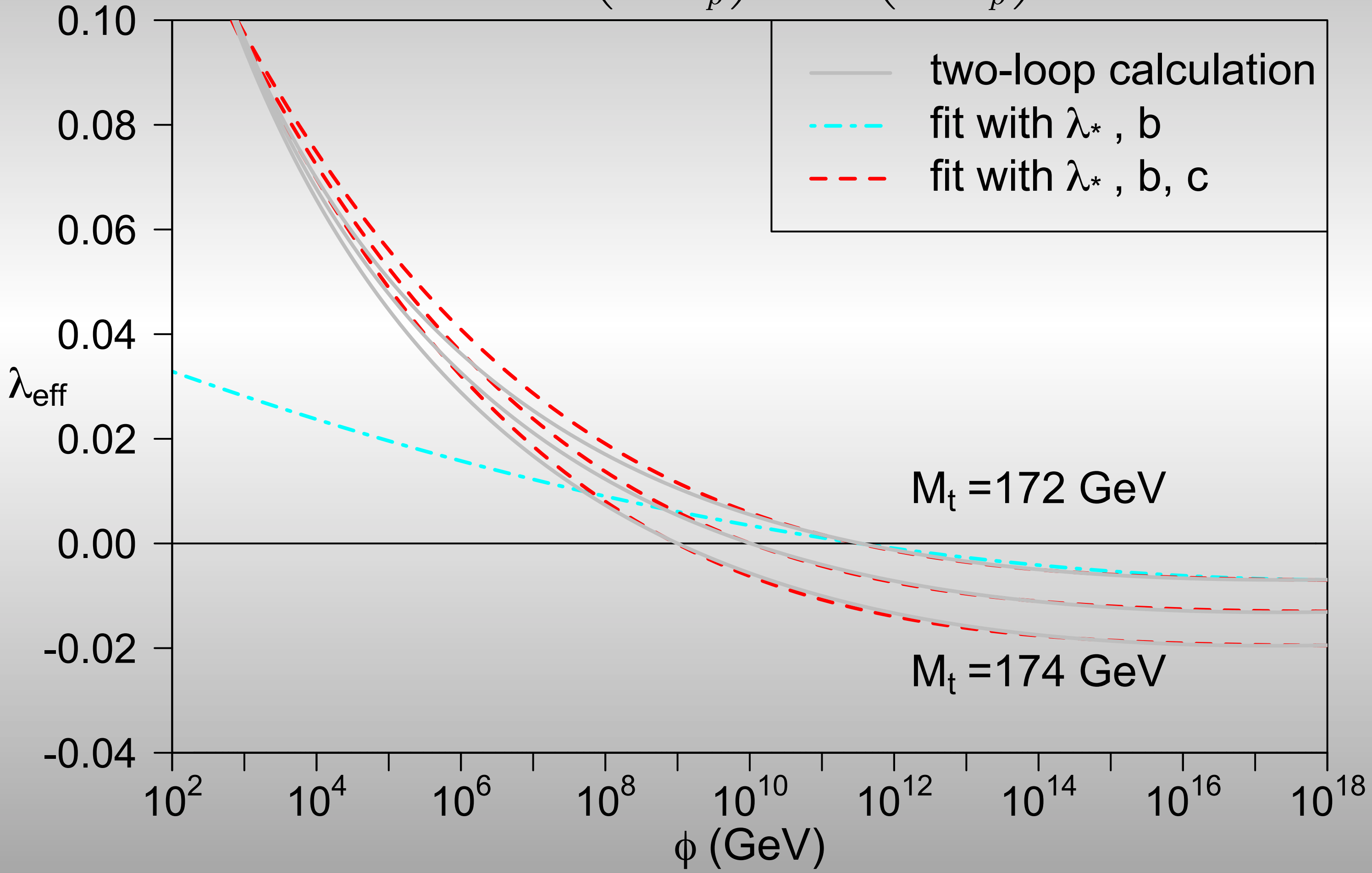


$$V(\phi) = \frac{1}{4}\lambda_{\rm eff}(\phi)\phi^4 + \frac{1}{6}\lambda_6\frac{\phi^6}{M_p^2}$$

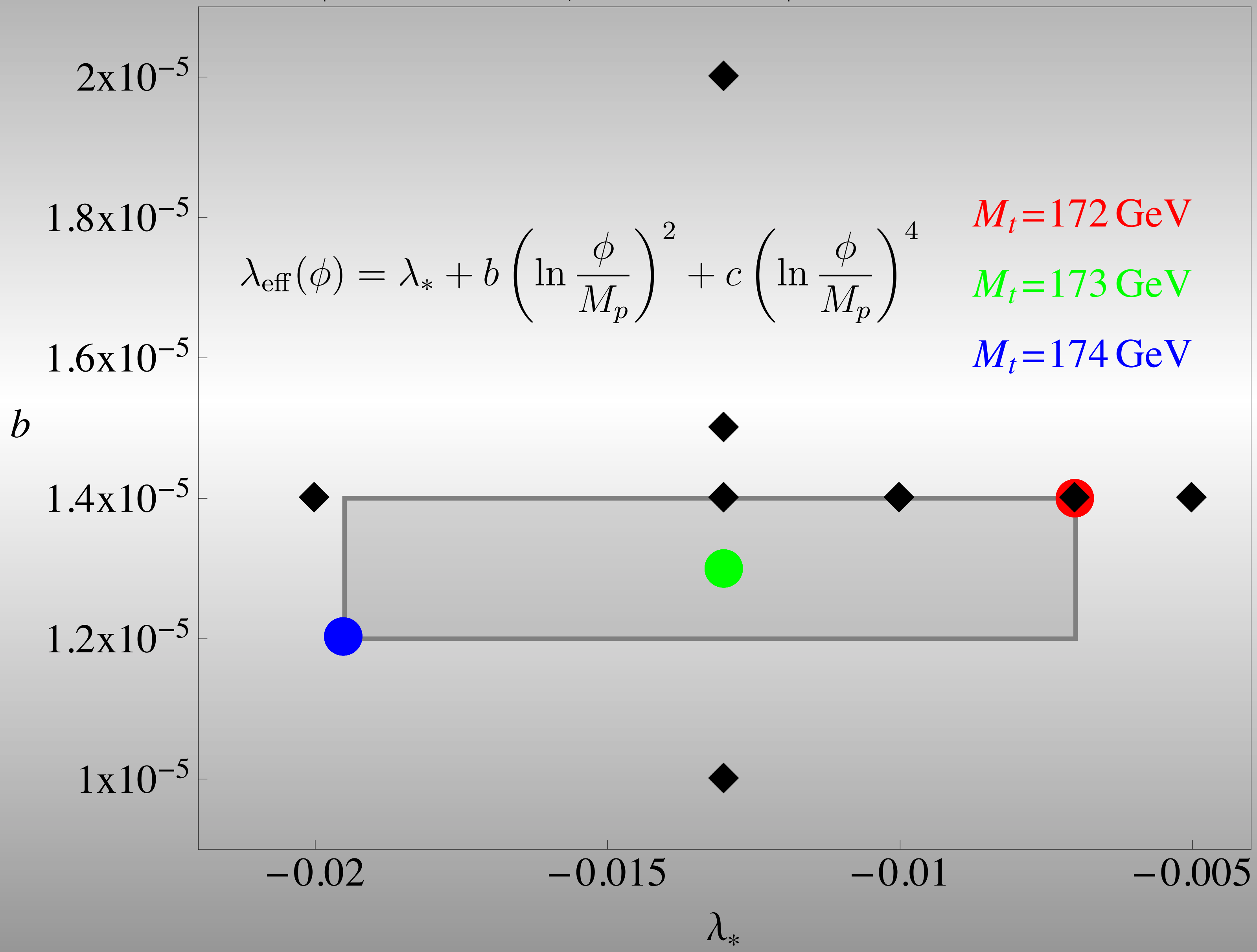
Analytic thin wall calculation uses a "QG corrected" potential, so tackle in detail for a realistic Higgs potential. Idea is to scan through parameter space (beyond standard model) to see how robust result it.

Fitting the Potential

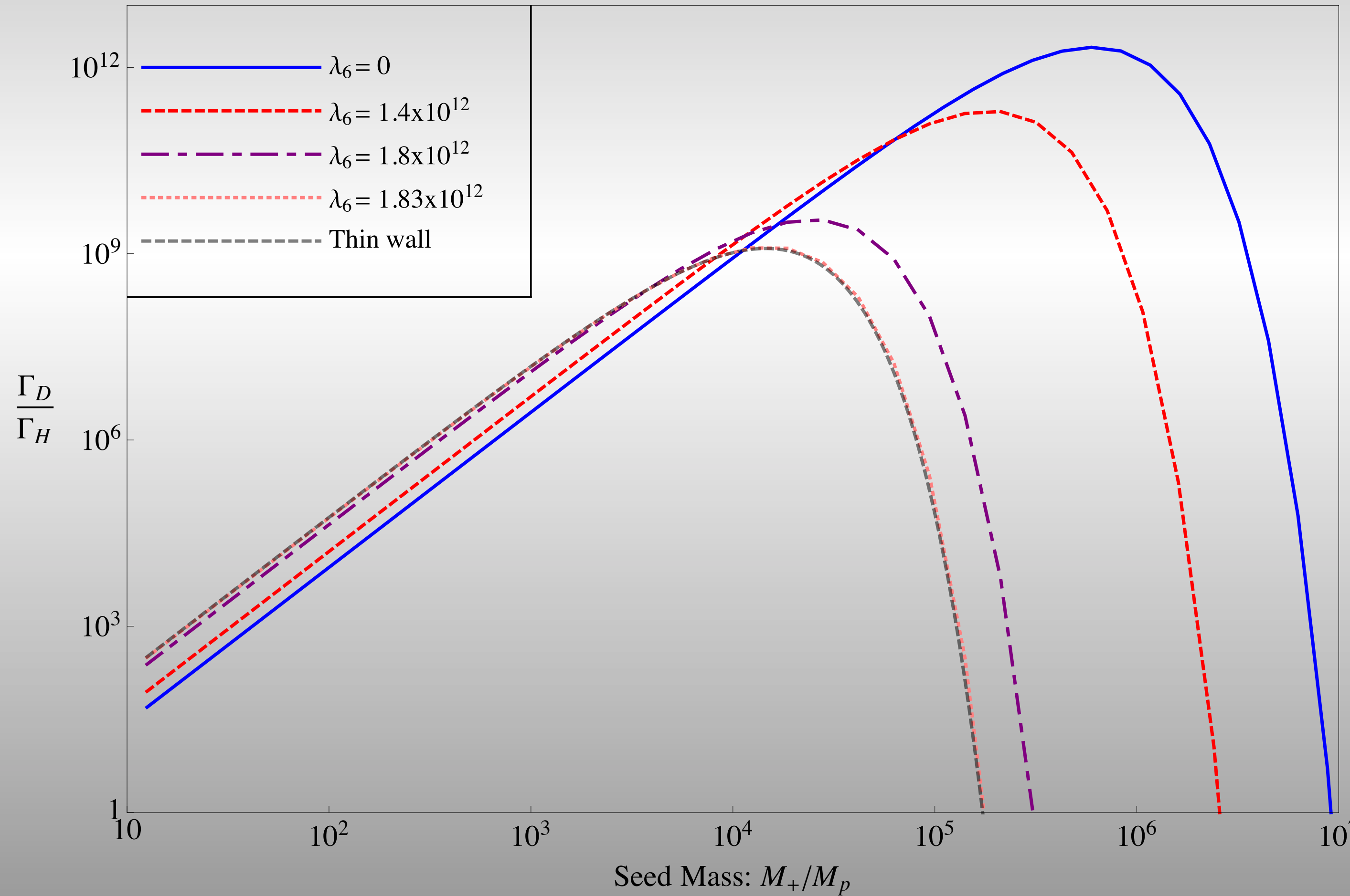
$$\lambda_{\text{eff}}(\phi) = \lambda_* + b\left(\ln\frac{\phi}{M_p}\right)^2 + c\left(\ln\frac{\phi}{M_p}\right)^4$$



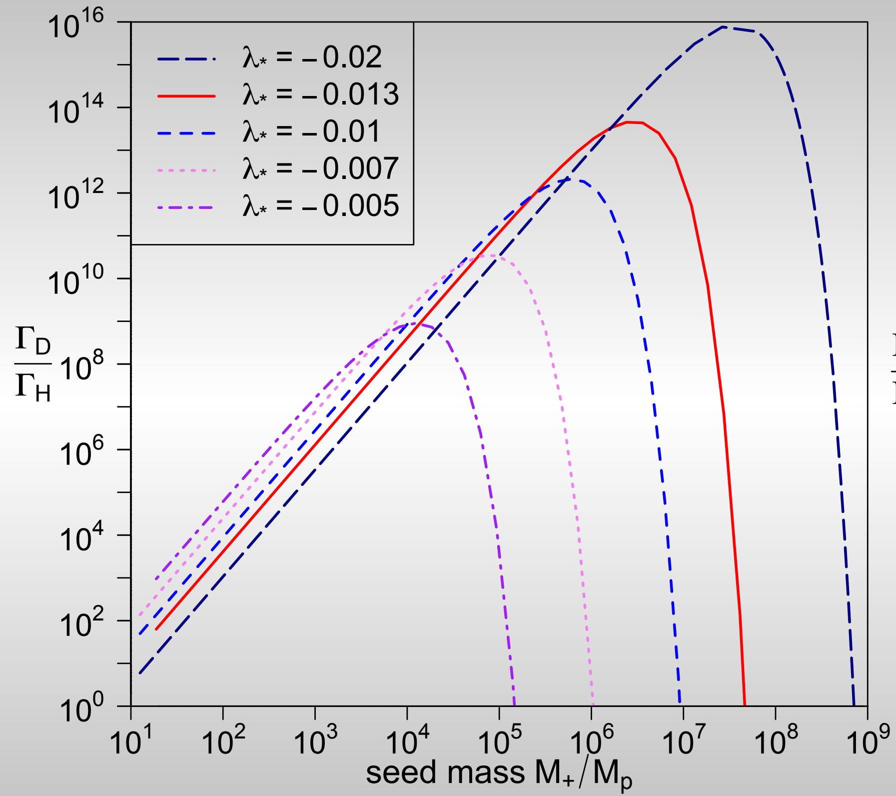
Numerical Integration

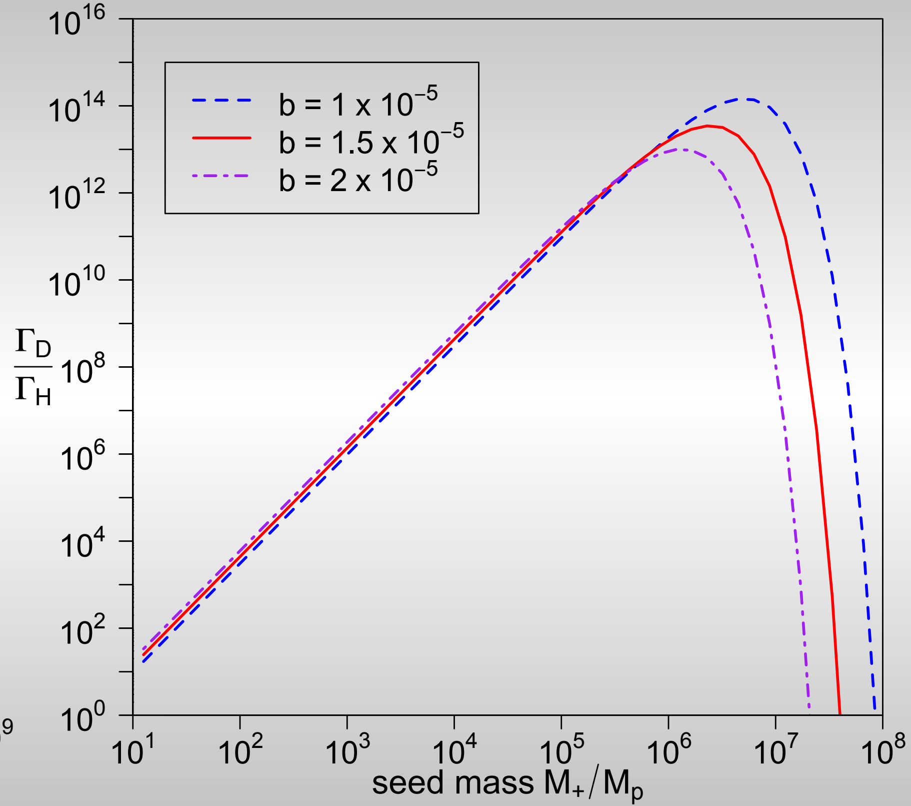


First check thin wall, by increasing **λ**$_6$. Thickening the wall increases the effectiveness of the instanton – the primordial black hole will hit the danger zone much sooner, and the decay will proceed rapidly.



Scanning through parameter space for pure SM potential shows main dependence on λ_*:

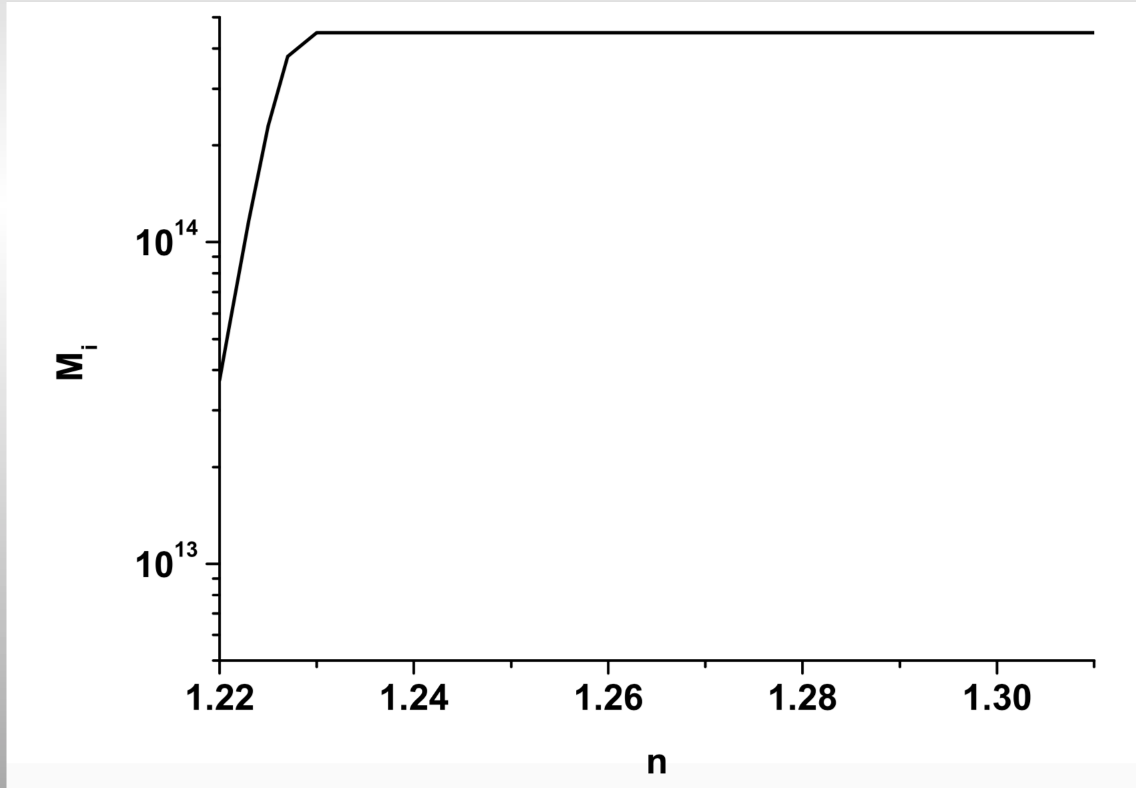




And because we are at such extreme scales, the lifetime of the universe drops to around 10^{-17}s!

Primordial black holes start out with small enough mass to evaporate and will eventually hit these curves.

Can view as a constraint on PBH's or (weak) on corrections to the Higgs potential.



Dai, RG, Stojkovic

Summary

- Vacuum decay is a nonperturbative but calculable process.
- Tunneling amplitude significantly enhanced in the presence of a black hole – very efficient for small black holes, so primordial black holes act as a trigger to change the state of the universe!
- While there is plenty of new physics that can happen – gravity, surprisingly, can have something to say about the SM!