Extrapolation to the High scale at LHC+ILC

Project : SFitter team + Jean-Loic Kneur, Claire Adam.

SFitter : `` Measuring Supersymmetry ''

Eur. Phys. J. C 54, 617–644 (2008), arXiv:0709.3985 [hep-ph].

Authors : R Lafaye, T. Plehn, M. Rauch, D. Zerwas

Reminder (1) : SFitter ?

Quoting an early paper (hep-ph/0512028) :

"If supersymmetry is discovered in the next generation of collider experiments, it will be crucial to determine its fundamental high-scale parameters from weak scale measurements."

SFitter is a complex tool, used to determine the underlying fundamental parameters :

- It uses as inputs sets of measurements (masses, mass differences, edges or thresholds) expected at LHC, ILC, or LHC+ILC.
- The expected errors are split between statistical (Gaussian, uncorrelated), experimental systematics (Gaussian, correlated) , and theory errors (flat distribution within allowed range) .
- For a given model (e.g. MSSM), the spectrum at the electroweak scale is calculated by, in particular, Suspect ("A Fortran code for the Supersymmetric and Higgs Particle Spectrum in the MSSM", hep-ph/0211331, *Abdelhak Djouadi, Jean-Loic Kneur and Gilbert Moultaka*)

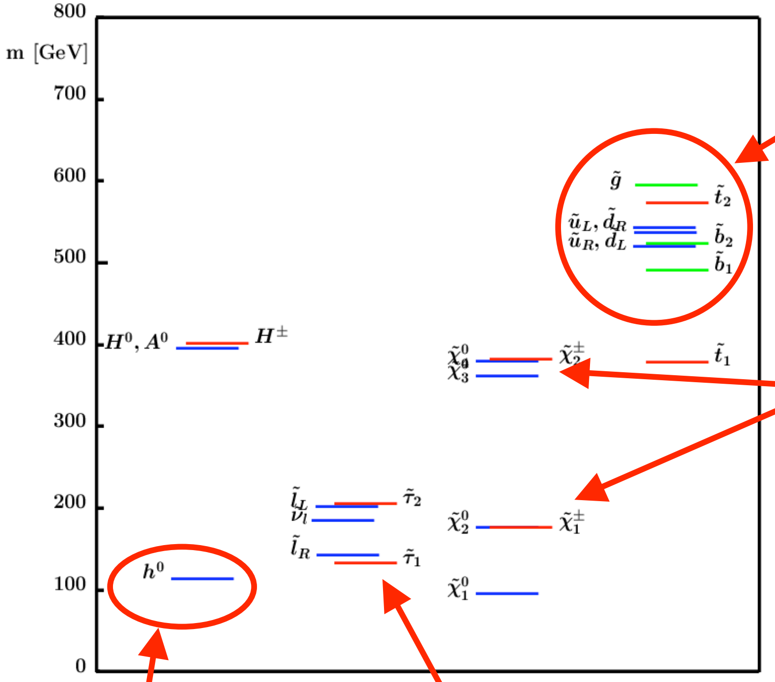
SFitter uses both to fit the parameters, using combination of Markov chains and Minuit.

Latest publication (arXiv:0709.3985 [hep-ph].) :

" For a "typical" point (SPS1a), and in two physics models (MSUGRA and MSSM), it was shown that a likelihood map could be built, a maxima identified, and that the parameters could be extracted with some errors, properly including experimental and theory errors."

Reminder (2) : SPS1a ?

$m_0 = 100GeV$ $m_{1/2} = 250GeV$ $A_0 = -100GeV$ $\tan\beta = 10$ sign(μ)=+
favourable for LHC and ILC (Complementarity)



"Physics Interplay of the LHC and ILC"
Editor G. Weiglein hep-ph/0410364

$\tilde{\tau}_1$ lighter than lightest $\chi^\pm$:

- $\chi^\pm$ BR 100% $\tau\tilde{\nu}$
- χ_2 BR 90% $\tau\tilde{\tau}$
- cascade:

$q_L \Rightarrow \chi_2\, q \Rightarrow \tilde{\ell}_R\, \ell\, q \Rightarrow \ell\, \ell\, q\chi_1$
visible

Reminder (3) : experimental inputs

SPS1a was studied in great detail and has a favorable phenomenology for both LHC and ILC, which are complementary :

- LHC measures kinematical endpoints and mass difference, and covers better the strongly interacting sparticle sector,
- ILC has an impressive accuracy for particles which are light enough to be produced in pairs, and a somewhat better precision in the gaugino sector.

type of measurement		nominal value	stat. error	LES error	JES error	theo. error
m_h		108.99	0.01	0.25		2.0
m_t		171.40	0.01		1.0	
$m_{\tilde{l}_L} - m_{\chi_1^0}$		102.45	2.3	0.1		2.2
$m_{\tilde{g}} - m_{\chi_1^0}$		511.57	2.3		6.0	18.3
$m_{\tilde{q}_R} - m_{\chi_1^0}$		446.62	10.0		4.3	16.3
$m_{\tilde{g}} - m_{\tilde{b}_1}$		88.94	1.5		1.0	24.0
$m_{\tilde{g}} - m_{\tilde{b}_2}$		62.96	2.5		0.7	24.5
$m_{ll}^{\max}$:	three-particle edge($\chi_2^0,\tilde{l}_R,\chi_1^0$)	80.94	0.042	0.08		2.4
$m_{llq}^{\max}$:	three-particle edge($\tilde{q}_L,\chi_2^0,\chi_1^0$)	449.32	1.4		4.3	15.2
m_{lq}^{low}:	three-particle edge($\tilde{q}_L,\chi_2^0,\tilde{l}_R$)	326.72	1.3		3.0	13.2
$m_{ll}^{\max}(\chi_4^0)$:	three-particle edge($\chi_4^0,\tilde{l}_R,\chi_1^0$)	254.29	3.3	0.3		4.1
$m_{\tau\tau}^{\max}$:	three-particle edge($\chi_2^0,\tilde{\tau}_1,\chi_1^0$)	83.27	5.0		0.8	2.1
m_{lq}^{high}:	four-particle edge($\tilde{q}_L,\chi_2^0,\tilde{l}_R,\chi_1^0$)	390.28	1.4		3.8	13.9
m_{llq}^{thres}:	threshold($\tilde{q}_L,\chi_2^0,\tilde{l}_R,\chi_1^0$)	216.22	2.3		2.0	8.7
m_{llb}^{thres}:	threshold($\tilde{b}_1,\chi_2^0,\tilde{l}_R,\chi_1^0$)	198.63	5.1		1.8	8.0

TABLE II: LHC measurements in SPS1a, taken from [19]. Shown are the nominal values (from SuSpect) and statistical errors, systematic errors from the lepton (LES) and jet energy scale (JES) and theoretical errors. All values are given in GeV.

In this study, will start with the most constraint combination : LHC+ILC

	m_{SPS1a}	LHC	ILC	LHC+ILC		m_{SPS1a}	LHC	ILC	LHC+ILC
h	108.99	0.25	0.05	0.05	H	393.69		1.5	1.5
A	393.26		1.5	1.5	$H+$	401.88		1.5	1.5
χ_1^0	97.21	4.8	0.05	0.05	χ_2^0	180.50	4.7	1.2	0.08
χ_3^0	356.01		4.0	4.0	χ_4^0	375.59	5.1	4.0	2.3
$\chi_1^\pm$	179.85		0.55	0.55	$\chi_2^\pm$	375.72		3.0	3.0
$\tilde{g}$	607.81	8.0		6.5					
$\tilde{t}_1$	399.10		2.0	2.0					
$\tilde{b}_1$	518.87	7.5		5.7	$\tilde{b}_2$	544.85	7.9		6.2
$\tilde{q}_L$	562.98	8.7		4.9	$\tilde{q}_R$	543.82	9.5		8.0
$\tilde{e}_L$	199.66	5.0	0.2	0.2	$\tilde{e}_R$	142.65	4.8	0.05	0.05
$\tilde{\mu}_L$	199.66	5.0	0.5	0.5	$\tilde{\mu}_R$	142.65	4.8	0.2	0.2
$\tilde{\tau}_1$	133.35	6.5	0.3	0.3	$\tilde{\tau}_2$	203.69		1.1	1.1
$\tilde{\nu}_e$	183.79		1.2	1.2					

TABLE I: Errors for the mass determination in SPS1a, taken from [19]. Shown are the nominal parameter values (from SuSpect), the error for the LHC alone, from the LC alone, and from a combined LHC+LC analysis. Empty boxes indicate that the particle cannot, to current knowledge, be observed or is too heavy to be produced. All values are given in GeV.

Latest paper (1) :

MSSM fit results, assuming vanishing theory errors

	LHC		ILC		LHC+ILC		SPS1a
$\tan\beta$	9.8±	2.3	17.6±	9.6	16.4±	7.0	10.0
M_1	101.5±	4.6	102.8±	0.72	102.7±	0.53	103.1
M_2	191.7±	4.8	192.3±	2.6	191.7±	1.7	192.9
M_3	575.7±	7.7	fixed 500		578.0±	6.3	577.9
$M_{\tilde{\tau}_L}$	196.2±$\mathcal{O}(10^2)$		185.4±	14.3	187.8±	13.6	193.6
$M_{\tilde{\tau}_R}$	136.2±	36.5	142.0±	16.4	139.0±	15.1	133.4
$M_{\tilde{\mu}_L}$	192.6±	5.3	194.4±	0.53	194.4±	0.51	194.4
$M_{\tilde{\mu}_R}$	134.0±	4.8	135.8±	0.26	135.7±	0.16	135.8
$M_{\tilde{e}_L}$	192.7±	5.3	194.4±	0.24	194.4±	0.22	194.4
$M_{\tilde{e}_R}$	134.0±	4.8	135.8±	0.17	135.7±	0.12	135.8
$M_{\tilde{q}3_L}$	478.2±	9.4	509.1±$\mathcal{O}(2\cdot10^2)$		489.6±	10.7	480.8
$M_{\tilde{t}_R}$	429.5±$\mathcal{O}(10^2)$		427.6±	$\mathcal{O}(10^2)$	402.9±	50.3	408.3
$M_{\tilde{b}_R}$	501.2±	10.0	fixed 500		494.4±	10.5	502.9
$M_{\tilde{q}_L}$	523.6±	8.4	fixed 500		526.7±	4.9	526.6
$M_{\tilde{q}_R}$	506.2±	11.7	fixed 500		508.2±	10.8	508.1
A_τ	fixed 0		2496.3±	$\mathcal{O}(10^4)$	2681.6±$\mathcal{O}(10^4)$		-249.4
A_t	-500.6±	58.4	-521.8±	160.1	-490.3±	166.8	-490.9
A_b	fixed 0		fixed 0		3084.9±$\mathcal{O}(10^4)$		-763.4
$A_{l1,2}$	fixed 0		fixed 0		fixed 0		-251.1
$A_{u1,2}$	fixed 0		fixed 0		fixed 0		-657.2
$A_{d1,2}$	fixed 0		fixed 0		fixed 0		-821.8
m_A	446.1±$\mathcal{O}(10^3)$		393.4±	1.1	393.4±	1.1	394.9
μ	350.9±	7.3	355.2±	2.5	355.2±	2.3	353.7
m_t	171.4±	1.0	171.4±	0.12	171.4±	0.12	171.4

TABLE VIII: Result for the general MSSM parameter determination in SPS1a assuming vanishing theory errors. As experimental measurements the kinematic endpoint measurements given in Tab. II are used for the LHC column, and the mass measurements given in Tab. I for the ILC column. In the LHC+ILC column these two measurements sets are combined. Shown are the nominal parameter values and the result after fits to the different data sets. All masses are given in GeV.

Latest paper (2) :

One step further : “bottom-up” extrapolation up to GUT scale, using Suspect

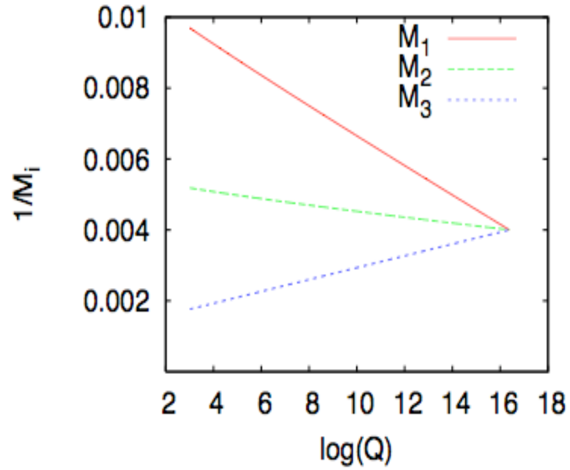


FIG. 10: SFitter/SuSpect output for the upward renormalization group running of the three gaugino masses in the MSSM. The central values are shown without error bars, a more detailed study of bottom–up running is beyond the scope of this paper [55].

= this work
(just starting…)

The problem : *due to the correlations, the EW scale error cannot be analytically extrapolated versus Q^2*
=> Use toy Monte-Carlos
and the RGE/suspect extrapolation

Error estimation

In the published results, the fit was performed in 2 steps :

- A scan of the 19-parameter MSSM space was done using Markov chains
- The "best" set of parameters was given as input to Minuit .

Here, given the "proof of principle" that the method works :

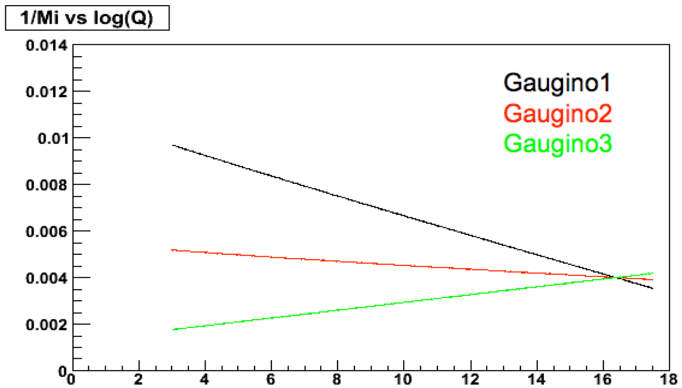
- start from the "true" low scale parameters, calculated suspect
- inputs are smeared according to their errors :

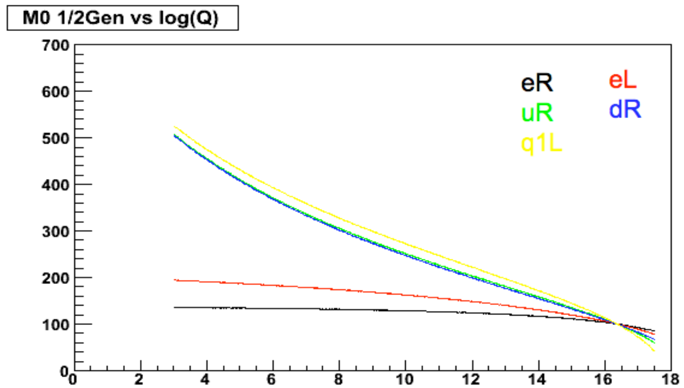
  100 toy inputs were used (as a starting point, will need more in future)
  each was fitted by SFitter
  each fit result was individually extrapolated "bottom-up", I.e. from electroweak up to GUT scale

⇒ *The distribution of each of the parameters versus Q^2 will give an estimate of the central values and uncertainty*

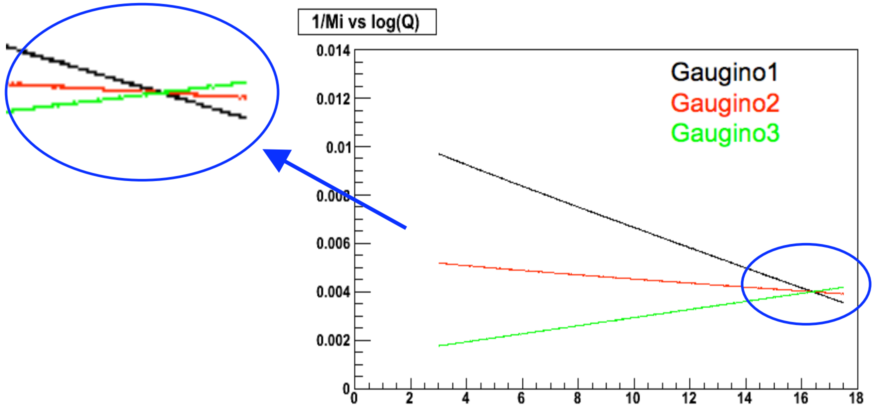
Freezing some MSSM parameters to help the fit ?

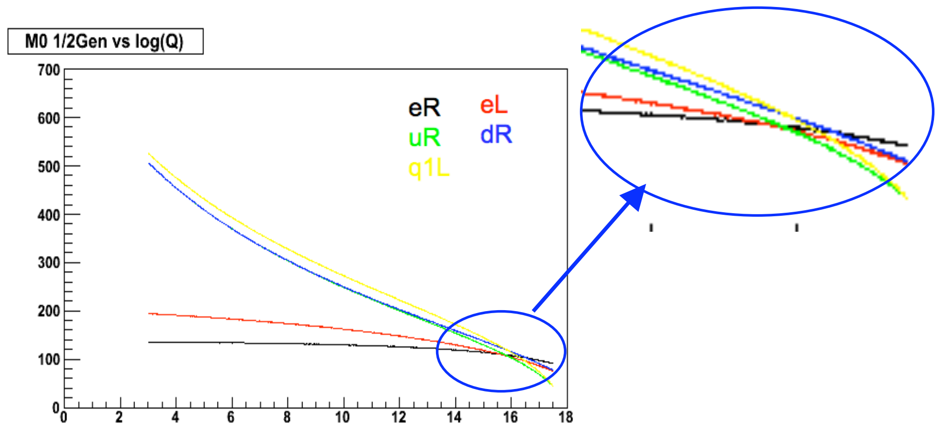
1) **$A_{l1,2}$ $A_{u1,2}$ $A_{d1,2}$** (trilinear couplings) are suppressed by the mass term (Yukawa coupling):
=> negligible and not measurable for the 1rst and second generation => set to zero
As expected, NO impact on the convergence :



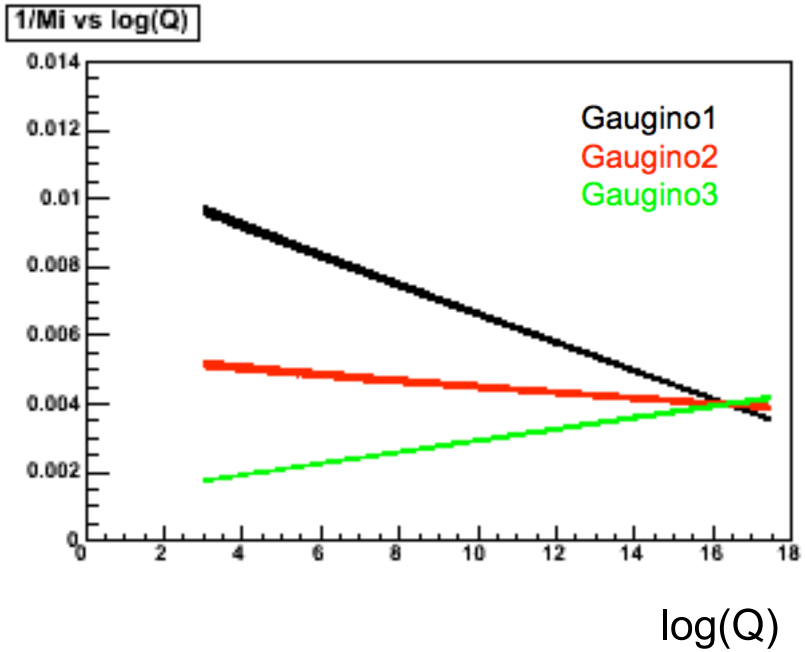


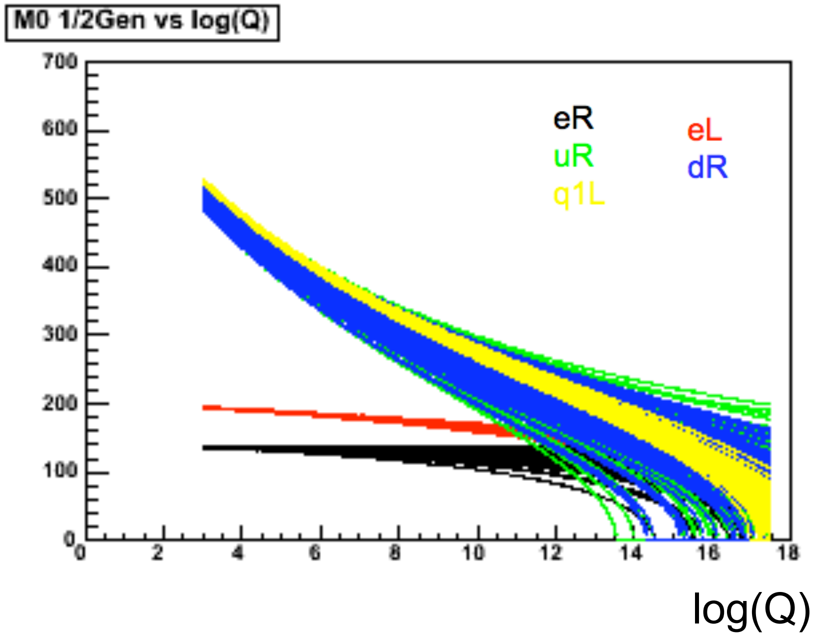
2) The light quark masses (u,d,s,c) are not experimentally distinguishable => use average mass for left and right handed.
Has a small effect on the convergence, we decided to live with it :





Results for 100 toys :



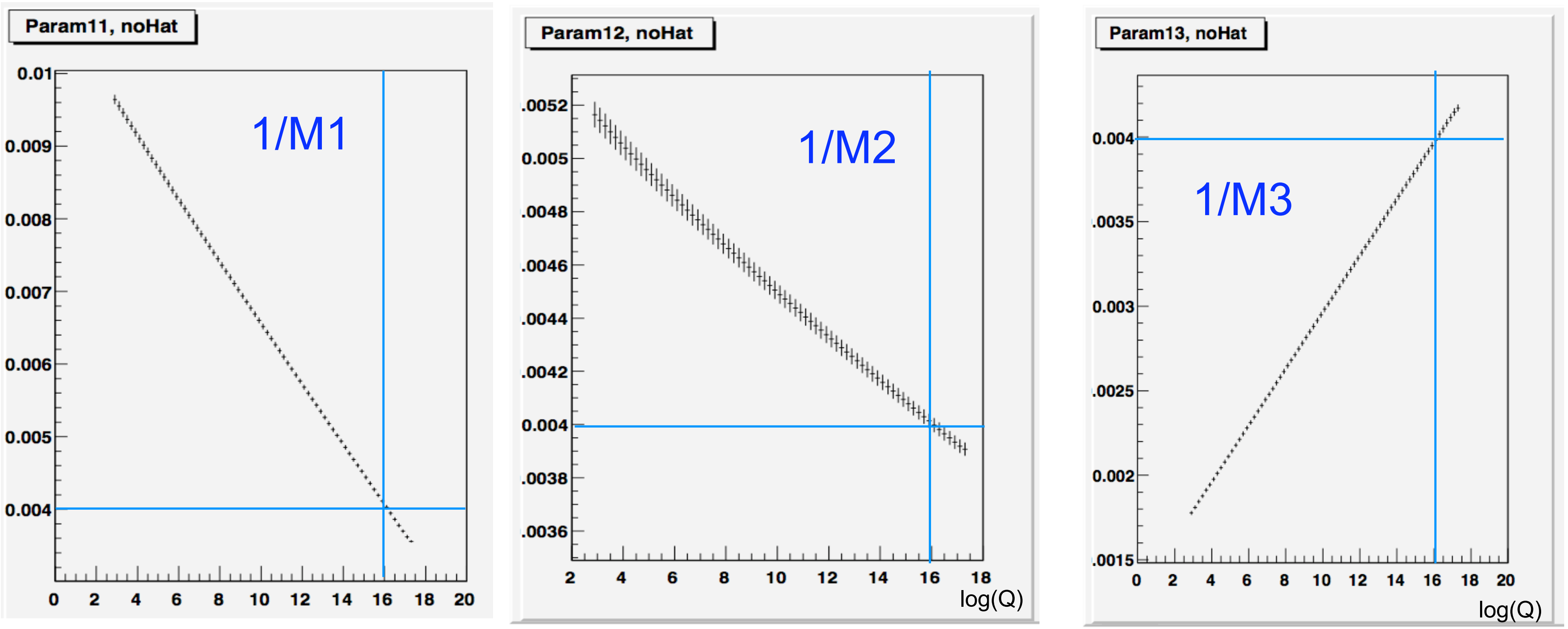


The correct way to estimate the errors would be to make a likelihood plot, but :

- *Most variables have a Gaussian like distribution and no tails*
- *A few (M_{eR} in particular) are strongly asymmetric, but also have very small tails*

Conclusion : the MEAN and RMS are good enough for a first look .

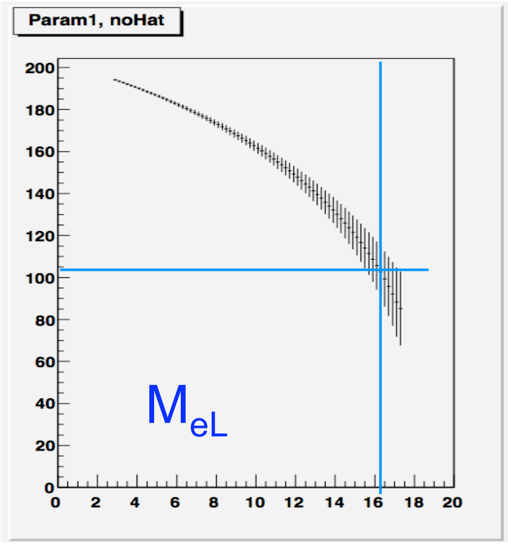
Profile of the gaugino masses :

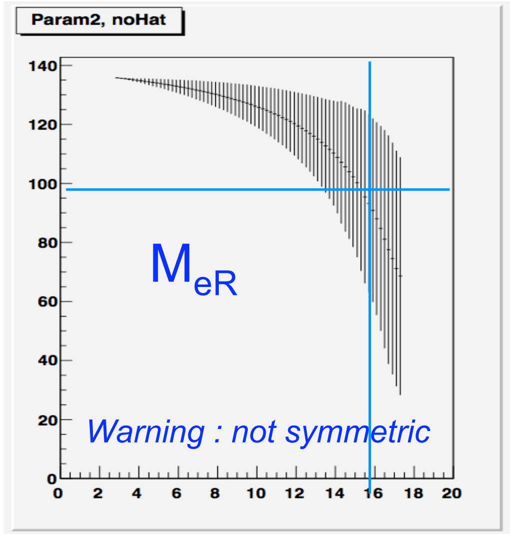


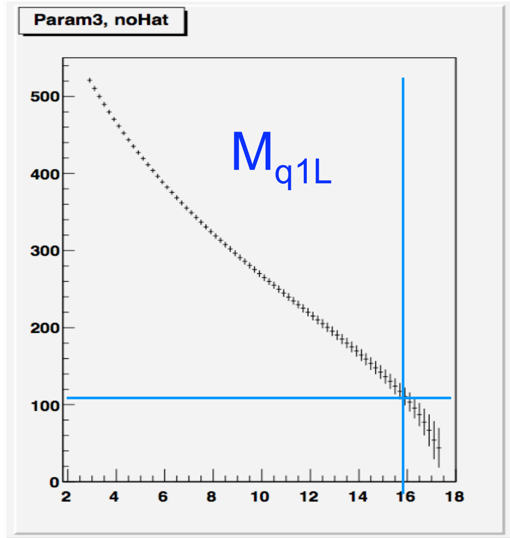
Error bars are RMS :

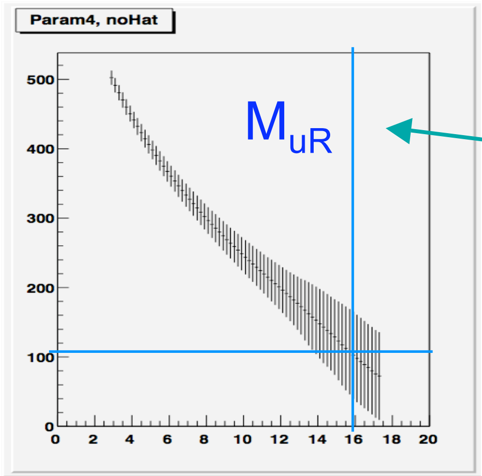
the evolution versus Q^2 is very smooth and stable

Profile of the 1rst generation Susy breaking term of sleptons/squarks

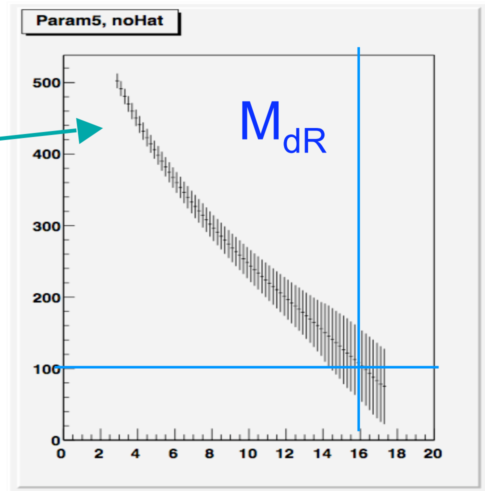








MuR = MdR at EW scale,
but RGE's do their job…

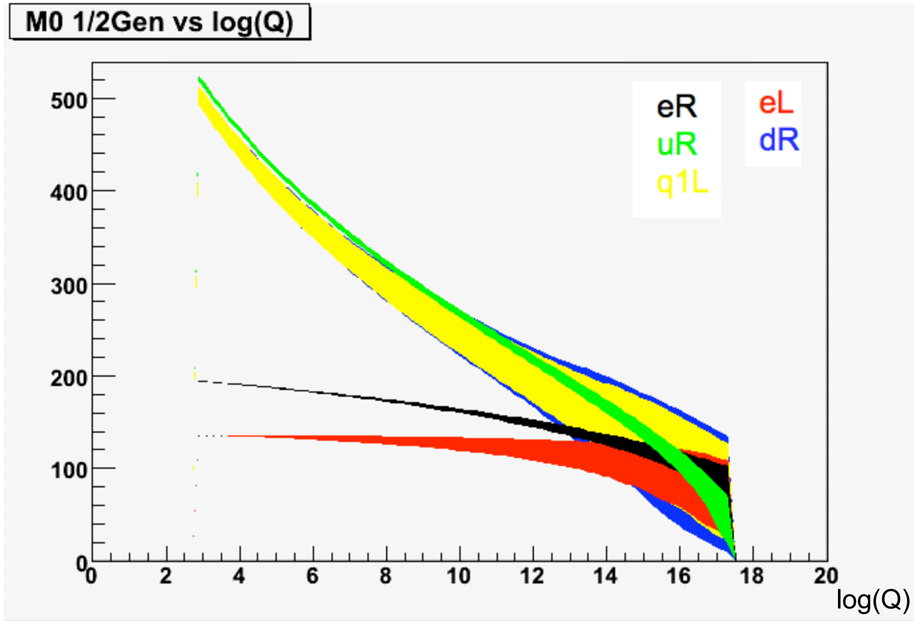


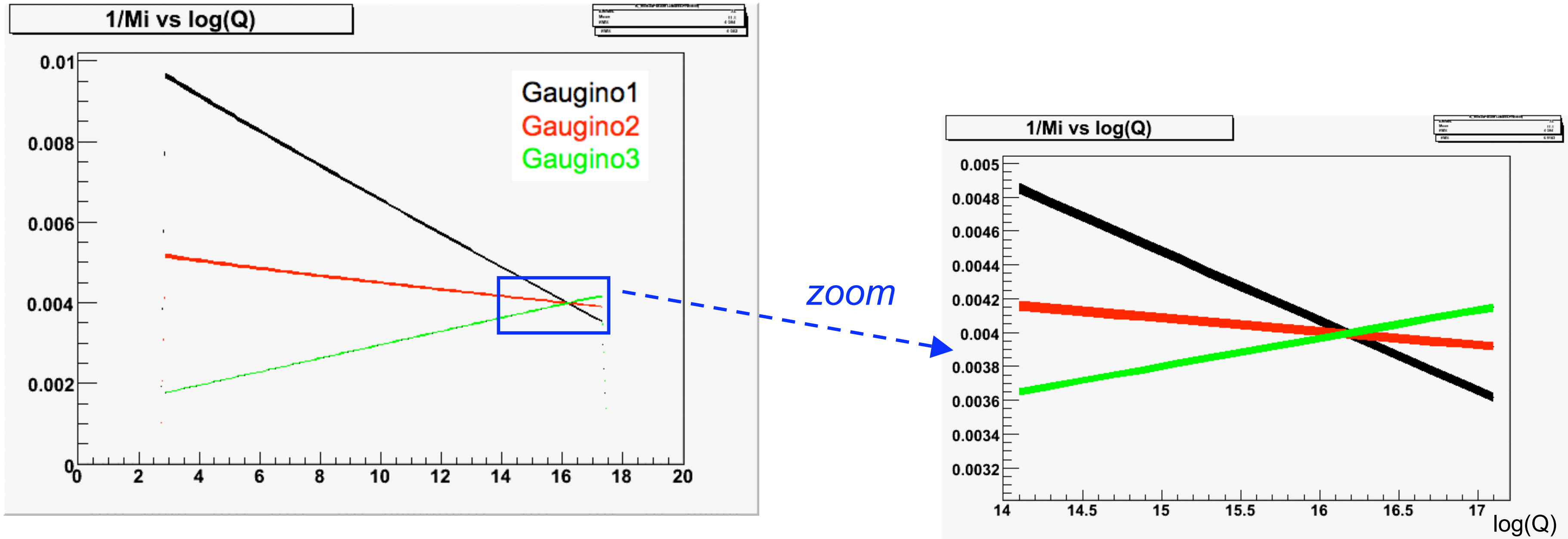
At ~10^{16} GeV (MGUT), all masses converge to ~100gev (M0) => plots "makes sense" !

Profile plots :

MEAN ± RMS

Reminder : theoretical errors are not included in this study (yet)

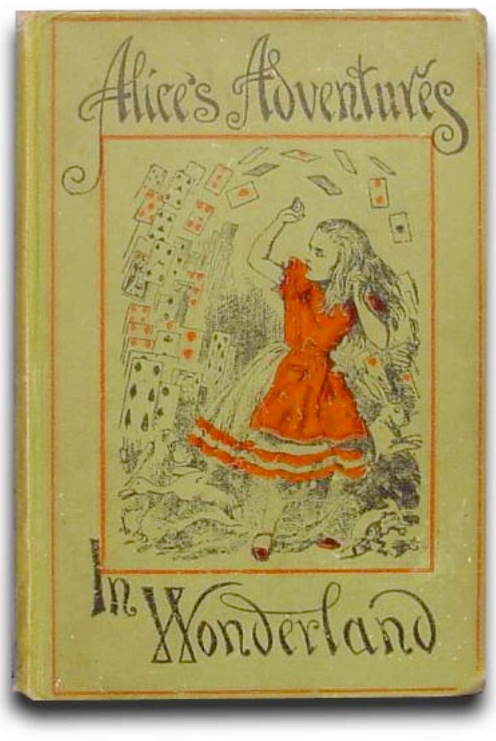




Conclusion :

This exercise should be seen

as a beginner's view !



Next steps include :

- *Include theoretical errors*
- *Use latest Suspect version*
- *Analyze also LHC alone*