Conformal Bootstrap Studies of Puzzles in Critical Phenomena

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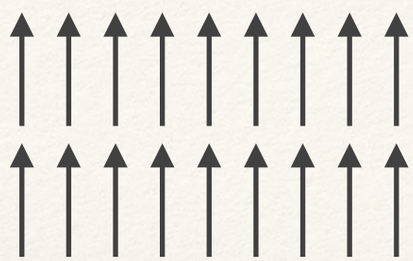
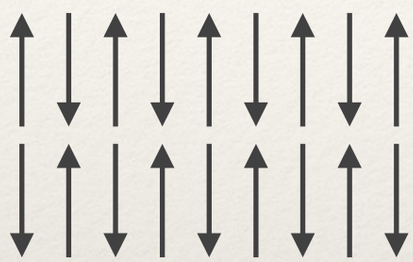
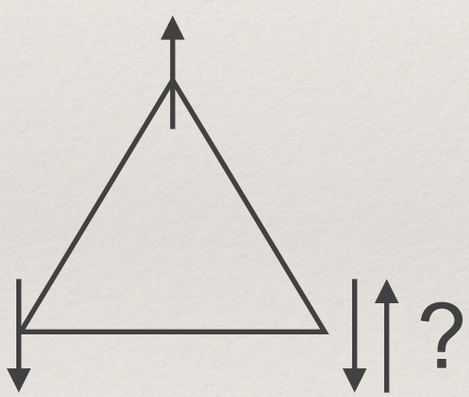
Frustrated Antiferromagnets



Ferromagnet



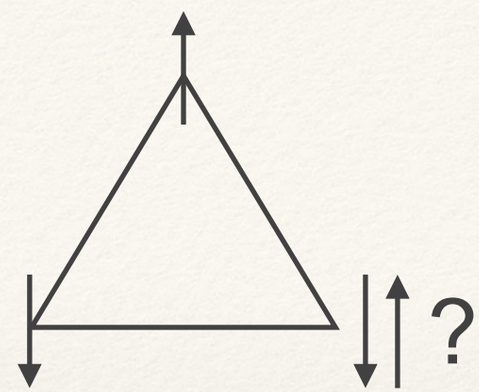
Antiferromagnet



Frustrated antiferromagnet

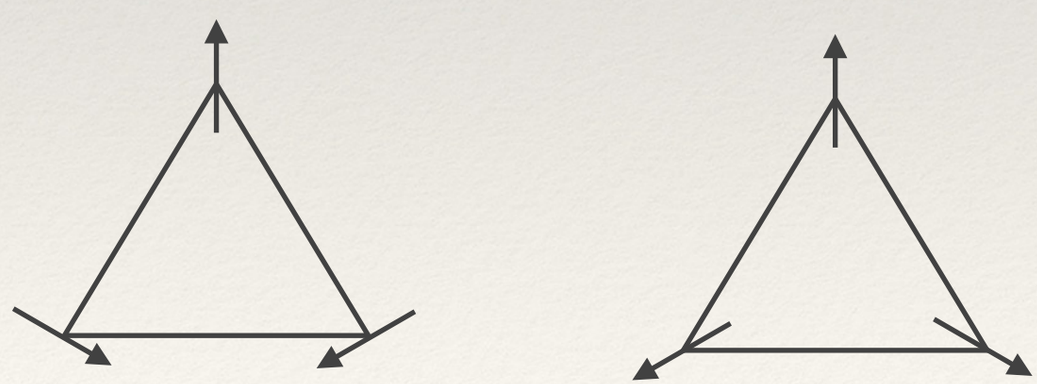
Frustration can arise due to the geometry of the lattice (stacked triangular antiferromagnets) or due to competition between near-neighbor and farther-neighbor interactions (helimagnets).

Frustrated Antiferromagnets

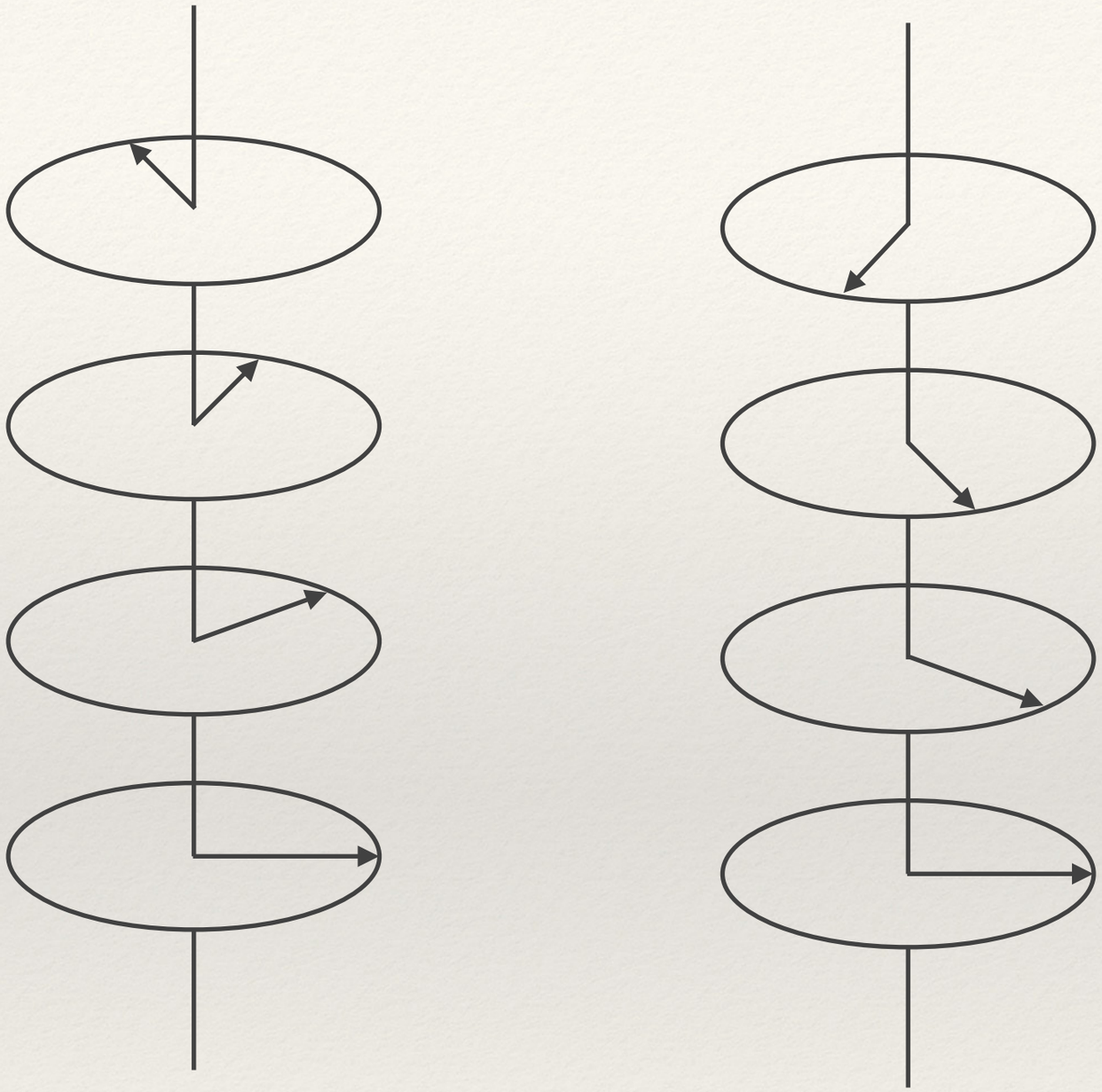


In the Ising case ($\mathbb{Z}_2$ symmetry), frustration leads to nontrivial degeneracy of the ground (ordered) state.

When the spin has continuous symmetry (e.g. O(2)), then we get noncollinear (canted) spin configurations with nontrivial ground-state degeneracy:



Helimagnets

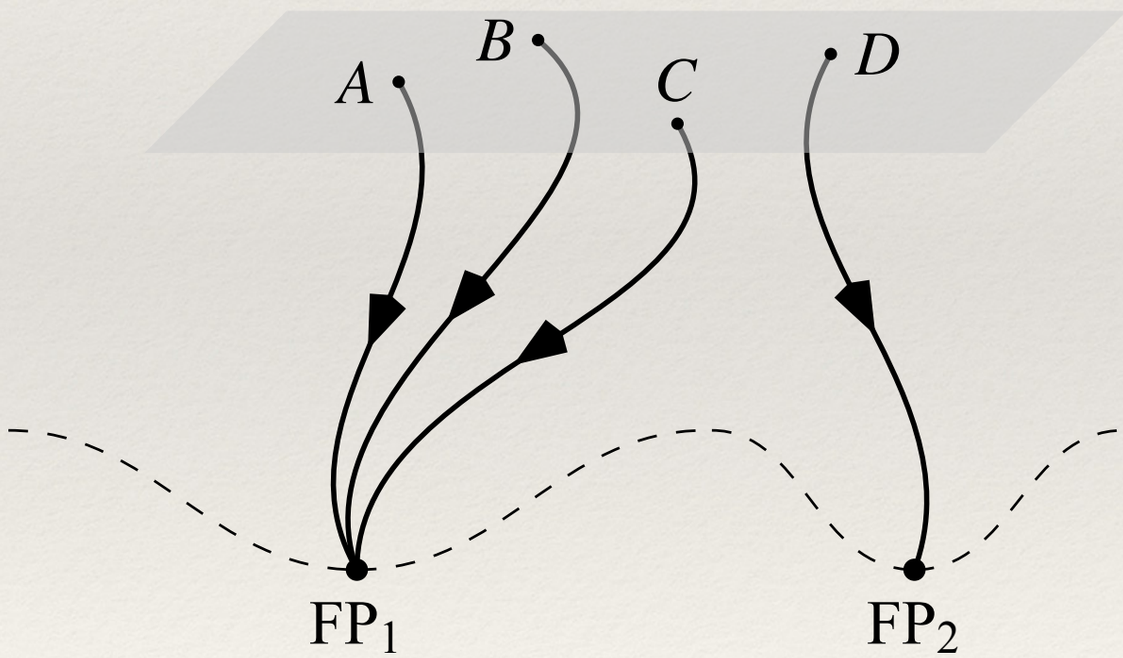


Critical Phenomena in Frustrated Antiferromagnets

Frustration leads to physics in magnetic systems that is different from the more well-known cases with no frustration.

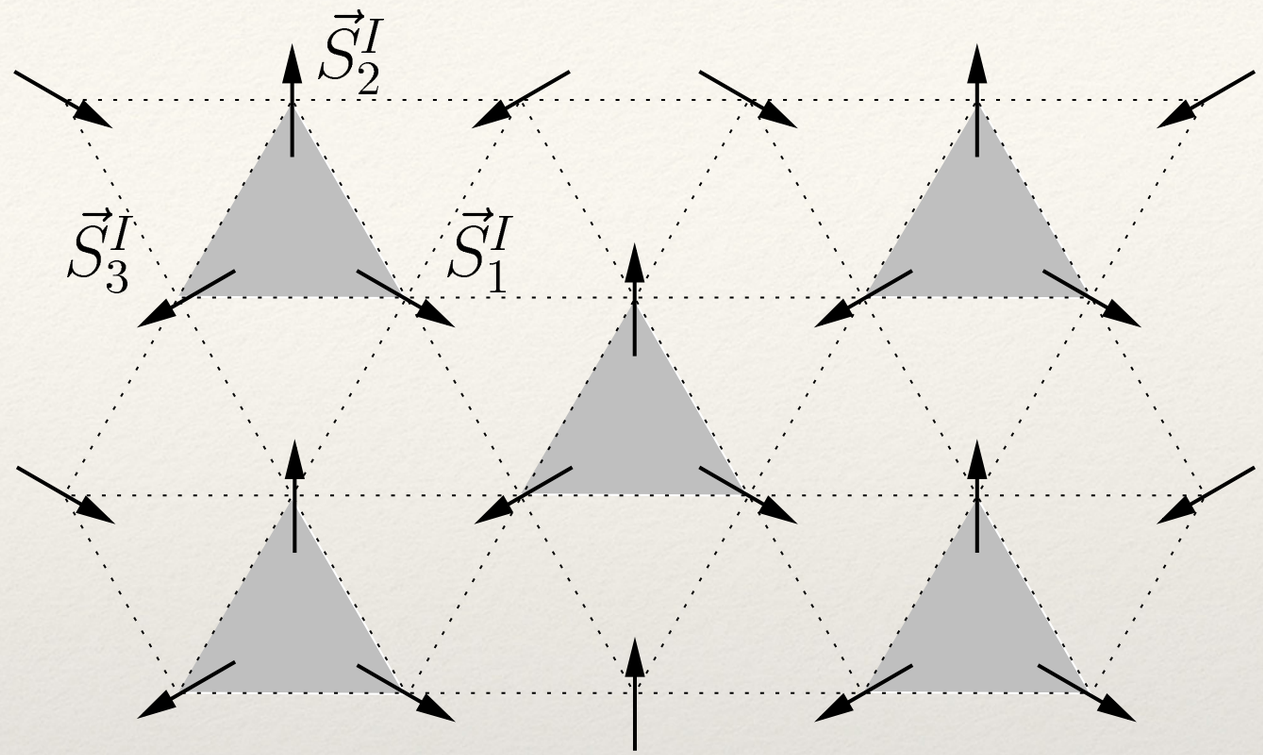
(Kawamura, 1979–1988)

Our interest is in the different universality classes that can be encountered in frustrated systems.



Order Parameter in Frustrated Antiferromagnets

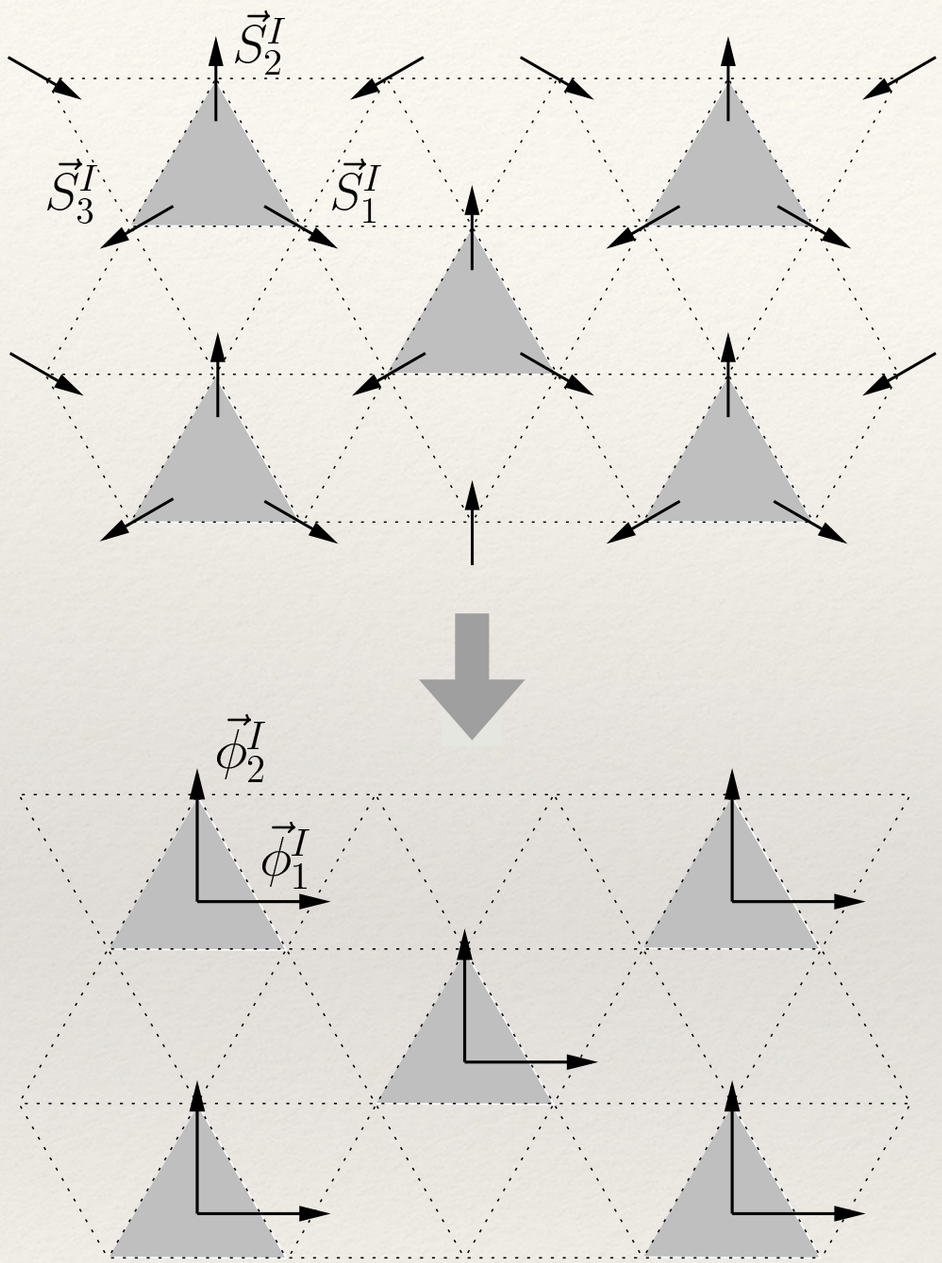
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\vec{\Sigma} \equiv \vec{S}_1^I + \vec{S}_2^I + \vec{S}_3^I = \vec{0}$$

We need to keep two vectors for each plaquette.

Order Parameter in Frustrated Antiferromagnets



Effective interactions:

$$H = H(\vec{\phi}_1, \vec{\phi}_2).$$

Symmetry transformations:

- $\begin{pmatrix} \vec{\phi}'_1 \\ \vec{\phi}'_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \pm\sin\theta & \pm\cos\theta \end{pmatrix} \begin{pmatrix} \vec{\phi}_1 \\ \vec{\phi}_2 \end{pmatrix}$
- $\vec{\phi}'_{1,2} = R\,\vec{\phi}_{1,2}\,,\ R \in O(2)$ or $O(3)$

Symmetry group:

$$O(2) \times O(2) \quad \text{or} \quad O(2) \times O(3)$$

Critical Phenomena in Frustrated Antiferromagnets

A general Lagrangian we can write down is

$$\mathscr{L} = \frac{1}{2}\sum_a \partial_\mu \vec{\phi}_a \cdot \partial^\mu \vec{\phi}_a + \frac{1}{24}u\left(\sum_a \vec{\phi}_a^{\,2}\right)^2 + \frac{1}{24}v\sum_{a,b}\left((\vec{\phi}_a \cdot \vec{\phi}_b)^2 - \vec{\phi}_a^{\,2}\vec{\phi}_b^{\,2}\right)$$

where $\vec{\phi}_a$ are m vectors of size n each.

The global symmetry is $O(m) \times O(n)/\mathbb{Z}_2$.

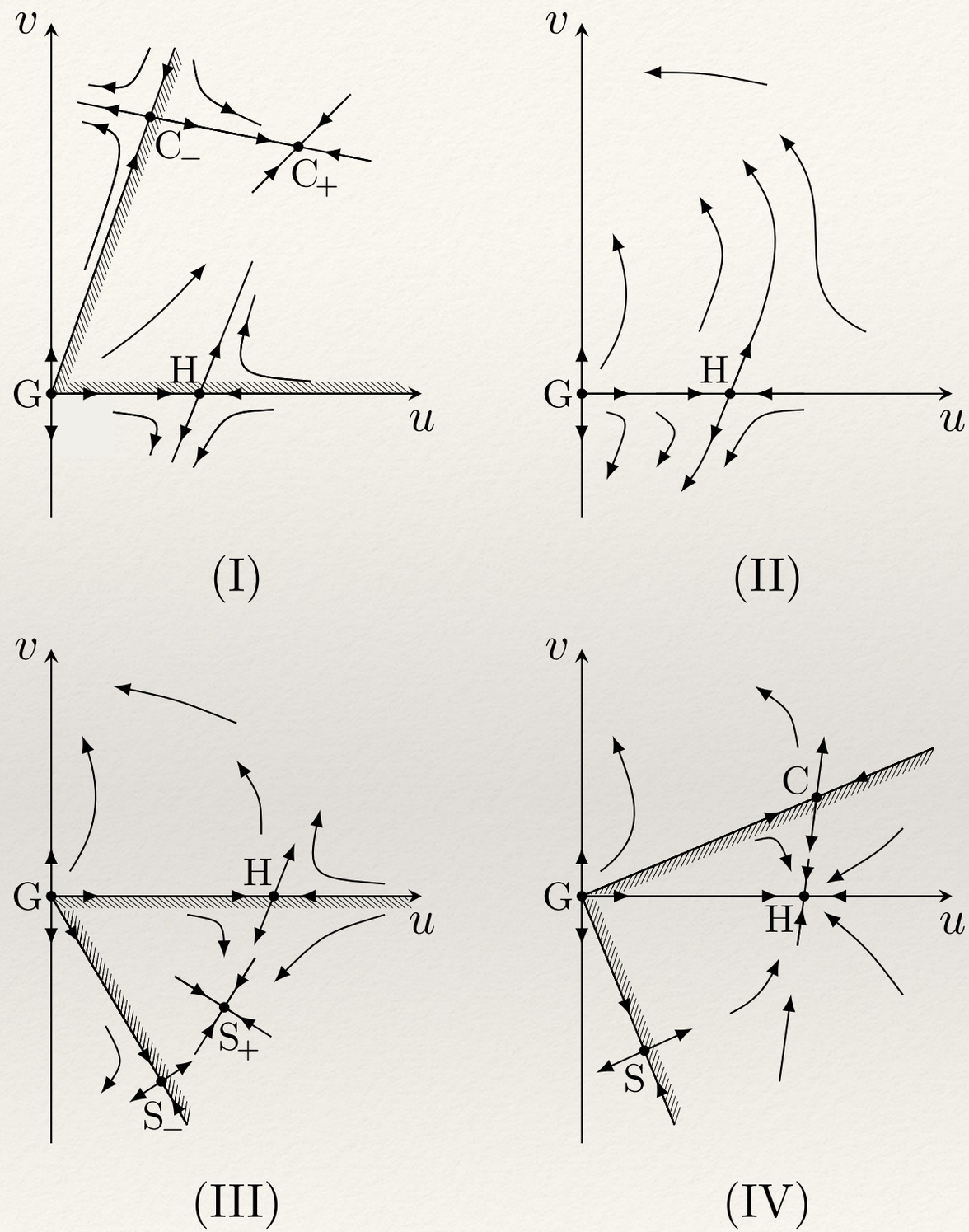
Physical cases realized in experiments have

$$m = n = 2, \qquad m = 2, n = 3.$$

ε Expansion

$\varepsilon = 4 - d$ expansion:

different phase diagrams depending on m, n.



$O(m) \times O(n)$ Irreps

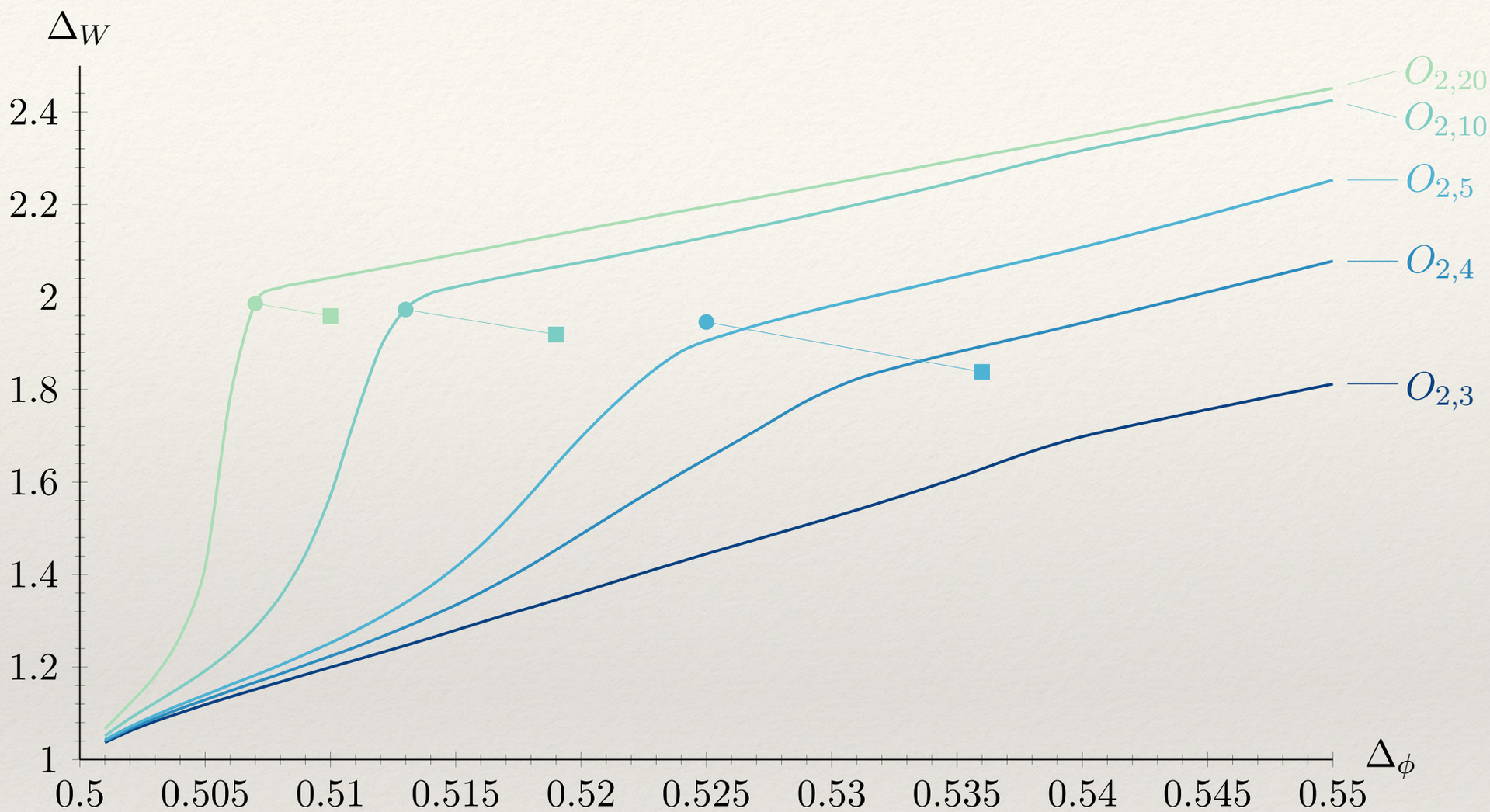
$$\phi \times \phi \sim \underbrace{SS + ST + TS + TT + AA}_{\text{even spin}} + \underbrace{SA + AS + TA + AT}_{\text{odd spin}}$$

Enough to bootstrap: four-point function of ϕ
four-point functions of ϕ and SS

In the following:

$$SS \to S$$
$$TS \to W$$
$$ST \to X$$
$$AA \to Z$$

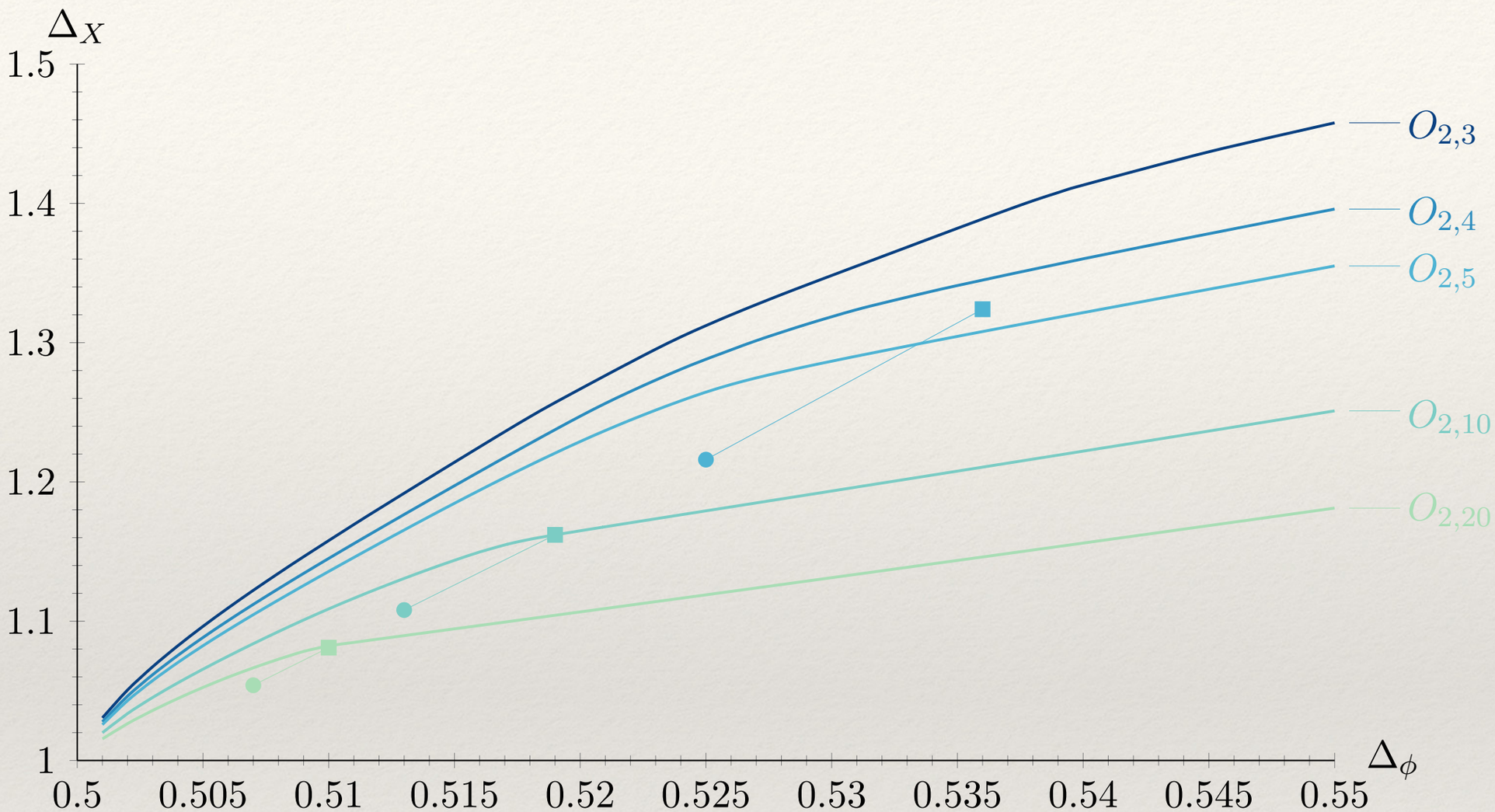
Single Correlator Bounds



(Henriksson, Kousvos & AS, 2020)
(see also Nakayama & Ohtsuki, 2014)

Squares: chiral fixed points
Circles: antichiral fixed points

Single Correlator Bounds



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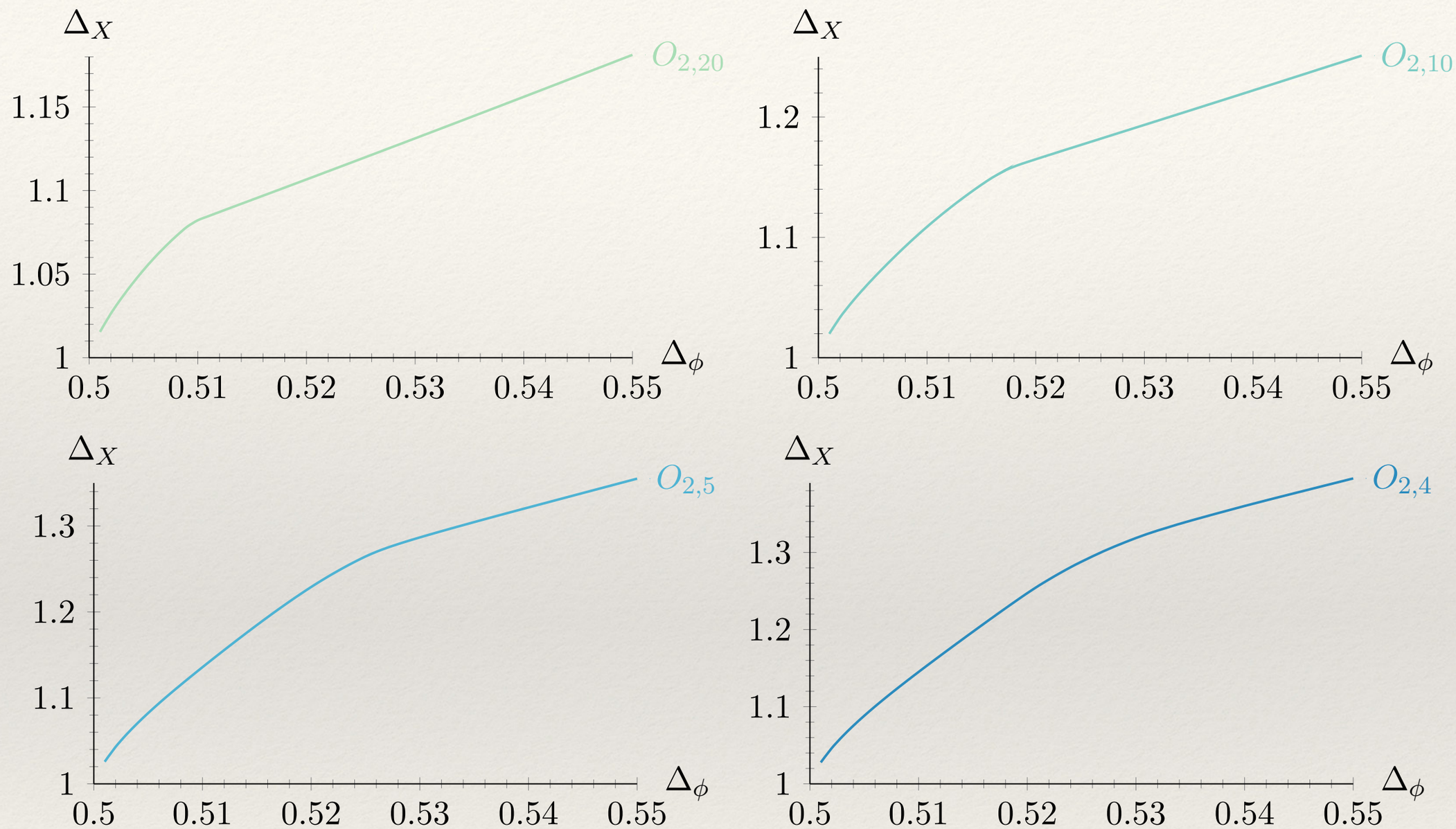
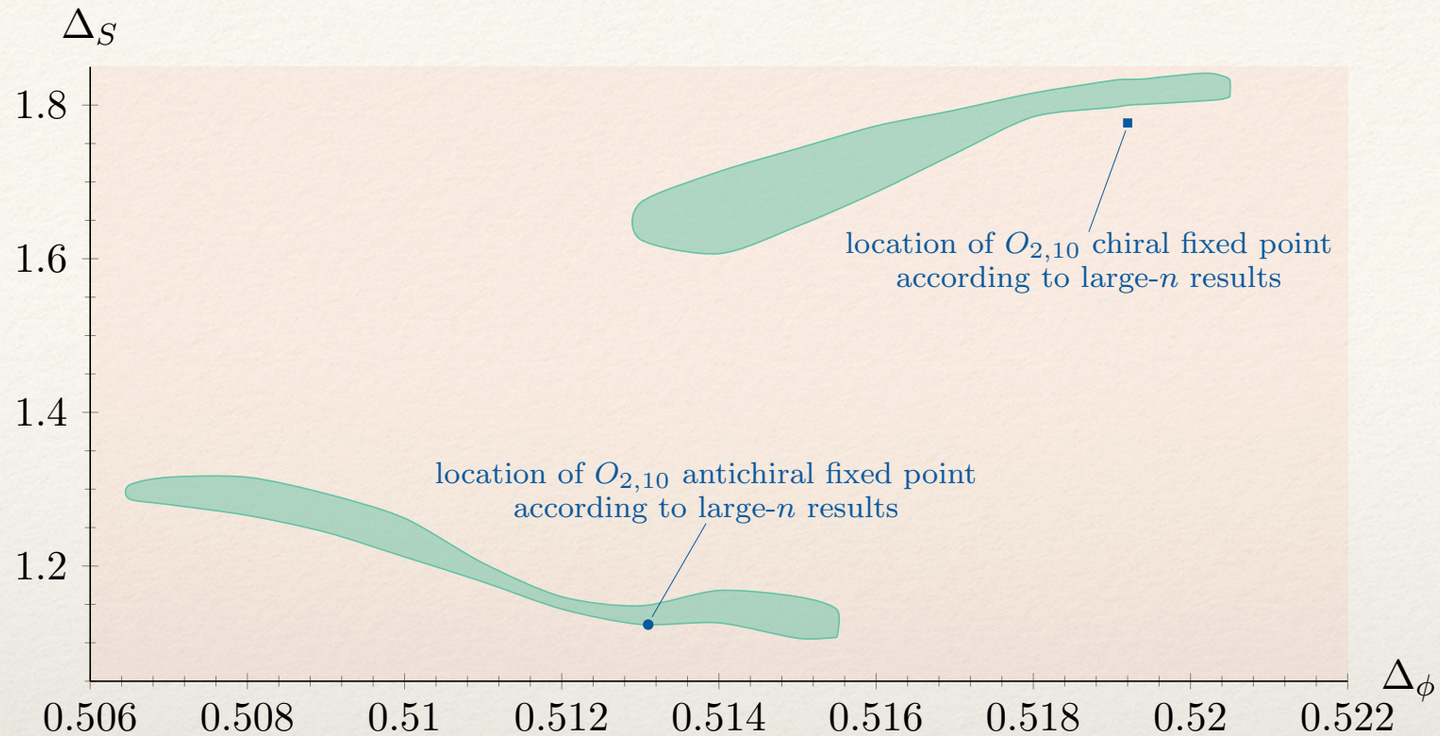
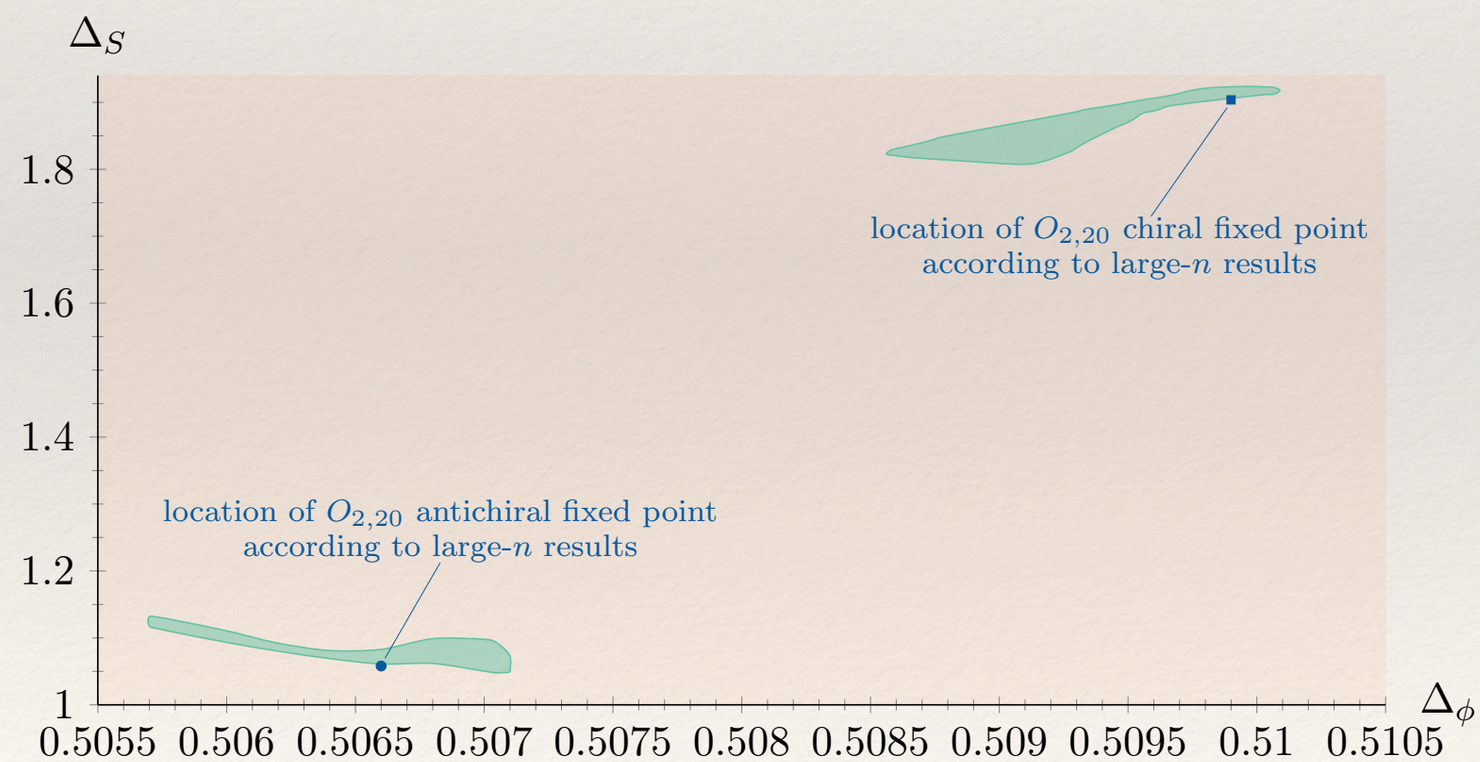


(Henriksson, Kousvos & AS, 2020)

Mixed Correlator Islands

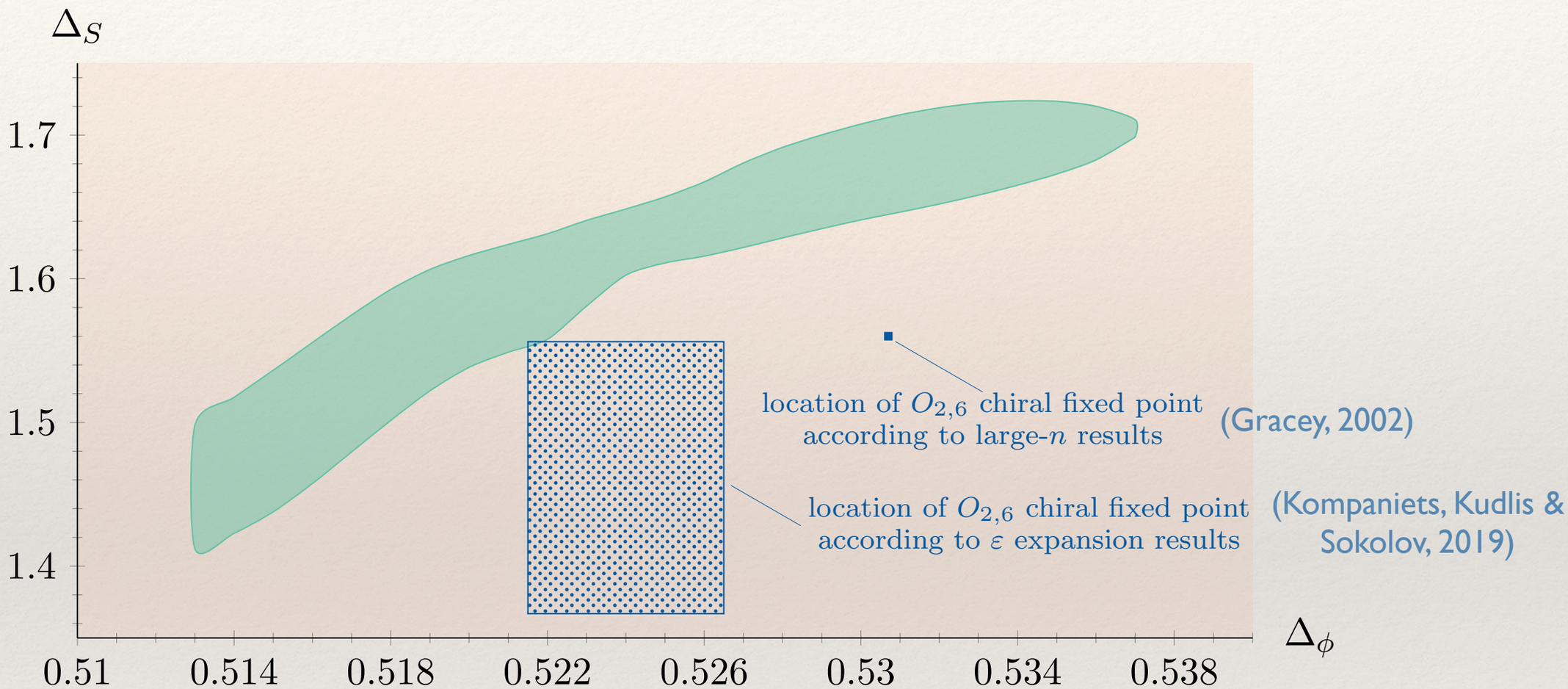


(large-n results from Gracey, 2002)



(Henriksson, Kousvos & AS, 2020)

Mixed Correlator Islands



(Henriksson, Kousvos & AS, 2020)

$O(2) \times O(2)$ Theory

$$\mathscr{L} = \frac{1}{2}\sum_a \partial_\mu \vec{\phi}_a \cdot \partial^\mu \vec{\phi}_a + \frac{1}{24}u\Big(\sum_a \vec{\phi}_a^{\,2}\Big)^2 + \frac{1}{24}v\sum_{a,b}\big((\vec{\phi}_a \cdot \vec{\phi}_b)^2 - \vec{\phi}_a^{\,2}\vec{\phi}_b^{\,2}\big)$$

For $m = n = 2$ there is a symmetry enhancement:

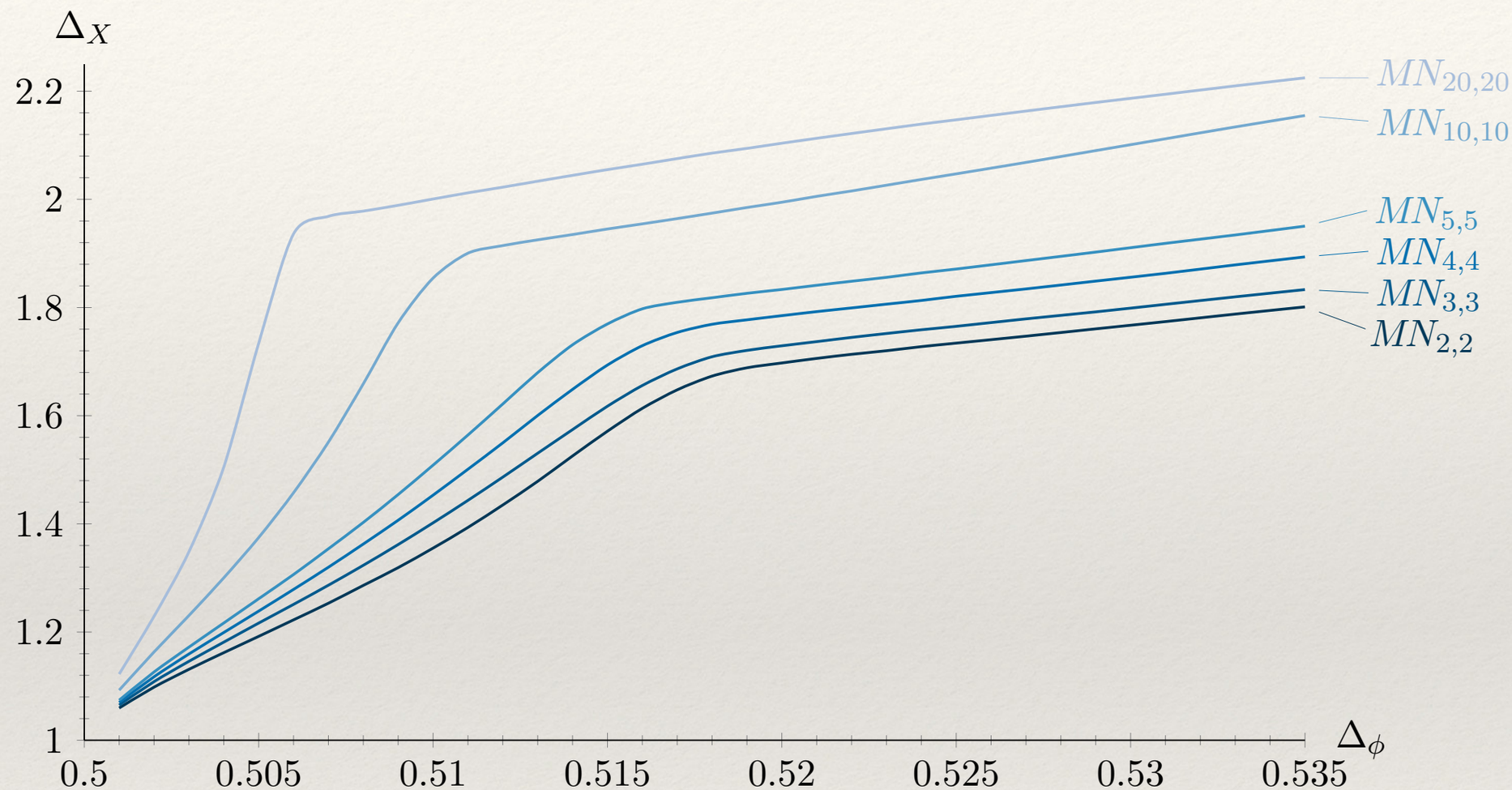
$$O(2) \times O(2) \to O(2)^2 \rtimes S_2$$

Consider the symmetry group $MN_{m,n} = O(m)^n \rtimes S_n$.

$O(2)^2 \rtimes S_2$: Describes XY STAs, NbO_2, Ho, Dy, Tb

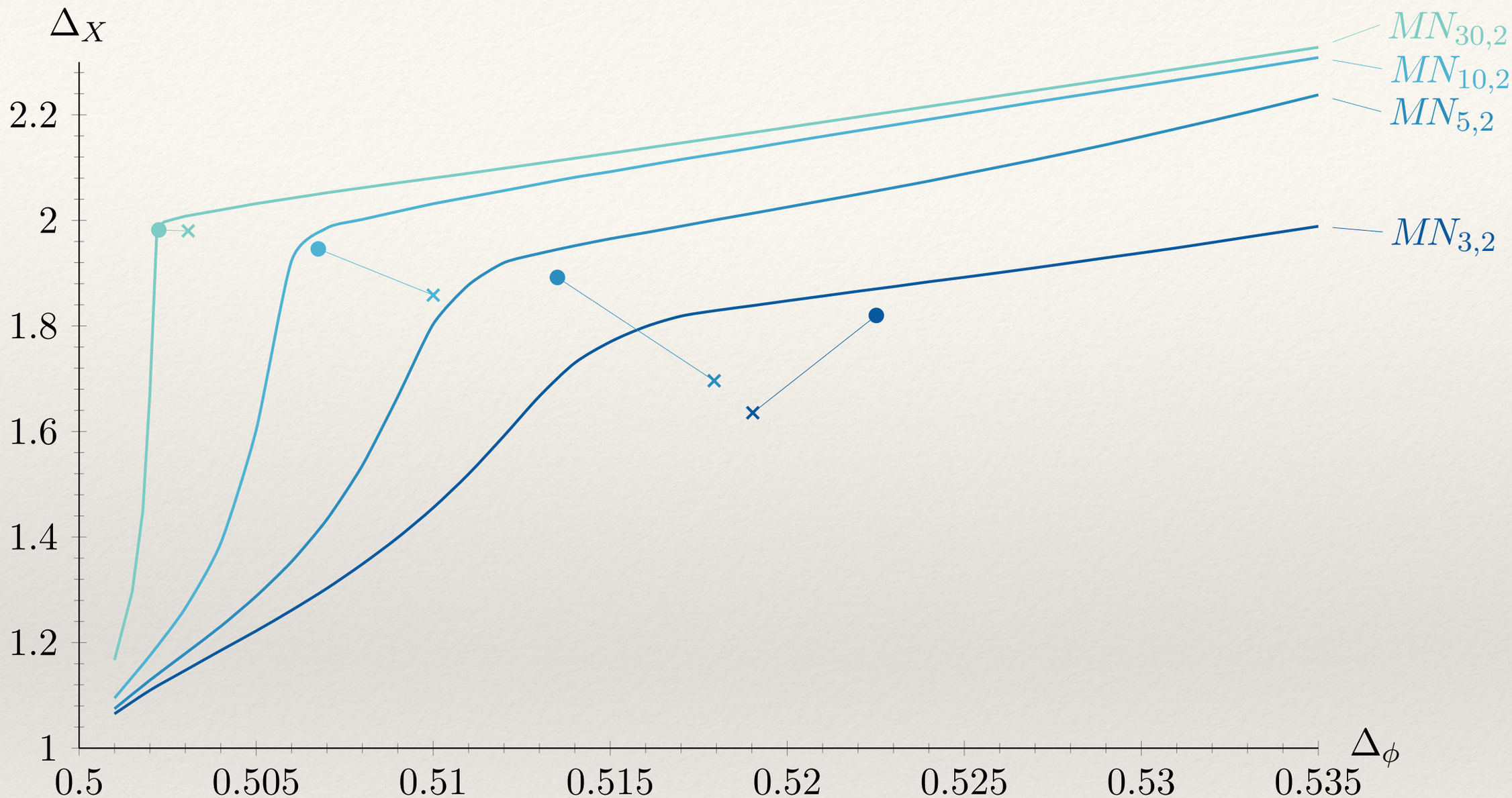
$O(2)^3 \rtimes S_3$: Describes K_2IrCl_6, TbD_2, Nd

Bootstrap Bounds for MN CFTs



(AS, 2019)

Bootstrap Bounds for MN CFTs



(AS, 2019; Henriksson & AS, 2021)

Dots: large *m* (leading order)
Crosses: ε expansion (up to order ε^3)

Experiments in Frustrated Systems

Compound	α	β	γ	ν
$CsMnBr_3$		0.21(1) 0.24(2) 0.21(2) 0.25(1) 0.22(2)	1.01(8)	0.54(3)
	0.39(9) 0.40(5) 0.44(5)		1.10(5)	0.57(3)
$CsNiCl_3$	0.37(8) 0.37(6) 0.342(5)	0.243 (5)		
$CsMnI_3$	0.34(6)			
$CsCuCl_3$	0.35(5)	0.23-0.25(2) 1st order		

XY STAs

Compound	α	β	γ	ν
Tb	0.20(3)	0.23(4) 0.21(2)		0.53
Ho	1^{st}order 0.27(2) 0.10-0.22	0.30(10) 0.37(10) 0.39(3) 0.39(2) 0.39(4) 0.39(4) 0.41(4) 0.38(1)	1.24(15) 1.14(10)	0.54(4) 0.57(4)
Dy	0.24(2)	0.335(10) $0.39^{+0.04}_{-0.02}$ 0.38(2) 0.39(1)	1.05(7)	0.57(5)

Helimagnets

(summarized in Delamotte et al., 2004)

There is also a structural phase transition in NbO_2 with critical exponents like those in the blue rectangle above.

Results from RG methods

Perturbative expansions (after resummations) for $m = n = 2$:

$\beta \approx 0.370\,, \quad \nu \approx 0.715 \quad (\nu < 0\,?)$ (ε expansion, Mudrov & Varnashev, 2001)

$\beta = 0.309(33)\,, \quad \nu = 0.571(29)$ (Pelissetto, Rossi & Vicari, 2000)

$\beta = 0.354(35)\,, \quad \nu = 0.65(6)$ (Calabrese, Parruccini, Pelissetto & Vicari, 2004)

Fixed point with $\nu > 0$

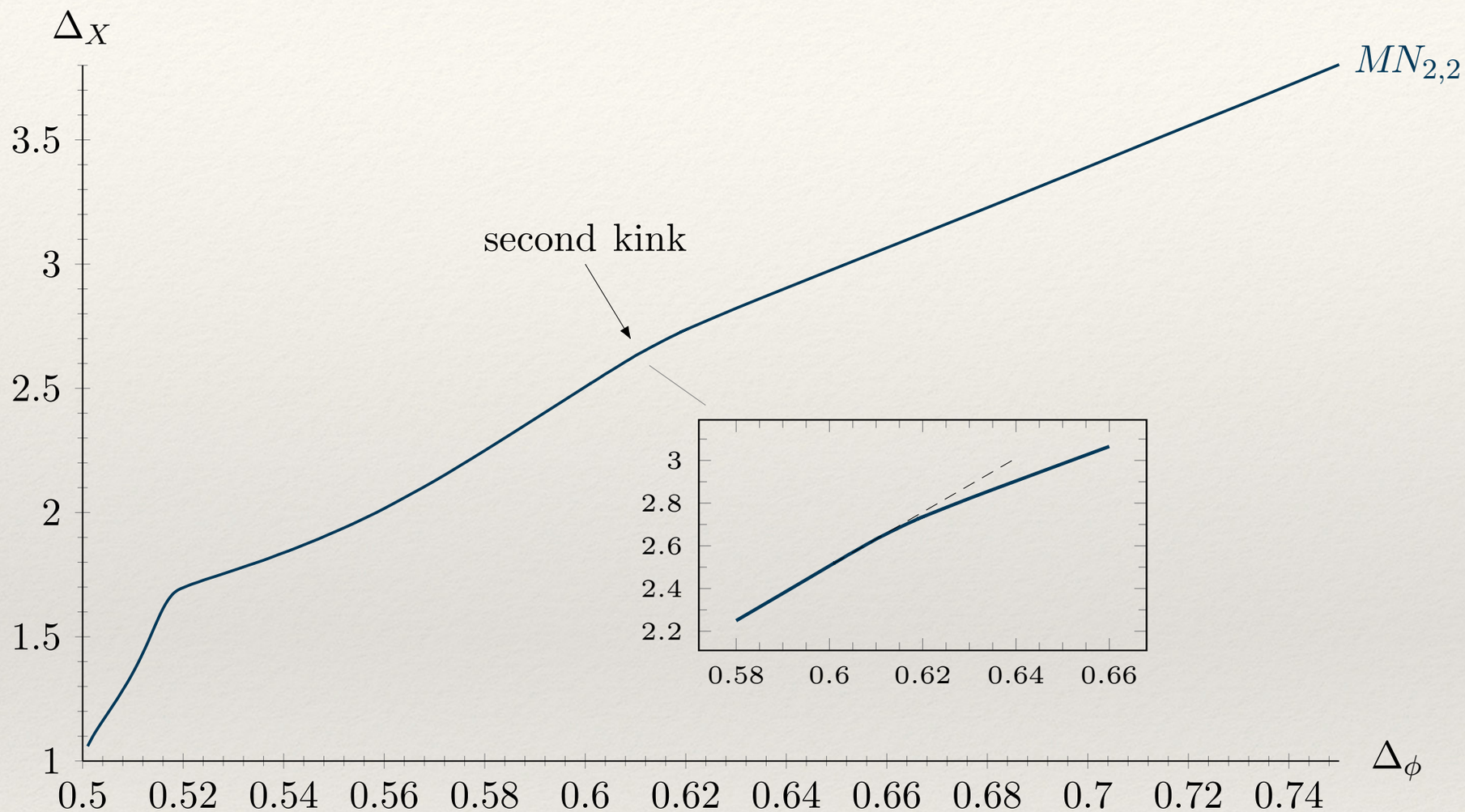
Monte Carlo for $m = n = 2$:

System	α	β	γ	ν	η
STA	0.34(6) 0.46(10) 0.43(10)	0.253(10) 0.24(2)	1.13(5) 1.03(4)	0.54(2) 0.50(1) 0.48(2)	-0.09(8) -0.06(4)
STA	1st order				

Unitarity?

(summarized in Delamotte et al., 2004)

$O(2)^2 \rtimes S_2$



(AS, 2019)

$O(2)^2 \rtimes S_2$

Experiments:

NbO_2	Ho and Dy	Tb and XY STAs
$\beta = 0.40^{+0.04}_{-0.07}$	$\beta = 0.39(4)$	$\beta = 0.23(4)$
	$\nu = 0.57(4)$	$\nu = 0.53(4)$

Unitarity?

RG methods:

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$\beta = 0.354(35)\,, \quad \nu = 0.65(6)$ (Calabrese, Parruccini, Pelissetto & Vicari, 2004)

$\beta = 0.245(30)\,, \quad \nu = 0.51(5)$ (Monte Carlo, averaged)

Bootstrap:

$\beta = 0.293(3)$ $\quad$ $\beta = 0.355(5)$

$\nu = 0.566(6)$ $\quad$ $\nu = 0.576(8)$

(AS, 2019)

Evidence for two distinct CFTs.

$O(2)^3 \rtimes S_3$

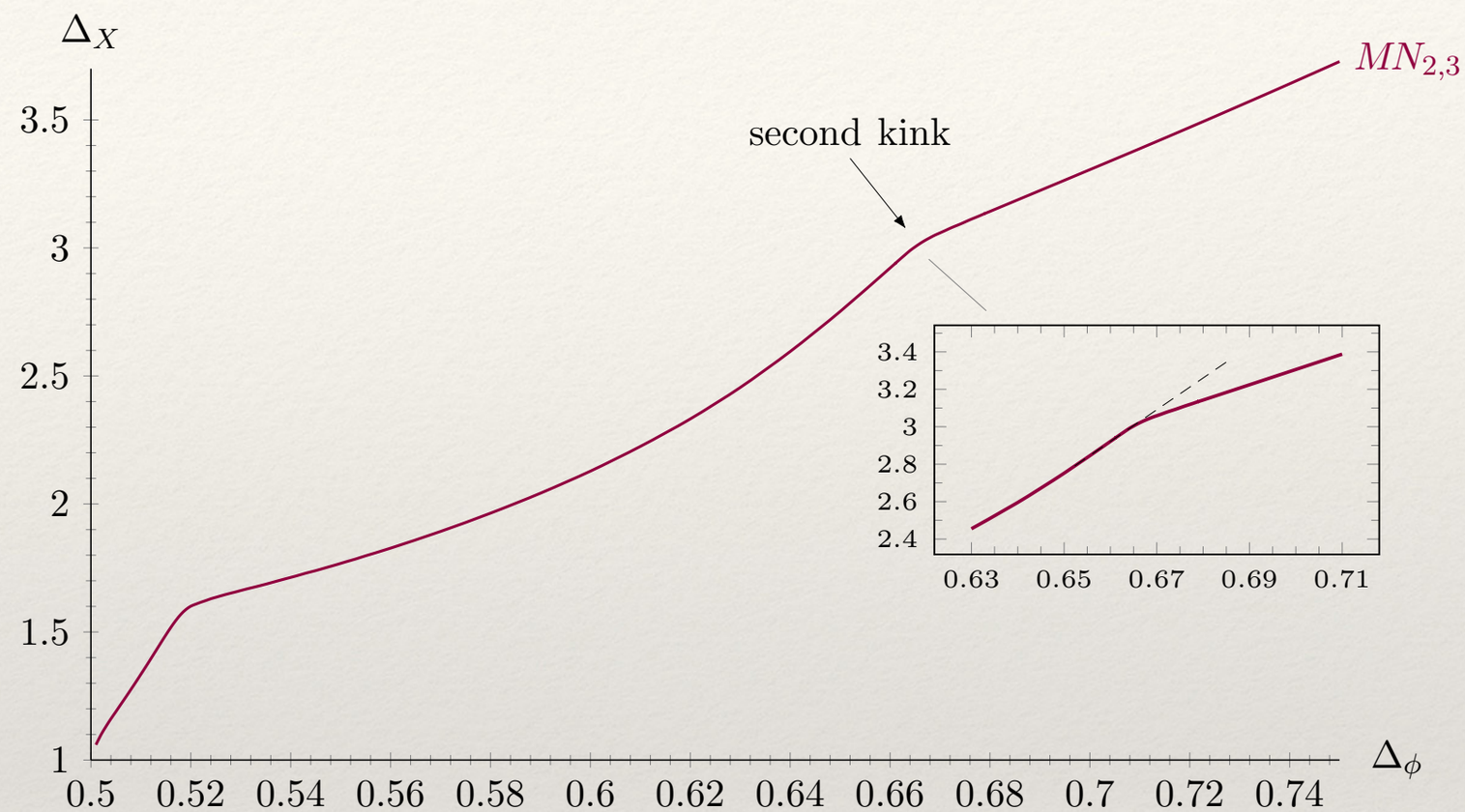
Experiments in Nd:

$$\beta = 0.36(2)$$

Perturbative methods:

$$\beta \approx 0.363\,, \qquad \nu \approx 0.702$$

(ε expansion, Mudrov & Varnashev, 2001)



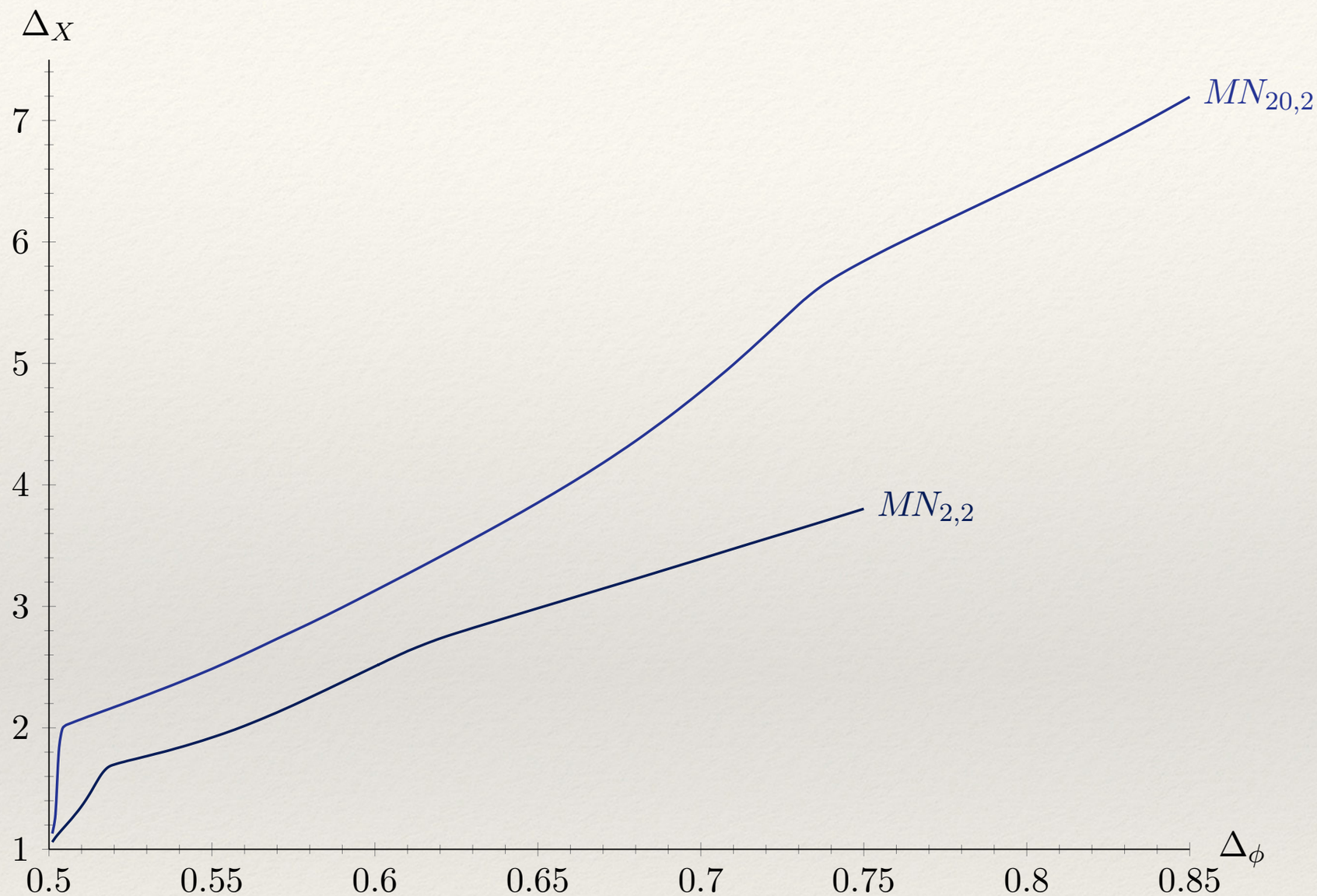
(AS, 2019)

Bootstrap:

$$\beta = 0.301(3) \qquad \beta = 0.394(5)$$

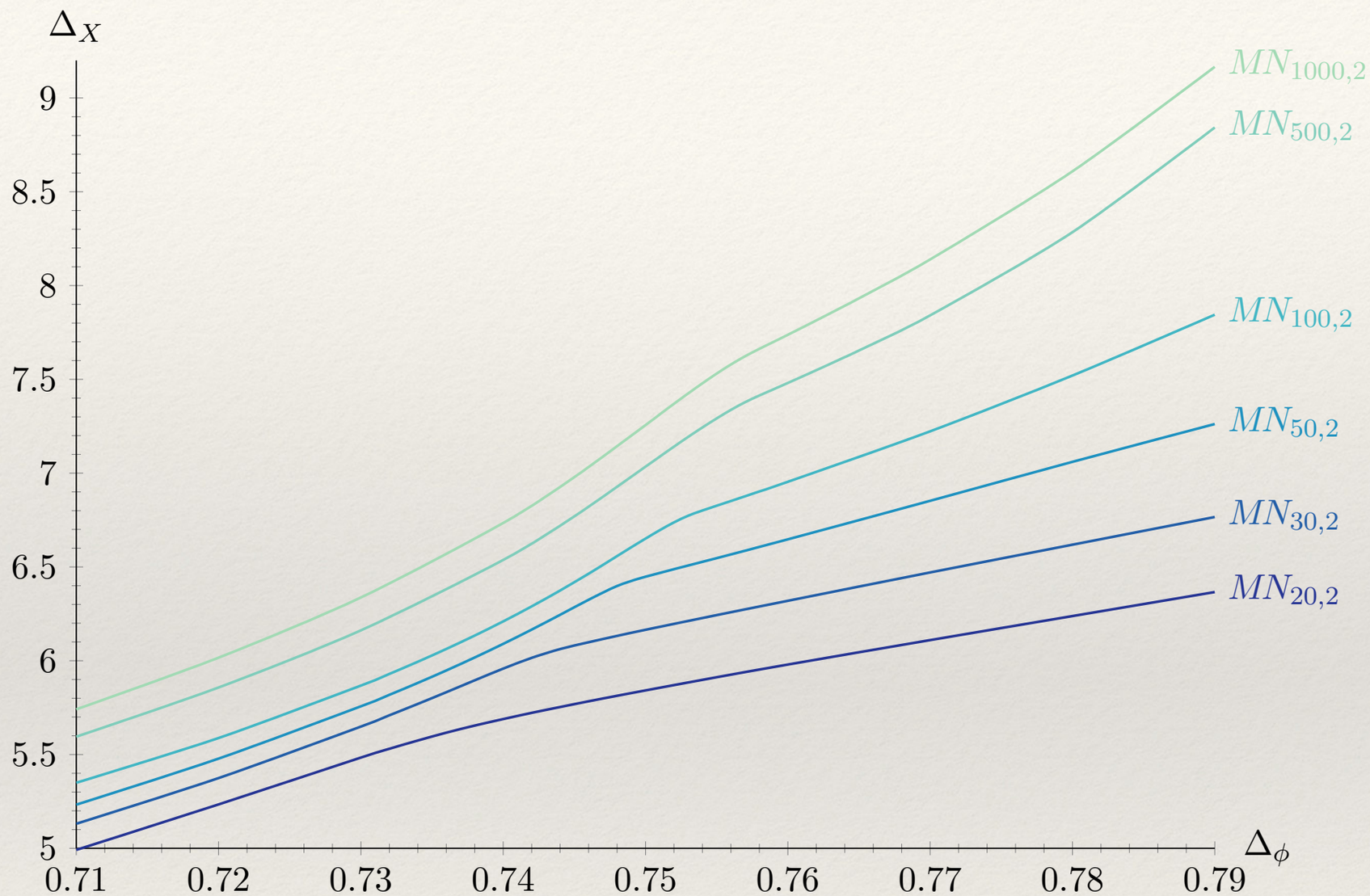
$$\nu = 0.581(6) \qquad \nu = 0.590(8)$$

Bootstrap Bounds for MN CFTs



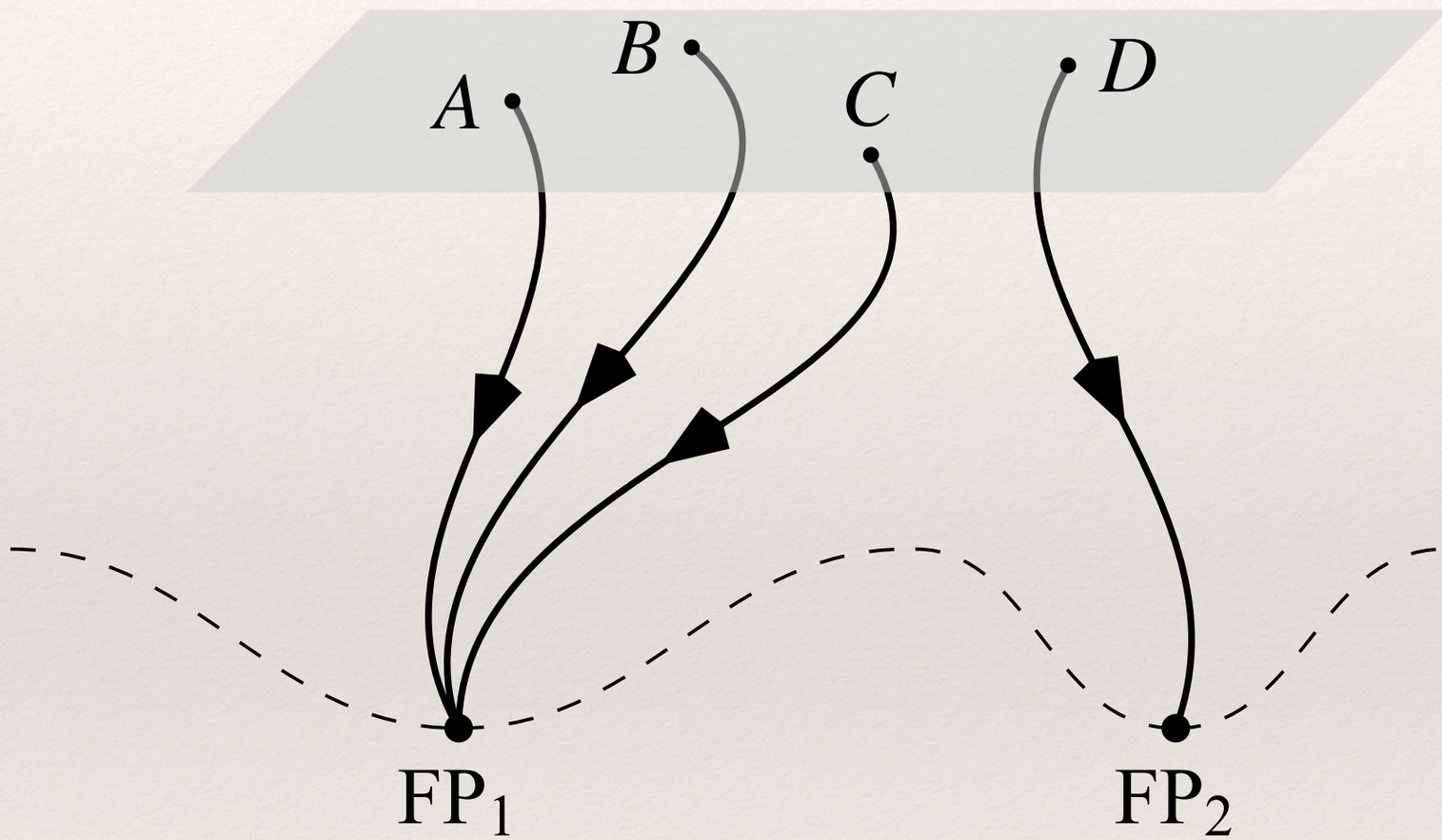
(Henriksson & AS, 2021)

Bootstrap Bounds for MN CFTs

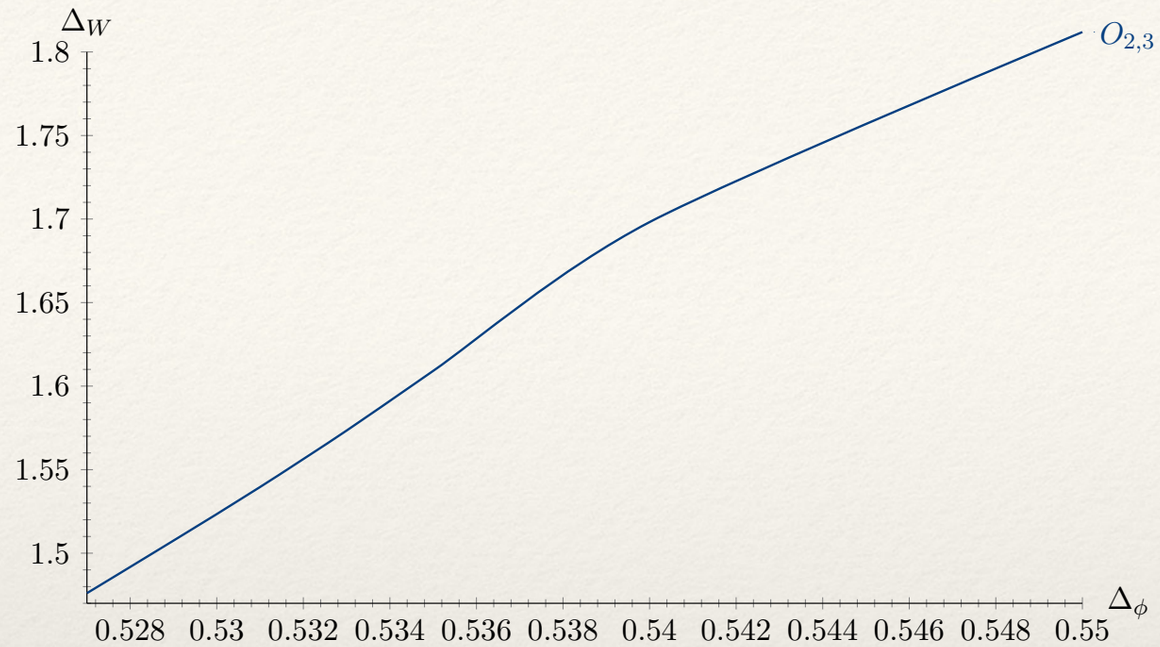


(Henriksson & AS, 2021)

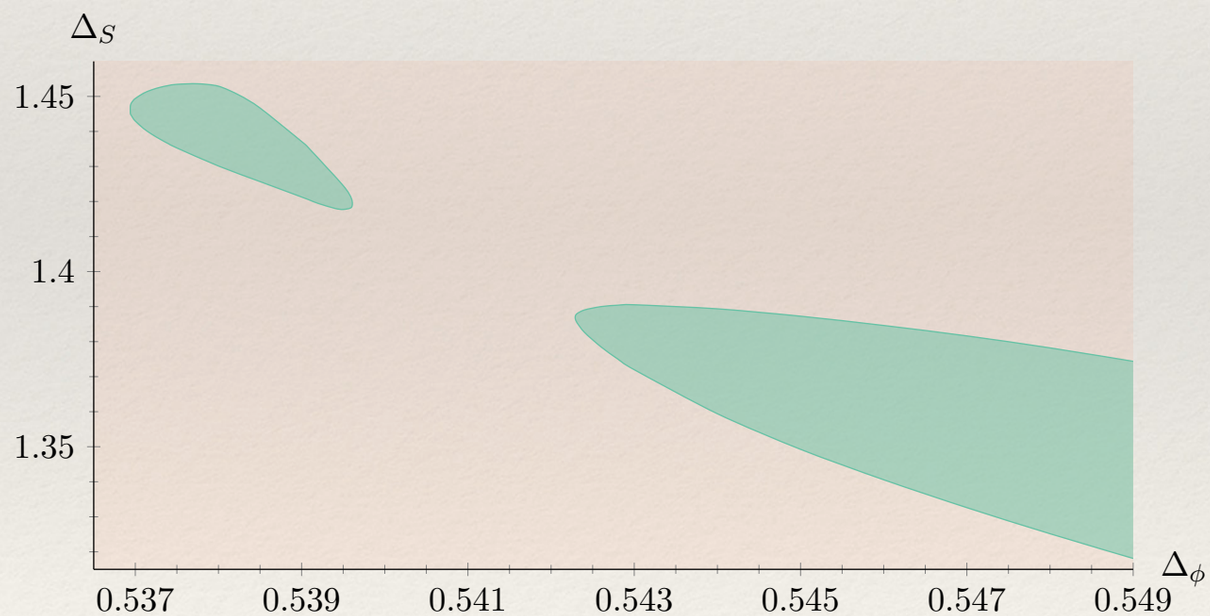
What determines fate in the IR?



$O(2) \times O(3)$ Chiral Fixed Point

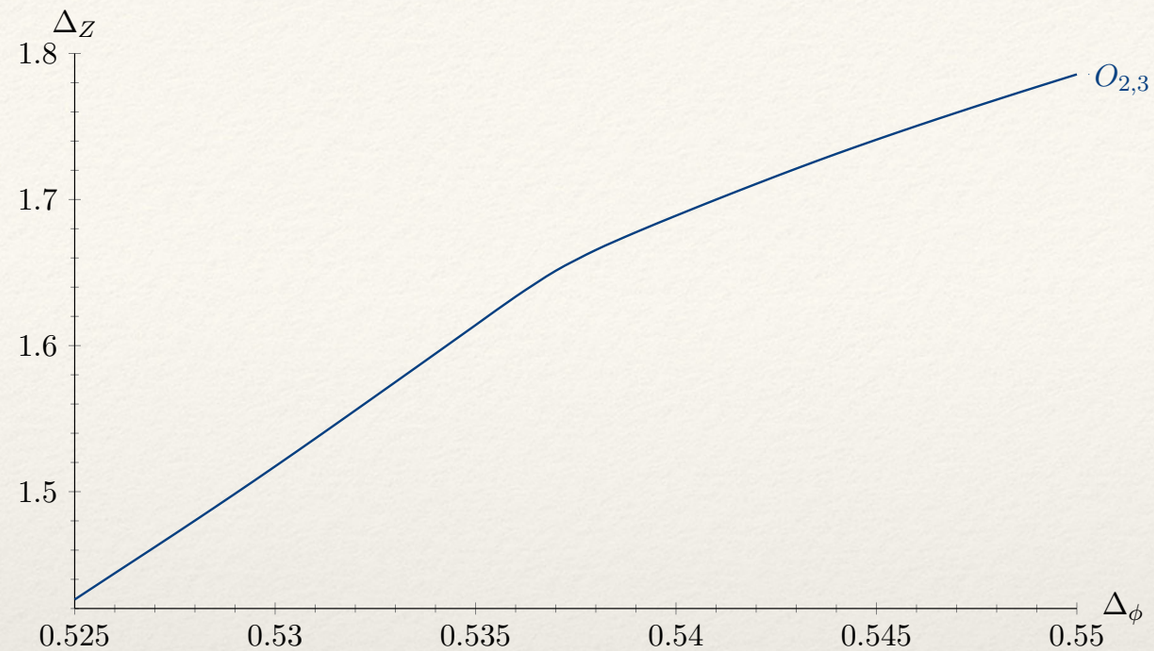


(Nakayama & Ohtsuki, 2014)

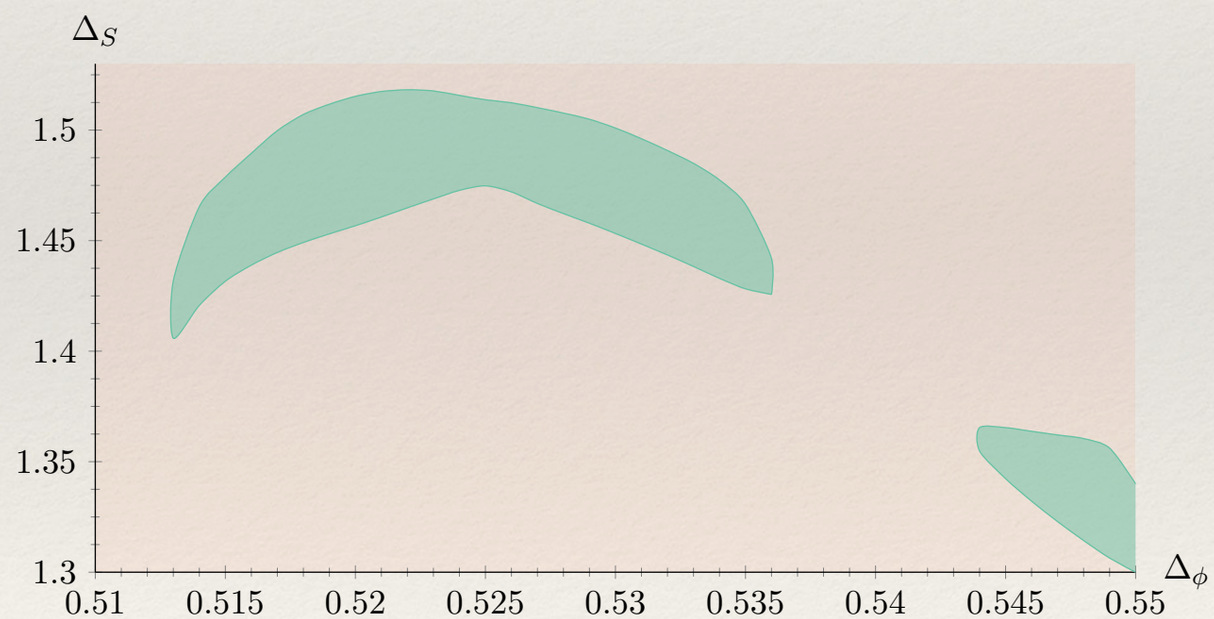


(Henriksson, Kousvos & AS, 2020)

$O(2) \times O(3)$ Collinear Fixed Point



(Nakayama & Ohtsuki, 2014)



(Henriksson, Kousvos & AS, 2020)

$O(2) \times O(3)$

Chiral:

Method	β	ν	ϕ_W
Bootstrap	0.344(5)	0.639(7)	0.818(16)
$\overline{\text{MS}}$ (Calabrese, Pelissetto & Vicari, 2004)	0.34(4)	0.63(5)	0.76(12)
MZM scheme (Calabrese, Pelissetto & Vicari, 2004)	0.30(4)	0.55(3)	0.58(6)
Monte Carlo (Nagano, Uematsu & Kawamura, 2019)	0.26(3)	0.52(1)	–

Experiments in STAs: $\beta = 0.28(3)$, $\nu = 0.62(5)$

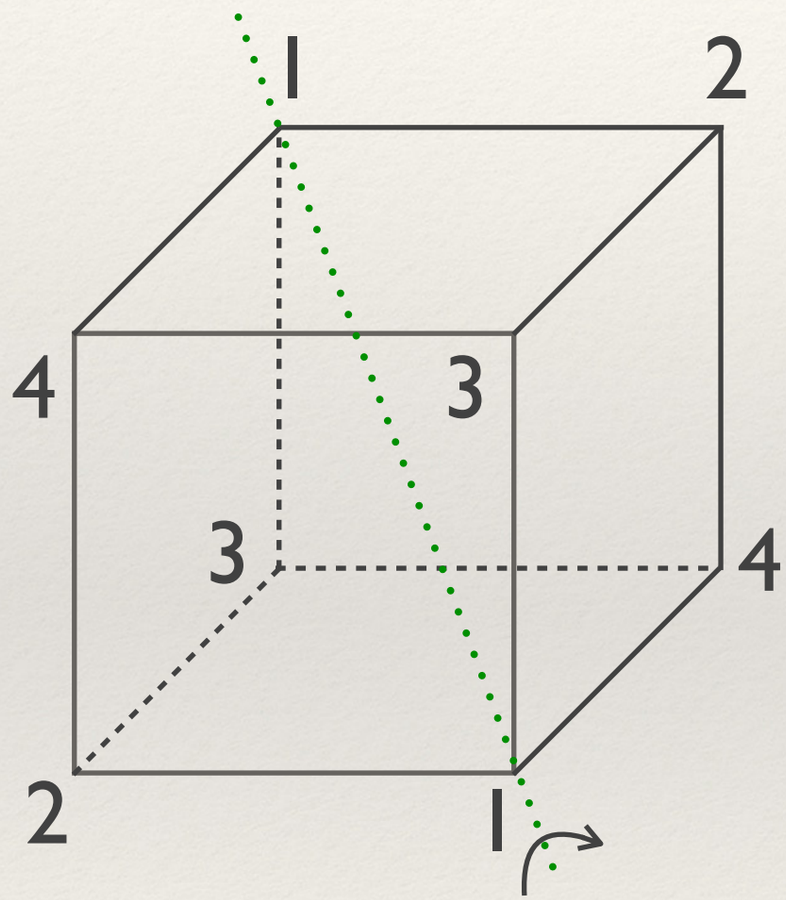
Collinear:

Method	β	ν	ϕ_Z
Bootstrap	0.341(19)	0.650(23)	0.89(4)
$\overline{\text{MS}}$ scheme (Calabrese, Pelissetto & Vicari, 2004)	0.34(5)	0.63(8)	0.75(16)
MZM scheme (Calabrese, Pelissetto & Vicari, 2004)	0.319(23)	0.59(4)	0.74(11)

Cubic CFTs in 3D

Cubic CFTs have a discrete global symmetry,

$$O_h = \mathbb{Z}_2^3 \rtimes S_3 \simeq S_4 \times \mathbb{Z}_2 \,.$$



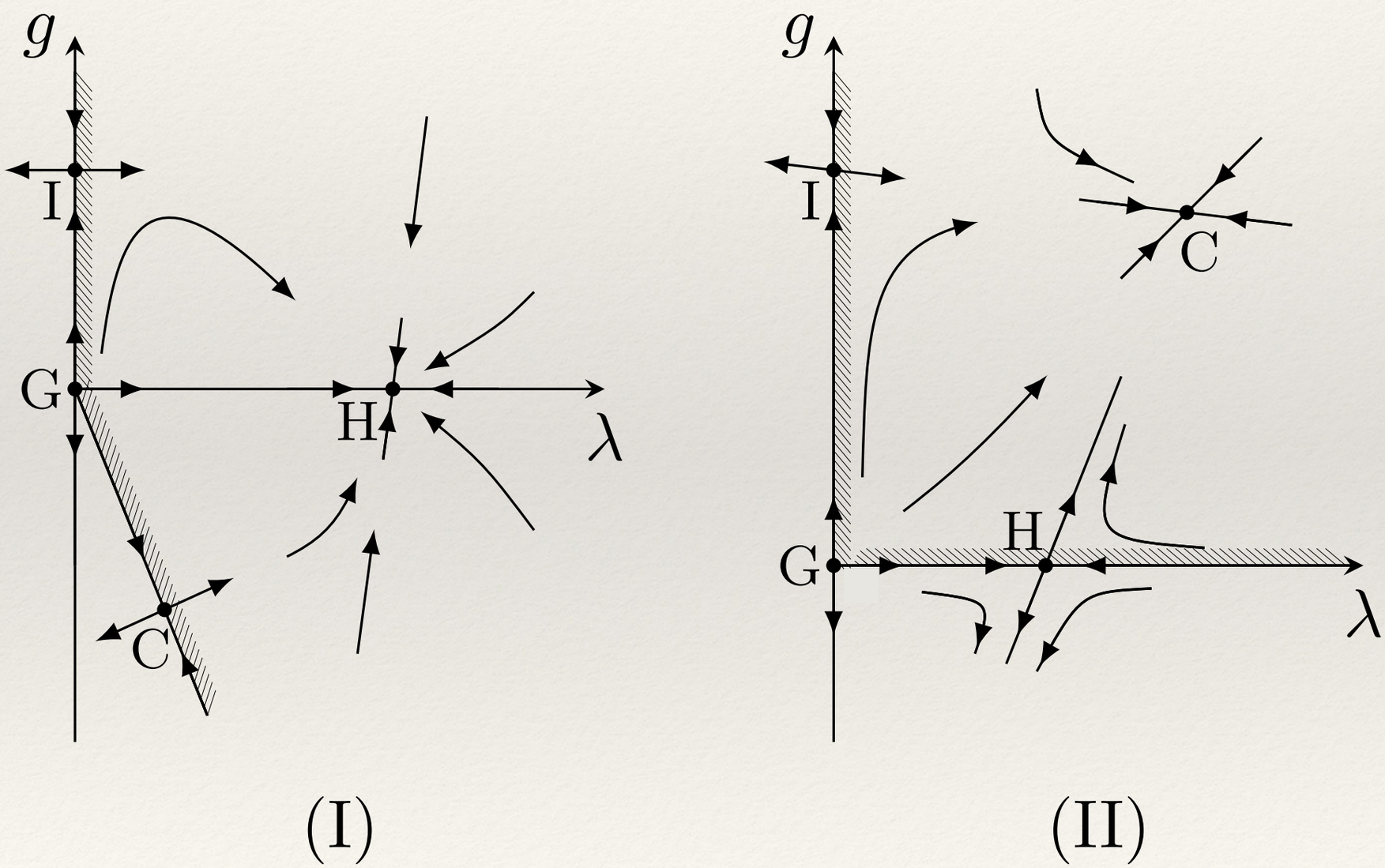
$$\vec{\phi} = (\phi_1, \phi_2, \phi_3)$$

$$V = \frac{1}{8}\lambda(\phi^2)^2 + \frac{1}{24} g \sum_{i=1}^{3} \phi_i^4$$

Relevant whenever we have cubic lattices, e.g. cubic magnets like Fe or Ni.

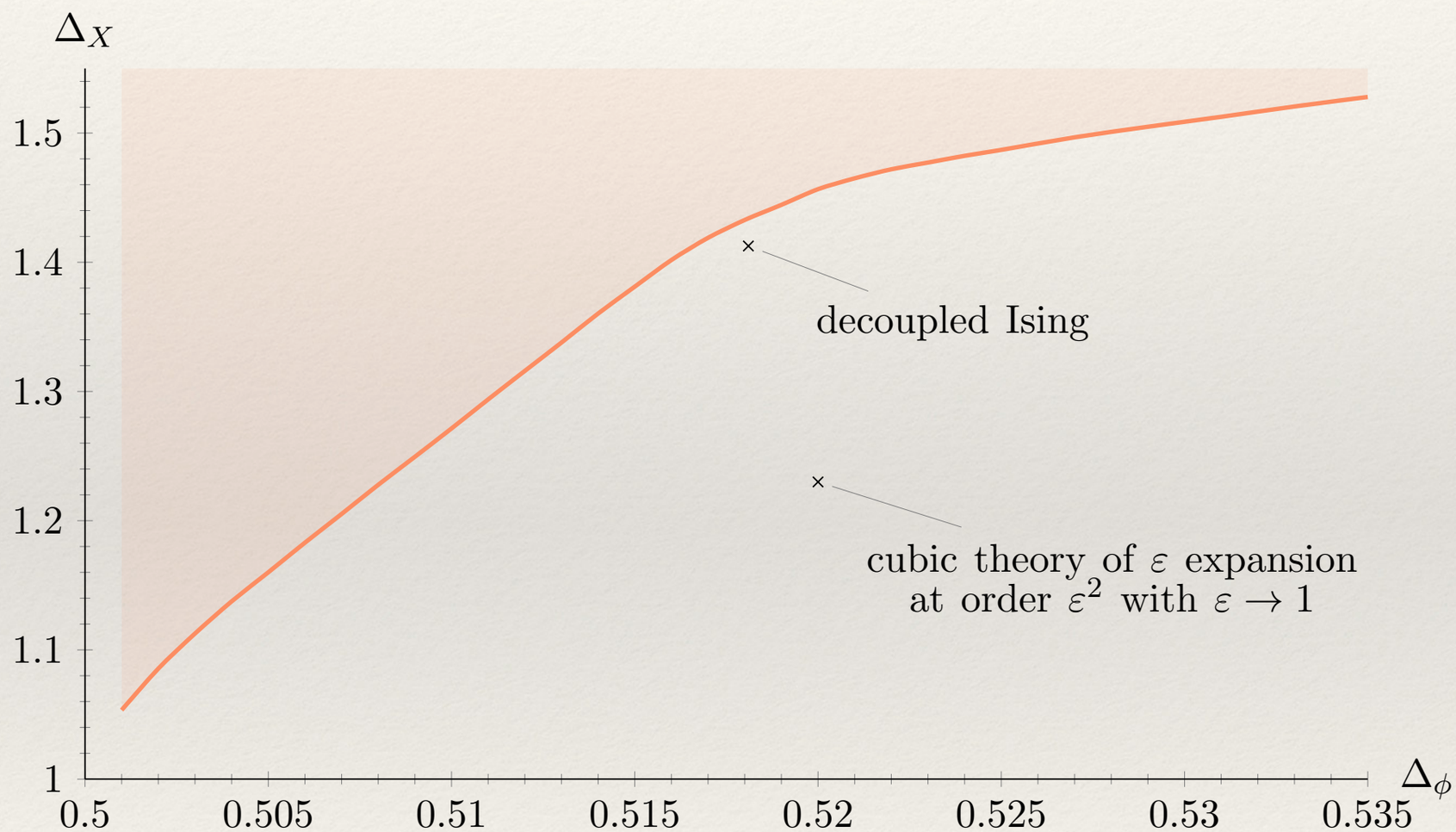
Cubic CFTs in 3D

Using the ε expansion one can find a fixed point with cubic symmetry, but the critical exponents are almost identical to those of the Heisenberg model.



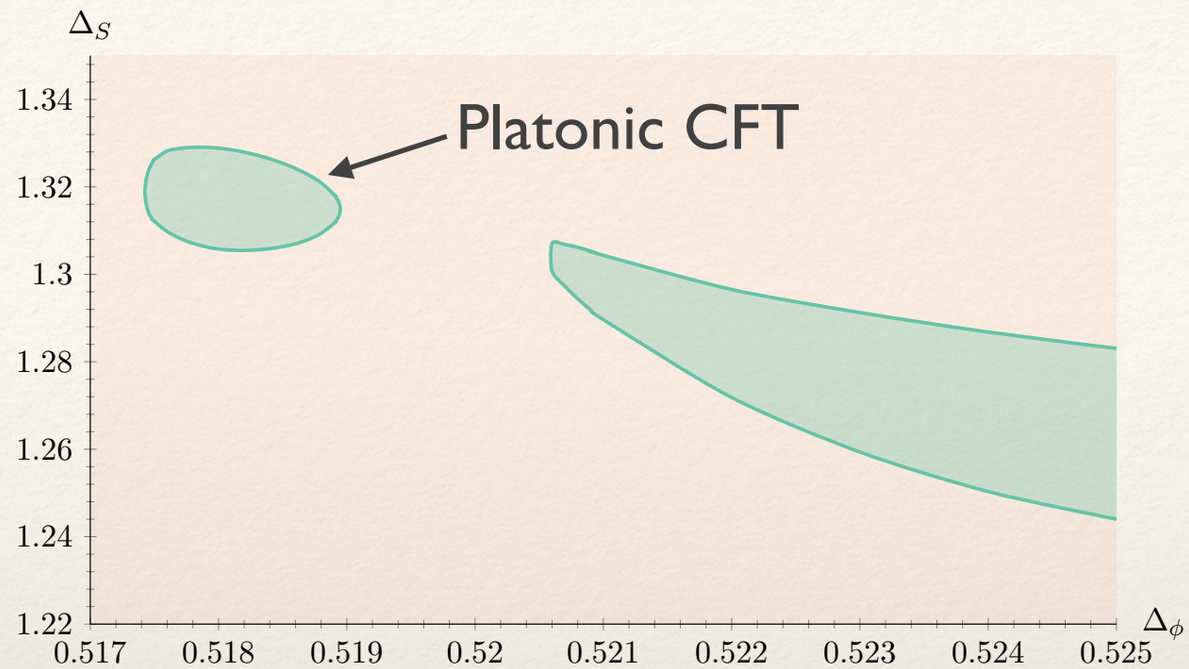
Bootstrapping Cubic CFTs in 3D

$$\phi_i \times \phi_j \sim \delta_{ij} S + X_{(ij)} + Y_{(ij)} + A_{[ij]}$$



(AS, 2018)

Bootstrapping Cubic CFTs in 3D



(Kousvos & AS, 2018)

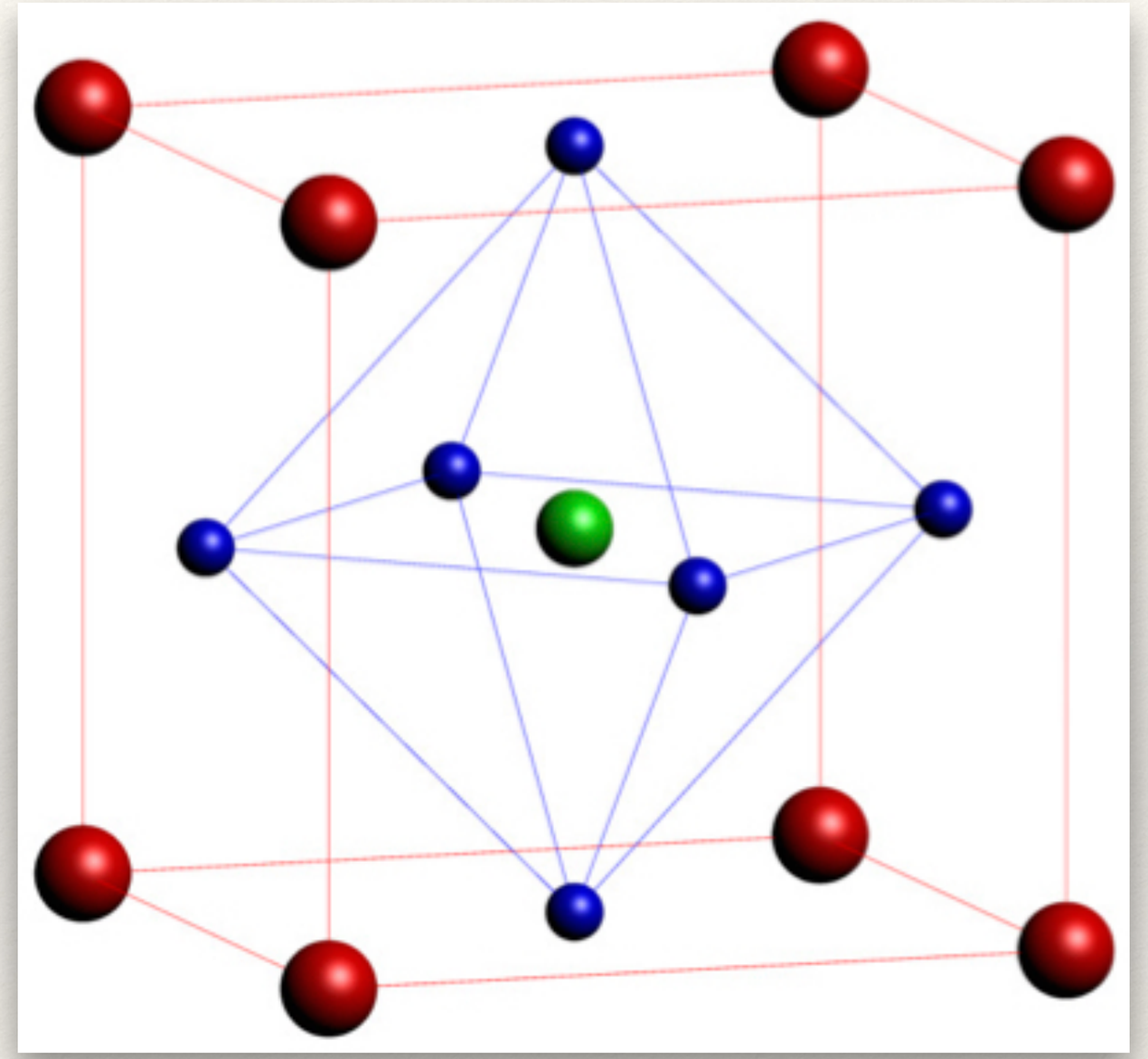
The critical exponents obtained differ significantly from those of the ε expansion:

$$\beta \approx 0.308 \pm 0.002\,, \quad \nu \approx 0.594 \pm 0.004,$$
$$\beta^{(\varepsilon)} \approx 0.368\,, \quad \nu^{(\varepsilon)} \approx 0.709.$$

This theory appears to be distinct from that of the ε expansion.

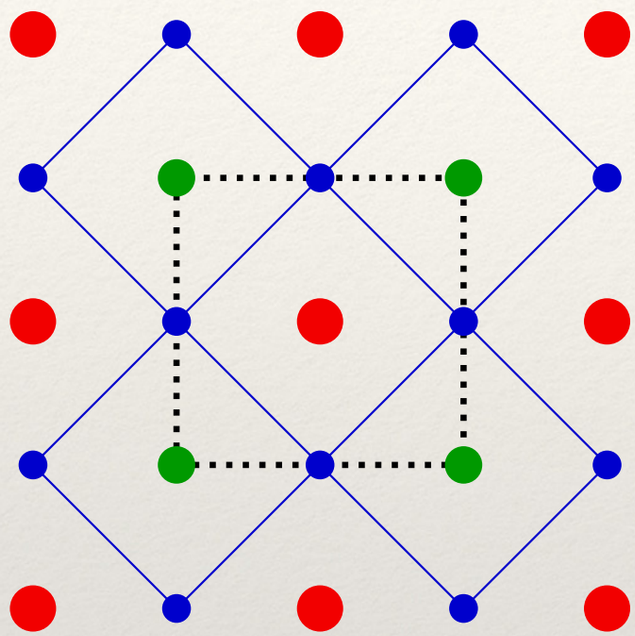
(Delamotte, Holovatch, Ivaneyko, Mouhanna, Tissier, 2006, 2008)

Structural Phase Transitions in Perovskites

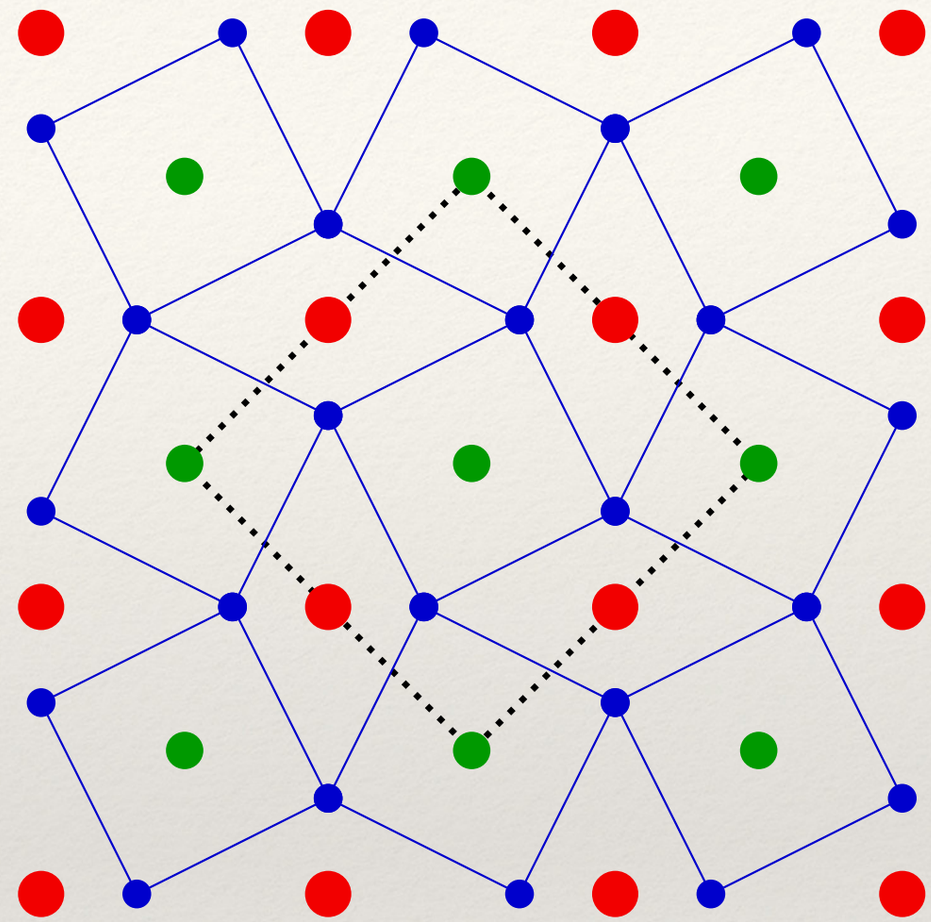


ABX_3

$SrTiO_3$



(a) $T > T_c$



(b) $T < T_c$

$T_c \approx 105$ K

$SrTiO_3$

Experimental measurements:

$$\beta = 0.33 \pm 0.02\,, \qquad \nu = 0.63 \pm 0.07$$

(Müller & Berlinger, 1971) (von Waldkirch, Müller, Berlinger & Thomas, 1972)

Platonic CFT:

$$\beta = 0.308 \pm 0.002\,, \qquad \nu = 0.594 \pm 0.004$$

3D Ising:

$$\beta = 0.326419 \pm 0.000003\,, \qquad \nu = 0.629971 \pm 0.000004$$

Role of strain? What happens if strain is removed?

Paramagnetic-Ferromagnetic Transitions

(Ben Hassine et al., 2017)

$La_{0.47}Eu_{0.2}Pb_{0.33}MnO_3$:

$$\beta = 0.306 \pm 0.002\,, \qquad \nu = 0.599 \pm 0.005$$

$La_{0.47}Y_{0.2}Pb_{0.33}MnO_3$:

$$\beta = 0.312 \pm 0.011\,, \qquad \nu = 0.600 \pm 0.009$$

Platonic CFT:

$$\beta = 0.308 \pm 0.002\,, \qquad \nu = 0.594 \pm 0.004$$

3D Ising:

$$\beta = 0.326419 \pm 0.000003\,, \qquad \nu = 0.629971 \pm 0.000004$$

Issue(?): Crystals have Pnma space group (orthorombic)

Summary

The numerical conformal bootstrap provides a widely-applicable method for the study of CFTs.

There is a host of experimental results pertaining to various types of phase transitions for which pre-bootstrap theoretical methods have given unsatisfactory results.

The bootstrap has suggested new non-perturbative universality classes, potentially relevant for physical systems.

Is the standard Landau-Ginzburg paradigm incomplete?

New tools presented in this workshop and discussions with experts across fields will hopefully enable further progress!