Informal Considerations on the Random Field Ising Model

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LPTMS

Glasses & RFIM

Freezing of fluctuations

Order parameter: overlap between configs.

$\delta\rho(x)$ local density fluctuations

$$\langle \delta\rho(x)\,\delta\rho'(x)\rangle \sim q_{EA}$$

$$q_{\rho,\rho'} = \int dx\, w(x-y)\, \delta\rho(x)\,\delta\rho'(y)$$

Order parameter: overlap between configurations

Landau free-energy for glasses

(I) Fix a conf. $\underline{X} = \{x_1 \dots x_N\}$

chosen with $\mu(\underline{X}) = e^{-\beta H(\underline{X})}/Z$

space

$$\underbrace{q_x(X,Y)}_{\text{local measure of similarity}}$$

(II) Consider

$$\mu(Y|X) = \frac{e^{-\beta H(\underline{Y}) + \beta \int dx \, \epsilon_x q_x(X,Y)}}{Z(\underline{X},\epsilon)}$$

(III) Large deviation function

$$V(q) = -T \int dx \, \mu(\underline{x}) \log Z(\underline{X},\epsilon)$$

$$\sum_x e^{-\beta H(x)} \qquad \log Z(x,q) \longrightarrow \sum e^{-\beta \sum_{a=1}^{n} H(x_a) + \beta \epsilon \sum_{a \geq 2} q(x_1, x_a)}$$

$$Z^{n-1}(x,q)$$

Glasses : $n \longrightarrow 1$ invariance S_{n-1}

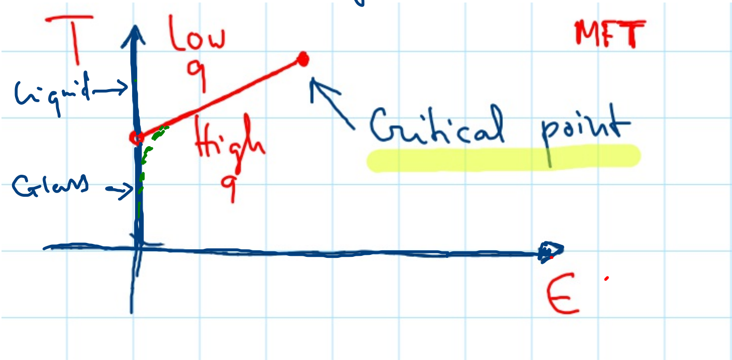
x : self-induced disorder

Fluctuations of overlap

thermal

sample-to-sample (choice of x)

Phase diagram



Critical point :

Critical overlap fluctuations in the Univ. Class of RFIM in pert. theory

Starting point :

special S_n symmetric point with ϕ_{ab}

RFIM (Random Field Ising Model)

Definition of the model

$$H = -J\sum_{\langle i,j\rangle} S_i S_j - \sum_i h_i S_i$$

h_i random $\qquad \overline{h_i h_j} = \Delta(i-j) = \Delta \times \delta_{ij}$

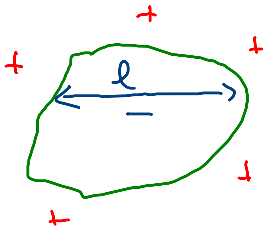
Para-Ferro transition

φ^4 version $\qquad F[\varphi] = \int dx \left[\frac{1}{2}(\nabla\varphi)^2 + V(\varphi) - h(x)\varphi(x)\right]$

Existence of the transition

Imry-Ma argument

Stability of magnetized phase



$$\Delta E = \sigma \ell^{d-1} - \sqrt{\Delta}\, \ell^{d/2}$$

transition for $d > 2$

$d = 2$ no-transition

$$G_c(r) = \overline{\langle \varphi(x)\varphi(0)\rangle} - \overline{\langle \varphi(x)\rangle\langle \varphi(0)\rangle} \sim \frac{1}{r^{d-2+\eta}}$$

$$G_d(r) = \overline{\langle \varphi(x)\rangle\langle \varphi(0)\rangle} - \overline{\langle \varphi(x)\rangle}\ \overline{\langle \varphi(0)\rangle} \sim \frac{1}{r^{d-4+\bar{\eta}}}$$

$\bar{\eta} = \eta = 0$ in MF

Pert. theory dominated by free.

(1) $\eta = \bar{\eta}$

(2) Dimensional reduction $\longrightarrow$ corr. functions (+ expon) of the pure model in dim $d' = d-2$

Solutions of Stochastic eq. :

$$\boxed{-\Delta\varphi + V'(\varphi) = h(x)}$$

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$$\overline{A[\{\varphi\}]} = \overline{\int d\varphi \;\delta(-\Delta\varphi + V' + h)\left|\det(-\Delta + V'')\right| A}$$

If the Stoch Eq has only one solution

$|\det| \longrightarrow \det \longrightarrow$

$$\delta(-\Delta\varphi + V' + h)\;\det(-\Delta + V'') = \int d\hat{\varphi}\, d\bar{\chi}\, d\chi\; e^{i\hat{\varphi}(-\Delta\varphi + V' + h)} \times$$

$$\times\; e^{-\frac{1}{2}\bar{\chi}(-\Delta + V'')\chi}$$

$S[\varphi, \hat{\varphi}, \bar{\chi}, \chi] \longrightarrow$ BRST Symmetric

$$i\hat{\varphi} = \frac{\partial}{\partial h}$$

$$\overline{e^{\int i\hat{\varphi}(x) h(x)}} = e^{\frac{\Delta}{2}\int (i\hat{\varphi})^2} \Longrightarrow \text{SuSy}$$

$$\Phi = \varphi + \theta\bar{\chi} + \bar{\theta}\chi + \theta\bar{\theta}\, i\hat{\varphi}$$

$\theta, \bar{\theta}$

Grassmann vars.

$$i\hat{\varphi}^2 = \int d\theta\, d\bar{\theta}\; \Phi \frac{\partial}{\partial\theta\partial\bar{\theta}} \Phi$$

$$S = \int dx\, d\theta\, d\bar{\theta} \left[\Phi \left(\partial_x^2 + \Delta\, \partial_\theta \partial_{\bar{\theta}} \right) \Phi + V(\Phi) \right]$$

$$\langle \Phi(x, \theta\bar{\theta})\, \Phi(0) \rangle = G(x^2 + \theta\bar{\theta}/\Delta) = G(x^2) + \frac{\theta\bar{\theta}}{\Delta} G'(x^2)$$

$$\|$$

$$\underbrace{\langle \varphi\varphi \rangle}_{G_d} + \theta \langle \bar{\chi}\varphi \rangle + \bar{\theta} \langle \chi\varphi \rangle + \theta\bar{\theta} \underbrace{\langle i\hat{\varphi}\varphi \rangle}_{G_c}$$

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$$= \langle\varphi(x)\varphi(0)\rangle + \theta\,\cancel{\langle\varphi(0)\bar\chi(x)\rangle} + \bar\theta\,\cancel{\langle\varphi(0)\chi(x)\rangle} + \theta\bar\theta\,\langle i\hat\varphi(x)\varphi(0)\rangle$$

$$\frac{1}{|x|^{d-4+\bar\eta}} \longrightarrow \boxed{\eta=\bar\eta} \longleftarrow \frac{1}{|x|^{d-2+\eta}}$$

Dimensional reduction: $d' = d-2$: integrals.

$$\int d^dx\, f'(x^2) \propto \int d^{d-2}x\, f(x^2)$$

RFIM in dim d $\sim$ IM in dim $d-2$

$\boxed{D_{uc} = 6}$ Evidently wrong in low d.

Failure of SuSy

$$-\Delta\varphi + V(\varphi) = h(x)$$

(I) Multiple solutions $|\det| \neq \det$

(II) Discontinuous jumps : $h \to h + \delta h$ Change of dominant solution

(III) Marginality : Solutions cannot be followed when $h(x) \to h(x) + \delta h(x)$

$$\frac{\delta\varphi}{\delta h} \neq (-\Delta + V'')^{-1} \to \text{Breaking of FDT (Ward Id of BRST)}$$

$\to$ Avalanches Large rearrangements

$$\boxed{\langle i\hat{\varphi}\varphi\rangle = \langle \bar{\chi}\chi\rangle \iff \frac{\delta\varphi}{\delta h} = (-\Delta + V'')^{-1} \quad \text{min. can be followed}}$$

Replicas

$$\overline{e^{-\beta\sum_{a=1}^{n} H(\varphi_a)}} = e^{\cdot S(\varphi_a)}$$

$$S[\varphi_a] = \int dx \sum_{a=1}^{n} -\varphi_a \Delta \varphi_a + V(\varphi_a) + \frac{\Delta}{2}\Big(\sum_a \varphi_a\Big)^2 \qquad n \to 0$$

Quadratic fluctuations

$$\langle \varphi_a^2 \rangle_c = \frac{1}{k^2+m^2} \quad ; \quad \langle \varphi_a \varphi_b \rangle = \frac{1}{k^2+m^4}$$

φ_a : not well defined scaling dimension

Cardy transformation

$$\varphi_a = \varphi + \delta_{a,1}\, i\hat{\varphi} + \chi_a \quad ; \quad \chi_1 = 0 \quad , \quad \sum_a \chi_a = 0$$

φ , $i\hat{\varphi}$, χ_a have well defined Scaling dimension

$$\Delta_\varphi = D/2 - 2 \quad ; \quad \Delta_{i\hat{\varphi}} = D/2 \quad ; \quad \Delta_\chi = D/2 - 1$$

keeping only the relevant terms leads .

$$S = \int dx \; i\hat{\varphi}\left[-\Delta\varphi + V'(\varphi)\right] + \frac{1}{2}\sum_a \chi_a\left(-\Delta + V''\right)\chi_a$$

$$\chi_a \longrightarrow n-2 \text{ indep. components} \longrightarrow \left(\det\left(-\Delta + V''\right)\right)^{-(n-2)/2}$$

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The cusp

Avalanches lead to singular contributions to correlation functions

Copies with different external fields h, $h + \delta h$ $\qquad \delta h \ll 1$

- Probability of meeting an avalanche of size S

$$\rho(S)\, dh$$

$$\overline{(m(h) - m(h+dh))^2}^{\,c} = |\delta h| \int dS\, \rho(S) \frac{S^2}{L^{2d}} + \text{reg.}$$

cusp

Tarjus - Tissier 2012

- Fractal dimension of avalanches

$$S \sim \ell^{d_f}$$

If $d_f < \frac{1}{2}(d+4+\bar{\eta})$ dim. reduction

$\longrightarrow$ $d_f = \frac{1}{2}(d+4+\bar{\eta})$ Susy Breaking

In $d \geq 6$ $d_f = 4$, $\bar{\eta} = 0$ $4 < \frac{d+4}{2}$ $\Longrightarrow$ Room for Susy below $d = 6$

The Cusp

Failure of FDT and avalanches

$G_d(x) = \overline{\langle \varphi(x)\, \varphi(0) \rangle} \longrightarrow$ Generating function of disconnected correlations

$$e^{W[h]} = \int d\varphi \; e^{-S(\varphi) + \int \varphi h \, dx}$$

$$\log \overline{e^{n_1 W(h+\delta h_1) + n_2 W(h+\delta h_2)}} = W_2[\delta h_1, \delta h_2]$$

$$\frac{\partial^2}{\partial n_1 \partial n_2}\Big|_{n_1=n_2=0} \frac{\delta W_2}{\delta h_1(x)\,\delta h_2(x)}\Big|_{\delta h_1=\delta h_2=0} = \underbrace{\overline{\langle \varphi(x) \rangle \langle \varphi(0) \rangle}^{\,c}}_{\text{singular if avalanches}}$$

Non perturbative RG

Uncontrolled Closure of exact RG eq.ns

$R_k(x)$ IR regulator $\Delta S_k[\varphi] = \int dx \, \frac{1}{2} \phi(x) R_k(x-y) \phi(y)$

$R_0(x) = 0 \qquad R_\Lambda(x) = \delta(x-y)\Lambda^2$

$$e^{W[h]} = \int d\varphi \, e^{-S - \Delta S + (h,\varphi)}$$

$$\Gamma(\varphi) = -W[h] + h\varphi - \Delta S(\phi)$$

$$\frac{\partial \Gamma}{\partial \log k} = \frac{1}{2} \mathrm{Tr} \left\{ \partial_{\log k} R \left[\Gamma_k^{(2)}(\varphi) + R \right]^{-1} \right\}$$

$$\boxed{\frac{\partial \Gamma}{\partial \log k} = \frac{1}{2} \mathrm{Tr} \left\{ \partial_{\log k} R \left[\Gamma_k^{(2)}(\varphi) + R \right]^{-1} \right\}} \;\; **$$

$$\Gamma^{(2)} = \frac{\delta \Gamma}{\delta\varphi\,\delta\varphi}$$

- Assume a given form for Γ
- Use the eq. under the restrict form

e.g. $$\Gamma(\varphi) = \int dx \; \frac{1}{2}(\nabla\varphi)^2 + U(\varphi)$$

Impose the validity of ** on spatially homogeneous configs. of φ

Combining Non Perturbative + Functional RG
replicas

Tissier-Tarjus + could find that

the RG flow gives rise to a cusp
at a finite time below $d_c \simeq 5.1$ ⟵ dim. reduction