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Quantum Generative Models for muonic force carriers events

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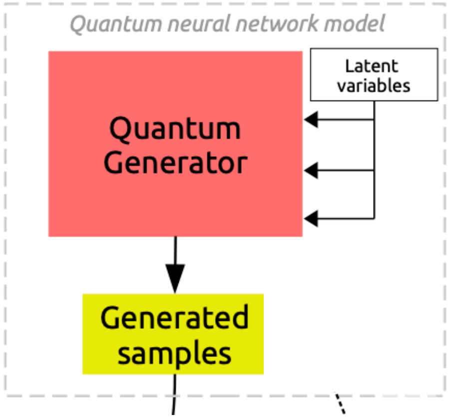
Key point: *We use a quantum circuit Born machine to generate multivariate and conditional distributions in HEP and evaluate it on actual quantum hardware*.

Quantum Generative models:

Quantum computing proposes an alternative to neural networks.

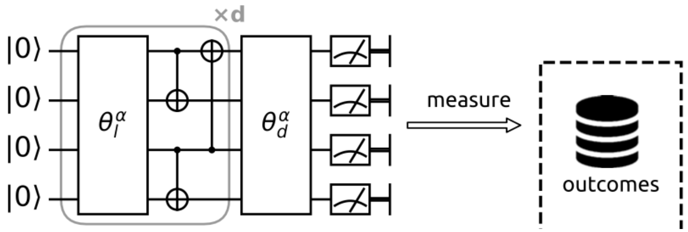
1 **QNN** with classical random source [1]



Pros: - continous
- low number of qubits

Cons: - high number of shots
- 1 sample = many shots
- no clear advantage

2 **Born machines** using quantum randomness [2,3]



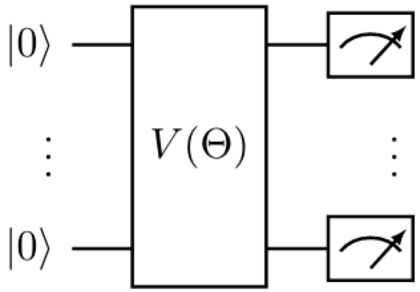
Pros: - uses true quantum randomness
- 1 shot = 1 sample
- fundamentaly different approach

Cons: - discrete
- needs more qubits

[1] *Style-based quantum generative adversarial networks for Monte Carlo events,* Carlos Bravo-Prieto et al (2021) https://arxiv.org/abs/2110.06933
[2] *Differentiable Learning of Quantum Circuit Born Machine,* Jin-Guo Liu and Lei Wang (2018) https://arxiv.org/abs/1804.04168
[3] *The Born supremacy: quantum advantage and training of an Ising Born machine,* Brian Coyle et al, (2021) https://www.nature.com/articles/s41534-020-00288-9

Quantum Circuit Born machine

1. ***Sample*** *from a variational wavefunction* $|\psi(\theta)\rangle$ *with probability given by the* ***Born rule****:*
   $p_\theta(x) = |\langle x|\psi(\theta)\rangle|^2$



n dimensional binary strings map to 2^n bins of the discretized dataset

2. ***Only*** *able to generate* ***discrete*** *PDFs (continuous in the limit #qubits* $\rightarrow \infty$*)*
3. ***Training****: metric (KL, MMD,…), trainable discriminator (quantum or classical NN), …*
4. ***Maximum Mean Discrepancy****: MMD(P,Q) =* $\mathbb{E}_{\substack{X\sim P\\Y\sim P}}[K(X,Y)] + \mathbb{E}_{\substack{X\sim Q\\Y\sim Q}}[K(X,Y)] - 2\mathbb{E}_{\substack{X\sim P\\Y\sim Q}}[K(X,Y)]$*, with K a gaussian kernel.*
5. ***Pros****: relativly easy to optimize,* ***Cons****: empircally less efficient than an adversarial approach*

Variational Quantum Algorithms

Use Classical computation in order to keep the circuit shallow making use of an **optimization based/learning based** approach. The gradient is exactly computed with 2*d (d=#parameters) forward (sampling) passes.

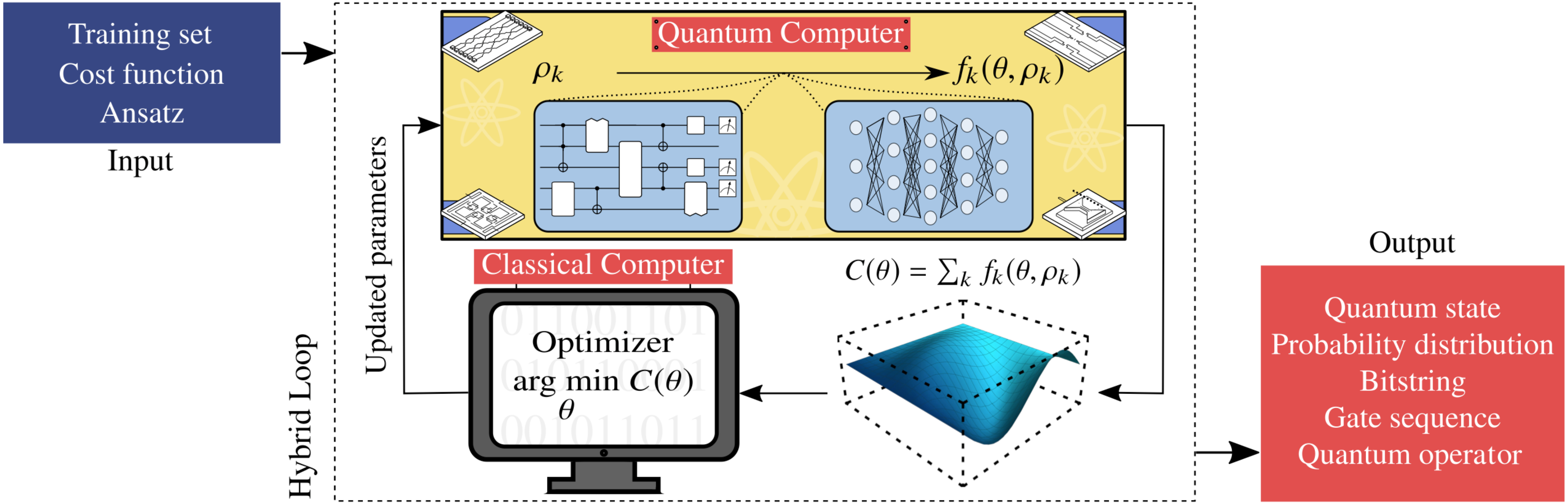
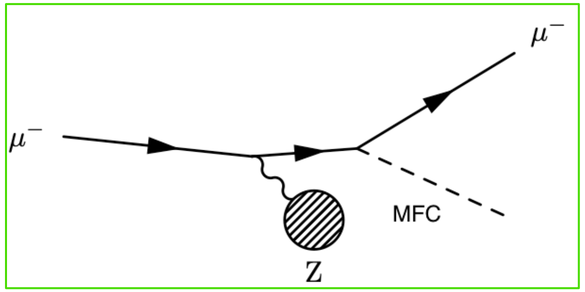


Figure 1. **Schematic diagram of a Variational Quantum Algorithm (VQA)** [4]

[4] Variational quantum algorithms, https://arxiv.org/pdf/2012.09265.pdf

Muonic Force Carriers

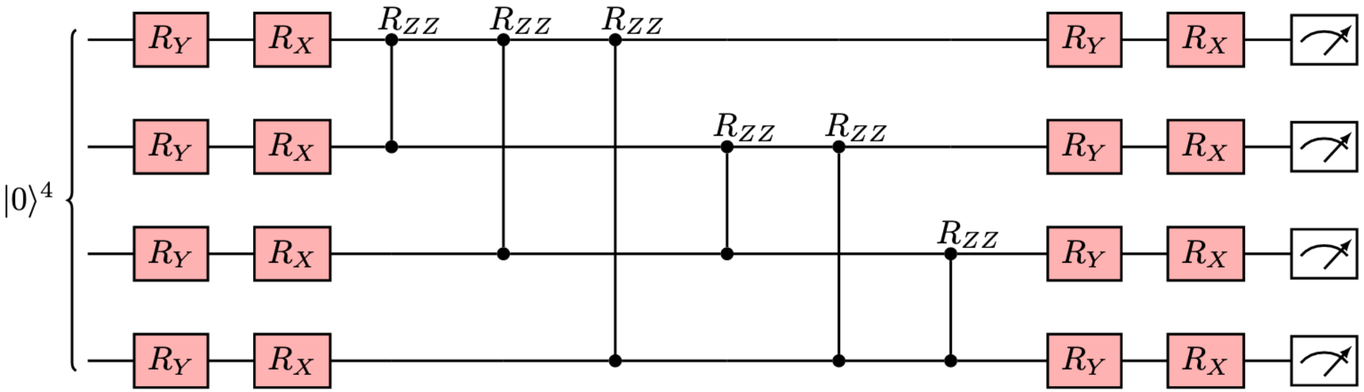


- MFCs appear in different theoretical hypothesis, as a constituent of **dark matter** and could explain the **anomalous magnetic dipole** moment of the muon or the anomaly in the measurement of the **proton radius.**
- 2 use cases: a) muon fixed-target collision (**FASER**).
  b) muon interactions in the **ATLAS**[5] calorimeter.
- We are interested to generates following features for the outgoing muon and MFC: **energy** (E), **transversal momentum** (pt) and **pseudorapidity** (η).

[5] Galon, I, Kajamovitz, E et al. "Searching for muonic forces with the ATLAS detector". In: *Phys. Rev. D 101, 011701* (2020)

One dimensional distribution

outgoing muon's energy at 50 GeV

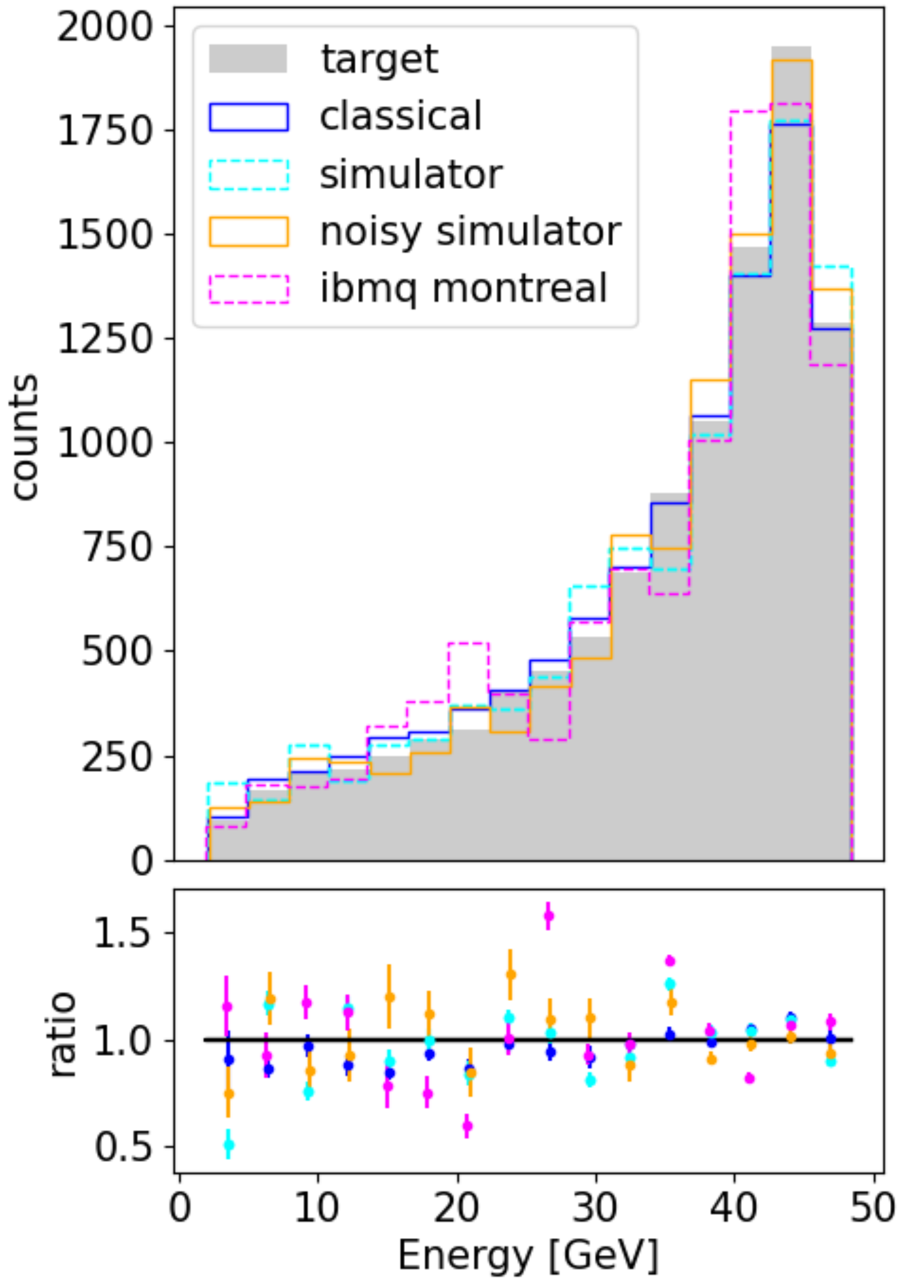


Comparison metric:

$$\mathrm{TV}(p,\pi) = \frac{1}{2}\sum_{x\in\Omega} |p(x) - \pi(x)|$$

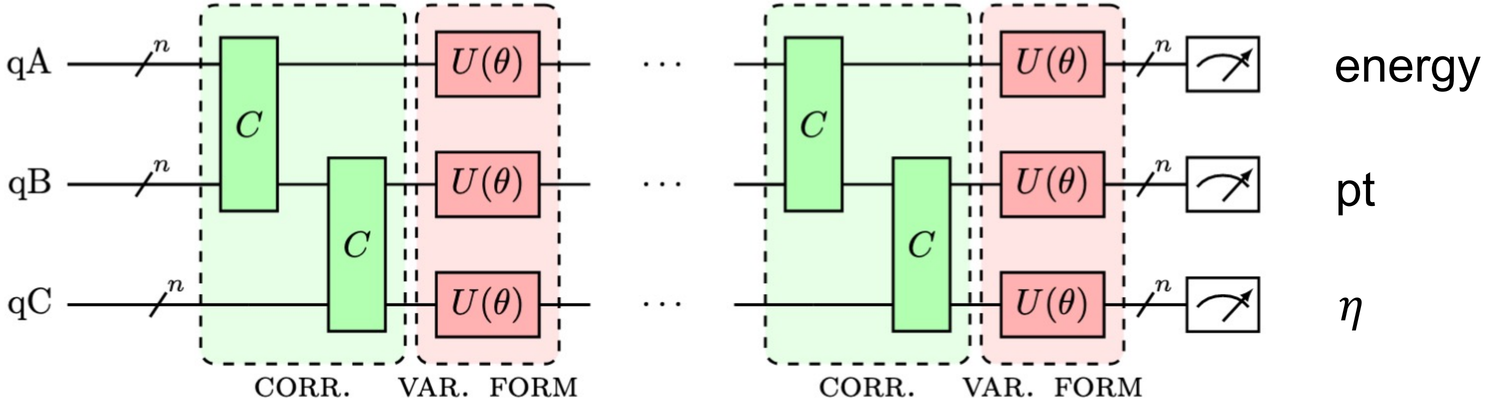
backend	TV
simulator	0.055
noisy simulator	0.043
ibmq montreal	0.074
GMMD	0.028
easy GMMD	0.290

Perfect simulator
Noisy simulator
IBMQ Montreal
Classical GMMD of size (15,128, 256,128,16,1)
Easy GMMD ~ QCBM in size

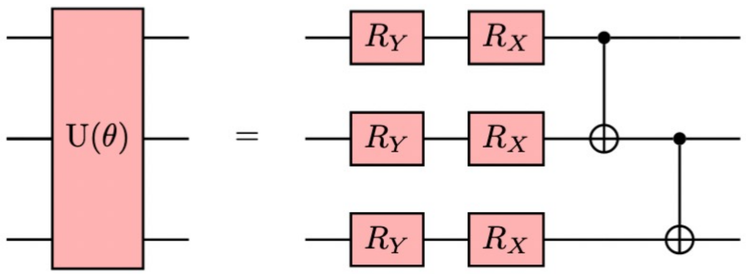


Born machine: multiples features[6]

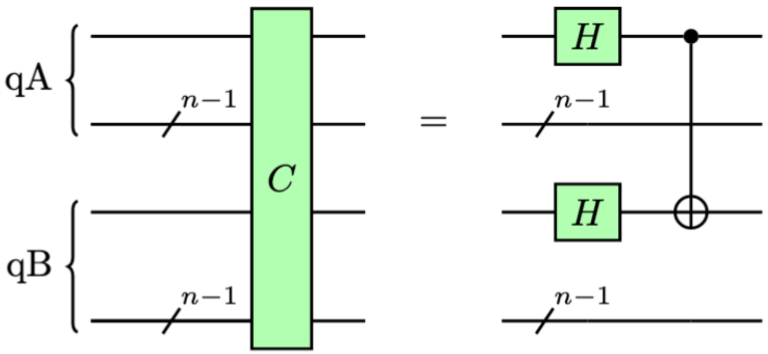
Use multiples quantum registers



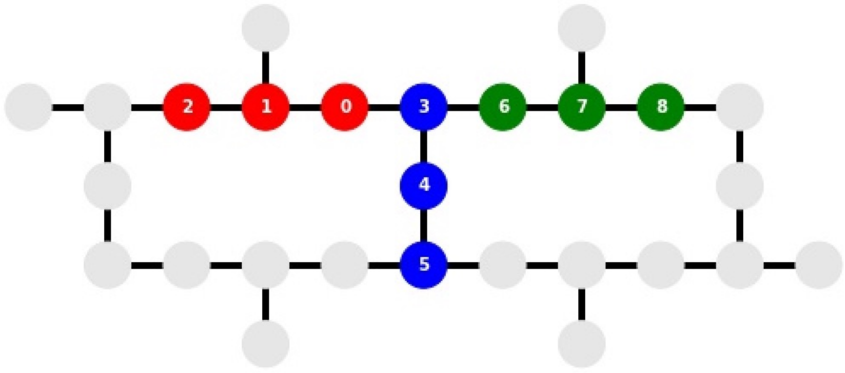
Local: learns the individual PDFs, time evolution of an Ising type Hamiltonian, conjectured to be difficult to simulate classicaly.



C: creates a corelated state between the first qubit in each register.



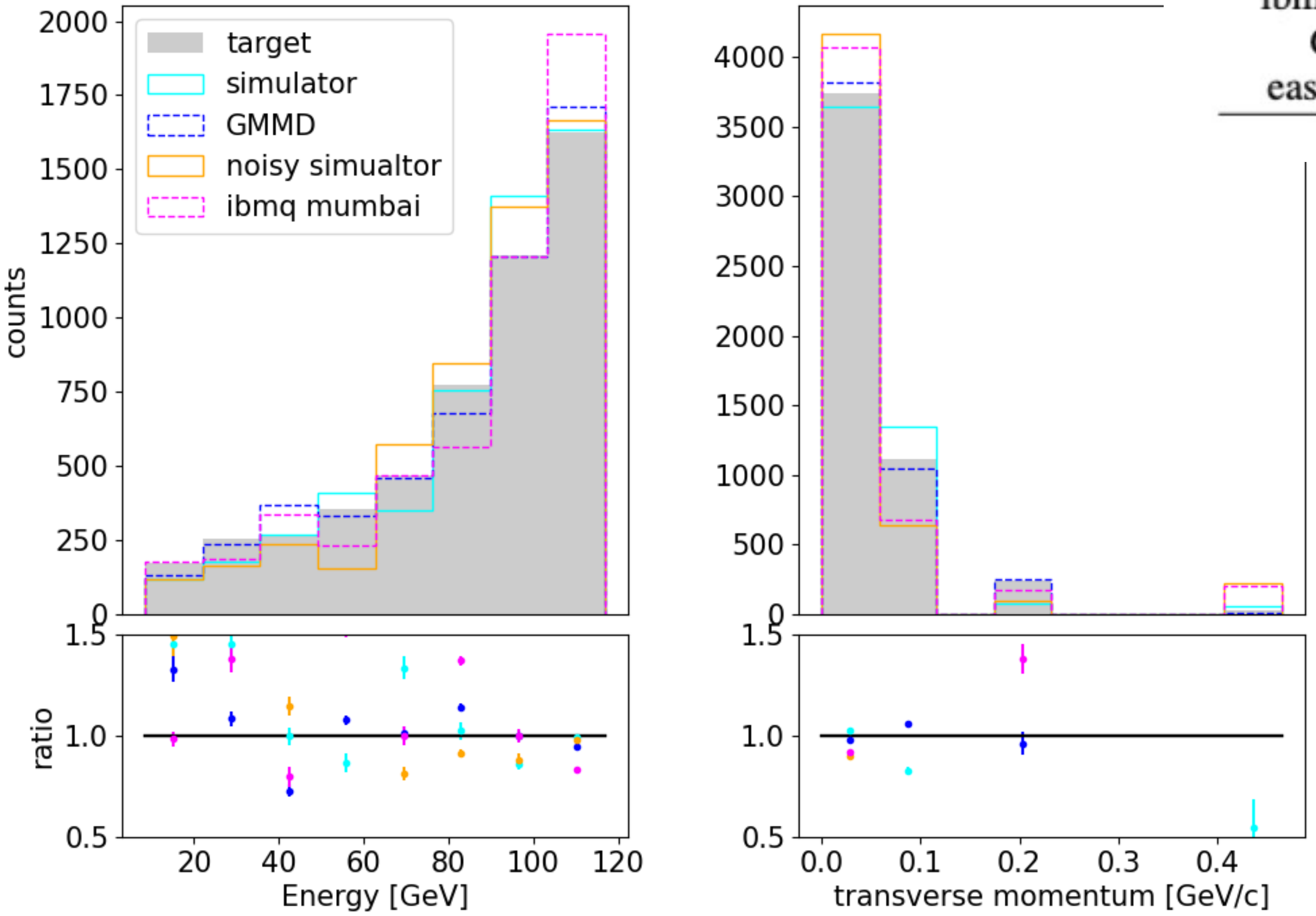
Can be mapped without swap gates on ibmq_mumbai

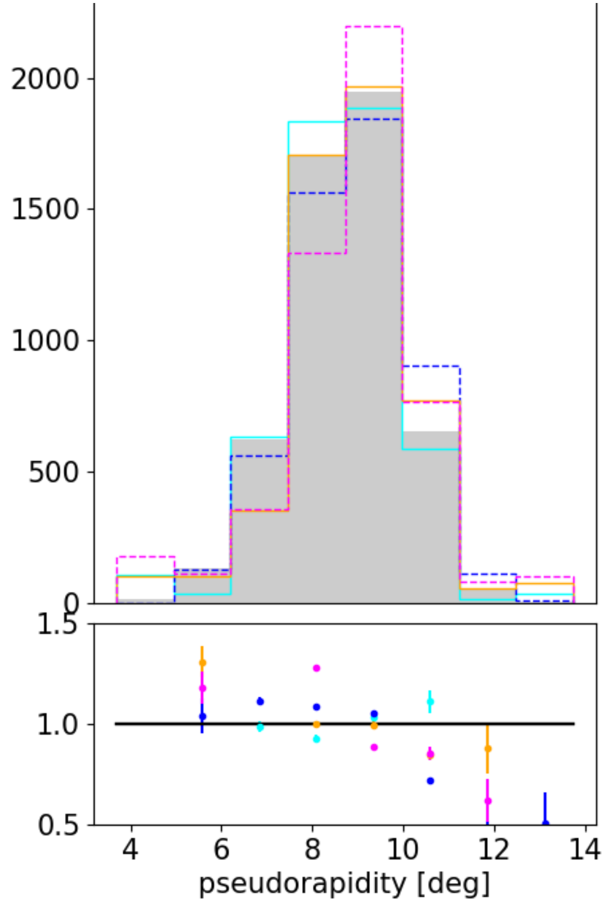


[6] Elton Yechao, Sonika Johri et al, "Generative Quantum Learning of Joint Probability Distribution Functions" In: arXiv 2109.06315

Results (125 GeV)

back-end	TV(E)	TV(pt)	TV(η)
simulator	0.055	0.05	0.052
noisy simulator	0.075	0.12	0.06
ibmq mumbai	0.078	0.097	0.13
GMMD	0.036	0.017	0.063
easy GMMD	0.360	0.040	0.110





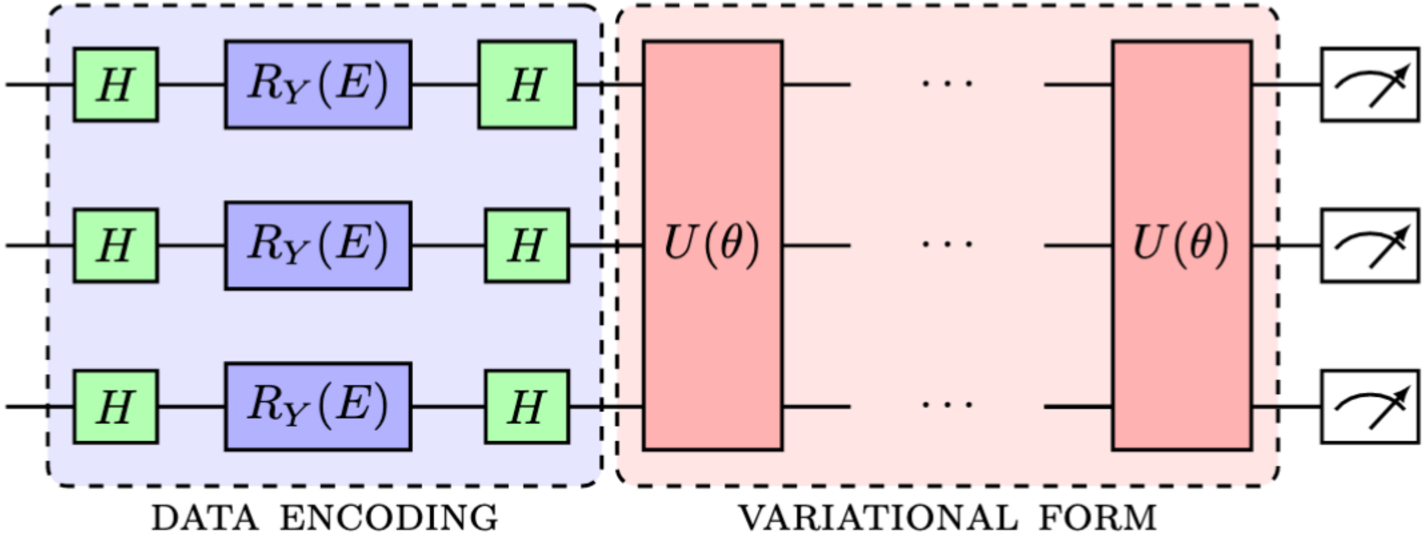
Conditional Born machine

green: fixed gates
blue: data encoding gates
red: trainable gates

***Data encoding**: via data-parametrized rotations*

***Input**: binning energy E (scaled between* $[0,1]$ *and preprcoessed with arcsin())*

***Interpolation**: train only on certain energy bins and the model should learn to predict in between.*



Data re-uploading makes the quantum circuit more expressive as function of the data.

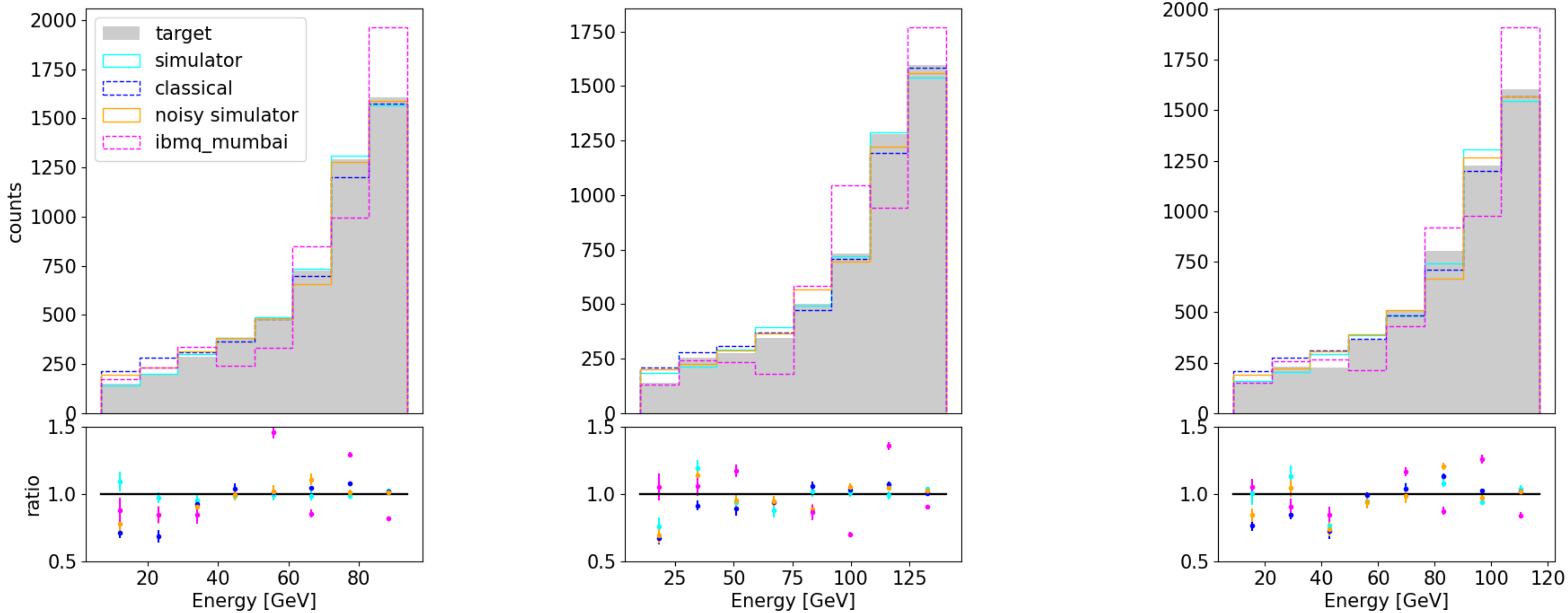
C-Born machine: results

back-end	TV(100)	TV(150)	TV(125)
simulator	0.033	0.016	0.033
noisy simulator	0.067	0.046	0.035
ibmq mumbai	0.15	0.13	0.094
GMMD	0.016	0.032	0.034

a) train: 100 GeV

b) train 150 GeV

c) test 125 GeV



Conclusion:

1. Use a quantum circuit Born machine to generate MFC events.
2. The Born machine is currently able to handle multivariate and conditional distributions and is competitive against GMMD of similar complexity.
3. Training on quantum hardware is important in order to assimilate the noise.
4. Futur work is devoted to scale the QCBM, in terms of number of qubits and register, and also to generate conditional multivariate distributions.

Thanks for your attention!

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