Nuclear incompressibility and the NS crust

GMR virtual workshop of the GDR RESANET 25-26/11/2020

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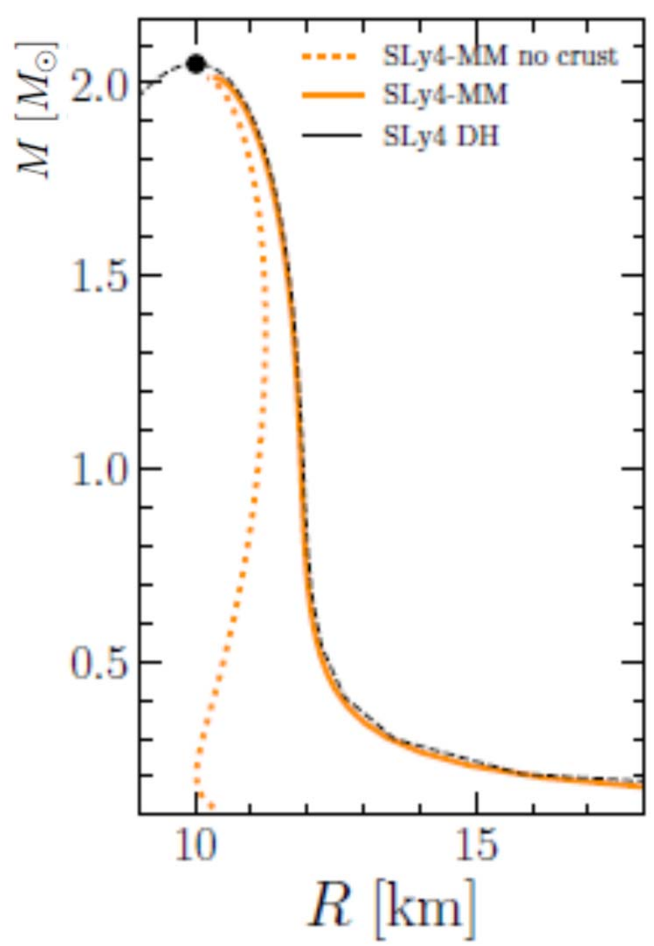
Plan

1. ***Incompressibility and other empirical parameters***
2. ***Modeling the NS crust***
3. ***Influence of incompressibility on crustal observables***



T.Carreau, PhD thesis 2020

The EoS empirical parameters

- $e = e_0 = e(\rho, \alpha = 0)$
  $\rightarrow X_n = \{E_{sat}, K_{sat}, Q_{sat}, Z_{sat}, \cdots\}$
- $e = e_{sym} = \frac{1}{2}\frac{\partial^2}{\partial\alpha^2} e(\rho, \alpha = 0)$
  $\rightarrow X_n = \{E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym} \cdots\}$

$$X_n = (3\rho_0)^n \frac{\partial^n e}{\partial \rho^n}(\rho = \rho_0, \alpha = 0)$$

K_{sat}, K_{sym} give the response to a compression mode from $\rho = \rho_0$

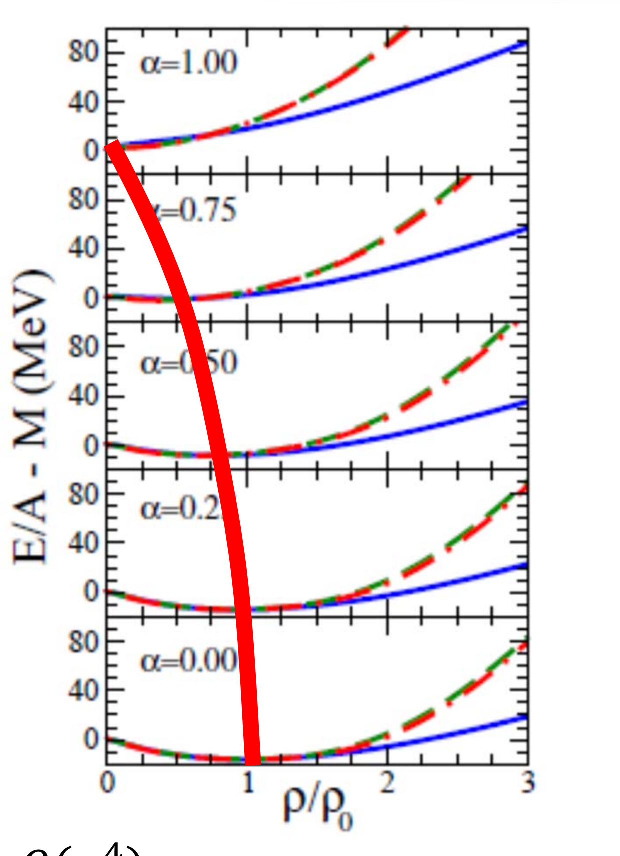
But if $\rho \neq \rho_0$, many parameters contribute!

Example: saturation density

$$\rho_0(\alpha) = \rho_0 - 3\rho_0 \frac{L_{sym}}{K_{sat}} \alpha^2 + O(\alpha^4)$$

$\Rightarrow$ Incompressibility of asymmetric matter

$$K_0(\alpha) = K_{sat} + K_\tau \alpha^2 + O(\alpha^4)$$
$$= K_{sat} + \left(K_{sym} - 6L_{sym} - L_{sym}\frac{Q_{sat}}{K_{sat}}\right)\alpha^2 + O(\alpha^4)$$



- J. Piekarewicz and M. Centelles, Phys.Rev.C 79, 054311 (2009).

Constraints from nuclear experiments…

TABLE I. Group A: saturation energy E_{sat}, density n_{sat}, incompressibility K_{sat}, and symmetry energy E_{sym} estimated from various analyses of experimental data. See text for more details.

Model	Ref.	E_{sat} (MeV)	n_{sat} (fm^{-3})	K_{sat} (MeV)	E_{sym} (MeV)
El. scatt.	Wang-99 [55]		0.1607	235 ±15	
LDM	Myers-66 [56]	−15.677	0.136[a]	295	28.06
LDM	Royer-08 [57]	−15.5704	0.133[a]		23.45
LSD	Pomorski-03 [58]	−15.492	0.142[a]		28.82
DM	Myers-77 [59]	−15.96	0.145[a]	240	36.8
FRDM	Buchinger-01 [60]		0.157 ±0.004		
INM	Satpathy-99 [61]	−16.108	0.1620	288 ±20	
DF-Skyrme	Tondeur-86 [62]		0.158		
DF-Skyrme	Klupfel-09 [63]	−15.91 ±0.06	0.1610 ±0.0013	222 ±8	30.7 ±1.4
DF-BSK2	Goriely-02 [64]	−15.79	0.1575	234	28.0
DF-BSK24, 28,29	Goriely-15 [65]	−16.045 ±0.005	0.1575 ±0.0004	245	30.0
DF-Skyrme	McDonnell-15 [66]	−15.75 ±0.25	0.160 ±0.005	220 ±20	29 ±1
DF-NLRMF	NL3* [67]	−16.3	0.15	258	38.7
DF-NLRMF	PK [68]	−16.27	0.148	283	37.7
DF-DDRMF	DDME1,2 [69,70]	−16.17 ±0.03	0.152 ±0.00	247 ±3	32.7 ±0.4
DF-DDRMF	PK [68]	16.27	0.150	262	36.8
Present Estimation		−15.8 ±0.3	0.155 ±0.005	230 ±20	32 ±2

[a]Value determined from r_0.

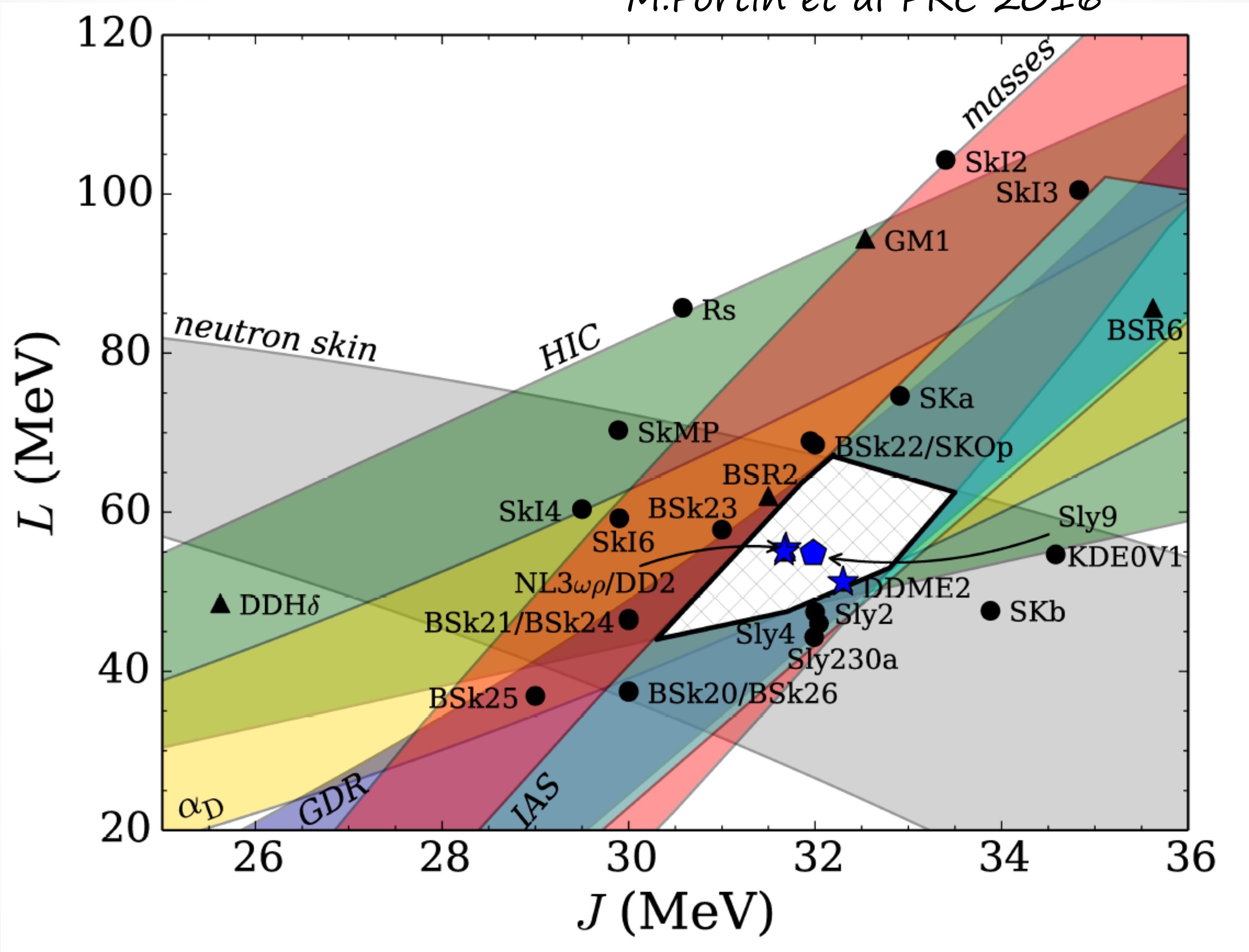
TABLE II. Group B parameters: isoscalar skewness Q_{sat}, slope of the symmetry energy L_{sym}, and isovector incompressibility K_{sym}. The parameter K_τ is defined as $K_\tau = K_{sym} - 6L_{sym} - Q_{sat}L_{sym}/K_{sat}$. See text for more details.

Model	Ref.	Q_{sat} (MeV)	L_{sym} (MeV)	K_{sym} (MeV)	K_τ (MeV)
DF-Skyrme	Berdichevsky-88 [71]	30	0		
DF-Skyrme	Farine-97 [72]	−700 ±500			
DF-Skyrme	Alam-14 [31]	−344 ±46	65 ±14	−23 ±73	−322 ±34
DF-Skyrme	McDonnell-15 [66]		40 ±20		
DF-NLRMF	NL3* [67]	124	123	106	−690
DF-NLRMF	PK [68]	−25	116	55	−630
DF-DDRMF	DDME1,2 [69,70]	400 ±80	53 ±3	−94 ±7	−500 ±7
DF-DDRMF	PK [68]	−119	79.5	−50	−491
Correlation	Centelles-09 [73]		70 ±40		−425 ±175
DF-RPA	Carbone-10 [74]		60 ±30		
Correlation	Danielewicz-14 [75]		53 ±20		
Correlation	Newton-14 [76]		70 ±40		
Correlation	Lattimer-14 [77]		53 ±20		
GMR	Sagawa-07 [78]				−500 ±50
GMR	Patel-14 [79]				−550 ±100
Present Estimation		300 ±400	60 ±15	−100 ±100	−400 ±100

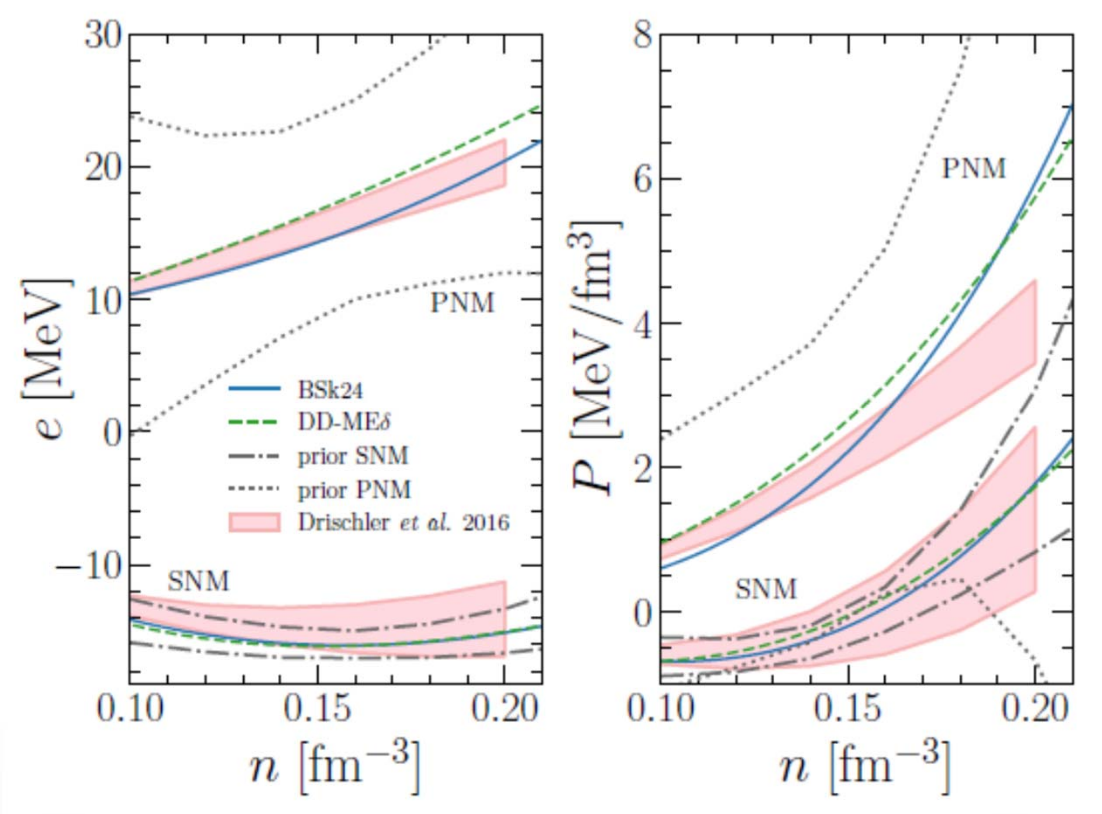
- J.Margueron et al PRC 2018

Constraints from nuclear experiments…

M.Fortin et al PRC 2016



.... and nuclear theory



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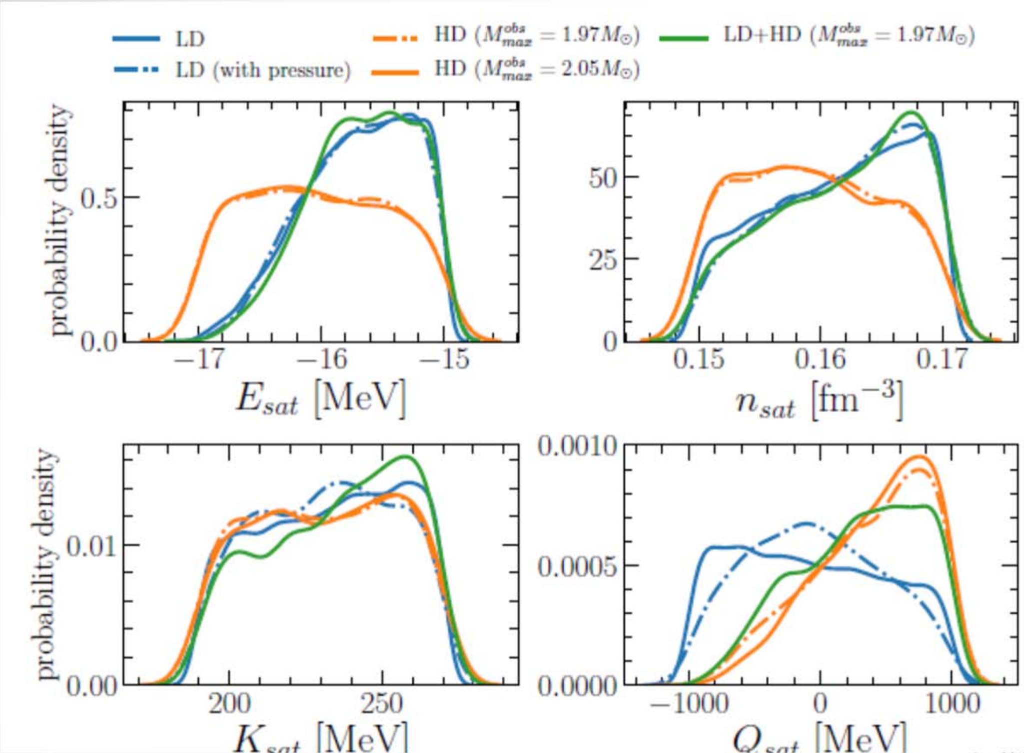
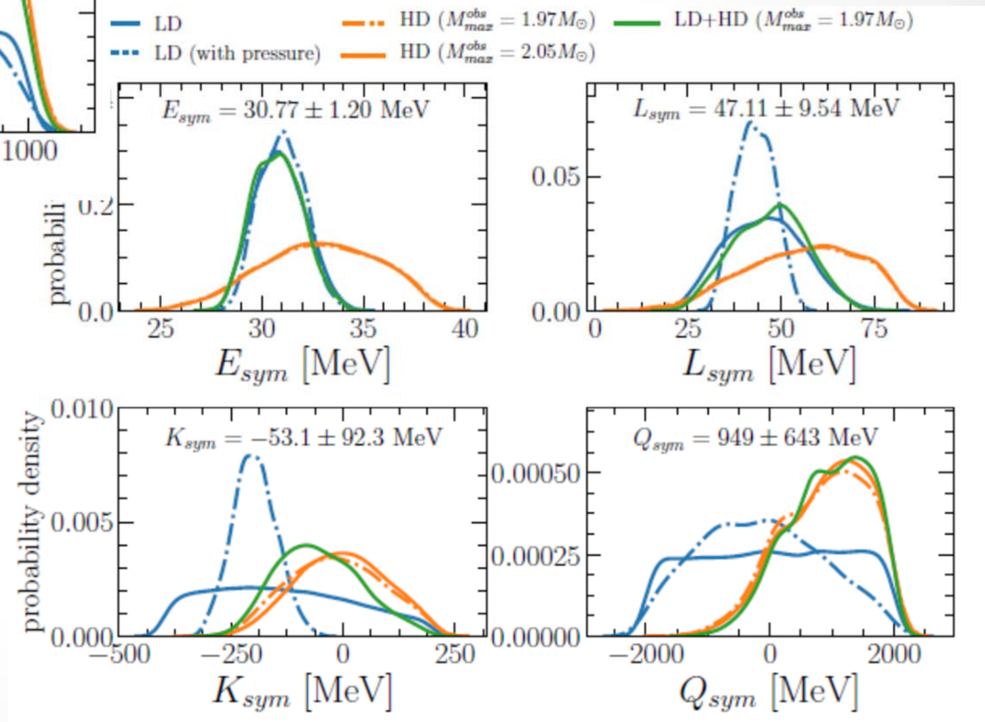
Present knowledge of the empirical parameters





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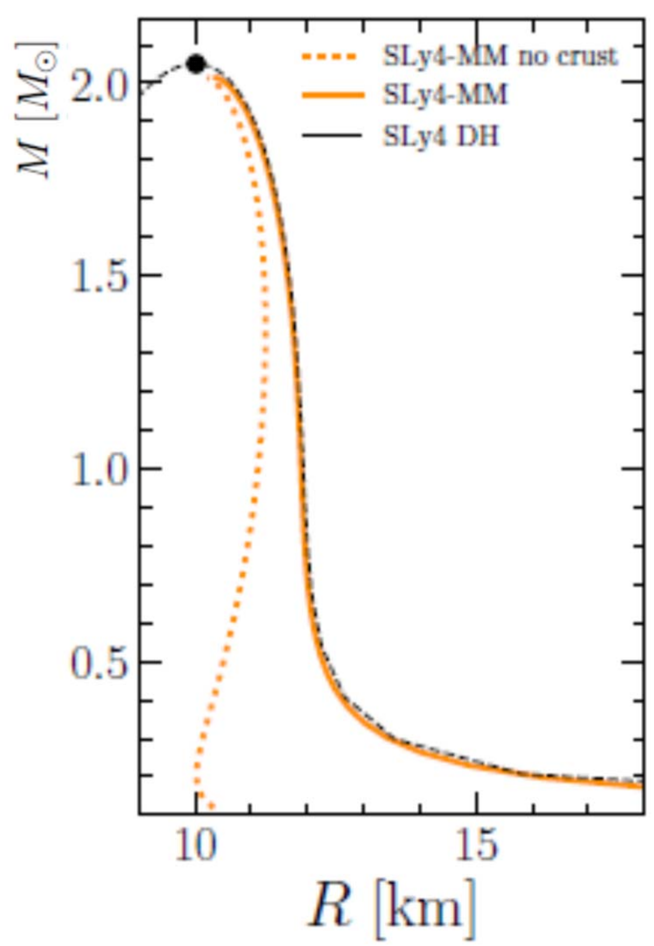

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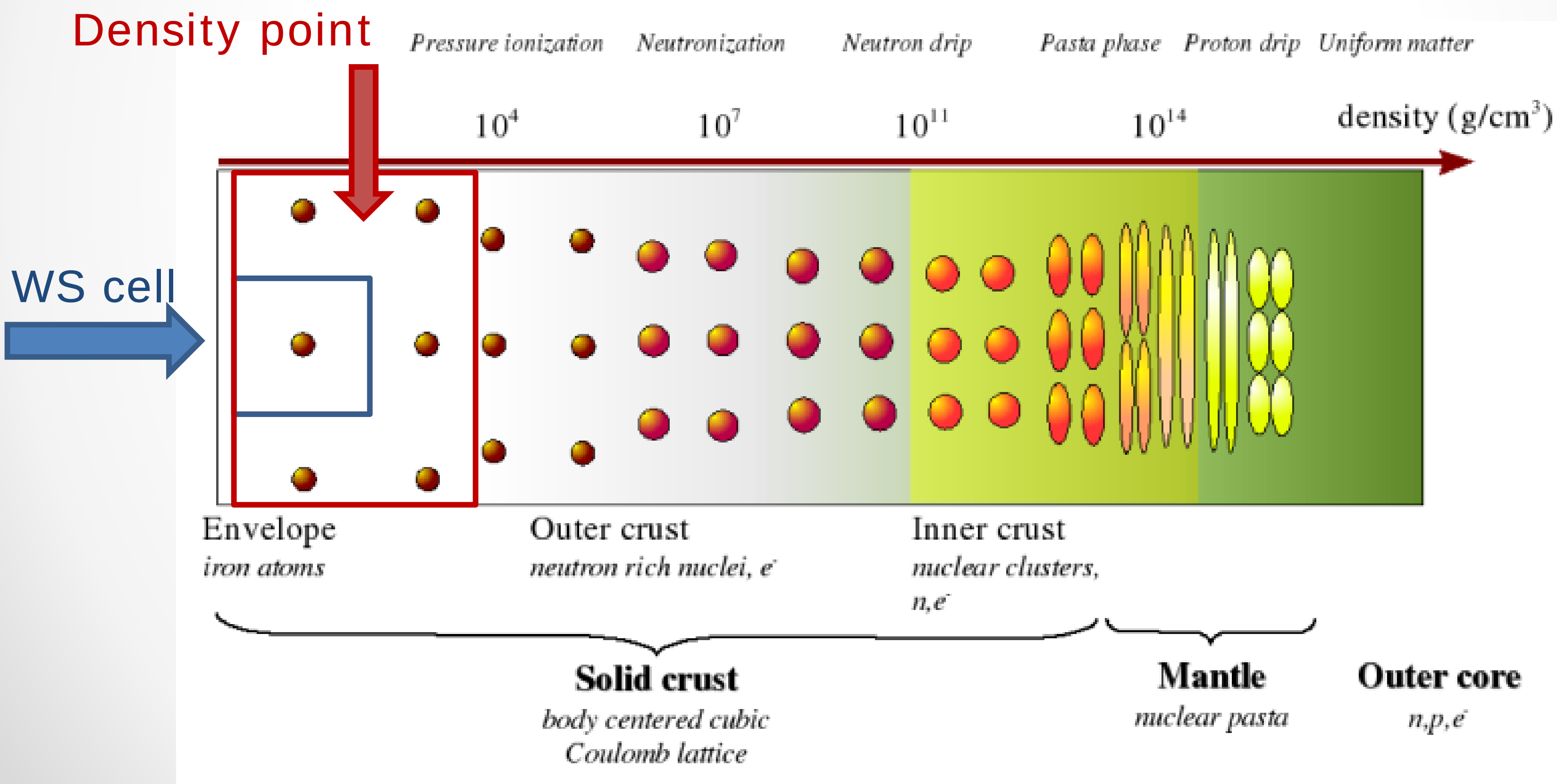
The catalyzed NS crust

- Below saturation matter is clusterized
- At T=0: solid state: BCC lattice
- Ground state energy density: $\varepsilon_{WS}(n_B) = \frac{\sum_i E_{WS}}{\sum_i V_{WS}} = \frac{E_{WS}}{V_{WS}}$ =min



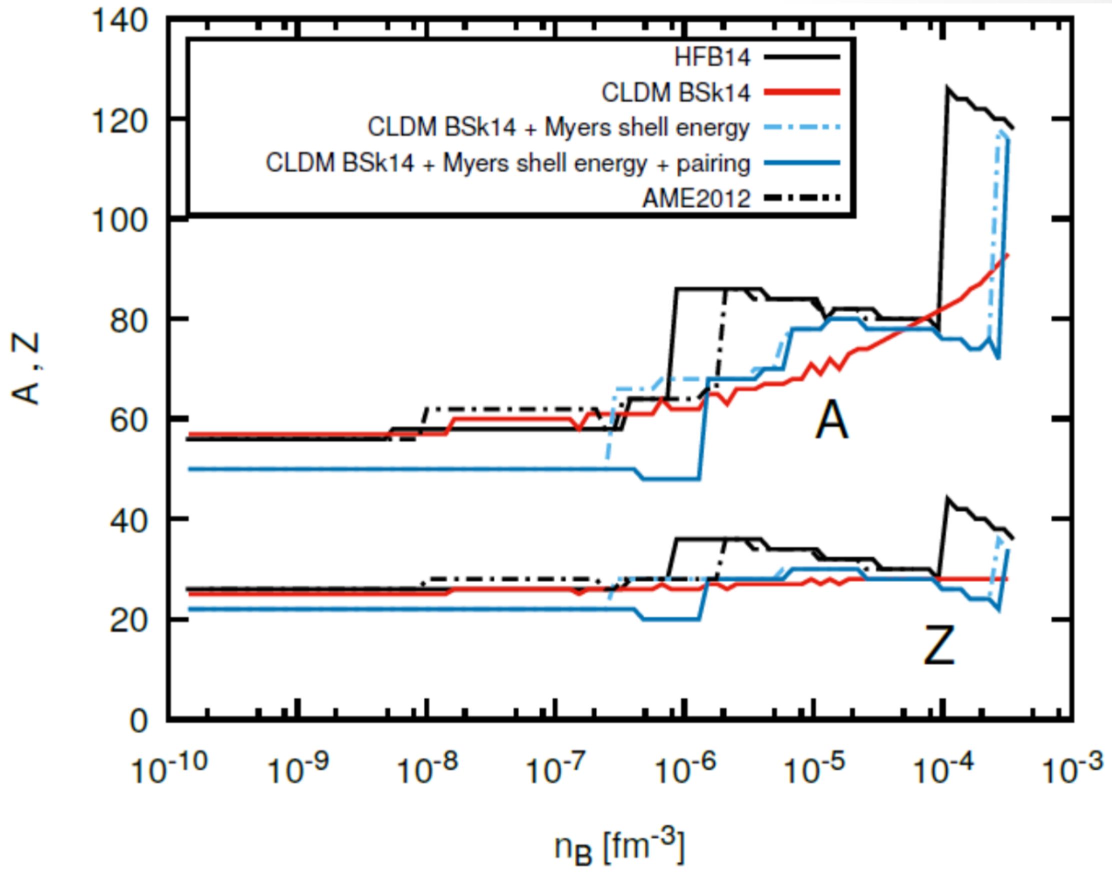
N.Chamel&P.Haensel, living reviews in Relativity 11 (2008)10

Nuclear modeling & crust composition

$\varepsilon_{WS}(n_B) = \min$

1. Choose an energy functional (= set of empirical parameters X_n)
2. Choose a many-body treatment (HFB, HF, ETF, CLDM…)

=> The inner and outer crust composition depends on the many-body treatment



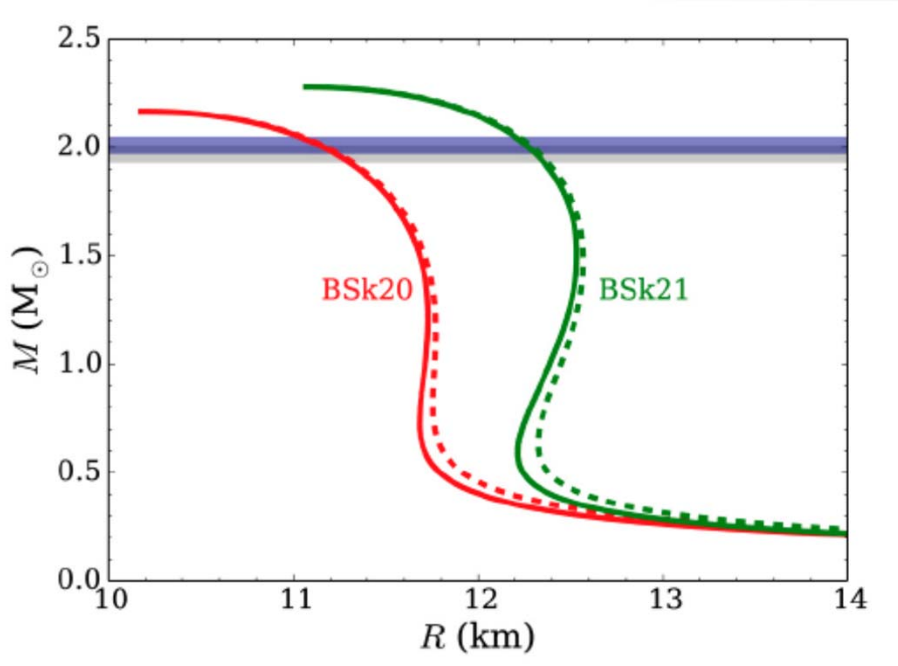
Nuclear modeling & crust composition

$\varepsilon_{WS}(n_B) = \min$

1. Choose an energy functional (= set of empirical parameters X_n)
2. Choose a many-body treatment (HFB, HF, ETF, CLDM…)

=> The inner and outer crust composition depends on the many-body treatment

=> But the uncertainty on astro observables due to the many body treatment is << than the error due to the EDF !!



M.Fortin et al. PRC 94, 035804 (2016)

Nuclear modeling & crust composition

$\varepsilon_{WS}(n_B) = \min$

1. Choose an energy functional (= set of empirical parameters X_n)
2. Choose a many-body treatment (HFB, HF, ETF, CLDM…)

- **Transition between crust and core:** $\boldsymbol{\varepsilon_{WS}^{hom.}(n_B) = \varepsilon_{WS}^{inhom.}(n_B)}$
- **All static observables can be calculated solving the TOV equation of hydrostatic equilibrium:**
- **in particular the crustal width and moment of inertia (connection to pulsar glitches)**

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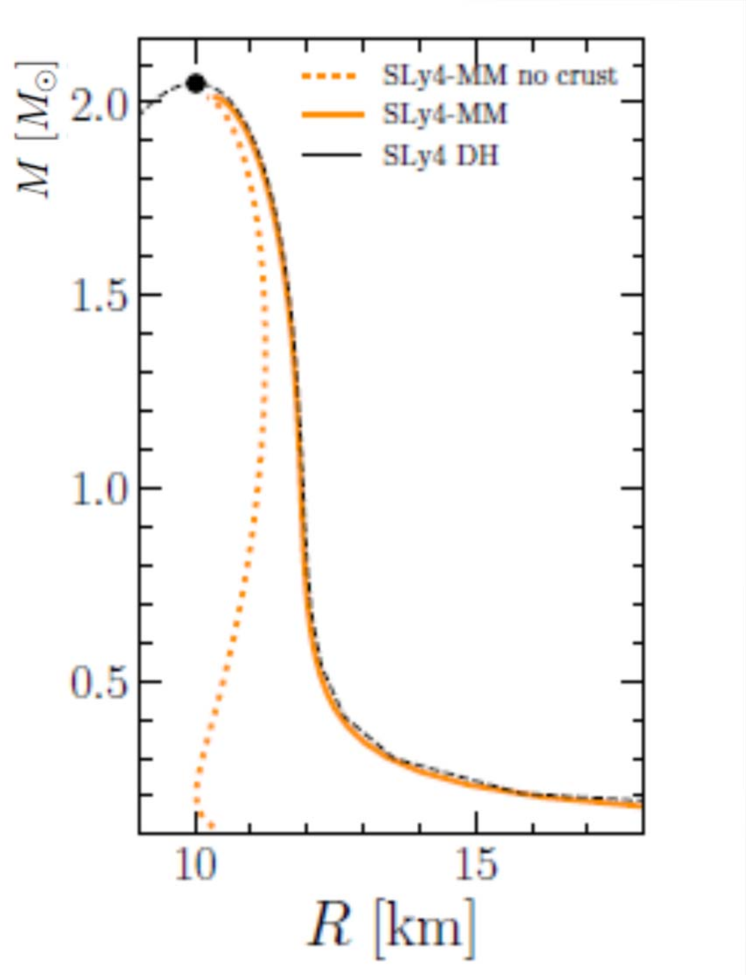

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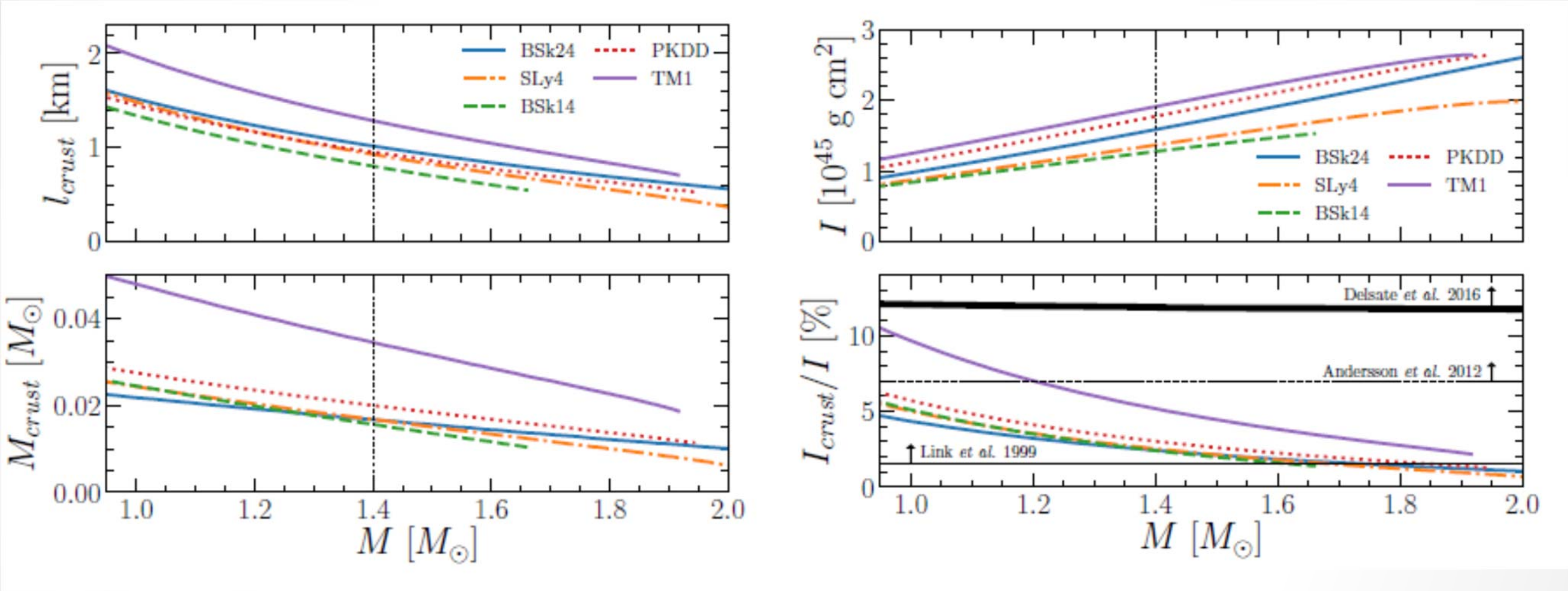
A chosen set of models



Which EoS parameters play a role?

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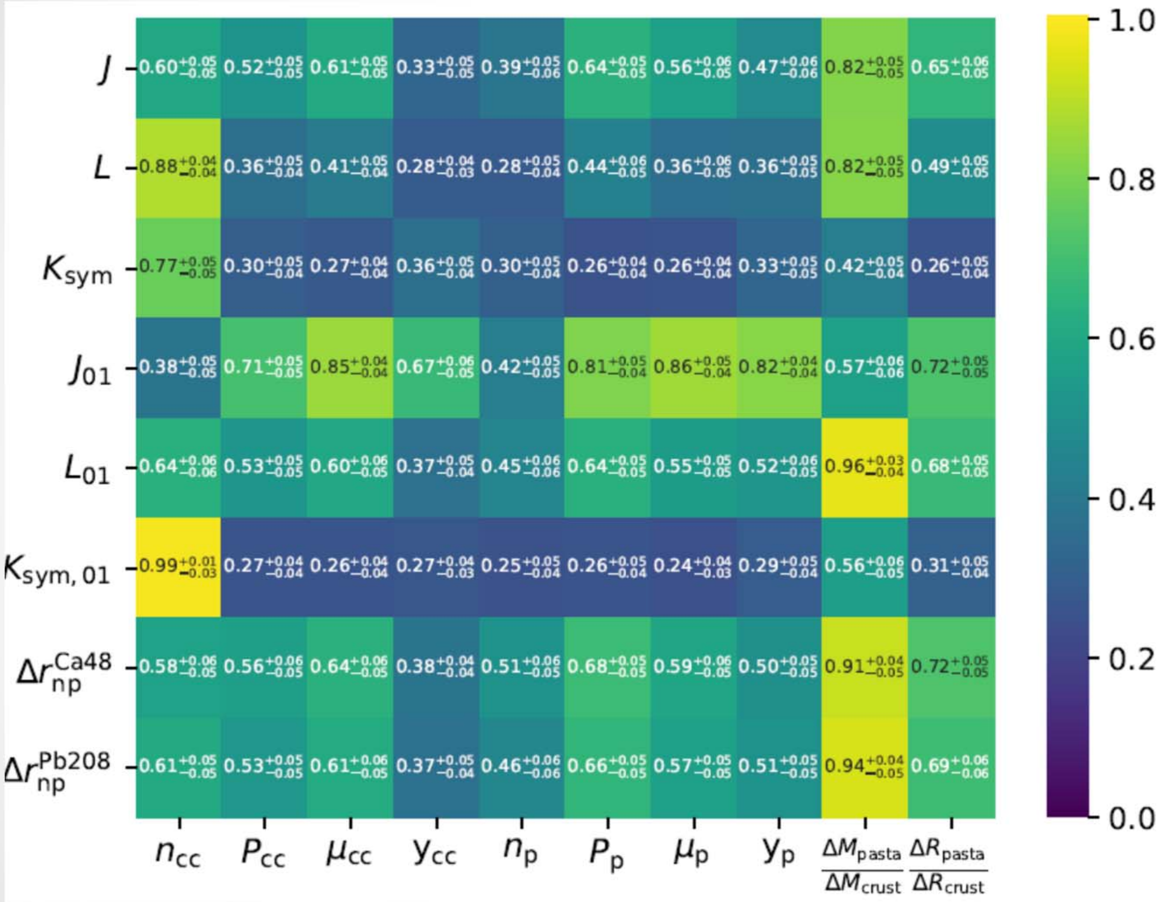
Bayesian analysis



- Skyrme interactions and CLDM modeling
- EFT constraint from Drischler et al. included

=> J, L (K_{sym}) are seen to rule crustal physics!

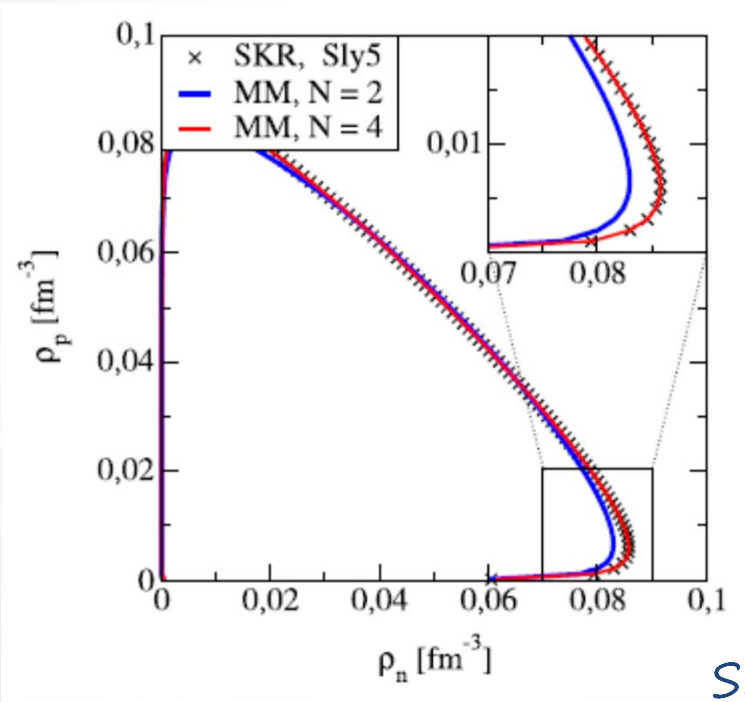
But there are complications…

L.Balliet et al, arXiv:2009.07696

Beyond J&L: a consistent Bayesian analysis of crust properties

1. The Skyrme functional is not flexible enough

- within Skyrme, Q_{sym}, Z_{sym}, Q_{sat}, Z_{sat} are uniquely set from low order parameters

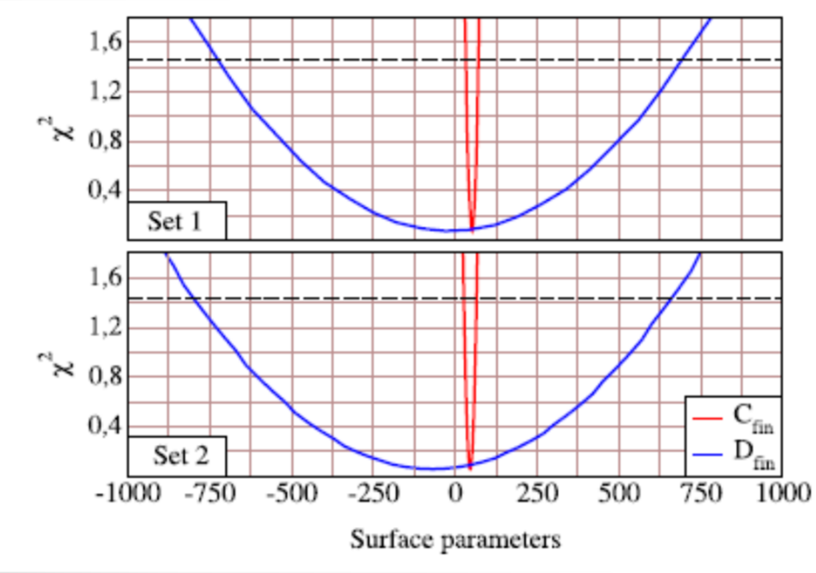


=> the uncertainty on the high order parameters must be included in the analysis

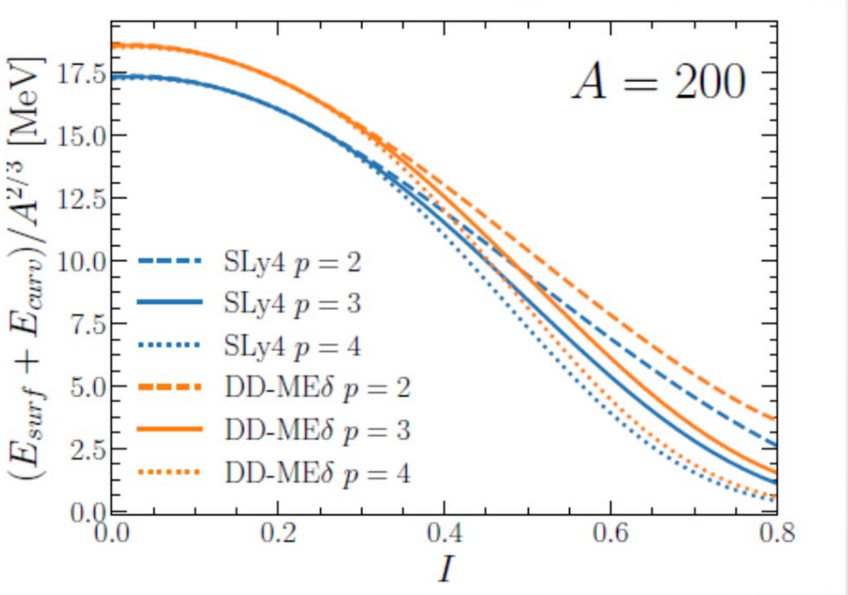
S.Antic et al, Jphys G 46 (2019) 065109

Beyond J&L: a consistent Bayesian analysis of crust properties

2. The energy of inhomogeneous matter depends on the surface tension (= gradient terms) in addition to the EoS



S.Antic et al, Jphys G 46 (2019) 065109



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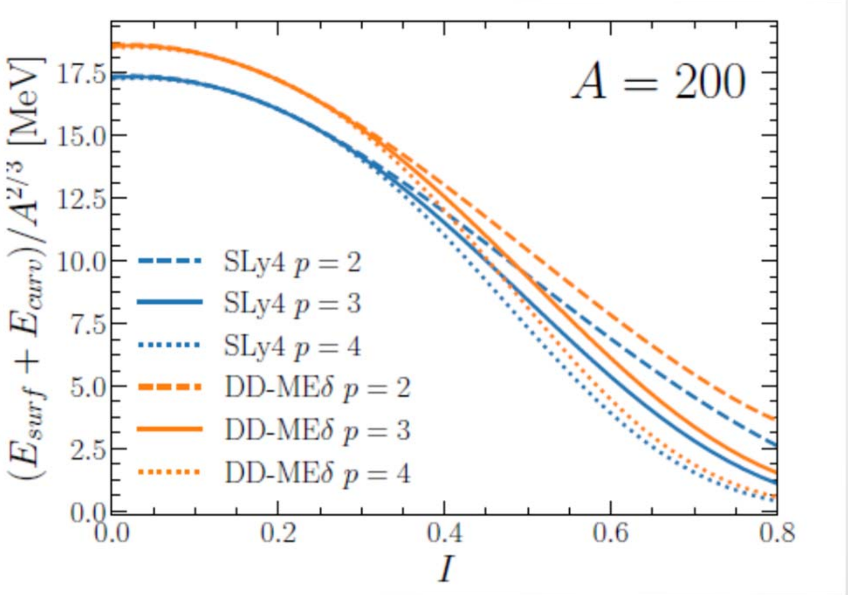
Beyond J&L: a consistent Bayesian analysis of crust properties

2. The energy of inhomogeneous matter depends on the surface tension (= gradient terms) in addition to the EoS

⇒ The surface tension must be calculated consistently with the bulk (fit on neutron rich nuclei)
⇒ The uncertainty at extreme I must be included in the Bayesian analysis



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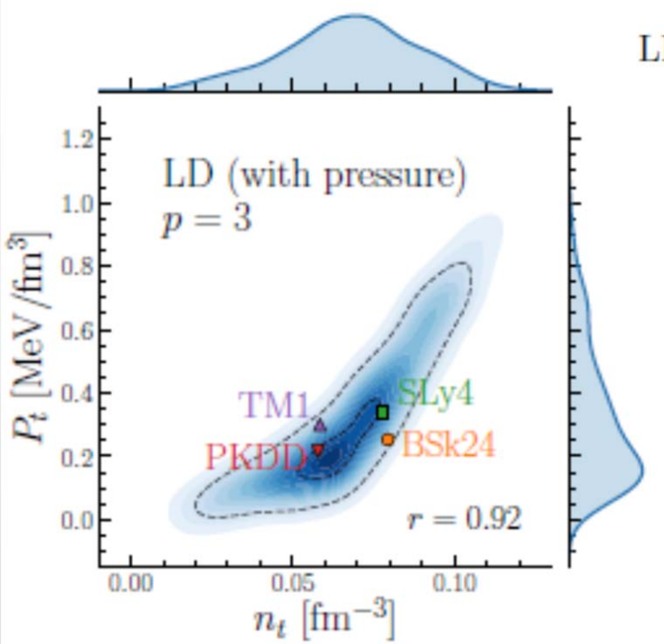
Results

n_t

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	p	σ_0	b_s	σ_{0c}	β
LD+HD	-0.11	-0.02	0.41	-0.51	0.25	-0.08	-0.26	-0.16	0.43	-0.14	0.11	0.11	0.15	-0.10	-0.08
LD+HD $p = 3$	-0.10	-0.02	0.42	-0.51	0.24	-0.11	-0.29	-0.17	0.46	-0.16	0.00	0.10	0.21	-0.10	-0.13
LD (w/ P) $p = 3$	-0.02	-0.05	0.12	-0.54	0.05	-0.06	-0.27	0.10	0.59	-0.11	0.00	0.02	0.09	-0.04	-0.06
LD $p = 3$	-0.08	0.02	0.29	-0.48	0.07	-0.05	-0.20	-0.04	0.47	-0.10	0.00	0.08	0.13	-0.05	-0.11
HD $p = 3$	0.23	0.07	0.30	-0.52	0.25	-0.16	-0.21	-0.06	0.33	-0.13	0.00	-0.23	-0.01	0.17	0.06
Prior $p = 3$	0.17	0.06	0.25	-0.52	0.06	-0.01	-0.22	-0.12	0.32	-0.04	0.00	-0.18	-0.10	0.21	-0.03

P_t

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	p	σ_0	b_s	σ_{0c}	β
LD+HD	-0.08	-0.00	0.31	-0.39	0.18	0.06	-0.26	-0.38	0.47	-0.14	0.12	0.07	0.11	-0.01	-0.07
LD+HD $p = 3$	-0.07	-0.01	0.32	-0.39	0.18	0.05	-0.29	-0.41	0.49	-0.16	0.00	0.07	0.03	-0.01	-0.10
LD (w/ P) $p = 3$	0.00	-0.02	0.11	-0.34	0.02	-0.09	-0.40	0.15	0.74	-0.13	0.00	-0.00	0.08	-0.00	-0.03
LD $p = 3$	-0.04	0.02	0.19	-0.31	0.06	0.10	-0.19	-0.23	0.46	-0.08	0.00	0.03	-0.05	0.04	-0.06
HD $p = 3$	0.22	0.05	0.27	-0.46	0.19	0.05	-0.16	-0.28	0.39	-0.14	0.00	-0.23	-0.17	0.26	-0.00
Prior $p = 3$	0.15	0.03	0.19	-0.42	0.04	0.15	-0.05	-0.24	0.36	-0.04	0.00	-0.16	-0.20	0.24	-0.06



The correlations move towards high order parameters….

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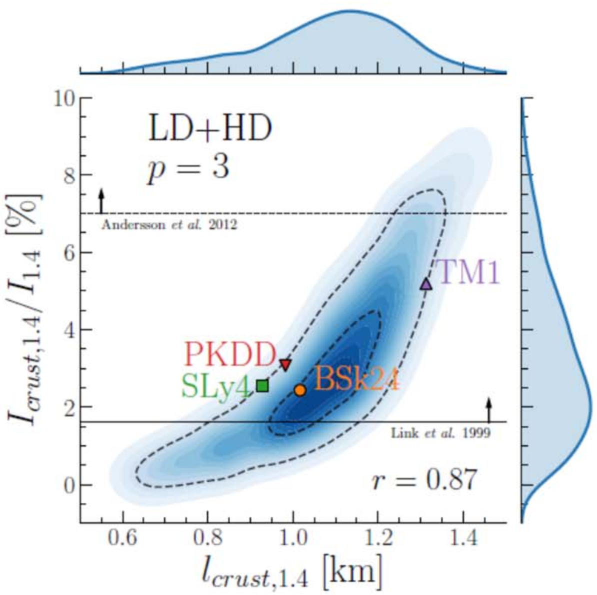
Results

$l_{crust,1.4}$

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	p	σ_0	b_s	σ_{0c}	β
LD+HD	-0.03	-0.13	0.37	-0.30	0.22	-0.05	-0.11	0.05	0.52	-0.11	0.10	0.04	0.11	-0.07	-0.04
LD+HD $p=3$	-0.02	-0.14	0.38	-0.29	0.20	-0.07	-0.14	0.04	0.54	-0.13	0.00	0.02	0.09	-0.06	-0.05
LD (w/ P) $p=3$	-0.00	-0.05	0.24	-0.17	0.18	0.00	-0.30	0.08	0.67	-0.17	0.00	0.00	0.02	0.01	-0.04
LD $p=3$	-0.03	-0.10	0.36	-0.20	0.12	0.02	-0.01	0.04	0.51	-0.09	0.00	0.03	0.03	-0.05	-0.07
HD $p=3$	0.26	-0.14	0.21	-0.27	0.22	0.35	0.09	0.16	0.35	-0.12	0.00	-0.28	-0.40	0.37	-0.07
Prior $p=3$	0.22	-0.12	0.23	-0.25	0.13	0.32	0.08	0.11	0.36	-0.09	0.00	-0.24	-0.36	0.34	-0.11

$I_{crust,1.4}/I_{1.4}$

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	p	σ_0	b_s	σ_{0c}	β
LD+HD	-0.06	-0.05	0.33	-0.35	0.18	0.05	-0.19	-0.28	0.52	-0.14	0.13	0.06	0.12	-0.03	-0.06
LD+HD $p=3$	-0.05	-0.06	0.34	-0.35	0.18	0.03	-0.22	-0.30	0.54	-0.16	0.00	0.05	0.03	-0.02	-0.08
LD (w/ P) $p=3$	-0.00	-0.03	0.23	-0.24	0.10	-0.04	-0.33	-0.02	0.73	-0.24	0.00	0.00	0.05	0.00	-0.04
LD $p=3$	-0.07	-0.03	0.30	-0.27	0.12	0.16	-0.13	-0.35	0.50	-0.08	0.00	0.07	-0.05	0.00	-0.11
HD $p=3$	0.24	-0.03	0.29	-0.44	0.20	0.11	-0.03	-0.16	0.44	-0.15	0.00	-0.25	-0.21	0.27	-0.00
Prior $p=3$	0.19	-0.03	0.25	-0.40	0.14	0.13	-0.01	-0.25	0.42	-0.09	0.00	-0.19	-0.21	0.24	-0.05



The correlations move towards high order parameters....

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Conclusions

- The NS crust properties depend in a complex way on the nuclear energy functional
- High order parameters Q_{sat}, Q_{sym} play an important role as well as surface properties at high isospin ratio
- How to progress?

Strategies for a better estimation of crustal properties

- **Theory:** Implementation of low energy constraints from ab-initio theory
  R. Somasundaram et al, arXiv:2009.04737
  S.Burrello, M.Grasso et al, in progress

- **Experiment:** looking for observables sensitive to the different unknowns:
  - global density variation in asymmetric matter (GMR in isotopic chains ?)
    $$\Rightarrow K_\tau = K_{sym} - 6L_{sym} - L_{sym}\frac{Q_{sat}}{K_{sat}}$$
  - isovector density variations at low density (soft monopole?)
    $$\Rightarrow K_{sym}, Q_{sym}$$
  - n-p density gradients (skin ?) $\Rightarrow L_{sym}, D_{fin}$

To be analyzed with the same meta-modeling approach