

CAT presentation: Study and optimization of a compact X-ray source

Alexandre Moutardier

IJCLab, Orsay, France

January 12th, 2021

ThomX



Figure: ThomX localisation

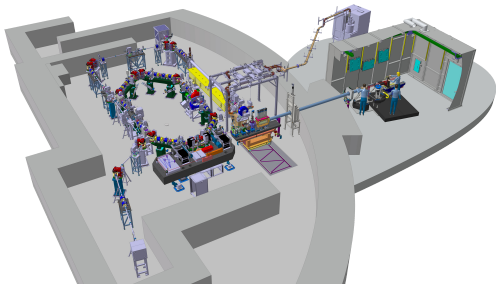


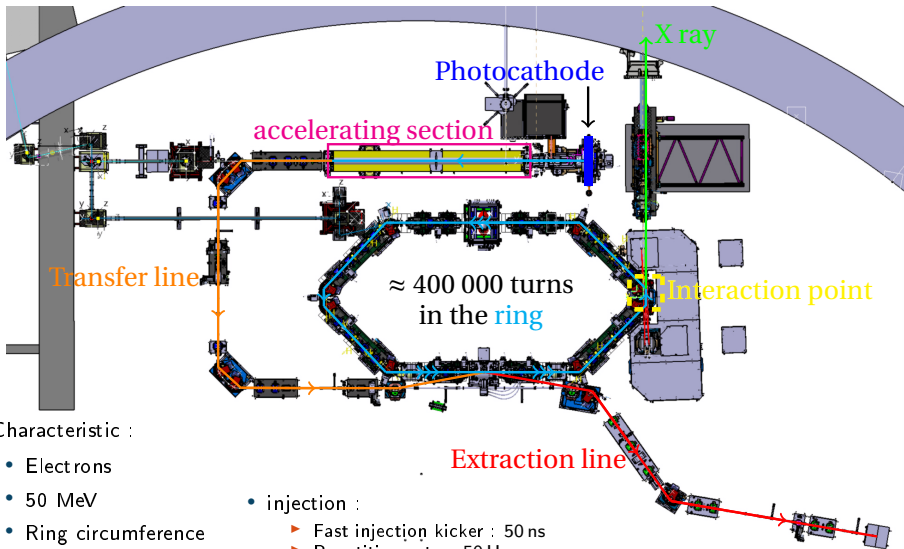
Figure: ThomX overview

ThomX goal :
Create high flux of X-rays
with a compact accelerator
using Compton
backscattering.

If you want to visit ThomX,
let me know.

- Funded in 2012 by an Equipex
- Reuse of the Igléx, a bunker of an old collider of the LAL (Orsay)
- Construction finish, ready for commissioning
- Waiting for authorisation of the ASN

ThomX view from above



Characteristic :

- Electrons
- 50 MeV
- Ring circumference : 18 m
- Beam charge : 1 nC

• injection :

- ▶ Fast injection kicker : 50 ns
- ▶ Repetition rate : 50 Hz

Figure: ThomX overview with principal component

Compton backscattering (CBS)

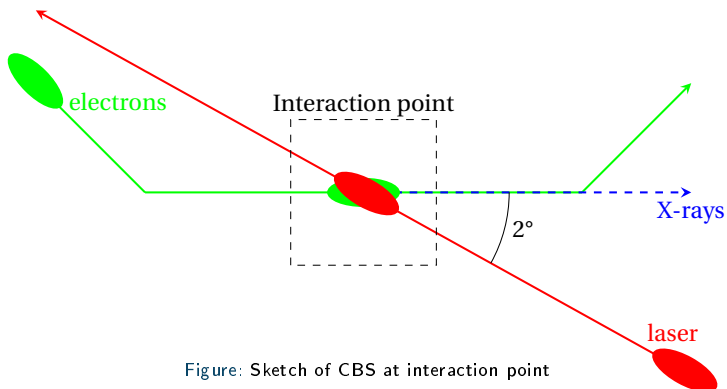


Figure: Sketch of CBS at interaction point

	Electron	Laser	X-ray
Energy	50 MeV	1 eV (1030 nm)	around 50 keV
Beam size	70 μm	70 μm	70 μm at creation, high divergence

Energy of x-ray is adaptable by changing the energy of the electrons.

Better overlap lead to more **X-ray**.

Position of beam in the ring must be **controlled precisely** (some 10^{th} of micrometer).

Injection must be perfectly **controlled**.

X-rays

Specification :

- Hard X-ray : ≈ 50 keV (90 keV achievable with 70 MeV electron)
- Adjustable energy
- Monochromatic by collimation
- Flux : $10^{11} - 10^{13}$ photon/s
- X-ray beam size in the experience room : $\sigma \approx 10$ cm

Application :

- Radiography
 - ▶ Study of artwork
 - ▶ Medical radiography (lungs radiography)
- Radiotherapy
 - ▶ Therapy by irradiation
 - ▶ Use monochromatic x-ray to excite specific target within tumour to "burn" them
- Absorption Tomography
- Crystallography
- Protein structure determination

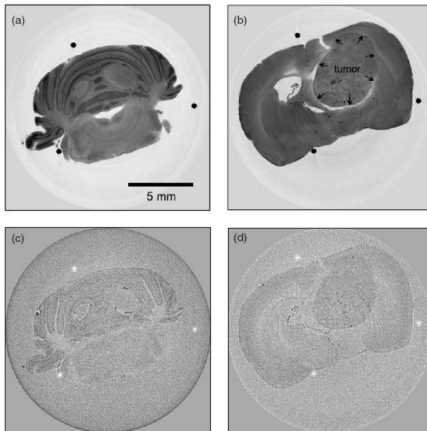


Figure 4 : Phase and attenuation-based tomography results. (a) Phase tomography slice through the rat's cerebellum showing a clear contrast between the white and gray brain matter. (b) Slice through a region of the brain containing a tumor (arrows indicate the tumor's 'pushing front', the border between the tumor-invaded and healthy brain tissue). (c) and (d) Corresponding slices through the absorption-based reconstruction of the specimen. All images are displayed on a linear gray scale corresponding to $\pm 2\sigma$, where σ is the standard deviation of the pixel gray values in the image.

Injection feedback

Injection process

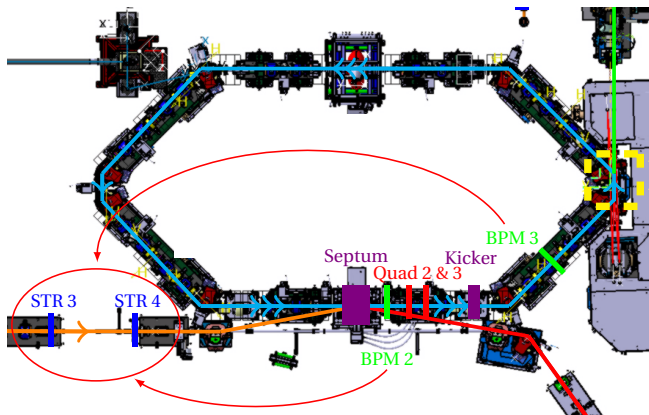


Figure: ThomX overview with principal component

To improve the injection :

- Measure the beam position in the first 2 Beam Position Monitors (BPM) of the ring (RI-C1/DG/BPM.02 and RI-C1/DG/BPM.03).
- Use this measure to calculate the correction that needs to be applied to the last 2 magnetic steerers of the transfer line (TL/AE/STR.03 and TL/AE/STR.04). The kicker strength is also modified to smooth the injection.

Test of the injection feedback code

- Protocol of code testing :
 - ▶ A random particle represented by $(x, px, y, py, z, pz)_{init}$ is randomly selected to symbolize a beam centroid
 - ▶ A simulation code is use to propagate it on ThomX and to evaluate position in the tow first BPM of the ring : $(x, y)_{BPM2}$ and $(x, y)_{BPM3}$
 - ▶ The correction code is used to compute correction of the injection, ie to pass through $(x, y)_{BPM2, wanted}$ and $(x, y)_{BPM3, wanted}$ ((0,0) in the frame of the reference particle)
 - ▶ A fraction of the correction are applied and simulation are done again several times

To evaluate the efficiency of an injection one can use the following estimator at each step :

$$E_v = \sqrt{\frac{\sum Ring's BPM v^2}{\# of BPM}}$$

Where v can be x , px , y or py and ring's BPM are all Beam Position Monitor of the ring except the first one exclude because of the off-axes travelling during the injection.

Evolution of estimator during iteration of feedback code

Convergence defined by :

- $E_x < 10\mu\text{m}$

- $E_y < 10\mu\text{m}$

- $E_{p_x} < 10\mu\text{rad}$

- $E_{p_y} < 10\mu\text{rad}$

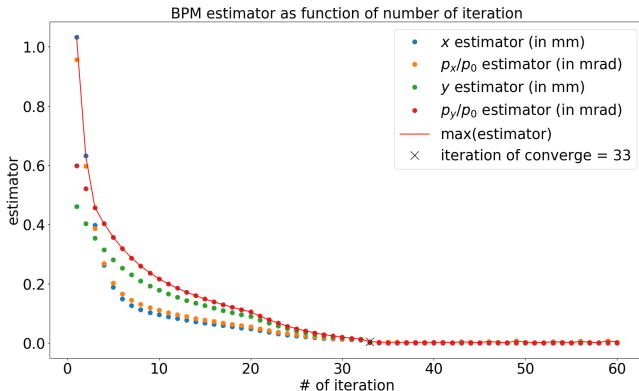


Figure: Evolution of the estimator at each iteration of the injection feedback code.

Test of iteration feedback system over 1000 simulation

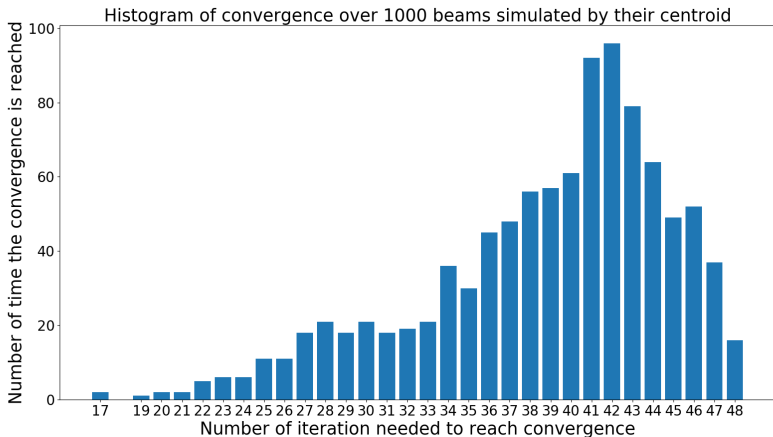


Figure: Histogram of the number of iteration needed to achieved convergence criterion. 1000 beams centroid injection has been simulated.

Convergence criterion achieved in less than 48 iteration each time, ie around some minute. Iteration never lead to losses.

Projected loss map

Study of losses

Goal :

- Simulate the propagation of particles from the beginning of the transfer line to the end of the first ring turn
- use aperture definition to check loss of particle
- Plot losses :
 - ▶ Along the accelerator
 - ▶ At one position

The beam size used to randomly selected particles is $\sigma = 1.5 \text{ mm}$.

Aperture definition :

- TL : circle of radius 17.5 mm
- Ring : Ellipse 40 mm large, 24 mm

The 12σ beam is the exactly pipe wide.

The maximum beam size use is 20σ

Losses along ThomX as function of the beam size

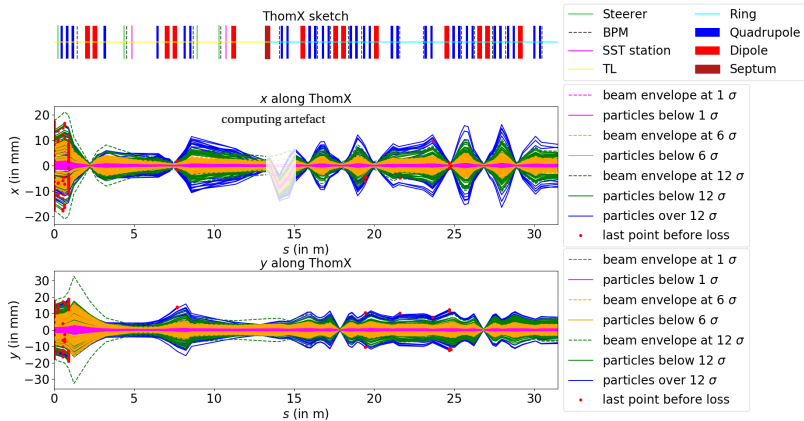


Figure: Losses along ThomX.

- Huge beam size \Rightarrow lot of losses at the beginning
- No losses for the 5σ beam
- 12σ beam \Rightarrow No losses after the first dipole

Losses map

The particles here are represented at their x-y position at the beginning of the TL, and the marker colour represents the localisation of particle lost.

- Square cut \Rightarrow Square shape
- First element cut (red dots)
 \Rightarrow Circular shape
- Losses in the 3rd quadrupole (TL_AR_QP03) mostly in upper and lower beam part
 \Rightarrow Large vertical beam (also see in previous' loss map at $s = 1.2$ m)
- The center = acceptable window
Loss = only particles without 12σ
Other particles (black cross) pass the first ring turn

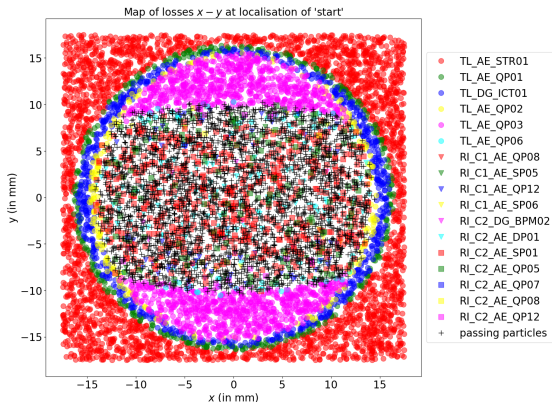


Figure: Histogram of the number of iteration needed to achieved convergence criterion. 1000 beams centroid injection has been simulated.

Acceptable window at the localisation of SST station may permit to check the possibility for a real beam to be transmitted until the end of the first ring turn even before to try it

Conclusion

- A code of injection feedback as been created.
- Primary testing are promising.
- Some fluctuation in the beam centroid and uncertainty in the measurement has to be added in the testing process.
- Loss map has been studies.
- Primary losses simulation show that beam losses are well controlled.
- Once beam measure may be done on ThomX, measured beam characteristic must be use to done again the study.

Thanks

Backup

Classical transfer matrices

- **Drift** of length L:

$$XX = YY = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$ZZ = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

$$M_{drift} = \begin{pmatrix} XX & 0 & 0 \\ 0 & YY & 0 \\ 0 & 0 & ZZ \end{pmatrix}$$

- **Quadrupole** of length L and strength k:

$$F = \begin{pmatrix} \cos(\sqrt{k} \times L) & \frac{1}{\sqrt{k}} \sin(\sqrt{k} \times L) \\ -\sqrt{k} \times \sin(\sqrt{k} \times L) & \cos(\sqrt{k} \times L) \end{pmatrix}$$

$$D = \begin{pmatrix} \cosh(\sqrt{k} \times L) & \frac{1}{\sqrt{k}} \sinh(\sqrt{k} \times L) \\ \sqrt{k} \times \sinh(\sqrt{k} \times L) & \cosh(\sqrt{k} \times L) \end{pmatrix}$$

if $k > 0$:

if $k < 0$:

$$M_{quad} = \begin{pmatrix} F & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & ZZ \end{pmatrix}$$

$$M_{quad} = \begin{pmatrix} D & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & ZZ \end{pmatrix}$$

- **Bending magnet** or **septum**:

Based on: TRACE 3-D Documentation, K. R. Crandall and D. P. Rusthoi, Third Edition (LA-UR-97-886), May 1997, Los Alamos National Laboratory, Page 14

<https://laacg.lanl.gov/laacg/services/traceman.pdf>

Backup

Other elements type

- **Change of frame:**

$$x_2 = x_1 + \delta x$$

$$px_2 = px_1 + \delta px$$

$$y_2 = y_1 + \delta y$$

$$py_2 = py_1 + \delta py$$

$$z_2 = z_1 + \delta z$$

$$pz_2 = pz_1 + \delta pz$$

- **Kicker** with kick in px:

Change of frame:

$$px_2 = px_1 + \frac{\text{kick}}{2}$$

Propagation within the kicker

$$M = M_{drift}(L_{kicker})$$

Change of frame:

$$px_2 = px_1 + \frac{\text{kick}}{2}$$

- **Steerer** with kick of Dev_x in px and Dev_y in py:

Propagation within half of the steerer:

$$M = M_{drift}\left(\frac{L_{str}}{2}\right)$$

Change of frame:

$$px_2 = px_1 + Dev_x$$

$$py_2 = py_1 + Dev_y$$

Propagation within half of the steerer:

$$M = M_{drift}\left(\frac{L_{str}}{2}\right)$$

In my presentation steerer have no length, hence it is simply a change of frame on px and py plane.

- **Screen, BPM:** specific element that does not have any effect on the beam
- **Start, end:** specific element to defined the beginning and the end of the line

Backup

Analytics code on MatLab: [structure](#)

Base on 2 classes:

acc_el

accelerator

Attributes

- type_el: string depending on element type (quad,bend...)
 - name: specific name (cf ThomX nomenclature)
 - at: position of beginning of the element
 - length: length of the element
 - save_calc: properties to save calculation after the element
 - variables: list of other useful parameter (depending on element type)
- name: Name of the accelerator
 - energy: Energy of the electron considered (50 MeV on my presentation)
 - list_el: list of object of type acc_el that characterise a line of an accelerator

acc_el

- `acc_el(type_el, name, position, length, variables, save_calc)`: constructor
- `eq(acc_el, acc_el2)`: Comparator
- `is_quasi_equal(acc_el, acc_el2)`: Comparator with tolerance of 10^{-14} on position and length of the element and without name comparison
- `matrix(acc_el, gamma_square, exact_val)`: return the transfer matrix of the corresponding element

accelerator

Method

- `accelerator(name, energy, list_el)`: Constructor
- `add(accelerator, acc_el)`: add `acc_el` at the end of the list of element
- `add_at_begining(accelerator, acc_el)`: add `acc_el` at the beginning of the list of element
- `take_part_of_line(accelerator, name_start_line, name_end_line)`: return an accelerator that begin at the beginning of `name_start_line` and end at the end of `name_end_line`
- `take_invert_of_line(accelerator)`: invert a line to permit calculation of inverse propagation
- `calcul_propagation(accelerator, exact_val, name_file)`: Compute propagation from the first element of `list_el` to the last one. Output are save on the file `name_file` and calculation may use ever exact value or approximate one (only small differences has been seen)

Computation of correction

- Why Python ?
 - ▶ To be implemented in ThomX cc and coupled with Taurus
- Goal:
 - ▶ Use position of beam measured on RI-C1/DG/BPM.02 and RI-C1/DG/BPM.03, constrain x, y and px to find correction of steerers TL/AE/STR.03, TL/AE/STR.04 and of kicker RI-C1/PE/KIC.01 needed to inject correctly the beam on the ring
The previous analytic software is used for that.
- Method:
 - ▶ extract $(x, px, y, py)_{BPM2}$ from $(x, y)_{BPM2}$ and $(x, y)_{BPM3}$ using analytic calculation from BPM2 to BPM3
 - ▶ extract $(x, px, y, py)_{BPM2, wanted}$ and $kick_{wanted}$ in kicker from $(x, y)_{BPM2, wanted}$, $(x, y)_{BPM3, wanted}$ and $px_{BPM3, wanted} = 0$ using analytic calculation from BPM2 to BPM3
 - ▶ compute $Dev_{3, wanted} = (Dev_{x3, wanted}, Dev_{y3, wanted})$ and $Dev_{4, wanted}$ such that:

$$M_{fromBPM2, toSTR3}(Dev_{3, w}, Dev_{4, w}) \times \begin{pmatrix} x \\ px \\ y \\ py \\ 0 \\ 0 \end{pmatrix}_{BPM2, w} = M_{fromBPM2, toSTR3}(Dev_3, Dev_4) \times \begin{pmatrix} x \\ px \\ y \\ py \\ 0 \\ 0 \end{pmatrix}_{BPM2}$$

Where w is for wanted

See [backup slide 22](#) for more details of calculation

Backup

Results returned by analytics code

All parameter of `acc_el` may be used as symbolic parameter.

The general solution is then:

$$\left\{ \begin{array}{l} x_2 = f_x(x_1, px_1, y_1, py_1, z_1, pz_1, \text{symbolic parameters}) \\ px_2 = f_{px}(x_1, px_1, y_1, py_1, z_1, pz_1, \text{symbolic parameters}) \\ y_2 = f_y(x_1, px_1, y_1, py_1, z_1, pz_1, \text{symbolic parameters}) \\ py_2 = f_{py}(x_1, px_1, y_1, py_1, z_1, pz_1, \text{symbolic parameters}) \\ z_2 = f_z(x_1, px_1, y_1, py_1, z_1, pz_1, \text{symbolic parameters}) \\ pz_2 = f_{pz}(x_1, px_1, y_1, py_1, z_1, pz_1, \text{symbolic parameters}) \end{array} \right.$$

Where f_i , for $i = x, px, y, py, z, pz$, is a linear function of its variable.

Backup

Detailed calculus of $(x, px, y, pz)_{BPM2}$ from $(x, y)_{BPM2}$ and $(x, y)_{BPM3}$

Hypothesis:

- Linear calculation
- X and Y plans uncoupled
- coupling between x and z because of bending magnet
- Symbolic parameters: Only the kick of the kicker in x and px planes from RI-C1/DG/BPM02 and RI-C1/DG/BPM03 of the transfer line
- No other change of frame, hence no constant value

Form of analytic output:

$$\left\{ \begin{array}{l} \mathbf{X}_{BPM3} = a_{x,x} \mathbf{X}_{BPM2} + a_{x,px} \mathbf{PX}_{BPM2} + a_{x,z} \mathbf{Z}_{BPM2} + a_{x,pz} \mathbf{PZ}_{BPM2} + a_{x,kick} \mathbf{KICK} \\ \mathbf{PX}_{BPM3} = a_{px,x} \mathbf{X}_{BPM2} + a_{px,px} \mathbf{PX}_{BPM2} + a_{px,z} \mathbf{Z}_{BPM2} + a_{px,pz} \mathbf{PZ}_{BPM2} + a_{px,kick} \mathbf{KICK} \\ \mathbf{Y}_{BPM3} = a_{y,y} \mathbf{Y}_{BPM2} + a_{y,py} \mathbf{PY}_{BPM2} + a_{y,z} \mathbf{Z}_{BPM2} + a_{y,pz} \mathbf{PZ}_{BPM2} \\ \mathbf{PY}_{BPM3} = a_{py,y} \mathbf{Y}_{BPM2} + a_{py,py} \mathbf{PY}_{BPM2} + a_{py,z} \mathbf{Z}_{BPM2} + a_{py,pz} \mathbf{PZ}_{BPM2} \\ \mathbf{Z}_{BPM3} = a_{z,x} \mathbf{X}_{BPM2} + a_{z,px} \mathbf{PX}_{BPM2} + a_{z,z} \mathbf{Z}_{BPM2} + a_{z,pz} \mathbf{PZ}_{BPM2} \\ \mathbf{PZ}_{BPM3} = a_{y,pz} \mathbf{PZ}_{BPM2} \end{array} \right.$$

Backup

Simplification and resolution for initial values

Assumption:

- $Z_{BPM2} = 0$
- $PZ_{BPM2} = 0$
- \mathbf{KICK}_{init} known (value use to track the particle)
- $\mathbf{X}_{BPM2,init}, \mathbf{Y}_{BPM2,init}, \mathbf{X}_{BPM3,init}$ and $\mathbf{Y}_{BPM3,init}$ known (computed or measured values)
- Unknown :
 - ▶ $\mathbf{PX}_{BPM2,init}$
 - ▶ $\mathbf{PY}_{BPM2,init}$

Hence:

$$\begin{cases} \mathbf{X}_{BPM3,init} &= a_{x,x}\mathbf{X}_{BPM2,init} + a_{x,px}\mathbf{PX}_{BPM2,init} + a_{x,kick}\mathbf{KICK}_{init} \\ \mathbf{Y}_{BPM3,init} &= a_{y,y}\mathbf{Y}_{BPM2,init} + a_{y,py}\mathbf{PY}_{BPM2,init} \end{cases}$$

Finally :

$$\begin{cases} \mathbf{PX}_{BPM2,init} &= \frac{\mathbf{X}_{BPM3,init} - a_{x,x}\mathbf{X}_{BPM2,init} - a_{x,kick}\mathbf{KICK}_{init}}{a_{x,px}} \\ \mathbf{PY}_{BPM2,init} &= \frac{\mathbf{Y}_{BPM3,init} - a_{y,y}\mathbf{Y}_{BPM2,init}}{a_{y,py}} \end{cases}$$

Backup

Simplification and resolution for wanted value

Assumption:

- $Z_{BPM2} = 0$
- $PZ_{BPM2} = 0$
- $PX_{BPM2,w} = 0$
- $\mathbf{X}_{BPM2,w}$, $\mathbf{Y}_{BPM2,w}$, $\mathbf{X}_{BPM3,w}$ and $\mathbf{Y}_{BPM3,w}$ known (wanted values, 0 in the particle reference frame)
- Unknown :
 - ▶ $\mathbf{PX}_{BPM2,w}$
 - ▶ \mathbf{KICK}_w
 - ▶ $\mathbf{PY}_{BPM2,w}$

Hence:

$$\begin{cases} \mathbf{X}_{BPM3,w} &= a_{x,x} \mathbf{X}_{BPM2,w} + a_{x,px} \mathbf{PX}_{BPM2,w} + a_{x,kick} \mathbf{KICK}_w \\ \mathbf{PX}_{BPM3,w} &= a_{px,x} \mathbf{X}_{BPM2,w} + a_{px,px} \mathbf{PX}_{BPM2,w} + a_{px,kick} \mathbf{KICK}_w = 0 \\ \mathbf{Y}_{BPM3,w} &= a_{y,y} \mathbf{Y}_{BPM2,w} + a_{y,py} \mathbf{PY}_{BPM2,w} \end{cases}$$

Finally :

$$\begin{cases} \mathbf{PX}_{BPM2,w} &= \frac{a_{x,kick} \times \mathbf{X}_{BPM3,w} - (a_{px,kick} \times a_{x,x} - a_{px,x} \times a_{x,kick}) \times \mathbf{X}_{BPM2,w}}{a_{px,px} \times a_{x,kick} - a_{x,x} \times a_{px,kick}} \\ \mathbf{KICK}_w &= \frac{a_{px,px} \times \mathbf{X}_{BPM3,w} - (a_{px,px} \times a_{x,x} - a_{px,x} \times a_{x,px}) \times \mathbf{X}_{BPM2,w}}{a_{x,x} \times a_{px,kick} - a_{px,px} \times a_{x,kick}} \\ \mathbf{PY}_{BPM2,w} &= \frac{\mathbf{Y}_{BPM3,w} - a_{y,y} \mathbf{Y}_{BPM2,w}}{a_{y,py}} \end{cases}$$

Backup

Detailed calculus of Dev_{x3} and Dev_{x3}

Hypothesis:

- Linear calculation
- X and Y plans uncoupled
- coupling between x and z because of bending magnet
- Symbolic parameters = $Dev_{x3}, Dev_{y3}, Dev_{x4}, Dev_{y4}$ from BPM RI-C1/DG/BPM02 to TL/AE/STR03

Form of analytic output:

$$\begin{cases} \mathbf{X}_2 &= a_{x,x}\mathbf{X} & +a_{x,px}\mathbf{PX} & +a_{x,z}\mathbf{Z} & +a_{x,pz}\mathbf{PZ} & +a_{x,devx3}\mathbf{Dev}_{x3} & +a_{x,devx4}\mathbf{Dev}_{x4} \\ \mathbf{PX}_2 &= a_{px,x}\mathbf{X} & +a_{px,px}\mathbf{PX} & +a_{px,z}\mathbf{Z} & +a_{px,pz}\mathbf{PZ} & +a_{px,devx3}\mathbf{Dev}_{x3} & +a_{px,devx4}\mathbf{Dev}_{x4} \\ \mathbf{Y}_2 &= a_{y,y}\mathbf{Y} & +a_{y,py}\mathbf{PY} & +a_{y,z}\mathbf{Z} & +a_{y,pz}\mathbf{PZ} & +a_{y,devy3}\mathbf{Dev}_{y3} & +a_{y,devy4}\mathbf{Dev}_{y4} \\ \mathbf{PY}_2 &= a_{py,y}\mathbf{Y} & +a_{py,py}\mathbf{PY} & +a_{py,z}\mathbf{Z} & +a_{py,pz}\mathbf{PZ} & +a_{py,devy3}\mathbf{Dev}_{y3} & +a_{py,devy4}\mathbf{Dev}_{y4} \\ \mathbf{Z}_2 &= a_{z,x}\mathbf{X} & +a_{z,px}\mathbf{PX} & +a_{z,z}\mathbf{Z} & +a_{z,pz}\mathbf{PZ} & +a_{z,devx3}\mathbf{Dev}_{x3} & +a_{z,devx4}\mathbf{Dev}_{x4} \\ \mathbf{PZ}_2 &= a_{y,pz}\mathbf{PZ} & & & & & \end{cases}$$

Where 2 index symbolise steerer's position and non index symbolise BPM position

For each variable there is also a constant term because of change of frame at the end of the septum. This term is not considered there because it would be simplify when one do "initial system" = "wanted system".

Backup

Simplification

Assumption:

- $Z_{BPM2} = 0$
- $PZ_{BPM2} = 0$
- $a_{x,devx3} = 0 = a_{y,devy3}$ because calculus are stop at the end of the steerer TL/AE/STR03 simulated by no-length change of variable in px and py

There is 2 systems:

- Initial system:

$$\begin{cases} \mathbf{X}_{str3,i} &= a_{x,x}\mathbf{X}_{BPM2,i} & + a_{x,px}\mathbf{PX}_{BPM2,i} & & + a_{x,devx4}\mathbf{Dev}_{x4,i} \\ \mathbf{PX}_{str3,i} &= a_{px,x}\mathbf{X}_{BPM2,i} & + a_{px,px}\mathbf{PX}_{BPM2,i} & + a_{px,devx3}\mathbf{Dev}_{x3,i} & + a_{px,devx4}\mathbf{Dev}_{x4,i} \\ \mathbf{Y}_{str3,i} &= a_{y,y}\mathbf{Y}_{BPM2,i} & + a_{y,py}\mathbf{PY}_{BPM2,i} & & + a_{y,devy4}\mathbf{Dev}_{y4,i} \\ \mathbf{PY}_{str3,i} &= a_{py,y}\mathbf{Y}_{BPM2,i} & + a_{py,py}\mathbf{PY}_{BPM2,i} & + a_{py,devy3}\mathbf{Dev}_{y3,i} & + a_{py,devy4}\mathbf{Dev}_{y4,i} \end{cases}$$

- Wanted system:

$$\begin{cases} \mathbf{X}_{str3,w} &= b_{x,x}\mathbf{X}_{BPM2,w} & + b_{x,px}\mathbf{PX}_{BPM2,w} & & + b_{x,devx4}\mathbf{Dev}_{x4,w} \\ \mathbf{PX}_{str3,w} &= b_{px,x}\mathbf{X}_{BPM2,w} & + b_{px,px}\mathbf{PX}_{BPM2,w} & + b_{px,devx3}\mathbf{Dev}_{x3,w} & + b_{px,devx4}\mathbf{Dev}_{x4,w} \\ \mathbf{Y}_{str3,w} &= b_{y,y}\mathbf{Y}_{BPM2,w} & + b_{y,py}\mathbf{PY}_{BPM2,w} & & + b_{y,devy4}\mathbf{Dev}_{y4,w} \\ \mathbf{PY}_{str3,w} &= b_{py,y}\mathbf{Y}_{BPM2,w} & + b_{py,py}\mathbf{PY}_{BPM2,w} & + b_{py,devy3}\mathbf{Dev}_{y3,w} & + b_{py,devy4}\mathbf{Dev}_{y4,w} \end{cases}$$

Backup

Simplification and resolution

- Unknown:
 - ▶ $\text{Dev}_{x3,w}$
 - ▶ $\text{Dev}_{y3,w}$
 - ▶ $\text{Dev}_{x4,w}$
 - ▶ $\text{Dev}_{y4,w}$
- Known:
 - ▶ All other variables

To ensure physical coherence, $(X, PX, Y, PY)_{str3,i} = (X, PX, Y, PY)_{str3,w}$

Finally:

$$\begin{cases} \text{Dev}_{x4,w} = \frac{a_{x,x}\mathbf{X}_i + a_{x,px}\mathbf{PX}_i + a_{x,devx4}\text{Dev}_{x4,i} - b_{x,x}\mathbf{X}_w - b_{x,px}\mathbf{PX}_w}{b_{x,devx4}} \\ \text{Dev}_{y4,w} = \frac{a_{y,y}\mathbf{Y}_i + a_{y,py}\mathbf{PY}_i + a_{y,devy4}\text{Dev}_{y4,i} - b_{y,y}\mathbf{Y}_w - b_{y,py}\mathbf{PY}_w}{a_{y,devy4}} \\ \text{Dev}_{x3,w} = \frac{a_{px,x}\mathbf{X}_i + a_{px,px}\mathbf{PX}_i + a_{px,devx3}\text{Dev}_{x3,i} + a_{px,devx4}\text{Dev}_{x4,i} - b_{px,x}\mathbf{X}_w - b_{px,px}\mathbf{PX}_w - b_{px,devx4}\text{Dev}_{x4,w}}{b_{px,devx3}} \\ \text{Dev}_{y3,w} = \frac{a_{py,y}\mathbf{Y}_i + a_{py,py}\mathbf{PY}_i + a_{py,devy3}\text{Dev}_{y3,i} + a_{py,devy4}\text{Dev}_{y4,i} - b_{py,y}\mathbf{Y}_w - b_{py,py}\mathbf{PY}_w - b_{py,devy4}\text{Dev}_{y4,w}}{b_{py,devy3}} \end{cases}$$

Where $\mathbf{U}_v = \mathbf{U}_{BPM2,v}$, with $\mathbf{U} = \mathbf{X}, \mathbf{Y}, \mathbf{PX}, \mathbf{PY}$ and $v = i, w$

Backup

Precision about choose of fraction of correction applied

Classical correction $\approx 3 \times 10^{-3}$

At one step:

- If $|dev_u - dev_c| > 10^{-4}$:

$$dev_a = dev_u + \frac{dev_c - dev_u}{10}$$

- If $|dev_u - dev_c| > 10^{-5}$:

$$dev_a = dev_u + \frac{dev_c - dev_u}{5}$$

- Else :

$$dev_a = dev_c$$

With :

- dev_u the correction use in last calculation
- dev_c the correction calculated at this step
- dev_a the correction apply at the next step