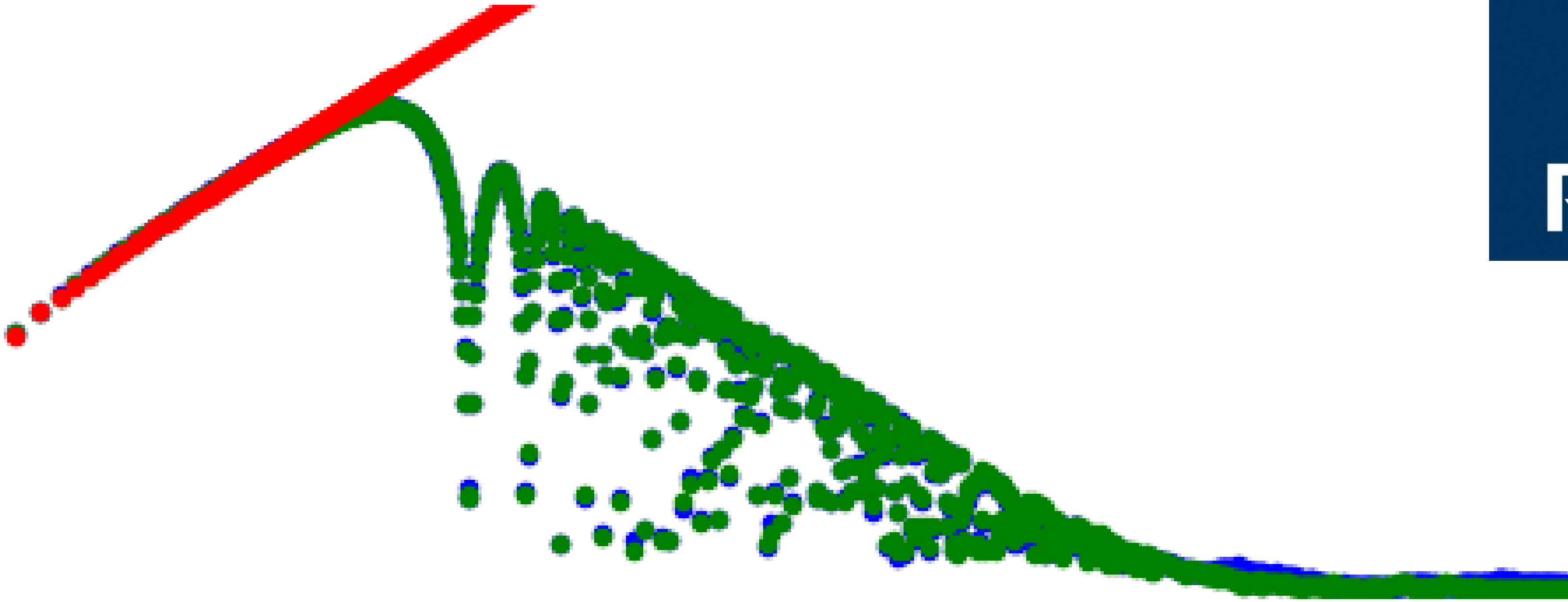

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Turbulence level dependent investigation of the cosmic ray diffusion coefficient

Paris Saclay AstroParticle Symposium 2021

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Diffusion

Diffusive particle propagation

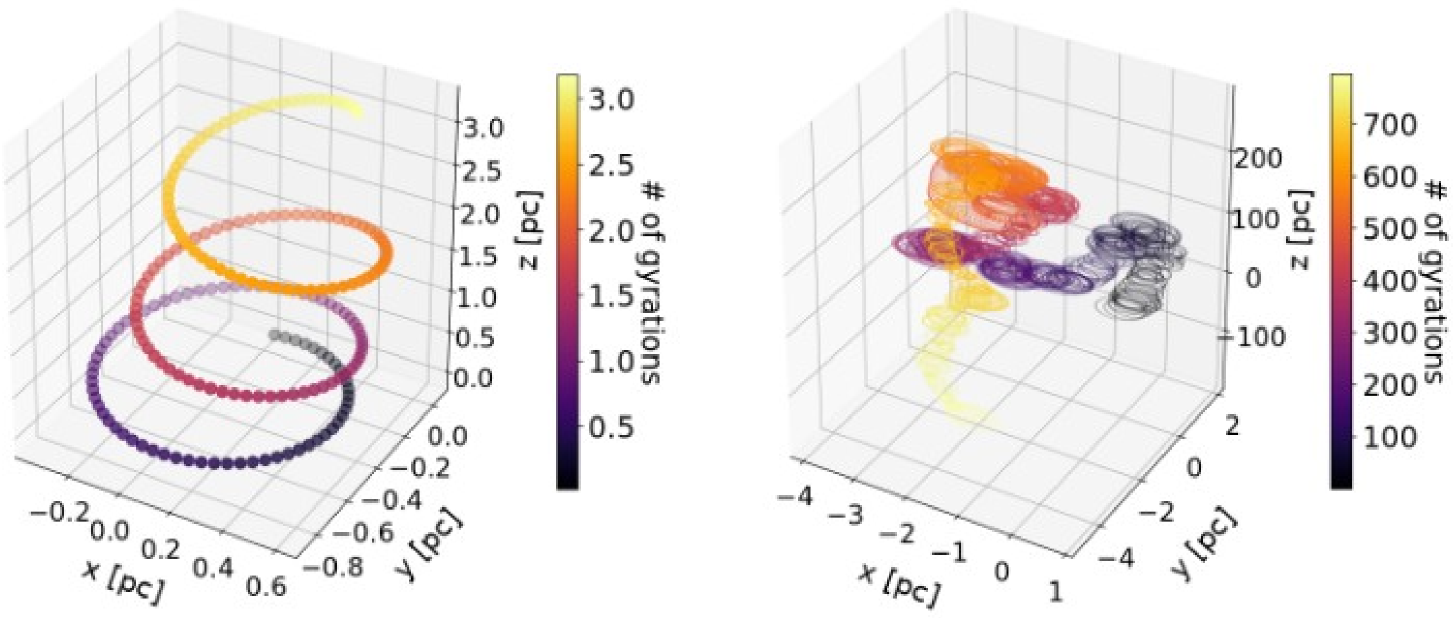


Fig. 1: Trajectory plots of the same particle propagating through a magnetic field with turbulent fluctuations. On the left the first 3 gyrations are shown, on the right the first 700 gyrations. Taken from „Closing in on the origin of Galactic cosmic rays using multimessenger information“, J.Tjus, and L.Merten, 2020.

Diffusive particle propagation

Considering particles in a regular field B, perturbed by a turbulent component b

Mean square displacement

$$\left\langle (\Delta x)^2 \right\rangle = \left\langle (x(t) - x(0))^2 \right\rangle$$

(Running) diffusion coefficient

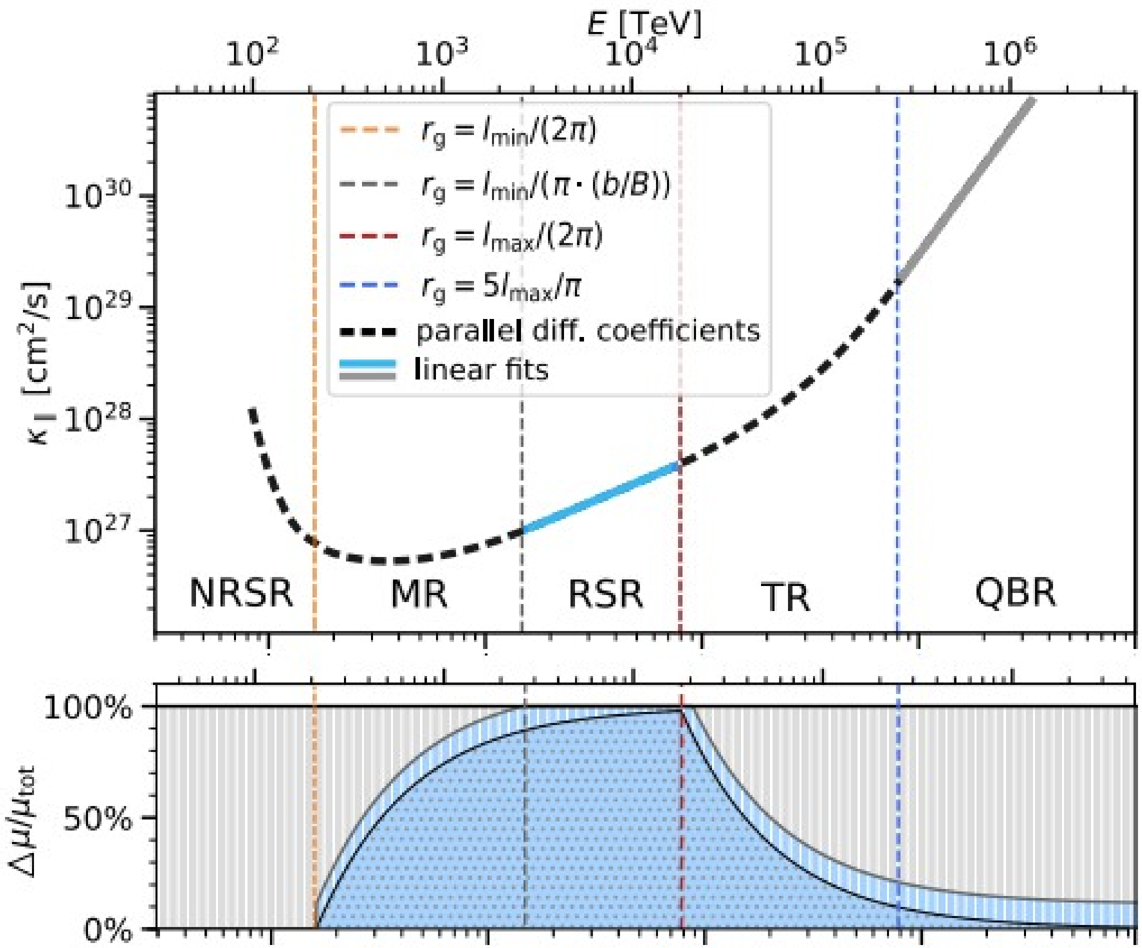
$$\kappa_{xx}(t) = \frac{\left\langle (\Delta x)^2 \right\rangle}{2t}$$

$$\kappa_{xx} = \lim_{t \to \infty} \frac{\left\langle (\Delta x)^2 \right\rangle}{2t}$$

Diffusion tensor

$$(\kappa_{ij}) = \begin{pmatrix} \kappa_\perp & 0 & 0 \\ 0 & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix} \qquad \vec{B} = B\vec{e}_z$$

Propagation regimes in turbulent magnetic fields

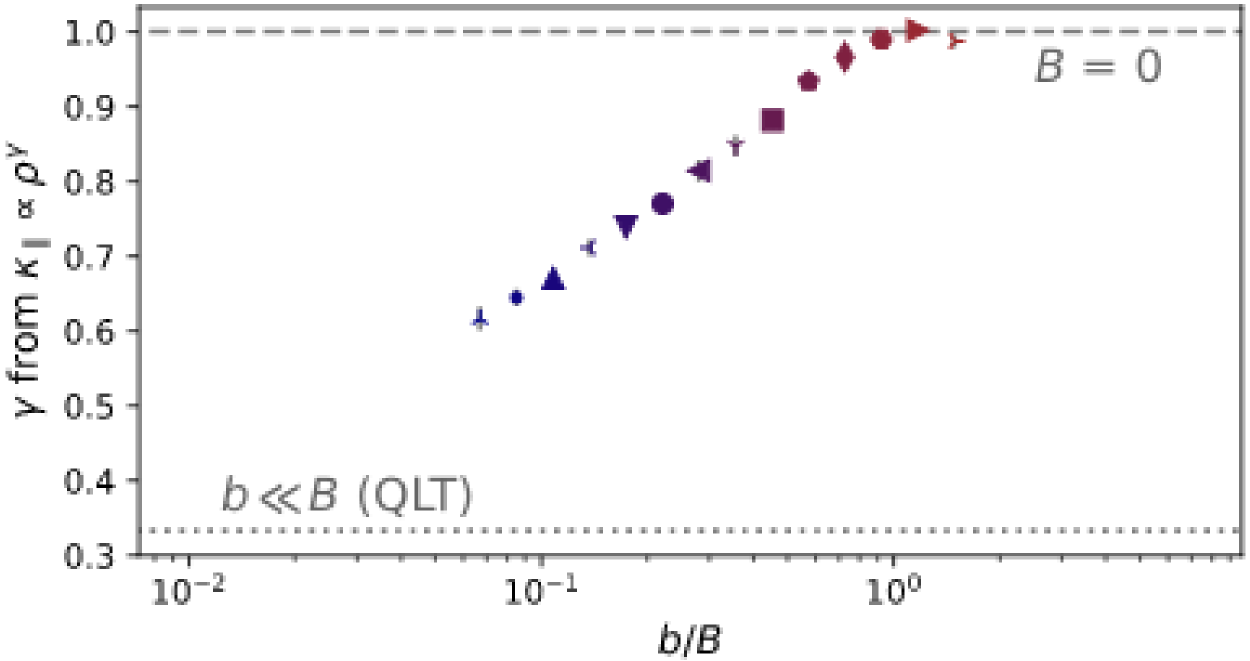


$$|\mu| = \frac{l}{2\pi r_g}$$

For a Kolmogorov spectrum the QLT (b<<B) predicts that κ par. scales in the RSR with **E^(1/3).**

Fig. 2: Overview of the different propagation regimes a particle of a certain rigidity can experience connected with the pitch angle, taken from „Turbulence-level dependence of cosmic-ray parallel diffusion“, P. Reichherzer et al 2020.

Turbulence dependent diffusion



In the work by Reichherzer et al. it was shown that the rigidity scaling of the diffusion coefficient is dependent on b/B.

Fig. 3: Spectral index of the parallel diffusion coefficient against the turbulence level, with the theoretical limits of 1/3 (QLT) and 1 (Bohm), taken from Reichherzer et al. 2020.

Method

Turbulence generation

Grid based method

Samplingtheorem: lmin >= 2d, lmax <= Nd
for a grid with N points and spacing d.

Interpolation becomes necessary.

Continuation of the volume becomes necessary.

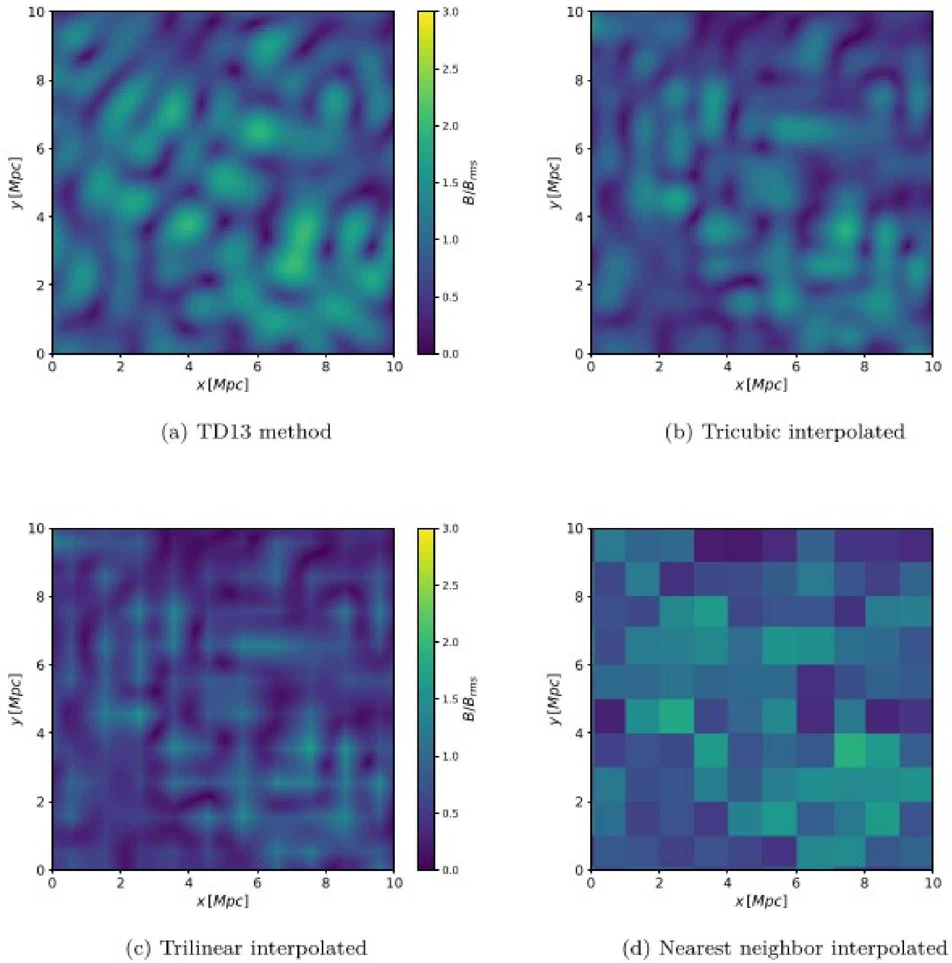
Gridless method

Turbulence is generated „on the fly“ in position space and therefore interpolation is not required, also field parameters can be chosen for a much broader range.

See:
J. Giacalone and J.R. Jokipii
„The Transport of Cosmic Rays across a turbulent magnetic field“, 1999.

RC Tautz and A. Dosch.
„On numerical turbulence generation for test-particle simulations“, 2013.

Motivation for a gridless method



(a) TD13 method

(b) Tricubic interpolated

(c) Trilinear interpolated

(d) Nearest neighbor interpolated

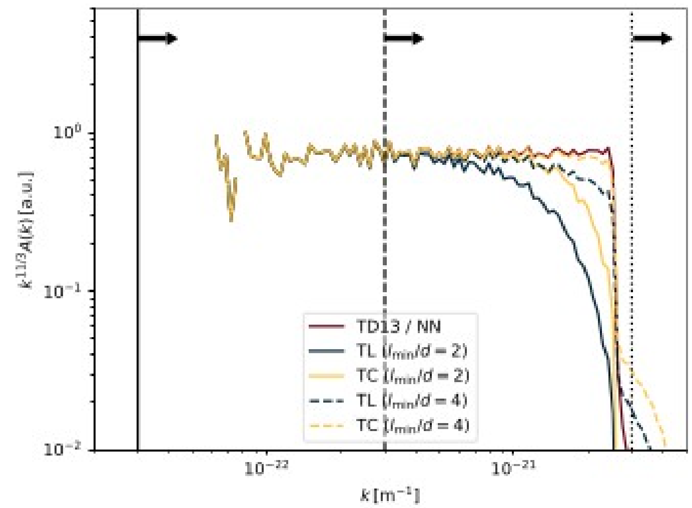


Fig. 4:
Left: Sliceplots of the same field (TD13 and Interpolated).
Right: Spectrum of the TD13-Field and spectra after Interpolation regarding different resolutions.

Both taken from „Interpolation of Turbulent Magnetic Fields and Its Consequences on Cosmic Ray Propagation“,L.Schlegel et al. 2020.

Simulation

Simulation setup

Using CRPropa 3.1.6 the following simulation setup was investigated:

- **Regular field B + turbulent field b (TD13 model, 250 wavemodes)**
- **Isotropic emission of protons with different energies E until a maximal trajectory length d_max(E,b/B) is reached**

$$d_{max} = n_{gyr} \cdot 2 \cdot \pi \cdot r_g = \frac{2 \cdot \pi \cdot n_{gyr} \cdot E}{|q| \cdot \sqrt{b^2 + B^2} \cdot c}$$

- **For turbulence levels b/B from 0 to 5 equidistantly spaced, energies corresponding to the RSR are calculated according to**

$$E_{\mathrm{high}} = \frac{l_{max} \cdot |q| \cdot B \cdot \sqrt{\left(\frac{b}{B}\right)^2 + 1} \cdot c}{2 \cdot \pi}, \qquad E_{\mathrm{low}} = \frac{l_{\mathrm{min}} \cdot |q| \cdot B \cdot \sqrt{\left(\frac{b}{B}\right)^2_{\mathrm{min}} + 1} \cdot c}{\left(\frac{b}{B}\right)_{\mathrm{min}} \cdot \pi}.$$

Analysis

Calculation of the running diffusion coefficient

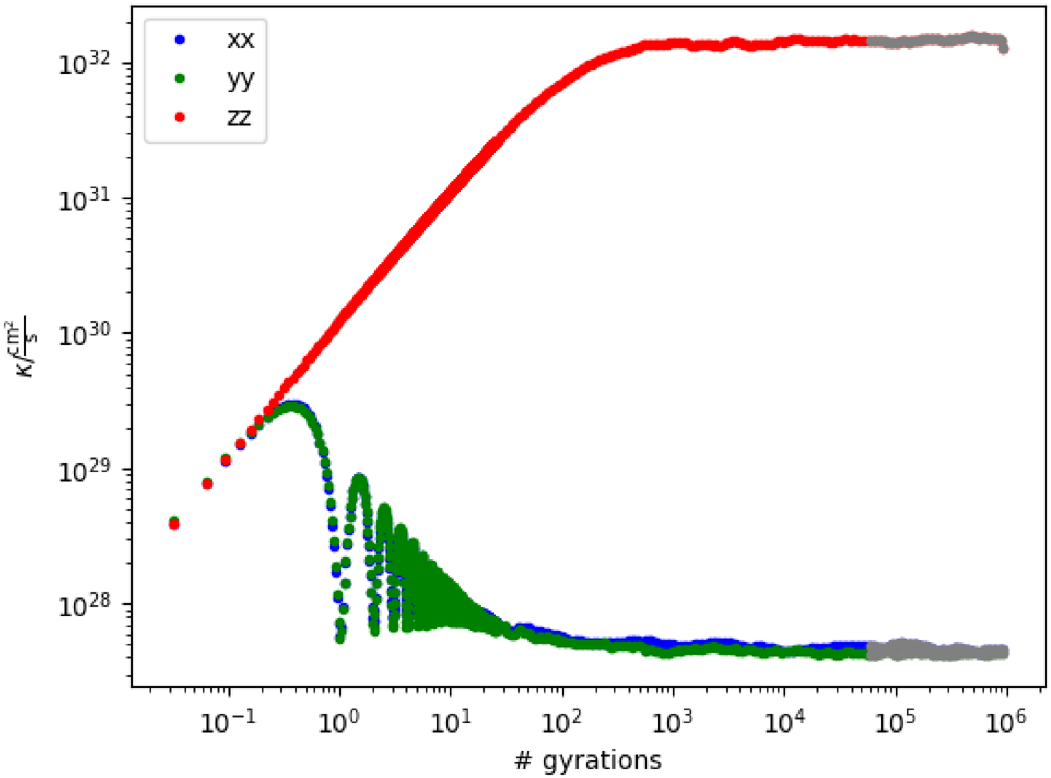


Fig. 5: Exemplarily plot of k against #gyr for E = 1.2e16 eV, b/B = 0.137.

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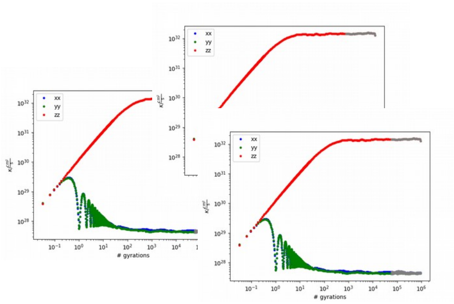
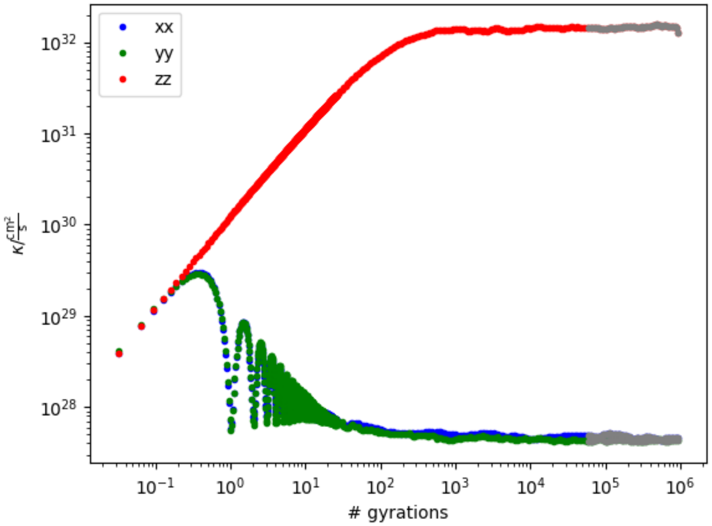
Datareduction and converged diffusion coefficient



Plot for every **E**,**b/B** combination and every Rnd-seed.



Arithmetic mean over the Rnd-seeds.

$\mathbb{K}$(**E**,**b/B**)

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Results

Energy dependence of the diffusion coefficient

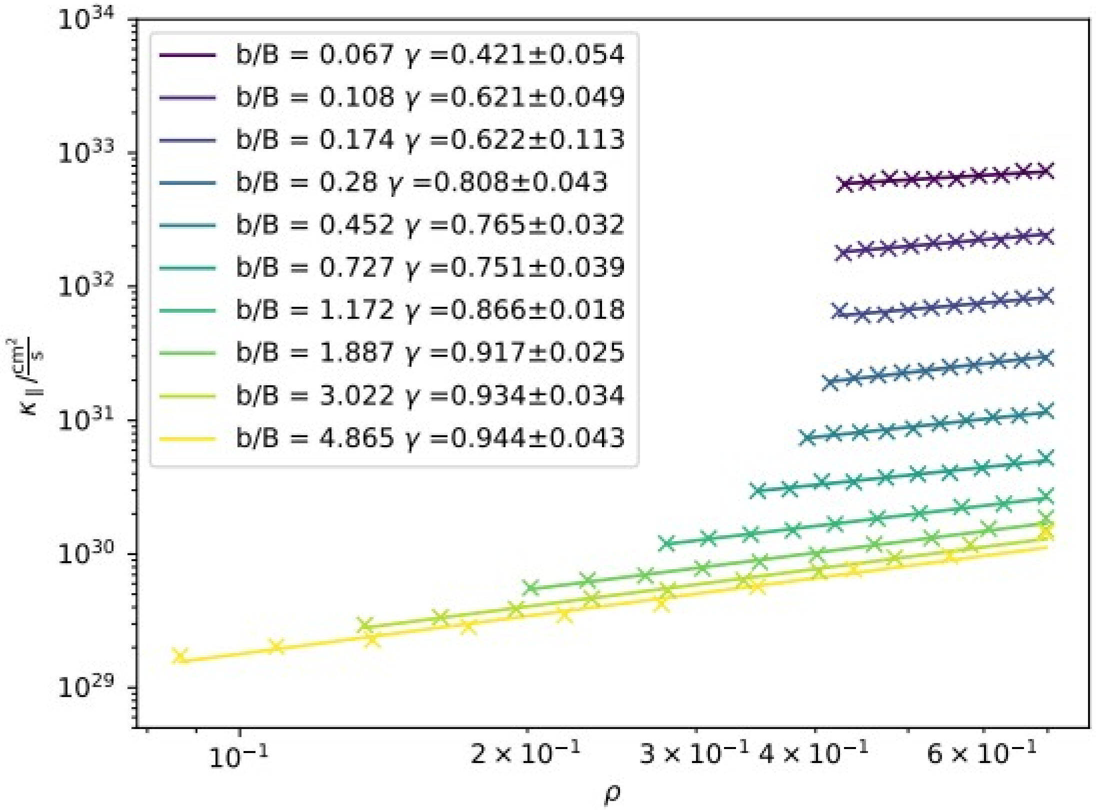

Fig. 7: Parallel diffusion coefficients plotted against the reduced rigidity for 10 different turbulent levels and corresponding linear fits.

Energy dependence of the diffusion coefficient

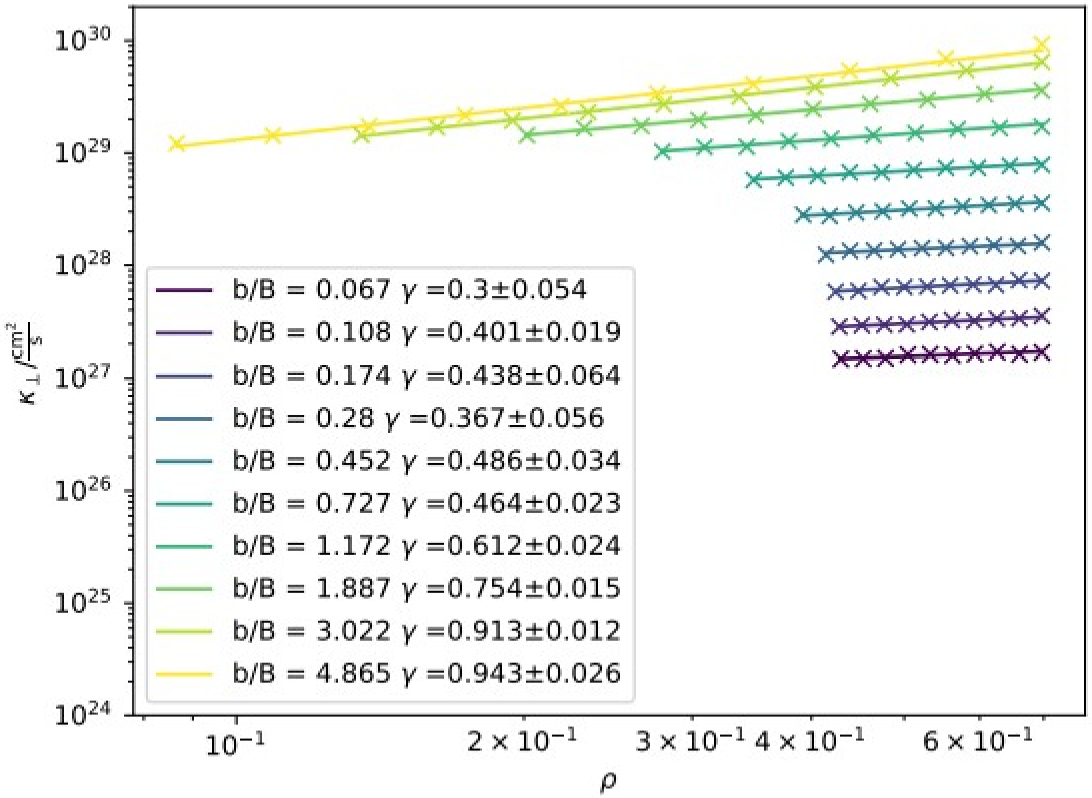

Fig. 8: Arithmetic mean of the perpendicular diffusion coefficients plotted against the reduced rigidity for 10 different turbulence levels and corresponding linear fits.

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Energy dependence of the diffusion coefficient

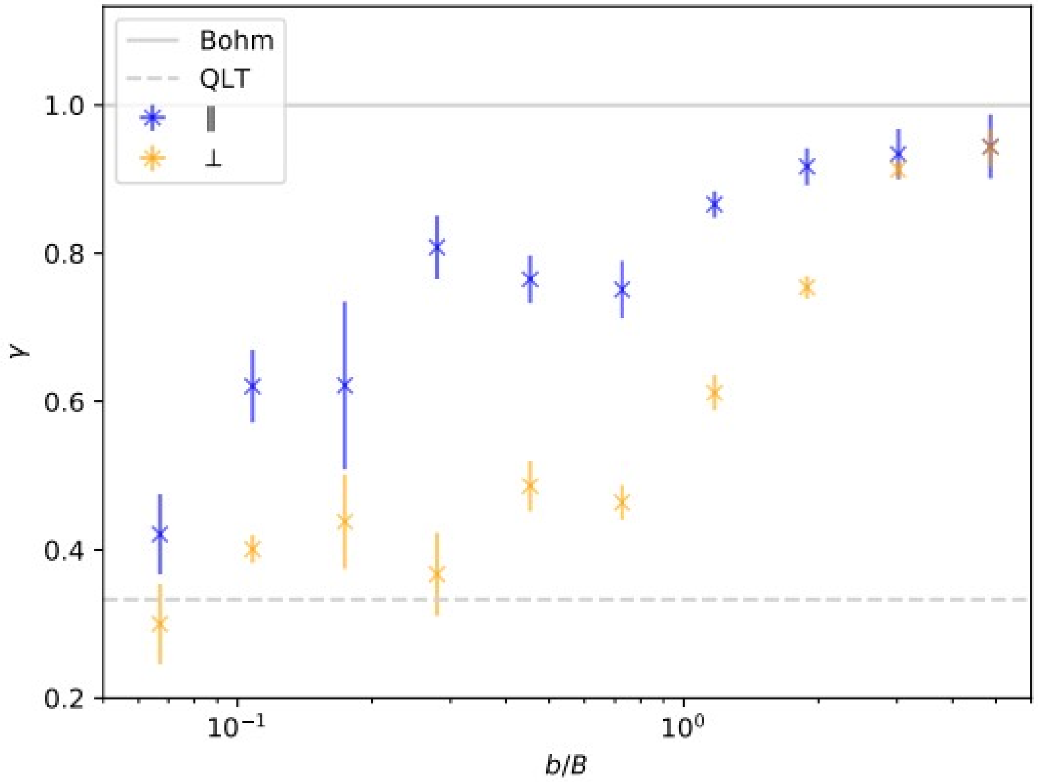

Fig. 9: Exponent γ plotted against the turbulence level for the parallel respective perpendicular diffusion coefficients.

Summary

- **QLT prediction of an energy scaling of the parallel diffusion coefficient in the RSR proportional to E^(1/3) is no longer valid for turbulence levels above 7 percent as numerically shown.**
- **Results previously obtained with a grid based approach could be reproduced with a gridless method, suitable for a further investigation of lower energies.**
- **Turbulence level dependence of diffusion could be relevant in the Galaxy assuming that the spectral index of the diffusion coefficients powerlaw changes with galactocentric radius (see talk by Patrick Reichherzer).**

References

- „Turbulence-level dependence of cosmic-ray parallel diffusion“, P. Reichherzer, J. Becker Tjus, E.G. Zweibel, L. Merten, M.J. Pueschel, 2020
- „Calculation of the Diffusion Tensor for the Galactic Magnetic Field and its Implementation in CRPropa“, M.Sc. Thesis P. Reichherzer, 2018
- „Interpolation of Turbulent Magnetic Fields and Its Consequences on Cosmic Ray Propagation“, L. Schlegel, A. Frie, B. Eichmann, P. Reichherzer, J. Becker Tjus, 2020
- „Investigation of the Influence of Magnetic Mirroring on the Cosmic-Ray Diffusion Coefficient“, M.Sc. Thesis L. Schlegel, 2021

Backup: Convergence tests for the turbulence

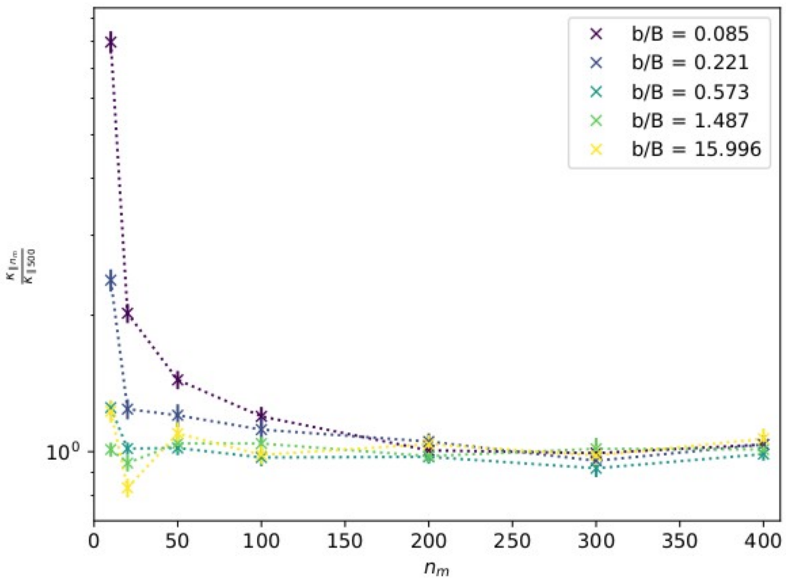

Fig. 10a: k(wm)/k(500) againts #wm for 1900 TeV.

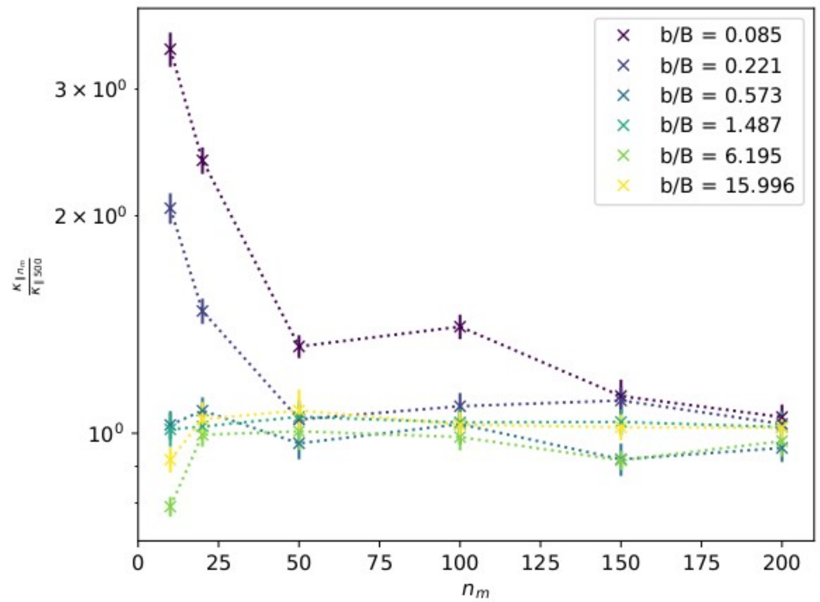

Fig. 10b: k(wm)/k(500) againts #wm for 5000 TeV.