Puzzles at the 3rd post-Minkowskian order

Gabriele Veneziano

based on recent work with:

P. Di Vecchia, C. Heissenberg, R. Russo

and earlier one w/ A. Gruzinov,

and w/ M. Ciafaloni, D. Colferai, F. Coradeschi

Introduction

- The recent detection of gravitational waves (GW) from coalescing binaries by LIGO/VIRGO has stimulated as well a lot of theoretical work on the subject.
- While the traditional methods for computing the expected waveforms (and interpret the signals):
  - Numerical Relativity (Pretorius, ...)
  - Post-Newtonian (PN) (Blanchet, ...)
  - Effective one body (EOB) (Buonanno-Damour, ...)

  are entirely classical, new approaches based on taking the classical limit of quantum-mechanical scattering amplitudes have been vigorously pursued.

- This brought together two theory communities:
  1. from Classical General Relativity;
  2. from High-Energy Particle Physics,

  generating a lot of synergy (e.g. GGI workshop 2021)
- Actually, the HE community has been interested in the gravitational 2-body problem since the late eighties ('t Hooft, Amati-Ciafaloni-GV*), Muzinich & Soldate,…) albeit w/ completely different motivations (see below)
- In that context high/transplanckian energy is crucial in order to make gravity relevant/dominant in the collision of two elementary particles (=> UR limit needed) and also to justify a semiclassical limit.

*) ACV hereafter

- What was completely missed at the time is that massive, astrophysical black holes can be thought of as elementary particles (no hair=> just mass and spin). If so those gedanken experiments become all but gedanken. However for BH's, the NR* (occasionally the R, but not the UR) regime is the most relevant one. Should we then forget about that earlier work? The answer is NO!
- In 1710.10599, Damour argued that useful input for the EOB can be obtained from the high-energy/UR regime of gravitational scattering and gave an example.
- Made sense, a posteriori: the Post-Minkowskian (i.e. loop) expansion could then be anchored to two limiting cases: the small-velocity PN expansion on the one hand, and the UR limit on the other.

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* not to be confused w/ Numerical Relativity

Outline

- The massless gravitational eikonal: a very quick reminder of ACV (1987-2007)
- The deflection puzzle (ACV90 vs BCRSSZ19)
  - The (sharpened) ACV90 argument
  - Solution via radiation reaction (=> CH's talk)
  - (A spin crisis? GV & G. Vilkovisky to appear)
- The radiation puzzle
  - ACV07's energy crisis at m=0
  - G&V-CC(C)V (partial) solution
  - The radiation puzzle @ large σ (=> RR's talk)
- (New challenges @ 4PM)

The massless gravitational eikonal: a quick reminder of ACV (1987-2007)

Motivations at the time were purely theoretical:

- The emergence of classical and quantum (string) gravity from gedanken experiments in flat spacetime (quite successful)

- The information paradox = checking unitarity of the resulting S-matrix even when the process is expected to lead to black-hole formation (not quite as successful)

Results in the weak gravity regime
($b \gg R = G\,E, l_s$)

- Restoring (elastic) unitarity via an eikonal resummation (trees violate p.w.u.)
- Gravitational deflection & time delay: emerging shock-wave metric at $O(G)$ (Cf. 'tHooft); extension up to $O(G^3)$ (ACV90, see later)
- t-channel "fractionation" and hard scattering (large Q) from large-distance (b) physics
- Tidal excitation of colliding strings, inelastic unitarity via an hermitian eikonal operator
- A first go at gravitational bremsstrahlung and an energy crisis (ACV07, see later)

Defining the elastic eikonal "phase" by

$$S(E,b) = \exp(2i\delta)\ ;\ \ \delta = \delta_0 + \delta_1 + \delta_2 + \ldots\ ;\ \ \delta_n = \mathcal{O}(G^{n+1})$$

the following results were found in ACV90 (D=4, GR, massless):

$$2\delta_0 = -\frac{Gs}{\hbar}\log\frac{b^2}{\lambda^2}$$

classical (IR div. Coulomb phase)

$$2Re\delta_1 = \frac{12G^2s}{\pi b^2}\log s\ ;\ \ Im\delta_1 = 0$$

quantum in massless case

$$2Re\delta_2 = \frac{4G^3s^2}{\hbar b^2}$$

deflection

$$Im\delta_2 \sim \frac{G^3s^2}{\hbar b^2}\log s \log\frac{b^2}{\lambda^2}$$

radiation

classical

The deflection puzzle

- In 1901.04424/1908.01493 an impressive calculation by BCRSSZ led to the first "complete" 3PM (i.e. 2-loop) result (in GR for two massive scalars).
- Checked to be consistent up to "5PN" (later also to "6PN") order but presented a puzzle.
- The high-energy (or just the massless) limit of the BCRSSZ result exhibited a logarithmic divergence in contrast with the perfectly finite ACV90 result.
- NB: gravity is supposedly free from m=0 (collinear) divergences. Quite some controversy -and a lot of confusion- was generated.

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BCRSSZ = Bern, Cheung, Roiban, Shen, Solon, Zeng

- Furthermore, both the massless limit of *ACV90* (in DNRVW-1911, BIP-MR-2002) and the BCRSSZ log-enhancement @ large s/m^2 (in P-MRZ-2005) were checked and claimed to be "universal" and yet they looked mutually incompatible.
- The prevailing attitude for a while was that the limits q << m (<< E) and m << q (<< E) are unrelated.
- But the solution of the puzzle turned out to be different...and more interesting.

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DNRVW = Di Vecchia, Naculich, Russo, GV, White

BIP-MR = Bern, Ita, Parra-Martinez, Ruf

P-MRZ = Parra-Martinez, Ruf, Zeng

Sharpening & solving the deflection puzzle

(DHRV 2008.12743)

Sharpening the ACV90 argument

- To put ACV90 on more rigorous and general grounds (e.g. @ $q < m$, extensions of GR) we used general properties of the scattering amplitude:
  - Real analyticity: $A^*(s^*,t) = A(s,t)$
  - Asymptotics in order to write subtracted fixed-t dispersion relations.
  - Xing symmetry: $A(s,t) = A(u,t)$, $u=-s-t+2m_1^2+2m_2^2$
  - Perturbative Unitarity
- From those we get informations on the high-energy limit of $\rho = \mathrm{Re}A(s,t)/\mathrm{Im}A(s,t)$ in analogy with what is done in high-energy soft hadronic physics.
- Popular in '70s, now again w/ LHC (soft) data.

For an asymptotic behavior $Im\ A \sim s^n \log^p s$, those model-independent constraints allow to express the asymptotic $Re\ \delta_2$ in terms of $Im\ \delta_2$, δ_0, and δ_1:

$$2Re\delta_2 = \frac{\pi p}{2\log s}(2Im\delta_2) - (4-3p)\frac{\delta_0}{s}(2\nabla\delta_0)^2 - (p-1)(2\delta_0)(2Im\delta_1)$$

non-universal quantum piece

For p generic we are left with a non-universal piece $\sim Im\delta_1$. Also, since neither $Im\delta_1$ nor δ_0 have a $\log s$, $Re\ \delta_2$ would not have a finite UR limit for $p > 1$.

And, indeed, $p=2$ is the BCRSSZ value...

$$2Re\delta_2 = \frac{\pi p}{2\log s}(2Im\delta_2) - (4-3p)\frac{\delta_0}{s}(2\nabla\delta_0)^2 - (p-1)(2\delta_0)(2Im\delta_1)$$

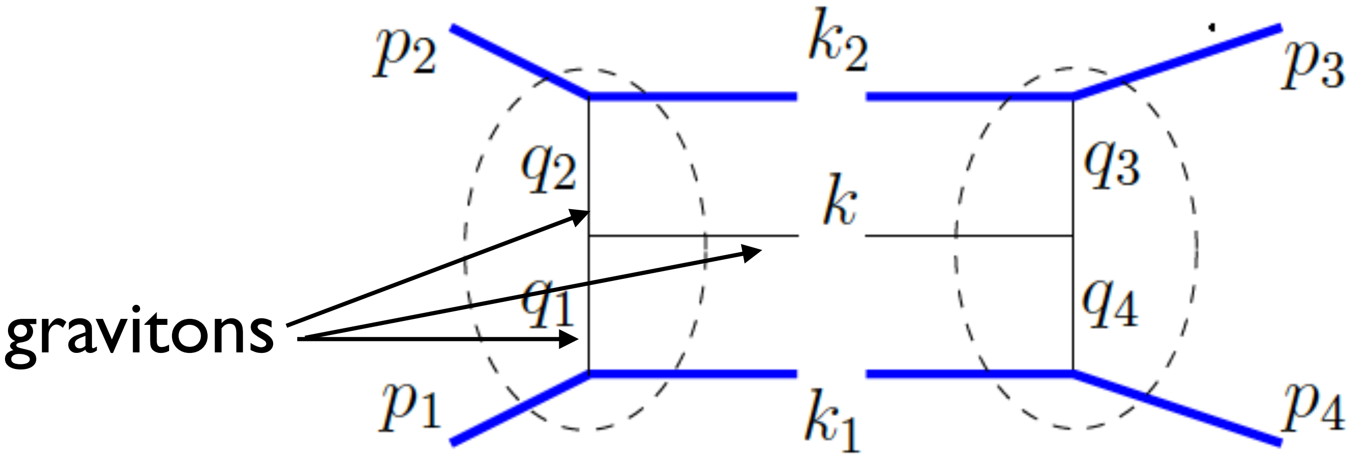
On the contrary, iff p=1, $Re\delta_2$ does not depend on the non-universal $Im\delta_1$ and approaches a finite UR limit. The above relation simplifies to:

$$2Re\delta_2 = \frac{\pi}{2\log s}(2Im\delta_2) - \frac{\delta_0}{s}(2\nabla\delta_0)^2$$

The logarithmically growing term in $Im\delta_2$ has an IR divergence which, however, cancels against the δ_0 term. This yields a finite result for $Re\delta_2$ whose large E/m limit agrees with the m=0 ACV90 result.

Shows the crucial role played by $Im\delta_2$ which, by unitarity, is directly related to the inelastic (3-particle) cut of the two-loop amplitude.

- Computed in ACV90 for m=0. We have redone the calculation for a generic q^2/m^2 ratio.
- Its kinematics is shown below. Called H diagram in both ACV and BCRSSZ, but not the same!
- In ACV the full 2->3 amplitude is squared.



It is less IR singular than the "H" diagram of BCRSSZ. We only get a single log s enhancement (p=1!) from k-rapidity integration.

- We then computed the full amplitude in N=8 SUGRA (massive ext. states introduced via KK) at arbitrary energy.
- We used IBP+DE, including contributions from the full soft (potential plus radiation) region.
- We found that, at high energy, the sum of planar and non-planar ladders gives a (subleading) contribution of the same order as the one coming from the H topology.
- The $\log^2 s$ terms in $\mathrm{Im}\, A_2$ cancel in the sum; same for the $\log s$ term in $\mathrm{Re}\, A_2$ (as implied by the A&X argument!)
- For the complete story see CH's talk!

3PM eikonal in N=8 SUGRA

$$\mathrm{Re}(\delta_2) = \frac{2G^3(2m_1m_2\sigma)^2}{\hbar b^2}$$

$$\left[\frac{\sigma^4}{\left(\sigma^2-1\right)^2} - \cosh^{-1}(\sigma)\left(\frac{\sigma^2}{\sigma^2-1} - \frac{\sigma^3\left(\sigma^2-2\right)}{\left(\sigma^2-1\right)^{5/2}}\right)\right]$$

P-MRZ/BCRSSZ

ACV-limit

New

cancel @ large σ

$$2m_1m_2\sigma = s - m_1^2 - m_2^2$$

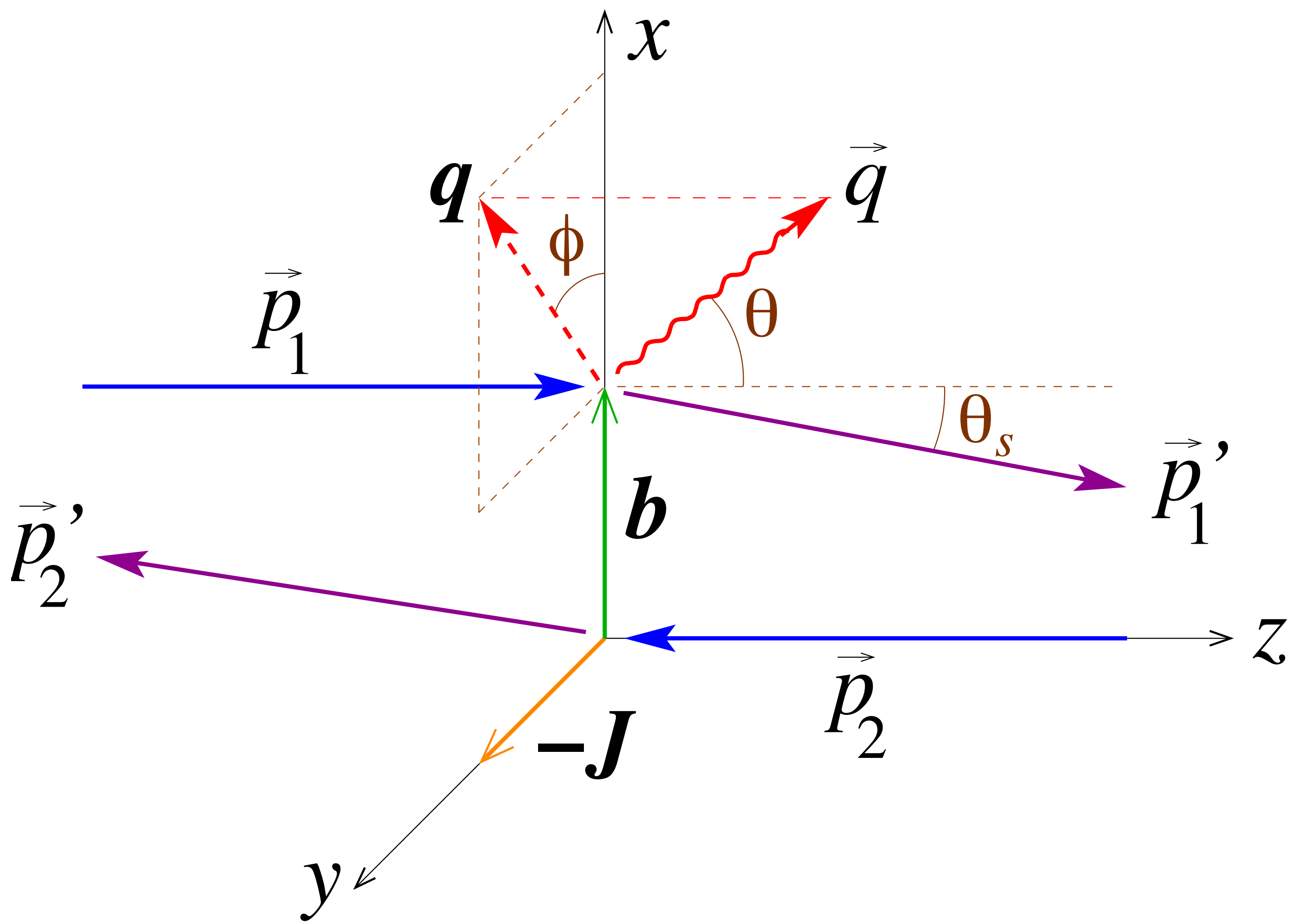
$$\cosh^{-1}(\sigma) \sim \log\sigma \text{ as } \sigma \to \infty$$

NB: new and old terms behave very differently in the NR limit, $\sigma \to 1$ ($s \to (m_1 + m_2)^2$), but cancel in UR

The radiation puzzle:

I. Gravitational bremsstrahlung from massless particle collisions

The process at hand



An old "energy crisis"

(ACV 0712.1209, J.Wosiek & GV 0805.2973)

Graviton spectum @ $\frac{Gs}{\hbar}\frac{R^2}{b^2} \sim \langle n_{gr} \rangle \gg 1$

$$R \equiv 2G\sqrt{s}\ ,\ \theta_s \sim \frac{2R}{b}$$

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp\left(-|k||b| - \omega\frac{R^3}{b^2}\right)\ ;\ \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim 1$$

even @ small θ_s => E-crisis. Instead, one would have hoped for a stronger cutoff in frequency, e.g.

$$\frac{dE_{gr}}{d^2k\ d\omega} = Gs\ R^2\ exp(-|k||b| - \omega R) \Rightarrow \frac{E_{gr}}{\sqrt{s}} \sim \frac{R^2}{b^2}$$

..and its (partial) resolution

Three methods

1. A classical GR approach
   (A. Gruzinov & GV, 1409.4555)
2. An amplitude-based approach
   (CC&Coradeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)
3. A ~~soft-theorem approach (Laddha & Sen, 1804.09193; Sahoo & Sen 1808.03288, 2105.08739; Addazi, Bianchi & GV, 1901.10986)~~

Comment: #2 goes over to #1 in the classical limit where they agree with #3 in overlap of their respective domains of validity (see, S&S 2105.08739 & R. Russo's talk this afternoon)

Domains of validity

- The classical GR and amplitude-based approaches cover a wide range of GW frequencies but were limited, so far, to small-angle scattering/radiation.
- The soft-theorems-based approach is not limited to small deflection/radiation angles but is only valid in a smaller frequency region (and thus cannot address the energy-crisis issue)

A classical GR approach

(A. Gruzinov & GV, 1409.4555)

Based on Huygens superposition principle in Fraunhofer's approximation (needs $\theta \ll 1$)

For gravity this includes in an essential way the gravitational time delay in AS's shock-wave metric.

A quantum-amplitudes approach (CCCV, 1512.00281, CCV,1812.08137)

Emission from external and internal legs throughout the whole ladder (with its suitable phase) has to be taken into account for not-so-soft gravitons.

One should also take into account the (finite) difference between the (infinite) Coulomb phase of the final 3-particle state and that of an elastic 2-particle state.

When this is done (so far again for $\theta \ll 1$), the GR result of G&V is recovered for $h\omega/E \to 0$!

The classical limit (NB: a resummation in G!)

Frequency + angular spectrum ($s = 4E^2$, $R = 4GE$)

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2\ ;\ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s\ ;\ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega \mathbf{x}\cdot\tilde{\boldsymbol{\theta}}}\left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad\qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

Re ζ^2 and Im ζ^2 correspond to the usual (+, x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones.

Analytic results: a Hawking knee & an unexpected bump

For $b^{-1} < \omega < R^{-1}$ the GW-spectrum is almost flat in ω

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi}\theta_s^2 E^2 \log(\omega R)^{-2}$$

Below $\omega = b^{-1}$ it "freezes", giving the expected zero-frequency limit (ZFL).

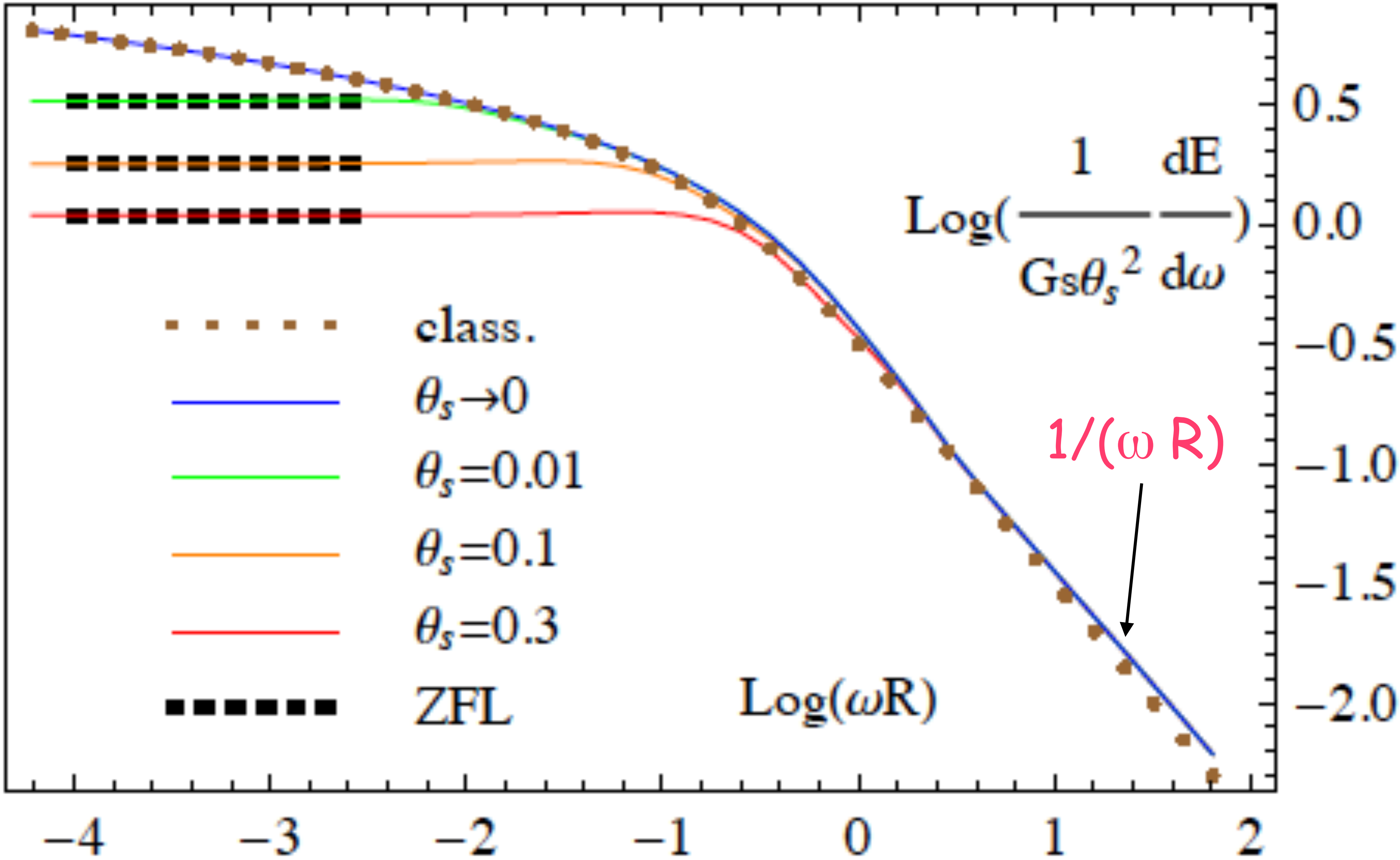
$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi}\ \theta_s^2 E^2\ \log(\theta_s^{-2})$$

Above $\omega = R^{-1}$ drops, becomes "scale-invariant"

Hawking knee!

$$\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}$$

(CCCV 1512.00281)



The "scale-invariant" spectrum gives a log ω^* sensitivity in the total radiated energy for a cutoff at $\omega = \omega^*$

Using, with some motivations (e.g. Dyson's bound on dE/dt), $\omega^* \sim b^2 R^{-3}$ (Cf. cutoff in ACV and VW) we find, to logarithmic accuracy,

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\,\theta_s^2\,\log(\theta_s^{-2})$$

The E-crisis is thus only partly solved: we need to go beyond some approximations made in G&V or CCCV, find the real value of ω^*, and extend the method to arbitrary θ.

Thank you now?

The fine spectrum at $\omega < 1/b$

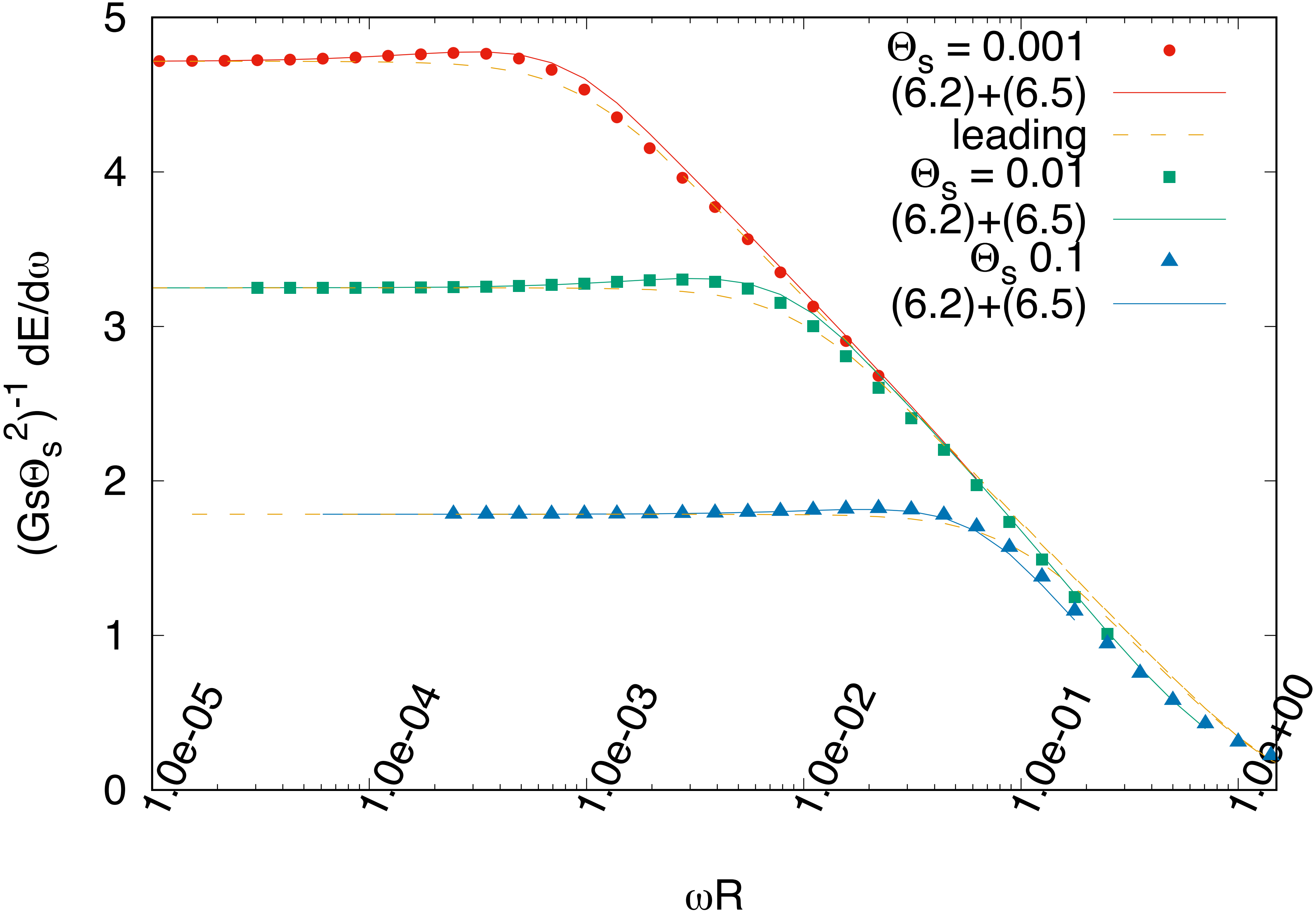
- At $\omega b < (\ll)\ 1$ there are corrections of order $(\omega b)\log(\omega b)$, $(\omega b)^2\log^2(\omega b)$.

- The ωb (both w/ and w/out $\log(\omega b)$) correction disappears after summing over polarizations, or after integration over the azimuthal angle.

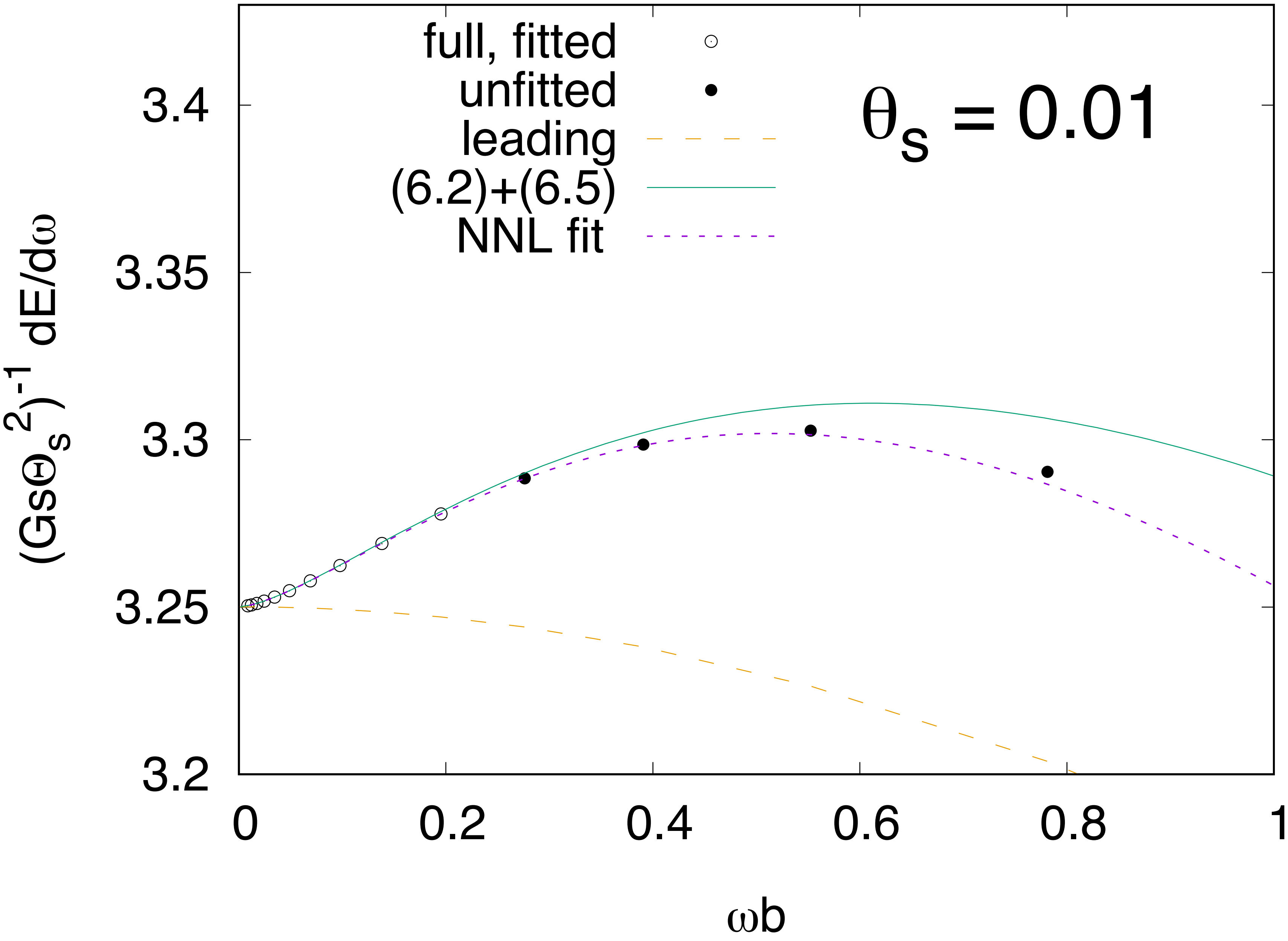
- The leading $(\omega b)^2\log^2(\omega b)$ correction to the total flux is positive and produces a (modest) bump at $\omega b \sim 0.5$.

- Anticipated/confirmed by soft-theorem techniques (Sen et al. Addazi-Bianchi-GV,...)

CCV 1812.08137



CCV 1812.08137



Can it become a healthy Hawking peak when b->R?

What about the massive UR case?

(to be discussed in RR's talk)

Waveforms and energy loss

- Many groups are still working on this. Several interesting results already available for the waveforms (see RR's talk).
- However the straight $O(G^3)$ result for the total E^{rad} (HP-MRZ, 2101.07255) leads (in the UR limit) to an "energy crisis" similar to the one found in ACV07 and VW08 (see also P. D'Eath's and Kovacs & Thorne's warnings on limits of validity).
- Furthermore, the UR limit is not Universal!

HP-MRZ, 2101.07255 (confirmed in DHRV, 2104.03256)

$$E^{rad} = \frac{\pi G^3 m_1^2 m_2^2 (m_1 + m_2)}{b^3 \sqrt{s}} \left[f_1(\sigma) + f_2(\sigma) \log \frac{\sigma + 1}{2} + f_3(\sigma) \frac{\sigma \cosh^{-1} \sigma}{2\sqrt{\sigma^2 - 1}} \right]$$

where in $\mathcal{N} = 8$

$$f_1 = \frac{8\sigma^6}{(\sigma^2-1)^{\frac{3}{2}}}, \qquad f_2 = -\frac{8\sigma^4}{\sqrt{\sigma^2-1}}, \qquad f_3 = \frac{16\sigma^4(\sigma^2-2)}{(\sigma^2-1)^{\frac{3}{2}}},$$

while in GR

$$f_1 = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2-1)^{\frac{3}{2}}},$$

$$f_2 = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2-1}},$$

$$f_3 = \frac{(2\sigma^2-3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2-1)^{\frac{3}{2}}},$$

$$\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \sqrt{\frac{\sigma}{\nu}} \;;\; \text{for } \sigma \to \infty$$

Another "energy crisis"?

For m=0 we saw that a resummation in G was necessary.

See RR's talk for the rest of the story

New challenges @ 4PM

- New challenges appear at 4PM (= 3 loop) order
- A partial result ("conservative part") has been obtained by Bern et al in 2101.07254.
- Unfortunately, it exhibits the same shortcomings as the 3PM conservative result, only worsened.
- Not only the UR (or zero mass) limit is even more singular than at 3 PM. Even at finite σ the result is IR divergent (coeff. related to E^{rad} !)
- Therefore at 4 PM adding RR (called the "tail contribution" in GR) is absolutely essential for recovering a finite result at any σ!
- Hard problem, now under study by several groups, but with partial PN-expanded results already available (e.g. Bini-Damour-Geralico, 2107.08896).

Beyond 3PM the deflection & radiation puzzles may well get fully entangled!

Thank you!