Drell-Yan process at intermediate energies with High-Energy Factorization and new unintegrated PDFs: pT spectra and angular distributions

M. A. Nefedov[1], V. A. Saleev[1,2]

3 June 2021,
GDR-QCD/QCD@short distances and
STRONG/2020/PARTONS/FTE@LHC/NLOAccess
joint workshop,
IJClab, Orsay

[1]Samara National Research University, Samara, Russia
[2]Bogolyubov Laboratory for Theoretical Physics, JINR, Dubna, Russia

The talk is based on:

- M.N., N. Nikolaev, V. Saleev, Phys.Rev. D**87** (2013) 1, 014022
- M.N., V. Saleev, Phys.Lett. B**790** (2019) 551; PoS DIS2019 (2019) 193; J.Phys.Conf.Ser. **1435** (2020) no.1, 012024].
- M.N., V. Saleev, Phys.Rev. D**102** (2020) 114018

Outline:

- *Parton Reggeization Approach* to restore QED gauge-invariance of hadronic tensor at $q_T \neq 0$
- New unintegrated PDF
- Results for pp and $p\bar{p}$ collisions
- Angular coefficients for pp-collisions with $\sqrt{S} = 110$ GeV

Introduction

Cross-section for the un-polarized Drell-Yan process ($S = (P_1 + P_2)^2$, $Q^2 = q^2 = (k_1 + k_2)^2$):

$$p(P_1) + p(P_2) \to \gamma^\star(q) + X \to l^+(k_1) + l^-(k_2) + X,$$

can be decomposed over *helicity structure functions* (HSFs) $F_{UU}^{(1,\dots)}$ as follows ($x_{A,B} = Qe^{\pm Y}/\sqrt{S}$):

$$\begin{aligned}\frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} &= \frac{\alpha^2}{4Q^2}\Big[F_{UU}^{(1)} \cdot \left(1 + \cos^2\theta\right) + F_{UU}^{(2)} \cdot \left(1 - \cos^2\theta\right) + \\ &+ F_{UU}^{(\cos\phi)} \cdot \sin(2\theta)\cos\phi + F_{UU}^{(\cos 2\phi)} \cdot \sin^2\theta\cos(2\phi)\Big],\end{aligned}$$

angular coefficients:

$$A_0 = \frac{F_{UU}^{(2)}}{F_{UU}^{(1)} + F_{UU}^{(2)}/2},\ A_1 = \frac{F_{UU}^{(\cos\phi)}}{F_{UU}^{(1)} + F_{UU}^{(2)}/2},\ A_2 = \frac{2F_{UU}^{(\cos 2\phi)}}{F_{UU}^{(1)} + F_{UU}^{(2)}/2}$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0},\ \mu = \frac{2A_1}{2 + A_0},\ \nu = \frac{2A_2}{2 + A_0}.$$

TMD-factorization

The TMD-factorization for structure functions:

$$
\begin{aligned}
F_{UU}^{(1,2,\dots)}(x_A, x_B, \mathbf{q}_T) &= \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1} + \mathbf{q}_{T2} - \mathbf{q}_T) \times \\
&\times \mathcal{F}_q(x_A, \mathbf{q}_{T1}) \mathcal{F}_{\bar{q}}(x_B, \mathbf{q}_{T2}) \times f_{q\bar{q}}^{(1,2,\dots)}(\mathbf{q}_{T1}, \mathbf{q}_{T2}) \\
&+ Y_{UU}^{(1,2,\dots)}
\end{aligned}
$$

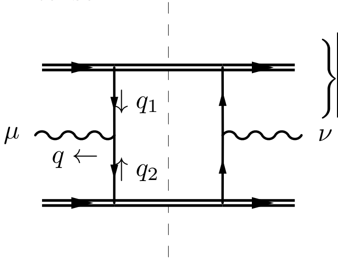
- Factorization for "TMD-term" is proven at leading power in q_T/Q
- "Y-term" is responsible for large q_T-behavior
- Typically "Y-term" is computed in *Collinear Parton Model* (CPM)
- *Does such a prescription correctly include all $O(q_T/Q)$ power corrections missing in "TMD-term"?*
- Possible problem is related with (lack of commonly accepted) **QED** gauge-invariant definition of "TMD-term" at $q_T \neq 0$.

TMD Parton-model

Leptonic ($L_{\mu\nu}$) and hadronic ($W_{\mu\nu}$) tensors:

$$d\sigma \sim L^{\mu\nu} W_{\mu\nu},$$

Parton model for hadronic tensor:



Decomposition for quark correlatior (un-polarized protons, $P_1^\mu = P_1^+ n_-^\mu/2$):

$$\Phi_{\bar{q}-}^{\alpha\beta} = \frac{q_1^+}{2} \left(\hat{n}_-^{\alpha\beta} f_1^{(q)} + \frac{q_{T1}^i \epsilon_{ij}}{\Lambda} \left(i\sigma^{-j}\gamma_5 \right)^{\alpha\beta} h_1^{(\perp q)} \right)$$

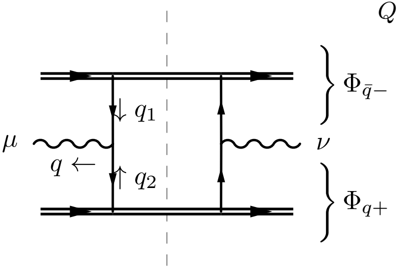
where $f_1^{(q)}(x, \mathbf{q}_T)$ – TMD quark number density, $h_1^{(\perp q)}(x, \mathbf{q}_T)$ – Boer-Mulders function [D. Boer, P. Mulders, 1998].

Idea: decompose Dirac structure of quark correlator in the proton rest frame:

$$\Phi_{\alpha\beta} = f_1^\mu \gamma^\mu_{\alpha\beta} + f_2^\mu (\gamma^\mu \gamma^5)_{\alpha\beta} + f_3^{\mu\nu} \left(i\sigma^{\mu\nu}\gamma^5 \right)_{\alpha\beta} + f_4 \delta_{\alpha\beta} + f_5 \left(i\gamma^5 \right)_{\alpha\beta},$$

then apply boost: $f_1^\mu \sim q_1^+ n_-^\mu$, $f_3^{\mu\nu} \sim q_1^+ n_-^\mu k_{T1}^\nu$, where $k_{T1}^i = \epsilon^{ij}(q_{T1}^j/\Lambda)$ – $\perp$ pseudo-vector, f_2, f_4, f_5 – drop-out in un-polarized case.

TMD Parton-model



$$
\begin{aligned}
\frac{W_{\mu\nu}}{Q^2} &= \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1}+\mathbf{q}_{T2}-\mathbf{q}_T) \\
&\times \frac{1}{Q^2}\,\mathrm{tr}\left[\Phi_{q+}\gamma_\mu \Phi_{\bar{q}-}\gamma_\nu\right] + O(|\mathbf{q}_T|/Q) \\
&= \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}\ \delta(\mathbf{q}_{T1}+\mathbf{q}_{T2}-\mathbf{q}_T) f_1^{(q)} f_1^{(q)} \\
&\times \underbrace{\frac{1}{4N_c Q^2}\,\mathrm{tr}\left[\left(\frac{q_2^-}{2}\hat{n}_+\right)\gamma_\mu \left(\frac{q_1^+}{2}\hat{n}_-\right)\gamma_\nu\right]}_{w_{\mu\nu}} \\
&+ (\mathrm{Boer-Mulders}) + O(|\mathbf{q}_T|/Q)
\end{aligned}
$$

$$F_{UU}^{(1)} = f_1^{(q)}(x_A,\mathbf{q}_{T1}) \otimes f_1^{(\bar{q})}(x_B,\mathbf{q}_{T2}),\ \ F_{UU}^{(2,\cos\phi)} \sim O(q_T^2/Q^2)$$

$$F_{UU}^{(\cos 2\phi)} = h_1^{(\perp q)}(x_A,\mathbf{q}_{T1}) \otimes h_1^{(\perp \bar{q})}(x_B,\mathbf{q}_{T2}) \otimes \frac{2(\mathbf{q}_T\mathbf{q}_{T1})(\mathbf{q}_T\mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1}\mathbf{q}_{T2})}{\mathbf{q}_T^2\Lambda^2}$$

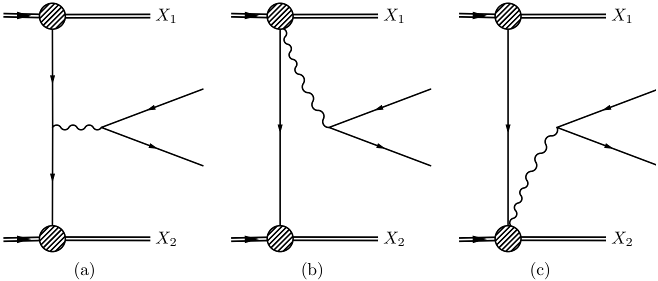
Problem: partonic tensor doesn't satisfy Ward identity at $q_T \neq 0$:

$$\boxed{q^\mu w_{\mu\nu} = O(q_T/Q),}$$

formally, GI is restored by $O(|\mathbf{q}_T|/Q)$-corrections to $w_{\mu\nu}$.

Full amplitude

In the full theory, not only t-channel ("Parton model") diagram but also diagrams with direct interaction of the photon with the proton and it's remnants are needed to restore gauge-invariance:



Is the contribution of non-Parton-model diagrams completely out of control or it can be factorized in some limit?
Let's consider the High-Energy limit:

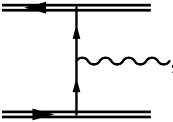
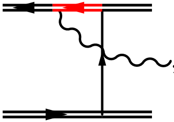
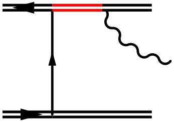
$$\boxed{S \gg Q^2, q_T^2}$$

Spectator model

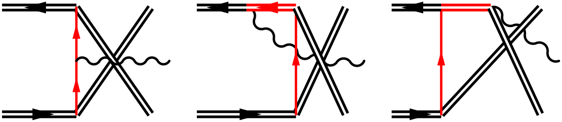
Let's consider the question of factorization in a concrete field-theoretic model, which includes (massless) proton fields, quarks, gluons and *spectator* fields of mass M_s. Protons, quarks and spectators carry $U(1)$ charge.
Let's consider the process:

$$\bar{p}(P_1) + p(P_2) \to \gamma^\star(q) + s(P_1') + s(P_2').$$

Most interesting diagrams in High-Energy limit (+ 2 similar diags.):

$\mathcal{D}_1 =$ , $\mathcal{D}_2 =$ , $\mathcal{D}_3 =$ 

Crossed-diagrams are doubly-suppressed:



Fadin-Sherman vertex

In the leading power in $\sqrt{S} = P_1^+ = P_2^-$, diags. 2 and 3 give:

$$\mathcal{D}_2^\mu \propto e_p \bar{v}(P_1)\gamma^\mu \frac{\hat{P}_1 - \hat{q}}{(P_1 - q)^2} \simeq e_p \bar{v}(P_1) \frac{P_1^+ \gamma^\mu \hat{n}_-}{2(-P_1^+ q^-)} = \bar{v}(P_1)\frac{i\hat{q}_1}{q_1^2}\left[ie_p \frac{\hat{q}_1 n_-^\mu}{q_-}\right],$$

$$\mathcal{D}_3^\mu \propto e_s \frac{(2P_1 + 2q_2 - q)^\mu}{(P_1 + q_2)^2}\bar{v}(P_1) \simeq \frac{P_1^+ n_-^\mu}{P_1^+ q^-}\bar{v}(P_1) = \bar{v}(P_1)\frac{i\hat{q}_1}{q_1^2}\left[-ie_s \frac{\hat{q}_1 n_-^\mu}{q_-}\right].$$

Collecting the contributions of all diagrams one obtains

$$\mathcal{M}_\mu \simeq (-\lambda_{spq}^2)\bar{v}(P_1)\frac{i\hat{q}_1}{q_1^2}\left(-i\Gamma_\mu(q_1, q_2)\right)\frac{-i\hat{q}_2}{q_2^2}u(P_2),$$

where **Fadin-Sherman vertex** [Fadin, Sherman, 1976]:

$$\Gamma_\mu(q_1, q_2) = e_q \gamma_\mu - (e_p - e_s)\hat{q}_1 \frac{n_\mu^-}{q_-} - (e_p - e_s)\hat{q}_2 \frac{n_\mu^+}{q_+},$$

depends only on e_q, since $\boxed{e_p - e_s = e_q}$ and it satisfies Ward identity: $\boxed{q^\mu \Gamma_\mu(q_1, q_2) = 0}$.

Helicity structure functions in PRA

The partonic tensor for **number-density** contribution in above-proposed *Parton Reggeization Approach* (PRA) reads:

$$w_{\mu\nu}^{\rm PRA} = \frac{1}{4}{\rm tr}\left[\left(\frac{q_2^-}{2}\hat{n}^+\right)\Gamma_\mu(q_1,q_2)\left(\frac{q_1^+}{2}\hat{n}^-\right)\Gamma_\nu(q_1,q_2)\right],$$

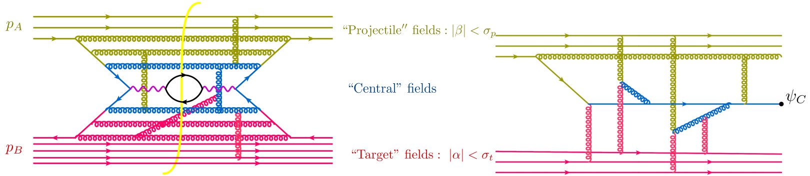
and it leads to the following partonic HSFs:

$$f_{\rm PRA}^{(1)} = 1 + \frac{q_T^2}{2Q^2},\ f_{\rm PRA}^{(2)} = \frac{(\mathbf{q}_{T1} - \mathbf{q}_{T2})^2}{Q^2},$$

$$f_{\rm PRA}^{(\cos\phi)} = \sqrt{\frac{Q^2}{q_T^2}}\frac{\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2}{Q^2},\ f_{\rm PRA}^{(\cos 2\phi)} = \frac{q_T^2}{2Q^2}.$$

Analysis by Ian Balitsky

In recent papers by I. Balitsky and A. Tarasov, the LO PRA-asatz for parity-even, number-density part of the hadronic tensor was confirmed in a very general analysis in a framework of *rapidity factorization.* Figures form hep-ph/2012.01588:

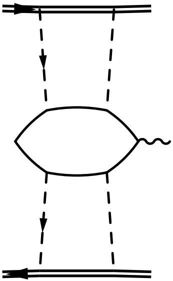


The calculation takes into account first q_T/Q-suppressed contribution to the hadronic tensor in leading $1/N_c$-approximation at LO in α_s for "central" fields but to all orders in "forward" and "backward" fields.

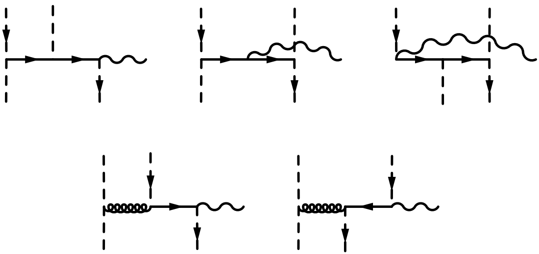
Further perturbative tests: two-Reggeon contribution

In QCD Boer-Mulders function is generated by additional gluon exchanges between spectators and hard process (see e.g. [D. Boer, *et.al.*, 2017]).

In PRA this corresponds to diagrams with more than one Reggeon in t-channel:



Diagrams for the central blob:



These diagrams should be computed to check if the contribution factorizes as we propose, especially at $q_T \sim Q$ (outside of TMD limit).

Work in progress...

New unintegrated PDF with exact normalization

- Was derived in [M.N., Saleev, 2020] from **Modified-MRK approxiamtion** for QCD matrix elements with additional real emissions, which smoothly interpolates between *Multi-Regge* and *Collinear* limits.
- Exactly satisfies the UPDF normalization condition for small-x and $x \sim 1$:

$$\int_0^{\mu^2} d\mathbf{q}_T^2 \; \Phi_i(x, \mathbf{q}_T^2, \mu^2) = \tilde{f}_i(x, \mu^2),$$

  where $\tilde{f}_i(x, \mu^2) = x f_i(x, \mu^2)$, to all orders in α_s for DGLAP-splitting functions $P_{ij}(z)$.
- We have checked, that for $\mathbf{q}_T^2 \ll Q^2$, our UPDF is consistent with *Collins-Soper-Sterman formula* up to **Next-to-Leading Logarithmic approximation.**

The perturbative shower part

Applicable for $t = \mathbf{q}_T^2 > t_0$, $t_0 \sim 1$ GeV:

$$\Phi_i^{(\text{pert. shower})}(x, t, \mu^2)$$

$$= \frac{\alpha_s(t)}{2\pi} \frac{T_i(t, \mu^2, x)}{t} \sum_{j=q,\bar{q},g} \int_x^1 dz\; P_{ij}(z) \tilde{f}_j\left(\frac{x}{z}, t\right) \theta\left(\Delta(t, \mu^2) - z\right),$$

with $\Delta(t, \mu^2) = \sqrt{\mu^2}/(\sqrt{\mu^2} + \sqrt{t})$ – rapidity-ordering [KMRW] cutoff,
Sudakov formfactor:

$$T_i(t, \mu^2, x) = \exp\left[-\int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \left(\tau_i(t', \mu^2) + \Delta\tau_i(t', \mu^2, x)\right)\right], \text{ where:}$$

$$\tau_i(t, \mu^2) = \sum_j \int_0^1 dz\; z P_{ji}(z) \theta(\Delta(t, \mu^2) - z),$$

$$\Delta\tau_i(t, \mu^2, x) = \sum_j \int_0^1 dz\; \theta(z - \Delta(t, \mu^2)) \left[z P_{ji}(z) - \frac{\tilde{f}_j\left(\frac{x}{z}, t\right)}{\tilde{f}_i(x, t)} P_{ij}(z) \theta(z - x)\right].$$

Non-perturbative shower and intrinsic q_T parts

Applicable for $t < t_0$:

$$\Phi_i^{(\text{non-pert. shower})}(x, t, \mu^2) = A t^\alpha (t_1 - t),$$

where parameters A, t_1 and α are determined by normalization, continuity and smoothness of UPDF at $t = t_0$.
Finally the shower-part is convoluted over $\mathbf{q}_T$ with Gaussian distribution of intrinsic $\mathbf{q}_T$ of a parton with some width σ_T:

$$\Phi_i(x, \mathbf{q}_T^2, \mu^2) = \int \frac{d^2\mathbf{k}_T}{\pi\sigma_{Ti}^2} e^{-\frac{\mathbf{k}_T^2}{\sigma_{Ti}^2}} \Phi_i^{(\text{shower})}(x, (\mathbf{q}_T - \mathbf{k}_T)^2, \mu^2).$$

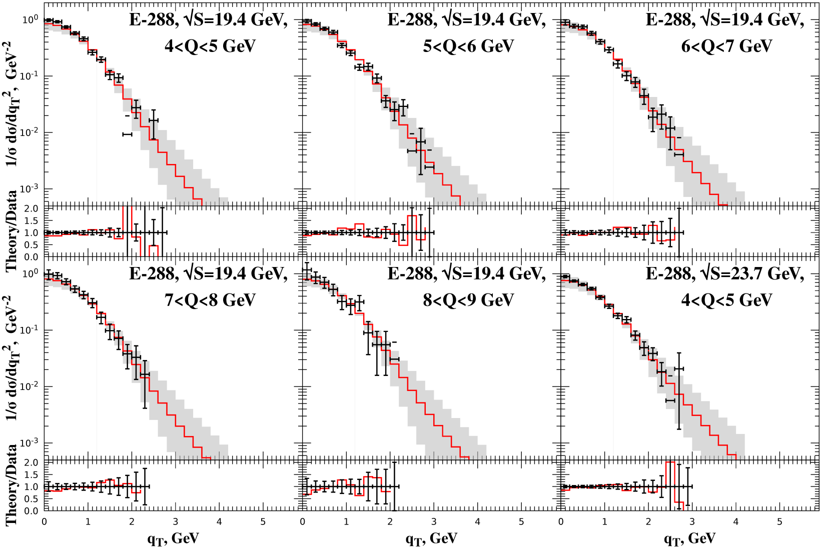
Parameters σ_T where taken equal for all quark flavors. The best-fit value $\sigma_T^{(\text{best fit})} \simeq 0.35$ GeV was found in a global fit of normalized $d\sigma/d\mathbf{q}_T^2/\sigma$ cross-sections for $\sqrt{S}$ from 20 up to 200 GeV.

Fit results

Dataset	Observable	$\sqrt{S}$(GeV)	Q(GeV)	$\frac{\sigma(\text{data})}{\sigma(\text{theory})}$ [+/- scale-uncert.] (+/- exp. uncert.)
E-288	$q^0 d\sigma/d^3q$	19.4	4-5	1.54[+0.63/-0.40](±0.20)
			5-6	1.50[+0.70/-0.45](±0.18)
			6-7	1.43[+0.73/-0.46](±0.18)
			7-8	1.22[+0.70/-0.43](±0.25)
			8-9	1.03[+0.64/-0.04](±0.35)
		23.7	4-5	1.64[+0.56/-0.35](±0.22)
			5-6	1.46[+0.57/-0.36](±0.14)
			6-7	1.47[+0.64/-0.42](±0.17)
			7-8	1.47[+0.70/-0.44](±0.20)
			8-9	1.43[+0.71/-0.45](±0.29)
		27.4	5-6	1.57[+0.55/-0.33](±0.13)
			6-7	1.47[+0.57/-0.36](±0.07)
			7-8	1.44[+0.60/-0.38](±0.08)
			8-9	1.35[+0.60/-0.38](±0.10)
E-605	$q^0 d\sigma/d^3q$	38.8	7-8	1.50[+0.55/-0.31](±0.18)
			8-9	1.42[+0.56/-0.33](±0.10)
			10.5-11.5	1.33[+0.60/-0.38](±0.11)
			11.5-13.5	1.40[+0.67/-0.40](±0.11)
			13.5-18	1.14[+0.60/-0.36](±0.17)
R-209	$d\sigma/d\mathbf{q}_T^2$	62	5-8	1.63[+0.40/-0.18](±0.29)
PHENIX	$q^0 d\sigma/d^3q$	200	4.8-8.2	1.50[+0.17/-0.10](±0.44)
CDF-1999	$d\sigma/d\|\mathbf{q}_T\|$	1800	66-116	2.07 [+0.23/-0.12](±0.11)
ATLAS-2019	$d\sigma/d\|\mathbf{q}_T\|$	13000	66-116	1.71 [+0.07/-0.06](±0.04)

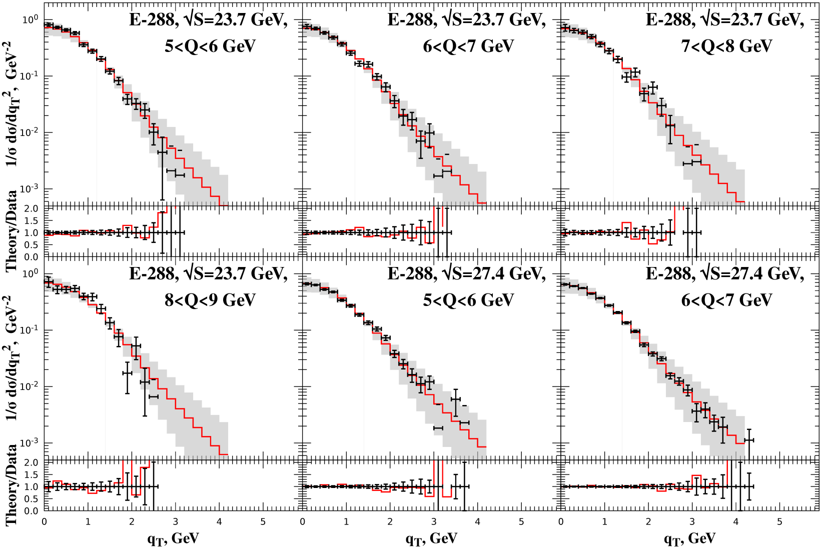
Fit results

UPDF generated from MSTW-2008 LO collinear PDF.



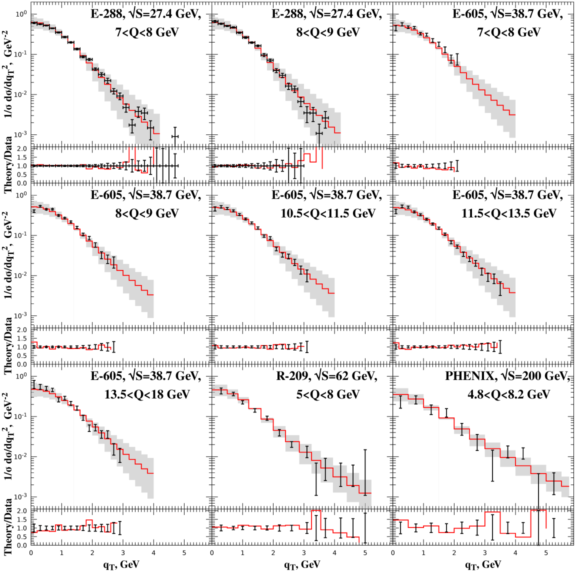
Fit results

UPDF generated from MSTW-2008 LO collinear PDF.



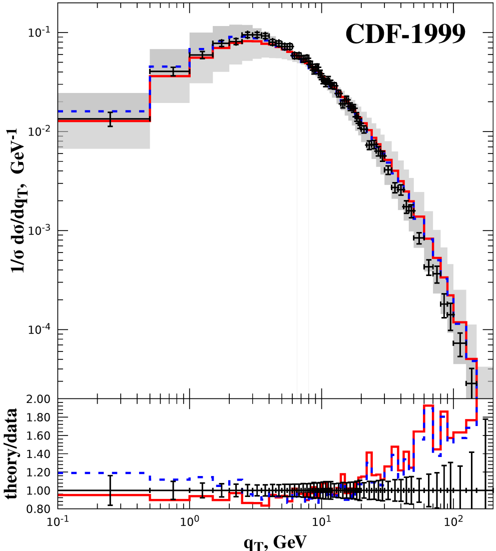
Fit results

UPDF generated from MSTW-2008 LO collinear PDF.



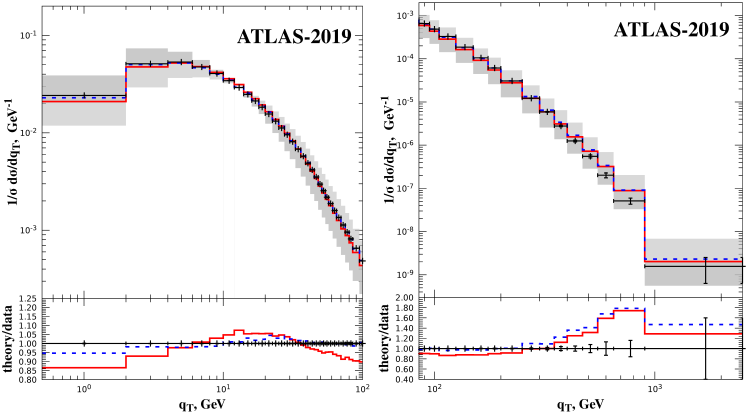
Z-boson $\mathbf{q}_T$ at $\sqrt{S} = 1.8$ TeV

The LO (generated form MSTW-2008 LO PDF) and NLO (genetrated from CT-18 NLO PDF using NLO P_{ij} in the shower formula and $\sigma_T = 0.35$ GeV).



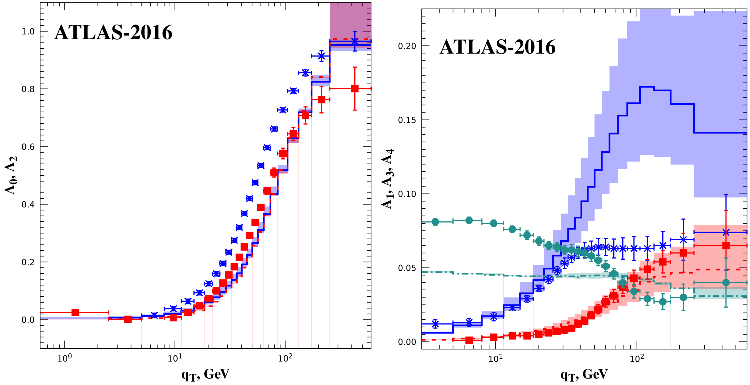
Z-boson $\mathbf{q}_T$ at $\sqrt{S} = 13$ TeV

The LO (generated form MSTW-2008 LO PDF) and NLO (genetrated from CT-18 NLO PDF using NLO P_{ij} in the shower formula and $\sigma_T = 0.35$ GeV).



Angular distributions at $\sqrt{S} = 8$ TeV and $Q = M_Z$

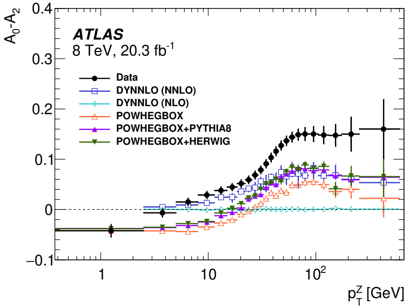
$$\frac{d\sigma}{dQ\,d\mathbf{q}_T^2\,dy\,d\Omega_l} = \frac{3}{16\pi}\frac{d\sigma}{dQ\,d\mathbf{q}_T^2\,dy}\left\{(1+\cos^2\theta_l) + \frac{A_0}{2}(1-3\cos^2\theta_l)\right.$$
$$+A_1\sin 2\theta_l\cos\phi_l + \frac{A_2}{2}\sin^2\theta_l\sin 2\phi_l + A_3\sin\theta_l\cos\phi_l + A_4\cos\theta_l$$
$$\left.+\,A_5\sin^2\theta_l\sin 2\phi_l + A_6\sin 2\theta_l\sin\phi_l + A_7\sin\theta_l\sin\phi_l\right\},$$

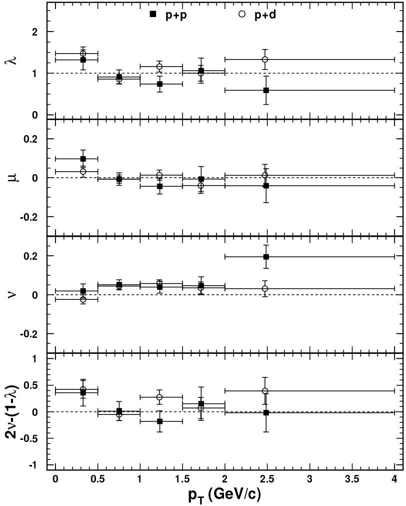


Left panel: A_0, A_2. Right panel: A_1, A_3, A_4.

Strong violation of Lam-Tung relation at $Q = M_Z$

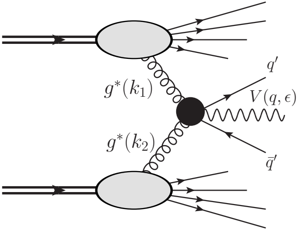
Lam-Tung relation: $\boxed{A_0 = A_2}$ or $\boxed{\lambda + 2\nu = 1}$, valid at LO and NLO in CPM. Left: figure from ATLAS paper [JHEP 08, 159 (2016)], right: figure form NuSea paper [PRL,102:182001,2009]

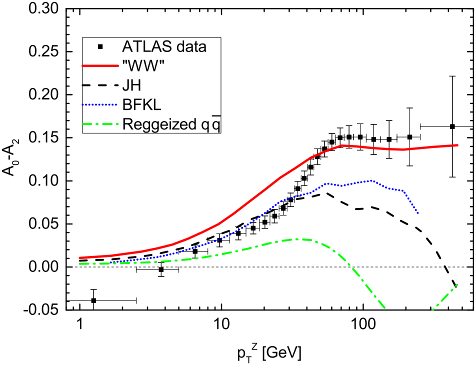




HEF with initial-state Reggeized gluons

Figures from [L.Motyka, *et.al.*, Phys.Rev., D**95**, 114025 (2017)]:

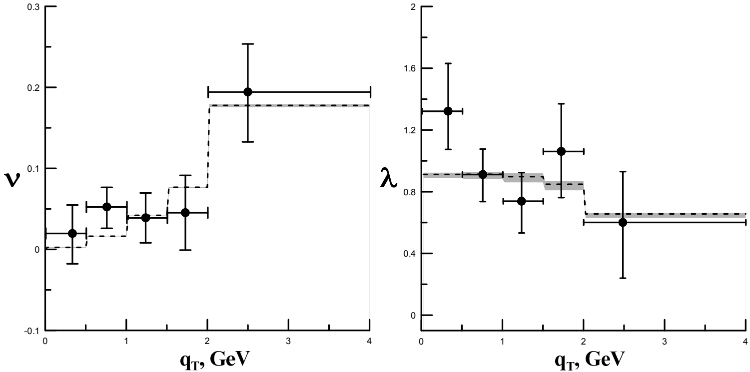




So there is a strong dependence of $A_0 - A_2$ on the gluon UPDF and spin-structure of the amplitude with additional emissions.

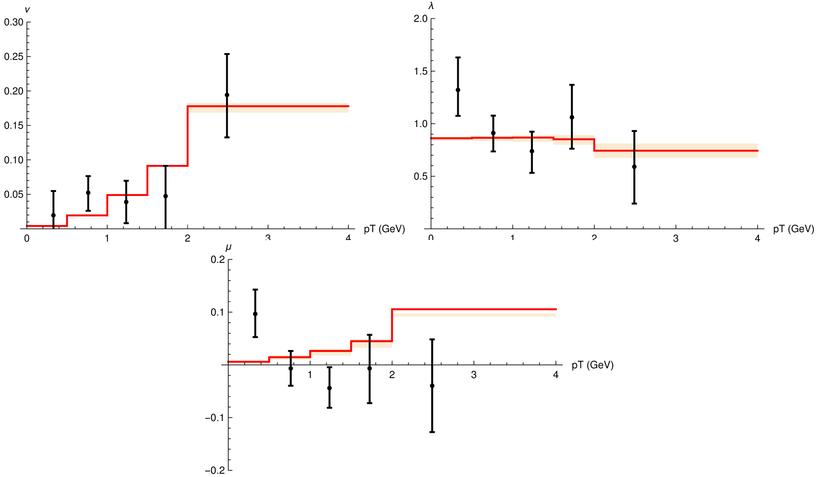
Description of NuSea data ($\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV) on angular coefficients

Description with “old” KMRW formula [M.N., Nikolaev, Saleev, 2013]:



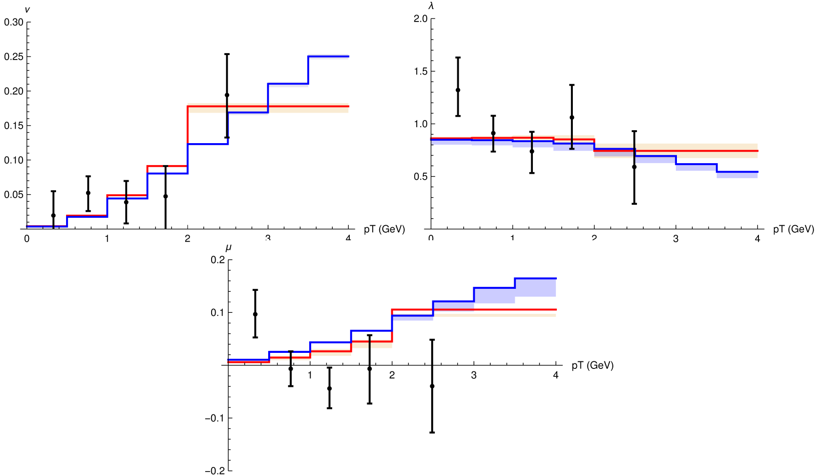
Description of NuSea data ($\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV) on angular coefficients

Description with "new" UPDF (genetrated from CT-18 NLO PDF using NLO P_{ij} in the shower formula and $\sigma_T = 0.35$ GeV):



What will change at LHCb-FT?

Comparison of NuSea ($\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV, $0 < x_F < 0.8$) and possible FT-LHCb ($\sqrt{S} = 110$ GeV) kinematics:



Conclusions

- High-energy factorization for Drell-Yan is applicable for $S \gg Q^2, \mathbf{q}_T^2$, more general than TMD factorization ($\mathbf{q}_T^2 \ll Q^2$).
- PRA leads to gauge-invariant hadronic tensor for $\mathbf{q}_T^2 \sim Q^2$
- LO PRA calculation with new KMRW-type UPDF *with exact normalization* nicely describes shapes of normalized $d\sigma/d|\mathbf{q}_T|$-distributions for nucleon-nucleon and pion-nucleon collisions
- Description of angular coefficients for Z-boson requires NLO in PRA
- NLO in PRA is needed to reduce theory uncertainty and *describe absolute (not normalized) cross-sections*
- Large violation of Lam-Tung relation for Z-boson is still a puzzle
- New measurements of Drell-Yan q_T-spectrum and angular coefficients are needed at $\sqrt{S} \sim 100$ GeV, as well as at high energies for $Q \neq M_Z$.

Thank you for your attention!