

Recent results in the computation of EW and QCD corrections for LHC processes

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Precision calculations — the key to fully exploit LHC measurements

Sample case: diboson production

- important SM test → trilinear couplings
- background for Higgs analyses and BSM searches
- very clean signatures in leptonic decay channels
- good statistics already with available data

All diboson processes available at NNLO QCD accuracy in the public **MATRIX** framework

[Grazzini, SK, Wiesemann (2018)]

- inevitable for data–theory agreement

Mandatory steps to match experimental precision also in the future

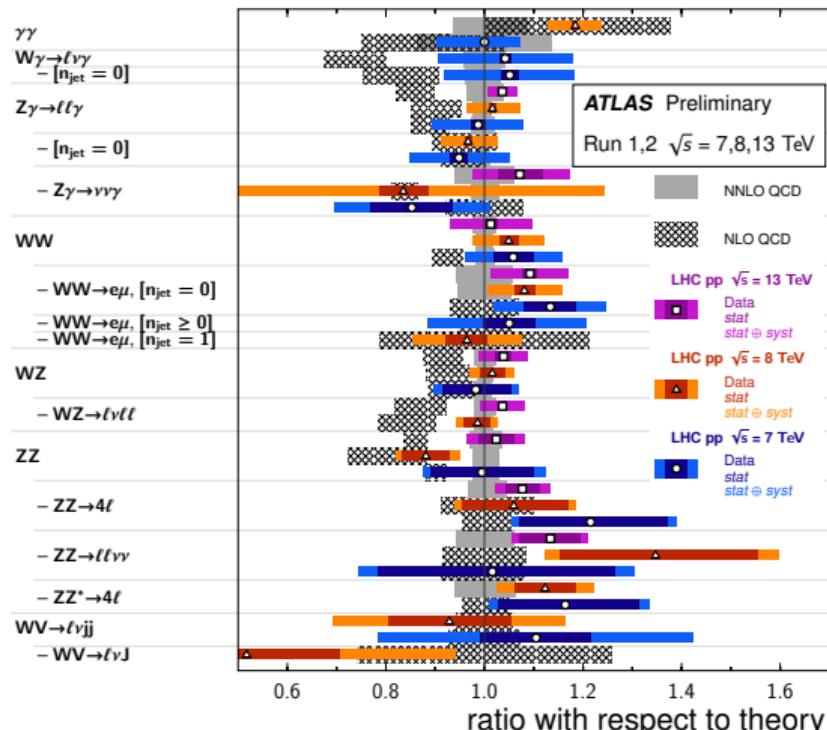
- leading QCD corrections beyond NNLO
- EW corrections and combination with QCD

⇒ **MATRIX v2** [Grazzini, SK, Wiesemann (to be released, beta version available)]

[ATLAS collaboration (2020)]

Diboson Cross Section Measurements

Status: May 2020



Outline

1 Motivation

2 Status and recent results in QCD calculations for LHC processes

- Triphoton production at NNLO QCD accuracy
- Heavy-quark pair production at NNLO QCD accuracy

3 Status and recent results in EW calculations for LHC processes

- Combination of NNLO QCD and NLO EW predictions for diboson production
- Mixed QCD–EW corrections for Drell–Yan production

4 Conclusions

NNLO QCD subtraction/slicing methods and implementations

Subtraction/slicing methods

- q_T subtraction [Catani, Grazzini (2007)]
- N -jettiness subtraction
[Boughezal, Focke, Liu, Petriello (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015)]
- Antenna subtraction [Gehrmann, Gehrmann-De Ridder, Glover (2005)]
- Sector-improved residue subtraction
[Czakon (2010); Boughezal, Melnikov, Petriello (2012)]
- ColorFul subtraction [Somogyi, Trocsanyi, Del Duca (2005)]
- Nested soft–collinear subtraction
[Caola, Melnikov, Röntsch (2017)]
- Analytic local sector subtraction
[Magnea, Maina, Pelliccioli, Signorile-Signorile, Torrielli, Uccirati (2018)]
- Projection to Born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)]
- Geometric subtraction [Herzog (2018)]
- ...

→ Extension beyond $2 \rightarrow 2$ conceptionally straightforward if amplitudes become available!

General (public) frameworks

- **MATRIX** (q_T slicing) [Grazzini, SK, Wiesemann]
 - $Z, W, H, \gamma\gamma, Z\gamma, W\gamma, WW, ZZ, WZ$
 - $ZH, WH, HH, t\bar{t}, b\bar{b}, \gamma\gamma\gamma, \dots$
- **MCFM** (N -jettiness slicing)
[Campbell, K. Ellis, Giele, Neumann, Williams]
 - $Z, W, H, ZH, WH, \gamma\gamma, Z\gamma$
 - $W\gamma, \gamma j, Zj, Wj, Hj, \dots$
- **NNLOJET** (antennna subtraction)
[Gehrmann, Gehrmann-de Ridder, Glover, Huss, Chen, Gauld, ...]
 - $jj, \gamma j, Zj, Wj, Hj, Zb, \dots$
- **STRIPPER** (sector-improved residue subtraction)
[Czakon, Mitov, Poncelet, Chawdhry, ...]
 - $t\bar{t}, jj, WW, Wc, \gamma\gamma\gamma, \gamma\gamma j, (jjj), \dots$
- ...

Recent achievements in (N)NNLO QCD calculations

First $2 \rightarrow 3$ calculations at NNLO QCD

- $\gamma\gamma\gamma$ [Chawdhry, Czakon, Mitov, Poncelet (2020), SK, Sotnikov, Wiesemann (2021)]
- $\gamma\gamma j$ [Chawdhry, Czakon, Mitov, Poncelet ('21)]
- (jjj) [Chawdhry, Czakon, Mitov, Poncelet (preliminary results at RADCOR 2021)]

Recent achievements in 2-loop $2 \rightarrow 3$ amplitudes

- leading-colour jjj
[Abreu, Page, Pascual, Sotnikov (2021), Abreu, Febres Cordero, Ita, Page, Sotnikov ('21)]
- full-colour $\gamma\gamma j$ [Agarwal, Buccioni, von Manteuffel, Tancredi ('21)]
- leading-colour $Wb\bar{b}$ [Badger, Hartanto, Zoia ('21)]

Heavy-quark loops for $gg \rightarrow$ diboson processes

- HH [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016), Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann (2019)]
- $\gamma\gamma$ [Maltoni, Mandal, Zhao (2019), Chen, Heinrich, Jahn, Jones, Kerner (2020)]
- ZZ [Agarwal, Jones, von Manteuffel ('20), Brønnum-Hansen, Wang ('20)]
- WW [Brønnum-Hansen, Wang ('20)]
- ZH [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk (2021)]

Inclusive $2 \rightarrow 1$ calculations at N^3LO QCD

- H [Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2015), + Furlan, Gehrmann, Lazopoulos (2016), Mistlberger (2018)]
- $b\bar{b} \rightarrow H$ [Duhr, Dulat, Mistlberger (2020), + Hirschi (2020)]
- W [Duhr, Dulat, Mistlberger (2020 & 2020)]
- γ^* [Duhr, Dulat, Mistlberger (2020)]

Fully differential $2 \rightarrow 1$ calculations at N^3LO QCD

- H [Dulat, Mistlberger, Pelloni (2019), Cieri, Chen, Gehrmann, Glover, Huss (2019)]
- $H (\rightarrow \gamma\gamma)$ [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni ('21)]
- Z/γ^* [Camarda, Cieri, Ferrera ('21)]

→ combination of local NNLO subtraction with slicing/projection methods promoted to N^3LO

First 3-loop amplitudes beyond $2 \rightarrow 1$

- leading-colour $\gamma\gamma$ [Caola, von Manteuffel, Tancredi (2021)]

Triphoton production at NNLO QCD accuracy

First MATRIX calculation for genuine $2 \rightarrow 3$ process at NNLO QCD

- q_T subtraction method for colourless final states directly applicable:

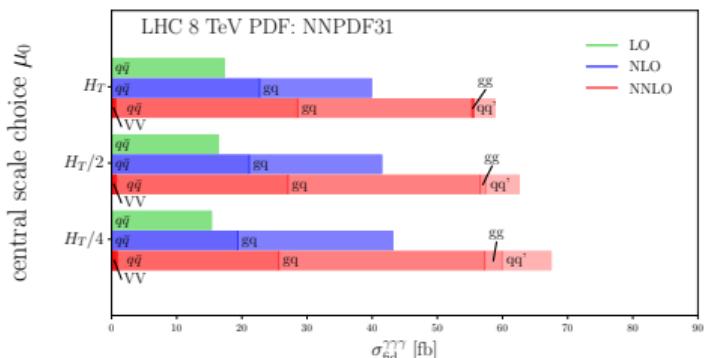
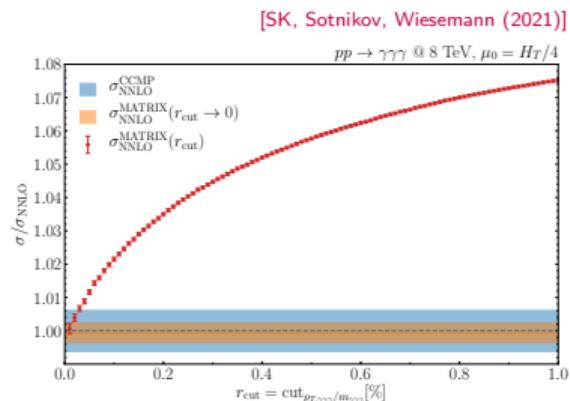
$$d\sigma_{\text{NNLO}}^{\gamma\gamma\gamma} = \mathcal{H}_{\text{NNLO}} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{\text{NLO}}^{\gamma\gamma\gamma+\text{jet}} - d\sigma_{\text{NNLO}}^{\gamma\gamma\gamma, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

- ↪ remarkable numerical control over slicing parameter dependence
- ↪ full agreement with independent calculation [Chawdhry, Czakon, Mitov, Poncelet (2020)]

Further important ingredients of the calculation

- fast and stable 2-loop amplitudes [Abreu, Page, Pascual, Sotnikov ('20)] generated with CARAVEL, using PENTAGONFUNCTIONS++ [Abreu et al. (2020)] [Chicherin, Sotnikov (2020)]
- fast and stable 1-loop amplitudes from OPENLOOP [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]
- highly efficient phase space integration in MUNICH [SK]

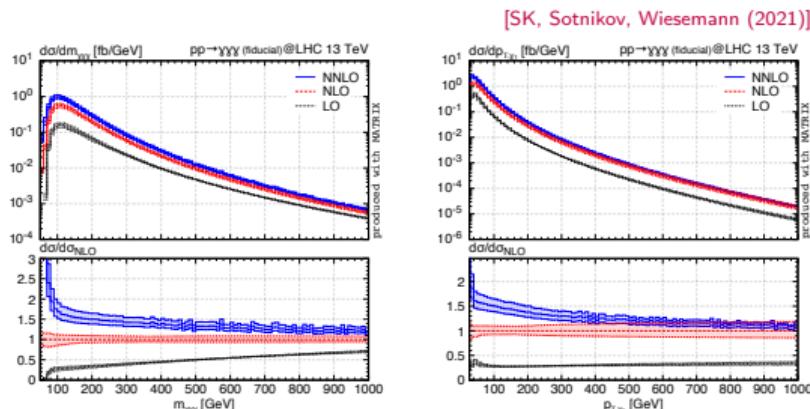
Huge NLO and NNLO QCD corrections, similar to diphoton production \Rightarrow split cross section into partonic channels



Triphoton production at NNLO QCD accuracy

Comparison with ATLAS data at 8 TeV

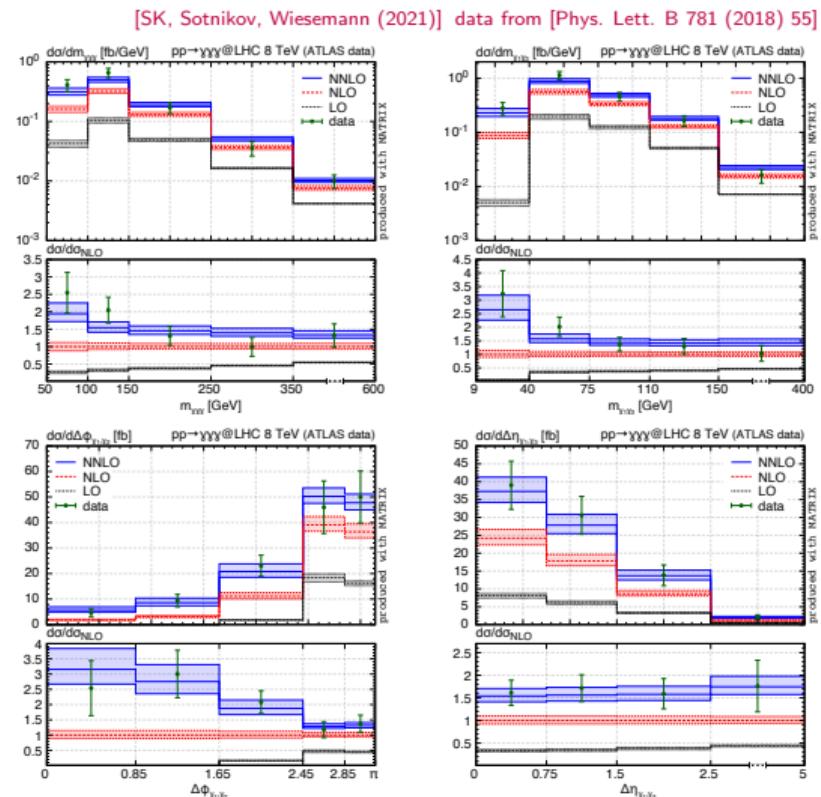
- perfect agreement with NNLO QCD predictions, due to both normalization and shape corrections
 - significant discrepancies at lower orders



- great numerical performance also with refined resolution and in suppressed phase space regions

↑

MATRIX fully suitable for triboson processes



Top-quark pair production at NNLO QCD accuracy

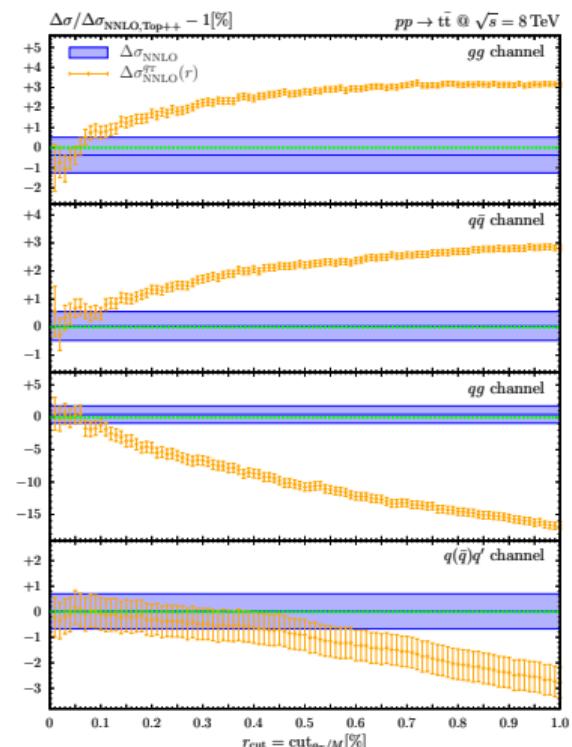
First MATRIX calculation for colourful final states at NNLO QCD

- extension of q_T subtraction method to production of heavy quarks:

$$d\sigma_{\text{NNLO}}^{t\bar{t}} = \mathcal{H}_{\text{NNLO}} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{\text{NLO}}^{t\bar{t}+\text{jet}} - d\sigma_{\text{NNLO}}^{t\bar{t}, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

- counterterm needs to account for IR behaviour of real contribution, including soft singularities related to final-state quarks
[Catani, Grazzini, Torre (2014), Ferroglio, Nuebert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
- $\mathcal{H}_{\text{NNLO}}^{t\bar{t}}$ contains remainder of integrated final-state soft singularities
[Catani, Devoto, Grazzini, Mazzitelli (to appear), Angeles-Martinez, Czakon, Sapeta (2018)]
- 2-loop amplitudes from numerical result [Bärnreuther, Czakon, Fiedler (2014)]
- massive NLO subtraction scheme required, e.g. dipole subtraction
[Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]
- slicing parameter dependence under good numerical control; investigation after splitting into partonic channels
 - full agreement with TOP++ [Czakon, Mitov (2014)]
 - validation also on the level of differential distributions
[Catani, Devoto, Grazzini, SK, Mazzitelli (2019), Czakon, Heymes, Mitov (2017)]

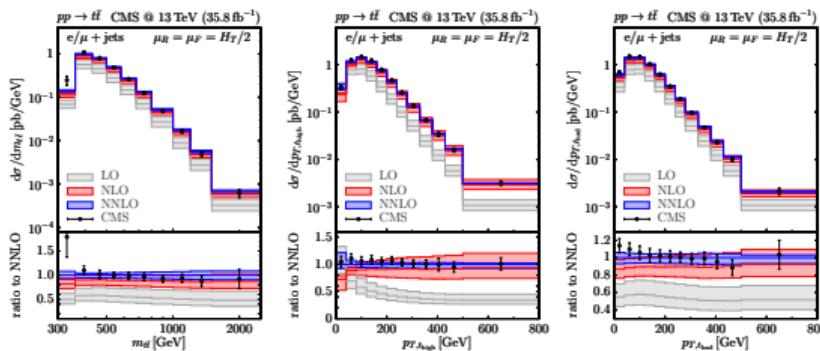
[Catani, Devoto, Grazzini, SK, Mazzitelli, Sargsyan (2019)]



Top-quark pair production at NNLO QCD accuracy

Good agreement with (multi)differential CMS data

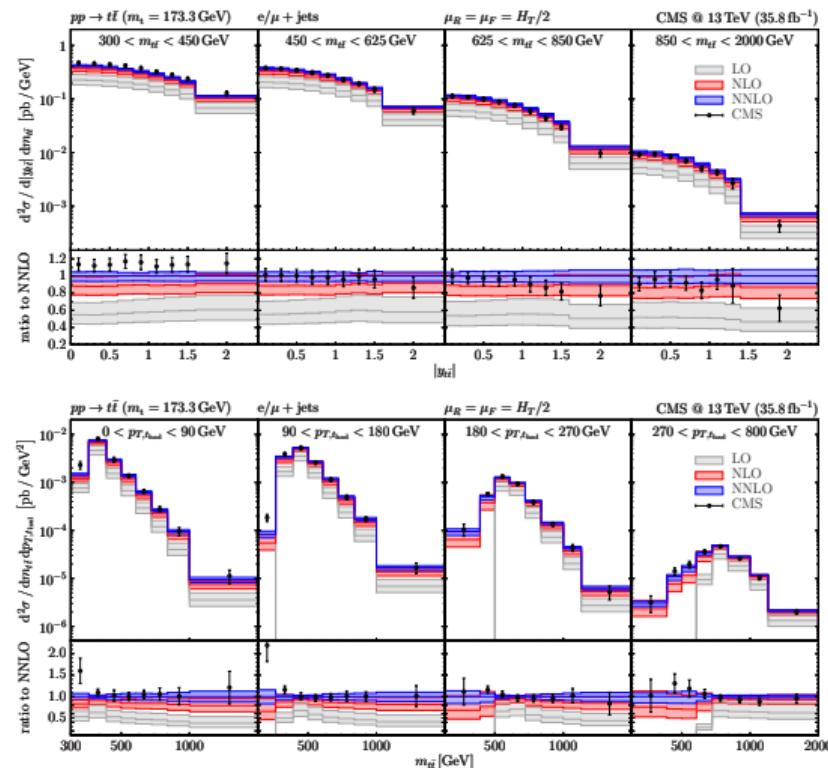
- lowest $m_{t\bar{t}}$ bin problematic: sensitivity to m_t value, threshold effects, extrapolation to stable tops, ...
- instabilities related to $p_{T,t\bar{t}} \rightarrow 0$ region
- would require resummation/shower matching



Indications for perturbative convergence

- widely overlapping bands from NLO to NNLO with reduced scale variation uncertainties

[Catani, Devoto, Grazzini, SK, Mazzitelli (2019)] data from [Phys. Rev. D 97 (2018) 112003]



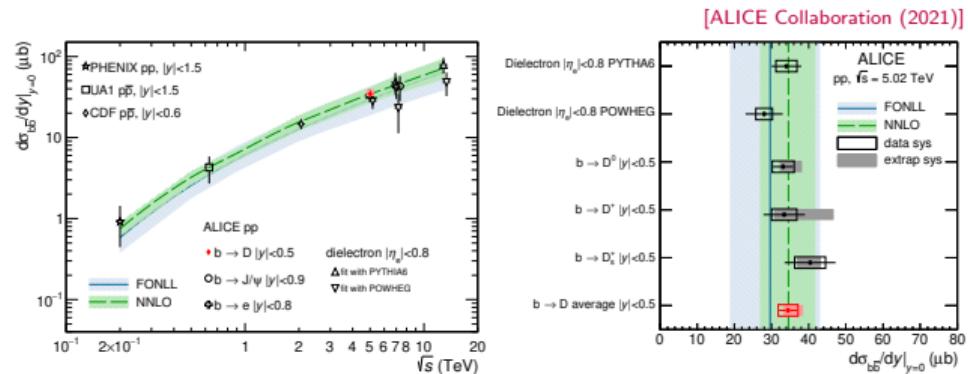
Bottom-quark pair production at NNLO QCD accuracy

Application to bottom quarks

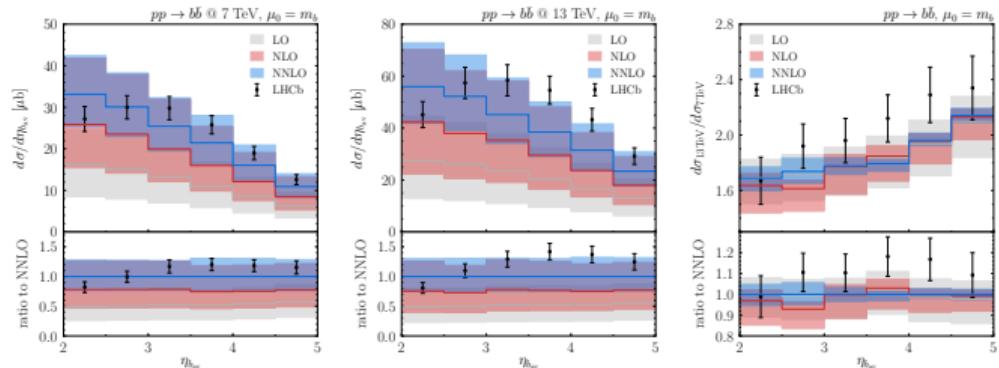
- conventionally similar to $t\bar{t}$ production
- applications in all LHC experiments
- larger uncertainties due to lower scales
 - reduction through ratios with partial cancellation of uncertainties
- numerically more challenging ($m_b \ll m_t$)
 - calculation still remarkably stable

MATRIX applications beyond $Q\bar{Q}$

- associated top-pair production
 - same QCD structure as $Q\bar{Q}$
 - more involved kinematics require numerical solutions in soft terms
- proof of principle for diagonal channels in $t\bar{t}H$ [Catani, Fabre, SK, Grazzini (2021)]

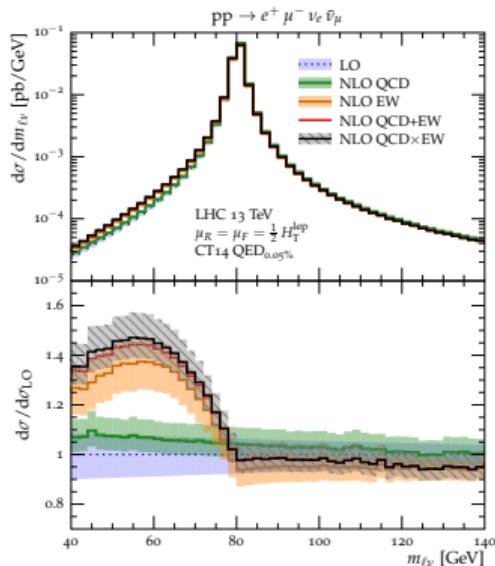


[Catani, Devoto, Grazzini, SK, Mazzitelli (2021)] data from [Phys. Rev. Lett. 118 (2017), no. 5 052002]



Main features of EW corrections

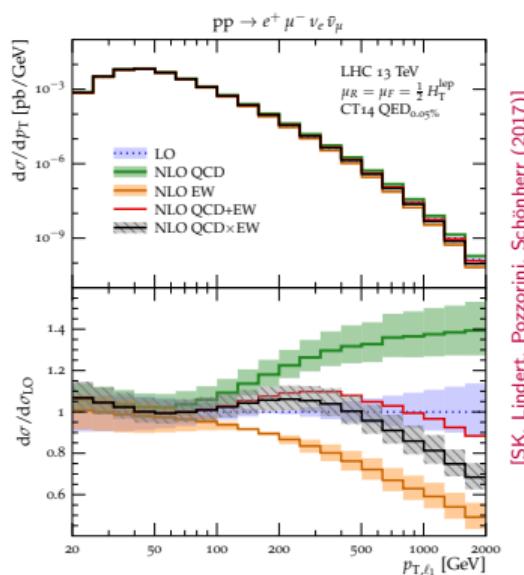
- Shape corrections in invariant-mass distributions



[SK, Lindert, Pozzorini, Schönherr (2017)]

Pure QED effect:
photon bremsstrahlung off decay
leptons (migration effect)

- Negative corrections in high-energy observables



[SK, Lindert, Pozzorini, Schönherr (2017)]

Genuine EW effect:
enhancement due to large universal
Sudakov logarithms

- Photon-induced processes

→ inclusion via LUXQED PDFs
[Manohar, Nason, Salam, Zanderighi (2016; 2017)]

- as Born processes,
e.g. $\gamma\gamma \rightarrow WW$
- as EW corrections,
from IS $\gamma \rightarrow q\bar{q}^*$ splittings

- Subdominant production modes (not maximal in α_s)

- e.g. $q\bar{q} \rightarrow Z^*/\gamma^* \rightarrow t\bar{t}$
- interferences between QCD and EW production modes
- corresponding tower of NLO contributions that cannot be uniquely qualified as QCD or EW corrections (in parts)

Status of NLO EW calculations

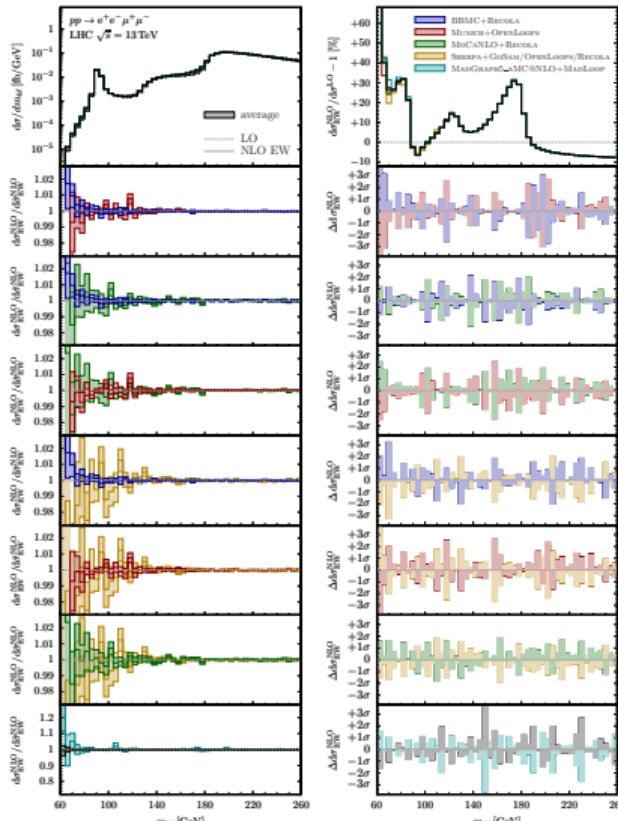
Dedicated comparison in Les Houches 2017 proceedings

- BBMC + RECOLA
- MUNICH/MATRIX + OPENLOOPs
- MoCANLO + RECOLA
- SHERPA + GoSAM/ OPENLOOPs/ RECOLA
- MADGRAPH5_AMC@NLO + MADLOOP

→ conceptionally solved, as for NLO QCD calculations

Recent highlights: high-multiplicity processes

- off-shell $t\bar{t}W$ production ($2 \rightarrow 8$) [Denner, Pelliccioli (2021)]
- off-shell $t\bar{t}H$ production ($2 \rightarrow 7$) [Denner, Lang, Pellen, Uccirati (2017)]
- off-shell WWW production ($2 \rightarrow 6$) [Dittmaier, Knippen, Schwan (2020)]
- vector boson scattering ($2 \rightarrow 6$)
 - $W^\pm W^\pm$ [Biedermann, Denner, Pellen (2017), Denner, Lang, Pellen, Uccirati (2017)]
 - WZ [Denner, Dittmaier, Maierhöfer, Pellen, Schwan (2019)]
 - ZZ [Denner, Franken, Pellen, Schmidt (2020)]
 - (W^+W^-) [Denner, Franken, Pellen, Schmidt (preliminary results at RADCOR 2021)]



Combination of QCD and EW corrections for diboson production – p_{T,V_2}

Both corrections sizable, particularly in high-energy tails of distributions

→ approximation of leading $\mathcal{O}(\alpha_s \alpha)$ effects desirable

Different combination approaches

- additive:

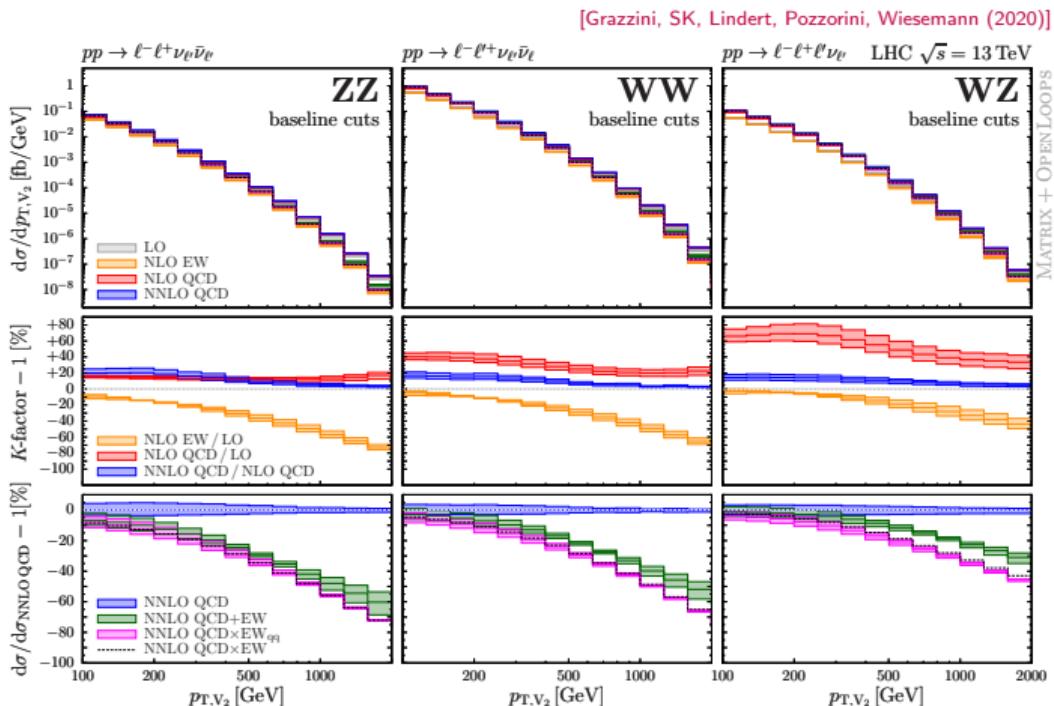
$$d\sigma^{\text{LO}}(1 + \delta_{\text{QCD}}^{(\text{N})\text{NLO}} + \delta_{\text{EW}}^{\text{NLO}})$$

- multiplicative:

$$d\sigma^{\text{LO}}(1 + \delta_{\text{QCD}}^{(\text{N})\text{NLO}})(1 + \delta_{\text{EW}}^{\text{NLO}})$$

- multiplicative (only $q\bar{q}$):

$$d\sigma_{q\bar{q}}^{\text{LO}}(1 + \delta_{q\bar{q}, \text{QCD}}^{(\text{N})\text{NLO}})(1 + \delta_{q\bar{q}, \text{EW}}^{\text{NLO}}) + \sigma_{\gamma\text{-ind., EW}}^{\text{NLO}}$$



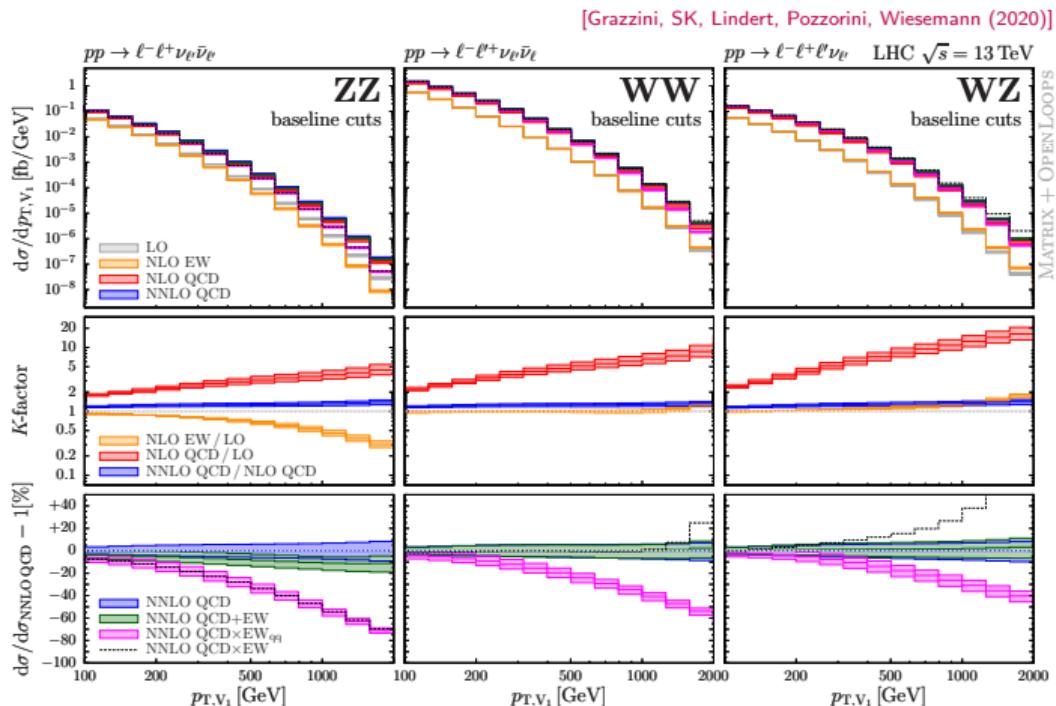
Factorized approaches well motivated for genuine VV observables (dominated by hard-VV topologies)

→ catch leading mixed QCD–EW effects and may thus be considered preferable.

Combination of QCD and EW corrections for diboson production – p_{T,V_1}

Situation more involved in presence of so-called **giant K-factors**

- ↪ QCD corrections in tails dominated by **hard V+jet topologies**
- ↪ also large positive EW corrections (photon-induced V+jet topologies)
- **additive** underestimates EW effects
- **multiplicative** combination multiplies large QCD and EW K-factors
 - ↪ **discarded**
- **multiplicative (only $q\bar{q}$)** shows expected Sudakov behaviour, but overestimates the EW effects (VV K-factor applied in V+jet region)



None of the approaches works perfectly well for **observables dominated by V+jet topologies**

- ↪ merged prediction, full mixed QCD–EW calculation, or phase space restriction to hard-VV topologies.

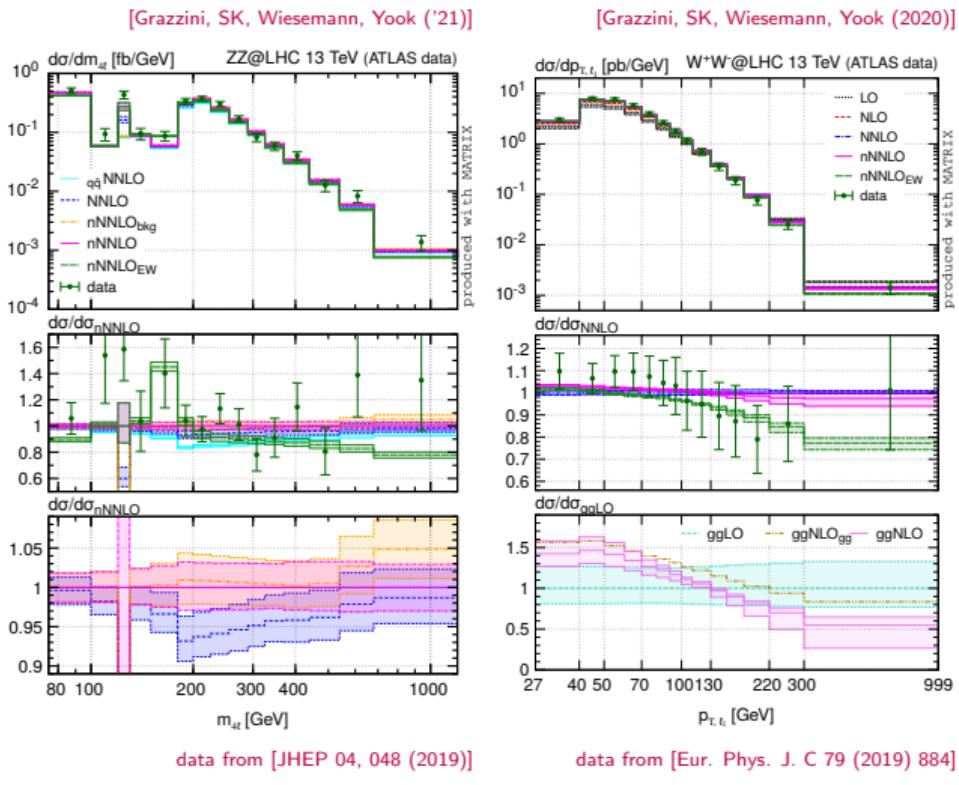
Best available fixed-order predictions for ZZ/WW production

- NNLO QCD for $q\bar{q}$ channel
- NLO EW combination for $q\bar{q}$ channel
- NLO QCD corrections for gg channel
(2-loop amplitudes from [ggVVamp](#),
[von Manteuffel, Tancredi (2015)]
improved by reweighting for m_t effects)

→ will be made publicly available for all VV processes in [MATRIX v2](#) [Grazzini, SK, Wiesemann]
(beta version already available for a while)

Diboson production beyond fixed order

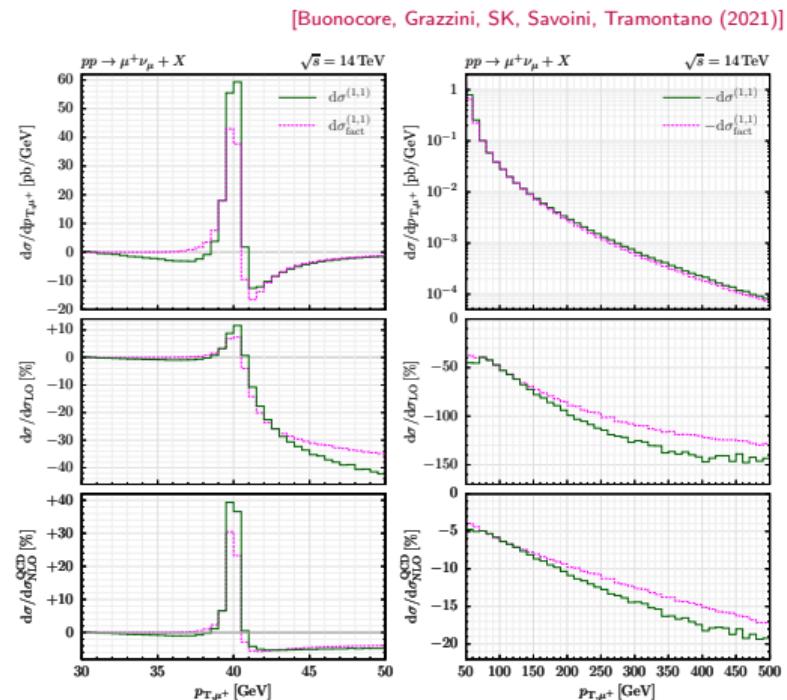
- resummation ($p_T, VV, p_{T,jet}^{\text{veto}}, \dots$)
[MATRIX+RADISH](#) [SK, Re, Rottoli, Wiesemann (2020)]
- event generation at NNLO QCD
[NNLOPS](#) [Re, Wiesemann, Zanderighi (2018)]
[MINNLO_{PS}](#) [Lombardi, Wiesemann, Zanderighi ('21)]
[GENEVA](#) [Alioli, Broggio, Gavardi, SK, Lim, Nagar, Napoletano, Rottoli (2021)]



Mixed NNLO QCD-EW calculation for off-shell W production

Calculation in the MATRIX framework

- subtraction of IR singularities by abelianisation of the q_T subtraction formalism for heavy quarks
→ massive leptons to regularize collinear singularities
- almost all contributions treated exactly
→ pole approximation for mixed 2-loop amplitude, improved by dedicated reweighting procedure
- qualitative agreement with the approximate result of [Dittmaier, Huss, Schwinn (2015)] around the Jacobian peak
→ also reliable in the remaining phase space
- Recent achievements in mixed 2-loop amplitudes
 - Z [Bonciani, Buccioni, Rana, Vicini (2020)]
 - W [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2021)]
 - off-shell Z/W at $\mathcal{O}(N_f \alpha_s \alpha)$ [Dittmaier, Schmidt, Schwarz (2020)]
 - off-shell Z [Heller, von Manteuffel, Schabinger, Spiesberger (2021)]



General implementation in the MATRIX framework (for colourless massive charged particles)

Conclusions

Status of higher-order QCD calculations

- $2 \rightarrow 2$ calculations at NNLO (almost) completed
- first $2 \rightarrow 3$ calculations at NNLO appeared
- $N^3\text{LO}$ calculations for $2 \rightarrow 1$ processes available
- great progress in 2-loop (and 3-loop) amplitudes

Status of EW calculations

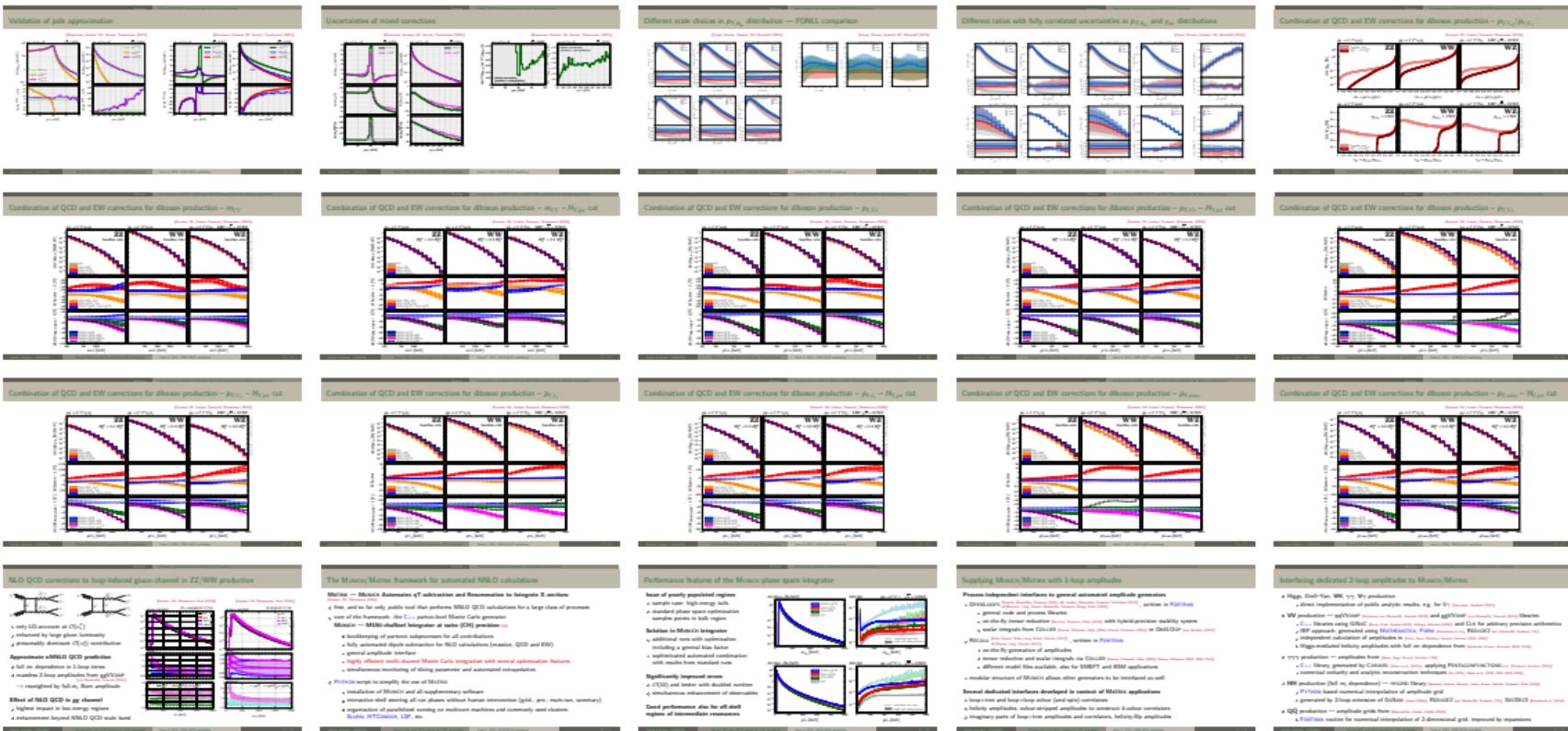
- NLO EW widely automated in several tools
- high-multiplicity processes calculated (up to $2 \rightarrow 8$)

Recent achievements in the MATRIX framework

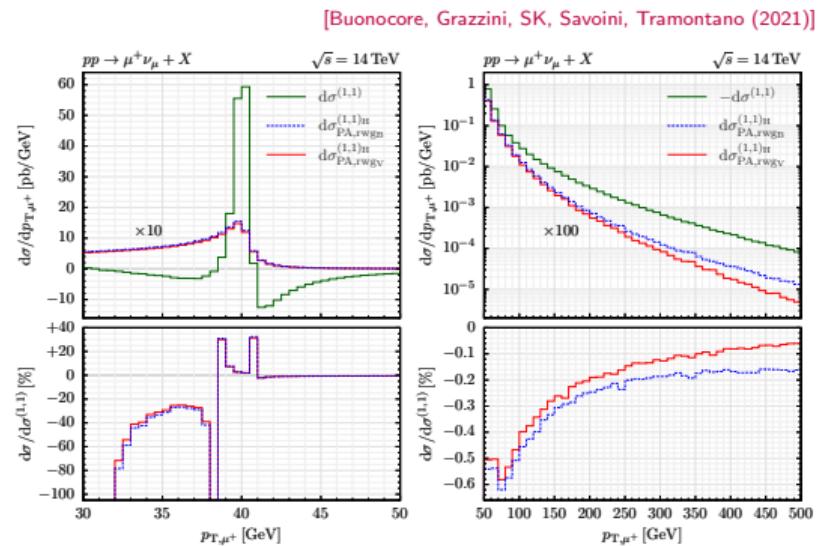
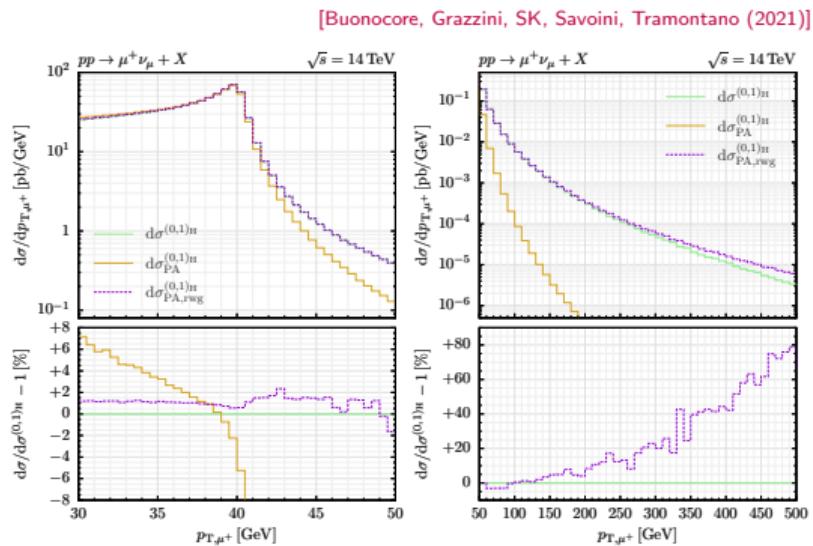
- triphoton production at NNLO QCD
- heavy-quark pair production at NNLO QCD
- combination of NNLO QCD and NLO EW corrections in diboson production
- mixed NNLO QCD-EW corrections for off-shell W production



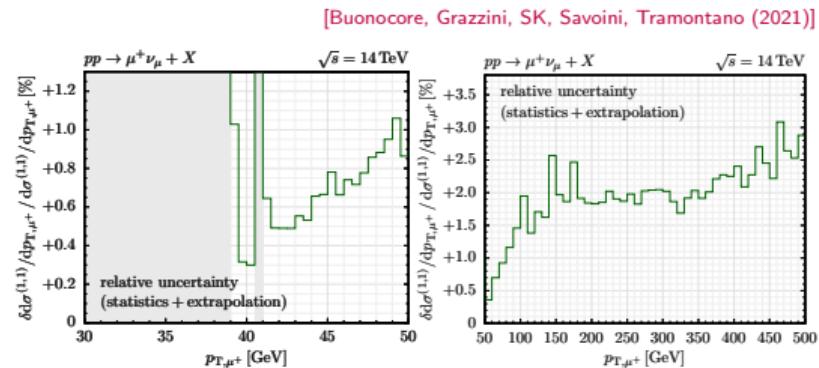
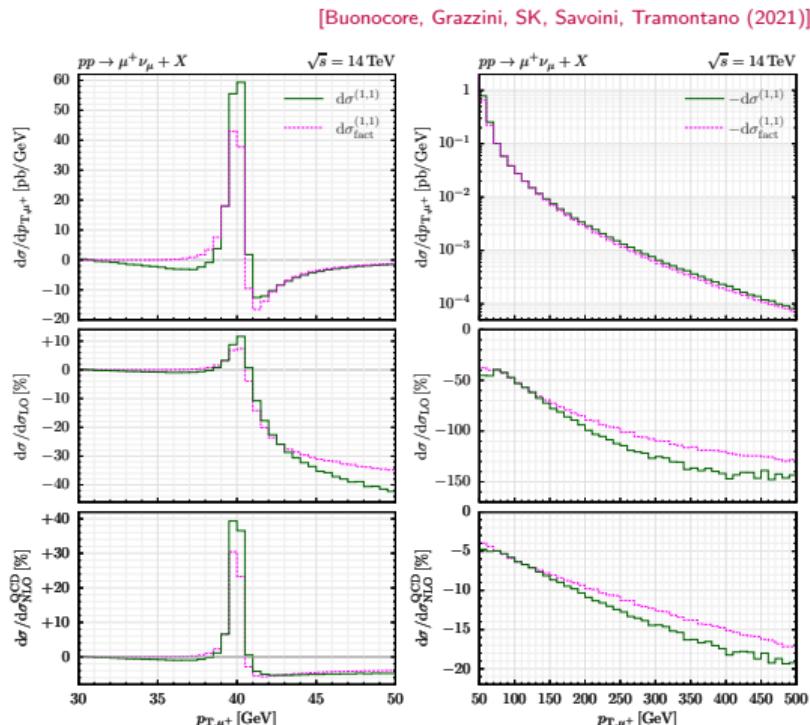
Backup



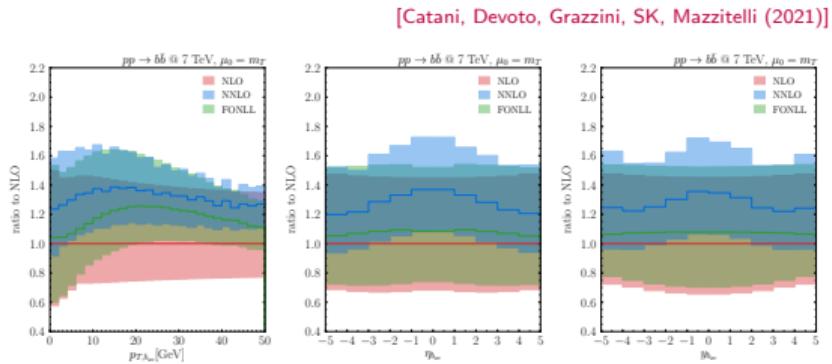
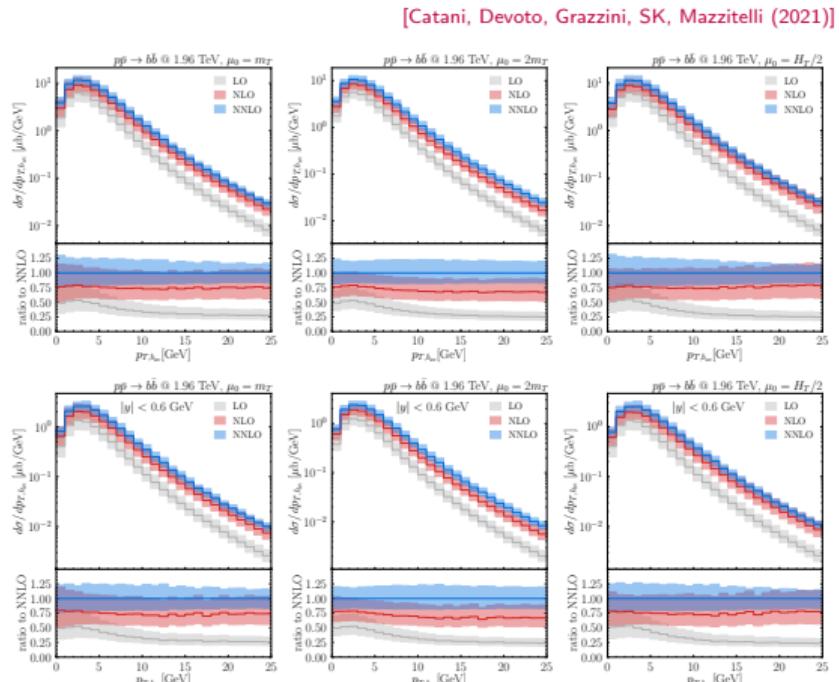
Validation of pole approximation



Uncertainties of mixed corrections

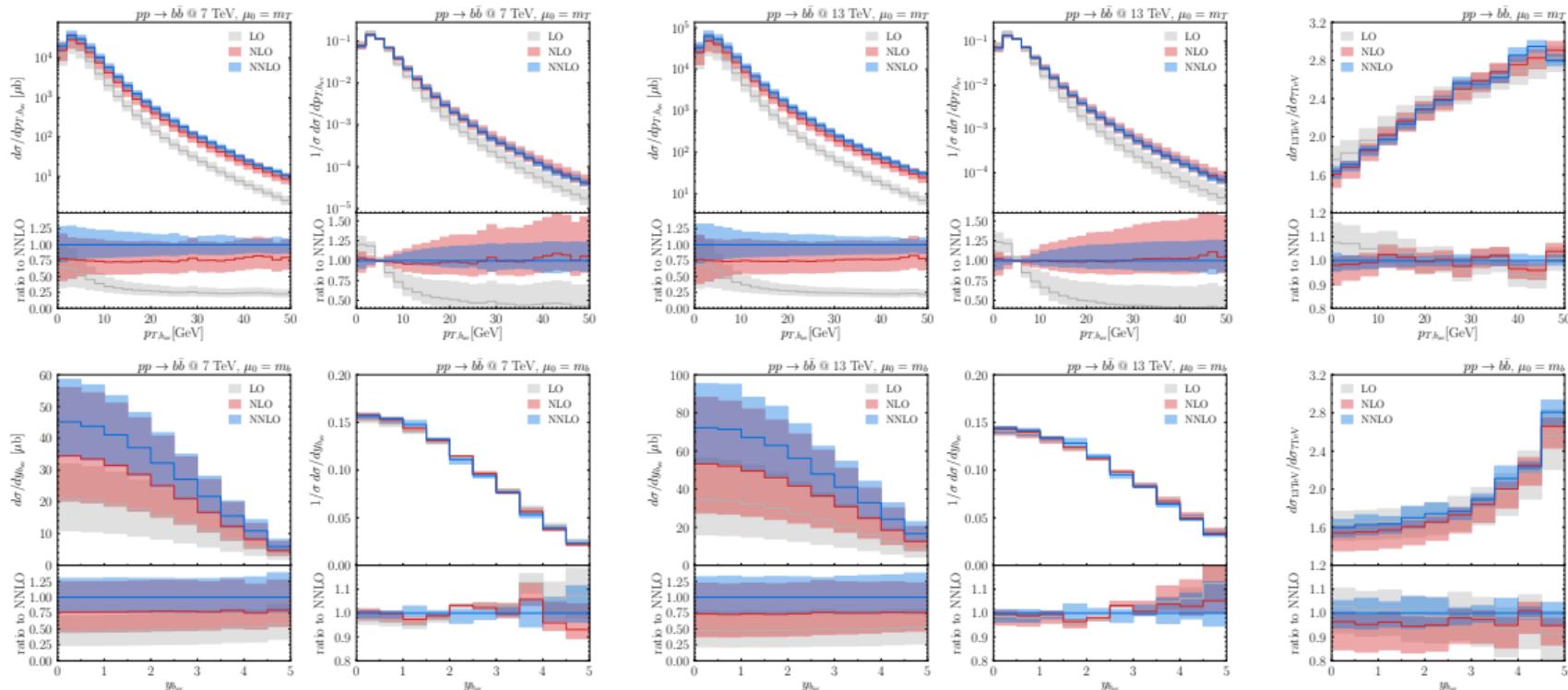


Different scale choices in $p_{T,b_{av}}$ distribution — FONLL comparison



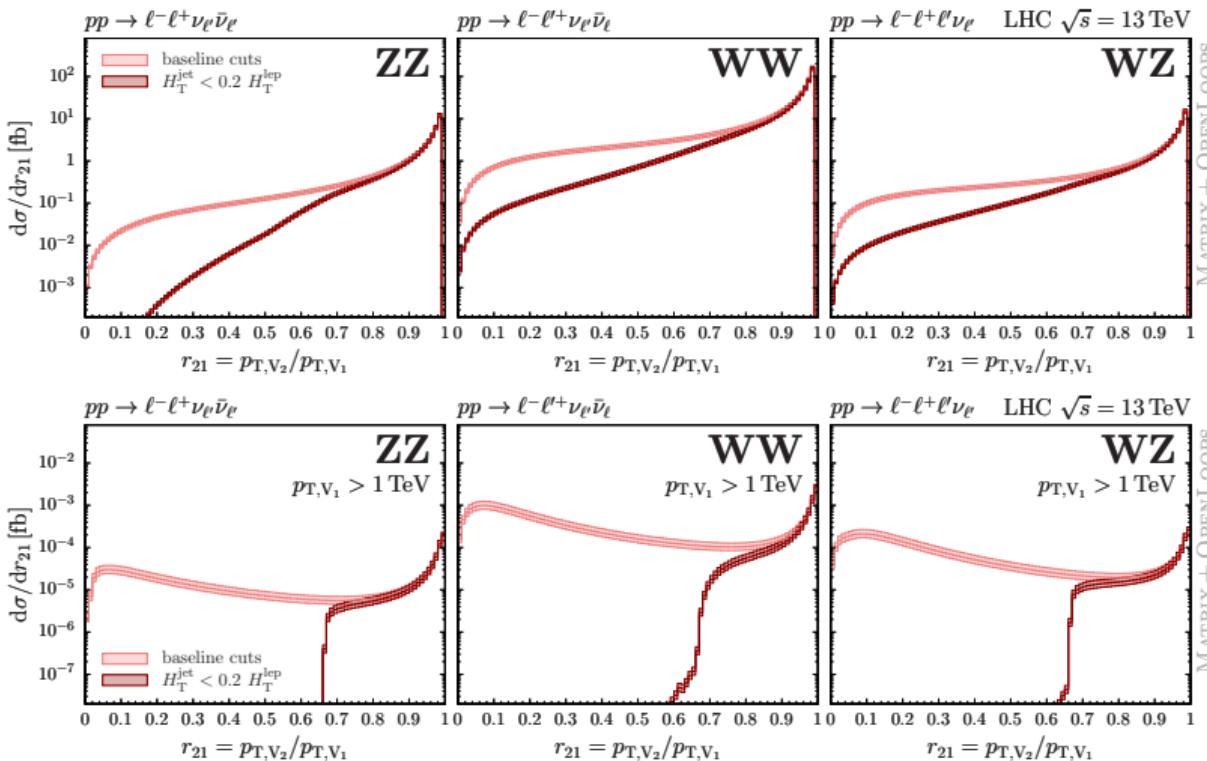
Different ratios with fully correlated uncertainties in $p_{T,b_{av}}$ and y_{av} distributions

[Catani, Devoto, Grazzini, SK, Mazzitelli (2021)]



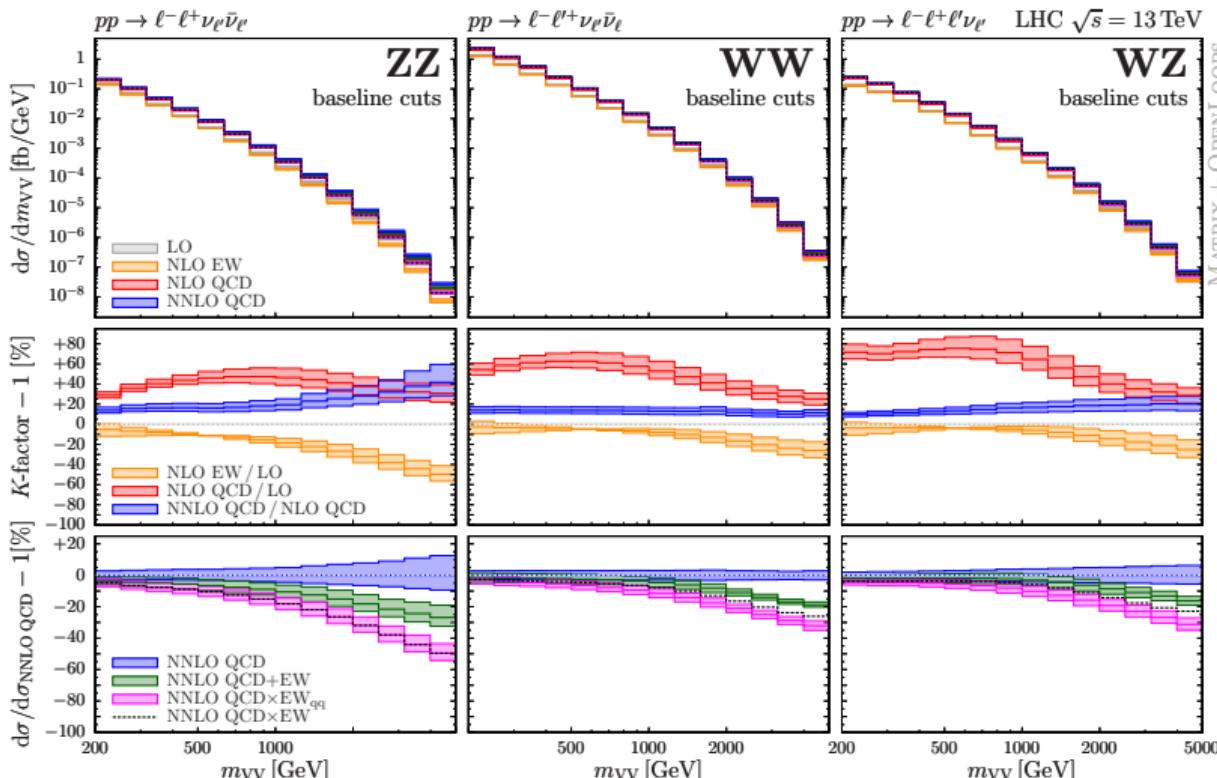
Combination of QCD and EW corrections for diboson production – $p_{\text{T},V_2}/p_{\text{T},V_1}$

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



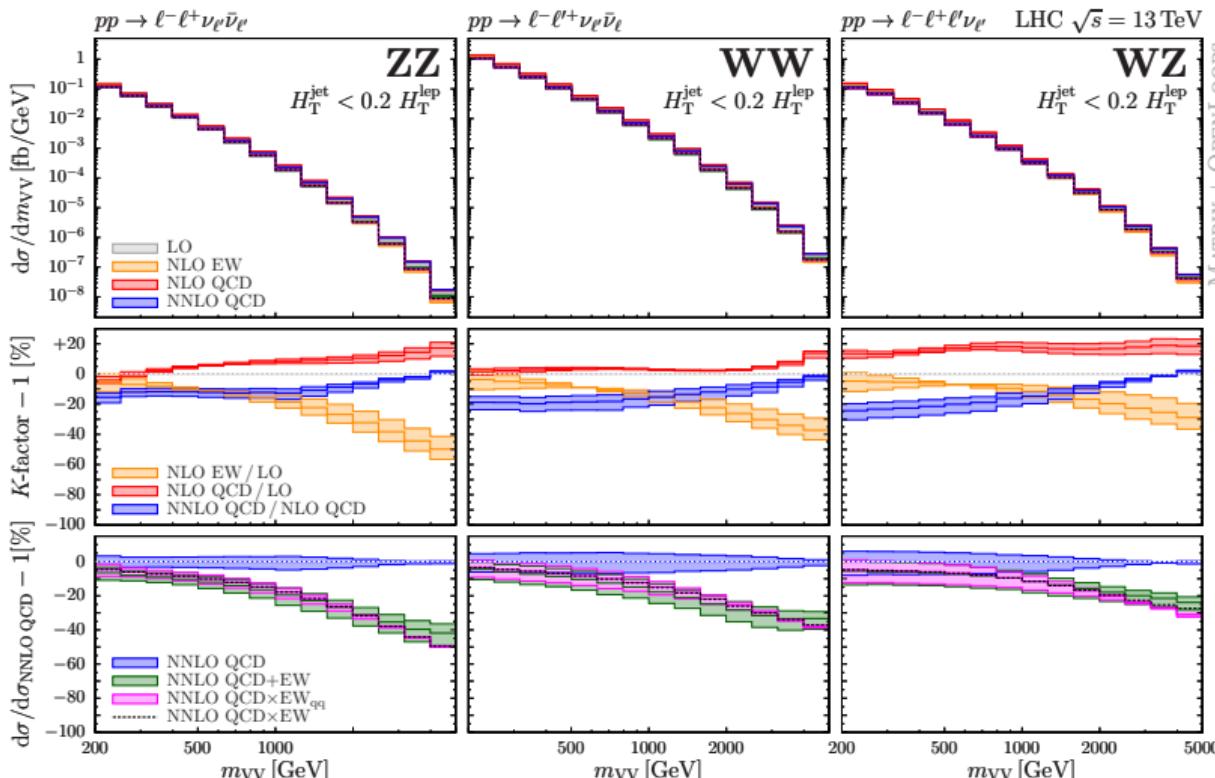
Combination of QCD and EW corrections for diboson production – m_{VV}

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



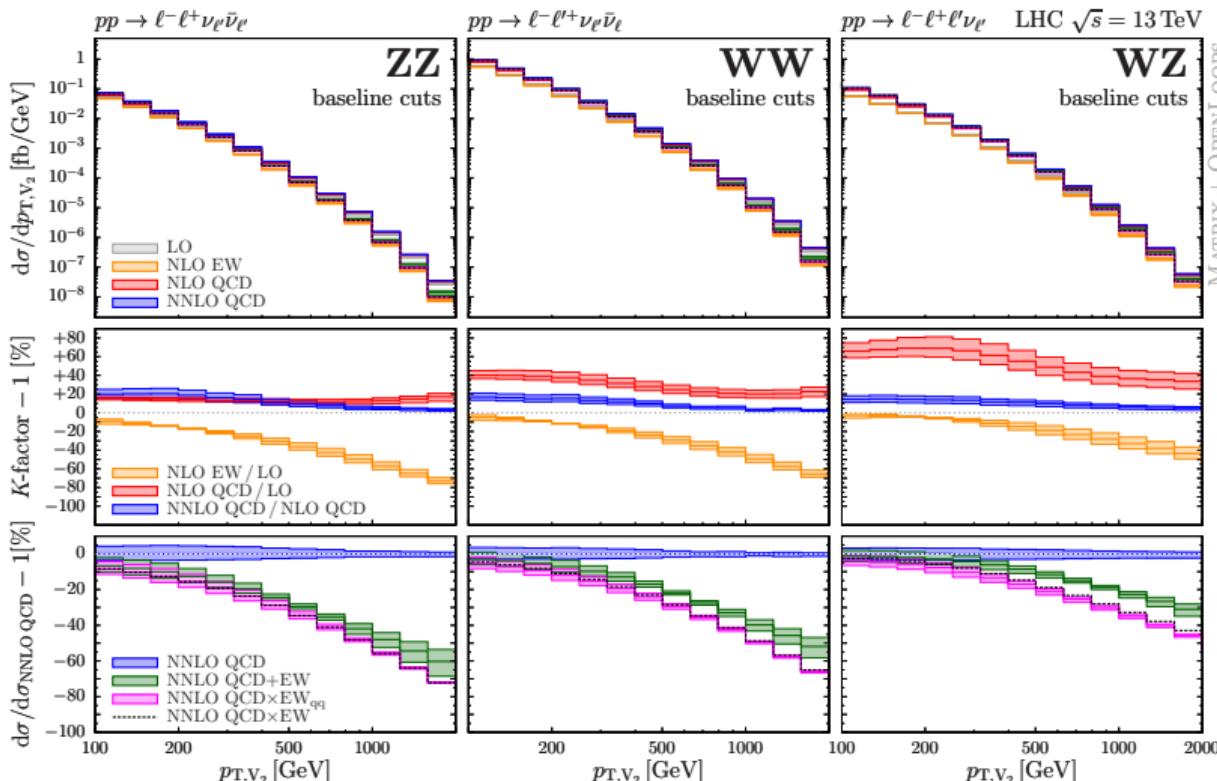
Combination of QCD and EW corrections for diboson production – m_{VV} – $H_{T,\text{jet}}$ cut

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



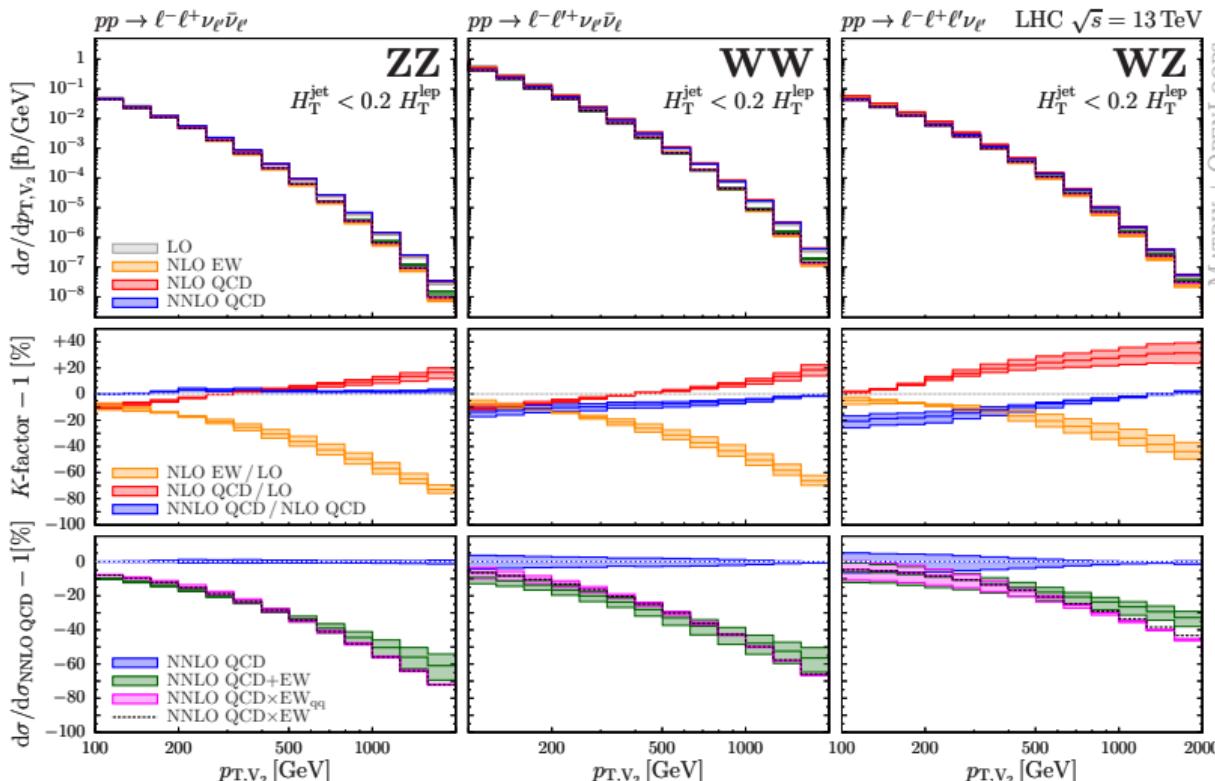
Combination of QCD and EW corrections for diboson production – p_{T,V_2}

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



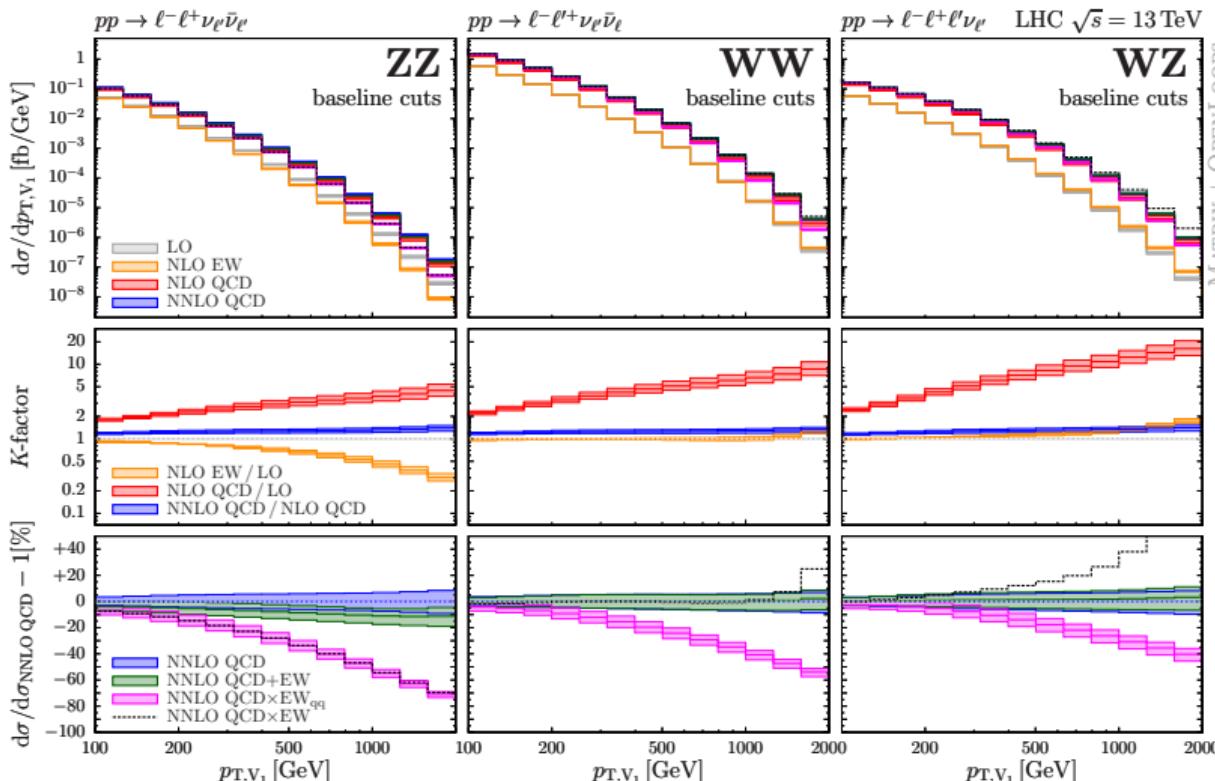
Combination of QCD and EW corrections for diboson production – $p_{\text{T},V_2} - H_{\text{T,jet}}$ cut

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



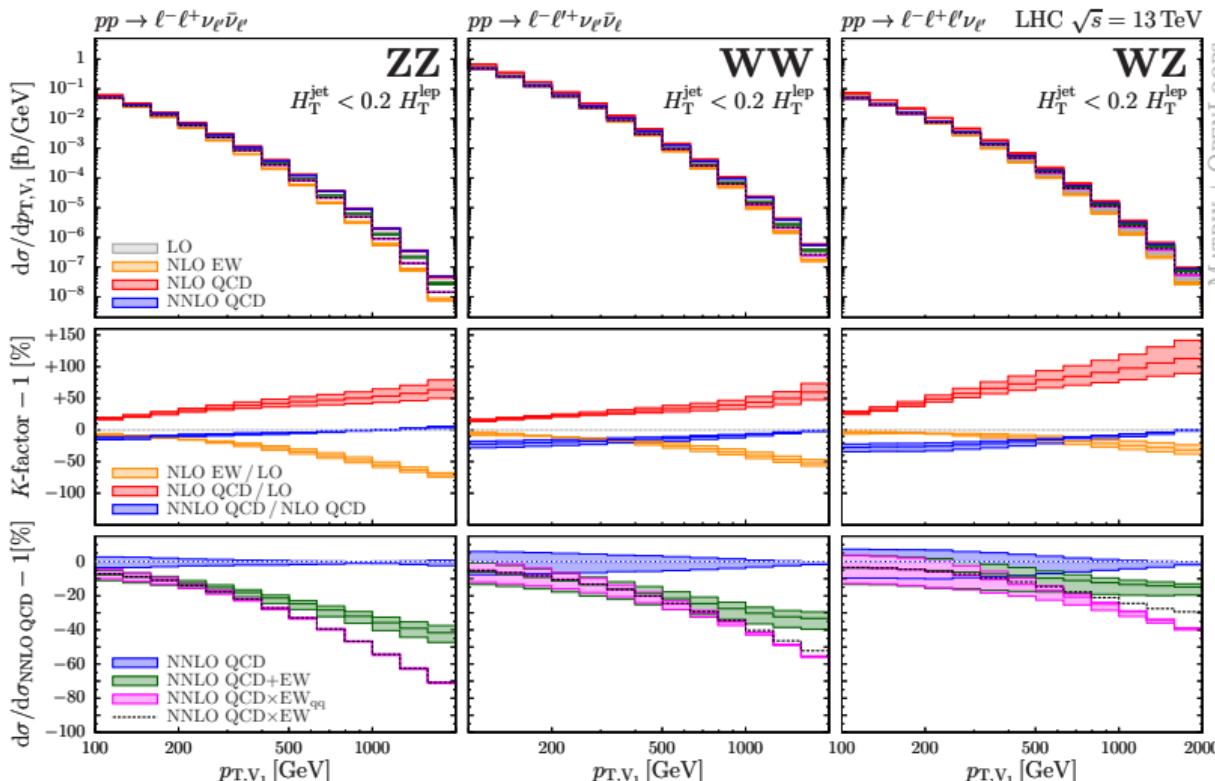
Combination of QCD and EW corrections for diboson production – p_{T,V_1}

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



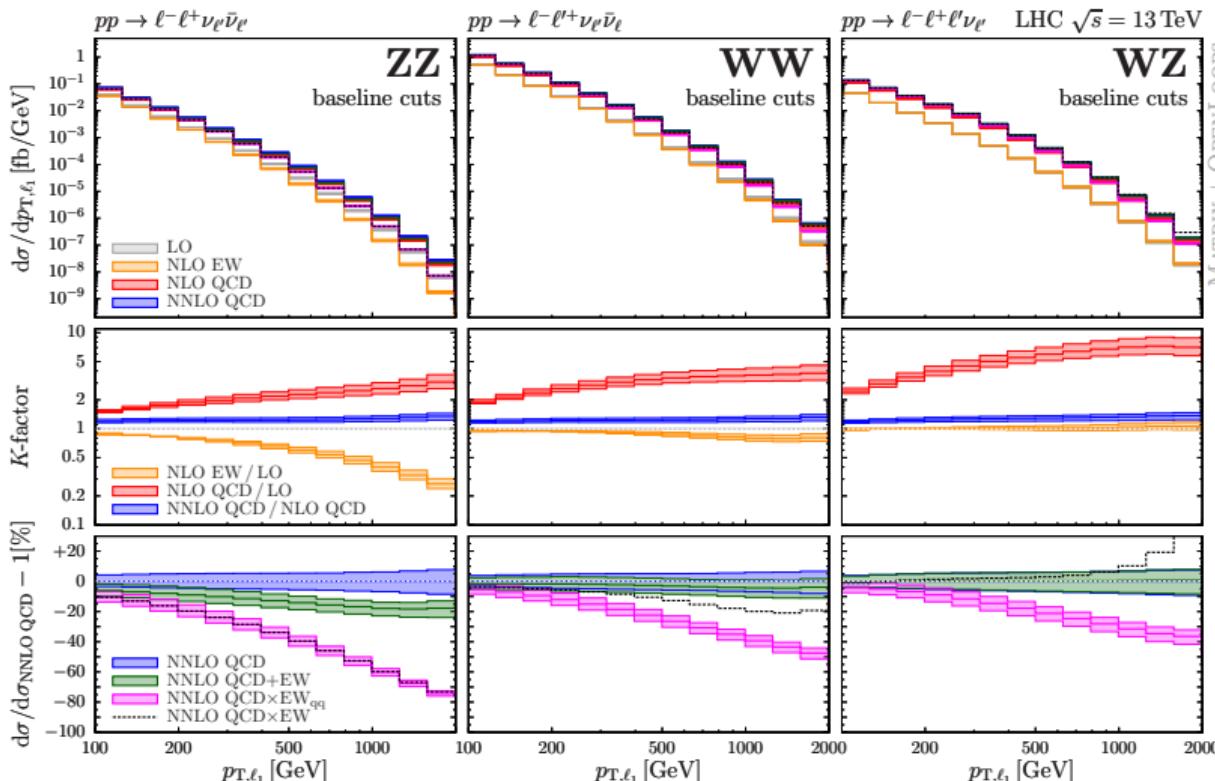
Combination of QCD and EW corrections for diboson production – $p_{\text{T},V_1} - H_{\text{T,jet}}$ cut

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



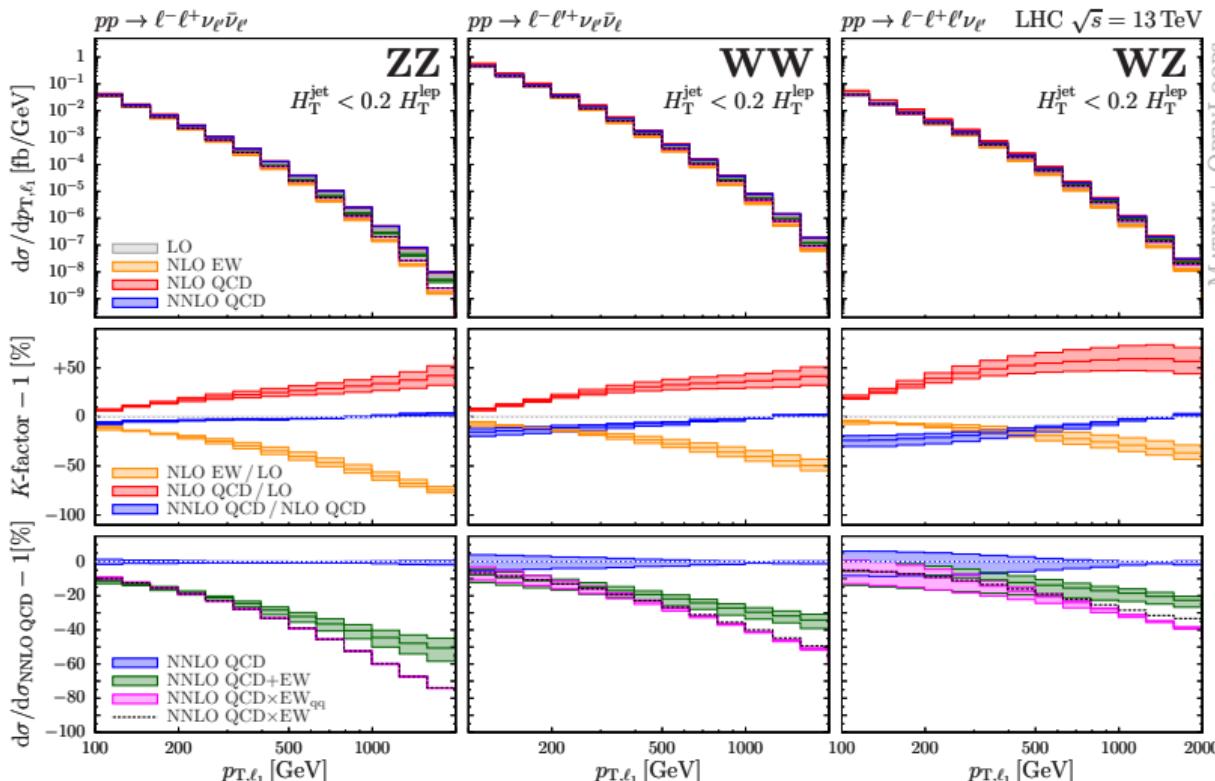
Combination of QCD and EW corrections for diboson production – p_{T,ℓ_1}

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



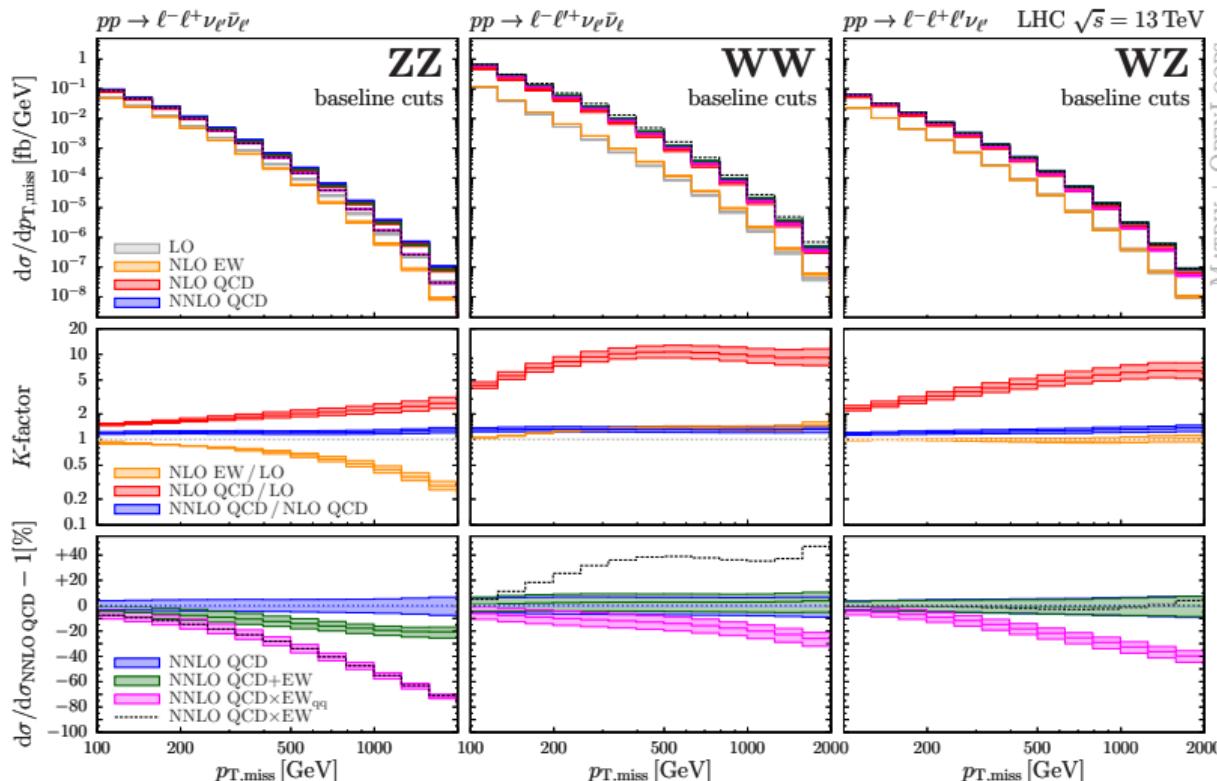
Combination of QCD and EW corrections for diboson production – $p_{\text{T},\ell_1} - H_{\text{T,jet}}$ cut

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



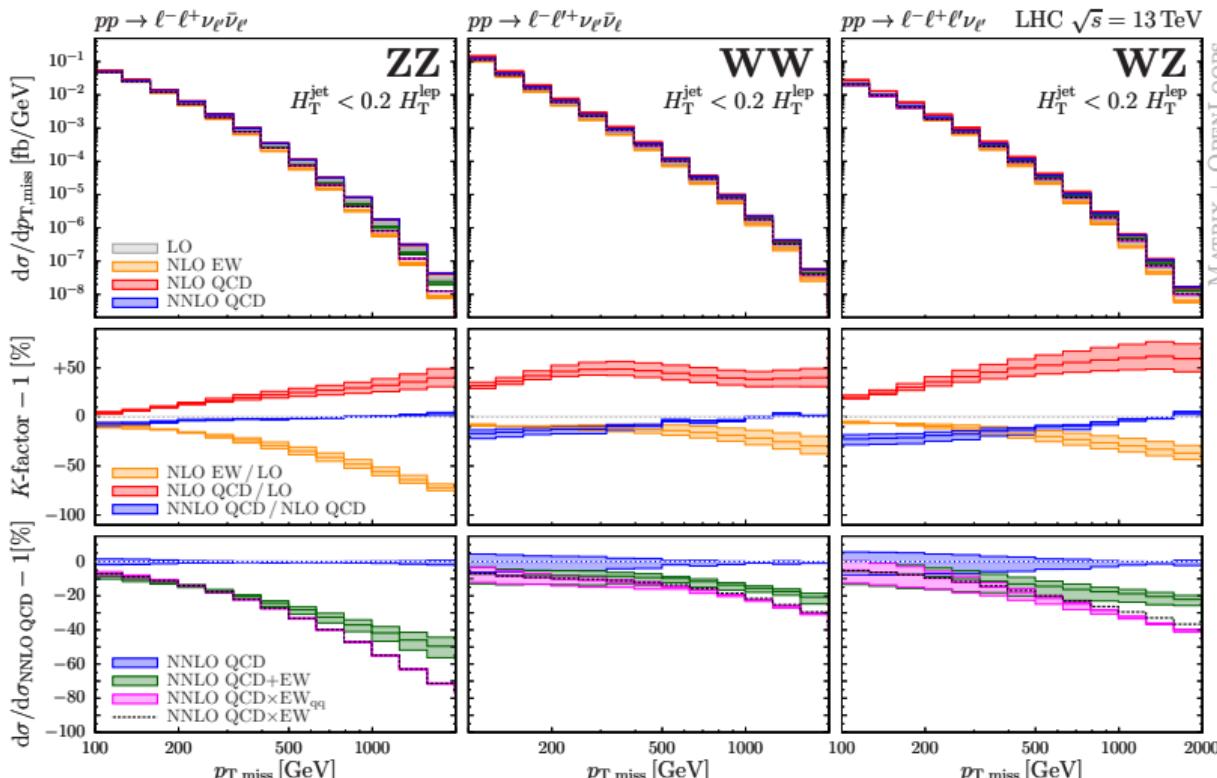
Combination of QCD and EW corrections for diboson production – $p_{\text{T,miss}}$

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]

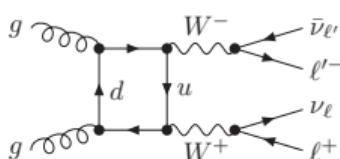
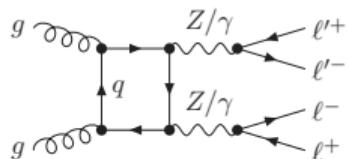


Combination of QCD and EW corrections for diboson production – $p_{\text{T},\text{miss}}$ – $H_{\text{T},\text{jet}}$ cut

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



NLO QCD corrections to loop-induced gluon channel in ZZ/WW production



- only LO-accurate at $\mathcal{O}(\alpha_s^2)$
- enhanced by large gluon luminosity
- presumably dominant $\mathcal{O}(\alpha_s^3)$ contribution

Approximate nNNLO QCD prediction

- full m_t dependence in 1-loop terms
- massless 2-loop amplitudes from ggVVAMP

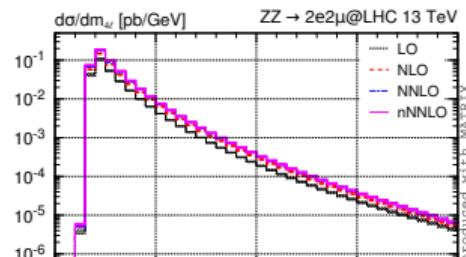
[von Manteuffel, Tancredi (2015)]

↪ reweighted by full- m_t Born amplitude

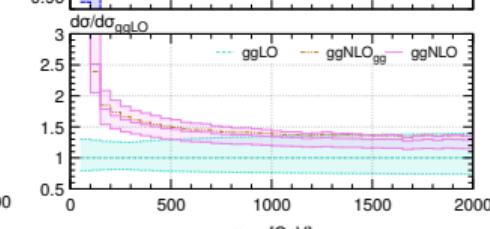
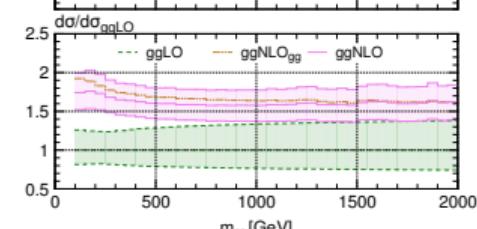
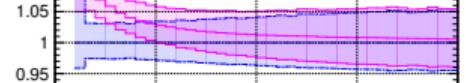
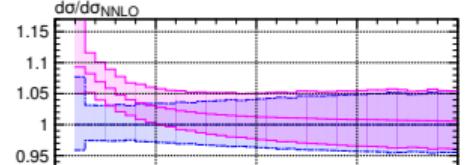
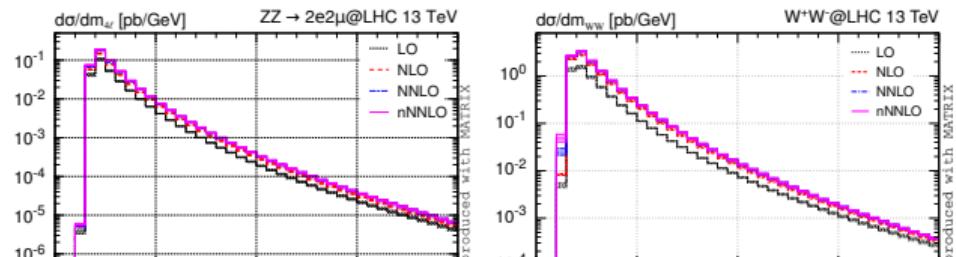
Effect of NLO QCD in gg channel

- highest impact in low-energy regions
- enhancement beyond NNLO QCD scale band

[Grazzini, SK, Wiesemann, Yook (2019)]



[Grazzini, SK, Wiesemann, Yook (2020)]



The MUNICH/MATRIX framework for automated NNLO calculations

MATRIX — MUNICH Automates qT-subtraction and Resummation to Integrate X-sections

[Grazzini, SK, Wiesemann (2018)]

- first, and so far only, public tool that performs NNLO QCD calculations for a large class of processes
- core of the framework: the C++ parton-level Monte Carlo generator

MUNICH — MUlti-chaNnel Integrator at swiss (CH) precision [SK]

- bookkeeping of partonic subprocesses for all contributions
 - fully automated dipole subtraction for NLO calculations (massive, QCD and EW)
 - general amplitude interface
 - highly efficient multi-channel Monte Carlo integration with several optimization features
 - simultaneous monitoring of slicing parameter and automated extrapolation
-
- PYTHON script to simplify the use of MATRIX
 - installation of MUNICH and all supplementary software
 - interactive shell steering all run phases without human intervention (grid-, pre-, main-run, summary)
 - organization of parallelized running on multicore machines and commonly used clusters:
SLURM, HTCONDOR, LSF, etc.

Performance features of the MUNICH phase space integrator

Issue of poorly populated regions

- sample case: high-energy tails
- standard phase space optimization samples points in bulk region

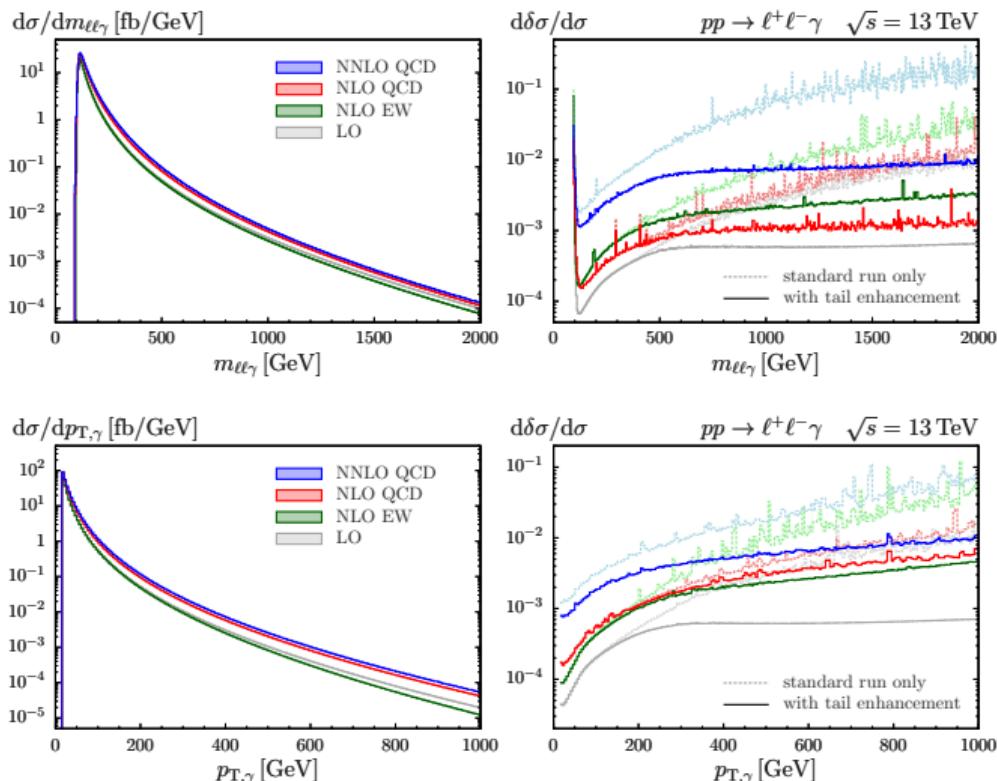
Solution in MUNICH integrator

- additional runs with optimization including a general bias factor
- sophisticated automated combination with results from standard runs

Significantly improved errors

- $\mathcal{O}(10)$ and better with doubled runtime
- simultaneous enhancement of observables

Good performance also for off-shell regions of intermediate resonances



Supplying MUNICH/MATRIX with 1-loop amplitudes

Process-independent interfaces to general automated amplitude generators

- OPENLOOPS [Cascioli, Maierhöfer, Pozzorini (2012); SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)], written in FORTRAN
 - general code and process libraries
 - on-the-fly tensor reduction [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)] with hybrid-precision stability system
 - scalar integrals from COLLIER [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2011)] or ONELOOP [van Hameren (2011)]
- RECOLA [Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)], written in FORTRAN
 - on-the-fly generation of amplitudes
 - tensor reduction and scalar integrals via COLLIER [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2003, 2006, 2011)]
 - different model files available, also for SMEFT and BSM applications
- modular structure of MUNICH allows other generators to be interfaced as well

Several dedicated interfaces developed in context of MATRIX applications

- loop×tree and loop×loop colour (and spin) correlators
- helicity amplitudes, colour-stripped amplitudes to construct 4-colour correlators
- imaginary parts of loop×tree amplitudes and correlators, helicity-flip amplitudes

Interfacing dedicated 2-loop amplitudes to MUNICH/MATRIX

- Higgs, Drell–Yan, **VH**, $\gamma\gamma$, $V\gamma$ production
 - direct implementation of public analytic results, e.g. for $V\gamma$ [Gehrmann, Tandredi (2012)]
- **VV** production — **qqVVAMP** [Gehrmann, von Manteuffel, Tancredi (2015)] and **ggVVAMP** [von Manteuffel, Tancredi (2015)] libraries
 - **C++** libraries using **GINAC** [Bauer, Frink, Kreckel (2002); Vollinga, Weinzierl (2005)] and **CLN** for arbitrary precision arithmetics
 - IBP approach, generated using **MATHEMATICA**, **FORM** [Vermaseren et al.], **REDUCE2** [von Manteuffel, Studerus ('12)]
 - independent calculation of amplitudes in [Caola, Henn, Melnikov, Smirnov, Smirnov (2015; 2016)]
 - Higgs-mediated helicity amplitudes with full m_t dependence from [Harlander, Prausa, Usovitsch (2019; 2020)]
- $\gamma\gamma\gamma$ production — amplitudes from [Abreu, Page, Pascual, Sotnikov ('20)]
 - **C++** library, generated by **CARAVEL** [Abreu et al. (2020)], applying **PENTAGONFUNCTIONS++** [Chicherin, Sotnikov (2020)]
 - numerical unitarity and analytic reconstruction techniques [Ita (2015); Abreu et al. (2018; 2018; 2019; 2019)]
- **HH** production (full m_t dependence) — **HHGRID** library [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
 - **PYTHON** based numerical interpolation of amplitude grid
 - generated by 2-loop extension of **GoSam** [Jones (2016)], **REDUCE2** [von Manteuffel, Studerus ('12)], **SECDEC3** [Borowka et al. (2015)]
- **Q \bar{Q}** production — amplitude grids from [Bärnreuther, Czakon, Fiedler (2014)]
 - **FORTRAN** routine for numerical interpolation of 2-dimensional grid, improved by expansions