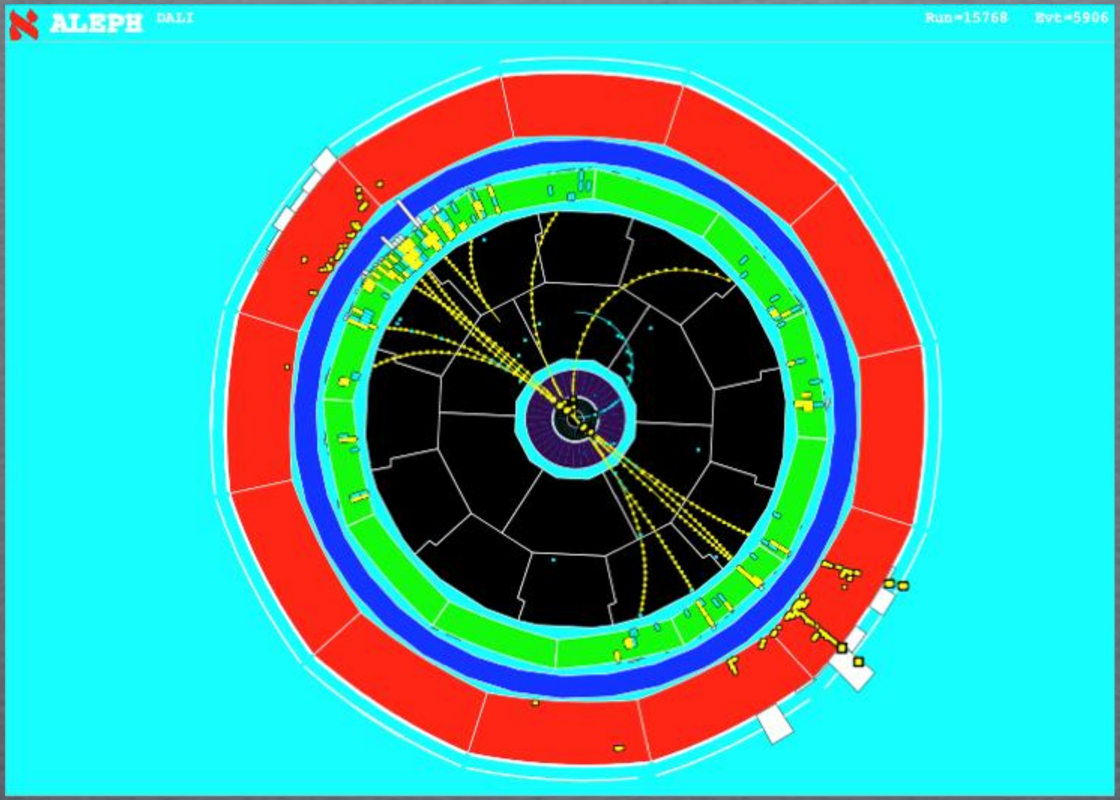
Resummation for collider phenomenology



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Joint workshop
"GDR-QCD/STRONG2020/PARTONS/FTE@LHC/NLOACESS"
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Outline

- Resummation: what it is and how you do it
- The precision frontier
- Spin correlations
- Jet substructure

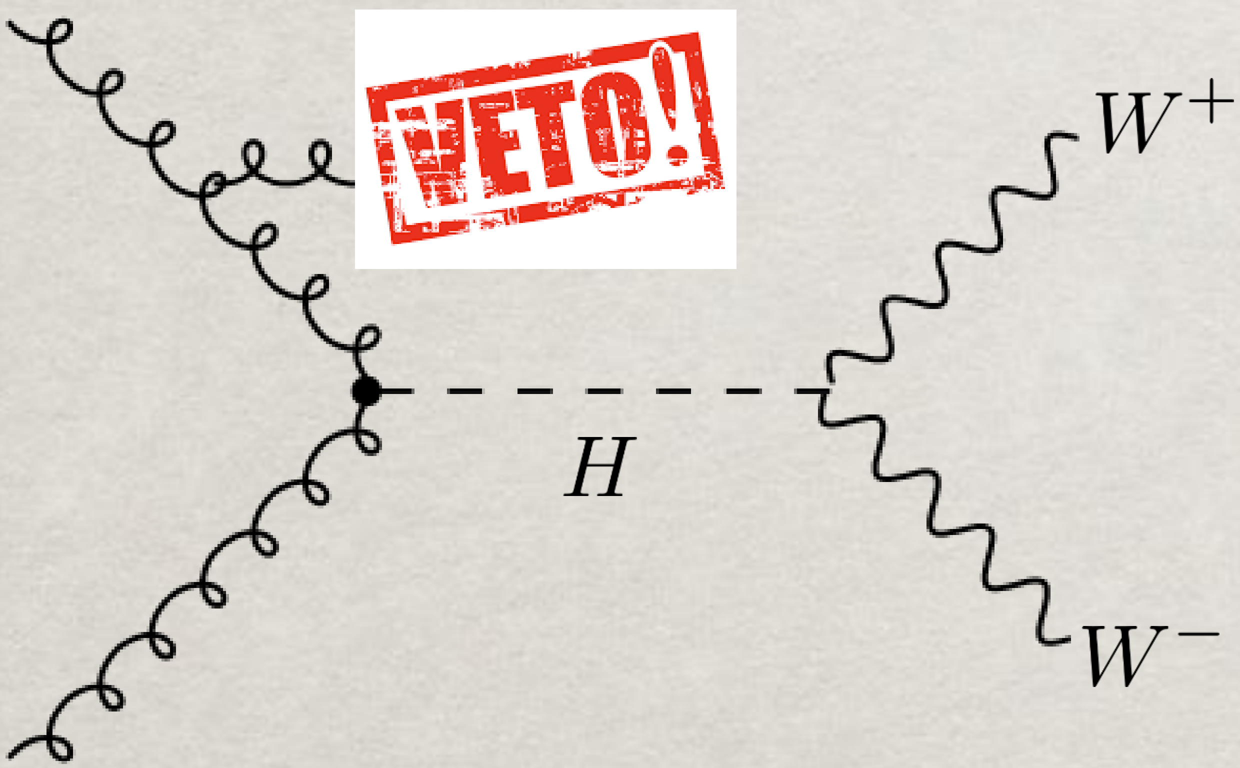
What is resummation?

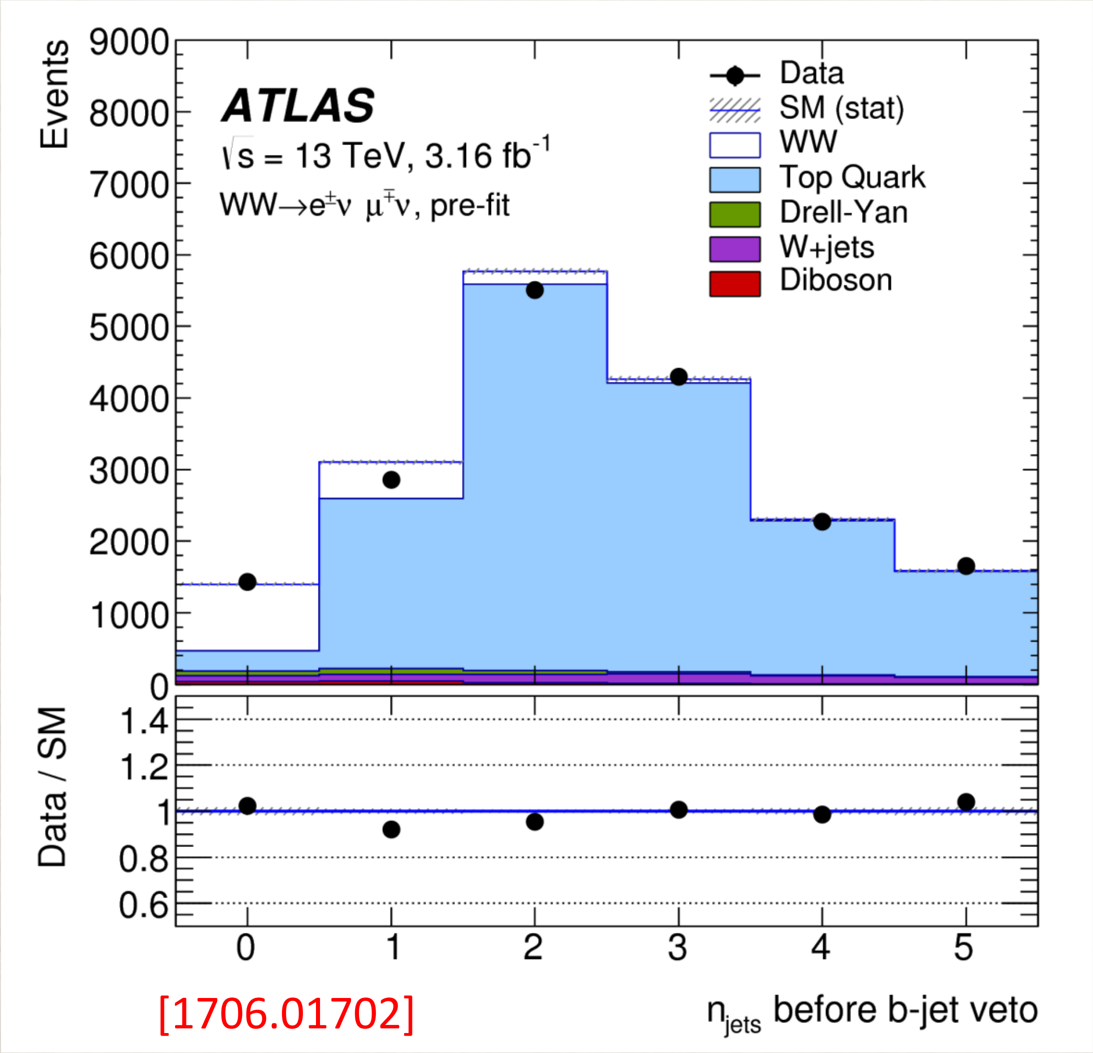
Two-scale problems

- Resummation is all about taming two-scale problems, where an object characterised by a “hard scale” Q is accompanied by softer objects at a scale $Q_0 \ll Q$

Two-scale problems in QCD

- Example: an object with a large invariant mass Q is produced and the transverse momentum of accompanying jets is below $Q_0 \ll Q$





- In QCD, large logarithm $\ln(Q/Q_0)$ appear whenever the phase space for the emission of soft and/or collinear gluons is restricted

Two-scale observables at FO

- Consider the cumulative distribution $\Sigma(Q, Q_0)$, the fraction of events such that an observable is below a given resolution Q_0 (e.g. $p_{\mathrm{t,jet}} < Q_0$)

- Logarithmic contributions become large whenever $Q_0 \ll Q$

$$\Sigma(Q, Q_0) \simeq 1 \quad - \quad C\,\frac{\alpha_s}{\pi}\ln^2\left(\frac{Q}{Q_0}\right) \quad + \quad \ldots$$

LO NLO

breakdown of perturbation theory!

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with $L \equiv \ln(Q/Q_0)$

$$\Sigma(Q, Q_0) \sim e^{\overbrace{Lg_1(\alpha_s L)}^{\text{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}} + \dots \right)$$

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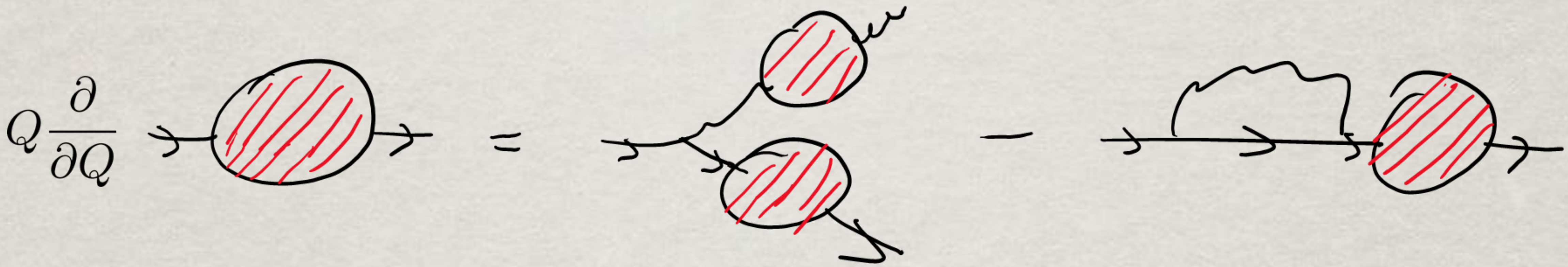
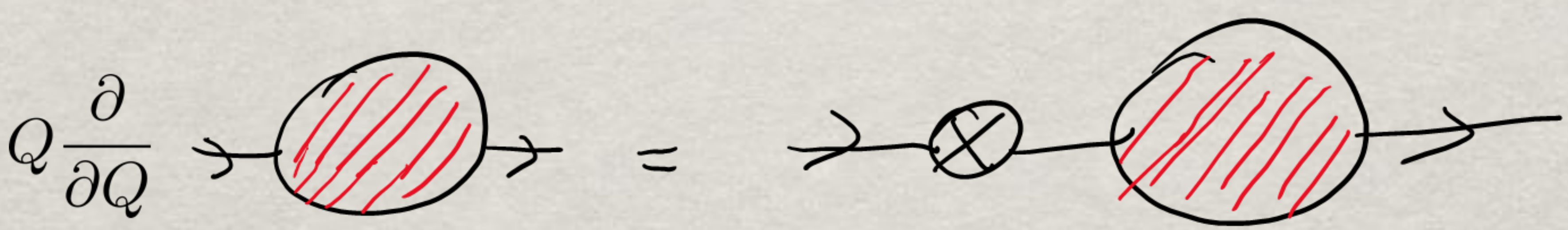
- The state-of-the art is NNLL for most observables, and for some even N^3LL
- The above representation holds only when LL factorise. For all other observables, there is no established way to define a resummation accuracy

QCD DYNAMICS IS NON-LINEAR

- Main problem of resummation: gluons and quarks radiate in the same fashion $\Longrightarrow$ QCD dynamics intrinsically non-linear



- The key feature of the most precise resummations is the possibility of following only one of the two branches of the splitting



- This results in a set of linear renormalisation group equations and resummation is mainly achieved by calculating anomalous dimensions

Factorisation is the key

- Resummation would never be possible if we could not separate process-dependent hard matrix elements from universal soft-collinear dynamics

  [Collins Soper Sterman hep-ph/0409313]

- Factorisation is enough to write general resummation formulae for suitable observables in QCD $\Longrightarrow$ CAESAR (NLL) / ARES (NNLL) approach

  [AB Salam Zanderighi hep-ph/0407286]

  [AB El-Menoufi Monni 1807.11487]

- Using the technique of strategy of regions, factorisation formulae can include observable constraints $\Longrightarrow$ Soft-collinear effective theory (SCET)

  [Bauer Fleming Pirjol Stewart hep-ph/0011336]

  [Becher Schwartz 0803.0342]

  [Becher Neubert 1007.4005]

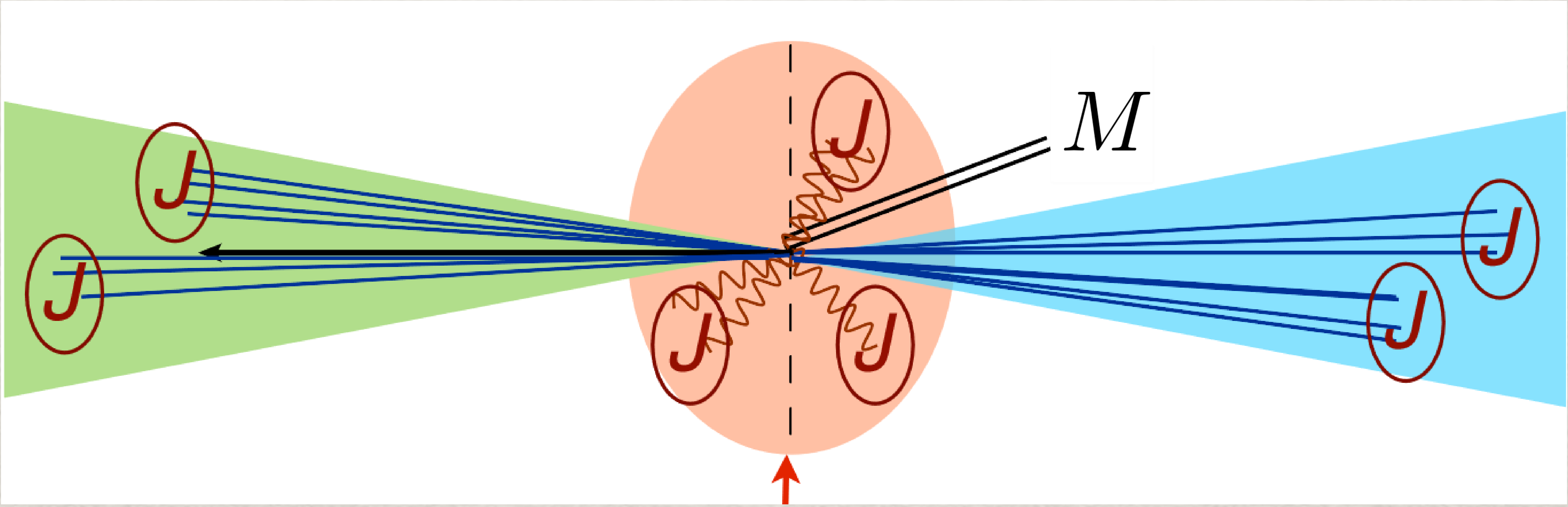
- Strategy of regions is the only way forward so far when no general principles such as gauge invariance can be invoked, e.g. quark-mass logarithms in loop amplitudes [Liu Mecaj Neubert Wang 2009.06779]

The precision frontier

Transverse momentum distributions

- The distribution in the transverse momentum of a colour singlet (e.g. Higgs, Z boson) admits an all order factorisation

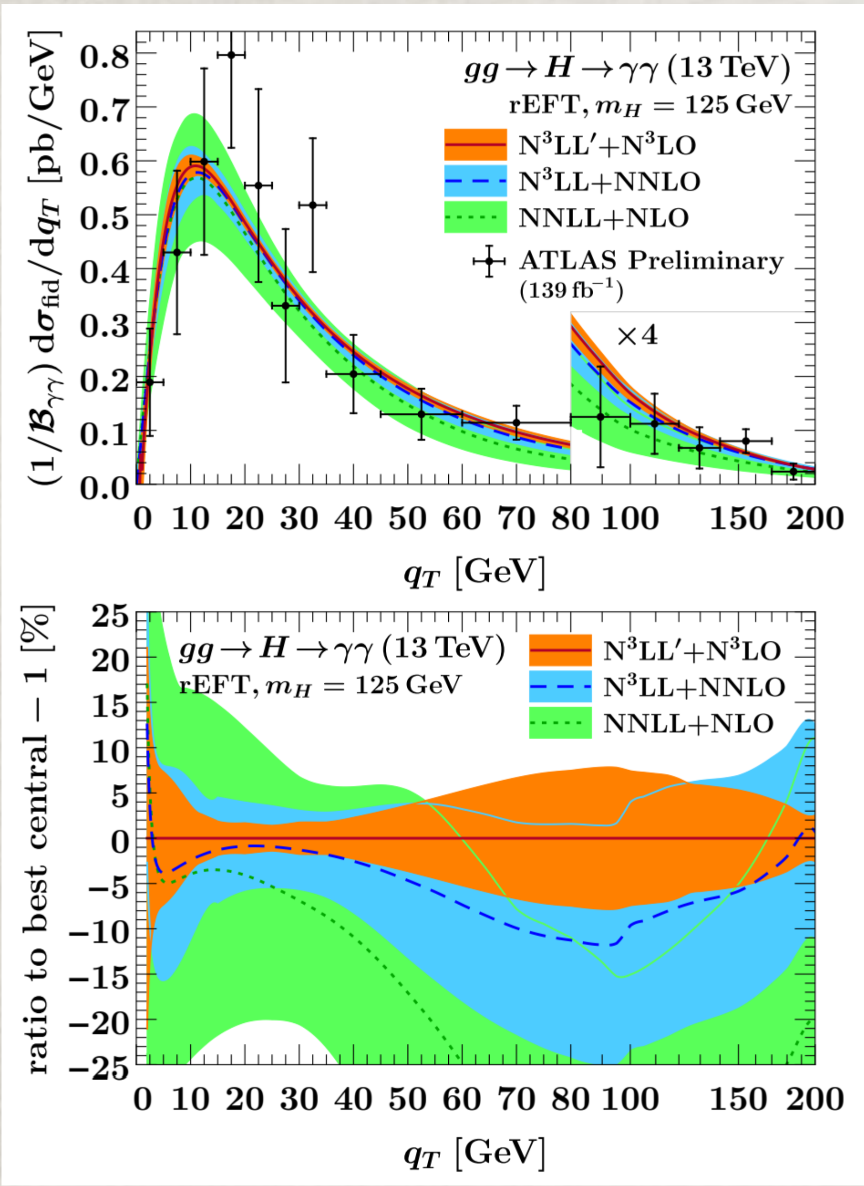
$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J) \otimes \mathcal{H}(M, \mu_H) \otimes \mathcal{B}(\mu_J)$$



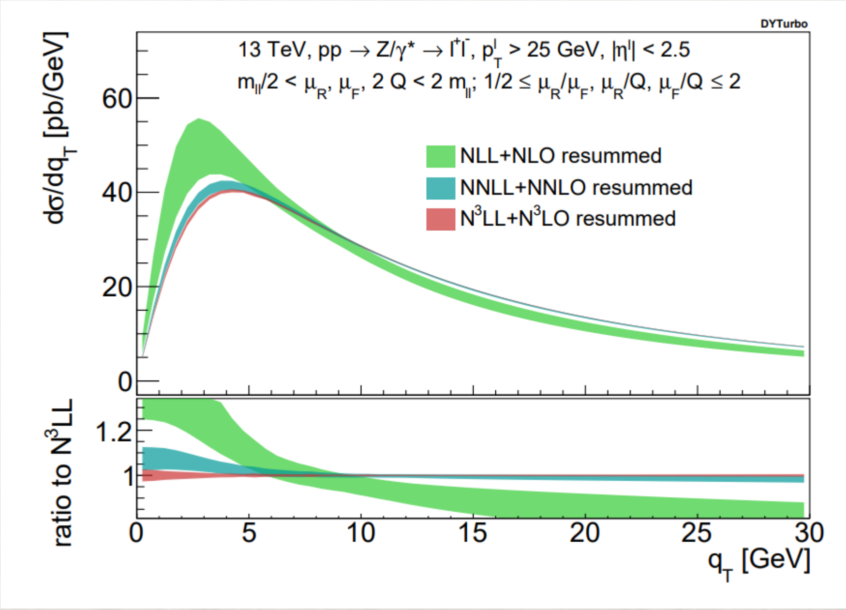
- Calculation of relevant anomalous dimensions makes it possible to achieve a remarkable N^{3}LL accuracy (times a constant at order α_s^3, a.k.a N^{3}LL')

B-space resummation

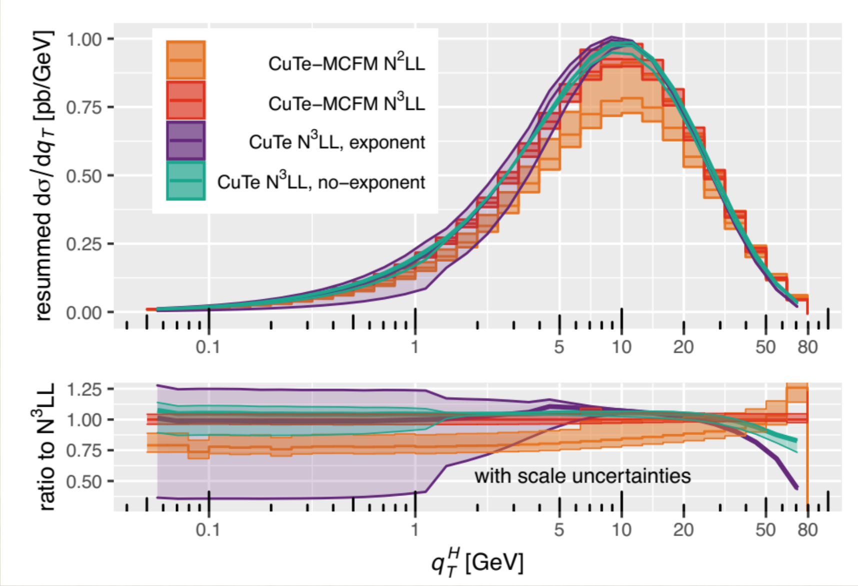
- Three calculations perform the resummation in impact-parameter or b-space, i.e. for the Fourier transform of $d\sigma/dq_T$



[Billis et al 2102.08039]



[Camarda et al 2103.04974]

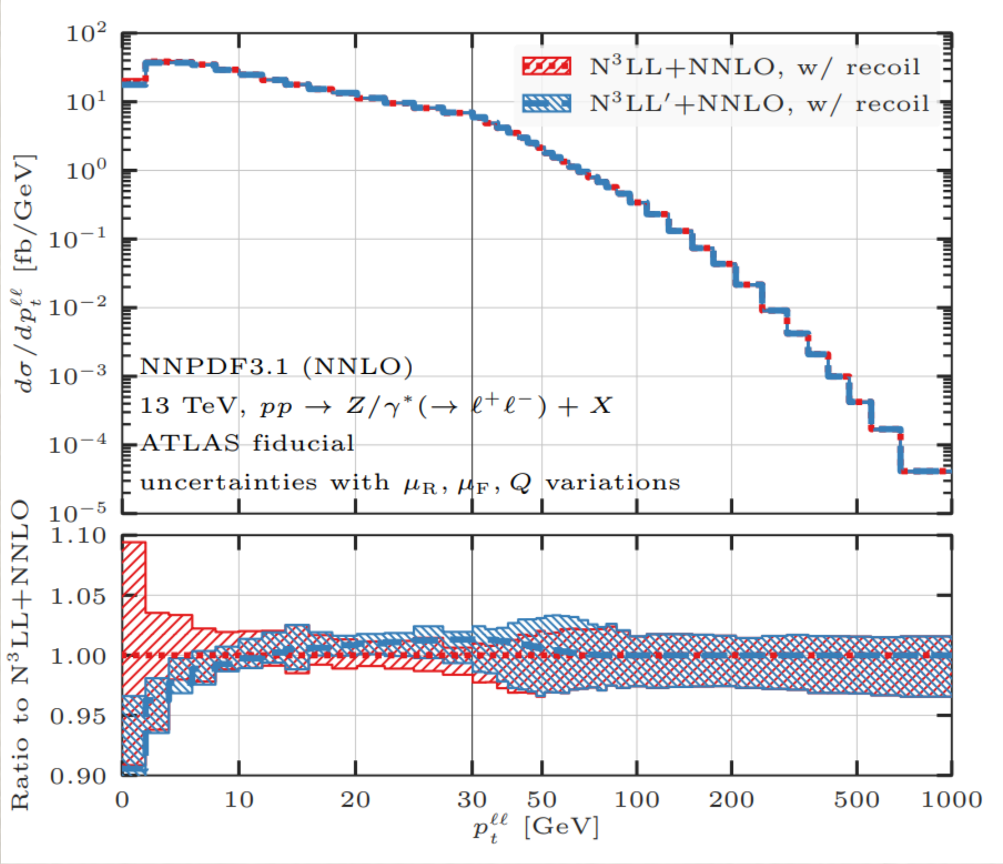


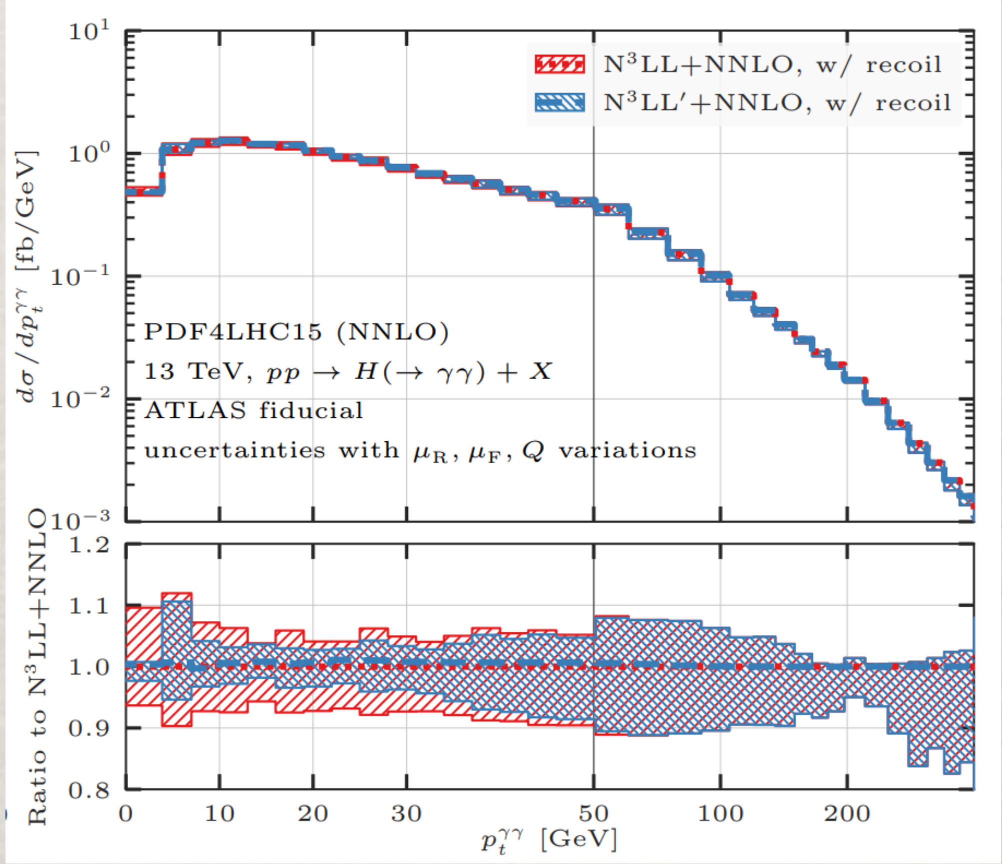
[Becher Neumann 2009.11437]

DIRECT SPACE RESUMMATION

- In the RadISH formalism, one resums $\ln(k_{t1}/M)$ where k_{t1} is the transverse momentum of the leading parton [Monni Re Torrielli 1604.02191]

- The resummation in $\ln(k_{t1}/M)$ is performed with the ARES philosophy and then the result is then integrated over k_{t1} and binned in q_T



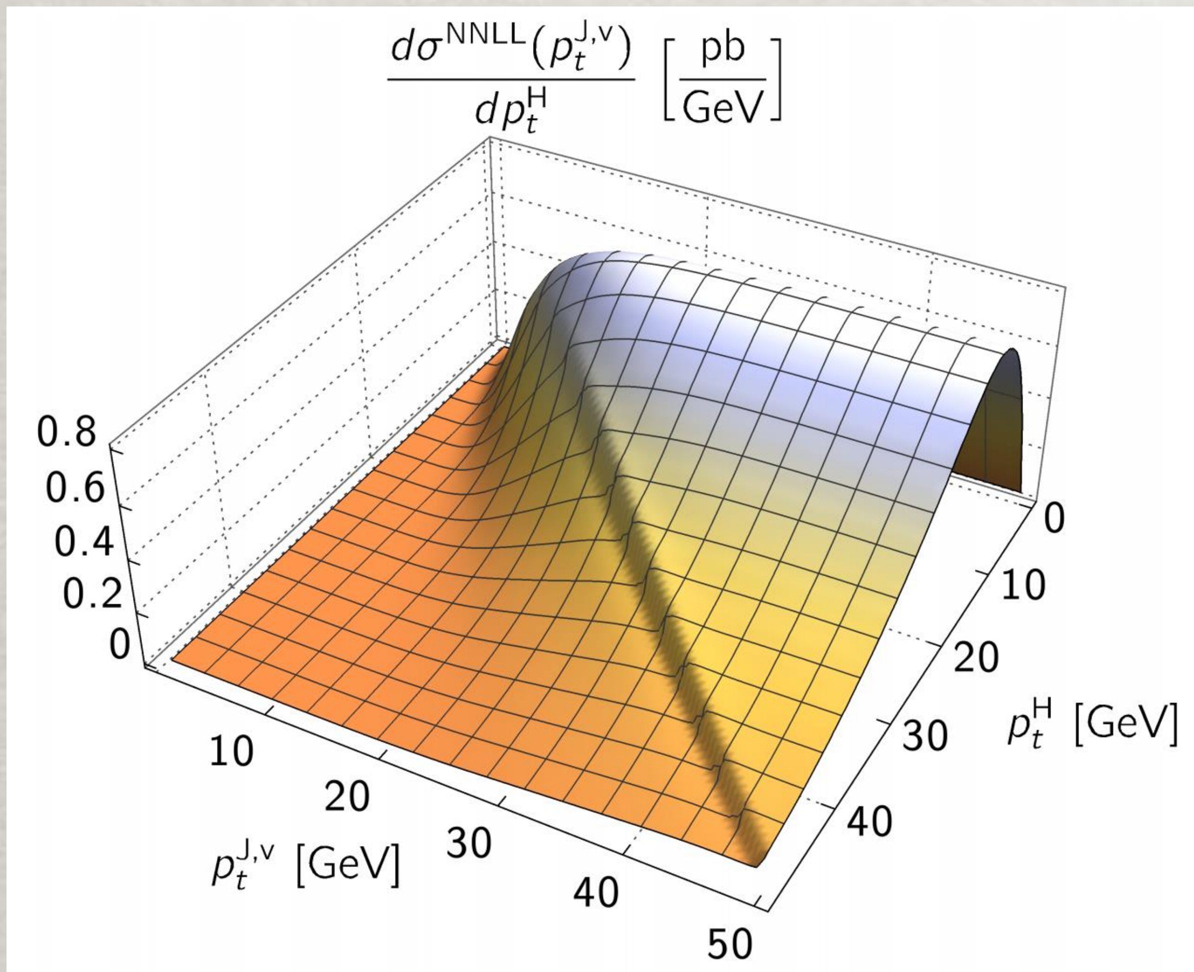


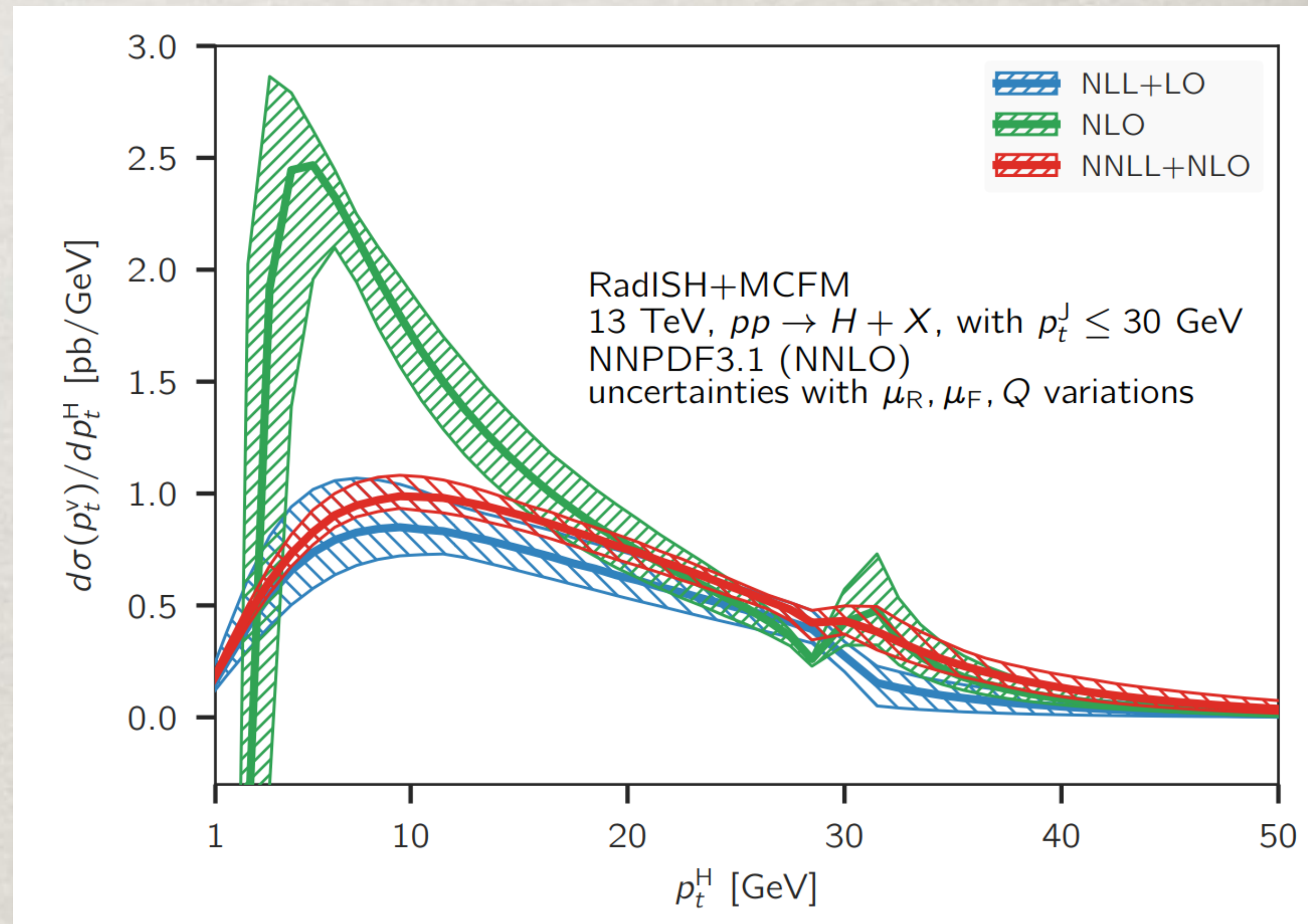
[Re Rottoli Torrielli 2104.07509]

- Uncertainties are reduced if one considers formally power-suppressed terms where the colour singlet acquires non-zero q_T via resummation

Adding constraints

- With the same RadISH formalism used for the q_T distribution, it is possible to add a constraint $p_{t,\mathrm{jet}} < p_{t,\mathrm{veto}}$ [Monni Rottoli Torrielli 1909.04704]





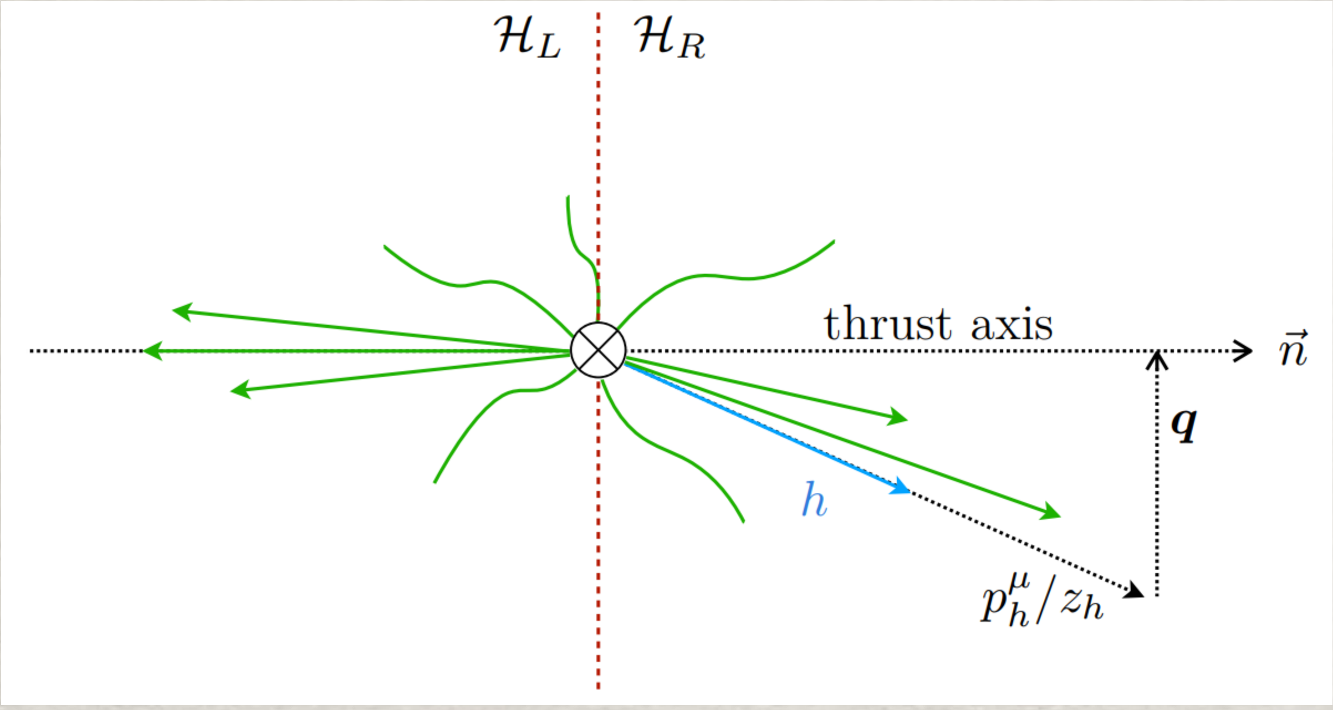
- The resummation accuracy here is “limited” to NNLL because of missing calculation of jet-veto effects

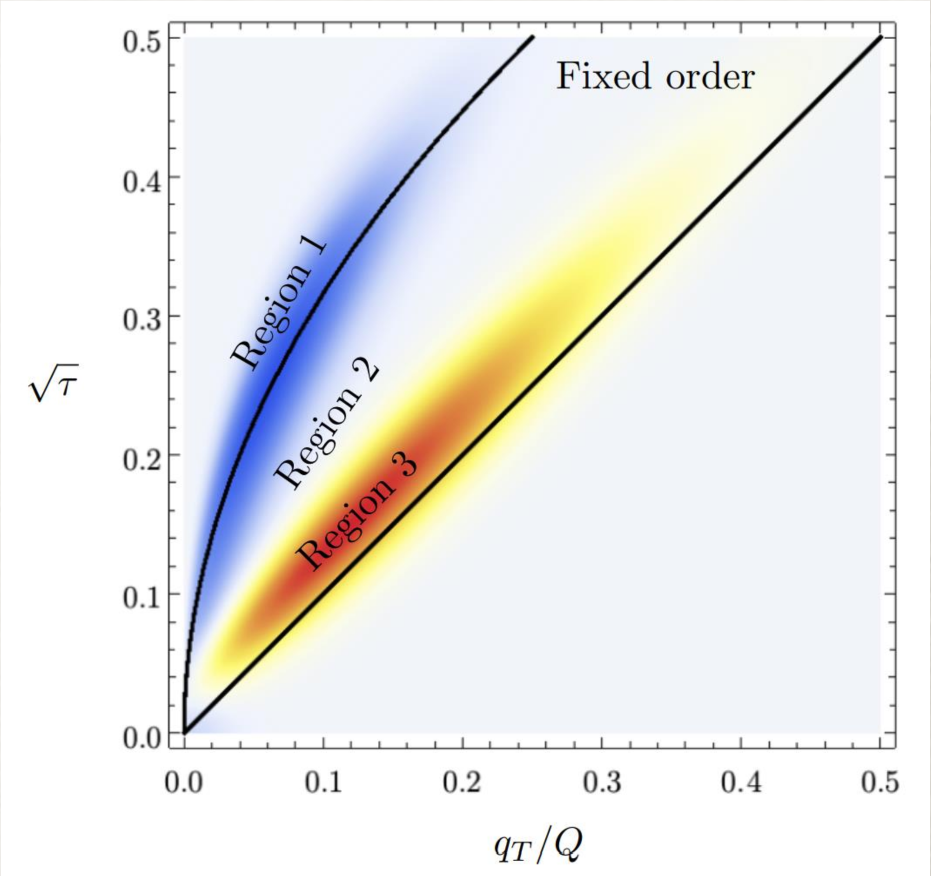
Adding constraints

- An observable similar to q_T in e^+e^- is the transverse momentum distribution of hadrons with respect to the thrust axis

- Within SCET, a simultaneous resummation of TMD fragmentation function and thrust distribution has been achieved at NNLL accuracy

[Makris Ringer Waalewijn 2009.11871]



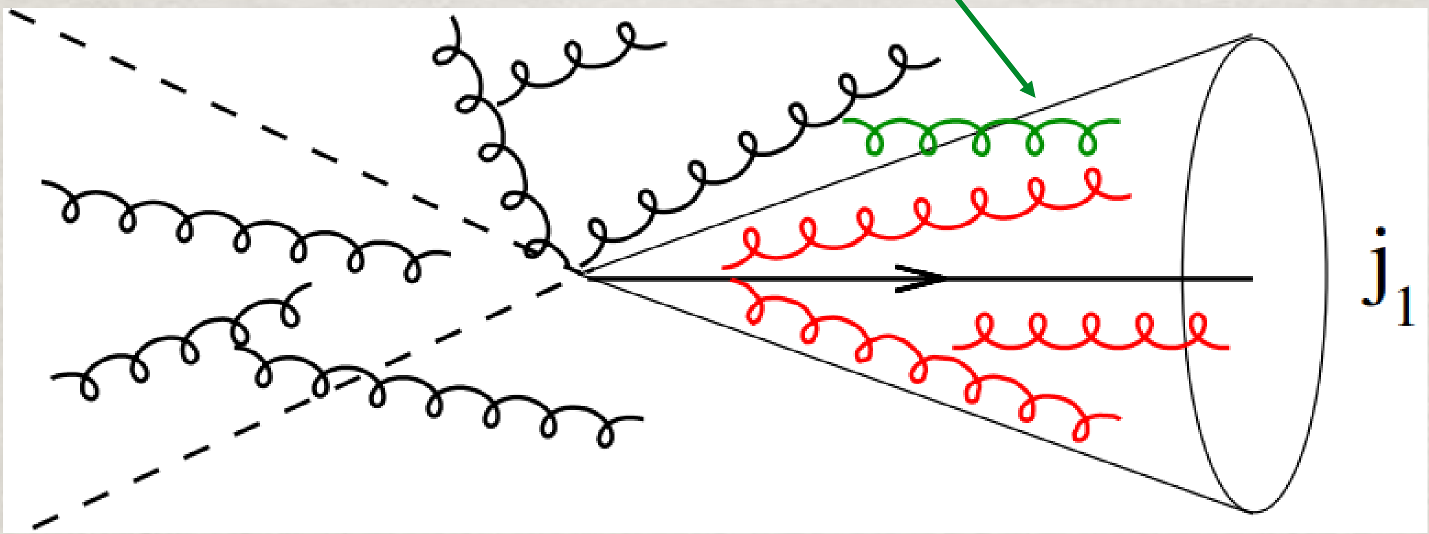


- For $q_T/Q \ll \tau$ one needs to consider non-global logarithms

Non-global logarithms

- Non-global logarithms (NGLs) arise whenever measurements are restricted to limited regions of phase space, e.g. single-jet mass distribution

- They originate when softest emission in a correlated cascade of soft gluons enters the measurement region [Dasgupta Salam hep-ph/0104277]

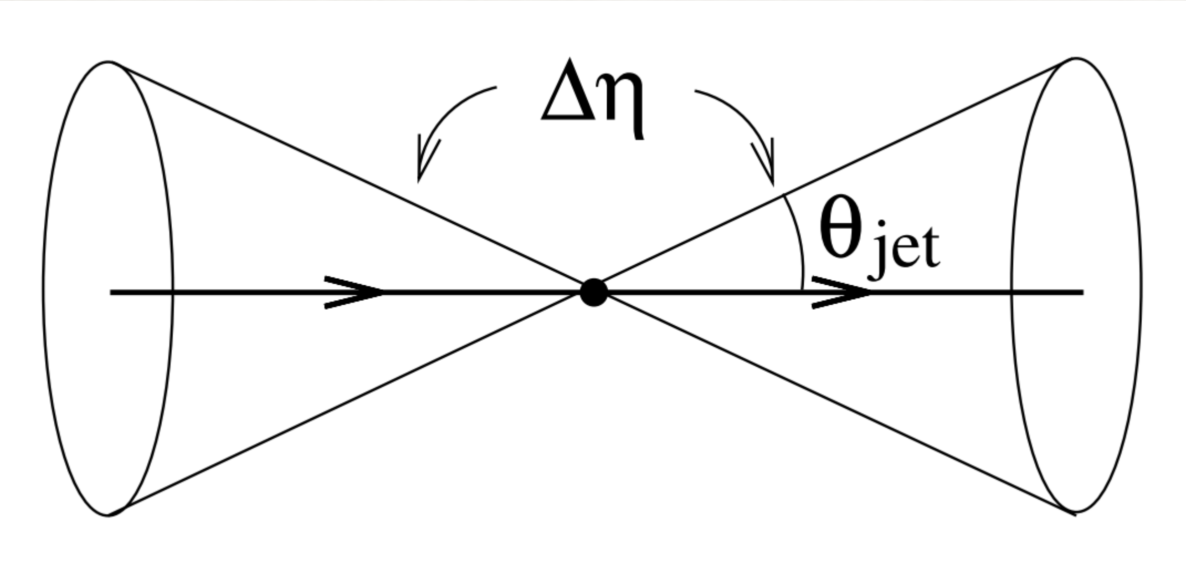


- Leading NGLs are single logarithms, of the form $\alpha_s^n \ln^n(Q/Q_0)$

- Leading NGLs in the limit of a large number of colours (large-N_c), they are resummed via the non-linear BMS equation [AB Marchesini Smye hep-ph/0206076]

NLL non-global logarithms

- Case study: veto emissions inside a rapidity slice



$$\Sigma(Q, Q_0) = \text{Prob}\left[\sum_{|\eta_i|<\Delta\eta} E_i < Q_0\right]$$

- All leading logarithms $\alpha_s^n \ln^n(Q/Q_0)$ in $\Sigma(Q, Q_0)$ are resummed by the BMS equation [AB Marchesini Smye hep-ph/0206076]

- At LL accuracy, it is also possible to include finite-N_c effects [Hatta Ueda 1304.6930]

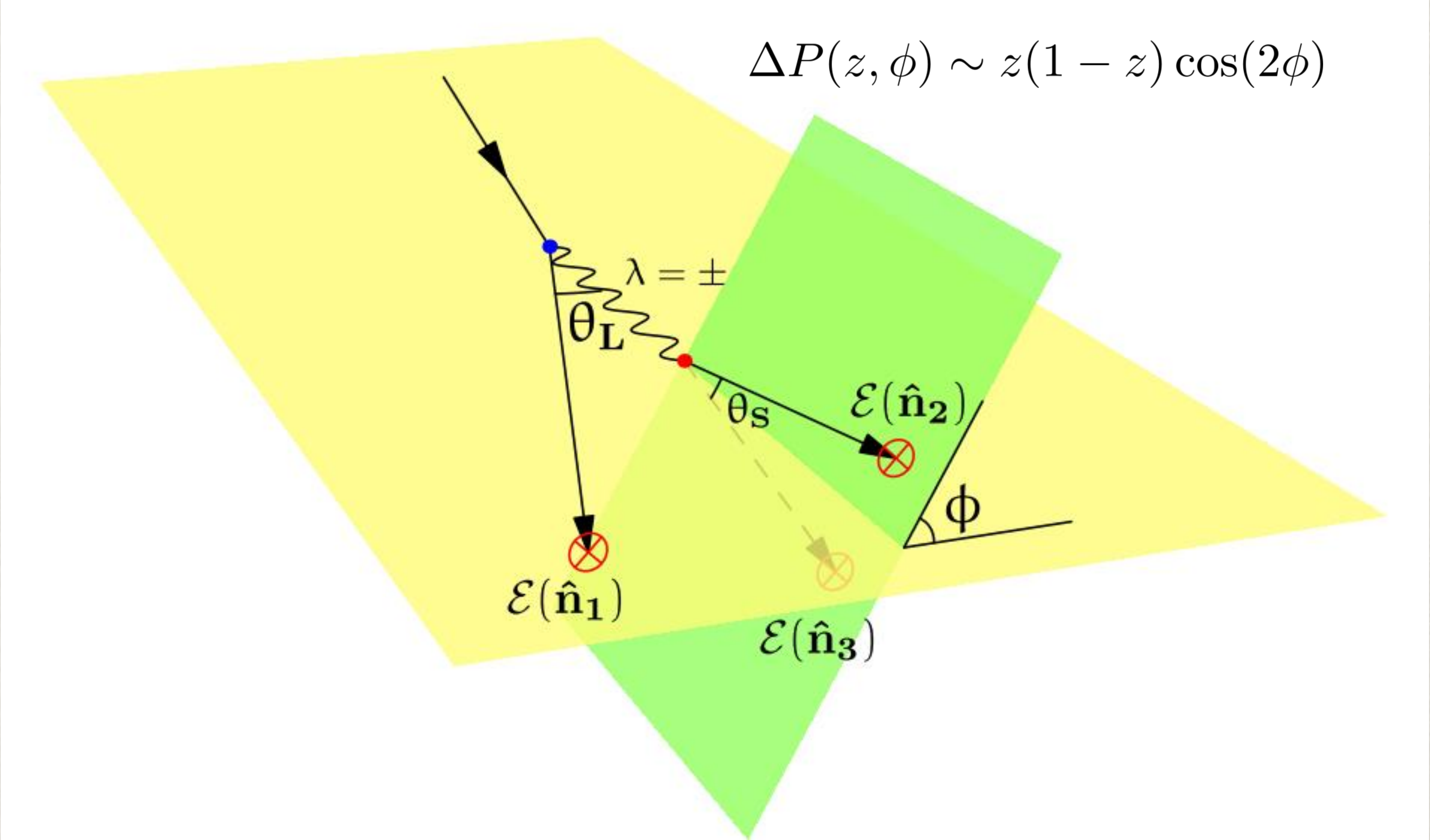
- The kernel of BMS equation has been recently improved at NLL accuracy in the large-N_c limit [AB Dreyer Monni 2104.06416]

- The new NLL resummation is suitable for Monte Carlo implementation

Spin correlations

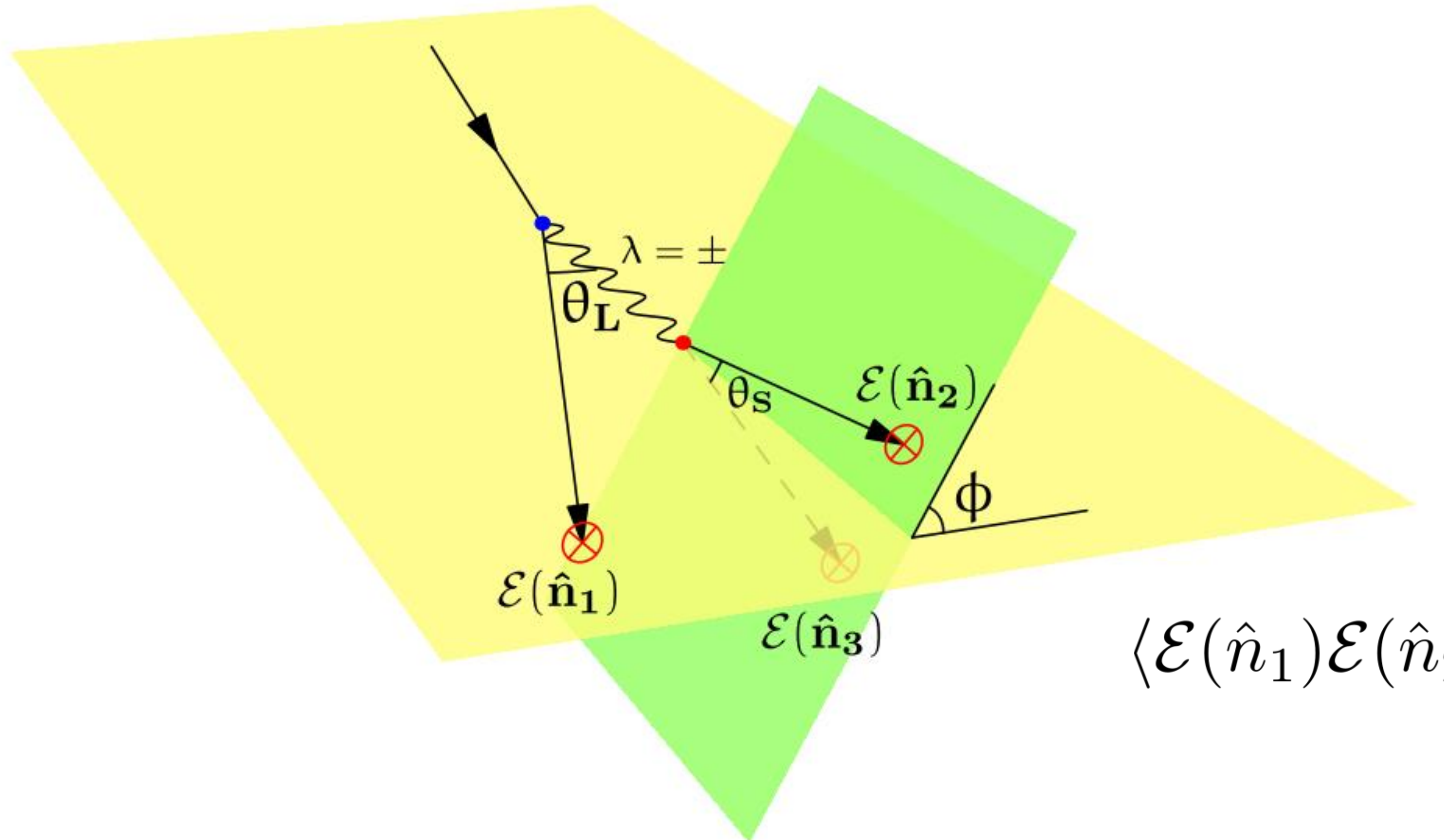
Gluon spin correlations

- Since gluons have spin-1, due to spin correlations their splittings depend on the azimuth with respect to a suitably defined plane



Triple-energy correlations

- Such spin correlations can be probed at single-logarithmic accuracy via triple-energy correlations



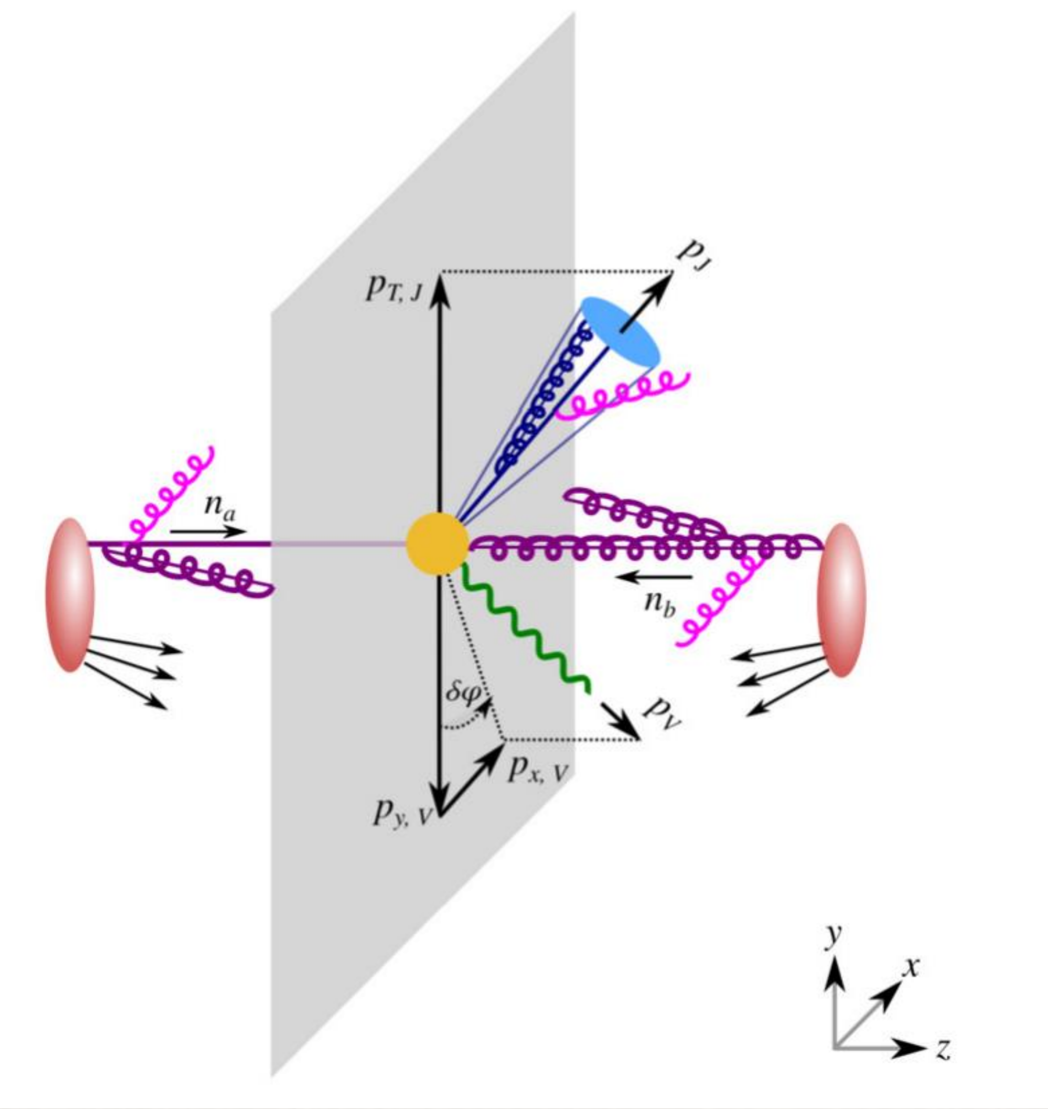
$$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3)\rangle \sim \frac{d\Sigma}{d\theta_L^2 d\theta_S^2 d\phi}$$

- Resummation of the multi-parton energy correlation is related to CFT light-ray operators $\vec{\mathbb{O}}^{[J]}$ of spin J and their anomalous dimensions [Chen Moult Zhu 2104.00009]

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = \frac{1}{(2\pi)^2}\frac{2}{\theta_L^2}\frac{2}{\theta_S^2}\vec{\mathcal{J}}\hat{C}(\phi)\left(\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right)^{\frac{\hat{\gamma}(3)}{\beta_0}}\left(\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right)^{\frac{\hat{\gamma}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1)$$

Azimuthal correlations

- The plane of the splitting can be fixed by the beam and a hard particle, e.g. a vector boson recoiling against a jet



- WTA jet axis $\Longrightarrow$ no NGLs $\Longrightarrow$ SCET all-order factorisation formula

[Chien et al 2005.12279]

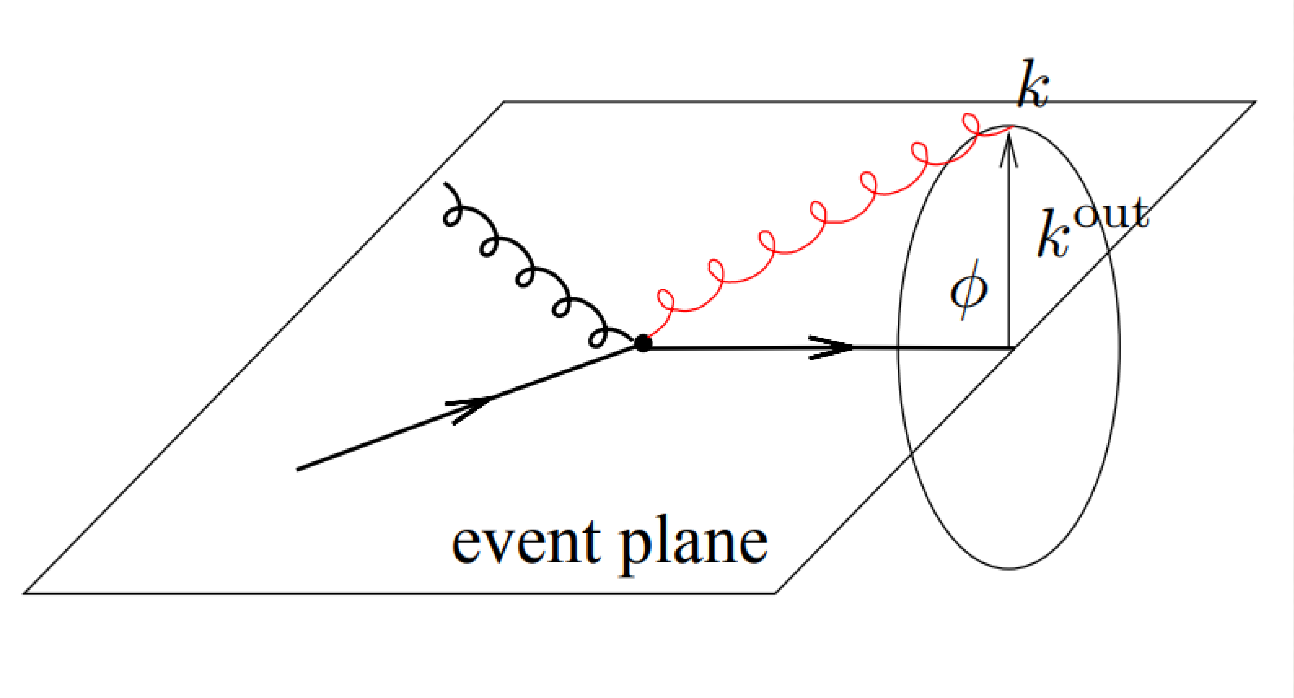
$$\frac{d\sigma}{dp_{x,V}\,dp_{T,J}\,dy_V\,d\eta_J} = \int \frac{db_x}{2\pi} e^{ip_{x,V}b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) \times \times S_{ijk}(b_x, \eta_J) \mathcal{H}_{ij\to Vk}(p_{T,V}, \eta_V - \eta_J) \times \times \mathcal{J}_k(b_x)$$

- Spin correlation effects occur at NNLL

- When the recoiling jet is a gluon jet, spin correlations give rise to a linearly polarised transverse momentum distribution $\mathcal{J}_g(b_\perp)$ in the final state

Near-to–planar shapes

- Out-of-plane event shapes, e.g. D-parameter, are also sensitive to spin correlations in gluon jets



$$D = \frac{27}{Q^3} \sum_{i<j<k} \frac{[\vec{p}_i \cdot (\vec{p}_j \times \vec{p}_k)]^2}{E_i E_j E_k}$$

- In the tree-jet region, spin correlations arise at NNLL

[Arpino AB El-Menoufi 1912.09341]

- In the two-jet region, when the hardest gluon is allowed to be soft and collinear, can spin correlation effects appear at NLL? [Larkoski Procita 1810.06563]

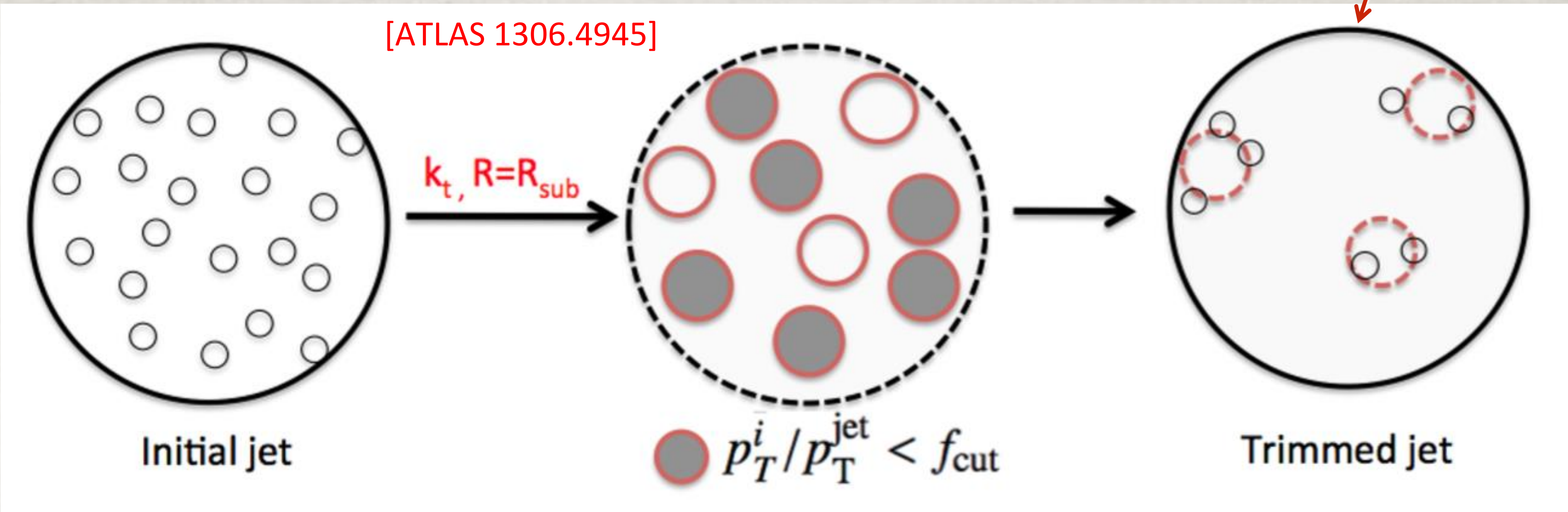
Jet substructure

Boosted objects

- Both HL-LHC and future colliders will produce boosted heavy objects, whose decay products fall in the same jet (e.g. boosted Higgs or tops)

[Butterworth Davison Rubin Salam 0802.2470]

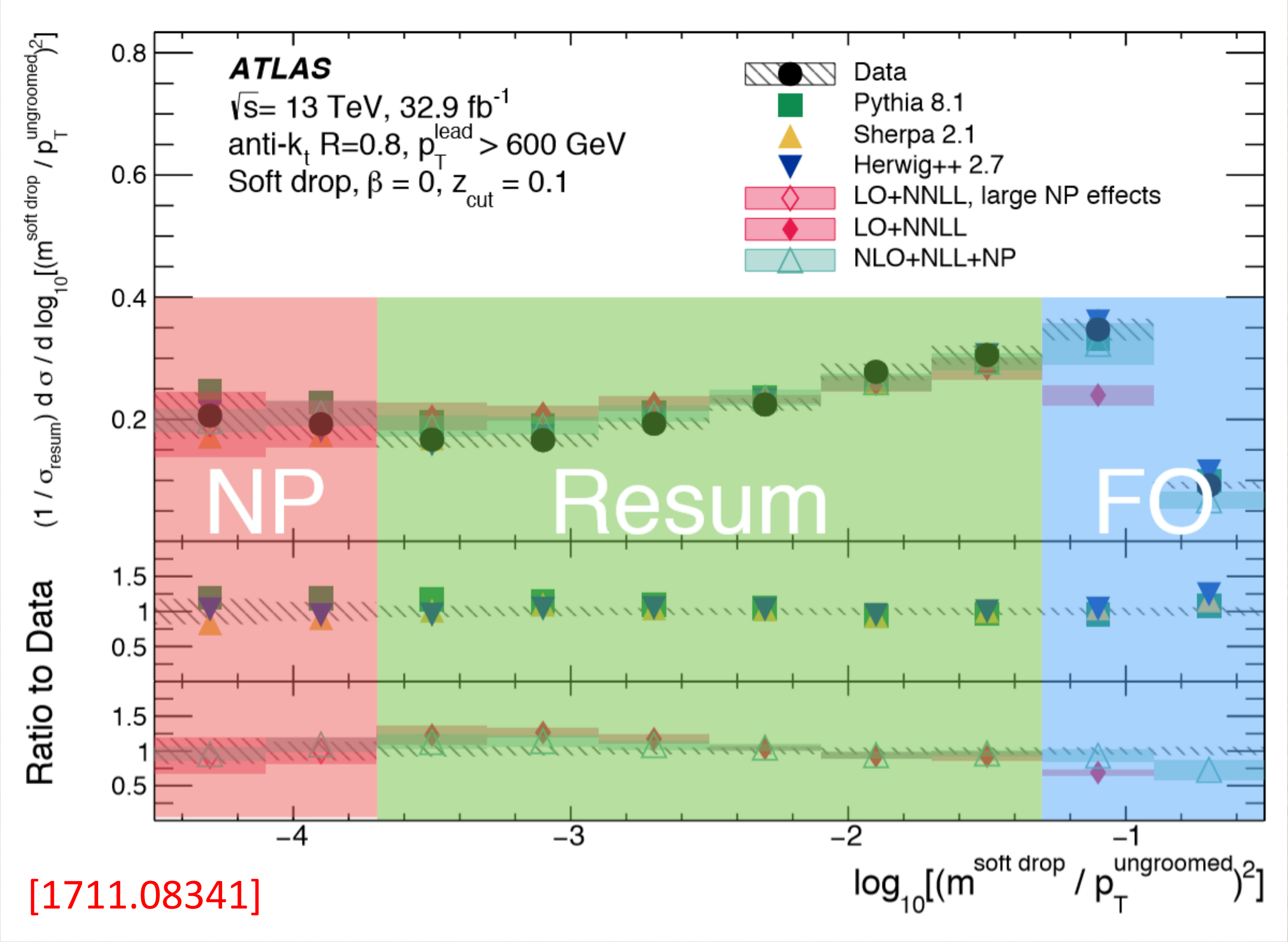
hadronic top candidate



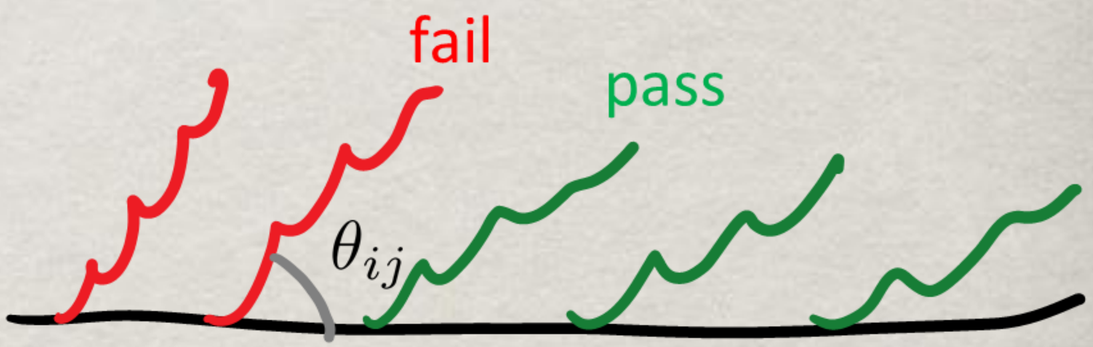
- Key feature of boosted object taggers are groomers, procedures that clean jets from soft constituents irrelevant for mass reconstruction

The soft-drop groomer

- Groomed jet-mass distributions cumbersome to model in QCD due to the presence of non-global logarithms (NGLs)
- New soft-drop groomers are free from NGLs $\Rightarrow$ accurate analytical modelling of different taggers now possible [Larkoski Marzani Soyez Thaler 1402.0007]



[1711.08341]

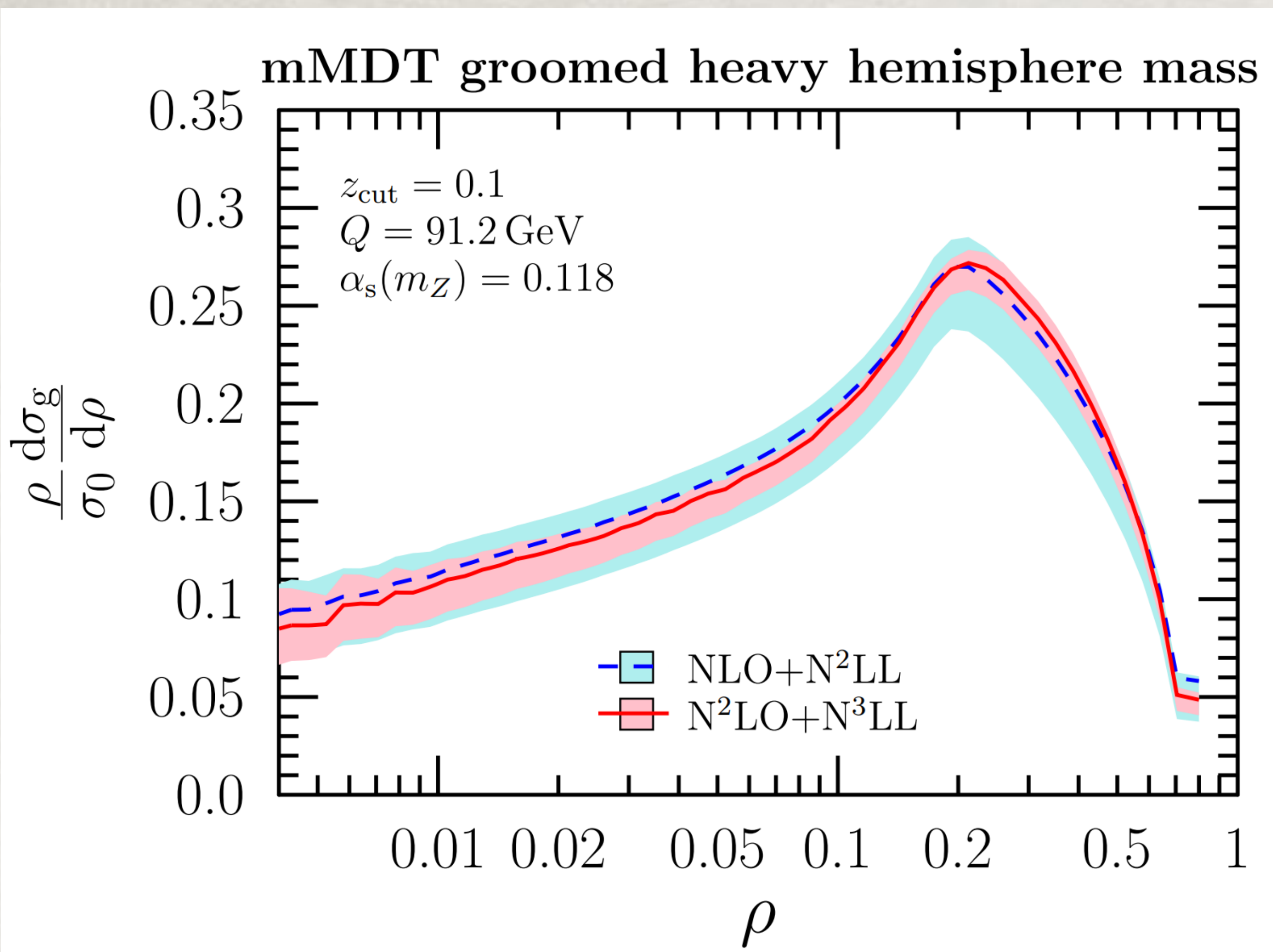


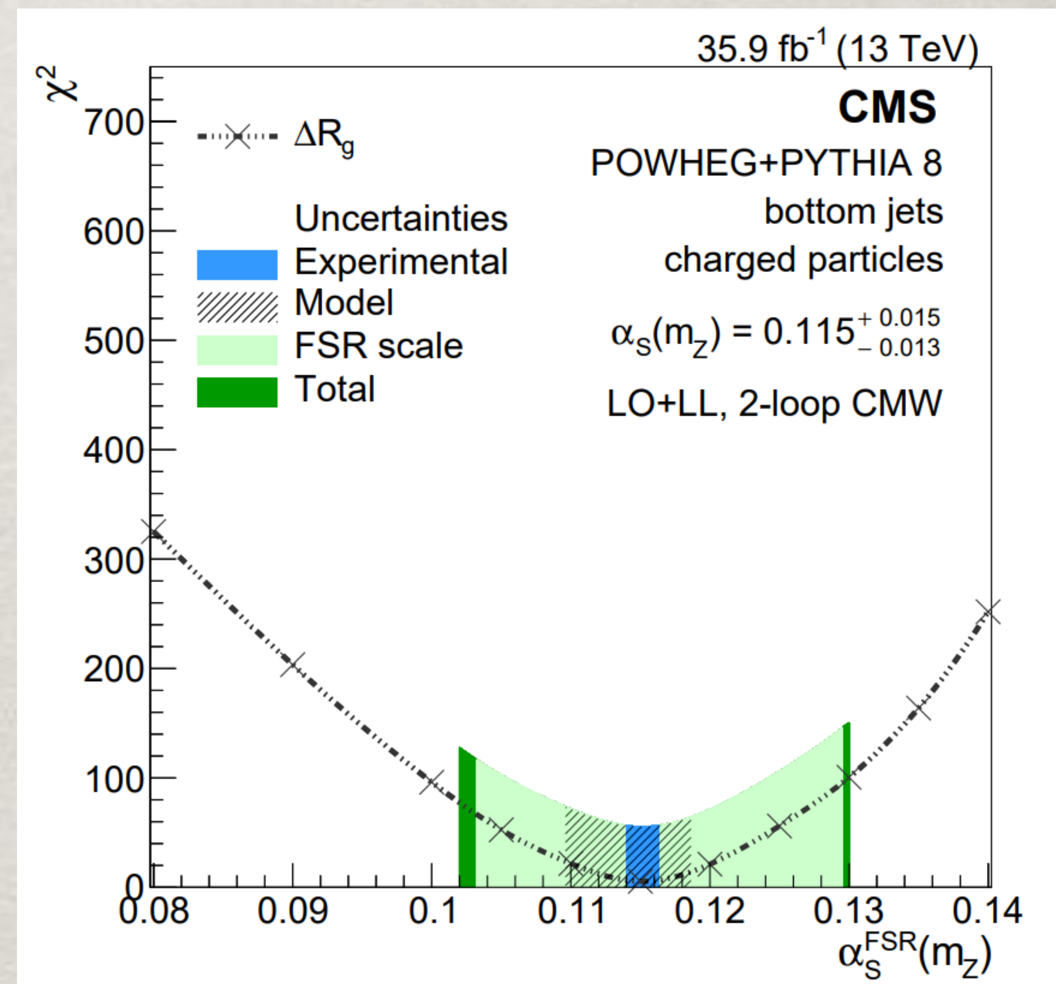
$$\frac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left(\frac{\theta_{ij}}{R}\right)^{\beta}$$

$\beta = 0$: mMDT

A NEW PRECISION ERA?

- The distribution in $\rho \equiv m_{\rm jet}^2/Q^2$ does obey a factorisation theorem in the region $m_{\rm jet}^2 \ll z_{\rm cut} Q^2 \ll Q^2$ [Frye Larkoski Schwartz Yan 1603.09338]
- Anomalous dimensions extracted from fixed-order calculations in e^+e^- annihilation $\Rightarrow$ N³LL resummation! [Kardos Larkoski Trocsanyi 2002.00942, 2002.05730]





- Preliminary studies of extraction of the strong coupling α_s at hadron colliders using jet-substructure observables [CMS 1808.07340]

Outlook

Resummation is a thriving field, with a plethora of new exciting results

- NNLL is the state-of-the-art, achievable both in QCD and with effective-theory methods
- Transverse momentum resummations have reached the impressive N^3LL accuracy
- First NLL resummation of non-global logarithms
- Renewed interest in spin-correlation effects
- New avenues for the use of jet-substructure observable for precision measurements, i.e. strong coupling

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Thank you for your attention!