Open heavy flavour production:
GM-VFNS & co

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Joint Workshop "GDR-QCD/QCD@short distances and STRONG2020/PARTONS/FTE@LHC/NLOAccess"
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Motivation

- Heavy quarks are copiously produced at the LHC and a good phenomenological understanding is needed:
    - **Elementary particle processes**
    - Important **hard probes of the QGP** in heavy ion collisions
    - **Background** to new physics
- **Multi-scale problem** serves as laboratory to understand other multi-scale problems in QCD and the SM (Higgs, W/Z, New Particles)
- **How to treat heavy quark masses consistently in pQCD?**
    - GM-VFNS/FONLL are used in modern **global analyses of PDF** to analyse heavy quark production in **deep inelastic scattering**
    - "pQCD with masses" should also work for **less inclusive observables**!
    - Heavy quark hadroproduction data provide constraints on the **gluon PDF**

Theoretical approaches

Theoretical Approaches

Heavy Flavour Production is a multi scale problem: p_T, m

- **FFNS** (Fixed Flavour Number Scheme):
  - Fixed Order Perturbtation Theory: LO, NLO, NNLO
- **ZM-VFNS** (Zero Mass Variable Flavour Number Scheme):
  - Resummed (RS): LL, NLL [Jean-Philippe Guillet, LAPTH]
- **Matched Fixed Order+Resummed**:
  - **GM-VFNS**: NLO+NLL [Ingo Schienbein, LPSC]
  - **FONLL**: NLO+NLL [Matteo Cacciari, LPTHE]
- **Matched Fixed Order+Parton Shower:**
  - MC@NLO: NLO+LL
  - **POWHEG:** NLO+LL [Emanuele Re, LAPTH]

Termes in the perturbation series

$L = \ln(m/p_T)$
$a = \alpha_s/(2\pi)$

Resummed ↓

Fixed Order →

	LL	NLL	NNLL	...
LO	1			
NLO	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...	...	...	...	...

FFNS/Fixed Order NLO

Resummed ↓

Fixed Order →

	LL	NLL	NNLL	...
LO $m \neq 0$	1			
NLO $m \neq 0$	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...	...	...	...	...

ZM-VFNS/Resummed NLL

Resummed ↓

Fixed Order →

	LL m=0	NLL m=0	NNLL	...
LO	1			
NLO	aL	a		
NNLO	$(aL)^2$	a(aL)	a^2	
...	...	...	...	...

GM-VFNS/FONLL (NLO+NLL)

Resummed ↓

Fixed Order →

	LL	NLL	NNLL	...
LO	1 m≠0			
NLO	aL m≠0	a m≠0		
NNLO	$(aL)^2$ m=0	a(aL) m=0	a^2	
...	... m=0	... m=0	...	...

FFNS/Fixed Order NNLO

Resummed ↓

Fixed Order →

	LL	NLL	NNLL	...
LO $m \neq 0$	1			
NLO $m \neq 0$	aL	a		
NNLO	$(aL)^2$	$a(aL)$	a^2	
...	...	...	...	...

Theoretical approaches:

Fixed Flavor Number Scheme (FFNS)

FFNS/Fixed Order

Factorization formula for inclusive heavy quark (Q) production:

PDFs

$$d\sigma^Q \simeq \sum_{a,b} f_a^A \otimes f_b^B \otimes d\tilde{\sigma}_{ab \to Q+X}$$

sum over all possible partonic subprocesses
NO heavy quark PDF

Calculable short distance cross section; log(pT/m) terms kept in **fixed order**

FFNS/Fixed Order

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Inclusive heavy-flavored hadron (H) production:

$$d\sigma^H = d\sigma^Q \otimes D_Q^H(z)$$

Convolution with a **scale-independent** FF

* non-perturbative
* describes hadronization
* not based on a fact. theorem

Theoretical approaches:
Zero Mass Variable Flavor Number Scheme (ZM-VFNS)

ZM-VFNS/RS

Factorization formula for inclusive heavy quark (Q) production:

$$d\sigma^{H+X} \simeq \sum_{a,b,c} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dz \; f_a^A(x_a, \mu_F) f_b^B(x_b, \mu_F) d\hat{\sigma}_{ab \to c+X} D_c^H(z, \mu_F') + \mathcal{O}(m^2/p_T^2)$$

- Same factorization formula as for inclusive production of pions and kaons
- Quark mass neglected in kinematics and the short distance cross section
- Allows to compute p_T spectrum for $p_T >> m$
- Needs **scale-dependent** FFs of quarks and gluons into the observed heavy-flavored hadron (H)

List of subprocesses in the ZM-VFNS

Massless NLO calculation: [Aversa,Chiappetta,Greco,Guillet,NPB327(1989)105]

1. $gg \to qX$
2. $gg \to gX$
3. $qg \to gX$
4. $qg \to qX$
5. $q\bar{q} \to gX$
6. $q\bar{q} \to qX$
7. $qg \to \bar{q}X$
8. $qg \to \bar{q}'X$
9. $qg \to q'X$
10. $qq \to gX$
11. $qq \to qX$
12. $q\bar{q} \to q'X$
13. $q\bar{q}' \to gX$
14. $q\bar{q}' \to qX$
15. $qq' \to gX$
16. $qq' \to qX$

$\oplus$ charge conjugated processes

- In the **VFNS** we need FFs into the heavy meson/baryon for:
  - Light quarks
  - Heavy quarks
  - Gluon
  - The entire VFNS can be extended to the one-particle inclusive case: evolution equations for PDFs and FFs and α_s; the matching conditions across the heavy flavor thresholds for PDFs and FFs and α_s; calculation of the short distance cross sections
- In the **FFNS** we only had one scale-independent FF of the heavy quark into the heavy meson/baryon

Cacciari, Mitov, Moch, ...

Fragmentation functions

Approach I: Perturbative FFs (PFFs)

Caccciari, Greco, Nason, Oleari, ...

$$D_i^H(z, \mu'_F) = D_i^Q(z, \mu'_F) \otimes D_Q^H(z)$$

PFF evolved with DGLAP; short distance; boundary condition calculable

Non-pert., scale-independent HF describing hadronization of heavy quark Q into heavy hadron H

Mellin-moments of $D_Q^H(z)$ determined from e^+e^- data

Approach II: treat FFs into H in the same way as FFs into pions or kaons

Binnewies, Kniehl, Kramer, ...

Non-pert. boundary conditions $D_i^H(z,m)$ from fit to e^+e^- data;
Determine FFs directly in x-space; evolved with DGLAP

PFF approach

Cacciari, Nason, PRL89(2002)122003

$$D_N \equiv \int D(z) z^N \frac{dz}{z}$$

$$\frac{d\sigma}{dp_T} = \int dz d\hat{p}_T \, D(z) \frac{A}{\hat{p}_T^n} \delta(p_T - z\hat{p}_T) = \frac{A}{p_T^n} D_n$$

n~3,4,5

Determine HF from N=2 moment in PFF approach; not from entire x-spectrum

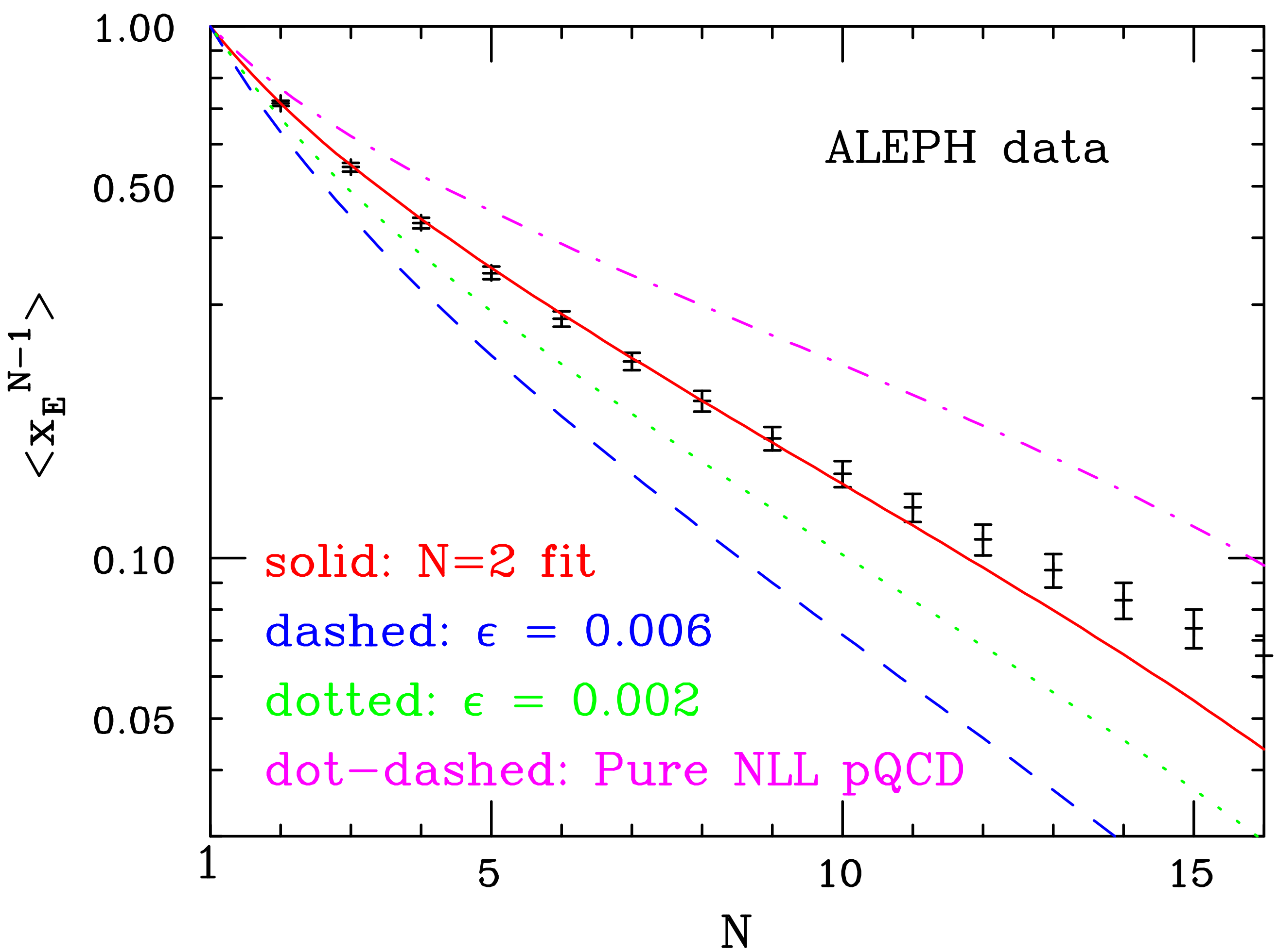
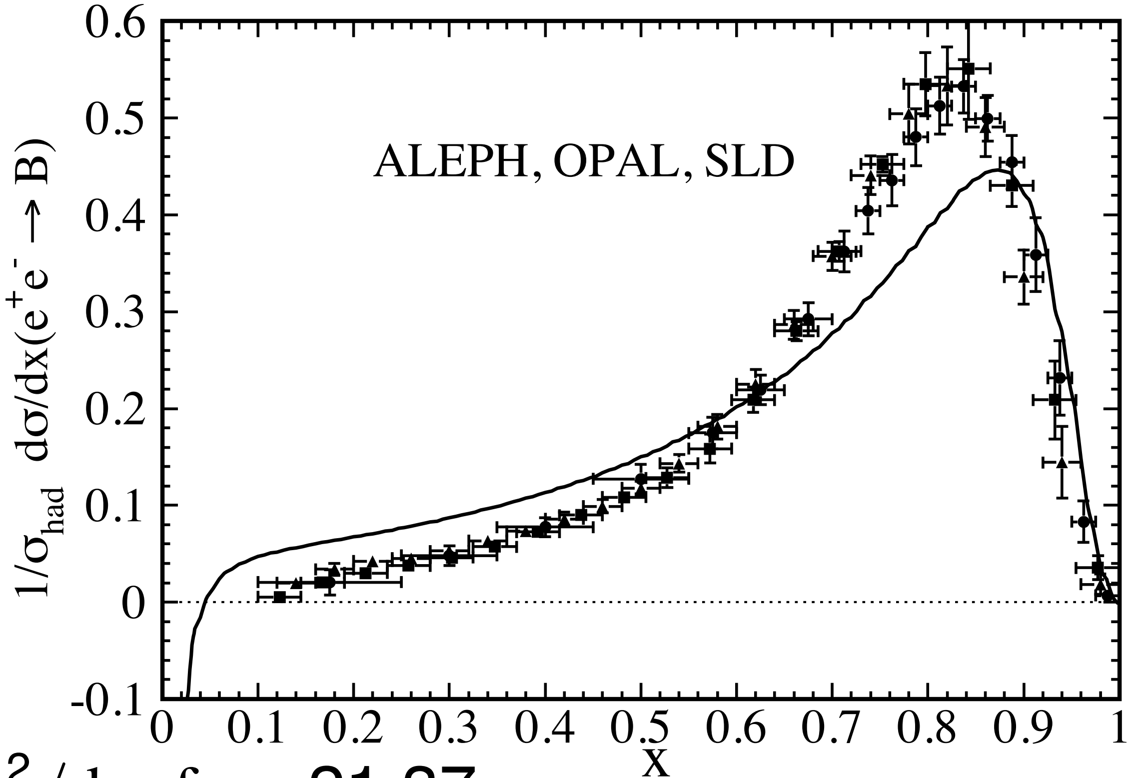


FIG. 1. Moments of the measured B meson fragmentation function, compared with the perturbative NLL calculation supplemented with different $D(z)$ non-perturbative fragmentation forms. The solid line is obtained using a one-parameter form fitted to the second moment.

FFs into B mesons [1] from LEP/SLC data [2]

Petersen

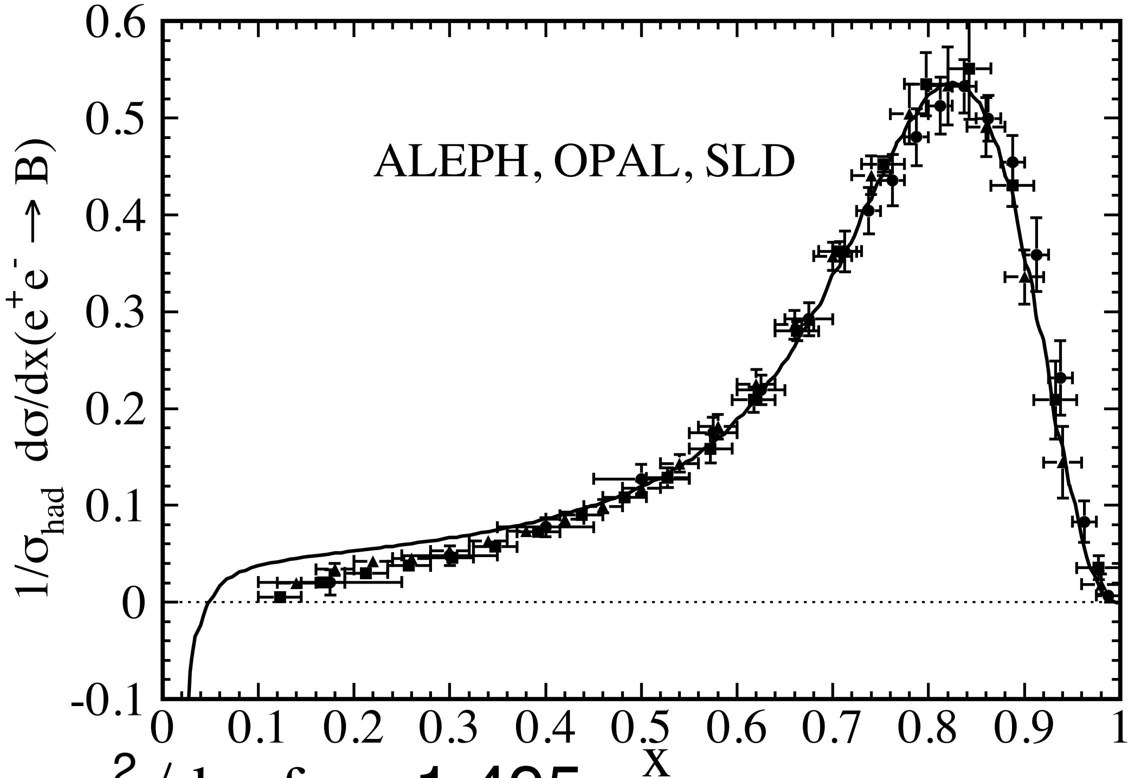
$$D(x,\mu_0^2) = N\frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2}$$



$\chi^2/\text{d.o.f.} = 21.37$

Kartvelishvili-Likhoded

$$D(x,\mu_0^2) = Nx^\alpha(1-x)^\beta$$



$\chi^2/\text{d.o.f.} = 1.495$

[1] Kniehl,Kramer,IS,Spiesberger,PRD77(2008)014011

[2] ALEPH, PLB512(2001)30; OPAL, EPJC29(2003)463; SLD, PRL84(2000)4300; PRD65(2002)092006

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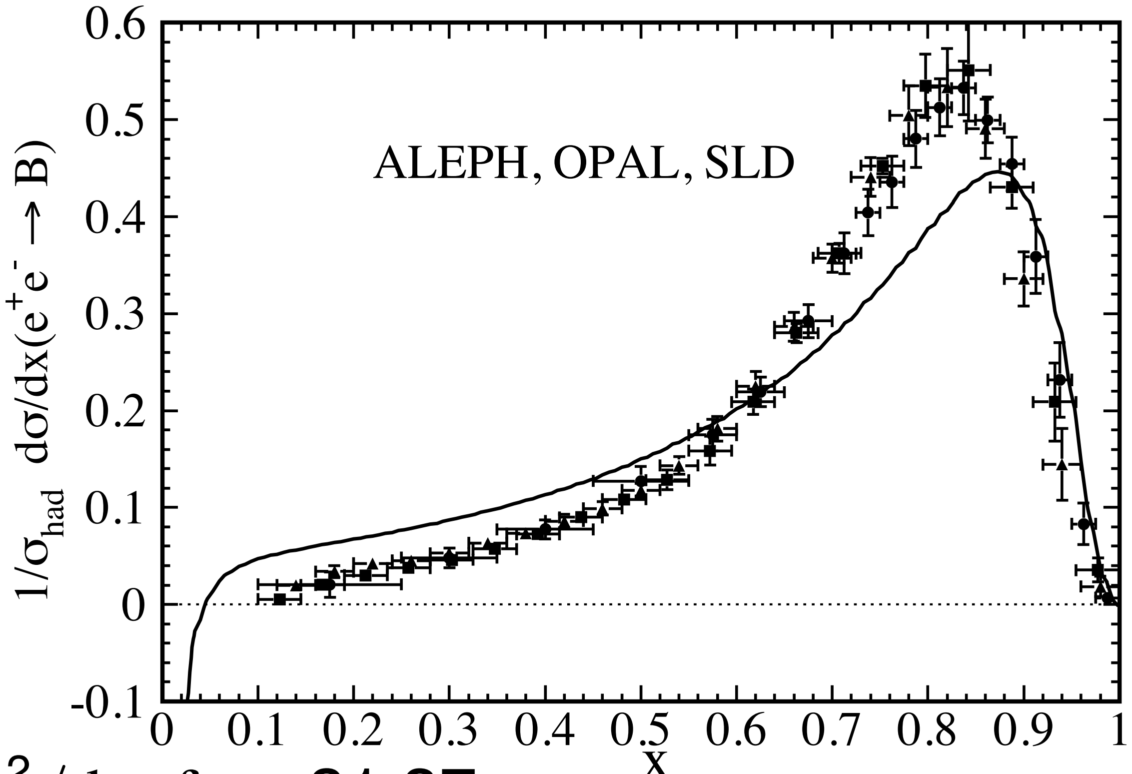

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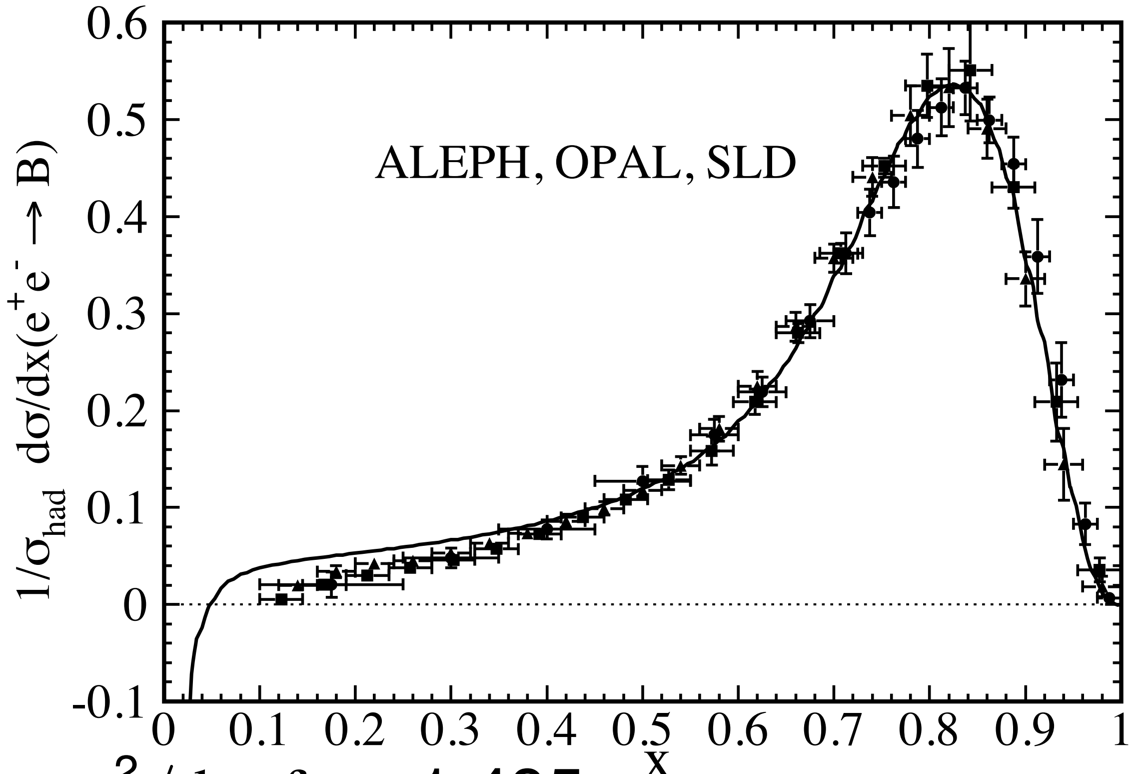


$\chi^2/\mathrm{d.o.f.} = 1.495$

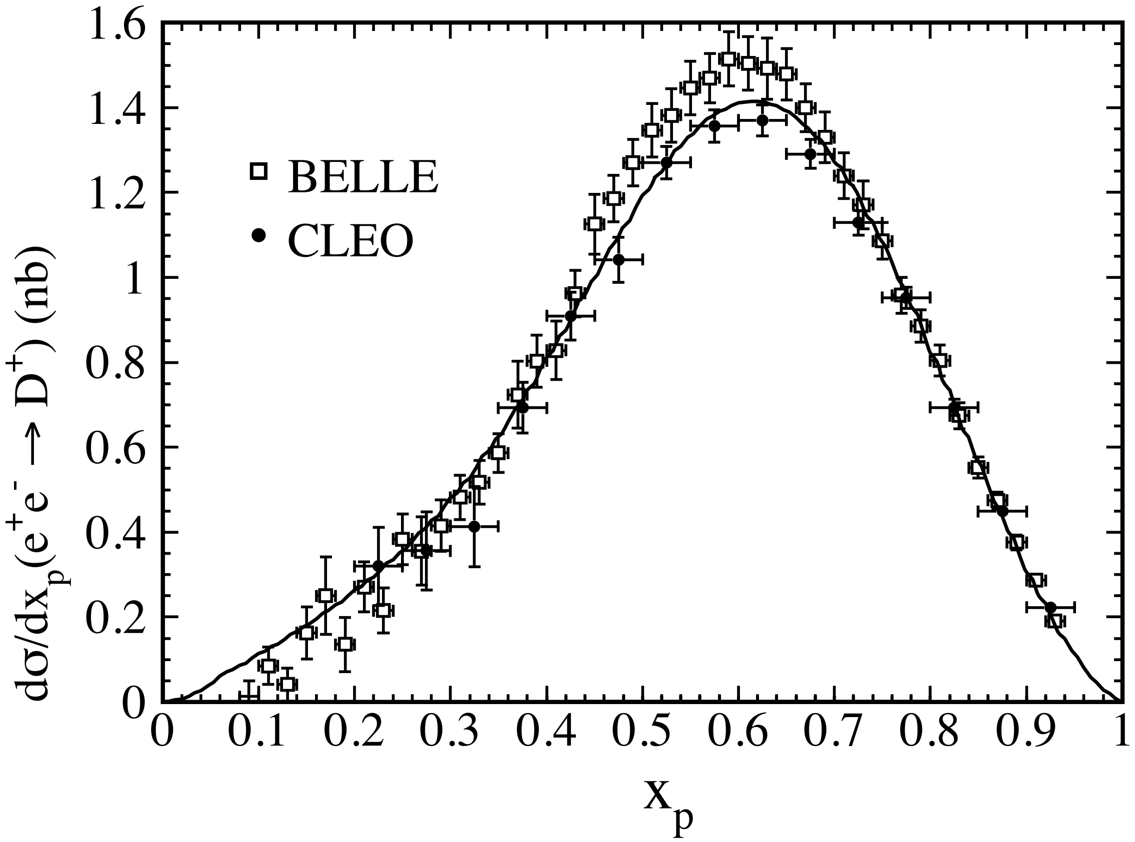
Note: The Petersen function or Kartvelishvili function is used here to parameterise the boundary condition for the heavy quark FF into the heavy meson which is then evolved.

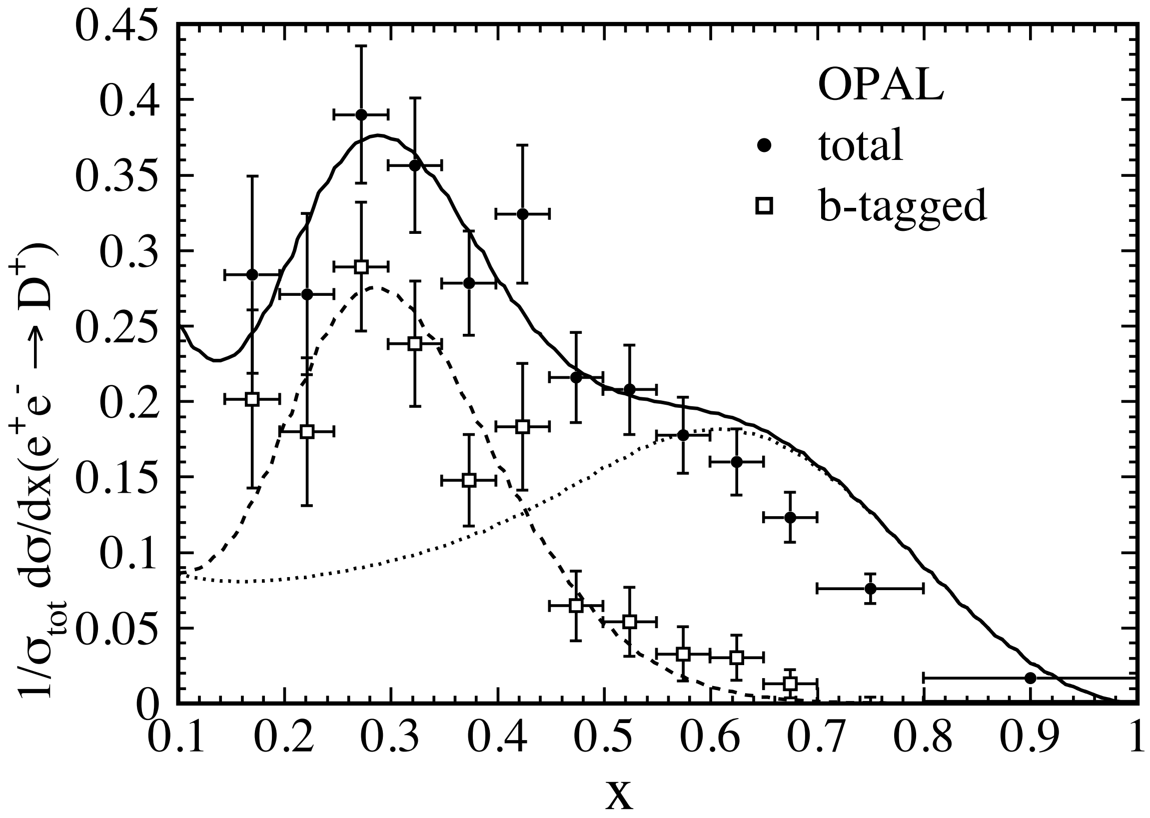
This is completely different from using a Petersen function as the scale independent "hadronization function".

[1] Kniehl,Kramer,IS,

[2] ALEPH, PLB512(... 00;

PRD65(2002)09200





FF for $c \to D^*$
from fitting to e^+e^- data

2008 analysis based on GM-VFNS
$\mu_0 = m$

global fit: data from
ALEPH, OPAL, BELLE, CLEO

BELLE/CLEO fit

[KKKS: Kneesch, Kramer, Kniehl, IS NPB799 (2008)]

tension between low and high energy data sets $\to$ speculations about non-perturbative (power-suppressed) terms

Theoretical approaches:
GM-VFNS/FONLL

GM-VFNS

- Similar factorization formula as in the ZM-VFNS, BUT:
  - Quark mass retained in kinematics and the short distance cross section
  - Allows to compute p_T spectrum for $p_T >> m$ and $p_T \sim m$
- Uses the same **scale-dependent** PFFs of quarks and gluons (in the MSbar scheme)
- the **scale-independent** hadronization function might a priori differ in FFNS, ZM-VFNS and GM-VFNS determinations but to make **connection to the fixed order calculation** it is usually assumed to be the same in all cases

List of subprocesses in the GM-VFNS

	Only light lines	Heavy quark initiated ($m_Q = 0$)	Mass effects: $m_Q \neq 0$
1	$gg \to qX$	-	$gg \to QX$
2	$gg \to gX$	-	-
3	$qg \to gX$	$Qg \to gX$	-
4	$qg \to qX$	$Qg \to QX$	-
5	$q\bar{q} \to gX$	$Q\bar{Q} \to gX$	-
6	$q\bar{q} \to qX$	$Q\bar{Q} \to QX$	-
7	$qg \to \bar{q}X$	$Qg \to \bar{Q}X$	-
8	$qg \to \bar{q}'X$	$Qg \to \bar{q}X$	$qg \to \bar{Q}X$
9	$qg \to q'X$	$Qg \to qX$	$qg \to QX$
10	$qq \to gX$	$QQ \to gX$	-
11	$qq \to qX$	$QQ \to QX$	-
12	$q\bar{q} \to q'X$	$Q\bar{Q} \to qX$	$q\bar{q} \to QX$
13	$q\bar{q}' \to gX$	$Q\bar{q} \to gX$, $q\bar{Q} \to gX$	-
14	$q\bar{q}' \to qX$	$Q\bar{q} \to QX$, $q\bar{Q} \to qX$	-
15	$qq' \to gX$	$Qq \to gX$, $qQ \to gX$	-
16	$qq' \to qX$	$Qq \to QX$, $qQ \to qX$	-

$\oplus$ charge conjugated processes

$$\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\mathrm{G}(\mathrm{m}, \mathrm{p_T})$$

FO: Fixed Order; FOM0: Massless limit of FO; RS: Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2} \simeq \begin{cases} 0.04 & : \quad p_T = m \\ 0.25 & : \quad p_T = 3m \\ 0.50 & : \quad p_T = 5m \\ 0.66 & : \quad p_T = 7m \\ 0.80 & : \quad p_T = 10m \end{cases}$$

$$\Rightarrow \mathrm{FONLL} = \begin{cases} \mathrm{FO} & : \quad \mathrm{p_T} \lesssim 3\mathrm{m} \\ \mathrm{RS} & : \quad \mathrm{p_T} \gtrsim 10\mathrm{m} \end{cases}$$

[1] Cacciari, Greco, Nason, JHEP05(1998)007

FONLL

- FFs in N-space in the PFF approach
- RS-FOM0 gets very large at small p_T:

  $G(m,p_T) = p_T^2/(p_T^2 + a^2 m^2)$ with $\mathbf{a=5}$

  needed to suppress this contribution sufficiently rapidly

  (GM-VFNS does the suppression via a **fine-tuned scale choice**; both solutions not really satisfactory!)
- Central scale choice for FO, RS, FOM0: m_T
- Error bands: $\mu_F = \mu_F'$ (only two scales varied)
- Predictions for LHC7 in arXiv:1205.6344

NLO Monte Carlo generators:
MC@NLO and POWHEG

NLO MC generators

- MC@NLO, POWHEG: hep-ph/0305252, arXiv:0707.3088
consistent matching of NLO matrix elements with parton showers (PS)

- Flexible simulation of hadronic final state
(PS, hadronization, detector effects)

  Note: FONLL and GM-VFNS only one-particle inclusive observables

- High accuracy: NLO+LL*
(FONLL and GM-VFNS have NLO+NLL accuracy)

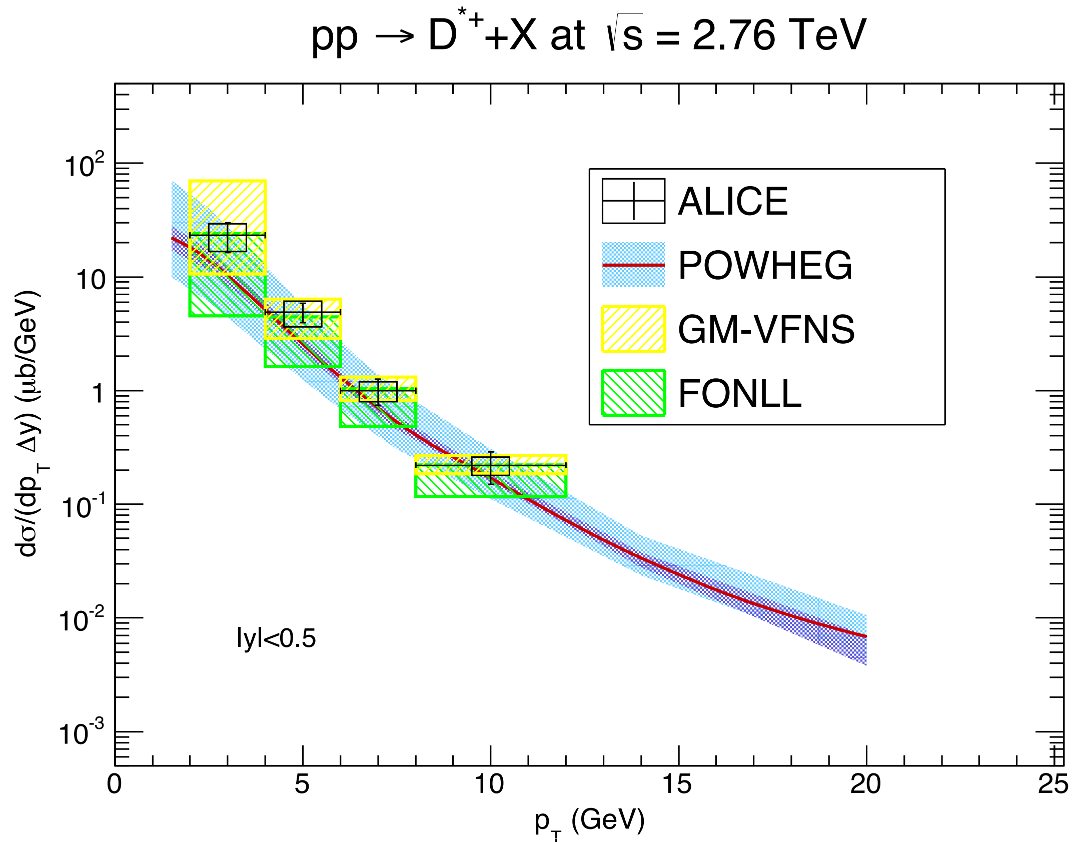
- Simulation of hadronic final state involves tuning;
NOT a pure theory prediction!

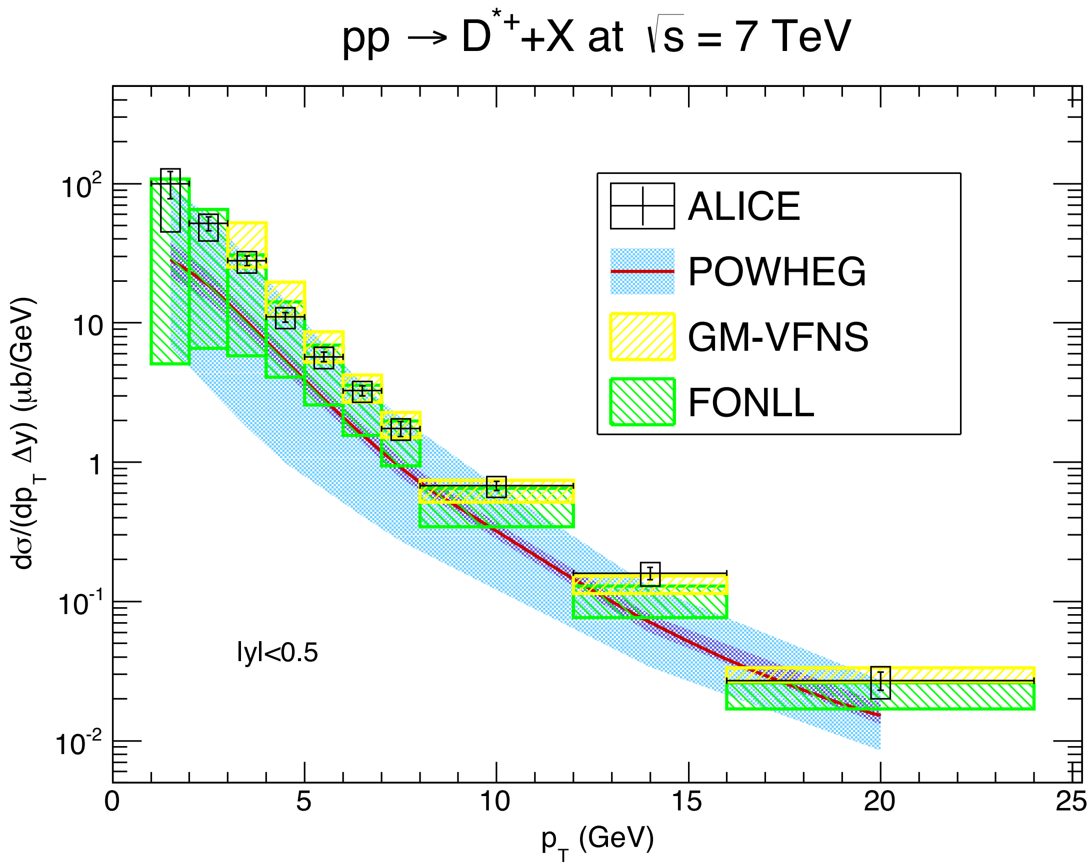
Current status

Comparison with LHC data

arXiv:1405.3083

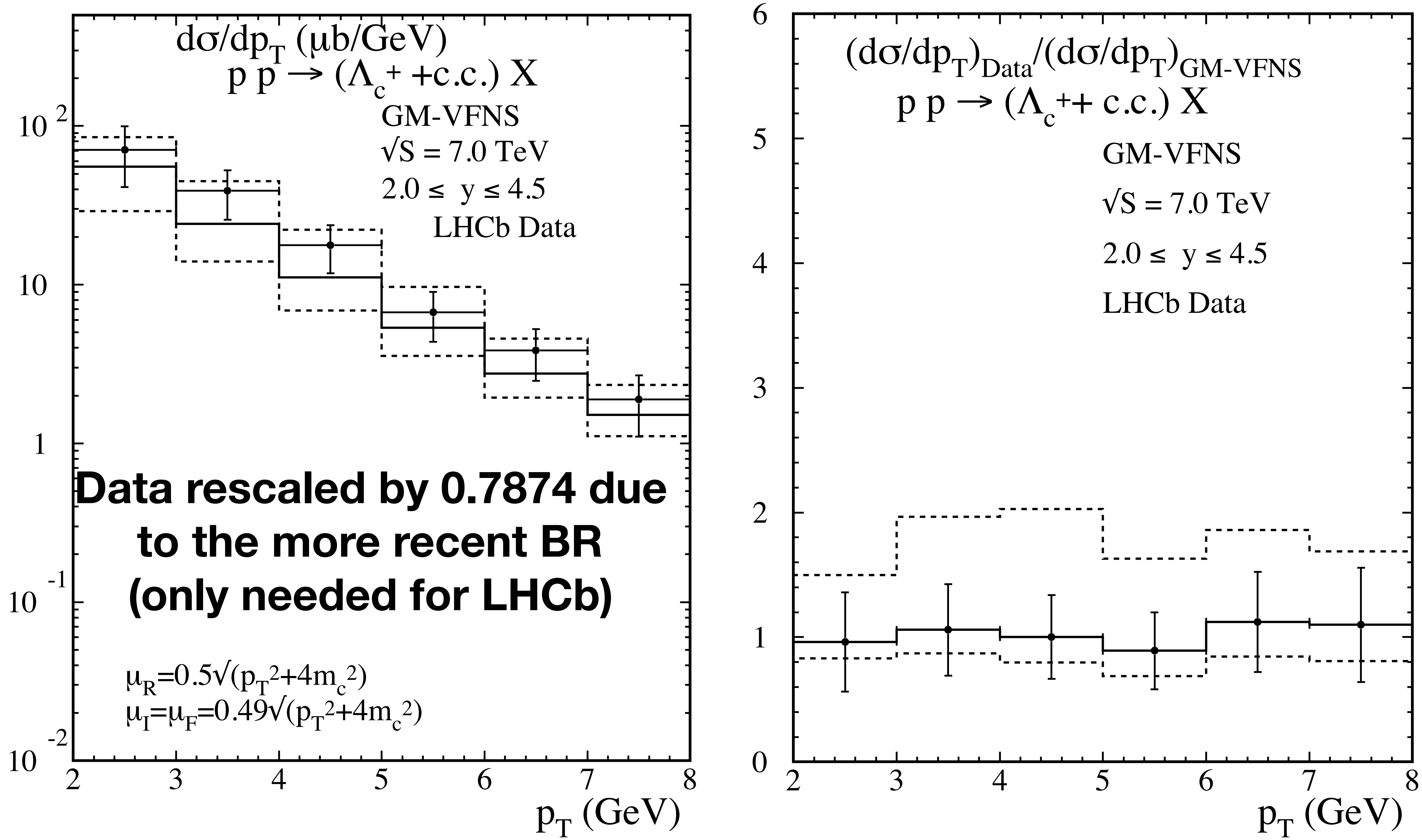
- NLO, NLL, NLO+NLL calculations available since many years
- Comparison with a large variety of data from ALICE, ATLAS, CMS and LHCb
- pp, pPb, (PbPb)
- D*,D0,D+,Ds,B,Lc
- Generally good agreement between data an GM-VFNS/ FONLL within large scale uncertainties





Comparison with LHC data [arXiv:2004.04213]

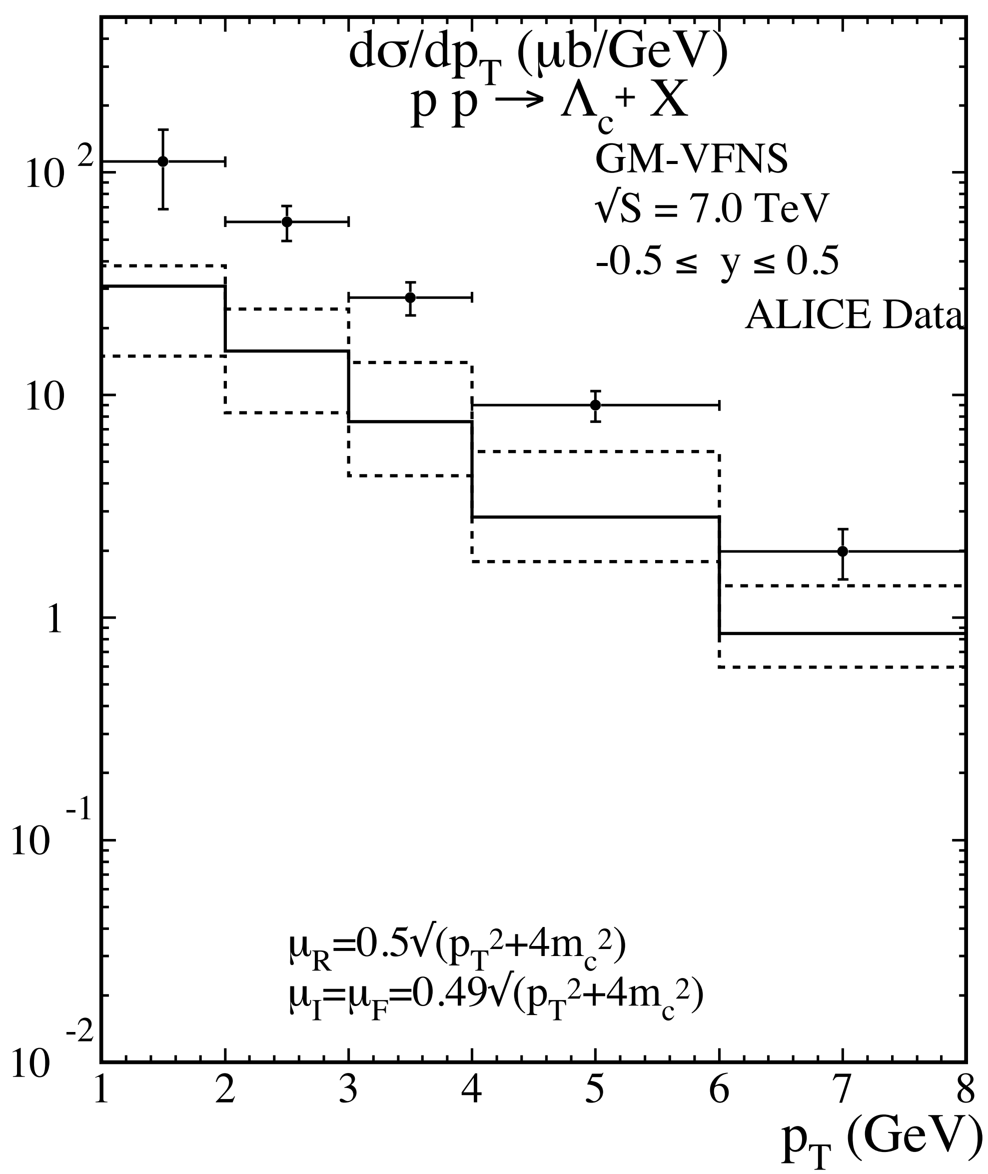
LHCb

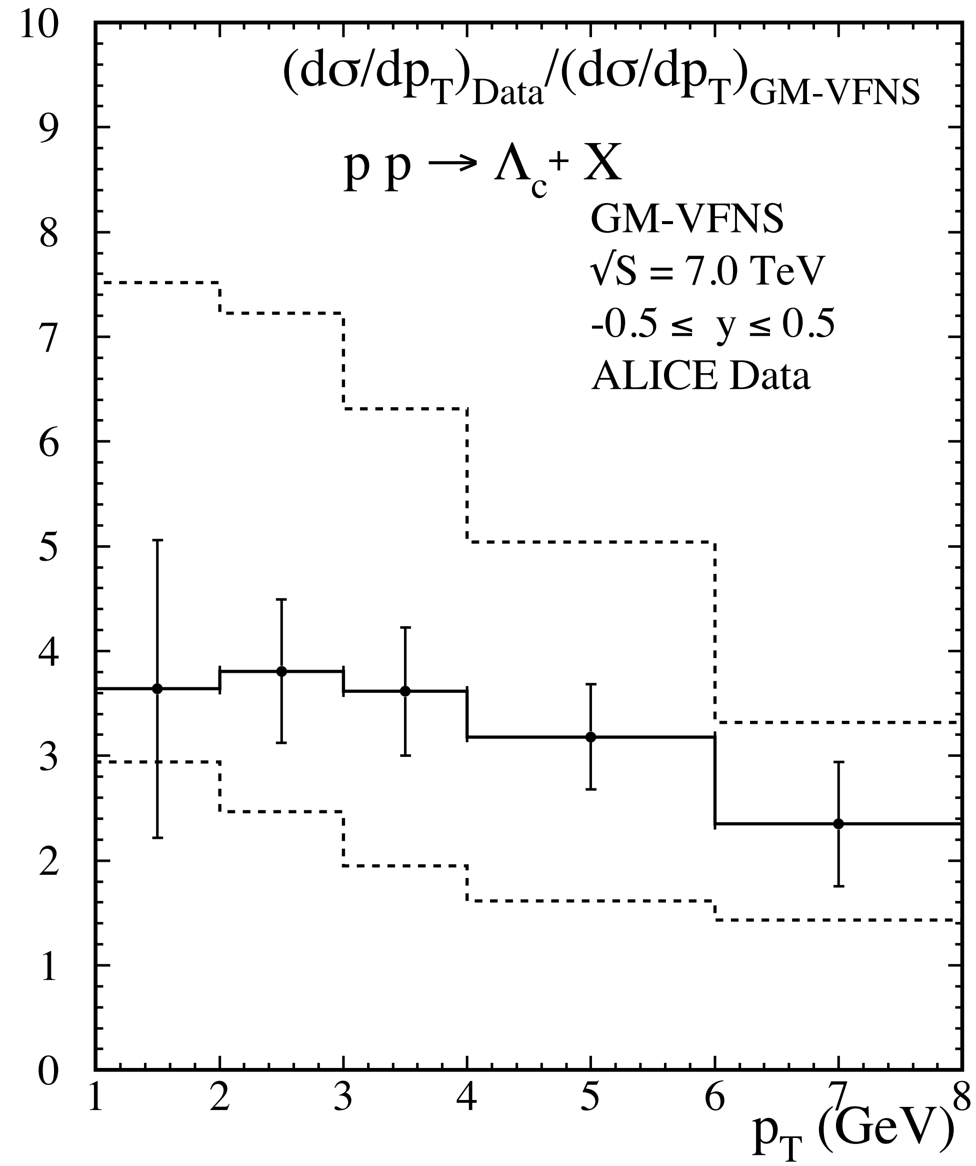


Results are shown with the old Λ_c FFs from 2006.
With the new FFs the cross sections are slightly lower(!) by 15% in the first p_T-bin to 35% in the last p_T-bin

Comparison with LHC data [arXiv:2004.04213]

ALICE





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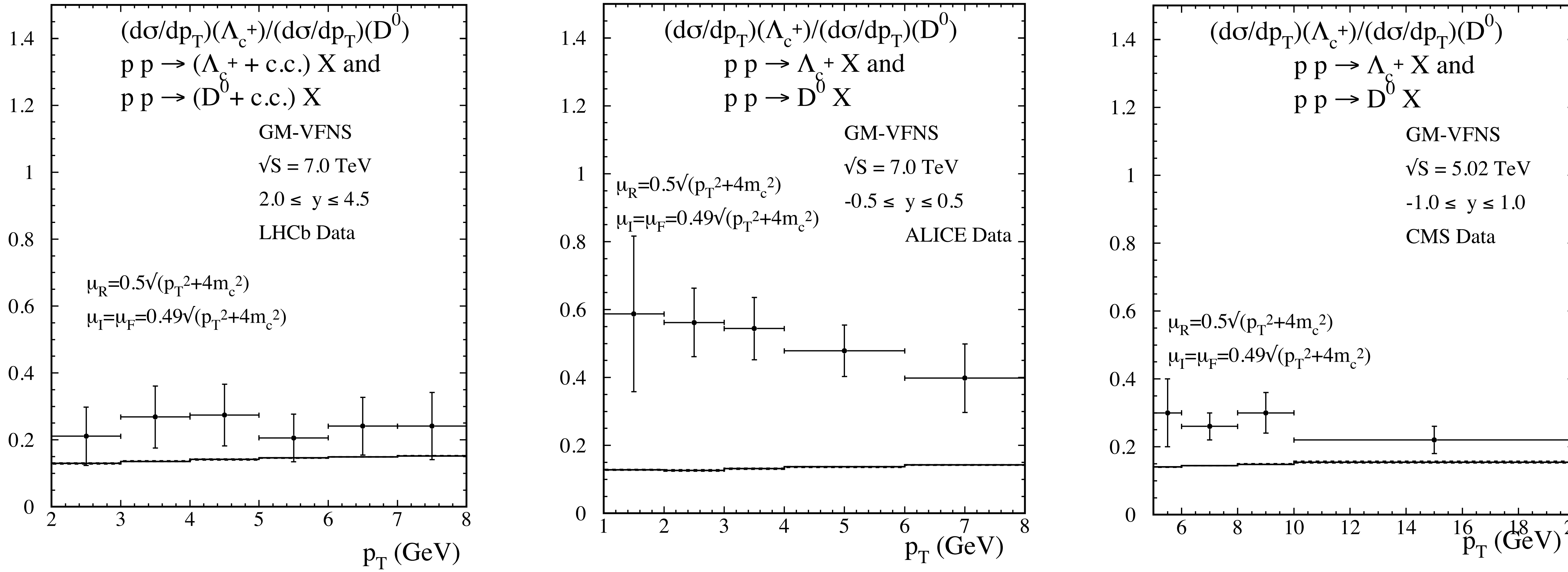
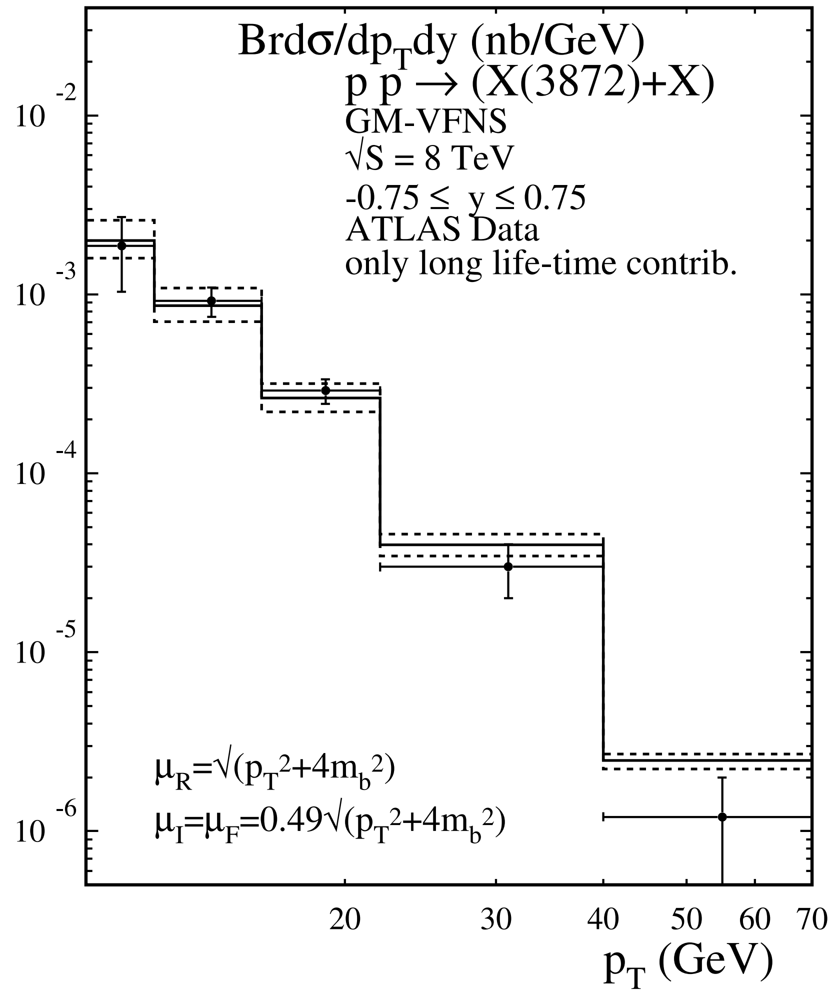
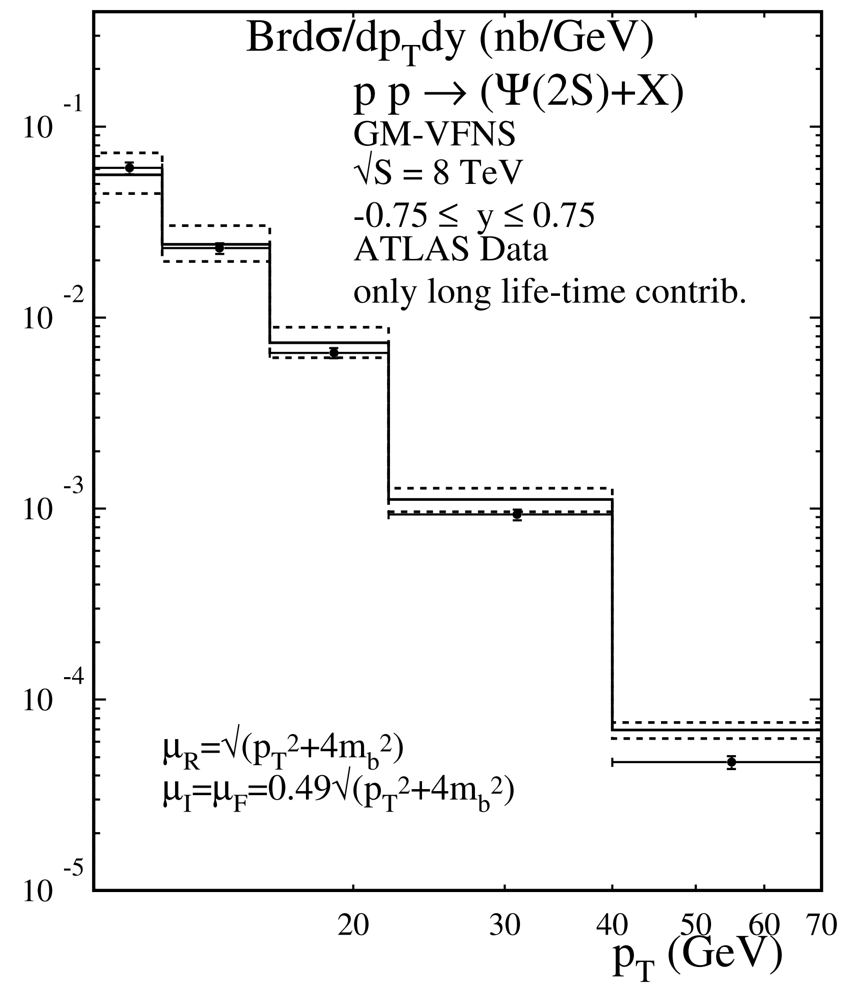
Λ_c/D^0 ratio [arXiv:2004.04213]



- LHCb: Theory < Data by about 1 sigma (scale uncertainty largely cancels)
- ALICE: Theory ~ 0.15, Data ~ 0.6 … 0.4; **clear disagreement due to Λ_c cross section**
- CMS: Theory ~ 0.15, Data ~ 0.3; Are ALICE and CMS data compatible at pT~7 GeV?
- Note: pQCD predicts a flat p_T dependence for p_T > ~$2m_c$

Predictions for Psi(2s) and X(3872) [arXiv:2103.00876]



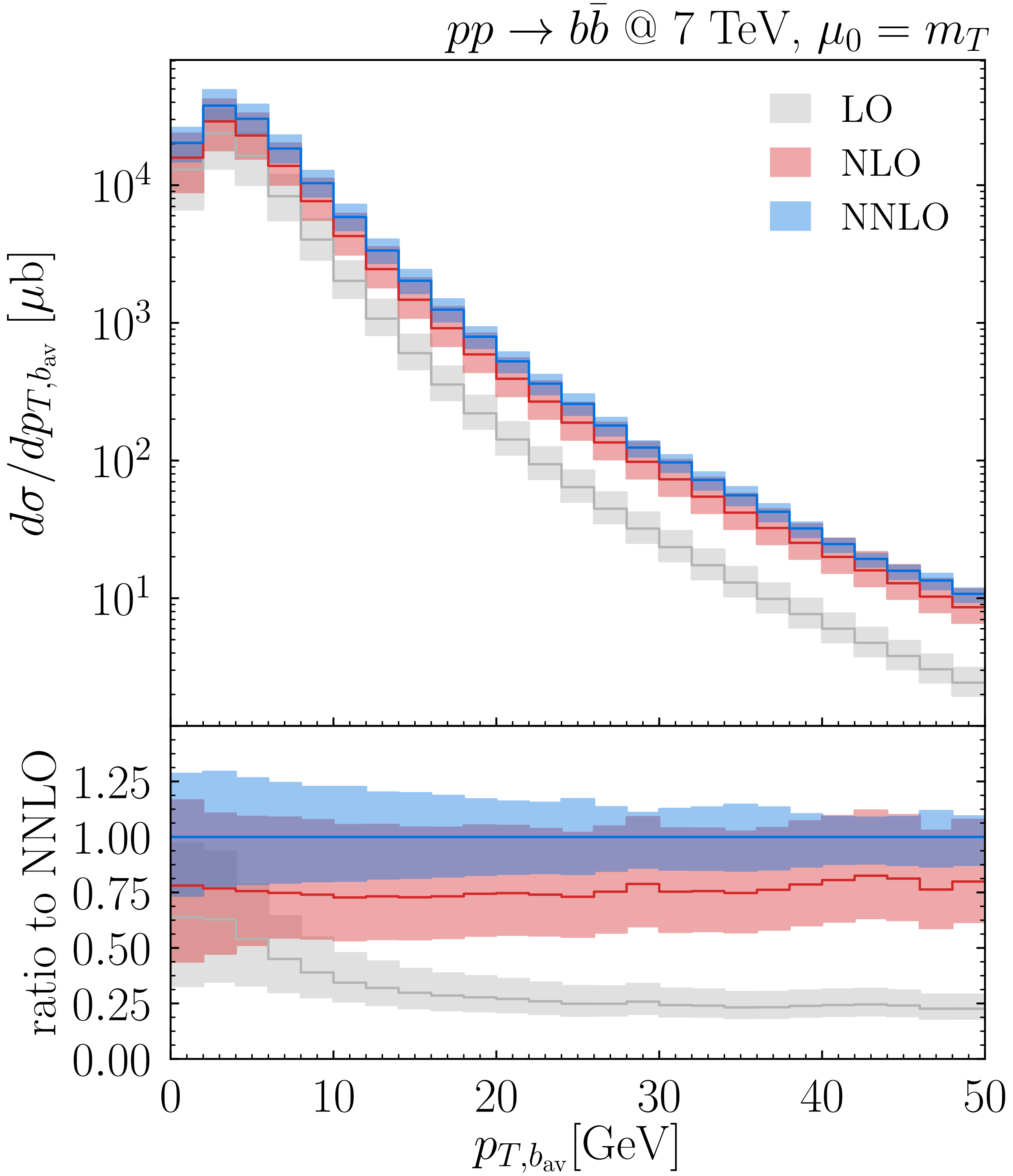


$$R_B = \frac{\mathrm{Br}(B \to X(3872) + X)\mathrm{Br}(X(3872) \to J/\psi\pi^+\pi^-)}{\mathrm{Br}(B \to \psi(2S) + X)\mathrm{Br}(\psi(2S) \to J/\psi\pi^+\pi^-)}$$

Name	$R_B \times 10^2$	Source
$R_B^{[20]}$	18 ± 8	Extracted from CDF II data [21] in Ref. [20]
R_B^{2L}	3.57 ± 0.348	ATLAS [12]
R_B^{LHCb}	$3.48 \pm 0.39 \pm 0.26$	Extracted from LHCb data [29] here
$R_B^{[30]}$	3.24 ± 0.29	Extracted from LHCb data [29] in Ref. [30]
R_B^{ATLAS}	$3.41 \pm 0.37\ ^{+0.63}_{-0.56}$	Our fit to ATLAS data [12]
R_B^{CMS}	$1.89 \pm 0.32\ ^{+0.38}_{-0.33}$	Our fit to CMS data [11]
$R_B^{\mathrm{ATLAS+CMS}}$	$2.54 \pm 0.33\ ^{+0.49}_{-0.43}$	Our joint fit to ATLAS [12] and CMS [11] data

NNLO

- Differential predictions for bbar@NNLO now available! [2010.11906]
- NNLO corrections are **sizable** (25% increase) and **reduce** perturbative uncertainties (which remain important)
- More phenomenological studies needed! Fragmentation Function effects to be included
- Future work: NNLO+NLL
- NNLL?



Conclusions

- NLO+NLL generally in agreement with data within large scale uncertainties
- Recent work for Lc, Psi(2S) and X(3872) production
- NNLO differential distributions available know
  - Cross section larger by ~25%
  - Scale uncertainty reduced
- More phone with NNLO necessary (pp, pA)
  - Including FFs
  - Inclusive hadrons
- NNLO+NLL? NNLO+NNLL?