FastJet

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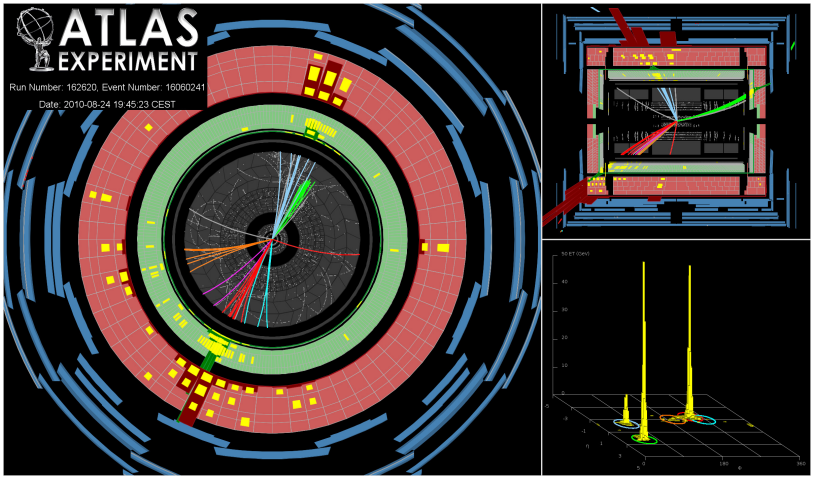
Some physics background



Particles/Energy flow organised in a few dominant directions $\Rightarrow$ JETS

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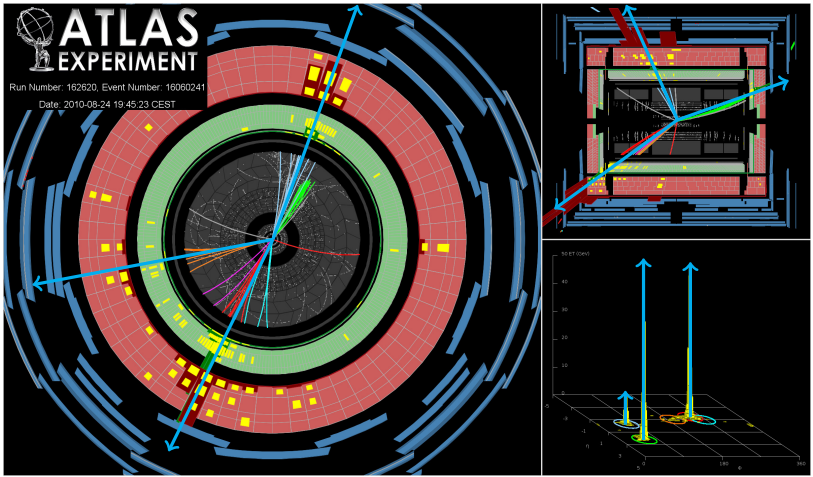

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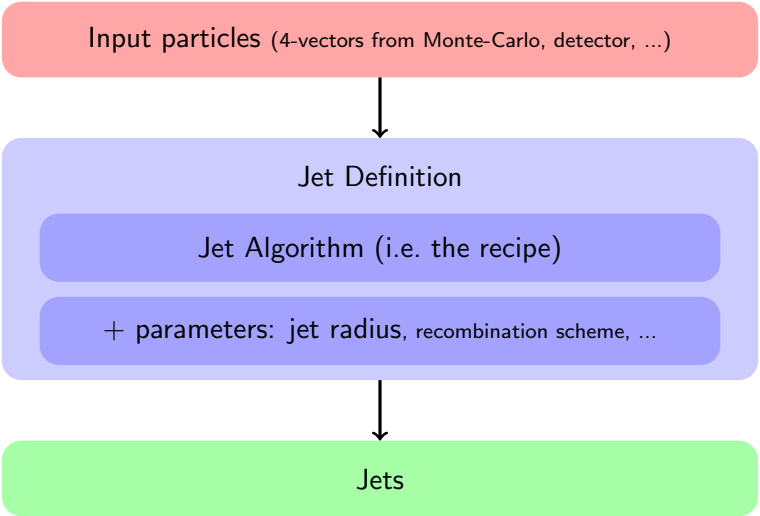
Jet algorithms and definitions



Most common approach today: recombination algorithms

Generalised-k_t algorithm

- From all the objects to cluster, define the distances

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2), \qquad d_{iB} = p_{t,i}^{2p} R^2$$

- repeatedly find the minimal distance
  if d_{ij}: recombine i and j into $k = i + j$
  if d_{iB}: call i a jet

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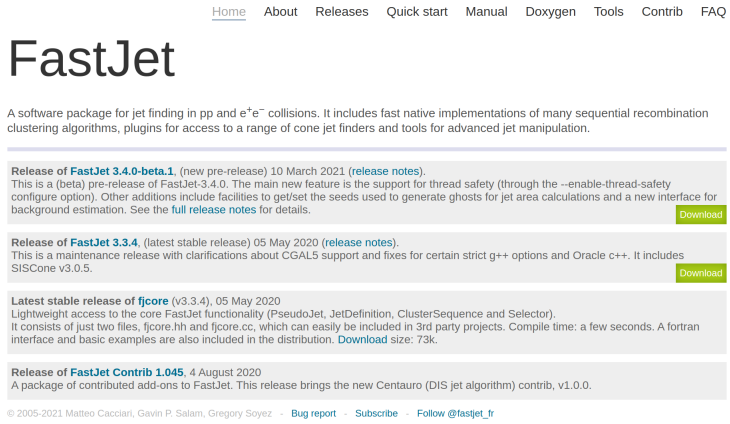
- Parameter p is (typically) one of
  - $p = 1$: k_t algorithm (closest to QCD) [Catani,Dokshitzer,Seymour,Weber,Ellis,Soper,1993]
  - $p = 0$: Cambridge/Aachen (geometrical distance) [Dokshitzer,Leder,Moretti,Webber,1997]
  - $p = -1$: anti-k_t (the LHC choice) [M.Cacciari,G.Salam,GS,2008]

FastJet is a C++ interface (*) for

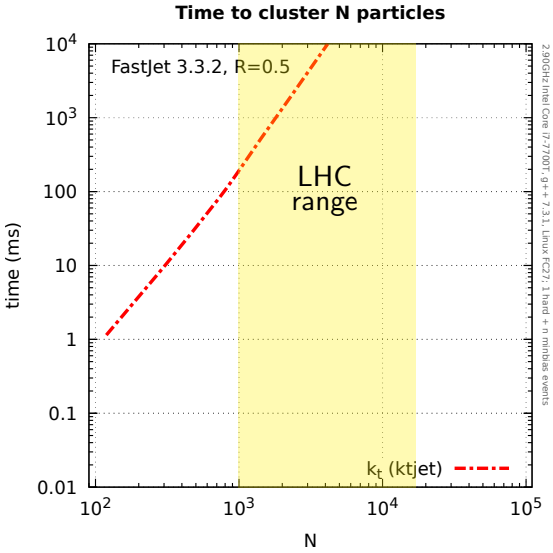
1. fast jet clustering
2. jet manipulations

See fastjet.fr
Cite arXiv:1111.6097
and hep-ph/0512210

[(*) also available in Python]



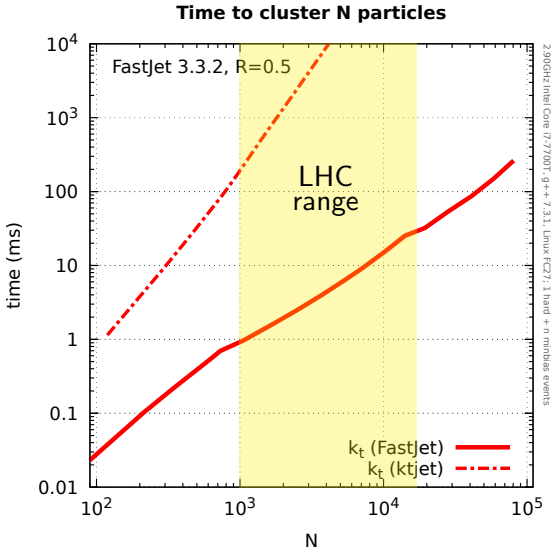
Fast clustering



- k_t before FastJet
  Complexity $\propto N^3$ (*)

(*) more on that later if enough time

Fast clustering



- k_t before FastJet
  Complexity $\propto N^3$ (*)
- k_t Fastjet's implementation
  gain 2-3 orders of magnitude
  Makes it usable at the LHC
  (Trigger spends $\sim$ 100ms/event)
  Complexity $\propto N^2$ or $N \ln N$ (*)

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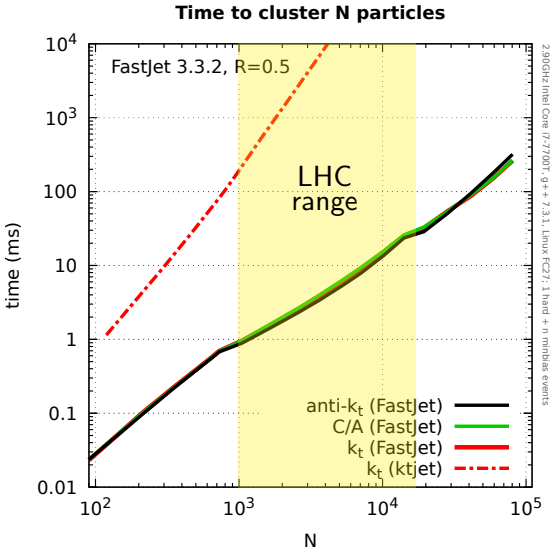

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  gain 2-3 orders of magnitude
  Makes it usable at the LHC
  (Trigger spends $\sim$ 100ms/event)
  Complexity $\propto N^2$ or $N \ln N$ (*)
- Similar time for anti-k_t and C/A

(*) more on that later if enough time

Simple code

All done in a few lines of code:

```
// list of input particles/objects
vector<PseudoJet> particles;

// Cluster with anti-k_t, R = 0.5
JetDefinition jet_def(antikt_algorithm, 0.5);

// Get the jets with p_t > 50 GeV, |y| < 2.5
Selector jet_selector = SelectorPtMin(50.0) * SelectorAbsRapMax(2.5);
vector<PseudoJet> jets = jet_selector(jet_def(particles));

// Simple manipulations
for (auto &jet :  jets){
  double pt = jet.pt(); // jet pt
  vector<PseudoJet> constituents = jet.constituents(); // particles that made up the jet
}
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In a nutshell: jet reconstruction/selection/operations made easy

Grown into framework for jet manipulation

Two main directions:

1. Background mitigation
2. Jet substructure

(Area-based) background subtraction (pileup or heavy-ion)

Often (in experimental contexts) events are polluted by large backgrounds

- Pileup in pp (superposition of several concurrent pp events)
- QGP medium/background in HI collisions

Basic approach

Based on the observation that backgrounds are (quasi-)homogeneous in rapidity-azimuth

1. Compute jet areas A_i (for each jet)
2. Estimate the background ρ for the event
3. Subtract each jet

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1. Compute jet areas A_i (for each jet)
2. Estimate the background ρ for the event
3. Subtract each jet

$$\rho = \text{median}_{i \in \text{jets}} \frac{p_{ti}}{A_i}$$

$$p_t^{(\text{sub})} = p_t - \rho A$$

(Area-based) background subtraction (pileup or heavy-ion)

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FastJet provides:

- A computation of the jet areas [ClusterSequenceArea]
- Tools to estimate the background density [Jet/GridMedianBackgroundEstimator]
- Tools to perform the subtraction [Subtractor]
- Facilities to implement user-defined subtraction methods

(Area-based) background subtraction (pileup or heavy-ion)

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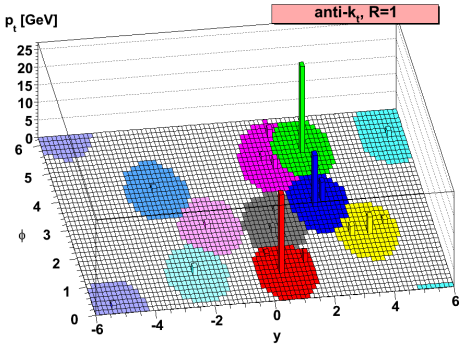

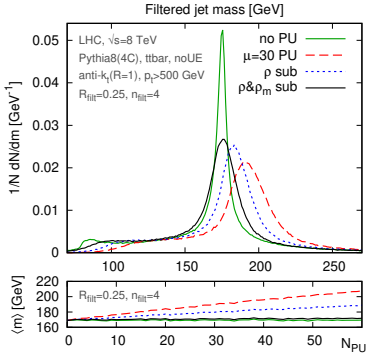

Core interface for substructure tools

One often need post-processing of jets, typically in jet substructure studies

FastJet provides

- Basic (historical) tools
- Common interface for user-defined tools

```
// apply a MassDropTagger
MassDropTagger md_tagger(0.667, 0.09);
PseudoJet tagged = md_tagger(jet);


// test if the tagger succeeded
if (tagged != 0){
  // apply a filter
  Filter filter(some_jet_def, some_selection);
  PseudoJet filtered = filter(tagged);
}
```

Many additional tools in **fastjet-contrib** [link]

Package	Version	Release date
Centauro	1.0.0	2020-08-04
ClusteringVetoPlugin	1.0.0	2015-05-04
ConstituentSubtractor	1.4.5	2020-02-23
EnergyCorrelator	1.3.1	2018-02-10
FlavorCone	1.0.0	2017-09-07
GenericSubtractor	1.3.1	2016-03-30
JetCleanser	1.0.1	2014-08-16
JetFFMoments	1.0.0	2013-02-07
JetsWithoutJets	1.0.0	2014-02-22
LundPlane	1.0.3	2020-02-23
Nsubjettiness	2.2.5	2018-06-06
QCDAwarePlugin	1.0.0	2015-10-08
RecursiveTools	2.0.0	2020-03-03
ScJet	1.1.0	2013-06-03
SoftKiller	1.0.0	2014-08-17
SubjetCounting	1.0.1	2013-09-03
ValenciaPlugin	2.0.2	2018-12-22
VariableR	1.2.1	2016-06-01

Core interface for substructure tools

One often need post-processing of jets, typically in jet substructure studies

FastJet provides

- Basic (historical) tools
- Common interface for user-defined tools

```
// apply a MassDropTagger
contrib::SoftDrop sd(2.0, 0.1);
PseudoJet tagged = sd(jet);


// test if the tagger succeeded
if (tagged != 0){
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  Filter filter(some_jet_def, some_selection);
  PseudoJet filtered = filter(tagged);
}
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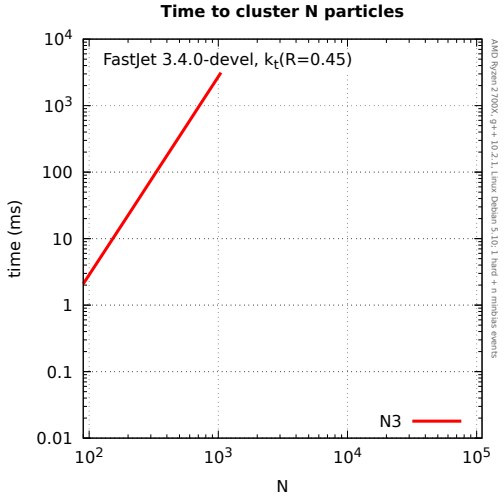
What makes FastJet fast?

Discussion about complexity

N particles to cluster $\Rightarrow \mathcal{O}(N)$ recombinations steps to get a finite number of jets



Basic approach ($\mathcal{O}(N^3)$)

Repeat $\mathcal{O}(N)$ times

- find minimum d_{ij} $[\mathcal{O}(N^2)]$
- recombine $[\mathcal{O}(1)]$

Total: $N \times N^2 = N^3$

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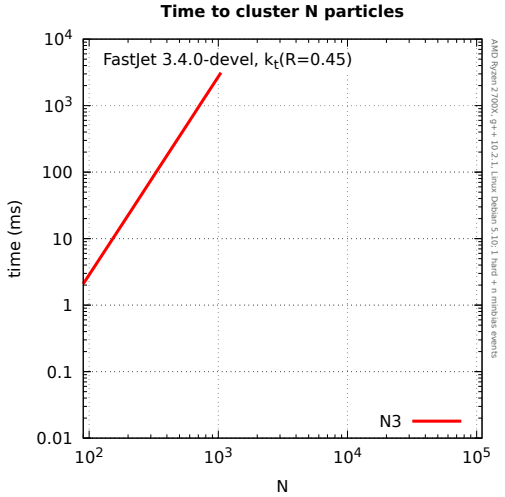

FastJet lemma

If i, j is the minimum $d_{ij} = \min(p_{ti}^2, p_{tj}^2)\Delta R_{ij}^2$ with $p_{ti} < p_{tj}$, then $\Delta R_{ij} < \Delta R_{ik}\ \forall k \neq i, j$.
I.e. the (gen-)k_t minimum is one of the geometrical nearest neighbours

Proof: Assume there is k s.t. $\Delta R_{ik} < \Delta R_{ij}$.
We have

$$\begin{aligned} d_{ik} &= \min(p_{ti}^2, p_{tk}^2)\Delta R_{ik}^2 \\ &< p_{ti}\Delta R_{ij}^2 = d_{ij} \end{aligned}$$

a contradiction

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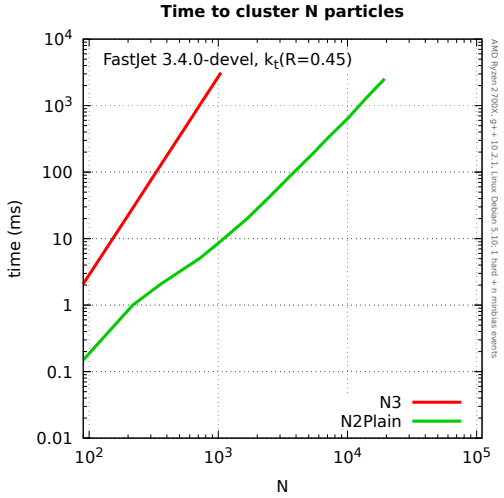

FastJet approach #1 ($\mathcal{O}(N^2)$)

- find NN_i nearest neighbour for each i $[\mathcal{O}(N^2)]$
- Repeat $\mathcal{O}(N)$ times
  - find minimum d_{ij} in NNs $[\mathcal{O}(N)]$
  - recombine $[\mathcal{O}(1)]$
  - update a few NN $[\mathcal{O}(N)]$

Total: $N^2 + N \times N = N^2$

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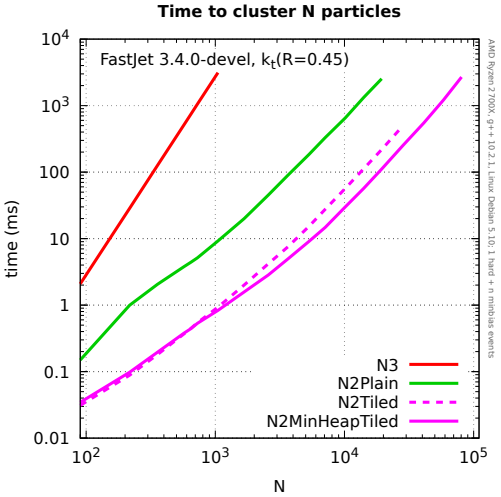

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more $\mathcal{O}(N^2)$ FastJet approaches

- Use tiling to limit to local search

Discussion about complexity

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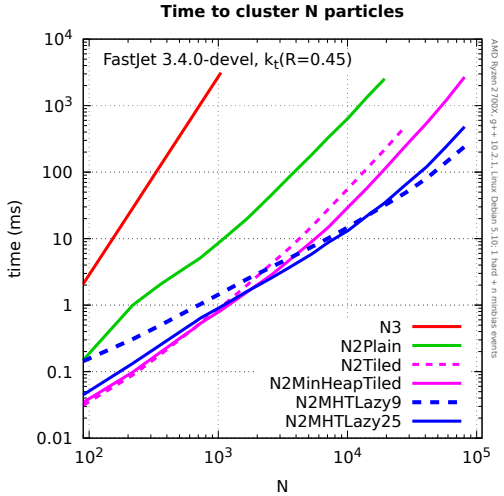

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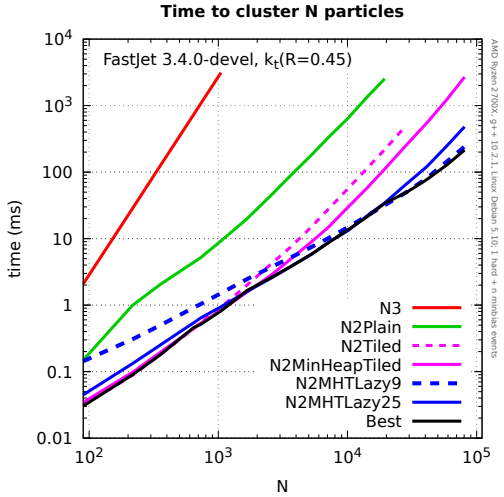
more $\mathcal{O}(N^2)$ FastJet approaches

- Use tiling to limit to local search
- Limit unnecessary searches

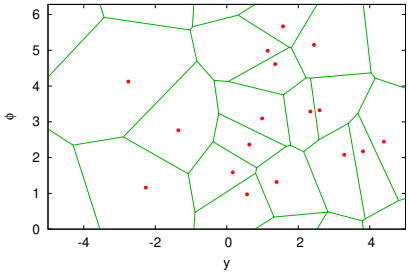
Discussion about complexity

N particles to cluster $\Rightarrow \mathcal{O}(N)$ recombinations steps to get a finite number of jets



FastJet ultimate: $\mathcal{O}(N \ln N)$

Use Voronoi graphs (Delaunay triangulations)
Limits the initialisation to $\mathcal{O}(N \ln N)$ and the search to $\ln N$. (Optimal at larger N)



Conclusions

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See fastjet.fr
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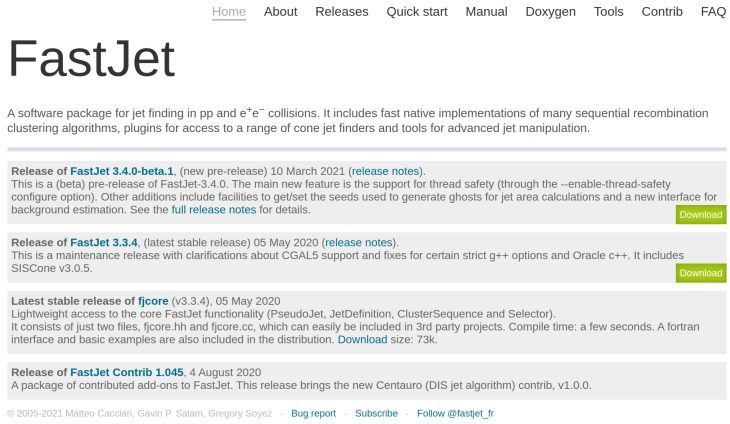
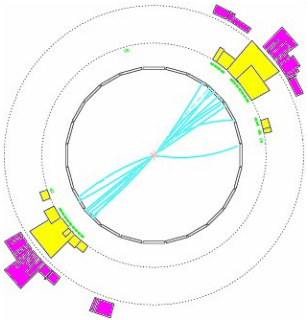


Jets and partons

"Jets" $\equiv$ bunch of collimated particles $\cong$ hard partons

How many jets?



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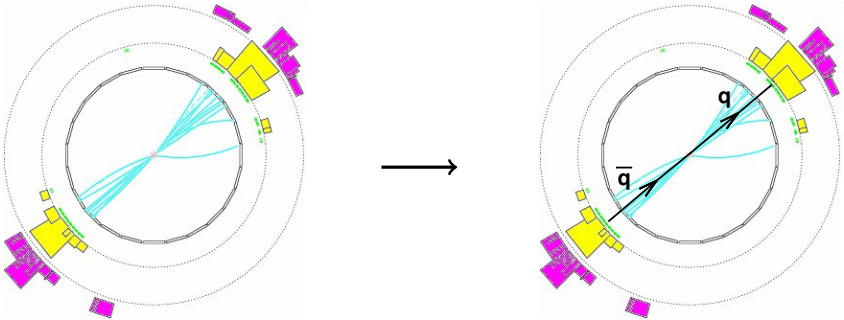
obviously 2 jets



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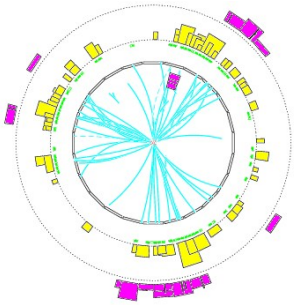

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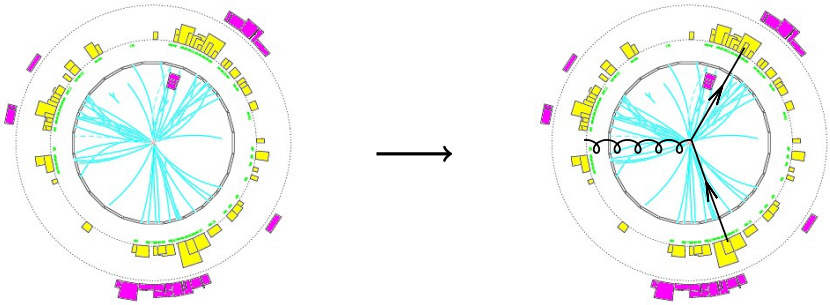
3 jets



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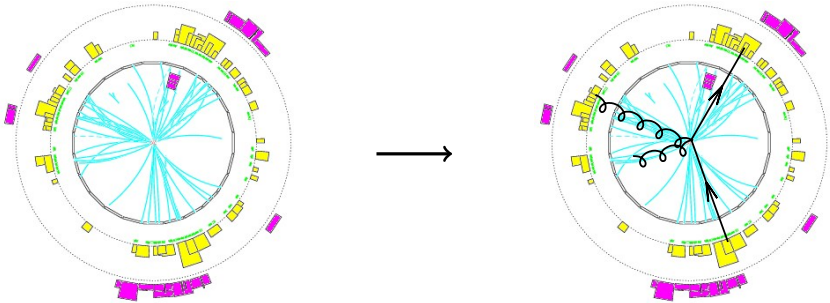
3 jets... or 4?



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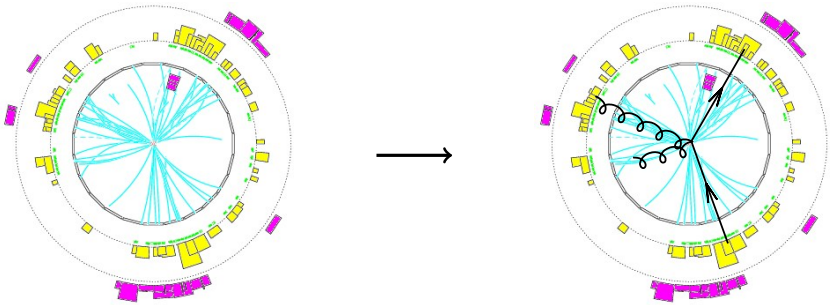

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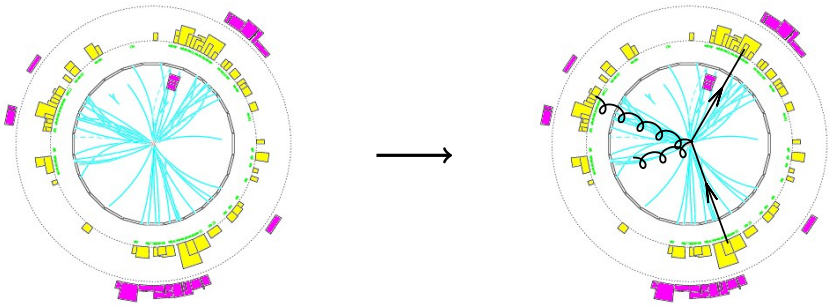

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- Define jets instead