DVCS off a pion target:

experimental assessment for pion GPDs

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Introduction

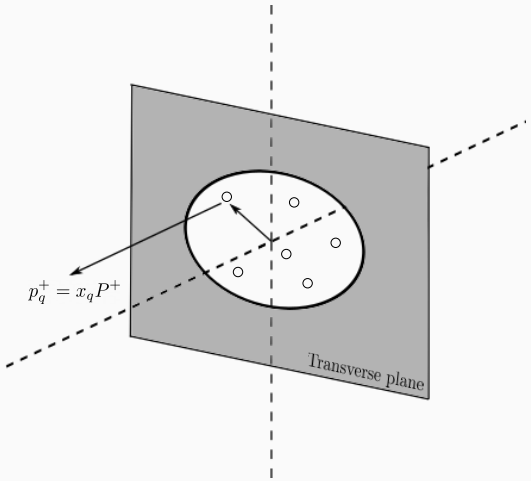
Introduction: GPDs and hadron's strcture

Question: *How can we gain insights into pion's strcture?*

Generalised parton distributions (GPDs)

Probabilistic interpretation:
probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction "x" of the hadron's light-cone momentum.

[M.Burkardt-PRD:071503(62)2020]



EMT: $\langle \pi(p') | T^{\mu\nu} | \pi(p) \rangle = 2P^\mu P^\nu \theta_2(t) + \frac{1}{2}(t g^{\mu\nu} - t^\mu t^\nu)\theta_1(t)$

GPD MM: $\int_0^1 dx x H^q(x,\xi,t) = A^q_{2,0}(t) + 4\xi^2 A^q_{2,2}(t)$

[X.Ji-PRL:610(78)1997]

Further properties:

1. **Parametrize DVCS amplitudes through CFFs.**
2. PDFs as forward limit.
3. Electromagnetic and gravitational FFs as Mellin moments.

Introduction

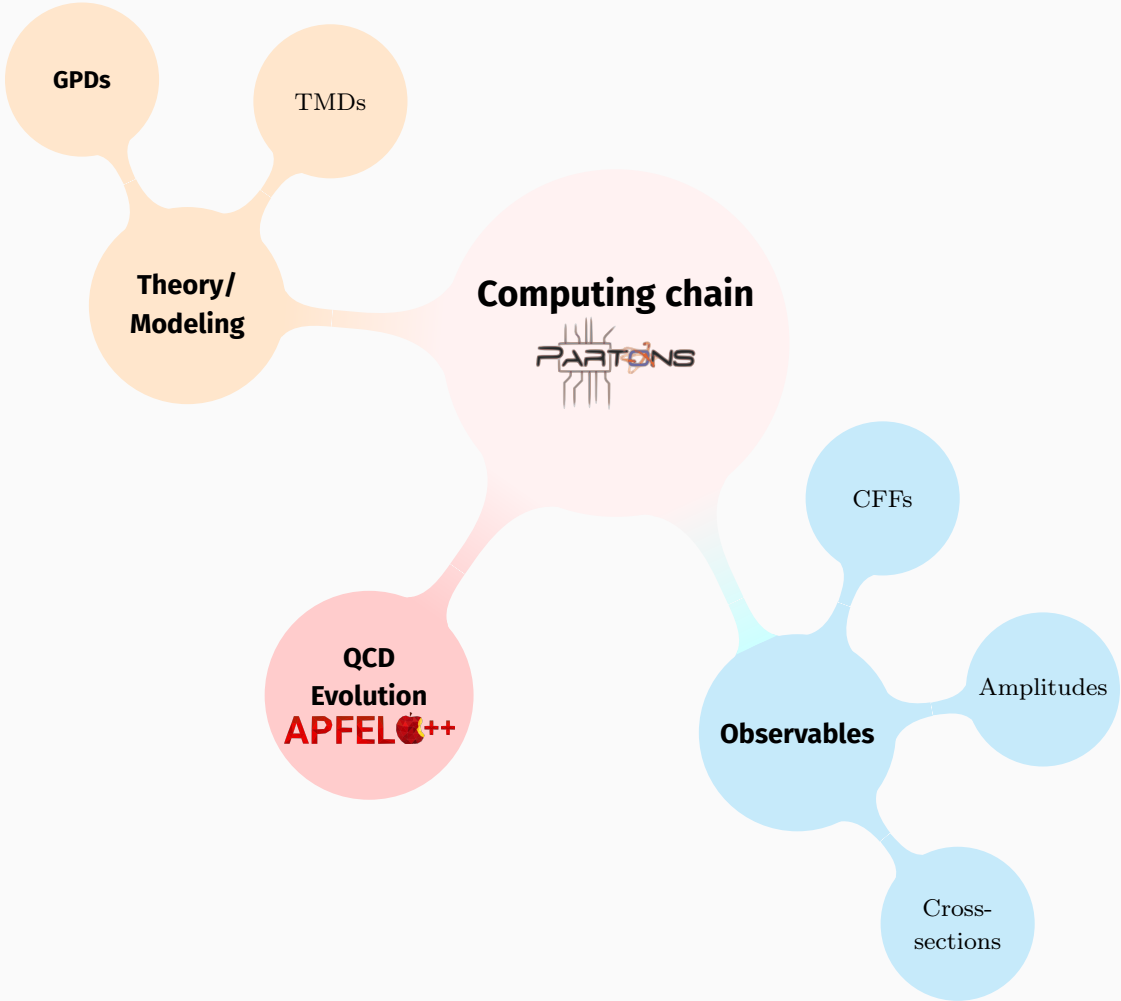
1. Pions: DCSB Nambu-Goldstone bosons
   - Clear window onto emergence of hadronic mass

2. Generalised parton distributions:
   - EMT gravitational form factors.
   - "3D" picture of hadron's structure

Two main questions guide this talk:

- Can we build "theoretically-complete" pion GPD models?
  Novel GPD models: positivity-saturated GPDs
- Can we probe them in experiment?
  Pion GPDs through Sullivan process:
  [D.Amrath et al.-EPJC:179(58)2008]

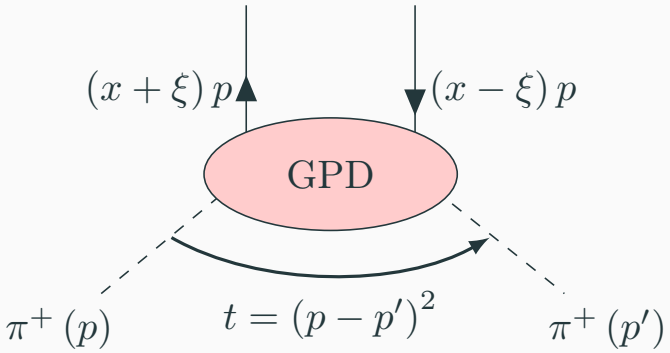
Introduction



GPD modelling

Can we build “theoretically-complete” pion GPD models?

GPD modelling: definition and properties



x: Momentum fraction of p.

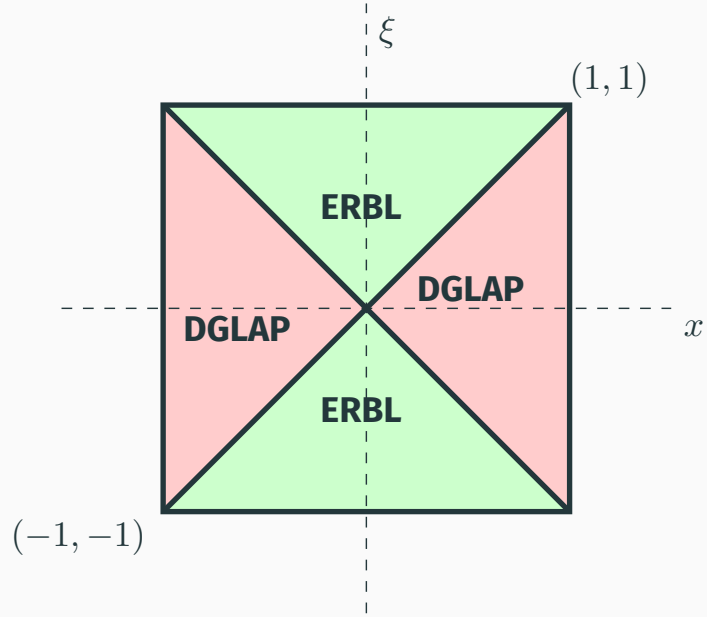
ξ: Fraction of momentum longitudinally tranfered.

t: Momentum transfer.

Kinematics:

[M.Diehl-Phys.Rept:41(388)2003]

- **DGLAP** ($|x| > |\xi|$):
  Emits/takes a quark ($x > 0$) or antiquark ($x < 0$).
- **ERBL:** ($|x| < |\xi|$):
  Emits pair quark-antiquark.



GPD modelling: definition and properties

- **Support:**
  [M.Diehl et al.-PLB:359(428)1998]

$$(x,\xi) \in [-1,1] \otimes [-1,1]$$

- **Polynomiality:** Order-m Mellin moments are degree-$(m+1)$ polynomials in ξ.
  [X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999]

$$\int_{-1}^{1} dx x^m H(x,\xi,t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t)\, \xi^k$$

Lorentz invariance

- **Positivity:**
  [P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999]

$$|H^q(x,\xi,t=0)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)} \qquad , \qquad |x| \geq \xi$$

Positivity of Hilbert space norm

- **Low energy soft-pion theorem**
  [M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

PCAC/Axial-Vector WTI

GPD modelling: general strategy

Goal: Build pion GPD models fulfilling all these constraints.

Problem: Different modelling strategies and different problems

1. **Overlap representation**
   [M.Diehl et al.-NPB:33(569)2001]
   Based on LFWFs, $\Psi^q\left(x, k_\perp^2\right)$
   → Polynomiality ?
   → Positivity ✓

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2. **Double Distribution representation**
   [D.Müller et al.-Fort.Phys:2(42)1994, JLAB-THY-00-33]
   Relying on Radon transform, $\mathcal{R}$
   → Polynomiality ✓
   → Positivity ?

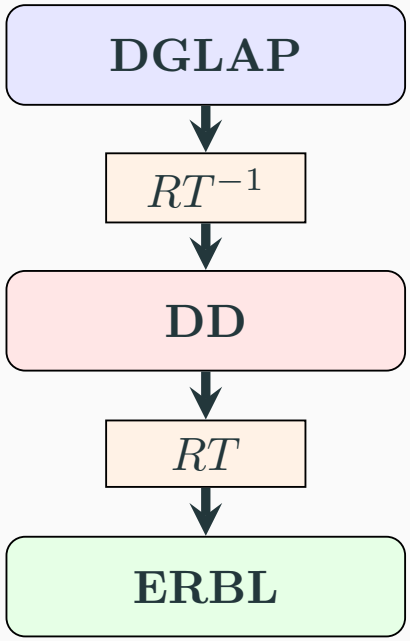
Solution:

Covariant extension: given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied.[N.Chouika et al.-EPJC:906(77)2017]

GPD modelling: covariant extension

Covariant extension: given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied. [N.Chouika et al.-EPJC:906(77)2017]

$$H(x,\xi,t) = \mathcal{R}\left[h(\beta,\alpha,t)\right] + \frac{1}{|\xi|} D^{+}\left(\frac{x}{\xi},t\right) + sgn(\xi)\, D^{-}\left(\frac{x}{\xi},t\right)$$

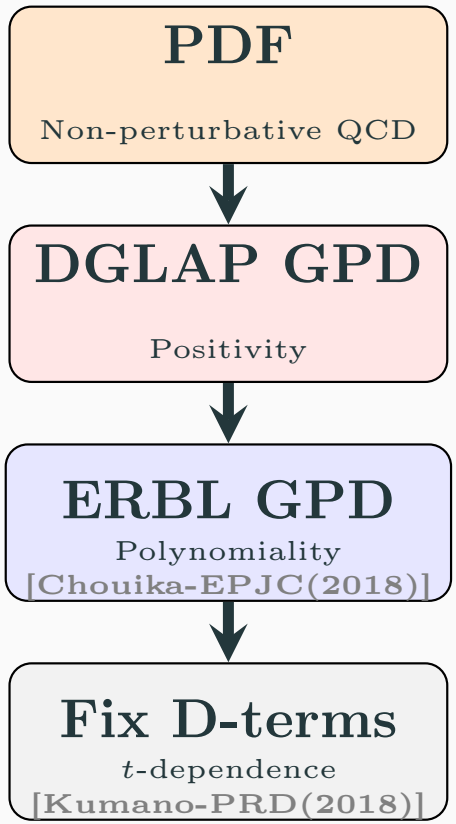


1. Build positive DGLAP GPD $\Rightarrow$ **How?**
2. Covariant extension: ERBL GPD
3. Soft pion theorem: fix $D^{\pm}(\alpha, 0)$

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Poby.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

GPD modelling: positivity saturated models

Question: *How can we build a positive DGLAP GPD?*



1. Overlap representation [M.Diehl-NPB:33(569)2001]

$$H^q(x,\xi,t)|_{|x|\geq\xi} = \int \frac{d^2k_\perp}{16\pi^3} \Psi^{q*}\left(x_-, k^2_{\perp,-}\right) \Psi^q\left(x_+, k^2_{\perp,+}\right)$$

2. Assume factorisation of the LFWF
[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi^q\left(x, k^2_\perp\right) \propto \varphi(x)\,\phi\left(k^2_\perp\right)$$

$$\downarrow \text{(Overalp rep.)}$$

$$H^q(x,\xi,t)|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right) q\left(\frac{x+\xi}{1+\xi}\right)}\,\Phi(x,\xi,t)$$

$$\downarrow (t=0)$$

$$H^q(x,\xi,0)|_{|x|\geq\xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right) q\left(\frac{x+\xi}{1+\xi}\right)}$$

Positivity saturated

Pion GPDs

Pion GPDs

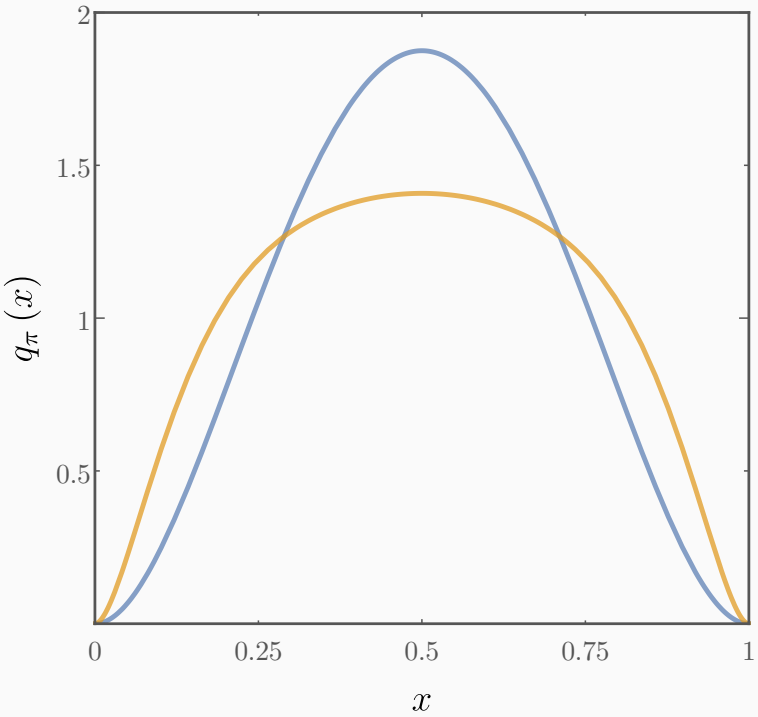
1. Under certain PTIR, chiral symmetry allows to factorize LFWF:
[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi_\pi^q\left(x,k_\perp^2\right) \propto \sqrt{q_\pi\left(x\right)}\frac{M^2}{\left(k_\perp^2+M^2\right)^2}$$

2. Pion GPD saturating positivity

$$H_\pi^q\left(x,\xi,t\right)|_{\mathrm{DGLAP}} = \frac{\sqrt{q_\pi\left(x_-\right)q_\pi\left(x_+\right)}}{\left(1+z^2\right)^2}\left[3+\frac{1-2z}{1+z}\frac{\operatorname{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}}\right]$$

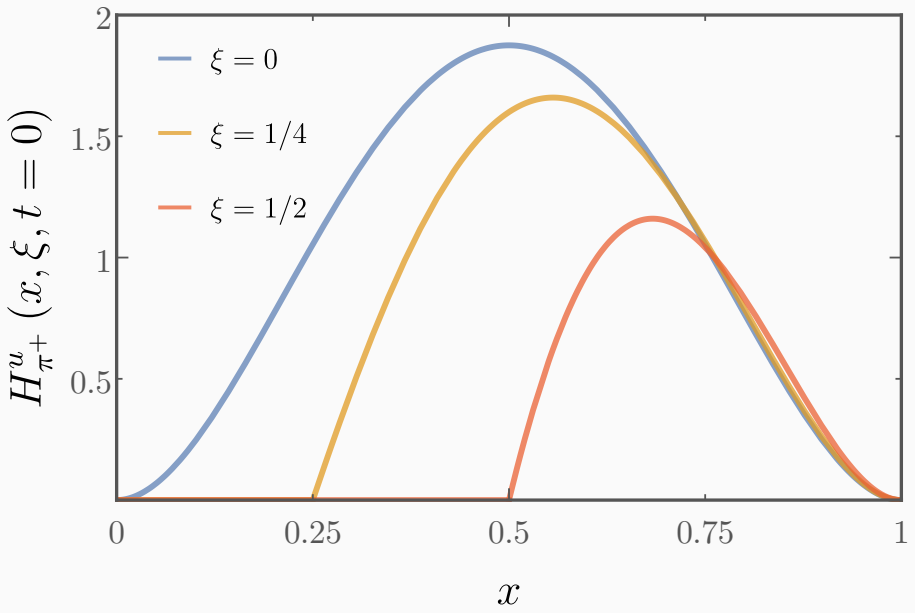
$$z=-t\left(1-x\right)^2/4M^2\left(1-\xi^2\right)$$

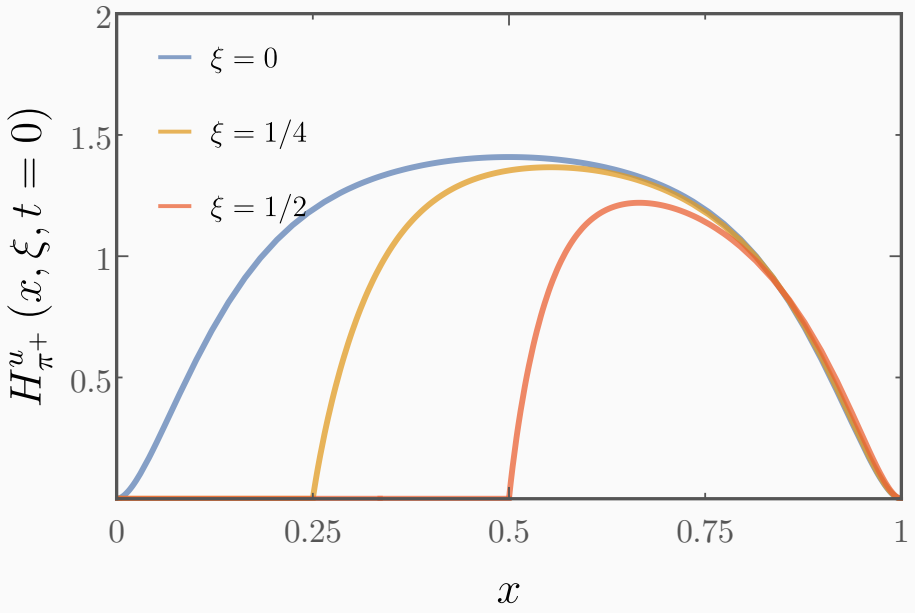


Two models:

- Algebraic model
  $q_\pi\left(x\right)=30x^2\left(1-x\right)^2$
- Realistic model (DSE)
  [M.Ding et al.-PRD:054014(101)2020]
  $q_\pi\left(x\right)=\mathcal{N}_q x^2\left(1-x\right)^2$
  $\times\left[1+\gamma x\left(1-x\right)+\rho\sqrt{x\left(1-x\right)}\right]$

Pion GPDs


$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
$H^u_{\pi^+}(x, \xi, t = 0)$
x
0
0.25
0.5
0.75
1
0.5
1
1.5
2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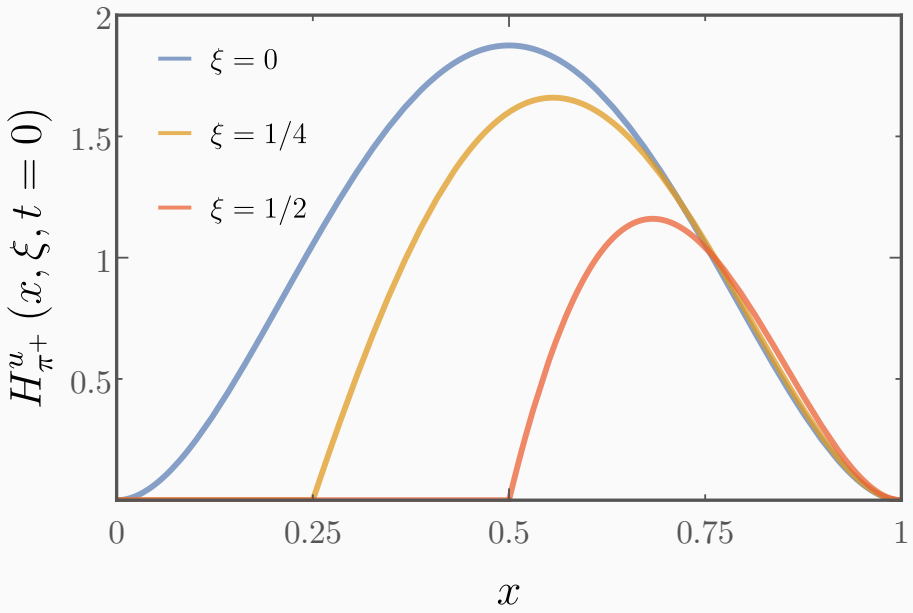
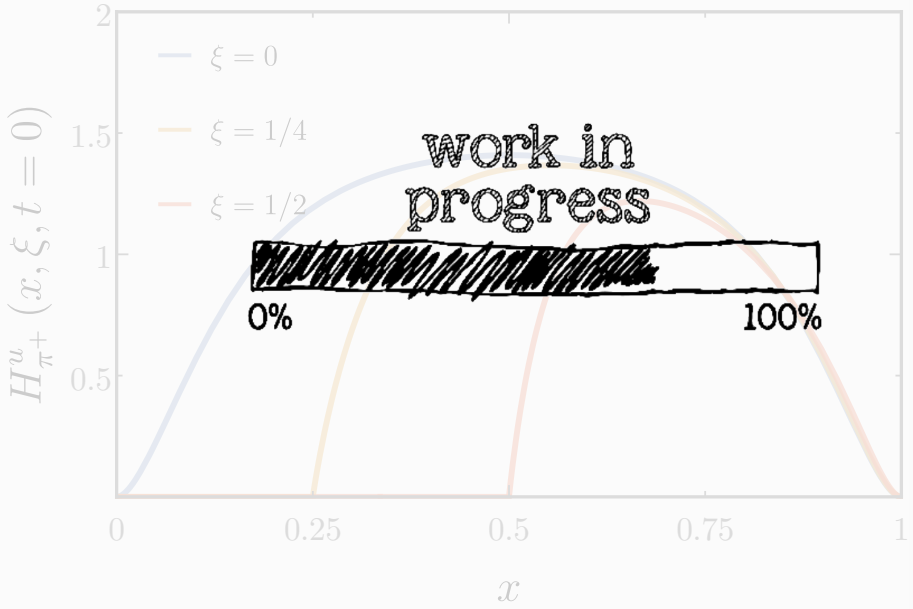
x
0
0.25
0.5
0.75
1
0.5
1
1.5
2

Pion GPDs


$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
$H^u_{\pi^+}(x, \xi, t = 0)$
x
0
0.25
0.5
0.75
1
0.5
1
1.5
2


work in progress
0%
100%
$\xi = 0$
$\xi = 1/4$
$\xi = 1/2$
$H^u_{\pi^+}(x, \xi, t = 0)$
x

Pion GPDs: covariant extension

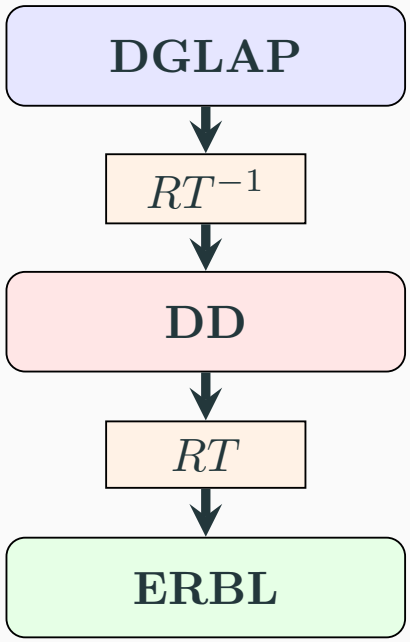
Covariant extension:

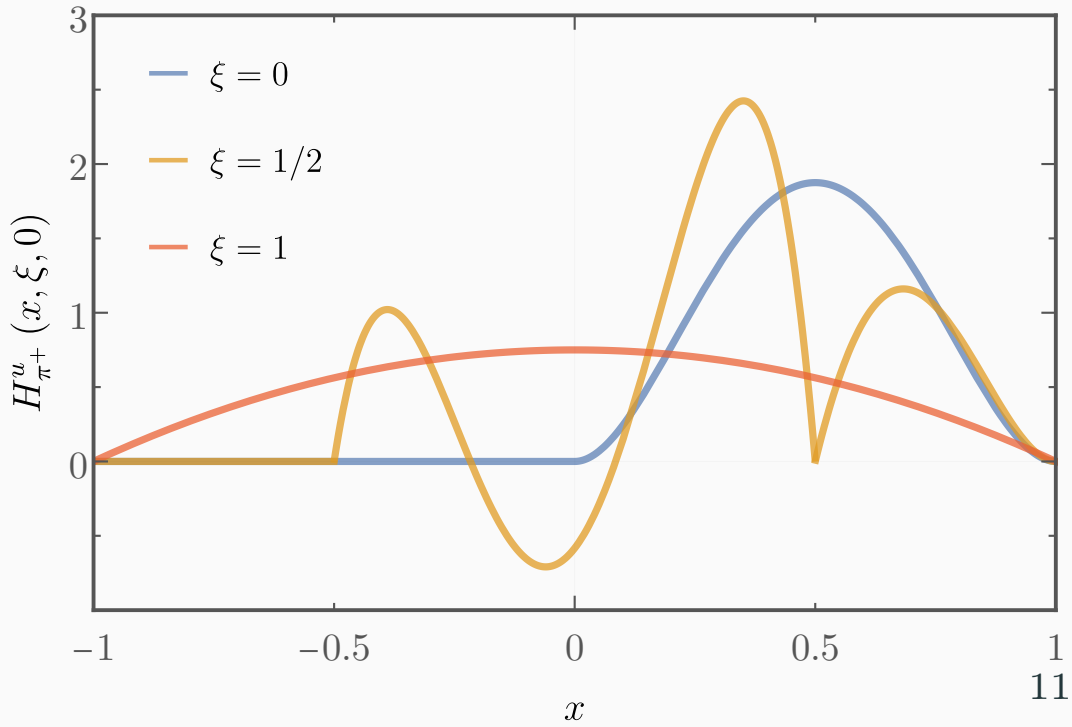
$$H^q(x,\xi,t) = \mathcal{R}\left[h(\beta,\alpha,t)\right] + \frac{1}{|\xi|} D^+\left(\frac{x}{\xi},t\right) + sgn(\xi)\, D^-\left(\frac{x}{\xi},t\right)$$

Fix D-terms with soft pion theorem:

[M.V.Polyakov-NPB:231(555)1999, C.Mezrag at al.-PLB:190(741)2015]

$$H^{I=0}_{\pi^+}(x,\xi,t)\big|_{\xi=1,t=0} = H_{\pi^+}(x,\xi,t) - H_{\pi^+}(-x,\xi,t)\big|_{\xi=1,t=0} = 0$$
$$H^{I=1}_{\pi^+}(x,\xi,t)\big|_{\xi=1,t=0} = H_{\pi^+}(x,\xi,t) + H_{\pi^+}(-x,\xi,t)\big|_{\xi=1,t=0} = \varphi\left(\tfrac{1+x}{2}\right)$$

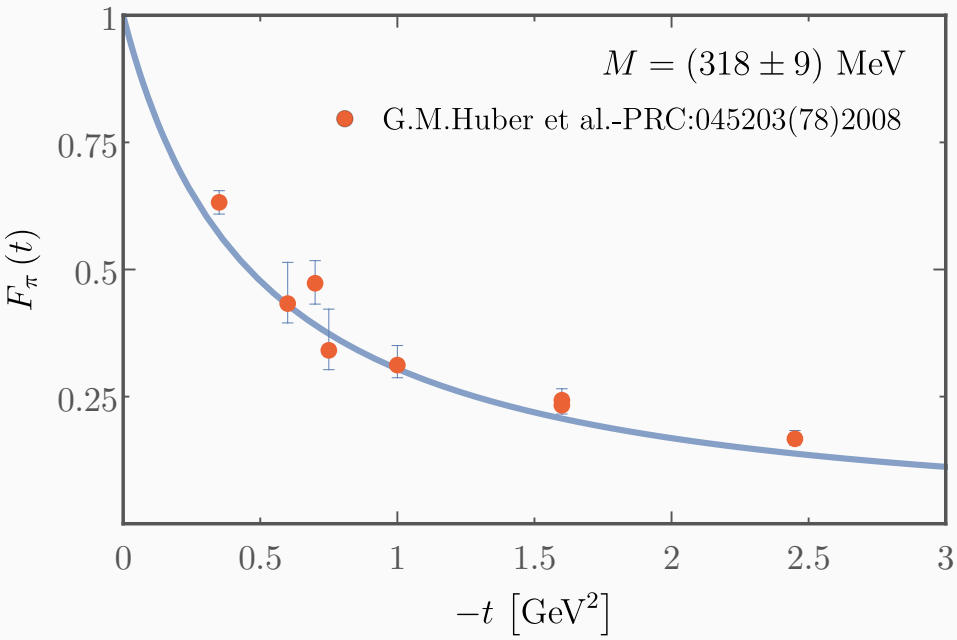


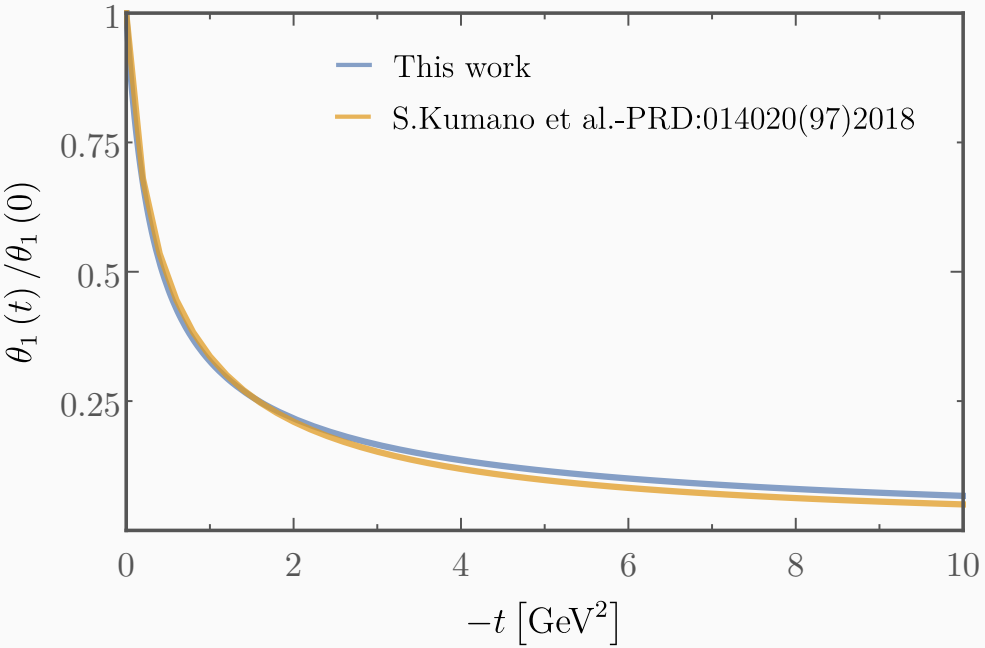


Pion GPDs: Mellin moments and form factors

D-term t-dependence:

$$d^{\pm}(t) = 1/\left(1 - t/4M^2\right)$$





Great agreement between experimental data for $F_\pi(t)$ and $\theta_1(t)$ and model prediction with one single free parameter.

Comparison with available data for $\theta_2(t)$ is not as good as for $\theta_1(t)$. This is currently being investigated.

Phenomenology of pion GPDs

Can we probe them in experiment?

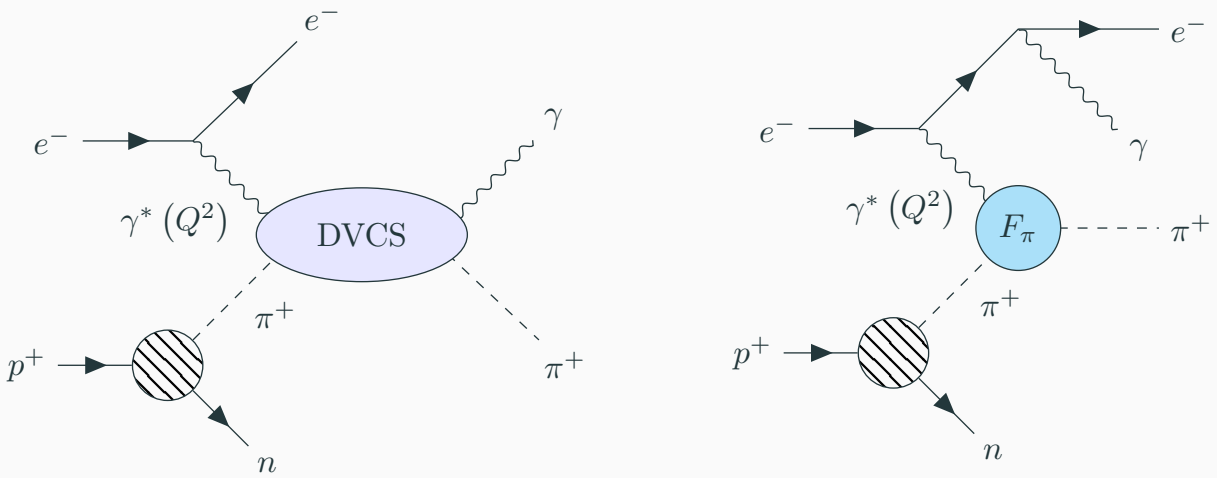
Phenomenology of pion GPDs: Sullivan process

We have stablished a way of bulding pion GPD models **fulfilling all of the QCD theoretical constraints**, so...

What about phenomenology?

DVCS amplitudes: parametrized by hadron GPDs.
[X.Ji-PRD:7114(55)1997]

Sullivan process [J.D.Sullivan-PRD:1732(5)1972]



The Sullivan process has already been employed for extracting π-EFFs.[G.M.Huber at al.-PRC:045203(78)2008]

Can we probe pion GPDs?
[D.Amrath at al.-EPJC:179(58)2008]

Phenomenology of pion GPDs: Sullivan process

In fact... this has been advocated in the recent EIC-Yellow report
[EICYR:phys.ins-det/2103.05419]

Science Question	Key Measurement	Key Requirements
What are the quark and gluon energy contributions to the pion mass?	Pion structure function data over a range of x and Q^2.	• Need to uniquely determine $e+p \to e'+X+n$ (low $-t$) • CM energy range ~10-100 GeV • Charged and neutral currents desirable
Is the pion full or empty of gluons as viewed at large Q^2?	Pion structure function data at large Q^2.	• CM energy ~100 GeV • Inclusive and open-charm detection
What are the quark and gluon energy contributions to the kaon mass?	Kaon structure function data over a range of x and Q^2.	• Need to uniquely determine $e+p \to e'+X+\Lambda/\Sigma^0$ (low $-t$) • CM energy range ~10-100 GeV
Are there more or less gluons in kaons than in pions as viewed at large Q^2?	Kaon structure function data at large Q^2.	• CM energy ~100 GeV • Inclusive and open-charm detection
Can we get quantitative guidance on the emergent pion mass mechanism?	Pion form factor data for Q^2 = 10-40 $(\mathrm{GeV}/c)^2$.	• Need to uniquely determine exclusive process $e+p \to e'+\pi^++n$ (low $-t$) • $e+p$ and $e+D$ at similar energies • CM energy ~10-75 GeV
What is the size and range of interference between emergent-mass and the Higgs-mass mechanism?	Kaon form factor data for Q^2 = 10-20 $(\mathrm{GeV}/c)^2$.	• Need to uniquely determine exclusive process $e+p \to e'+K+\Lambda$ (low $-t$) • L/T separation at CM energy ~10-20 GeV • Λ/Σ^0 ratios at CM energy ~10-50 GeV
What is the difference between the impacts of emergent- and Higgs-mass mechanisms on light-quark behavior?	Behavior of (valence) up quarks in pion and kaon at large x.	• CM energy ~20 GeV (lowest CM energy to access large-x region) • Higher CM energy for range in Q^2 desirable
What is the relationship between dynamically chiral symmetry breaking and confinement?	Transverse-momentum dependent Fragmentation Functions of quarks into pions and kaons.	• Collider kinematics desirable (as compared to fixed-target kinematics) • CM energy range ~20-140 GeV
More speculative observables		
What is the trace anomaly contribution to the pion mass?	Elastic J/Ψ production at low W off the pion.	• Need to uniquely determine exclusive process $e+p \to e'+J/\Psi+\pi^++n$ (low $-t$) • High luminosity ($\geq 10^{34}\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) • CM energy ~70 GeV
Can we obtain tomographic snapshots of the pion in the transverse plane? What is the pressure distribution in a pion?	Measurement of DVCS off pion target as defined with Sullivan process.	• Need to uniquely determine exclusive process $e+p \to e'+\gamma+\pi^++n$ (low $-t$) • High luminosity ($\geq 10^{34}\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) • CM energy ~10-100 GeV
Are transverse momentum distributions universal in pions and protons?	Hadron multiplicities in SIDIS off a pion target as defined with Sullivan process.	• Need to uniquely determine SIDIS off pion $e+p \to e'+h+X+n$ (low $-t$) • High luminosity ($10^{34}\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) • $e+p$ and $e+D$ at similar energies desirable • CM energy ~10-100 GeV

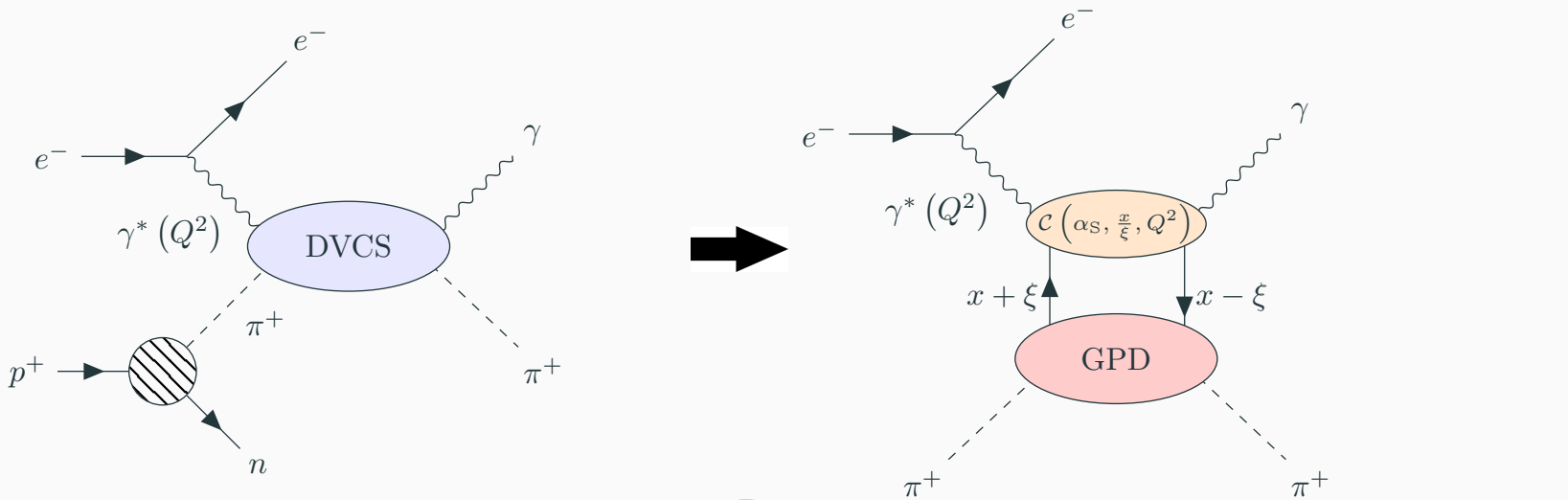
Let us see if that would be feasible in a future electron-ion collider.

Phenomenology of pion GPDs: Sullivan process

One pion exchange approximation: [D.Amrath et al.-EPJC:179(58)2008]

- $-t < 0,6\,\mathrm{GeV}^2$
- $\sigma_L >> \sigma_\perp$

} Met at EIC [EICYR:phys.ins-det/2103.05419]



Phenomenology

1. σ-DVCS [D.Amrath et al.-EPJC(2008)]
2. Asymmetry
3. ...

=

QCD Evolution

APFEL++

[V.Bertone et al.CPComm(2014), V.Bertone et al.:hep-ph/1708.00911]

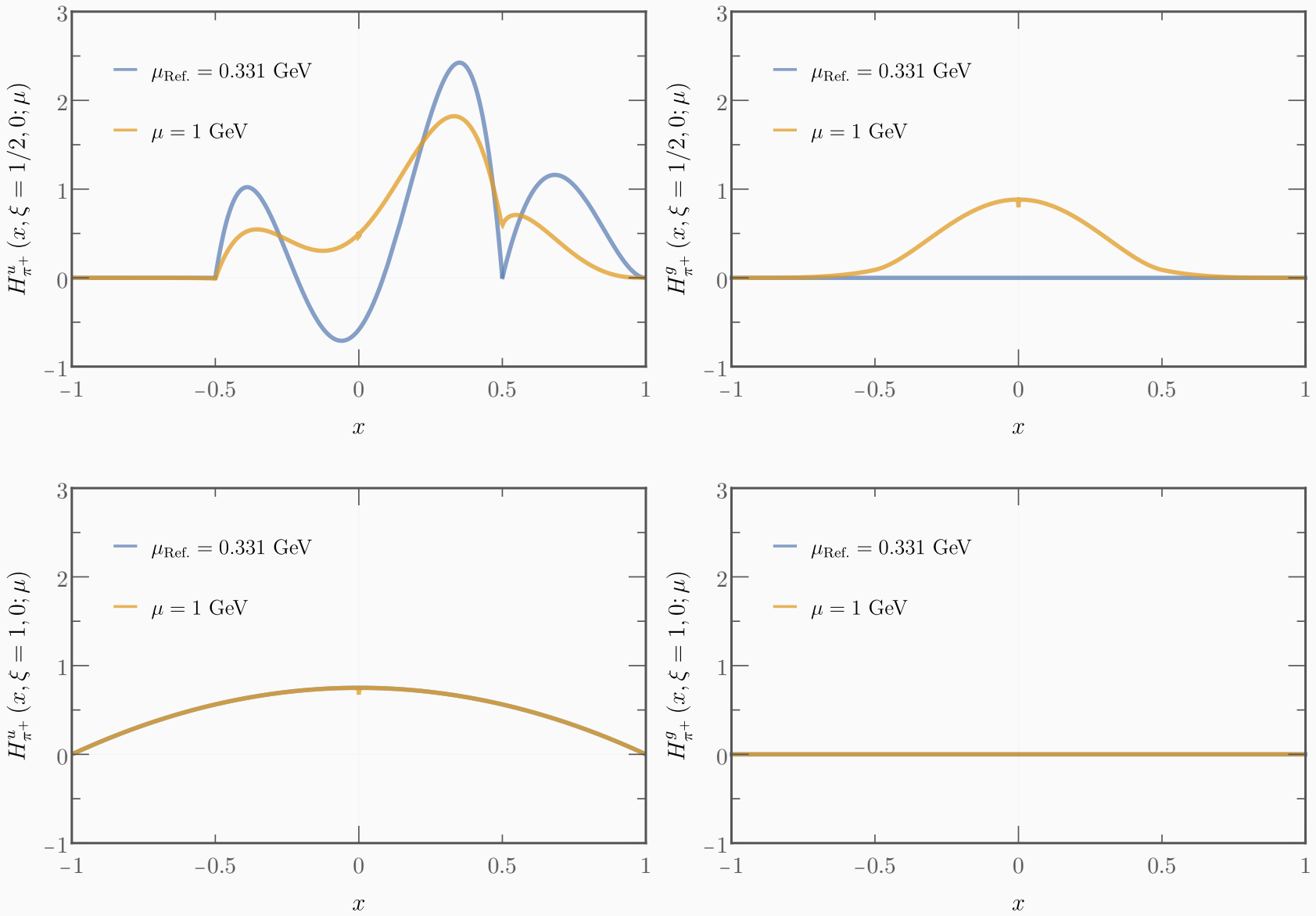
+

LO/NLO CFFs

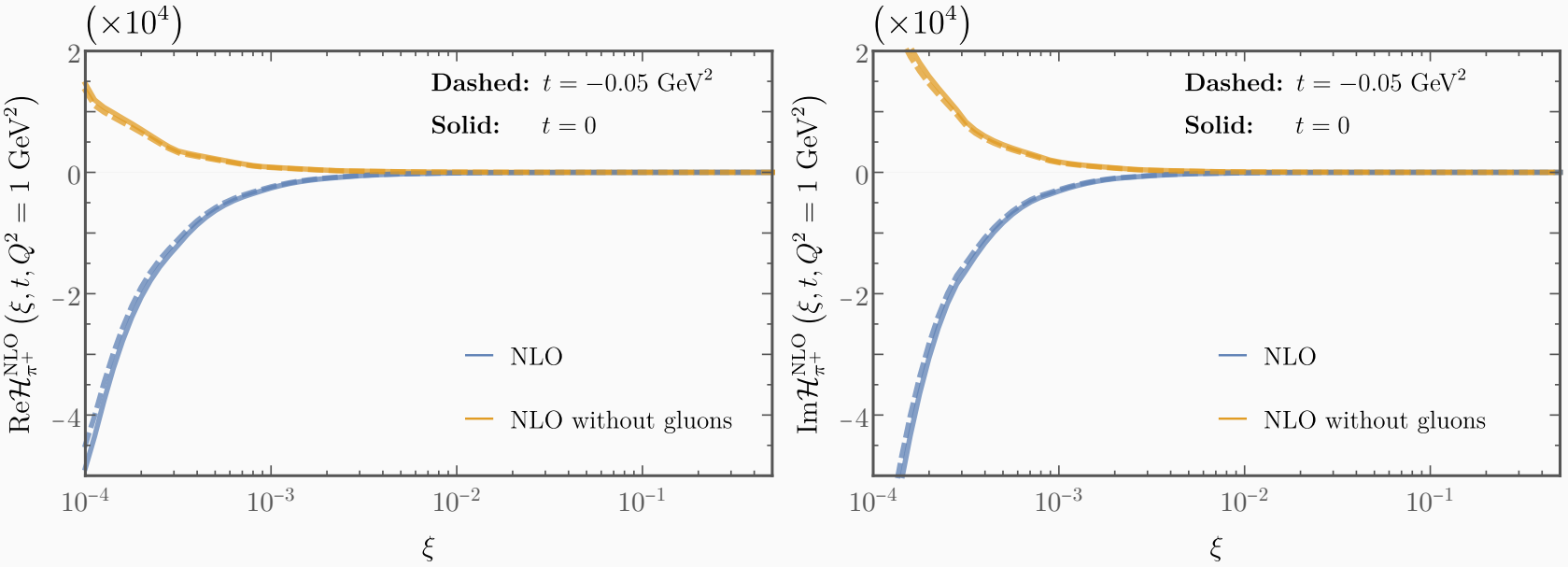
PARTONS

[B.Berthou et al.-EPJC(2018)]

Phenomenology of pion GPDs: QCD evolution



Phenomenology of pion GPDs: Compton Form Factors



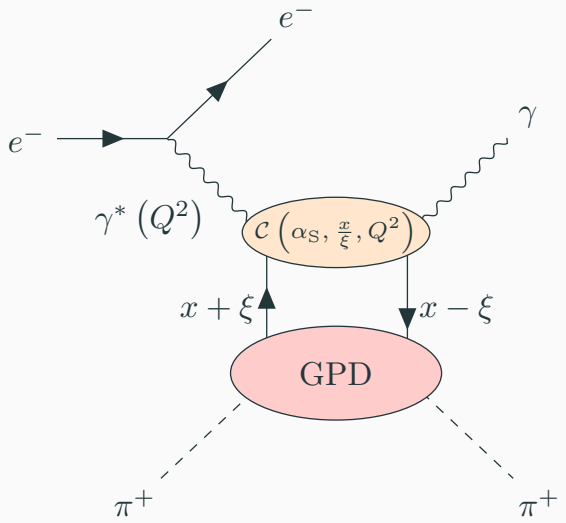
Gluon dominance makes essential at least NLO accuracy in any phenomenlogical analysis of DVCS at an EIC.

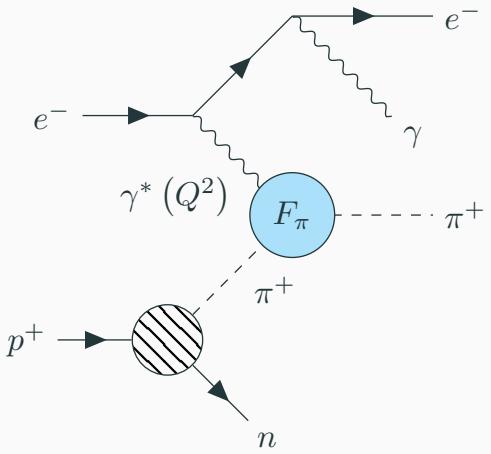
Phenomenology of pion GPDs: DVCs and Sullivan process

Can we measure DVCS?

One pion exchange approximation: [D.Amrath at al.-EPJC:179(58)2008]

$-t < 0,6\,\mathrm{GeV}^2$ $\qquad$ $\sigma_L >> \sigma_\perp$

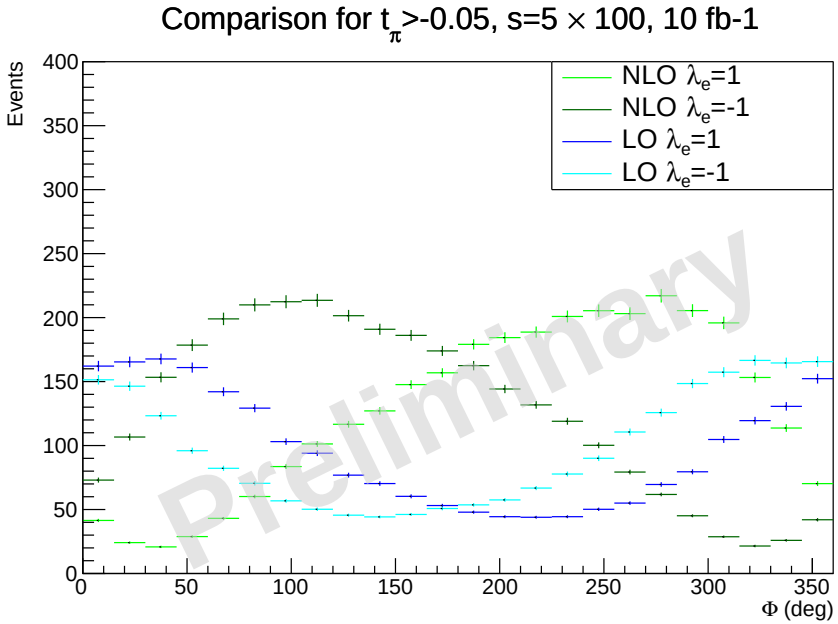


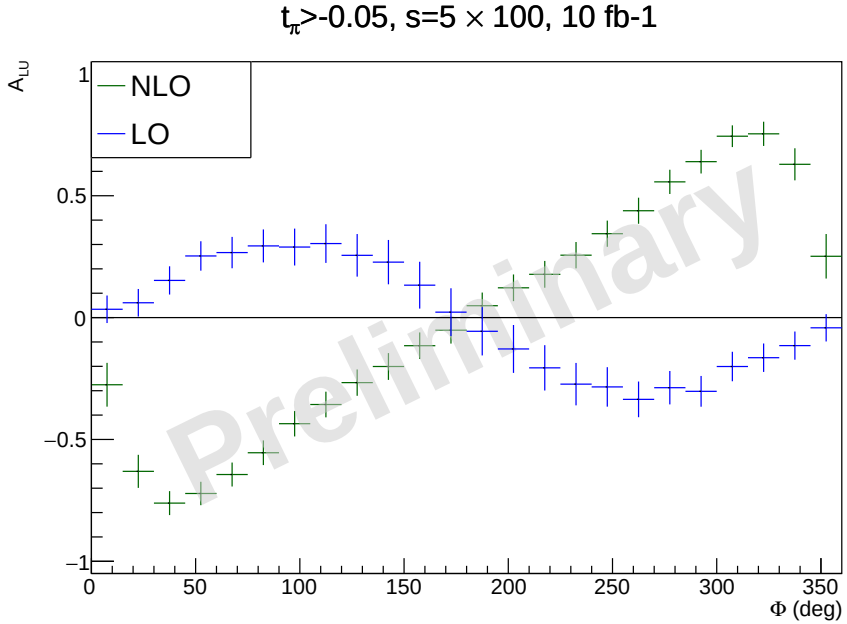


Changing lepton polarisation one can (formally) access interference between DVCS and BH amplitudes.

Is it experimentally feasible?

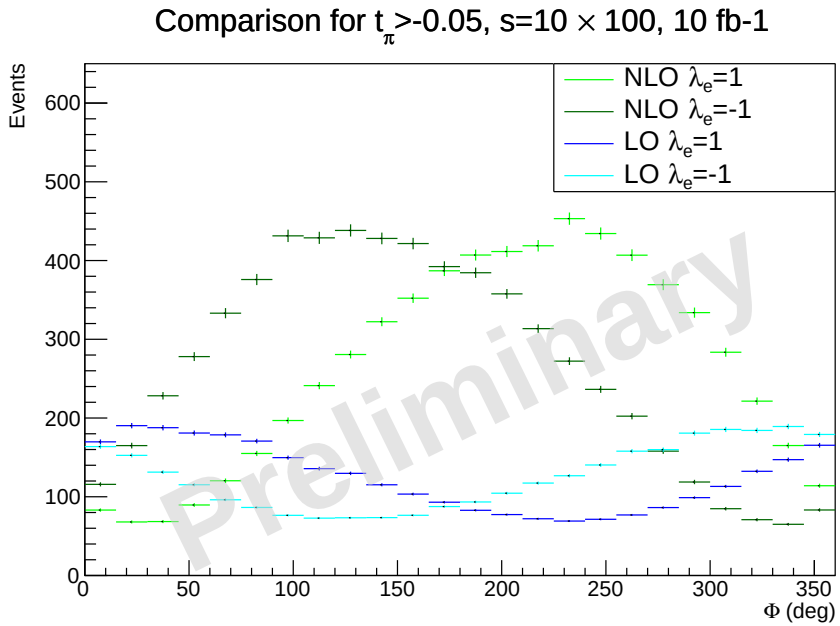
Phenomenology of pion GPDs: Asymmetry (EIC)

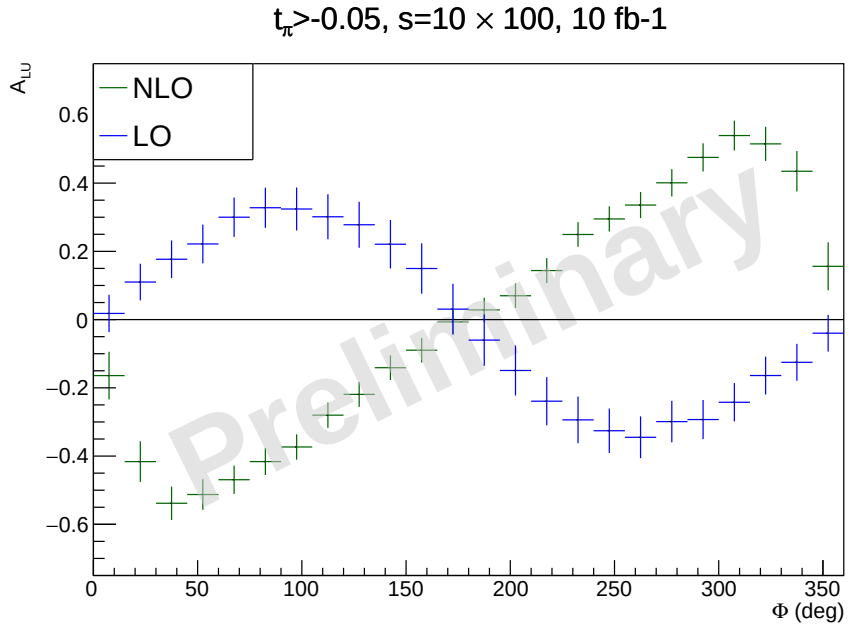




Non-zero asymmetry: optimism about measuring DVCS on pions at future EIC.

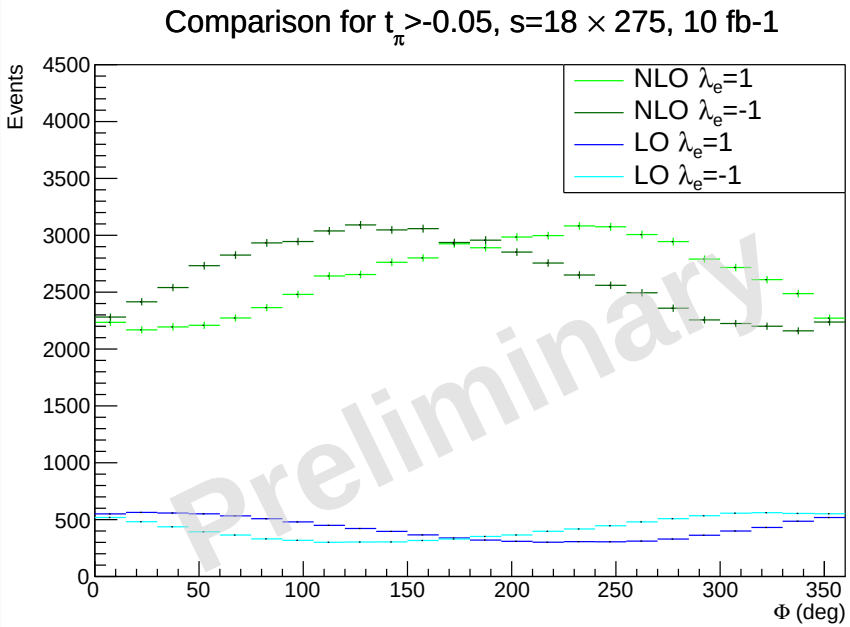
Phenomenology of pion GPDs: Asymmetry (EIC)

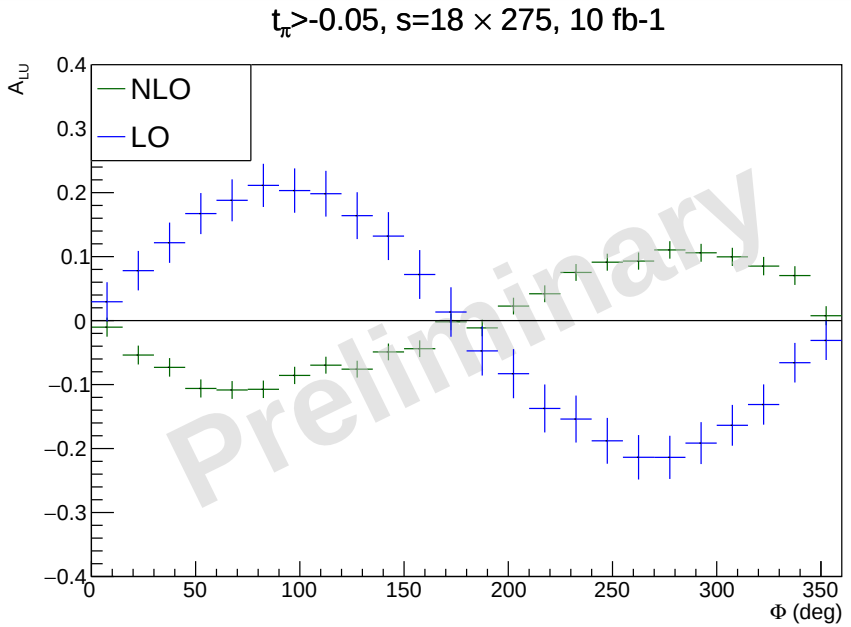




Non-zero asymmetry: optimism about measuring DVCS on pions at future EIC.

Phenomenology of pion GPDs: Asymmetry (EIC)





Non-zero asymmetry: optimism about measuring DVCS on pions at future EIC.

Summary and perspectives

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Summary

1. Pion GPD models fulfilling every theoretical constraint
   - Polynomiality: Covariant extension.
   - Positivity
   - PCAC/AV-WTI: Soft pion theorem.
   - Agreement with experimental data for EFFs and GFFs.
2. `PARTONS` implements complete computing chain
   - From GPDs to DVCS CFFs
   - From DVCS CFFs to observables
3. DVCS on virtual pions influenced by gluon content
   - Higher order analysis needed for phenomenology.
4. Pion structure to be tested at future electron-ion colliders
   - Insights into EHM could be gained experimentally.

Summary and perspectives

Perspectives

Currently at work

1. Exploit realistic pion PDF
2. Comparison with previous studies
    - Original paper by D. Amrath, M. Diehl and J. P. Lansberg
      [D.Amrath et al.-EPJC:179(58)2008]

Forthcoming developments

3. Extension of the computating chain
    - Higher order analysis
    - Baryons

Thank you!