

# TMD phenomenology with the **NangaParbat** code

Valerio Bertone

IRFU, CEA, Université Paris-Saclay

In collaboration with:

R. Abdul Khalek, A. Bacchetta, C. Bissolotti, G. Bozzi, M. Cerutti, F. Delcarro, E. R. Nocera, F. Piacenza, M. Radici, A. Signori

université  
PARIS-SACLAY



June 1, 2021, STRONG2020 joint workshop 2021

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement № 824093

# Factorisations

🍏 The  $q_T$  distribution of a generic **high-mass** ( $Q$ ) system produced, for example, in hadronic collisions has two main regimes:

🍏 for  $q_T \gtrsim Q$  **collinear factorisation** at *fixed perturbative order* is appropriate:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{coll.}} = \int_0^1 dx_1 \int_0^1 dx_2 f_1(x_1, Q) f_2(x_2, Q) \frac{d\hat{\sigma}}{dq_T} + \mathcal{O}\left[\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^n\right]$$

🍏 for  $q_T \ll Q$  **transverse-momentum-dependent (TMD) factorisation** at *fixed logarithmic accuracy* is appropriate:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{TMD}} = \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2) + \mathcal{O}\left[\left(\frac{q_T}{Q}\right)^m\right]$$

🍏 Collinear and TMD factorisations may eventually be **matched** to produce accurate results over the the full  $q_T$  spectrum.

# TMD factorisation

🍏 TMD factorisation introduces two independent *artificial* scales:

- 🍏 the **renormalisation scale**  $\mu$ , originating from UV renormalisation,
- 🍏 the **rapidity scale**  $\zeta$ , originating from the cancellation of rapidity divergencies.

🍏 The respective **evolution equations** are:

$$\frac{\partial \ln F}{\partial \ln \sqrt{\zeta}} = K(\mu)$$

$$\frac{\partial \ln F}{\partial \ln \mu} = \gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta}}{\mu}$$

with: 
$$\frac{\partial K}{\partial \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

🍏 In addition, for small  $b_T$ , TMDs can be matched onto coll. distributions:

$$F(\mu, \zeta) = C(\mu, \zeta) \otimes f(\mu)$$

🍏 The solution is:

$$F(\mu, \zeta) = \exp \left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} C(\mu_0, \zeta_0) \otimes f(\mu_0)$$

🍏 Anomalous dims. and matching funcs. **perturbatively** computable.

# TMD factorisation

🍏 TMD factorisation introduces two independent *artificial* scales:

- 🍏 the **renormalisation scale**  $\mu$ , originating from UV renormalisation,
- 🍏 the **rapidity scale**  $\zeta$ , originating from the cancellation of the rapidity divergencies.

🍏 The respective **evolution equations** are:

$$\frac{\partial \ln F}{\partial \ln \sqrt{\zeta}} = K(\mu)$$

$$\frac{\partial \ln F}{\partial \ln \mu} = \gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta}}{\mu}$$

with: 
$$\frac{\partial K}{\partial \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

🍏 In addition, for small  $b_T$ , TMDs can be matched onto coll. distributions:

Matching  
onto collinear

$$F(\mu, \zeta) = C(\mu, \zeta) \otimes f(\mu)$$

🍏 The solution is:

Evolution (Sudakov) factor

$$\mu_b = b_0 / b_T$$

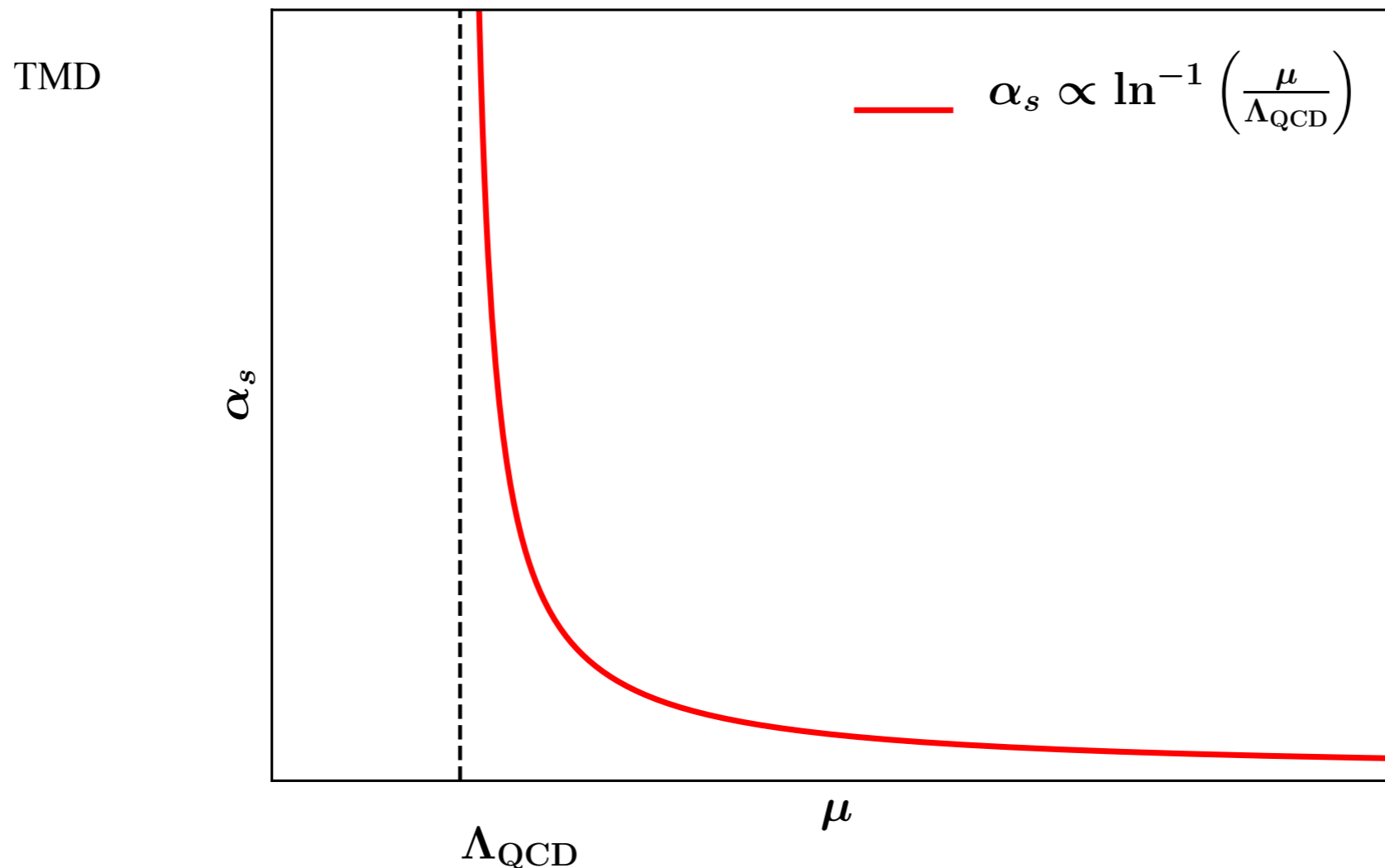

$$F(\mu, \zeta) = \exp \left\{ K(\mu_0) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu')) \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} C(\mu_0, \zeta_0) \otimes f(\mu_0)$$

🍏 Anomalous dims. and matching funcs. **perturbatively** computable.

# TMD factorisation

- 🍏 When integrating over  $b_T$ , **large values of  $b_T$**  give rise to low scales in the **non-perturbative** region:

$$\frac{d\sigma}{dq_T} \propto \int_0^\infty db_T \alpha_s^p \left( \frac{1}{b_T} \right) \dots \sim \int_0^Q dk_T \alpha_s^p (k_T) \dots$$



- 🍏 Blindly integrating over the full phase space would give a **divergent** result.

# TMD factorisation

🍏 Introduce the so-called  **$b^*$ -prescription**:

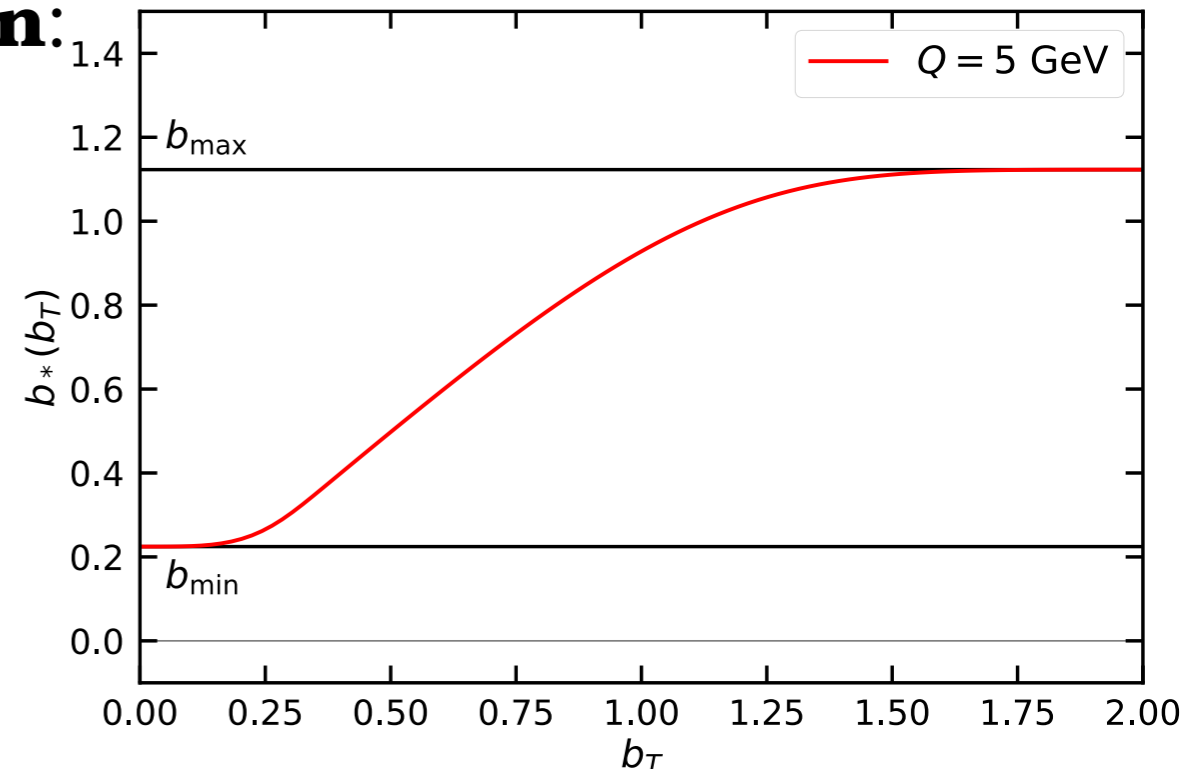
$$b_*(b) = b_{\max} \left( \frac{1 - \exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E} / Q$$

🍏 and rewrite (CSS approach):

$$F(x, b_T, \mu, \zeta) = \left[ \frac{F(x, b_T, \mu, \zeta)}{F(x, b_*(b_T), \mu, \zeta)} \right] F(x, b_*(b_T), \mu, \zeta) \equiv f_{\text{NP}}(x, b_T, \zeta) F(x, b_*(b_T), \mu, \zeta)$$



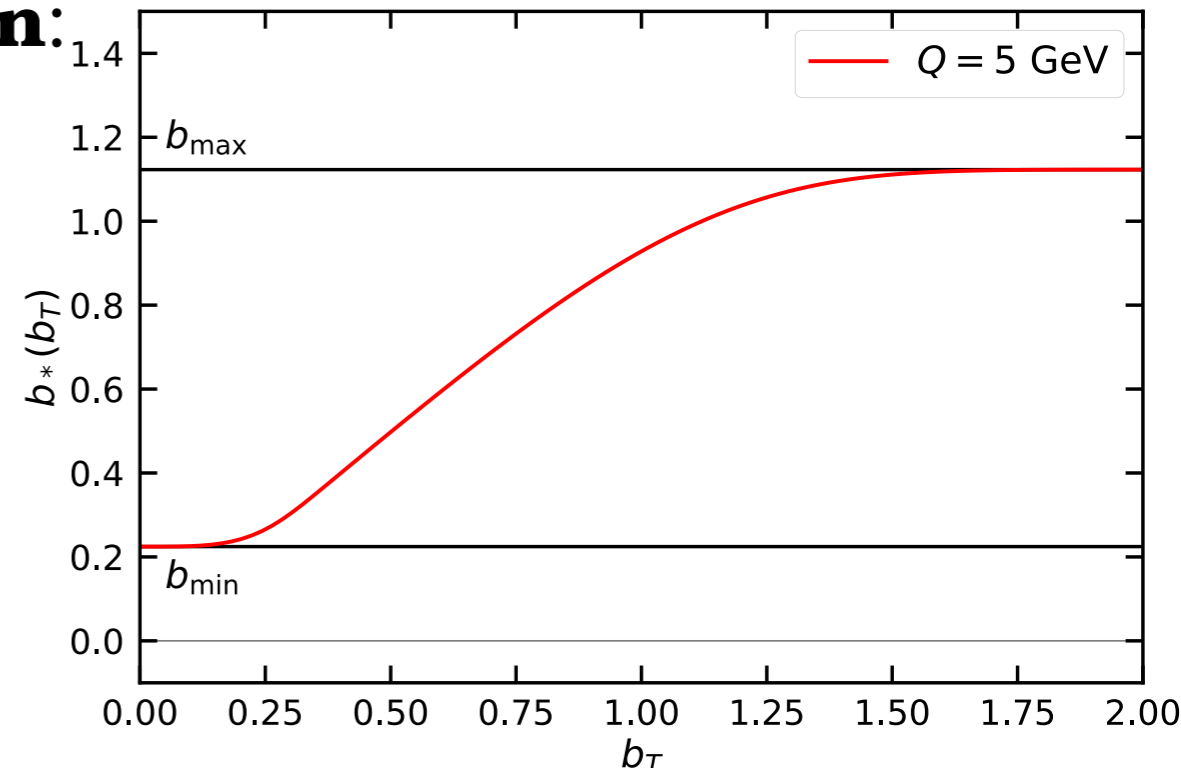
# TMD factorisation

🍏 Introduce the so-called  **$b^*$ -prescription**:

$$b_*(b) = b_{\max} \left( \frac{1 - \exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1 - \exp\left(-\frac{b^4}{b_{\min}^4}\right)} \right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E} / Q$$



🍏 and rewrite (CSS approach):

$$F(x, b_T, \mu, \zeta) = \left[ \frac{F(x, b_T, \mu, \zeta)}{F(x, b_*(b_T), \mu, \zeta)} \right] F(x, b_*(b_T), \mu, \zeta) \equiv \underbrace{f_{\text{NP}}(x, b_T, \zeta)}_{\text{Non-perturbative}} \underbrace{F(x, b_*(b_T), \mu, \zeta)}_{\text{Purely perturbative}}$$

🍏 Properties of  $f_{\text{NP}}$ :

🍏 has to go to **one** as  $b_T$  goes to zero: reproduce the fully perturbative regime,

🍏 has to go to **zero** as  $b_T$  becomes large: mimic the Sudakov suppression.

🍏 Bottom line: avoidance of the non-perturbative region upon integration in  $b_T$  implies the presence of **both**  $b^*$ -prescription and  $f_{\text{NP}}$ .

# TMD factorisation

 Final expression:

$$\begin{aligned} F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\ &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\ &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C \end{aligned}$$



# TMD factorisation

🍏 Final expression:

$$\begin{aligned}
 F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
 &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
 &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C
 \end{aligned}$$

- matching onto the collinear region at  $b_T \ll 1/\Lambda_{\text{QCD}}$ ,
- factorises as *hard* (perturbative) and *longitudinal* (i.e. collinear, non-perturbative).

# TMD factorisation

🍏 Final expression:

$$\begin{aligned}
 F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
 &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
 &\times \exp \left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} && : C
 \end{aligned}$$

- matching onto the collinear region at  $b_T \ll 1/\Lambda_{\text{QCD}}$ ,
- factorises as *hard* (perturbative) and *longitudinal* (i.e. collinear, non-perturbative).

- CS and RGE evolution,
- evolution to large scales,
- perturbative.

# TMD factorisation

🍏 Final expression:

$$\begin{aligned}
 F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) &= \sum_j C_{f/j}(x, b_*; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) && : A \\
 &\times \exp \left\{ K(b_*; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} && : B \\
 &\times \exp \left\{ \underbrace{g_{j/P}(x, b_T)}_{\text{green}} + \underbrace{g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}}}_{\text{blue}} \right\} && : C
 \end{aligned}$$

- matching onto the collinear region at  $b_T \ll 1/\Lambda_{\text{QCD}}$ ,
- factorises as *hard* (perturbative) and *longitudinal* (i.e. collinear, non-perturbative).

- avoid the Landau pole through  $b_*$ ,
- $f_{\text{NP}}$  accounts for the introduction of  $b_*$ ,
- $f_{\text{NP}}$  is non-perturbative thus **fitted** to data.

- CS and RGE evolution,
- evolution to large scales,
- perturbative.

# Logarithmic counting

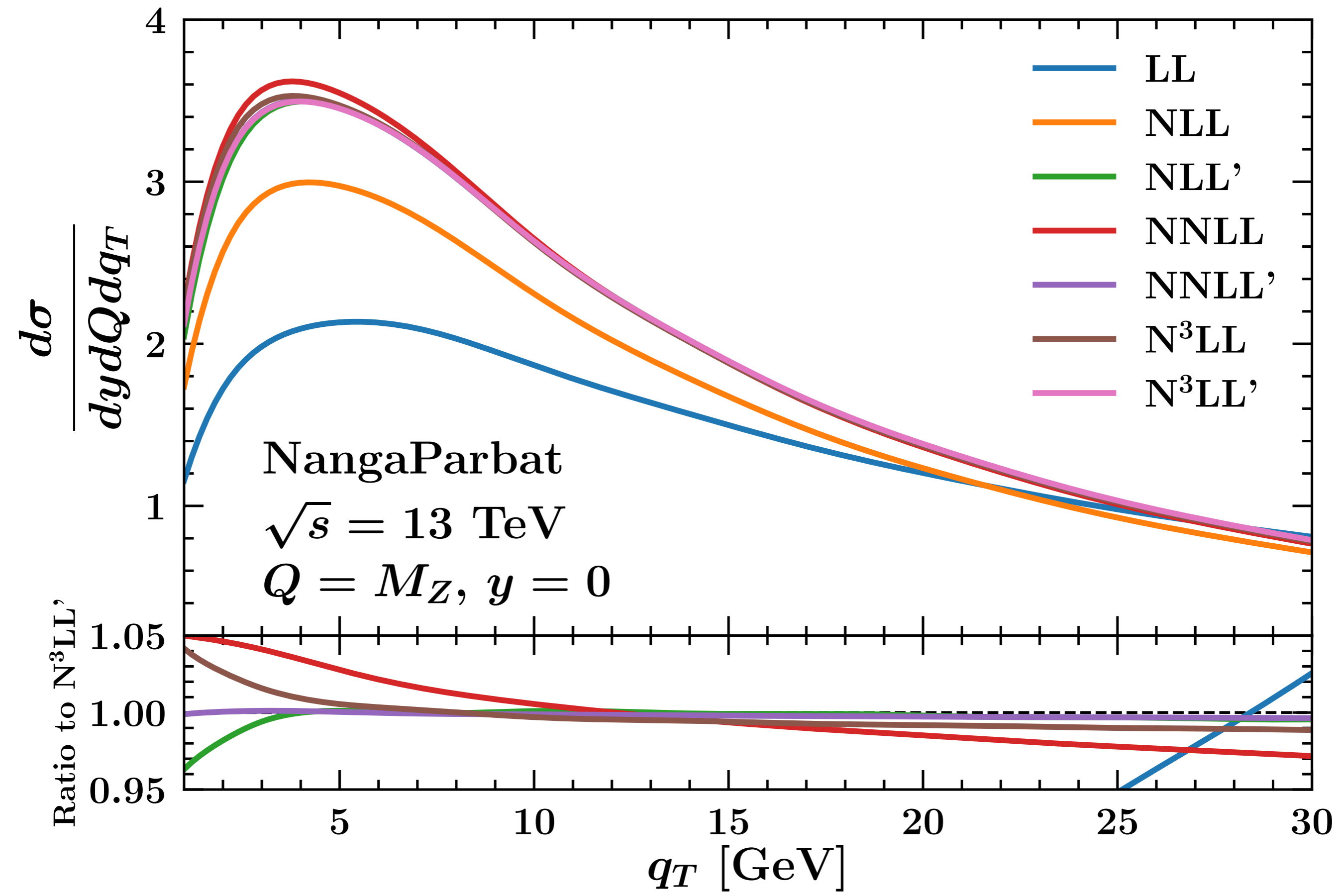
🍏 TMD factorisation allows us to **resum large logarithms**:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{TMD}} = \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2)$$

$$F_f(x, \mathbf{b}_T, \mu, \zeta) = \sum_j C_{f/j}(c, \mathbf{b}_T; \mu_b, \zeta) \otimes f_j(x, \mu_b) \times \exp \left\{ K(\mathbf{b}_T, \mu_b) \ln \frac{\sqrt{\zeta}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\}$$

Accuracy	$\gamma_K$	$\gamma_F$	$K$	$C_{f/j}$	$H$
LL	$\alpha_s$	-	-	1	1
NLL	$\alpha_s^2$	$\alpha_s$	$\alpha_s$	1	1
NLL'	$\alpha_s^2$	$\alpha_s$	$\alpha_s$	$\alpha_s$	$\alpha_s$
N <sup>2</sup> LL	$\alpha_s^3$	$\alpha_s^2$	$\alpha_s^2$	$\alpha_s$	$\alpha_s$
N <sup>2</sup> LL'	$\alpha_s^3$	$\alpha_s^2$	$\alpha_s^2$	$\alpha_s^2$	$\alpha_s^2$
N <sup>3</sup> LL	$\alpha_s^4$	$\alpha_s^3$	$\alpha_s^3$	$\alpha_s^2$	$\alpha_s^2$
N <sup>3</sup> LL'	$\alpha_s^4$	$\alpha_s^3$	$\alpha_s^3$	$\alpha_s^3$	$\alpha_s^3$

# Logarithmic counting



# Matching TMD and collinear

- Accurate predictions for all  $q_T$ 's by **additive matching**, order by order in perturbation theory, of collinear and TMD calculations:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{matched}} = \left(\frac{d\sigma}{dq_T}\right)_{\text{TMD}} + \left(\frac{d\sigma}{dq_T}\right)_{\text{coll.}} - \left(\frac{d\sigma}{dq_T}\right)_{\text{d.c.}}$$

- In order for the match to actually take place:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{TMD}} \xrightarrow{\text{f.o.}} \left(\frac{d\sigma}{dq_T}\right)_{\text{d.c.}} \xleftarrow{q_T \ll Q} \left(\frac{d\sigma}{dq_T}\right)_{\text{coll.}}$$

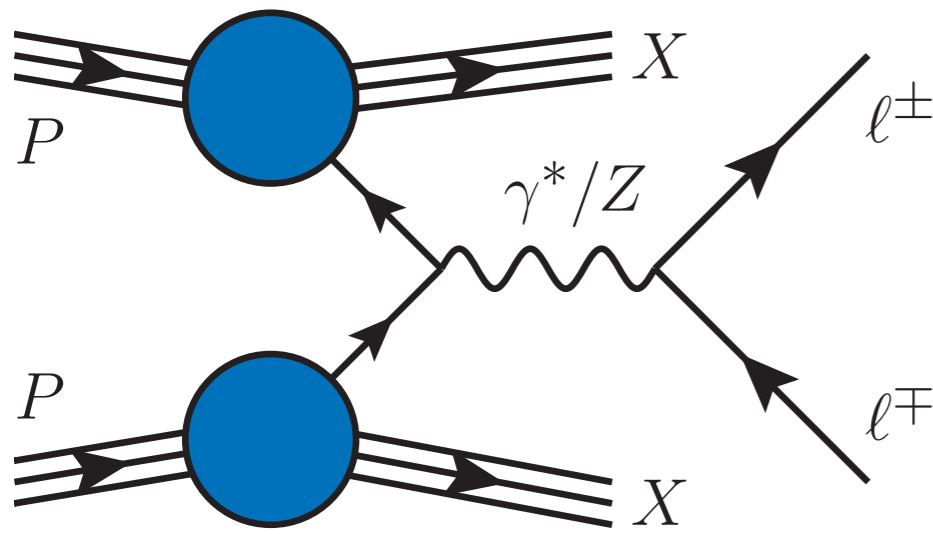
- Therefore, the “fixed-order” parts have to match in the relevant limits:

Logarithmic accuracy	Minimal f.o. accuracy
NLL'	$\alpha_s$ (LO)
N <sup>2</sup> LL	$\alpha_s$ (LO)
N <sup>2</sup> LL'	$\alpha_s^2$ (NLO)
N <sup>3</sup> LL	$\alpha_s^2$ (NLO)
N <sup>3</sup> LL'	$\alpha_s^3$ (NNLO)

# Factorising processes

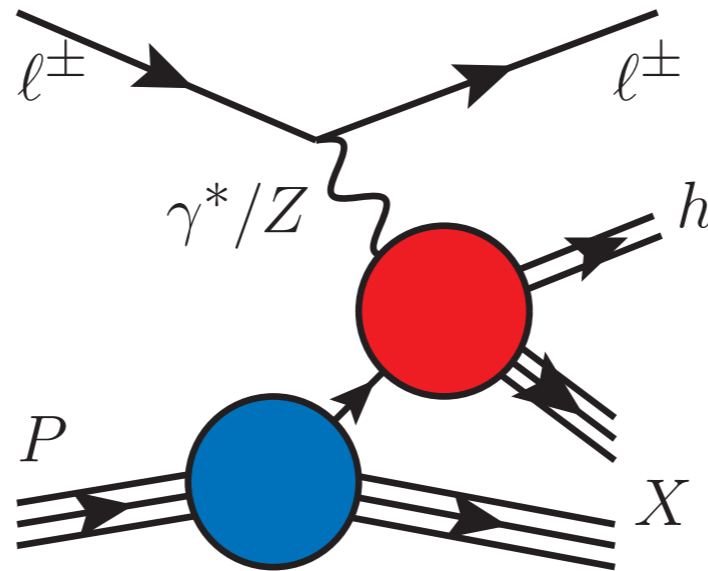
Processes for which leading-power factorisation has been **proven**:

Drell-Yan



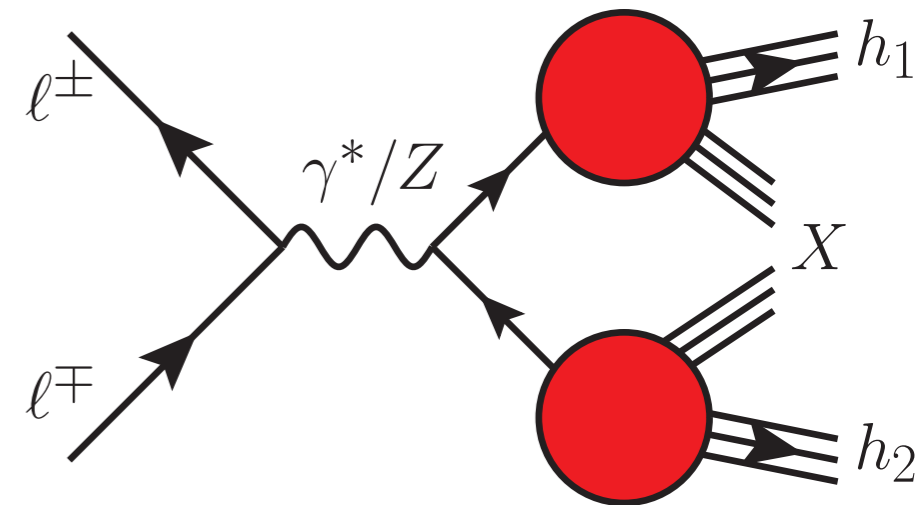
$$PP \longrightarrow l^\pm l^\mp X$$

Semi-inclusive DIS



$$Pl^\pm \longrightarrow l^\pm h X$$

$e^+e^-$  annihilation



$$l^\pm l^\mp \longrightarrow h_1 h_2 X$$

Two PDFs:

One PDF and one FF:

Two FFs:

Lots of data:

many precise data points:

di-hadron prod. from:

low-energy: FNAL,

HERMES at DESY,

BELLE at KEK,

mid-energy: RHIC,

COMPASS at CERN.

BABAR at SLAC.

high-energy:  
Tevatron, LHC.

# A framework for TMD analyses

## *NangaParbat*

- 🍏 Public implementation of TMD factorisation and CSS formalism:
  - 🍏 **Drell-Yan** with fiducial cuts operative,
  - 🍏 validating **semi-inclusive DIS**.
- 🍏 Main focus on fast and accurate computations aimed at TMD fits:
  - 🍏 exploitation of **interpolation** techniques.

<https://github.com/MapCollaboration/NangaParbat>



### Nanga Parbat: a TMD fitting framework

---

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.



# A framework for TMD analyses

## *NangaParbat*

🍏 The numerical computation of a cross section can be reduced to:

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{TMD}} \simeq \sum_{n,\alpha,\tau} W_{n\alpha\tau} f_{\text{NP}}^{(1)}(x_1^{(\alpha,\tau)}, b_T^{(n)}, \zeta^{(\tau)}) f_{\text{NP}}^{(2)}(x_2^{(\alpha,\tau)}, b_T^{(n)}, \zeta^{(\tau)})$$

🍏 The weights  $W$  can be **precomputed** and **stored**:

🍏 perturbative ingredients,

🍏 non-perturbative ingredients: collinear distributions and Landau pole regularisation,

🍏 integration over the final-state phase space including fiducial cuts.

🍏 The cross section is thus obtained by “convoluting” the weights with the non-perturbative function(s)  $f_{\text{NP}}$ :

🍏 **fast computation** that can thus be used during a fit.

# Pavia 2019 (PV19): the settings

[Bacchetta et al., JHEP 07 (2020) 117, arXiv:1912.07550]

🍏 Functional form of the non-perturbative function:

$$f_{\text{NP}}(x, b_T, \zeta) = \left[ \frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp \left( -g_{1B}(x) \frac{b_T^2}{4} \right) \right] \exp \left[ - (g_2 + g_{2B} b_T^2) \ln \left( \frac{\zeta}{Q_0^2} \right) \frac{b_T^2}{4} \right]$$

$$g_1(x) = \frac{N_1}{x\sigma} \exp \left[ -\frac{1}{2\sigma^2} \ln^2 \left( \frac{x}{\alpha} \right) \right] \quad \text{and} \quad g_{1B}(x) = \frac{N_{1B}}{x\sigma_B} \exp \left[ -\frac{1}{2\sigma_B^2} \ln^2 \left( \frac{x}{\alpha_B} \right) \right]$$

🍏 a total of 9 free parameters.

🍏 Complete treatment of the experimental uncertainties:

🍏 **correlated** systematics (additive and multiplicative) properly treated,

🍏 uncertainties deriving from **collinear PDFs** also included.

🍏 Fits using all the available perturbative orders: **from NLL to N<sup>3</sup>LL**.

🍏 **Full integration** over  $q_T$ ,  $Q$  and  $y$  when required:

🍏 no narrow-width nor “middle-point” approximations.

🍏 No *ad hoc* **normalisation**:

🍏 fit both shape and normalisation.

🍏 **Monte Carlo** method for the experimental error propagation.

# PV19 fit: Drell-Yan data

Fixed target

RHIC

Tevatron

LHC



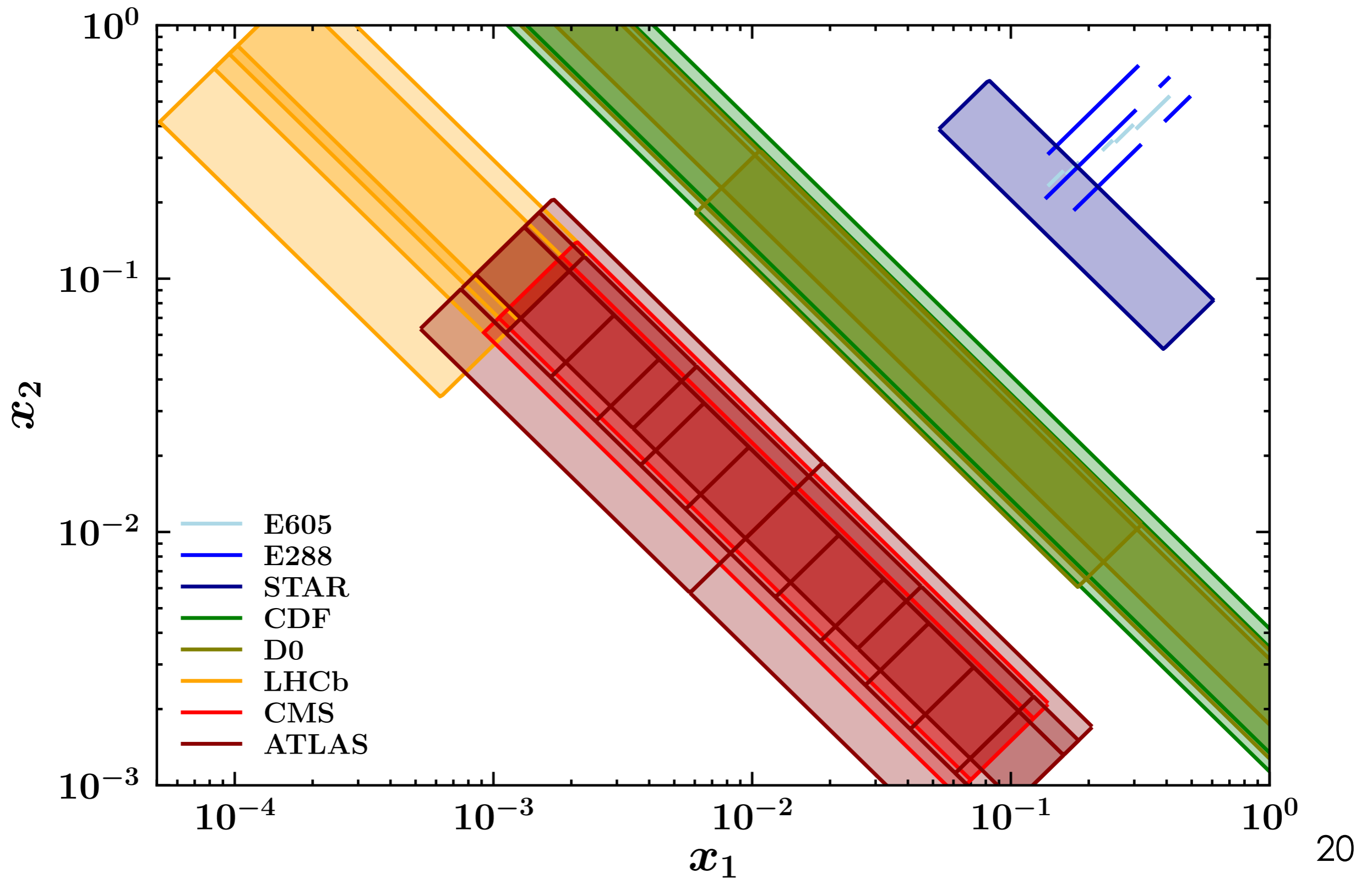
Experiment	$N_{\text{dat}}$	Observable	$\sqrt{s}$ [GeV]	$Q$ [GeV]	$y$ or $x_F$	Lepton cuts	Ref.
E605	50	$Ed^3\sigma/d^3q$	38.8	7 - 18	$x_F = 0.1$	-	[79]
E288 200 GeV	30	$Ed^3\sigma/d^3q$	19.4	4 - 9	$y = 0.40$	-	[80]
E288 300 GeV	39	$Ed^3\sigma/d^3q$	23.8	4 - 12	$y = 0.21$	-	[80]
E288 400 GeV	61	$Ed^3\sigma/d^3q$	27.4	5 - 14	$y = 0.03$	-	[80]
STAR 510	7	$d\sigma/dq_T$	510	73 - 114	$ y  < 1$	$p_{T\ell} > 25$ GeV $ \eta_\ell  < 1$	-
CDF Run I	25	$d\sigma/dq_T$	1800	66 - 116	Inclusive	-	[81]
CDF Run II	26	$d\sigma/dq_T$	1960	66 - 116	Inclusive	-	[82]
D0 Run I	12	$d\sigma/dq_T$	1800	75 - 105	Inclusive	-	[83]
D0 Run II	5	$(1/\sigma)d\sigma/dq_T$	1960	70 - 110	Inclusive	-	[84]
D0 Run II ( $\mu$ )	3	$(1/\sigma)d\sigma/dq_T$	1960	65 - 115	$ y  < 1.7$	$p_{T\ell} > 15$ GeV $ \eta_\ell  < 1.7$	[85]
LHCb 7 TeV	7	$d\sigma/dq_T$	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[86]
LHCb 8 TeV	7	$d\sigma/dq_T$	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[87]
LHCb 13 TeV	7	$d\sigma/dq_T$	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$	[92]
CMS 7 TeV	4	$(1/\sigma)d\sigma/dq_T$	7000	60 - 120	$ y  < 2.1$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.1$	[88]
CMS 8 TeV	4	$(1/\sigma)d\sigma/dq_T$	8000	60 - 120	$ y  < 2.1$	$p_{T\ell} > 15$ GeV $ \eta_\ell  < 2.1$	[89]
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/dq_T$	7000	66 - 116	$ y  < 1$ $1 <  y  < 2$ $2 <  y  < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.4$	[93]
ATLAS 8 TeV on-peak	6 6 6 6 6	$(1/\sigma)d\sigma/dq_T$	8000	66 - 116	$ y  < 0.4$ $0.4 <  y  < 0.8$ $0.8 <  y  < 1.2$ $1.2 <  y  < 1.6$ $1.6 <  y  < 2$ $2 <  y  < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.4$	[90]
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/dq_T$	8000	46 - 66 116 - 150	$ y  < 2.4$	$p_{T\ell} > 20$ GeV $ \eta_\ell  < 2.4$	[90]
Total	353	-	-	-	-	-	-

[Bacchetta et al., JHEP 07 (2020) 117, arXiv:1912.07550]

🍏 Only data with  $q_T / Q < 0.2$  (TMD factorisation region).

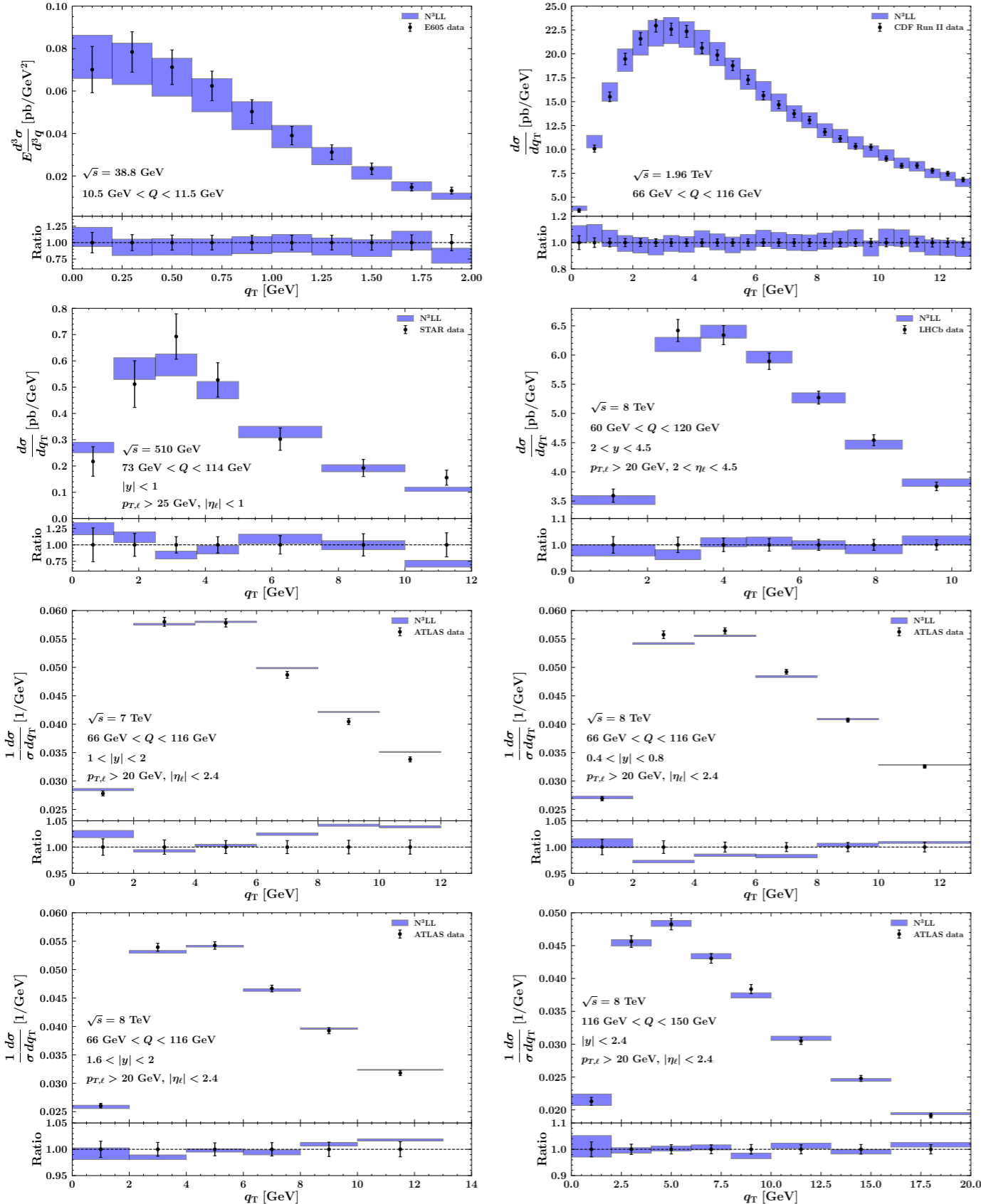
# PV19 fit: Drell-Yan data

$$\left(\frac{d\sigma}{dq_T}\right)_{\text{TMD}} = \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T \cdot \mathbf{q}_T} F_1(x_1, \mathbf{b}_T, Q, Q^2) F_2(x_2, \mathbf{b}_T, Q, Q^2)$$



# PV19 fit

## Fit quality at $\mathcal{N}^3LL$



Experiment		$\chi_D^2/N_{\text{dat}}$	$\chi_\lambda^2/N_{\text{dat}}$	$\chi^2/N_{\text{dat}}$
E605	7 GeV $< Q < 8$ GeV	0.419	0.068	0.487
	8 GeV $< Q < 9$ GeV	0.995	0.034	1.029
	10.5 GeV $< Q < 11.5$ GeV	0.191	0.137	0.328
	11.5 GeV $< Q < 13.5$ GeV	0.491	0.284	0.775
E288 200 GeV	4 GeV $< Q < 5$ GeV	0.213	0.649	0.862
	5 GeV $< Q < 6$ GeV	0.673	0.292	0.965
	6 GeV $< Q < 7$ GeV	0.133	0.141	0.275
	7 GeV $< Q < 8$ GeV	0.254	0.014	0.268
E288 300 GeV	8 GeV $< Q < 9$ GeV	0.652	0.024	0.676
	4 GeV $< Q < 5$ GeV	0.231	0.555	0.785
	5 GeV $< Q < 6$ GeV	0.502	0.204	0.706
	6 GeV $< Q < 7$ GeV	0.315	0.063	0.378
E288 400 GeV	7 GeV $< Q < 8$ GeV	0.056	0.030	0.086
	8 GeV $< Q < 9$ GeV	0.530	0.017	0.547
	11 GeV $< Q < 12$ GeV	1.047	0.167	1.215
	5 GeV $< Q < 6$ GeV	0.312	0.065	0.377
E288 400 GeV	6 GeV $< Q < 7$ GeV	0.100	0.005	0.105
	7 GeV $< Q < 8$ GeV	0.018	0.011	0.029
	8 GeV $< Q < 9$ GeV	0.437	0.039	0.477
	11 GeV $< Q < 12$ GeV	0.637	0.036	0.673
E288 400 GeV	12 GeV $< Q < 13$ GeV	0.788	0.028	0.816
	13 GeV $< Q < 14$ GeV	1.064	0.044	1.107
	STAR	0.782	0.054	0.836
	CDF Run I	0.480	0.058	0.538
CDF Run II	0.959	0.001	0.959	
D0 Run I	0.711	0.043	0.753	
D0 Run II	1.325	0.612	1.937	
D0 Run II ( $\mu$ )	3.196	0.023	3.218	
LHCb 7 TeV	1.069	0.194	1.263	
LHCb 8 TeV	0.460	0.075	0.535	
LHCb 13 TeV	0.735	0.020	0.755	
CMS 7 TeV	2.131	0.000	2.131	
CMS 8 TeV	1.405	0.007	1.412	
ATLAS 7 TeV	$0 <  y  < 1$	2.581	0.028	2.609
	$1 <  y  < 2$	4.333	1.032	5.365
	$2 <  y  < 2.4$	3.561	0.378	3.939
	$0 <  y  < 0.4$	1.924	0.337	2.262
ATLAS 8 TeV on-peak	$0.4 <  y  < 0.8$	2.342	0.247	2.590
	$0.8 <  y  < 1.2$	0.917	0.061	0.978
	$1.2 <  y  < 1.6$	0.912	0.095	1.006
	$1.6 <  y  < 2$	0.721	0.092	0.814
	$2 <  y  < 2.4$	0.932	0.348	1.280
	ATLAS 8 TeV off-peak	116 GeV $< Q < 150$ GeV	0.501	0.003
<b>Global</b>		<b>0.88</b>	<b>0.14</b>	<b>1.02</b>

# PV19 fit

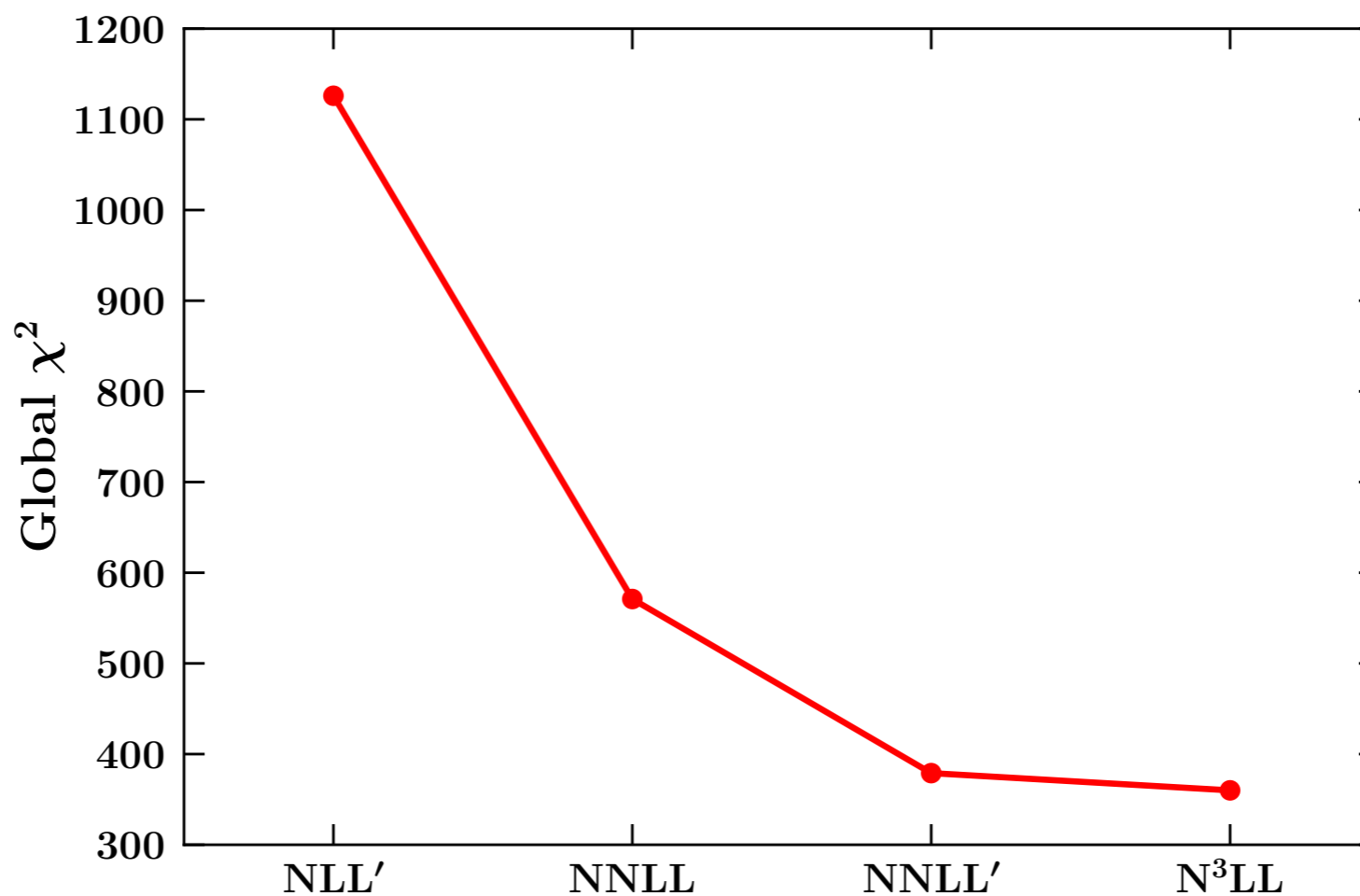
## *Perturbative convergence*

🍏 Global  $\chi^2$  as a function of the perturbative accuracy:

Order	NLL	NLL'	NNLL	NNLL'	N <sup>3</sup> LL
$\chi^2$ / n.d.p.	<b>~20</b>	<b>3.19</b>	<b>1.62</b>	<b>1.07</b>	<b>1.02</b>



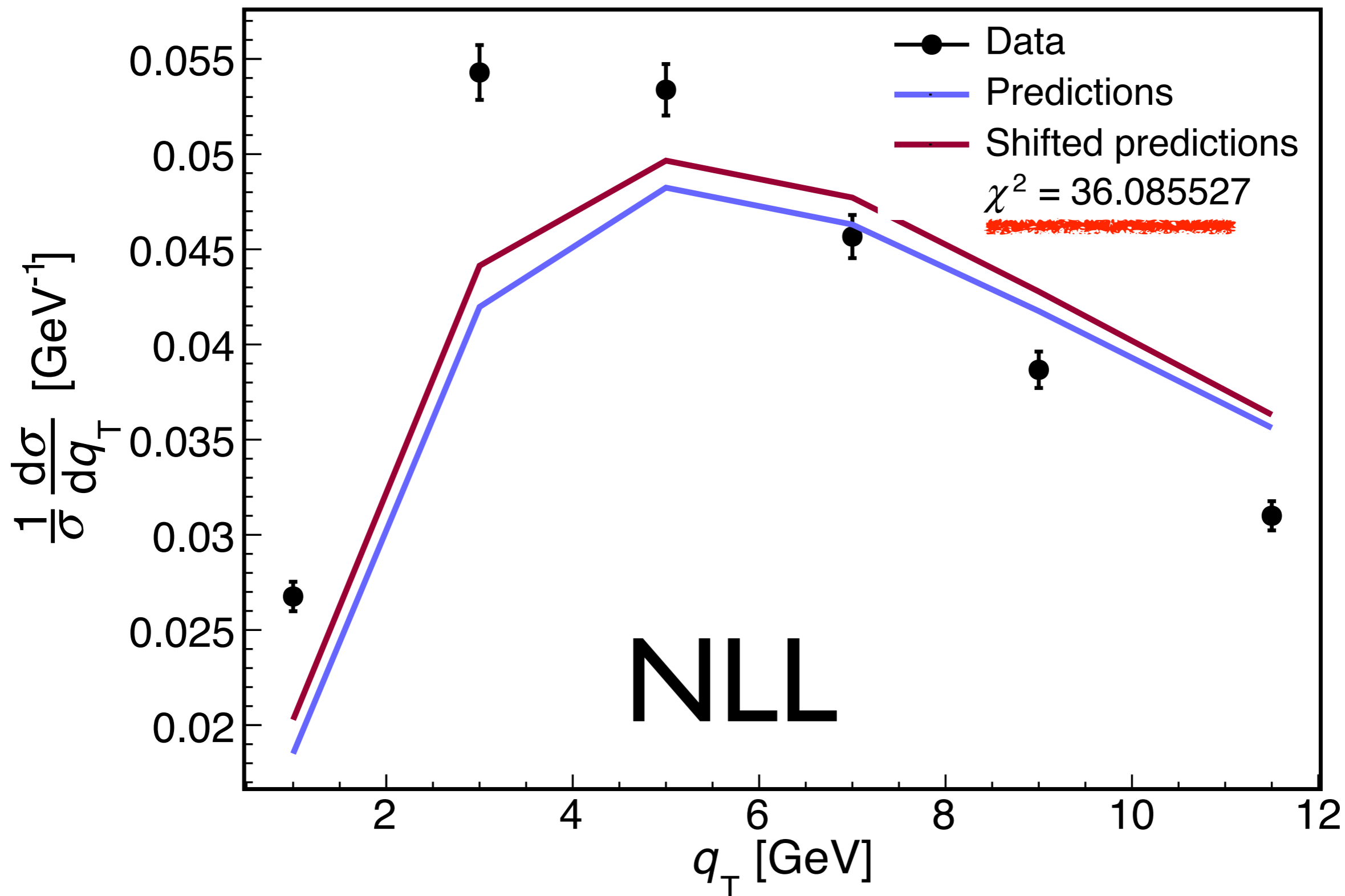
🍏 Clear perturbative **convergence**.



# PV19 fit

## *Perturbative convergence*

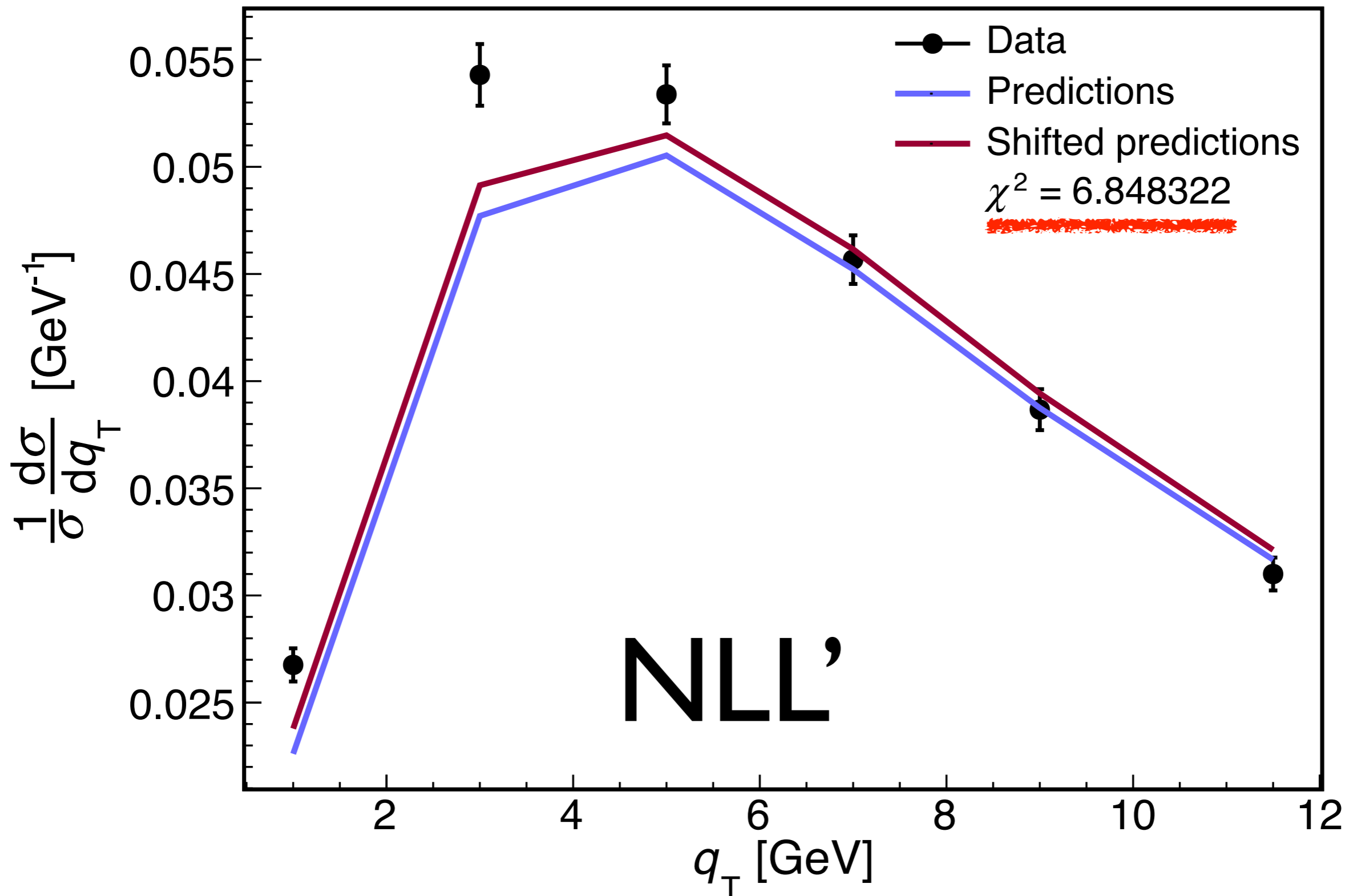
ATLAS at 8 TeV,  $66 \text{ GeV} < Q < 116 \text{ GeV}$ ,  $2 < |y| < 2.4$



# PV19 fit

## *Perturbative convergence*

ATLAS at 8 TeV,  $66 \text{ GeV} < Q < 116 \text{ GeV}$ ,  $2 < |y| < 2.4$

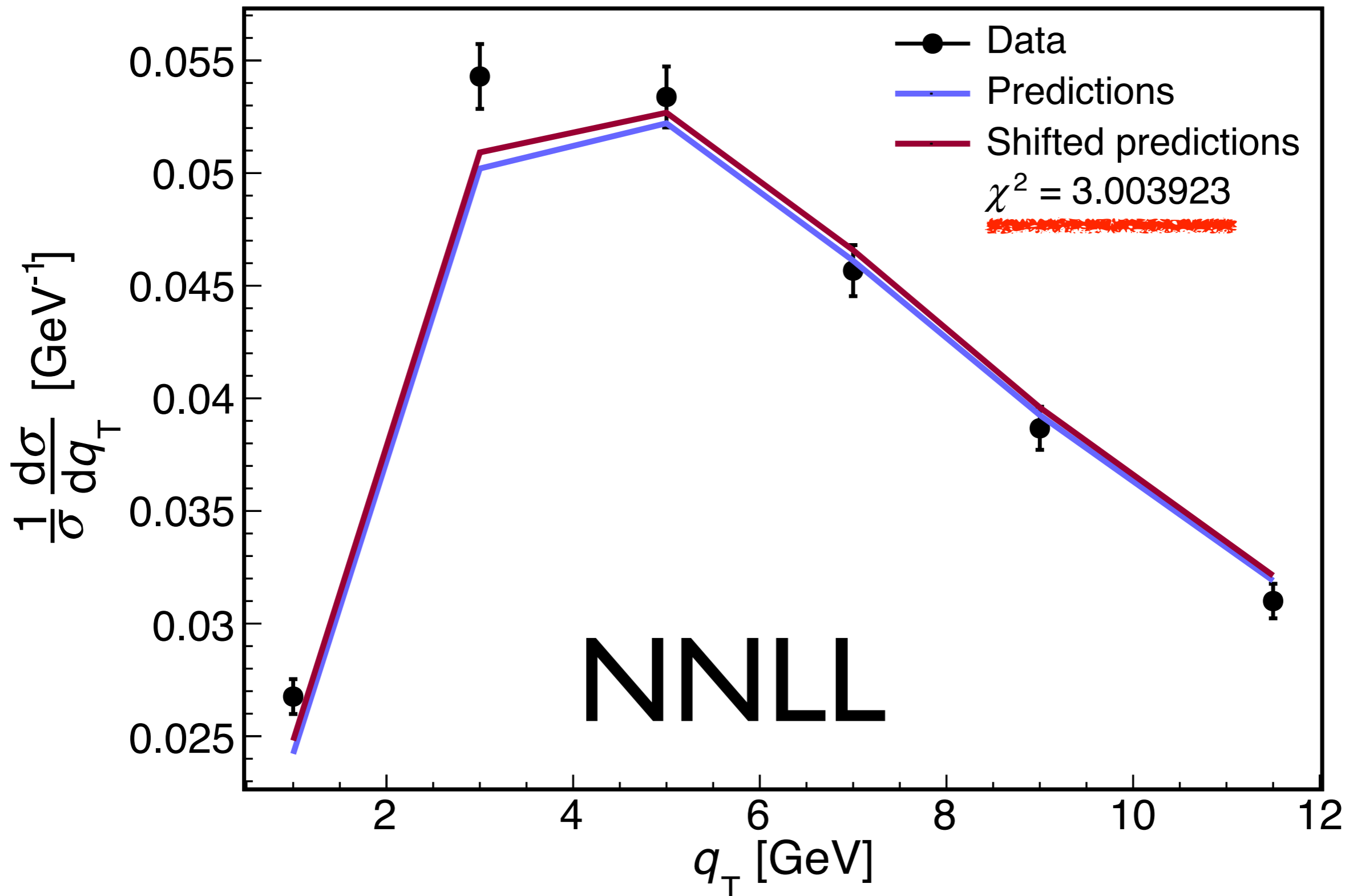




# PV19 fit

## *Perturbative convergence*

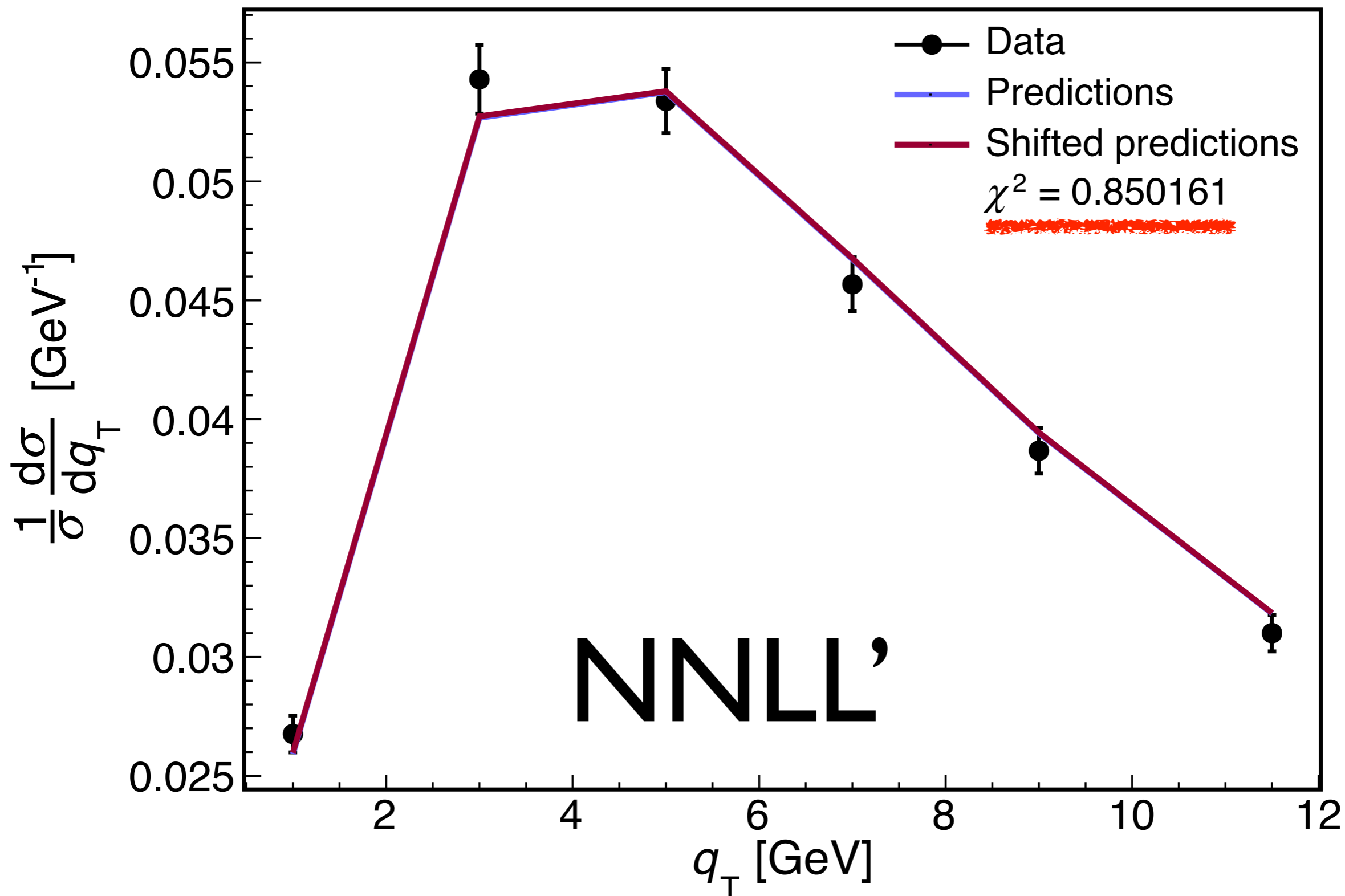
ATLAS at 8 TeV,  $66 \text{ GeV} < Q < 116 \text{ GeV}$ ,  $2 < |y| < 2.4$



# PV19 fit

## *Perturbative convergence*

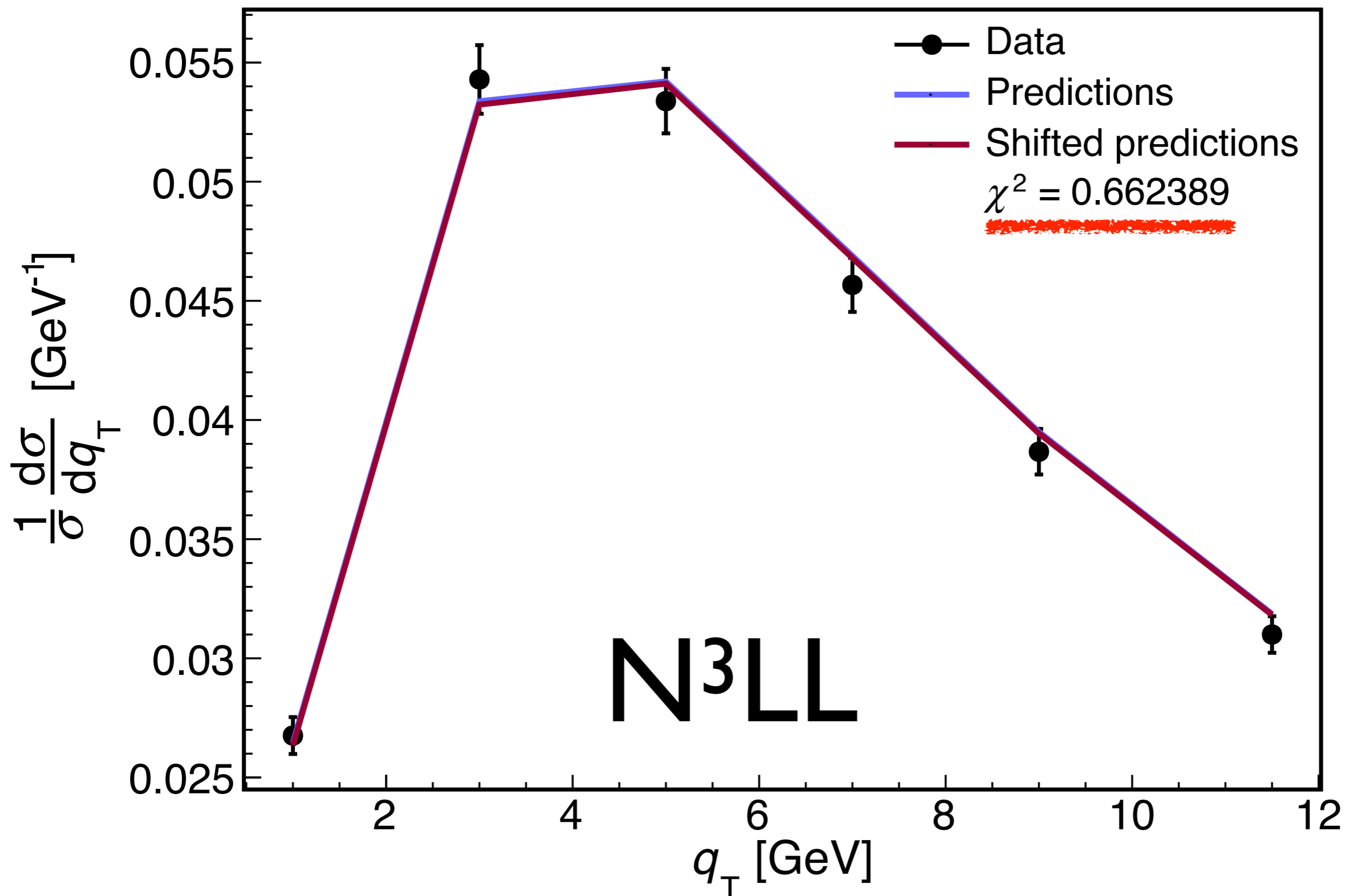
ATLAS at 8 TeV,  $66 \text{ GeV} < Q < 116 \text{ GeV}$ ,  $2 < |y| < 2.4$



# PV19 fit

## *Perturbative convergence*

ATLAS at 8 TeV,  $66 \text{ GeV} < Q < 116 \text{ GeV}$ ,  $2 < |y| < 2.4$



# Recent results

## *The LHC electroweak precision working group*

- 🍏  **$W$  mass** measurement now possible to increasing precision at the LHC, **utilises  $Z$   $q_T$  spectrum.**
- 🍏 Necessitates increased accuracy in theory predictions - many development in this area.
- 🍏 Sudakov **double** logarithms ( $L = \ln(Q^2/q_T^2)$ ) are left over from the cancellation of IR divergences.
- 🍏 At low  $q_T$ ,  $\alpha_s L^2 \sim 1$ , perturbative expansion breaks down  $\Rightarrow$  **resummation.**  
$$\frac{d\sigma}{dq_T} \underset{q_T \rightarrow 0}{\simeq} 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots$$
- 🍏 Resum these large logs up to given order - Possible up to  **$\mathbf{N^3LL}$** .
- 🍏 **Many different approaches** - the goal is to compare them to understand their **differences, uncertainties** and **accuracy.**

# Recent results

## *The LHC electroweak precision working group*

### 2. Different Approaches to Resummation

## Groups and Codes involved

- $q_T$  resummation
  - ▶ DYRes/DYTURBO  
Camarda et al., '19
  - ▶ reSolve  
Coradeschi, T.C., '17
- TMD
  - ▶ NangaParbat  
Bacchetta et al., '19
  - ▶ arTeMiDe  
Scimemi, Vladimirov, '17
- SCET
  - ▶ SCETLib  
Ebert et al. '17
  - ▶ (CuTe)  
Becher et al. '11,'20
- Parton Shower-like/Branching
  - ▶ RadISH  
Monni et al. '16,'17
  - ▶ (PB-TMD)  
Martinez et al. '20

Many groups, well spread across the several different approaches.

# Recent results

## *The LHC electroweak precision working group*

### 2. Different Approaches to Resummation

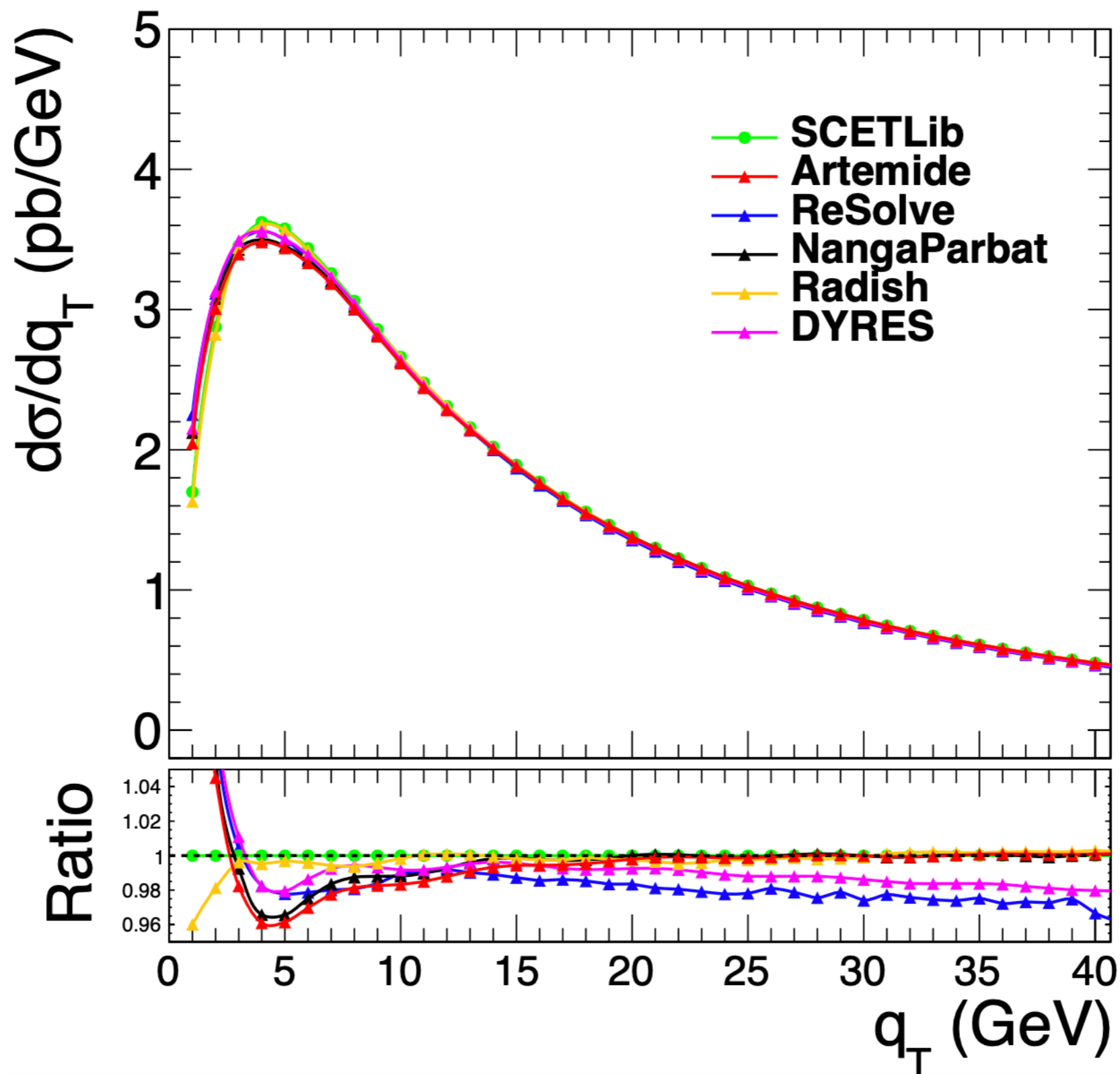
## Groups and Codes involved

- $q_T$  resummation
  - ▶ DYRes/DYTURBO  
Camarda et al., '19
  - ▶ reSolve  
Coradeschi, T.C., '17
- TMD
  - ▶ NangaParbat  
Bacchetta et al., '19
  - ▶ arTeMiDe  
Scimemi, Vladimirov, '17
- SCET
  - ▶ SCETLib  
Ebert et al. '17
  - ▶ (CuTe)  
Becher et al. '11,'20
- Parton Shower-like/Branching
  - ▶ RadISH  
Monni et al. '16,'17
  - ▶ (PB-TMD)  
Martinez et al. '20

Many groups, well spread across the several different approaches.

# Recent results

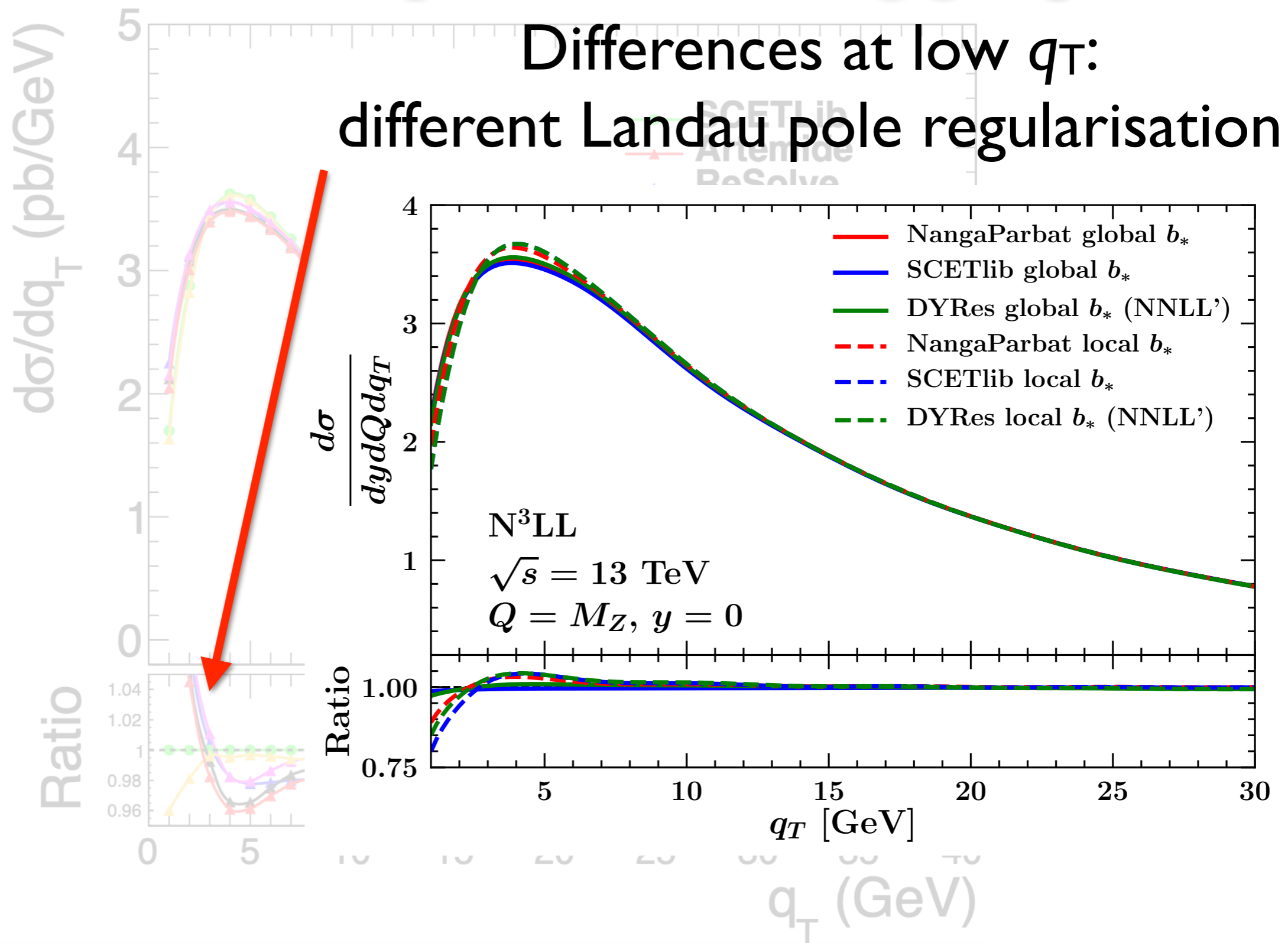
*The LHC electroweak precision working group*



🍏 Small differences and well understood.

# Recent results

*The LHC electroweak precision working group*



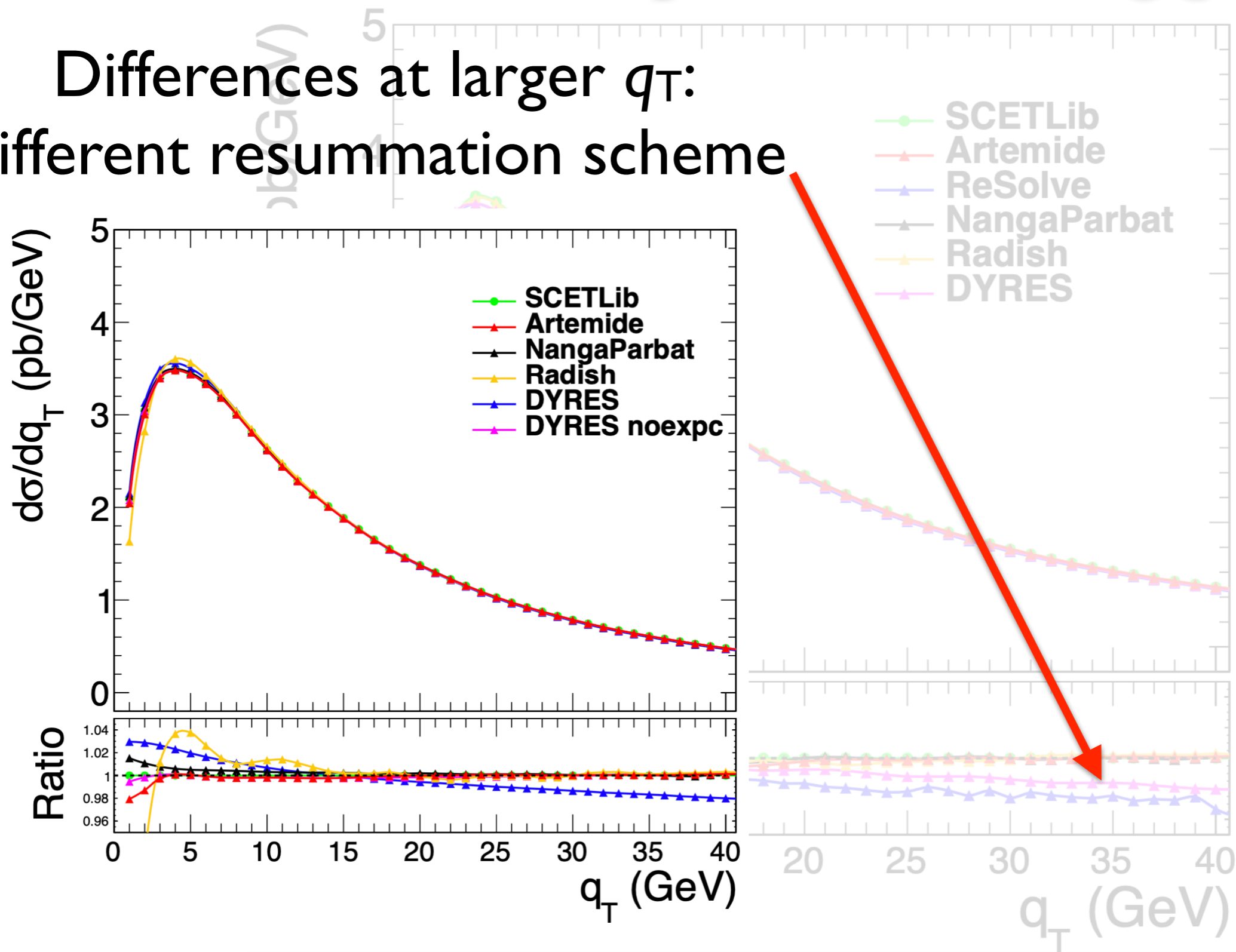
🍏 Small differences and well understood.



# Recent results

*The LHC electroweak precision working group*

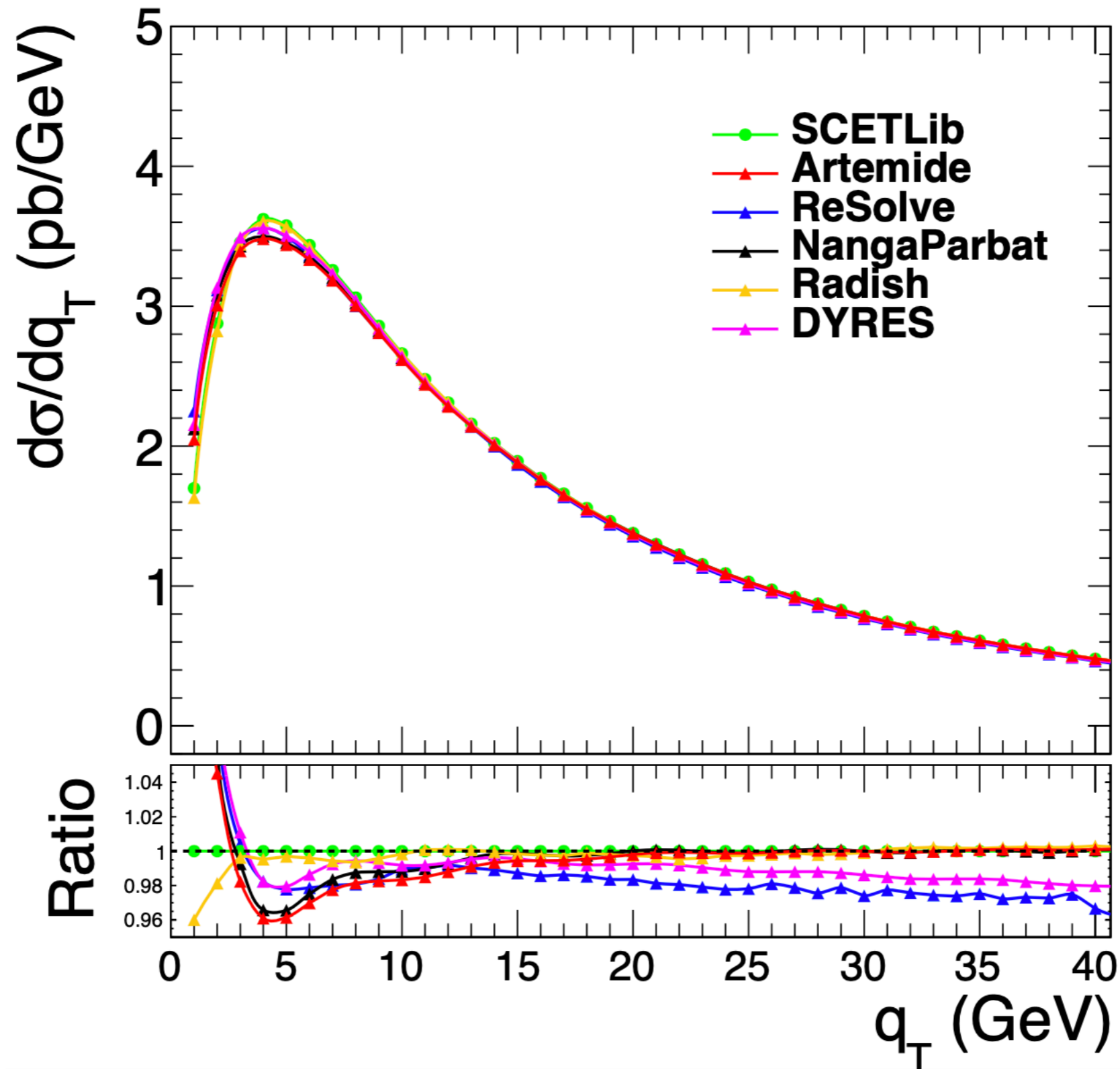
Differences at larger  $q_T$ :  
different resummation scheme



🍏 Small differences and well understood.

# Recent results

*The LHC electroweak precision working group*



🍏 Benchmark extremely **successful!**

# Conclusions

- 🍏 **TMD factorisation** *à la* CSS well-established and phenomenologically successful.
- 🍏 Implementation within **NangaParbat**:
  - 🍏 fast  $\Rightarrow$  suitable for **TMD fits**,
  - 🍏 already used for the **PV19** TMD PDF extraction,
  - 🍏 currently being extended to analyse **SIDIS** data and determine TMD FFs,
  - 🍏 accurate  $\Rightarrow$  state-of-the-art perturbative accuracy (**N<sup>3</sup>LL**) necessary for precision physics (involved in the LHC electroweak precision working group).
  - 🍏 Extensive comparisons against other formalisms and codes make it extremely solid.
  - 🍏 Used for impact studies for the **Electron-Ion Collider**.
- 🍏 NangaParbat can be employed for a fundamental study of TMDs as non-perturbative objects defined in terms of light-cone operators:
  - 🍏 interface to **PARTONS** being planned.