MAPFF1.0 — Pion fragmentation functions using single-inclusive annihilation and semi-inclusive DIS data

Paper	arXiv:2105.08725
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Public code	https://github.com/MapCollaboration/MontBlanc
LHAPDF sets	https://lhapdf.hepforge.org/

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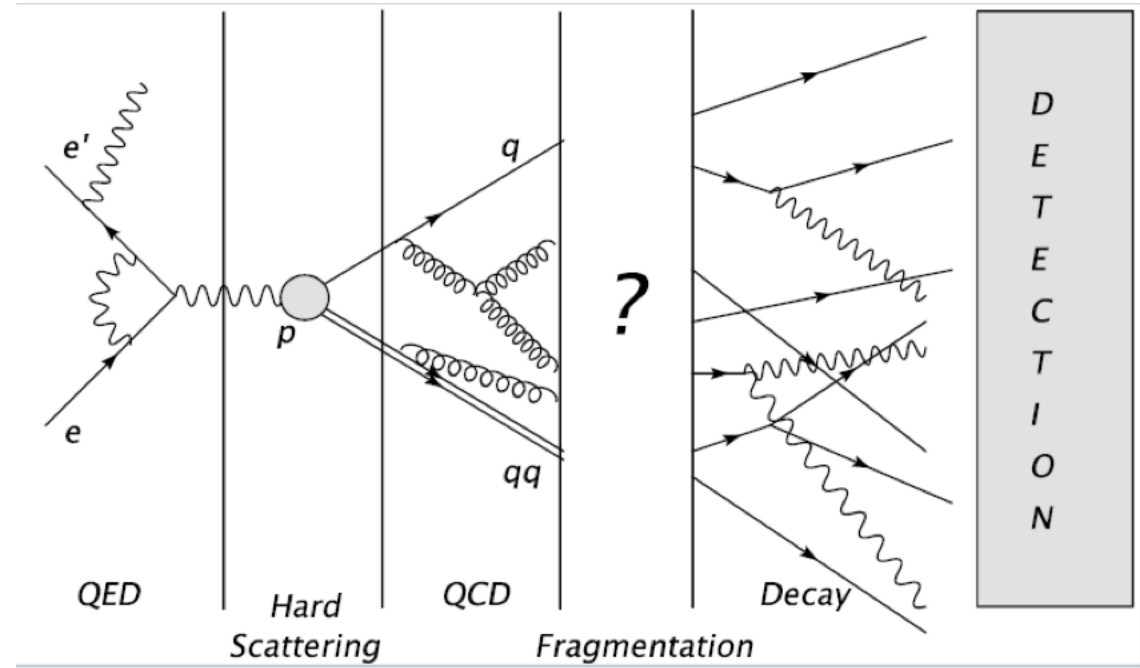
GDR-QCD/QCD@short distances and STRONG2020/PARTONS/FTE@LHC/NLOAccess

Outline

Introduction

Definitions

- Fragmentation refers to the inclusive production of hadrons without any assumptions concerning the hadronisation mechanisms.
- Whenever a specific hadron is identified in the final state, its associated fragmentation functions (FFs) appear as non-perturbative ingredient of factorised cross-section.



From A. Vossen's talk

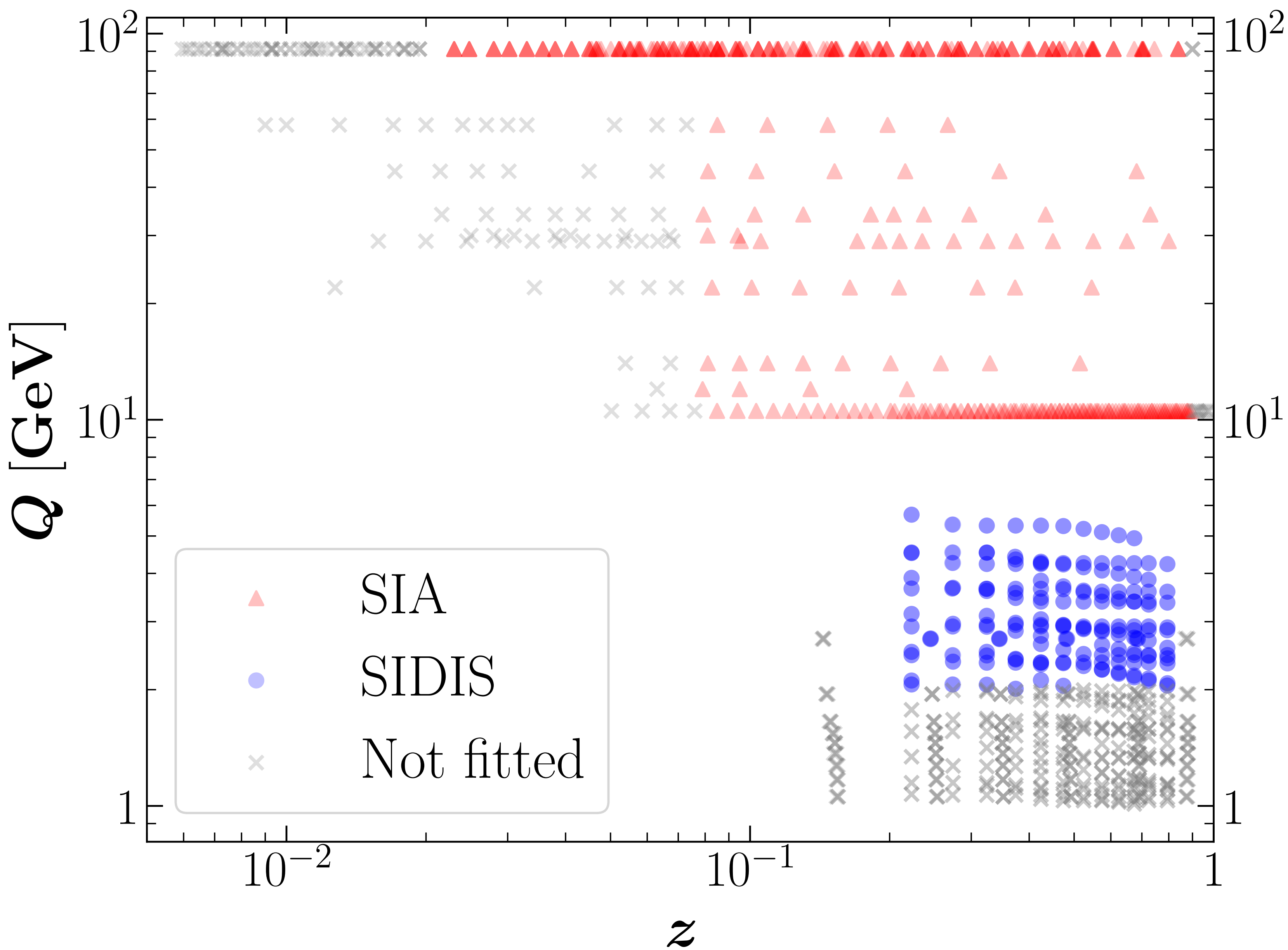
- FFs denoted by $D_i^h(z)$ encodes the hadronisation of a parton of type i into a hadron of type h, where the hadron carries the fraction z of the parton momentum.

Processes considered

- Single-inclusive charged pion production in electron-positron annihilation (SIA)
  $e^+ + e^- \to \pi^\pm + X$
- Semi-inclusive deep-inelastic lepton-nucleon scattering (SIDIS)
  $e^- + N \to e^- + \pi^\pm + X$

Experimental data

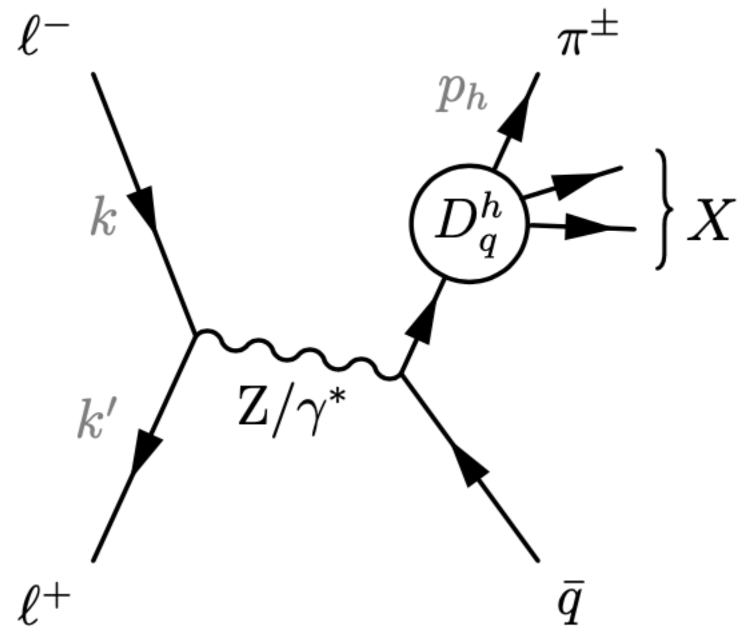
Experiment	Prc	Obs
ALEPH	SIA	M
BABAR	SIA	M
BELLE	SIA	X
DELPHI	SIA	M and TM
OPAL	SIA	M
SLD	SIA	M and TM
TASSO	SIA	M and X
TOPAZ	SIA	M
TPC	SIA	M
COMPASS	SIDIS	M
HERMES	SIDIS	M



X: Cross section
M: Multiplicity
TM: Tagged Multiplicity

Single-Inclusive annihilation

$$e^+(k_1) + e^-(k_2) \to \pi^\pm(p_h) + X\,.$$



SIA LO Kinematics

$$Q^2 = q^2$$

$$z = \frac{2p_h \cdot q}{Q^2}$$

$$s = Q^2$$

- **Cross section:**

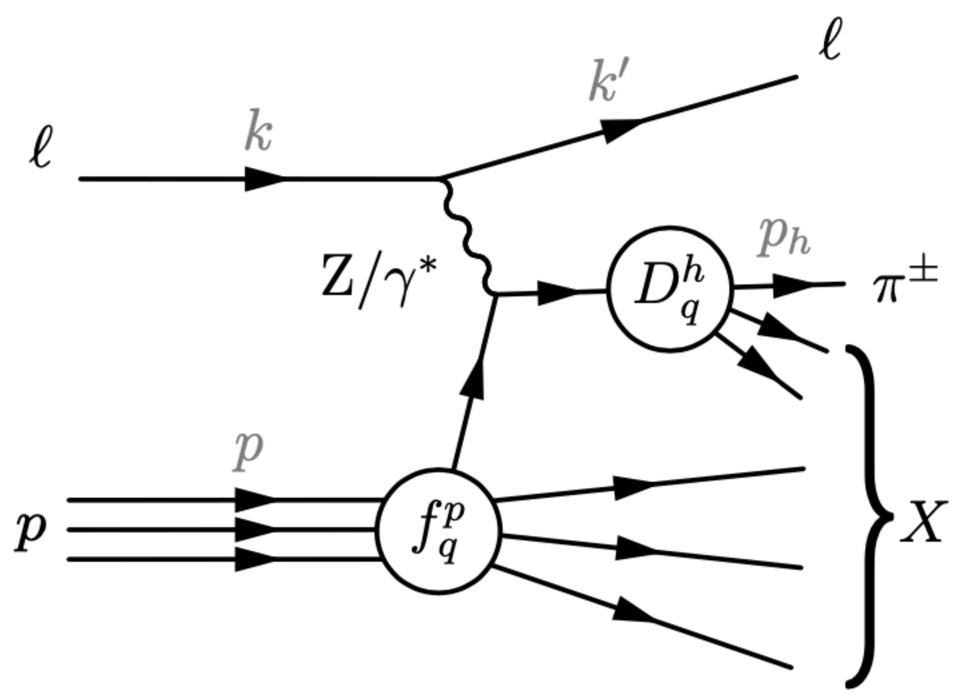
$$\frac{d\sigma^h}{dz}(z,Q) = \frac{4\pi\alpha(Q)}{Q^2} F_2^h(z,Q)$$

- **Structure functions:** $F_2^{\mathrm{NC},\pi^\pm} = \frac{1}{n_f}\left(\sum_j^{n_f} \hat{e}_{q_j}^2\right) F_{2,\mathrm{S}}^{\mathrm{NC},\pi^\pm} + F_{2,\mathrm{NS}}^{\mathrm{NC},\pi^\pm}$

$$F_{2,\mathrm{S}}^{\mathrm{SIA},\pi^\pm} = C_{2,\mathrm{S}}^{\mathrm{SIA}} \otimes D_{\underline{\Sigma}}^h + C_{2,g}^{\mathrm{SIA}} \otimes D_{\underline{g}}^h, \qquad F_{2,\mathrm{NS}}^{\mathrm{SIA},\pi^\pm} = C_{2,\mathrm{NS}}^{\mathrm{SIA}} \otimes \sum_j^{n_f} \hat{e}_{q_j} D_{j,\mathrm{NS}}^h$$

Semi-inclusive DIS

$$\ell(k) + N(p) \to \ell(k') + \pi^{\pm}(p_h) + X\,.$$



SIDIS LO Kinematics

$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

$$z = \frac{p \cdot p_h}{p \cdot q}$$

$$y = \frac{Q^2}{xs}$$

$$s = (k^2 + p^2)$$

- **Cross section ($Q \ll M_Z$):** $\dfrac{d^3\sigma}{dx dQ dz} = \dfrac{4\pi\alpha^2(Q)}{xQ^3}\left[\left(1+(1-y)^2\right) F_2(x,z,Q^2) - y^2 F_L(x,z,Q^2)\right]$

$$F_i(x,z,Q) = x\sum_{q\bar{q}} e_q^2 \Big\{ [C_{i,qq}(x,z,Q) \otimes \underline{f_q(x,Q)} + C_{i,qg}(x,z,Q) \otimes \underline{f_g(x,Q)}] \otimes D_q^{\pi^{\pm}}(z,Q)$$
$$+ \; [C_{i,gq}(x,z,Q) \otimes \underline{f_q(x,Q)}] \otimes D_g^{\pi^{\pm}}(z,Q) \Big\}, \qquad i = 2, L\,.$$

$$C(x,z) \otimes f(x) \otimes D(z) = \int_x^1 \frac{dx'}{x'} \int_z^1 \frac{dz'}{z'} C(x',z') f\left(\frac{x}{x'}\right) D\left(\frac{z}{z'}\right)$$

Theoretical setup

- **Coefficient functions (NLO, n =1):** $$C(x,z,Q)=\sum_{n=0}\left(\frac{\alpha_s(Q)}{4\pi}\right)^n C^{(n)}(x,z)$$

- **Integrated multiplicities:**

$$\frac{dM}{dz}=\left[\int_{Q_{\rm min}}^{Q_{\rm max}}dQ\int_{x_{\rm min}}^{x_{\rm max}}dx\int_{z_{\rm min}}^{z_{\rm max}}dz\,\frac{d^3\sigma}{dxdQdz}\right]\Big/\left[\Delta z\int_{Q_{\rm min}}^{Q_{\rm max}}dQ\int_{x_{\rm min}}^{x_{\rm max}}dx\,\frac{d^2\sigma}{dxdQ}\right]$$

- **Exact charge conjugation:** $$D_{q(\overline{q})}^{\pi^-}(x,Q)=D_{\overline{q}(q)}^{\pi^+}(x,Q)\,,\quad D_g^{\pi^-}(x,Q)=D_g^{\pi^+}(x,Q)$$

- **ZM-VFNS with inactive flavours NOT set to zero below their threshold (inactive = do not evolve with DGLAP below their threshold) — $m_c=1.51$ GeV and $m_b=4.92$ GeV.**

- **NNPDF31_nlo_pch_as_0118 PDF set used for SIDIS predictions.**

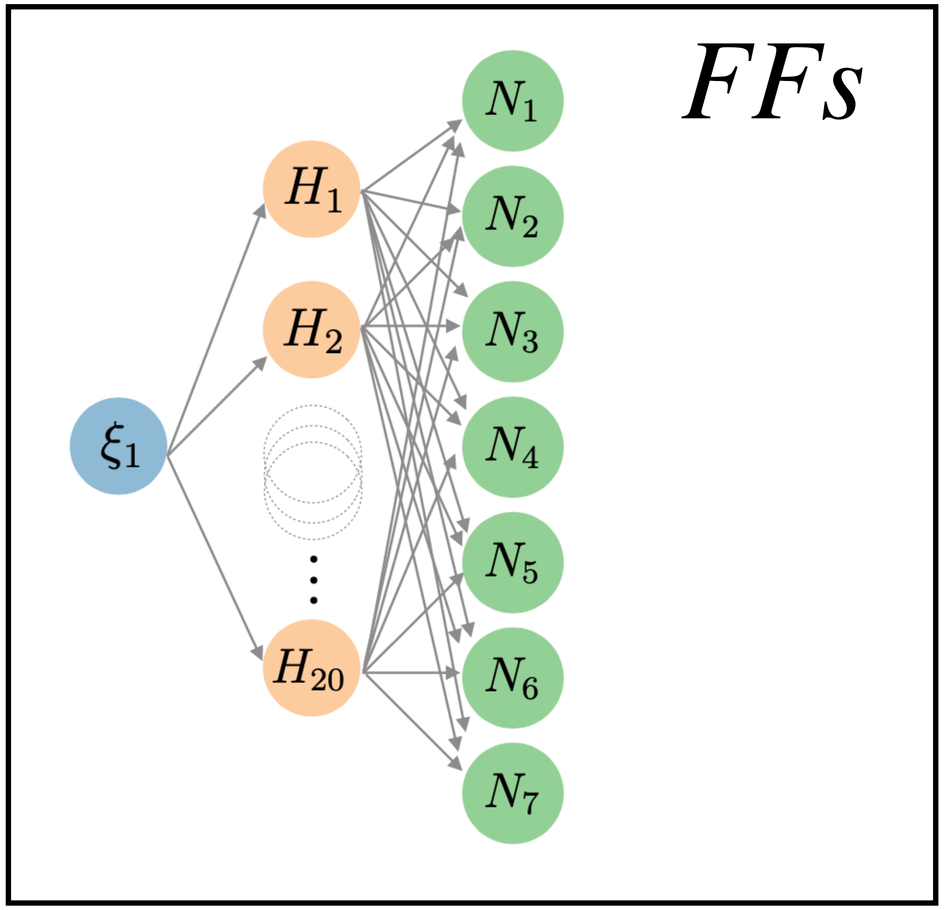
Methodology

NangaParbat
(chi-square, covariance, sampling, etc.)

APFEL++

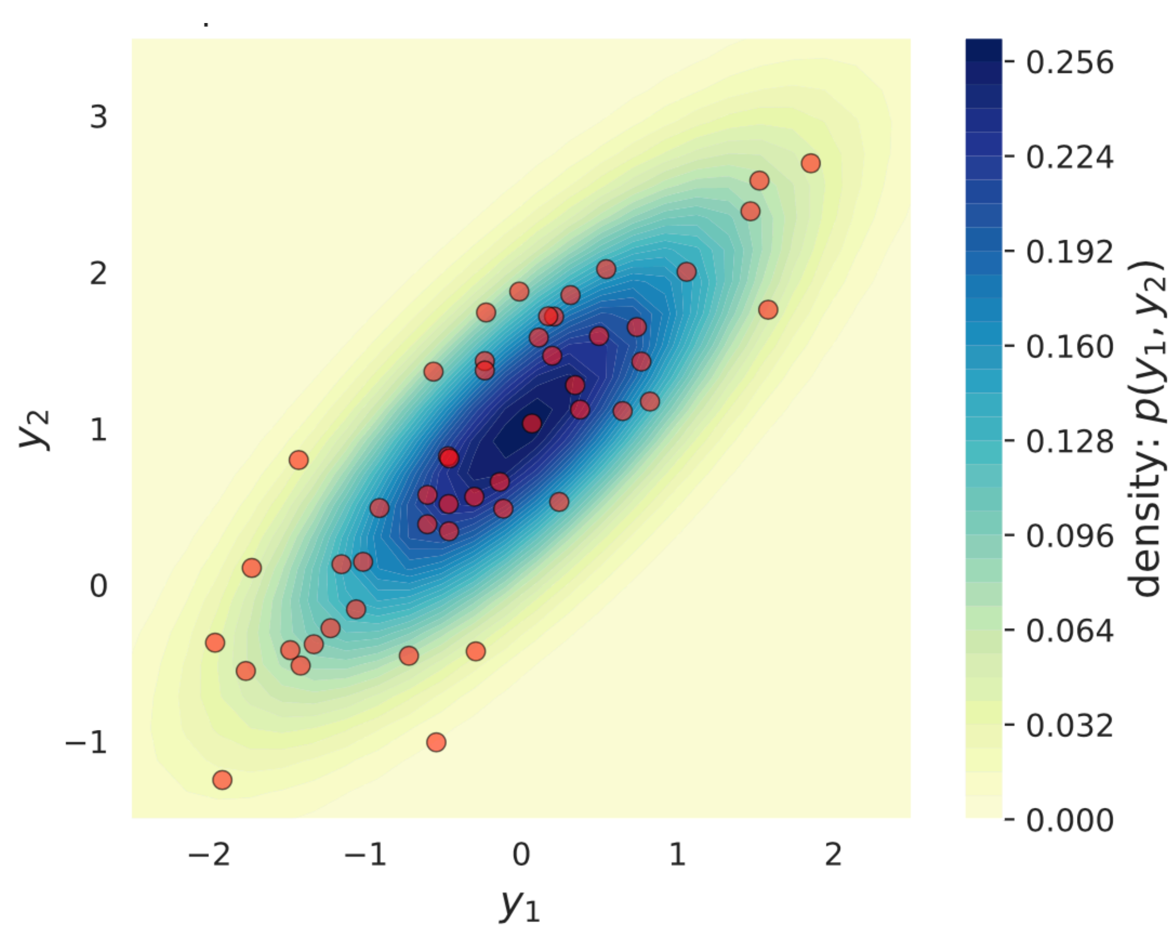
Preds $= \hat{\sigma} \otimes PDF \otimes$

MAP_FF1.0: NNAD & ceres-solver



Methodology

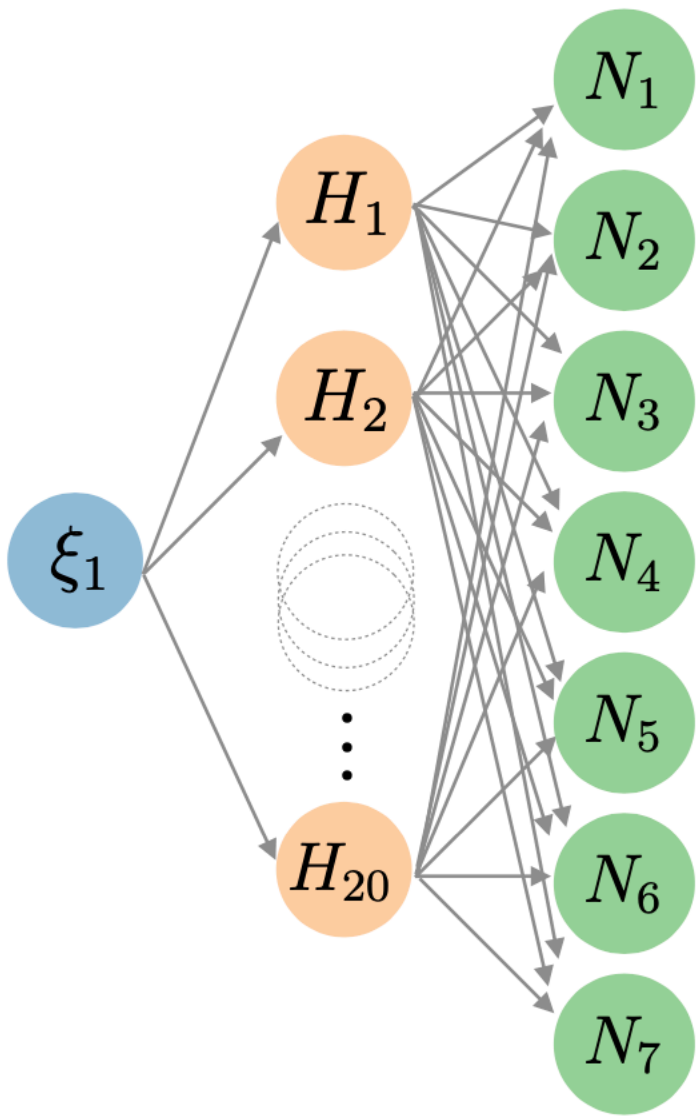
- **The Gaussian assumption:** $$\mathcal{G}\left(\boldsymbol{x}^{(k)}\right) \propto \exp\left[\left(\boldsymbol{x}^{(k)}-\boldsymbol{\mu}\right)^T \cdot \boldsymbol{C}^{-1} \cdot \left(\boldsymbol{x}^{(k)}-\boldsymbol{\mu}\right)\right]$$
- **Covariance Matrix:** $$C_{ij} = \delta_{ij}\sigma^2_{i,\,\mathrm{unc}} + \sum_\beta \sigma^{(\beta)}_{i,\,\mathrm{corr}}\sigma^{(\beta)}_{j,\,\mathrm{corr}}$$
- **Replica generation:** $$\boldsymbol{x}^{(k)} = \boldsymbol{\mu} + \boldsymbol{L}\cdot\boldsymbol{r}^{(k)} \longrightarrow \frac{1}{N_{\mathrm{rep}}}\sum_k^{N_{\mathrm{rep}}} x_i^{(k)} \simeq \mu_i\,, \qquad \frac{1}{N_{\mathrm{rep}}}\sum_k^{N_{\mathrm{rep}}} x_i^{(k)}x_j^{(k)} \simeq \mu_i\mu_j + C_{ij}$$



Methodology

- **Parameterisation:**

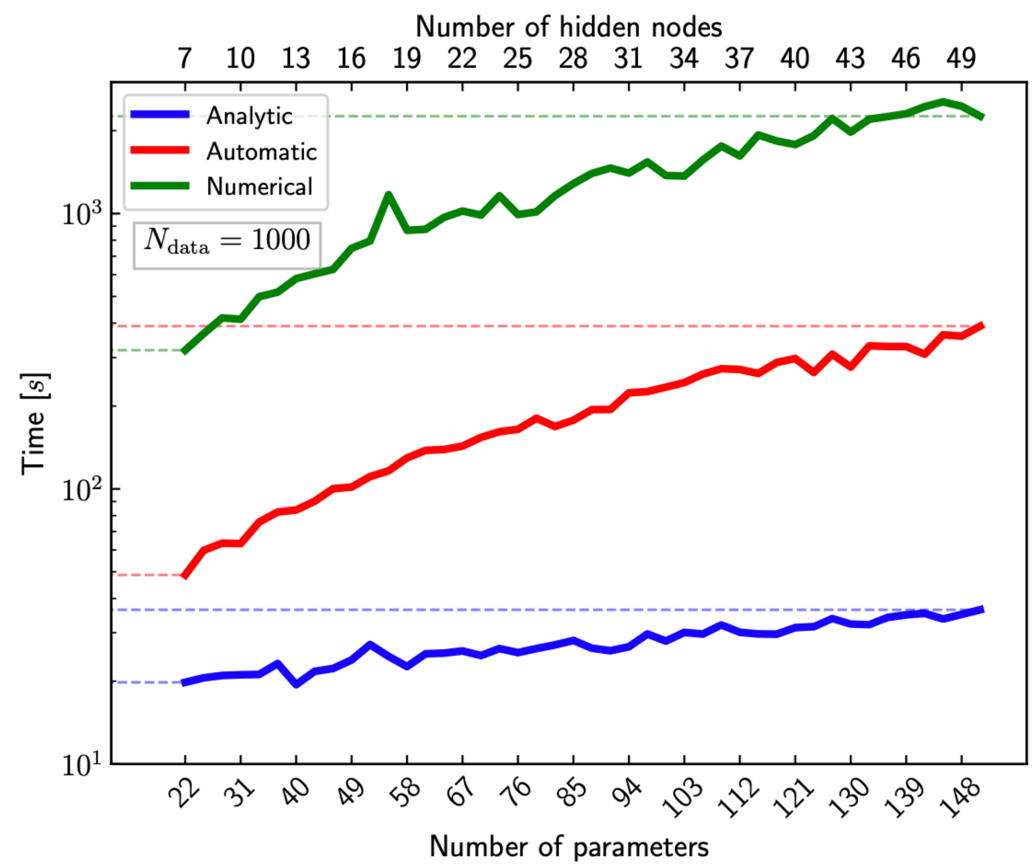
$$zD_i^{\pi^+}(z, \mu_0 = 5\,\text{GeV}) = \left(N_i(z;\boldsymbol{\theta}) - N_i(1;\boldsymbol{\theta})\right)^2$$



7 flavours assuming a partially symmetric sea

$$\{D_u^{\pi^+},\ D_{\bar{d}}^{\pi^+},\ D_d^{\pi^+} = D_{\bar{u}}^{\pi^+},\ D_s^{\pi^+} = D_{\bar{s}}^{\pi^+},\ D_c^{\pi^+} = D_{\bar{c}}^{\pi^+},\ D_b^{\pi^+} = D_{\bar{b}}^{\pi^+},\ D_g^{\pi^+}\}$$

Analytic derivatives for minimisation (NNAD)



- **6 flavours [SU(2) isospin symmetry + partially symmetric sea] → deterioration of the fit quality**
- **11 flavours [no symmetries] → overly redundant**

Methodology

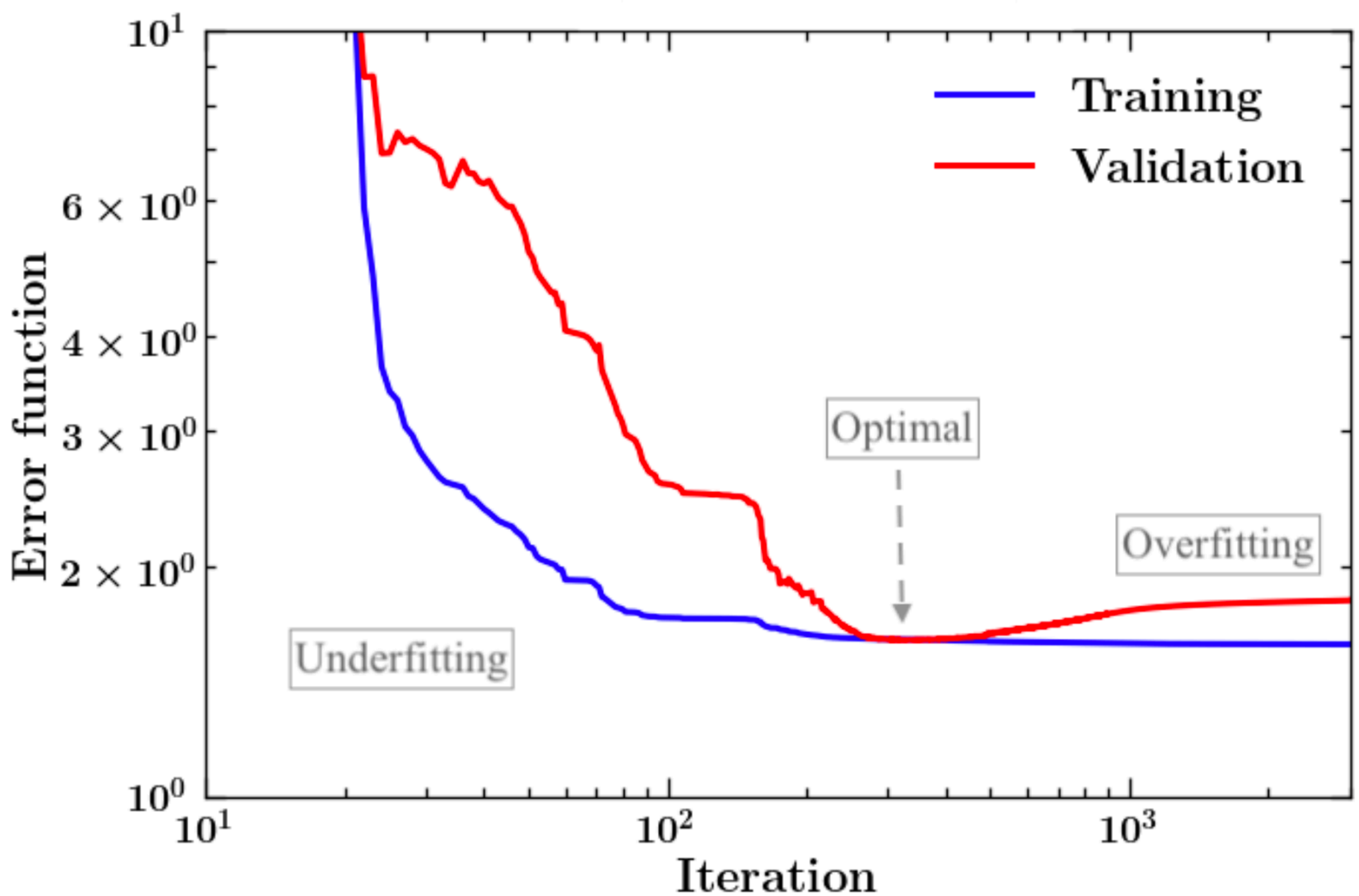
- **Maximum Log-likelihood:**

$$\chi^{2(k)} \equiv \left(\boldsymbol{T}(\boldsymbol{\theta}^{(k)}) - \boldsymbol{x}^{(k)}\right)^T \cdot \boldsymbol{C}^{-1} \cdot \left(\boldsymbol{T}(\boldsymbol{\theta}^{(k)}) - \boldsymbol{x}^{(k)}\right)$$

Every fit of a data replica is performed with a different PDF MC member in the SIDIS theory prediction
Ensuring the propagation of PDF correlated uncertainties.

However, the contribution of these uncertainties to the fit turned out to be very mild (see appendix)

- **cross-validation and stopping:**



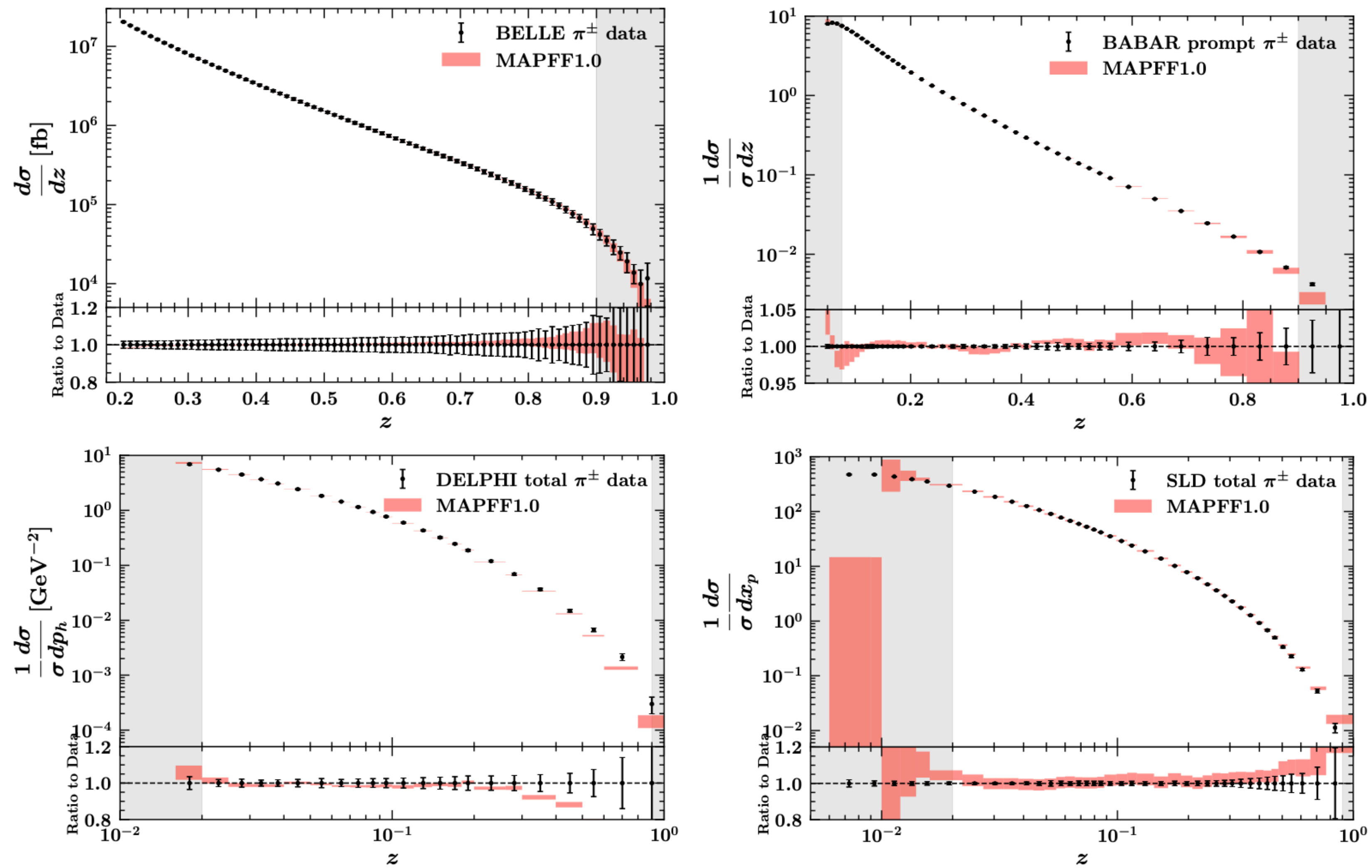
The minimisation is carried out by means of *ceres-solver*

in particular the trust region Levenberg–Marquardt algorithm

Relying on the knowledge of the analytic derivatives of the neural network itself via *NNAD*

Results — Fit Quality

Experiment	χ^2 per point	N_{dat} after cuts
HERMES π^- deuteron	0.60	2
HERMES π^- proton	0.02	2
HERMES π^+ deuteron	0.30	2
HERMES π^+ proton	0.53	2
COMPASS π^-	0.80	157
COMPASS π^+	1.07	157
Total SIDIS	0.78	322
BELLE $\pi^\pm$	0.09	70
BABAR prompt $\pi^\pm$	0.90	39
TASSO 12 GeV $\pi^\pm$	0.97	4
TASSO 14 GeV $\pi^\pm$	1.39	9
TASSO 22 GeV $\pi^\pm$	1.85	8
TPC $\pi^\pm$	0.22	13
TASSO 30 GeV $\pi^\pm$	0.34	2
TASSO 34 GeV $\pi^\pm$	1.20	9
TASSO 44 GeV $\pi^\pm$	1.20	6
TOPAZ $\pi^\pm$	0.28	5
ALEPH $\pi^\pm$	1.29	23
DELPHI total $\pi^\pm$	1.29	21
DELPHI *uds* $\pi^\pm$	2.84	21
DELPHI bottom $\pi^\pm$	1.67	21
OPAL $\pi^\pm$	1.72	24
SLD total $\pi^\pm$	1.14	34
SLD *uds* $\pi^\pm$	2.05	34
SLD bottom $\pi^\pm$	0.55	34
Total SIA	1.10	377
Global data set	**0.90**	**699**



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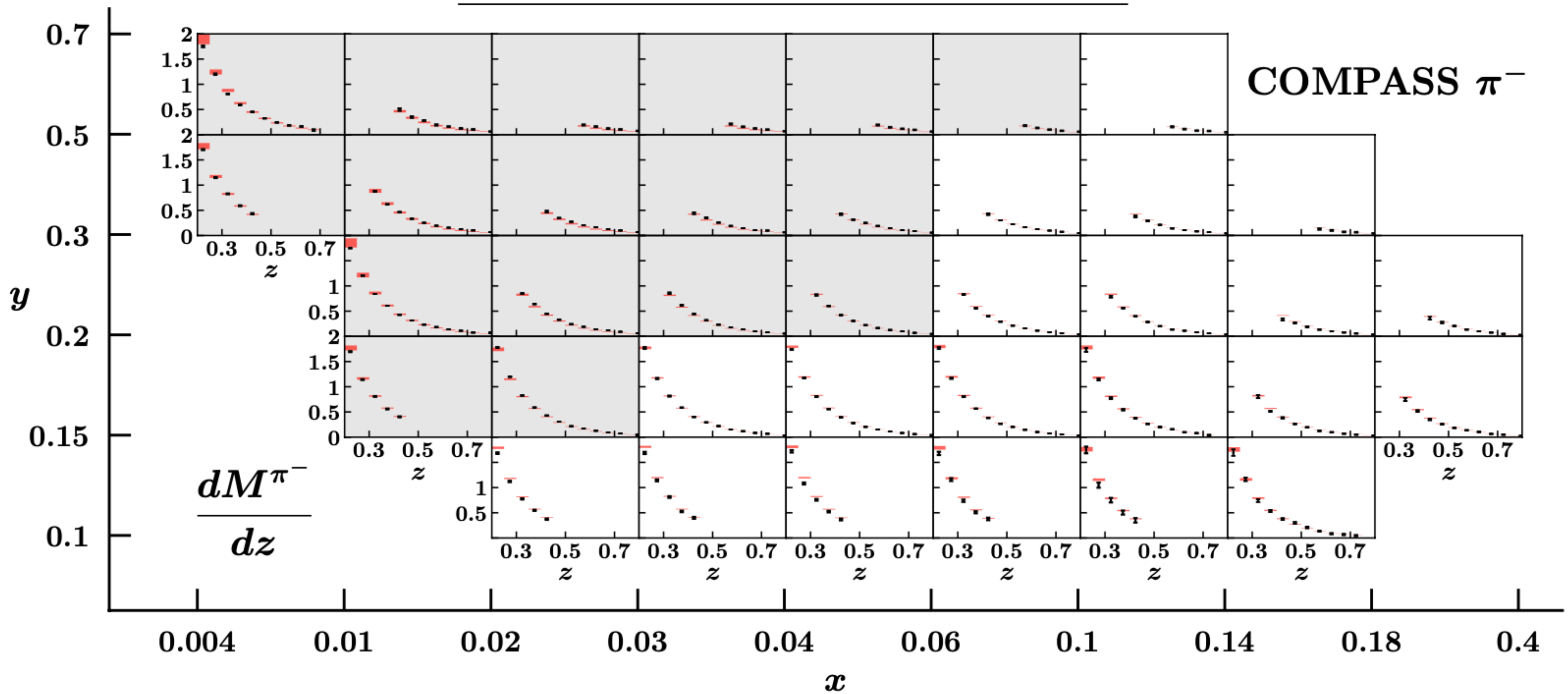

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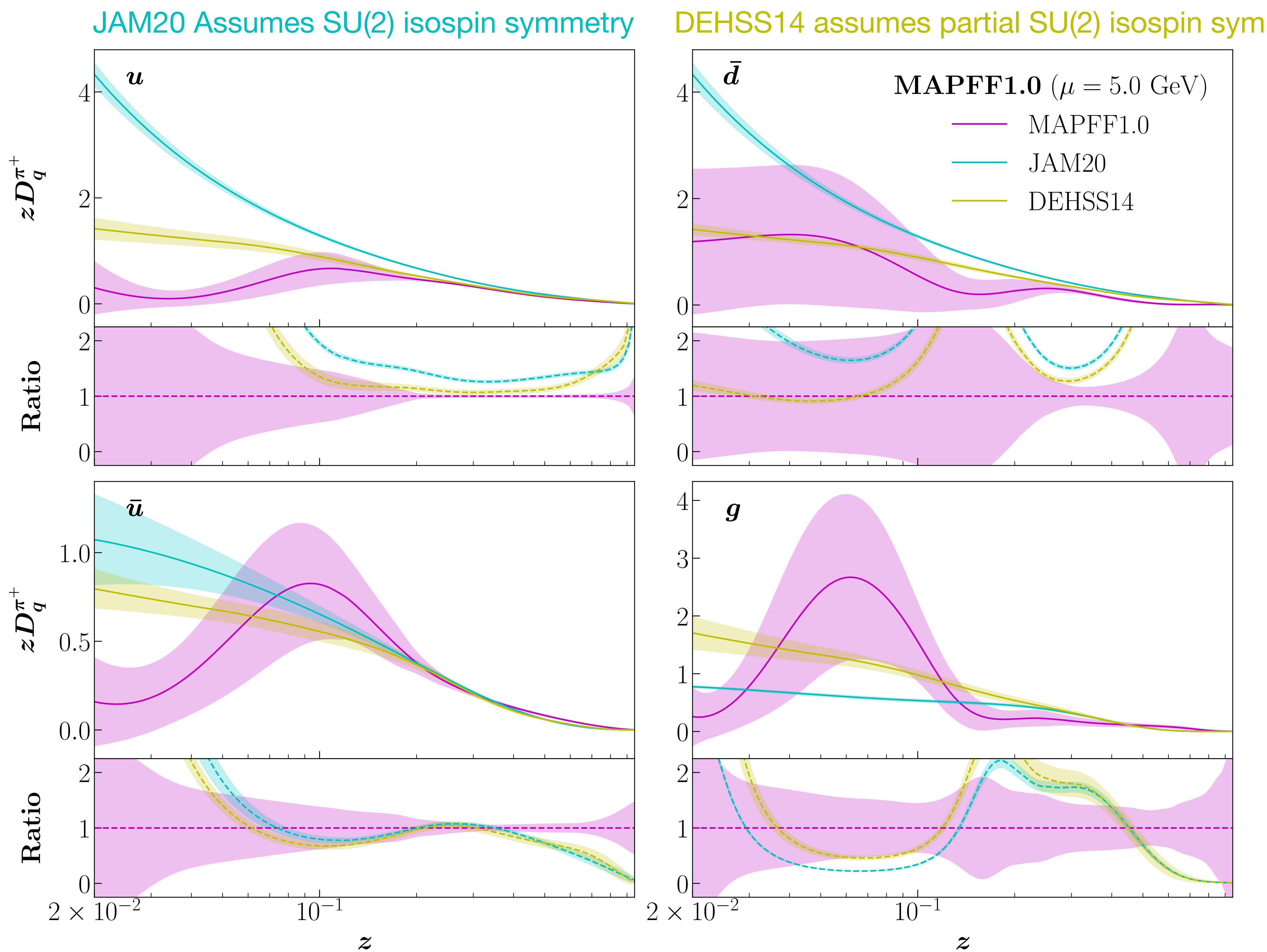
Results — FFs

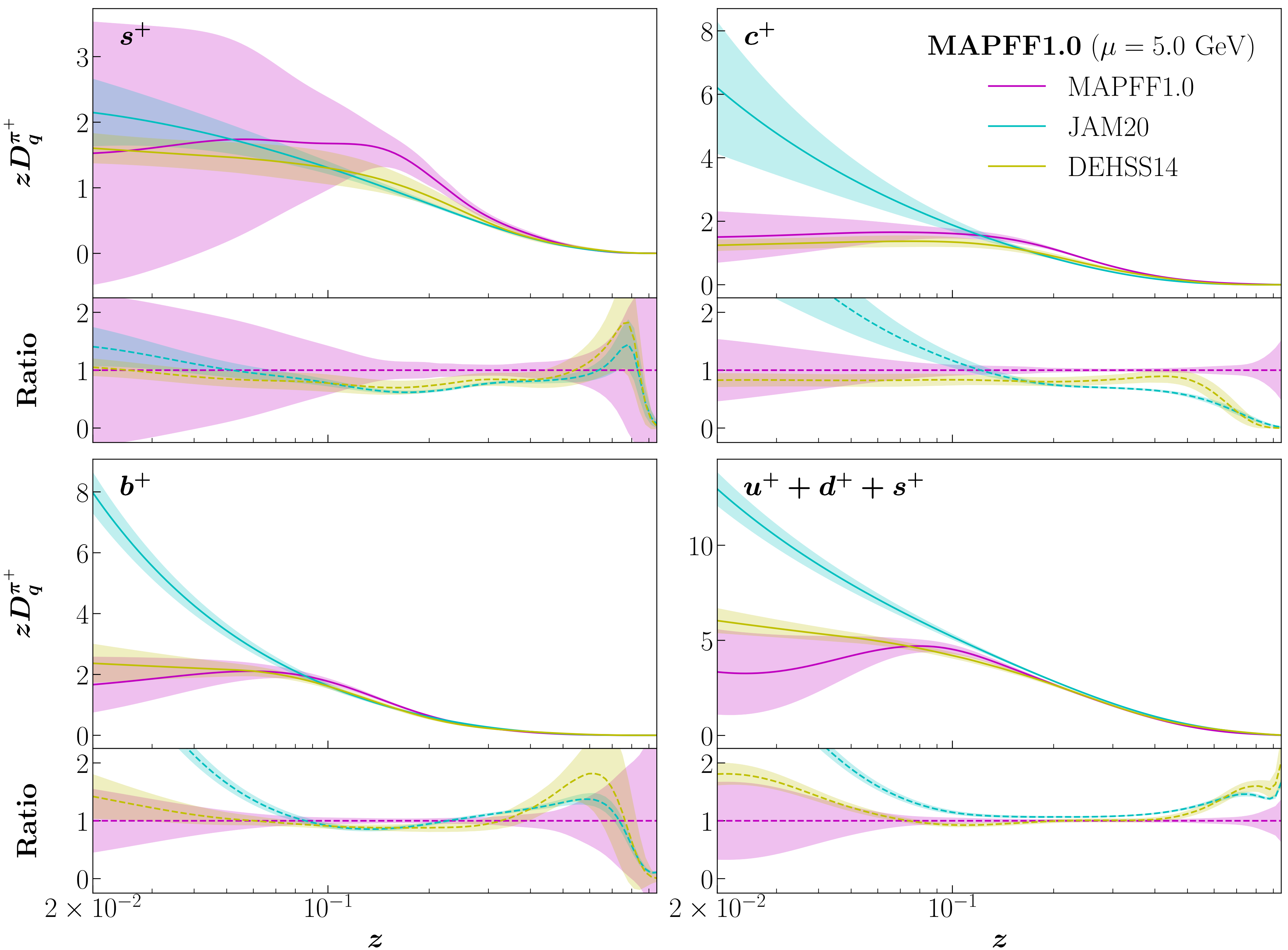


JAM20 Assumes SU(2) isospin symmetry
DEHSS14 assumes partial SU(2) isospin sym
MAPFF1.0 ($\mu = 5.0$ GeV)
MAPFF1.0
JAM20
DEHSS14
u
$\bar{d}$
$\bar{u}$
g
$zD_q^{\pi^+}$
Ratio
2×10^{-2}
10^{-1}
z

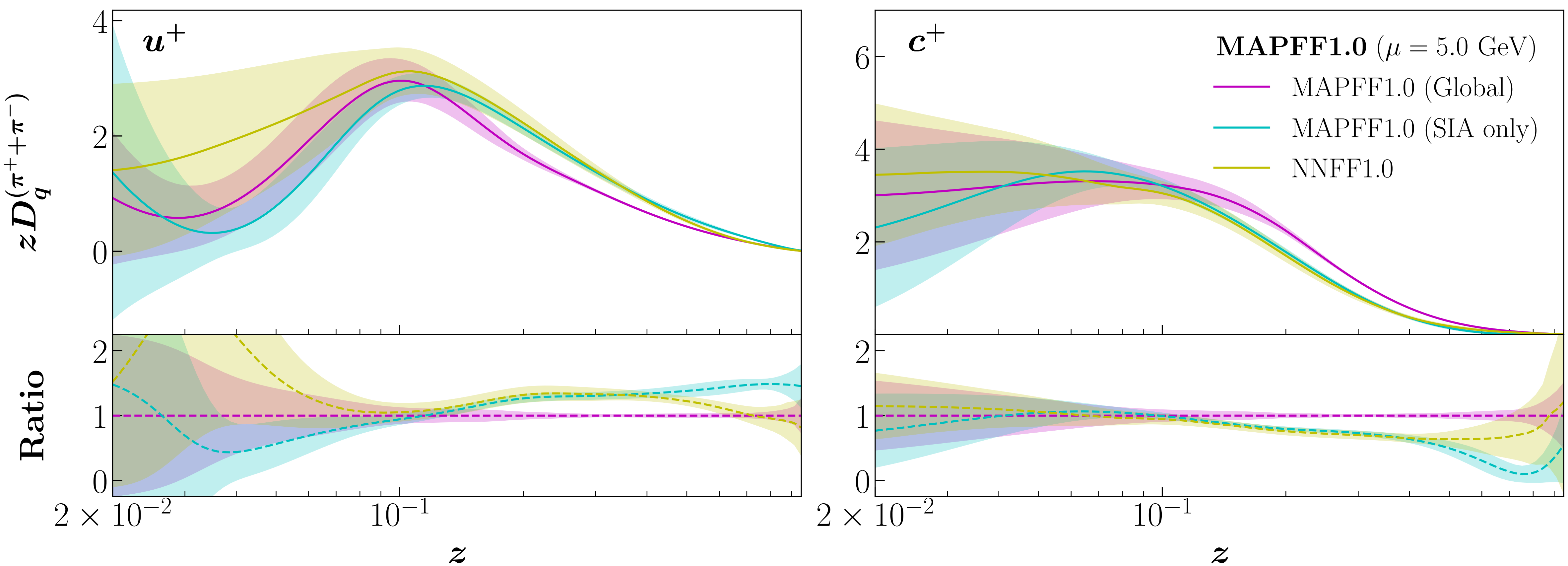
Results — FFs

JAM20 fits COMPASS

DEHSS14 fits an old version of COMPASS



Results — Tension with charm

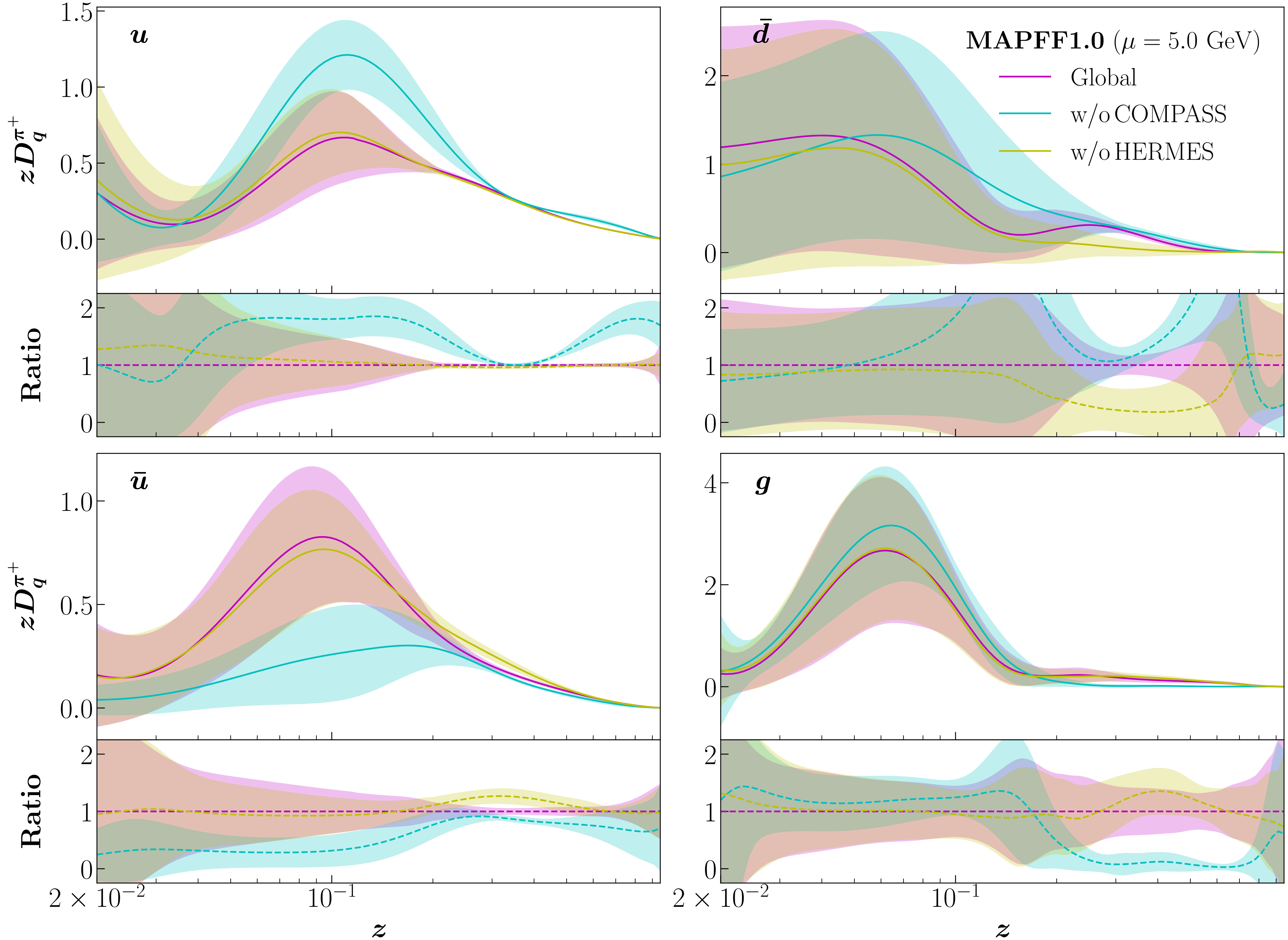


- **If the COMPASS data is excluded from the fit, the SLD charm-tagged data can be satisfactory fitted.**
- **The tension is due to the enhancement of the charm FF caused by COMPASS leading to a $\chi^2/N \simeq 6$ for SLD charm-tagged data.**
- **The suppression of u^+, also leads to a deterioration of the uds-tagged measurement from both DELPHI ($\chi^2/N = 2.84$) and SLD ($\chi^2/N = 2.05$) but these are moderate.**

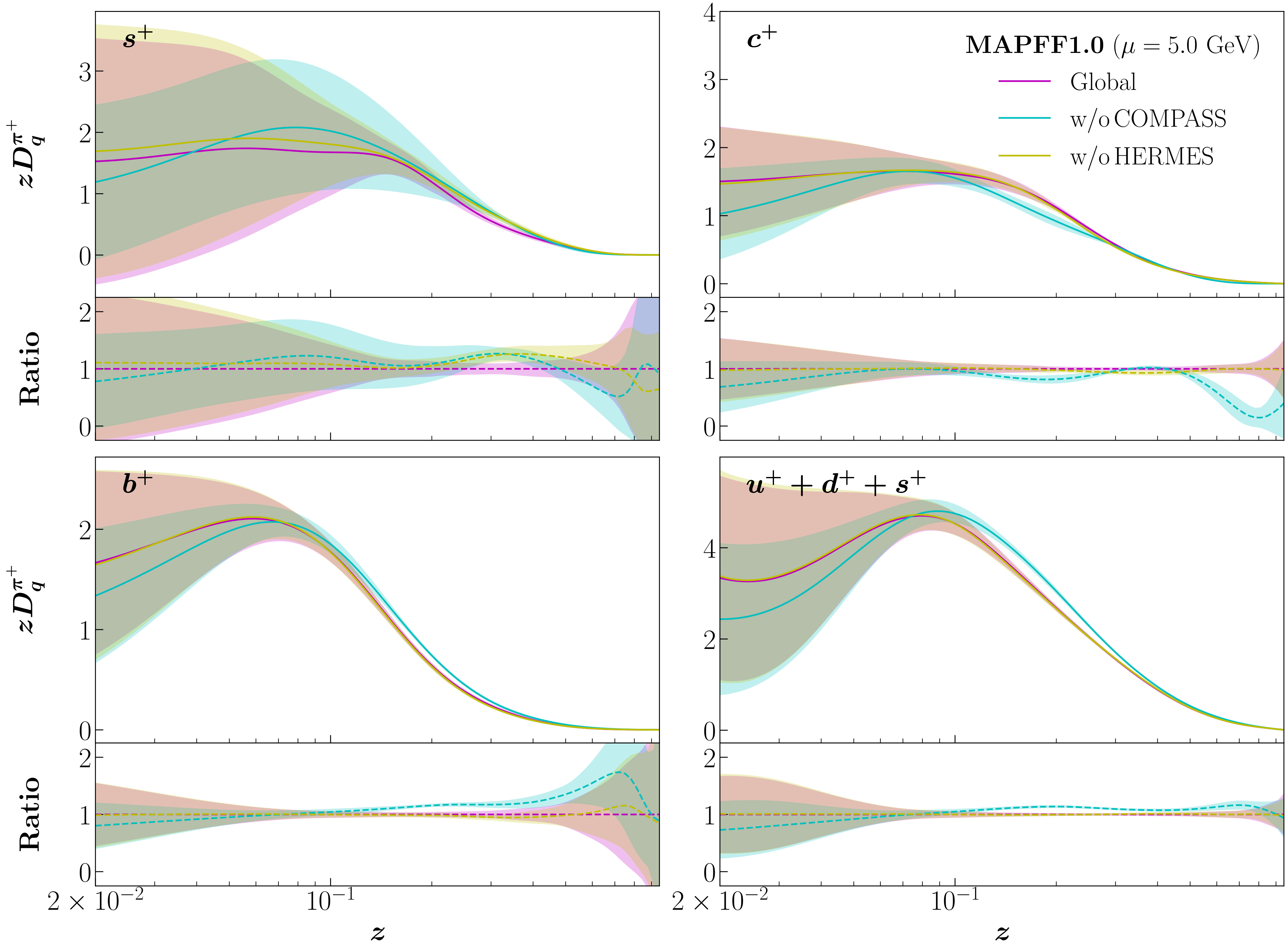
Results — Impact of SIDIS

COMPASS dominates for most flavours

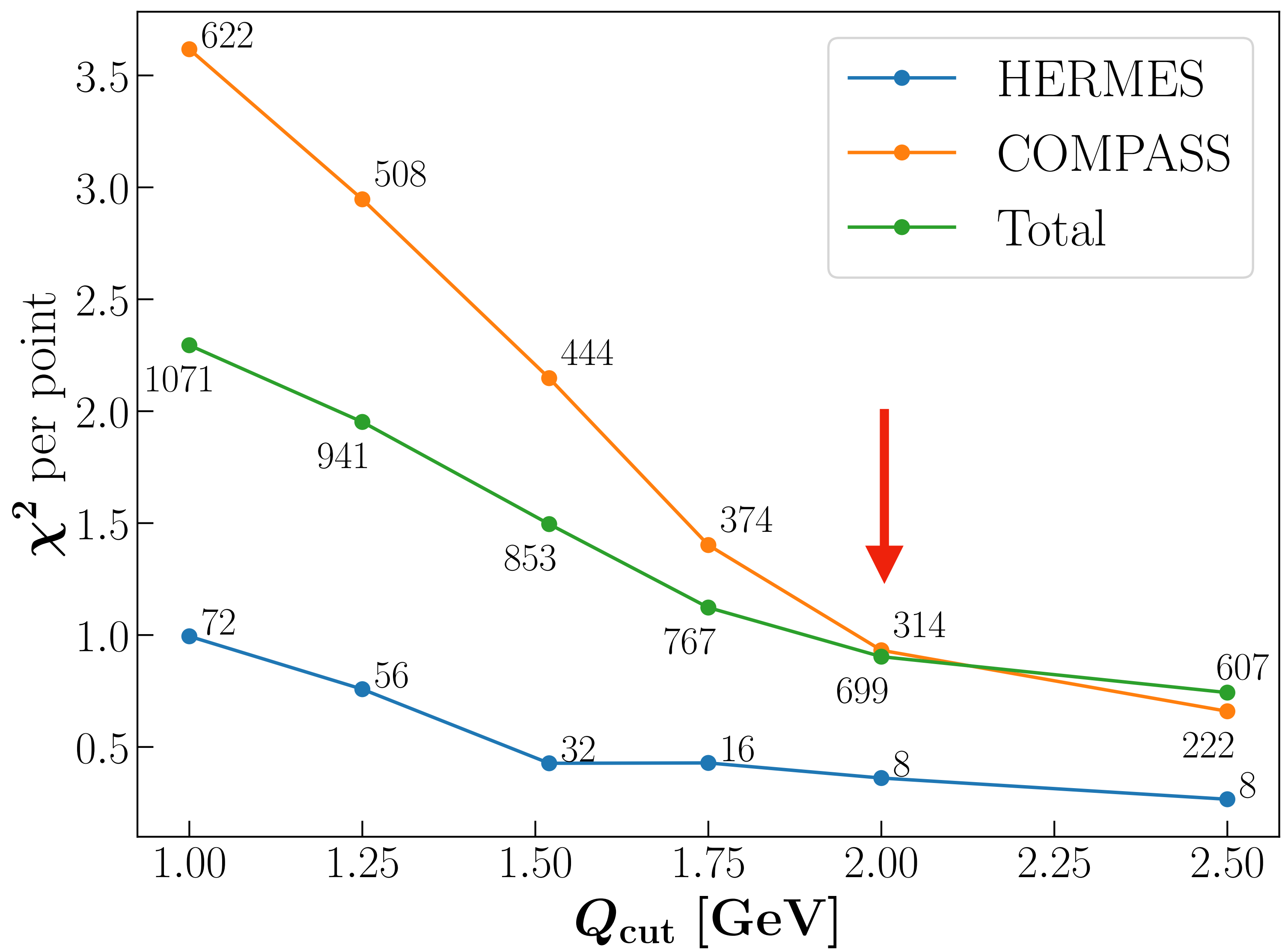
HERMES (8pts) still have a significant impact



Results — Impact of SIDIS



Results — SIDIS validity region



- χ^2 **decreases with** Q_{cut}**, confirming that perturbation theory works better for larger values of the hard scale** Q**.**

- **HERMES can be described down to** $Q = 1$ **GeV, but COMPASS quickly deteriorates which is particularly emphasised by the large number of data points the latter has.**

- **Based on this investigation we chose to restrict the SIDIS phase space by** $Q > 2$ **GeV in the fit.**

Summary

Framework

- MAPFF1.0 is a determination of charged pions collinear FFs from SIA and SIDIS data based on the Monte Carlo method at NLO accuracy
- 7 independent flavours are parameterised in terms of a neural network and fitted to data by means of a trust-region algorithm relying on the analytic derivatives of the neural network itself.

Results

- The global χ^2 as well as the χ^2 of all the single data sets included in the fits are fully satisfactory.
- the resulting FFs are almost insensitive to the treatment of PDFs uncertainties and central values.
- The inclusion of SIDIS data worsened the description of SLD charm-tagged data, which was removed.
- COMPASS plays a vital role in constraining and separating FFs flavours. However, HERMES, despite the limited number of points, has a noticeable impact on FFs.
- The $Q_{\text{cut}} = 2$ on SIDIS, guarantees a reliable applicability of NLO accurate predictions to the SIDIS data set.

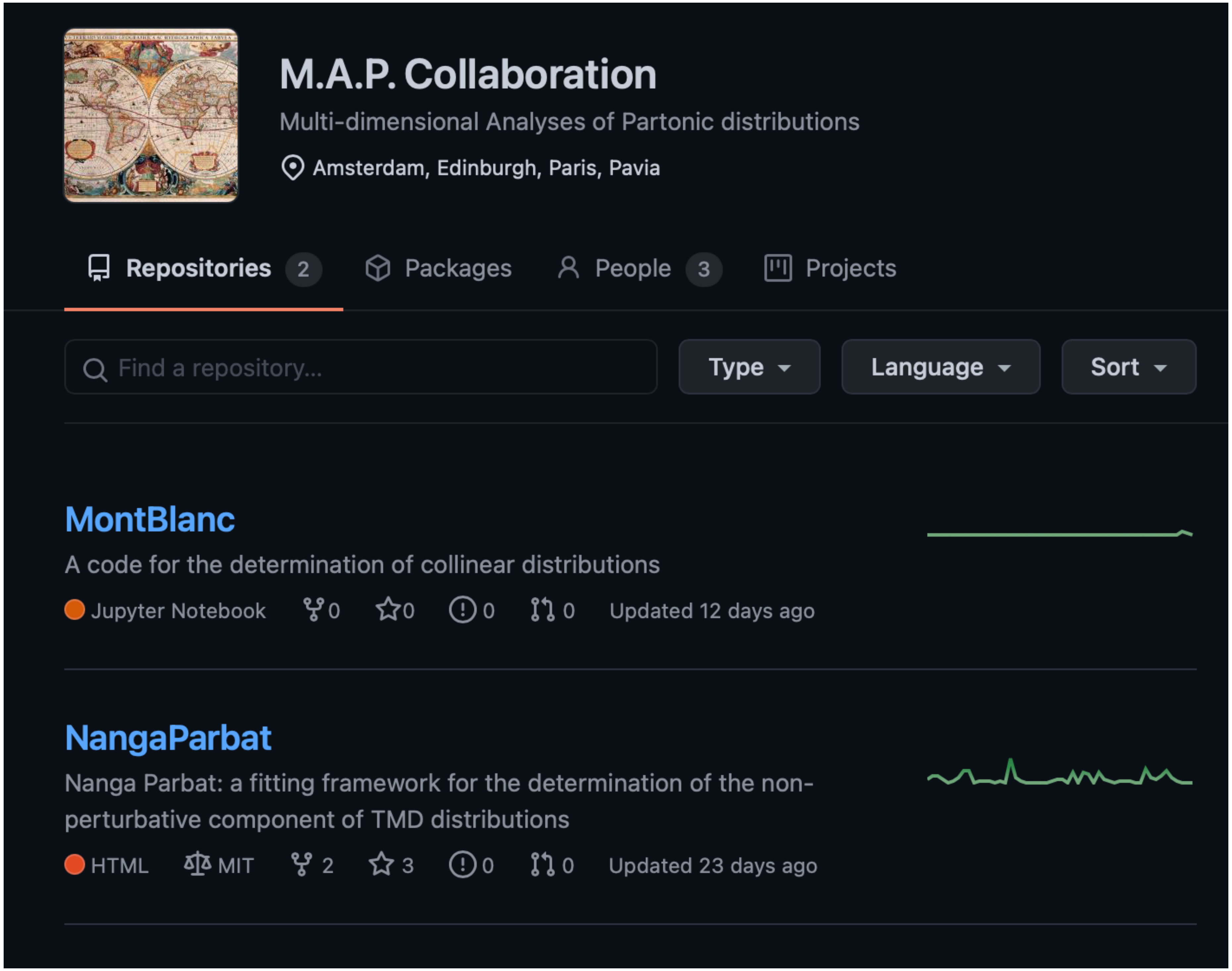
Public code	https://github.com/MapCollaboration/MontBlanc
LHAPDF sets	https://lhapdf.hepforge.org/

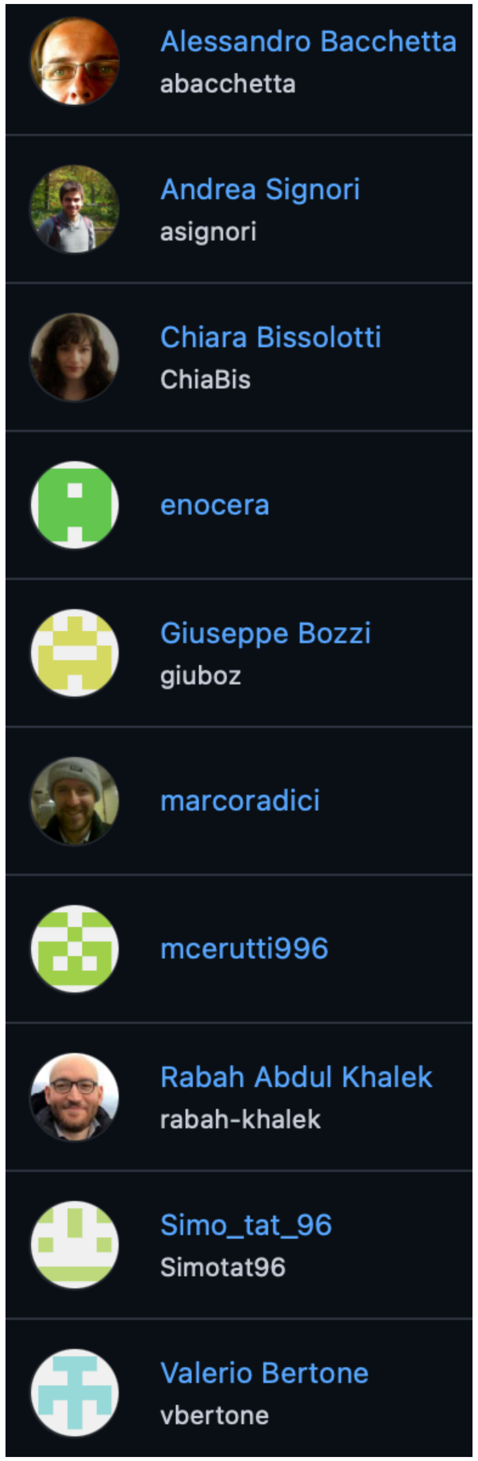
Future steps

- Extension to charged Kaons, proton/anti-proton and charged unidentified hadrons FFs.
- An accurate determination of the collinear FFs is instrumental to a reliable determination of transverse-momentum-dependent (TMD) distributions (See Valerio's previous talk)… This connection will be potentially explored in future work.

Thank you!

https://github.com/MapCollaboration





Impact of PDF uncertainties