DQMP Department of Quantum Matter Physics

Impurities in quantum baths

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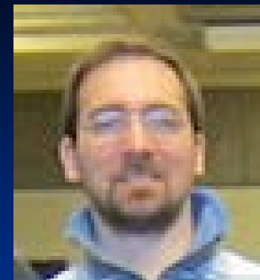
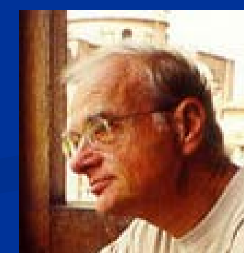


M. Zvonarev A. Kantian N. Kamar

F. Massel P. Torma P. LeDoussal








A.M. Visuri A. Daley B. Horovitz




F. Minardi T. Fukuhara S. Kuhr I. Bloch

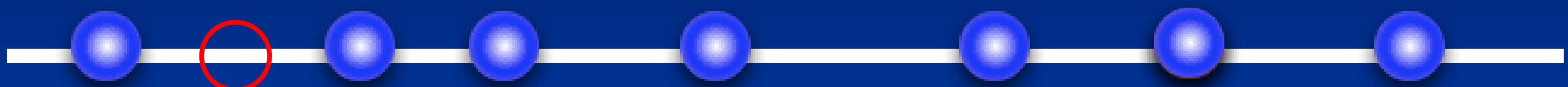
Problem

- One impurity coupled to a 1D quantum bath

$$H = \frac{P^2}{2M} + H_{1D} + V\rho(X)$$

- Equilibrium physics: ``polaron”
- Not so ``simple” if out of equilibrium: Anderson orthogonality catastrophy

Global vs local quench



- Local quench
- $M = \infty$ X-ray edge problem

$$\langle GS \,|\, d_{0,t} d^{\dagger}_{0,0} \,|\, GS \rangle = \langle GS \,|\, e^{iH_0 t} e^{-iH_1 t} \,|\, GS \rangle \sim \left(\frac{1}{t} \right)^{\nu}$$

- M finite: destroyed by recoil, but not in 1D !

How to treat ?

- Bosonization

$$H = \frac{\hbar}{2\pi} \int dx [\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

- u,K : depend on interactions, density

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right] \sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$$\langle \nabla\phi(x)\nabla\phi(0)\rangle \sim \frac{1}{x^2} \qquad \langle e^{i2\phi(x)} e^{-i2\phi(0)} \rangle \sim \frac{1}{x^{2K}}$$

``Bosonic" type behavior (K >1)

Mobile impurity

M. B. Zvonarev, V. V. Cheianov, TG, PRL 99 240404 (2007);

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} [g\delta(x_i - x_j) + U(x_i - x_j)]$$

$$\gamma = mg/\hbar^2 \rho_0$$

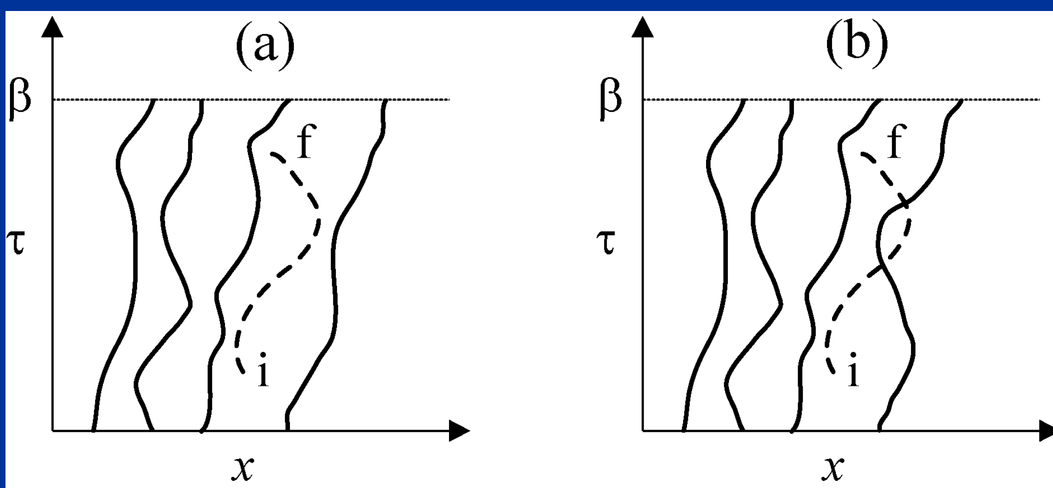
$$\frac{m^*}{m} = \frac{3\gamma}{2\pi^2}$$

Single spin down particle $G_\perp(x,t) = \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$

$$\sum_q \epsilon(q) c_q^\dagger c_q + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b_{-k}^\dagger) \rho_\downarrow(-k)$$

Trapped regime

U large: particle cannot cross



$$N(x,t) = \rho_0 x - \frac{1}{\pi}[\phi(x,t) - \phi(0,0)]$$

Trapped regime

$$G_{\perp}(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_F)}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho_0 x)^2}{2\ln(t/t_F)}\right\}.$$

- Subdiffusive behavior
- No power law or Lorentz invariance
- No light cone singularity

Full solution

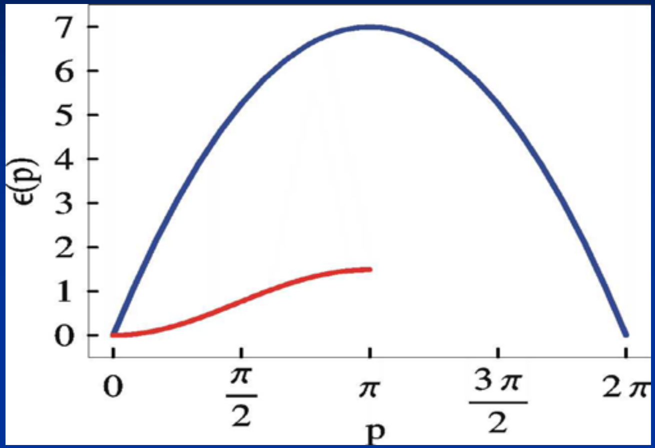
No exact solution available

$$G_{\perp}(x,t) = \int \frac{dk}{2\pi} e^{ikx} \int \frac{d\omega}{2\pi} e^{-i\omega t} A(k,\omega),$$

$$A(k,\omega) = \sum_{\nu} \delta(\hbar\omega - E_{\nu}(k)) |\langle \nu, k | s_{-}(k) | \Uparrow \rangle|^2.$$

Set of minimal assumptions on the theory

- 1D: minimum in the spectrum



supported by exact solutions

- A(k,ω) is scale free close to threshold

$$A(k,\omega) \simeq c(k)[\hbar\omega - \varepsilon(k)]^{\Delta(k)}, \qquad \hbar\omega \geq \varepsilon(k),$$

$$\epsilon(k) = \frac{k^2}{2m^*}$$

$$\Delta(k) = \alpha - 1 + \beta k^2 + \cdots,$$

$$c(k) = c_0 + c_1 k^2 + \cdots,$$

General result

$$G_{\perp}(x,t) \simeq t^{-\alpha}\left[\beta \ln\left(\frac{t}{t_F}\right) + \frac{it\hbar}{2m_*}\right]^{-1/2} \times \exp\left\{\frac{i m_* x^2}{2t\hbar - 4i\beta m_* \ln(t/t_F)}\right\}.$$

Comparison with trapped regime:

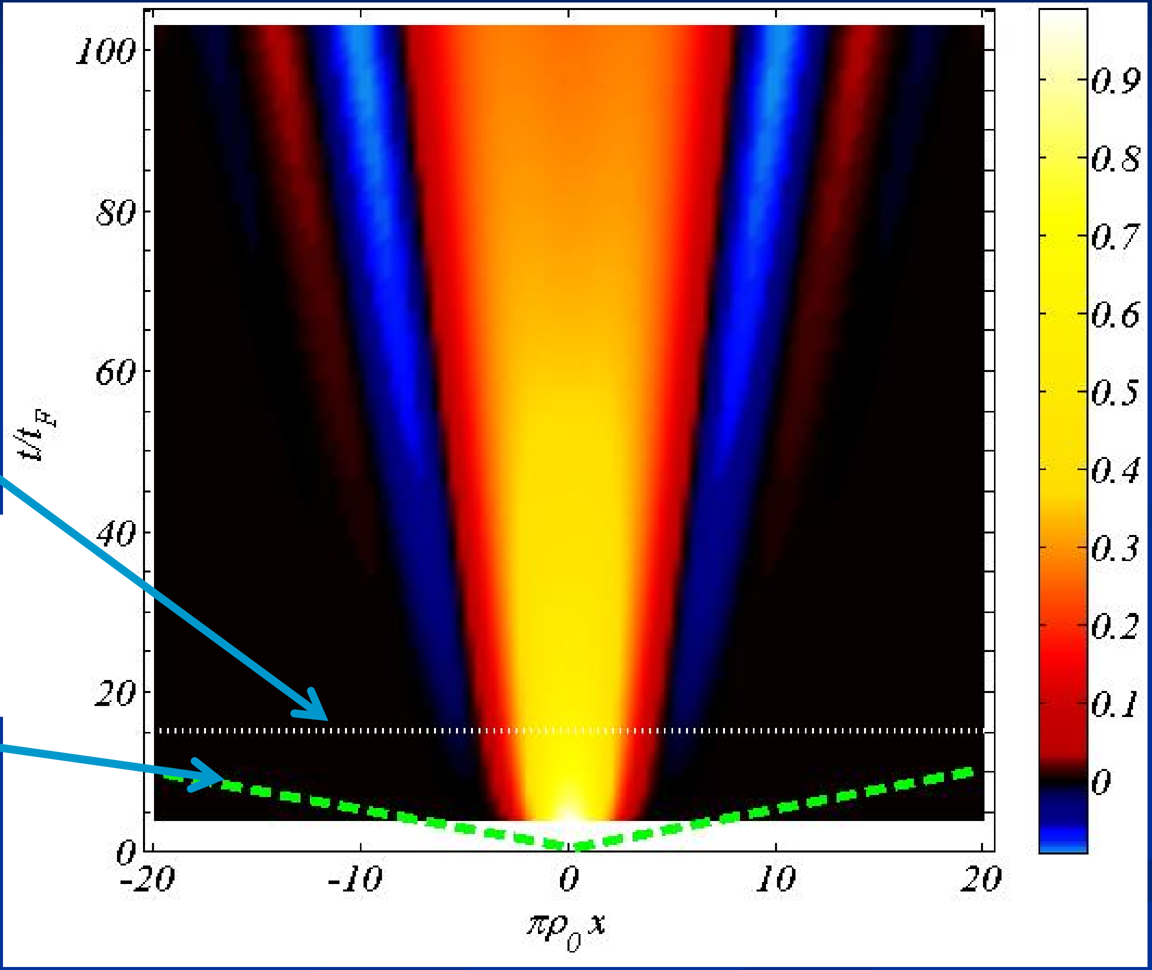
$$\alpha = 0, \qquad \beta = \frac{K}{2(\pi\rho_0)^2}$$

Exponent depends only on K, new universality class : Ferromagnetic liquid

Propagation of the impurity

Trapped/open regimes

Light cone of spinless bosons



$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_F)}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho_0 x)^2}{2\ln(t/t_F)}\right\}.$$

$$G_\perp \simeq e^{-(x^2/2\ell^2)} t^{-\alpha} G_\perp^H, \qquad \ell(t) = \frac{2K^{-(1/2)}}{\pi\rho_0} \frac{t/t_F}{\sqrt{\ln t/t_F}} \frac{m}{m_*}.$$

Theories: Akhanjee, Tserkovnyak, Matveev, Furusaki, Kamenev, Glazman, Gangardt, Lamacraft, ……

Experiments: Cambridge, Florence, Innsbruck, Munich, …

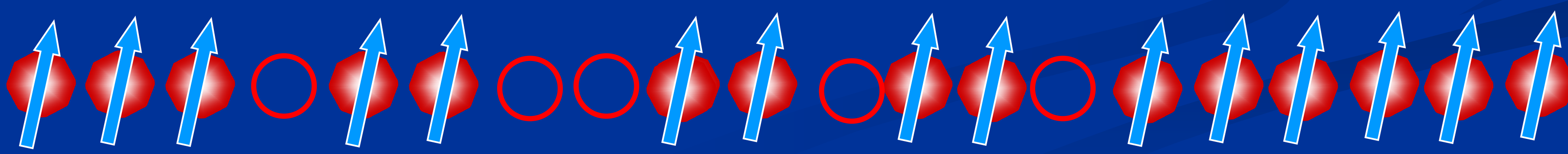
Hubbard model

M. B. Zvonarev, V. V. Cheianov, TG, PRL 104 110401 (2009)

$$H = -t_h \sum_{\substack{j=1 \\ \alpha=\uparrow,\downarrow}}^{M} (b^\dagger_{\alpha j} b_{\alpha j+1} + \text{H.c.}) + U \sum_{j=1}^{M} \varrho_j(\varrho_j - 1).$$

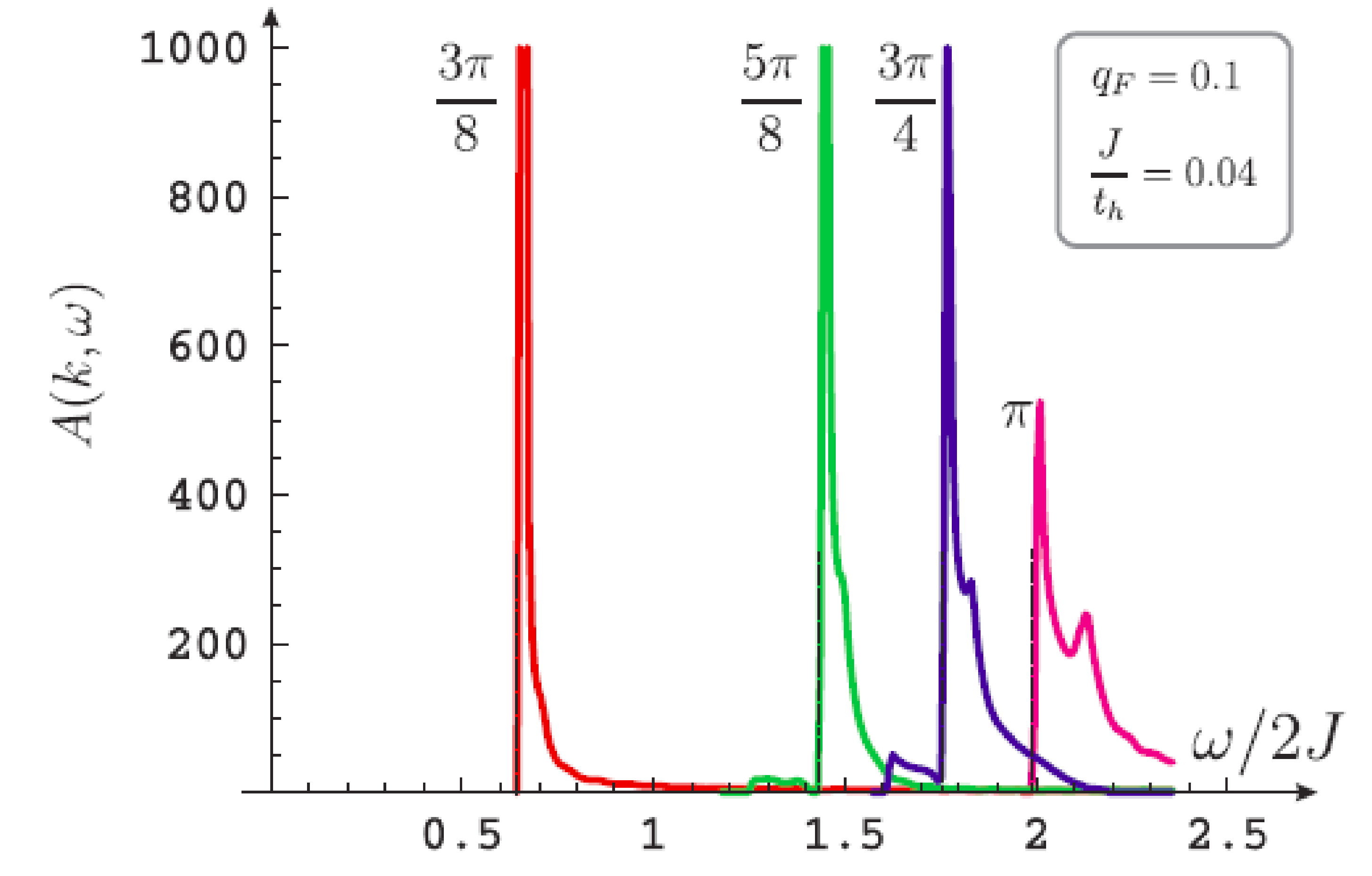
$$H_c = -t_h \sum_{j=1}^{M} (c_j^\dagger c_{j+1} + \text{H.c.}),$$

$$H_s = -2J \sum_{j=1}^{N} \left[\ell(j)\ell(j+1) - \frac{1}{4} \right],$$



$$s_-(j) = \varrho_j \ell_-(\mathcal{N}_j)$$

$$G_\perp(j,t) = \int_{-\pi}^{\pi} d\lambda G_H(\lambda,t) D_\nu(\lambda; j,t).$$



Obeys: $\alpha = 0, \qquad \beta = \frac{K}{2(\pi\rho_0)^2}$

Yang-Gaudin model

M. B. Zvonarev, V. V. Cheianov, TG, PRB 80 201102® (2009)

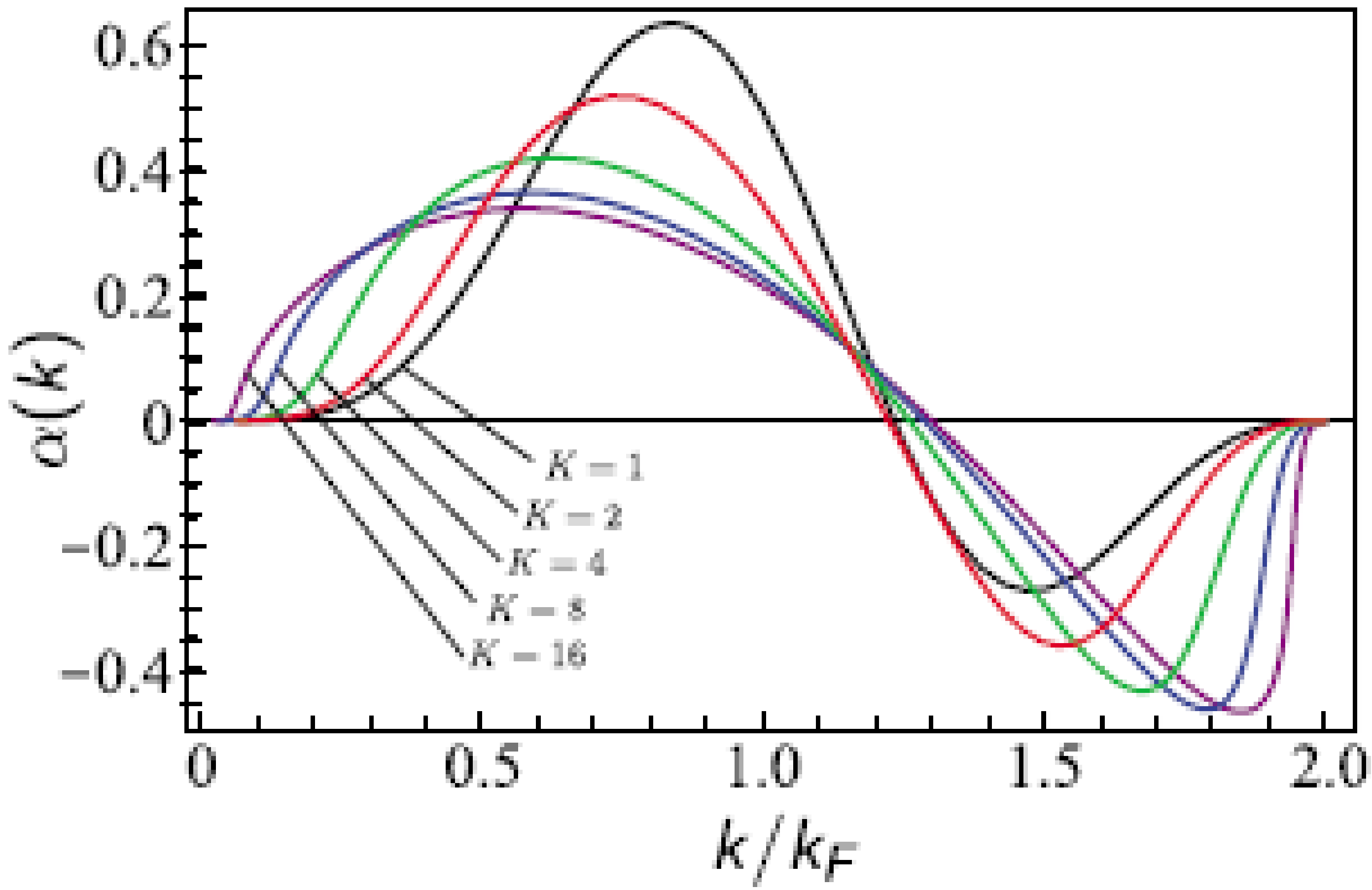
$$H = \int_0^L dx[\partial_x \psi_\uparrow^\dagger \partial_x \psi_\uparrow + \partial_x \psi_\downarrow^\dagger \partial_x \psi_\downarrow + g\rho^2],$$

Effective field theory :

$$H_{LL} +$$

$$H_i = -\sum_{r=\pm} \frac{v_r \beta_r}{2\pi} \int_0^L dx\, \partial_x \varphi_r(x) \tilde{s}_z(x).$$

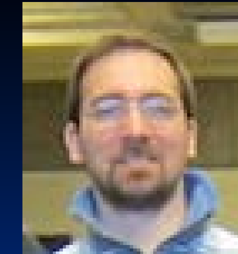
$$\Delta(k) = -1 + \frac{1}{4\pi^2}(\beta_+^2 + \beta_-^2).$$



$$\Delta(k) = -1 + \frac{K}{2}\left(\frac{k}{k_F}\right)^2 + \frac{(K-1)^2}{K}\alpha(k),$$

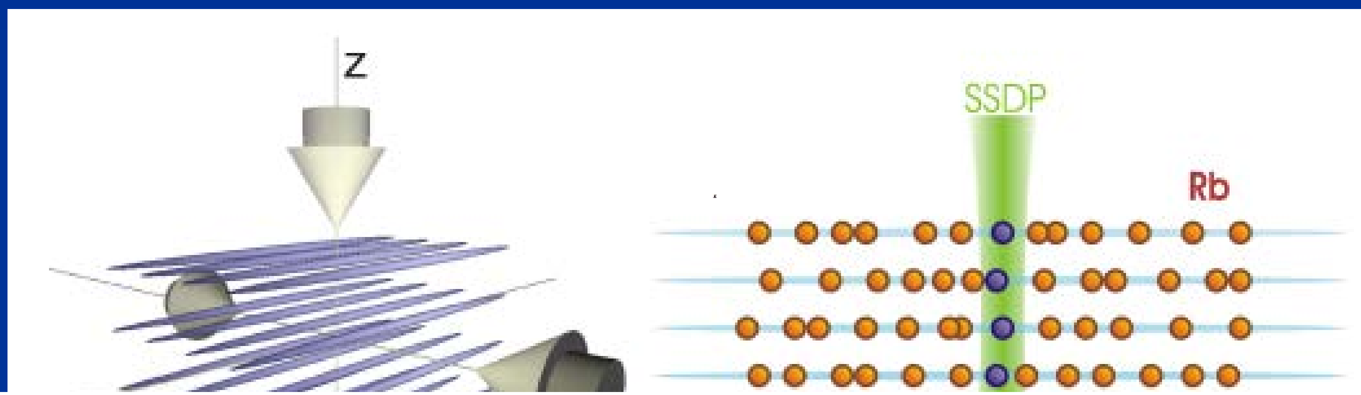
Experiments

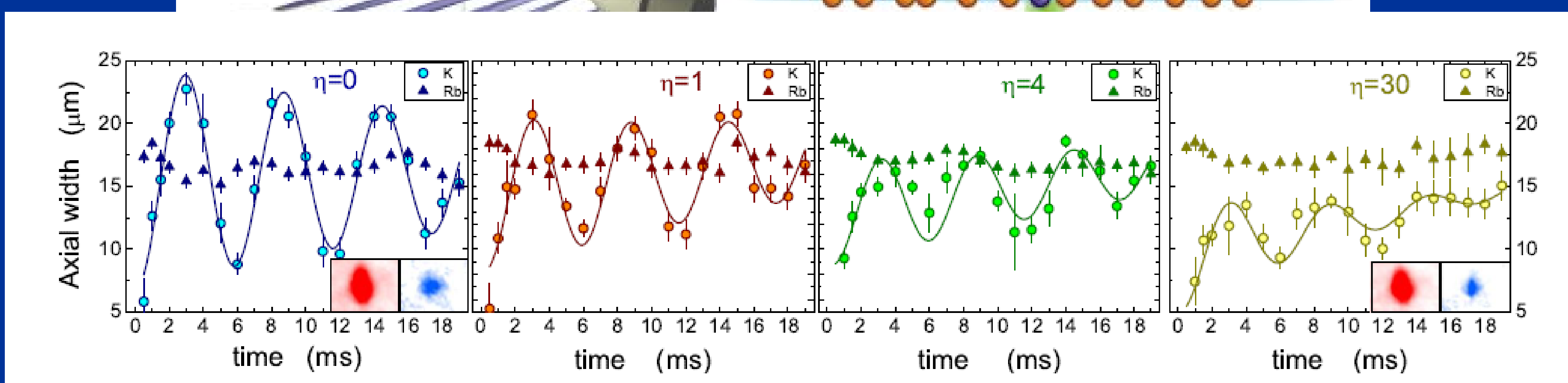


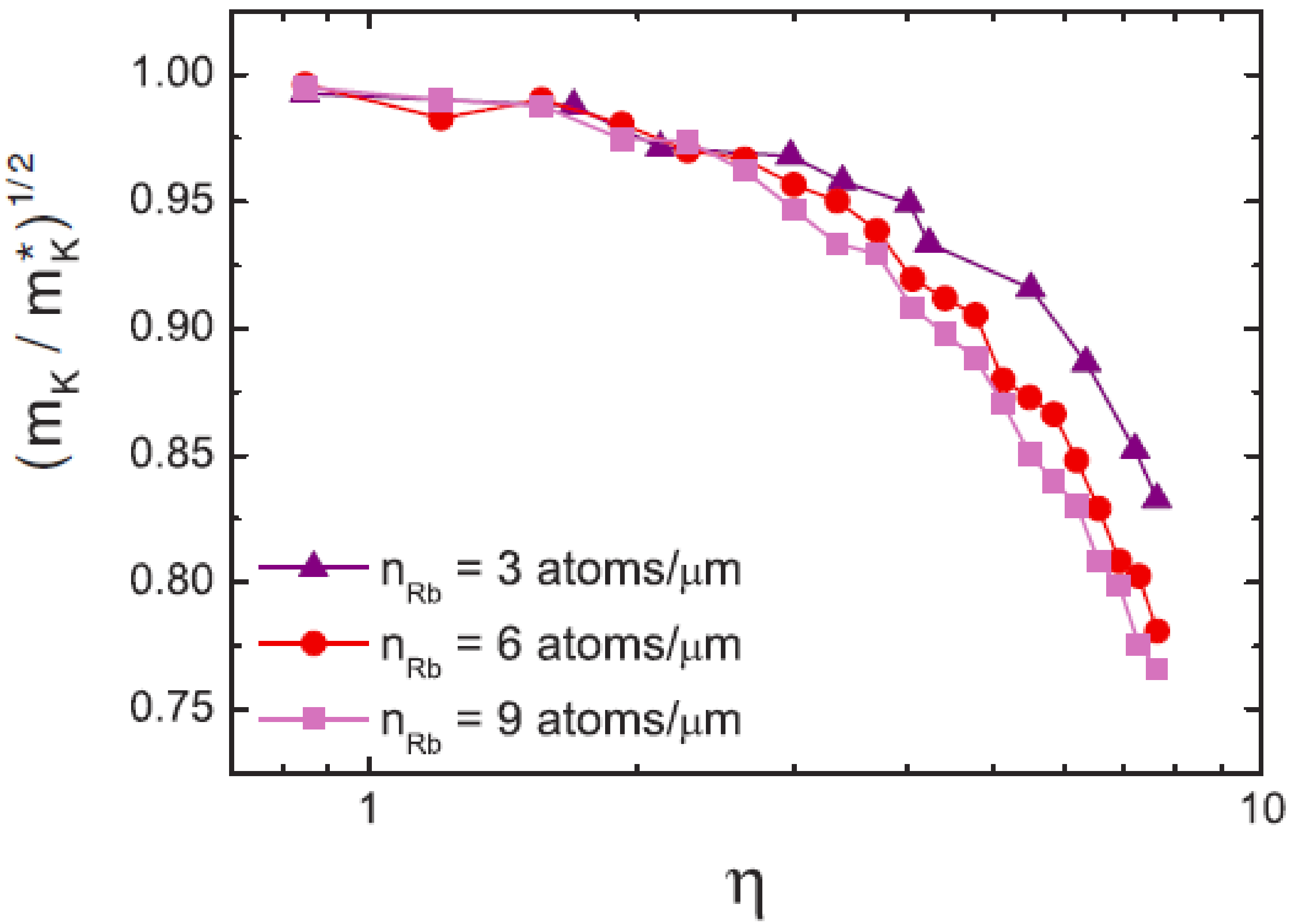
Diffusive impurity

J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kantian, TG
PRA 85 023623 (2012)





Polaronic effect

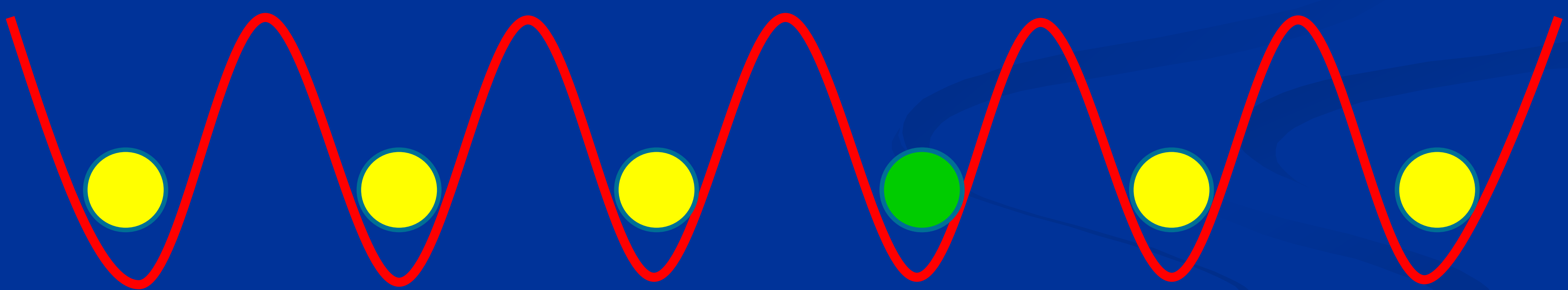


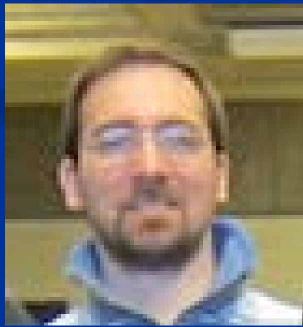
$S_0 =$

$)]^2,$

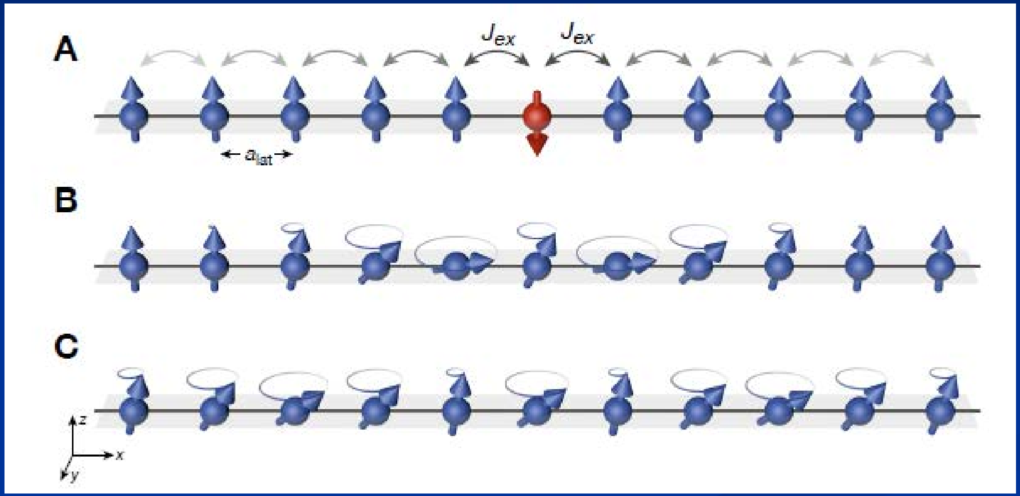
Mobile impurity

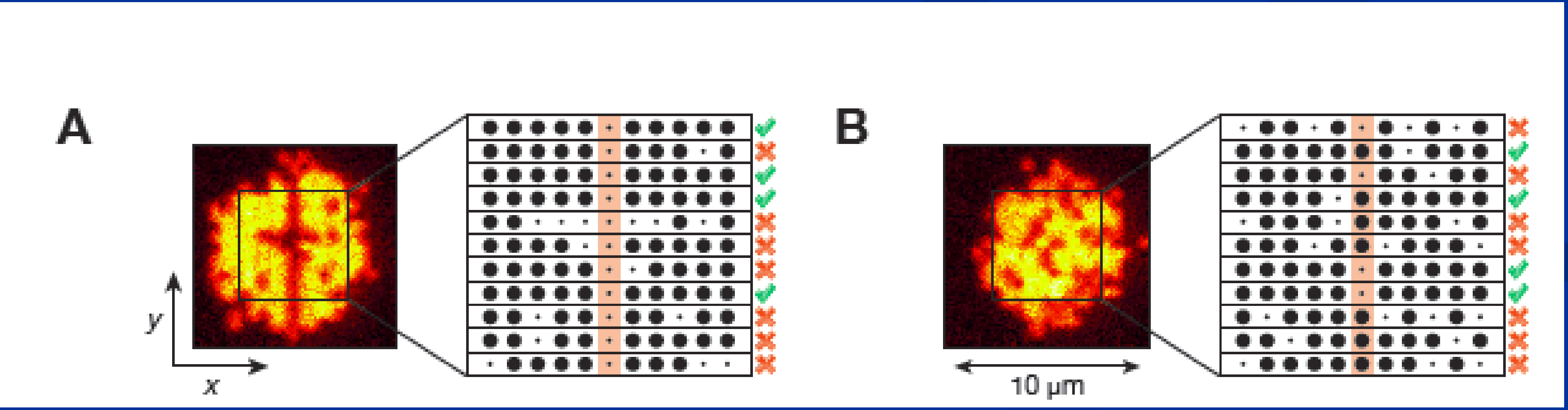
T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schauss, S. Hild, D. Bellem, U. Schollwock, TG, C. Gross, I. Bloch, S. Kuhr , Nat. Phys. (2013)



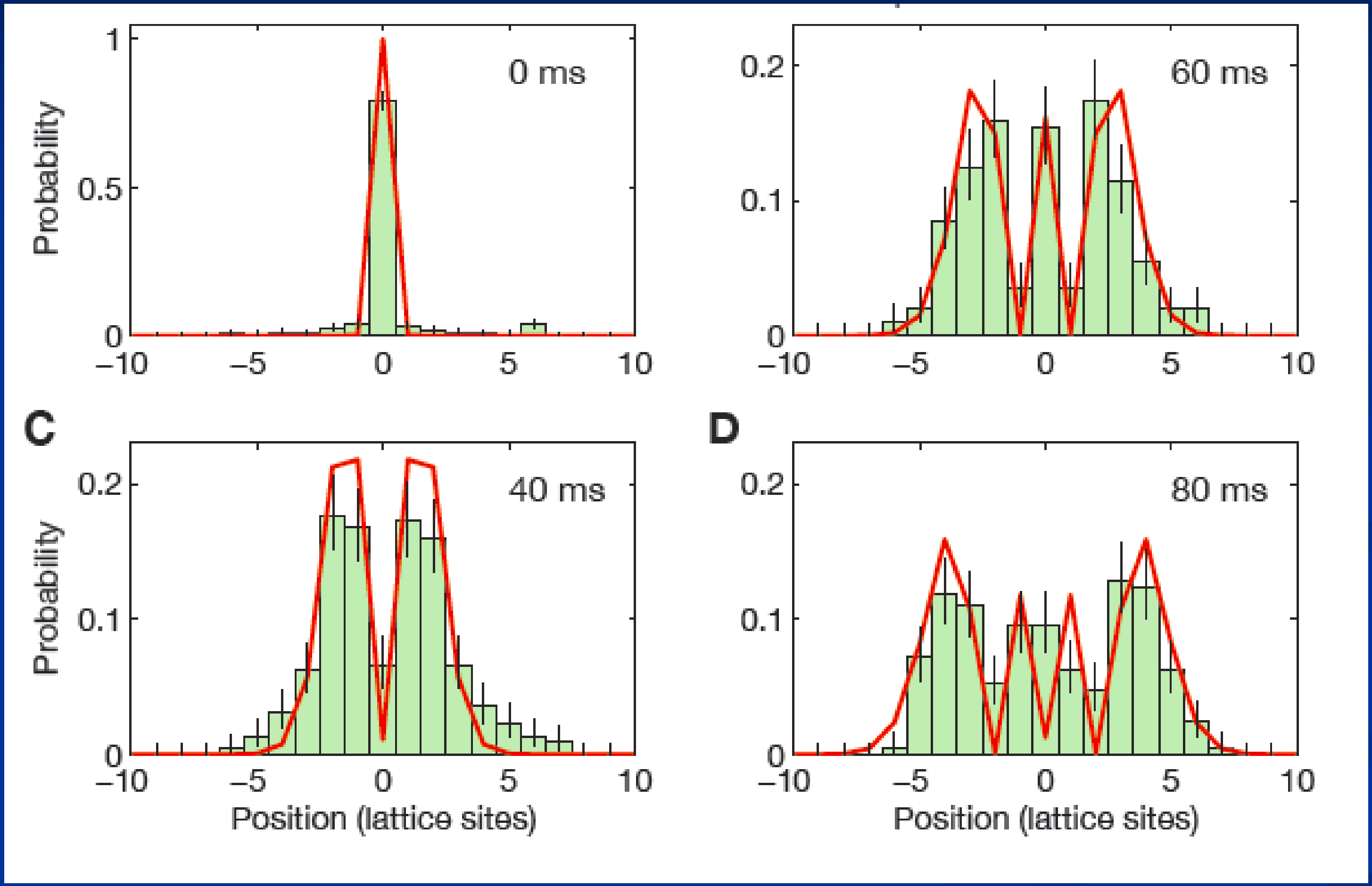


Ferromagnetic Heisenberg



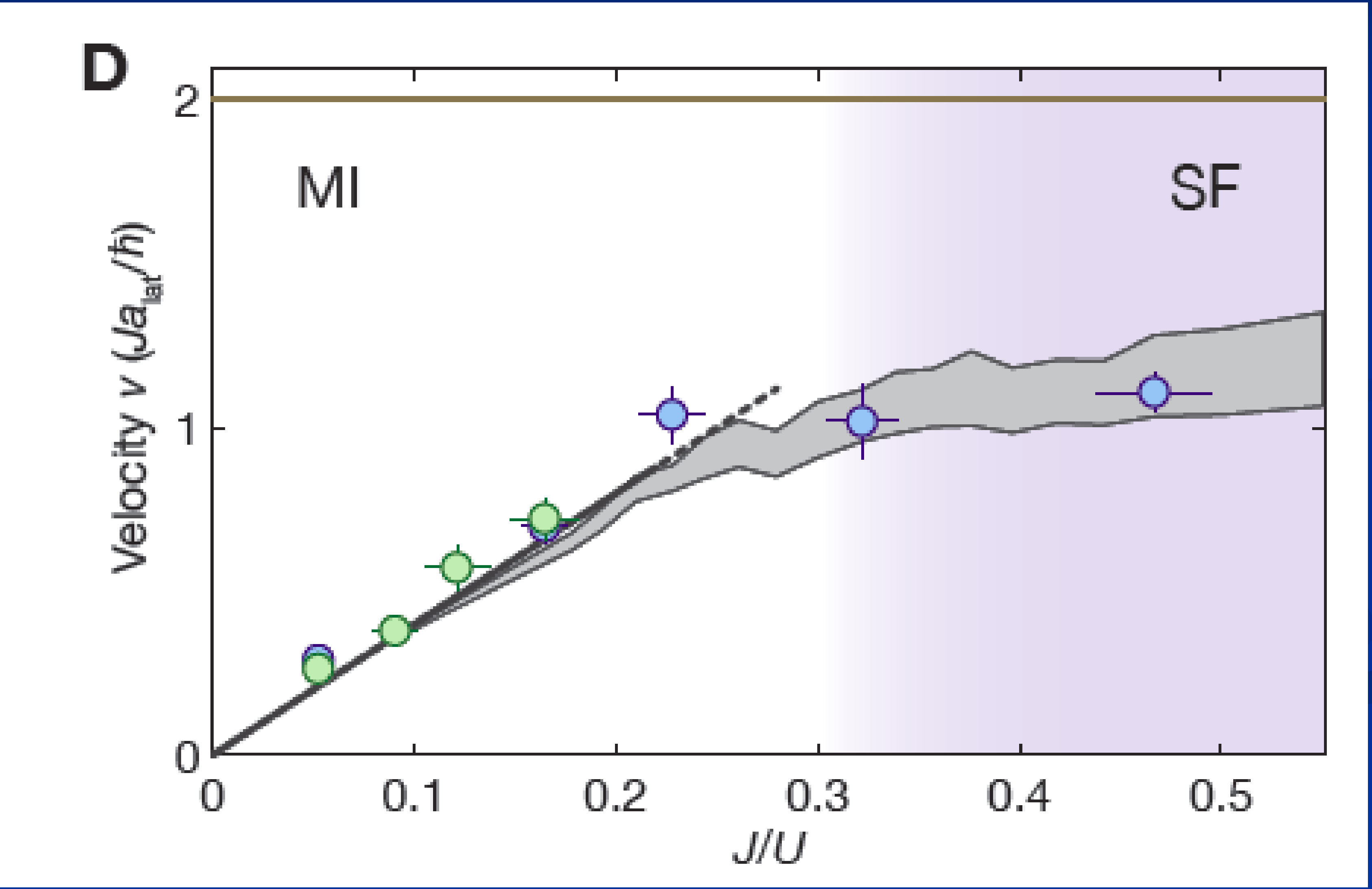


Coherent propagation of a magnon

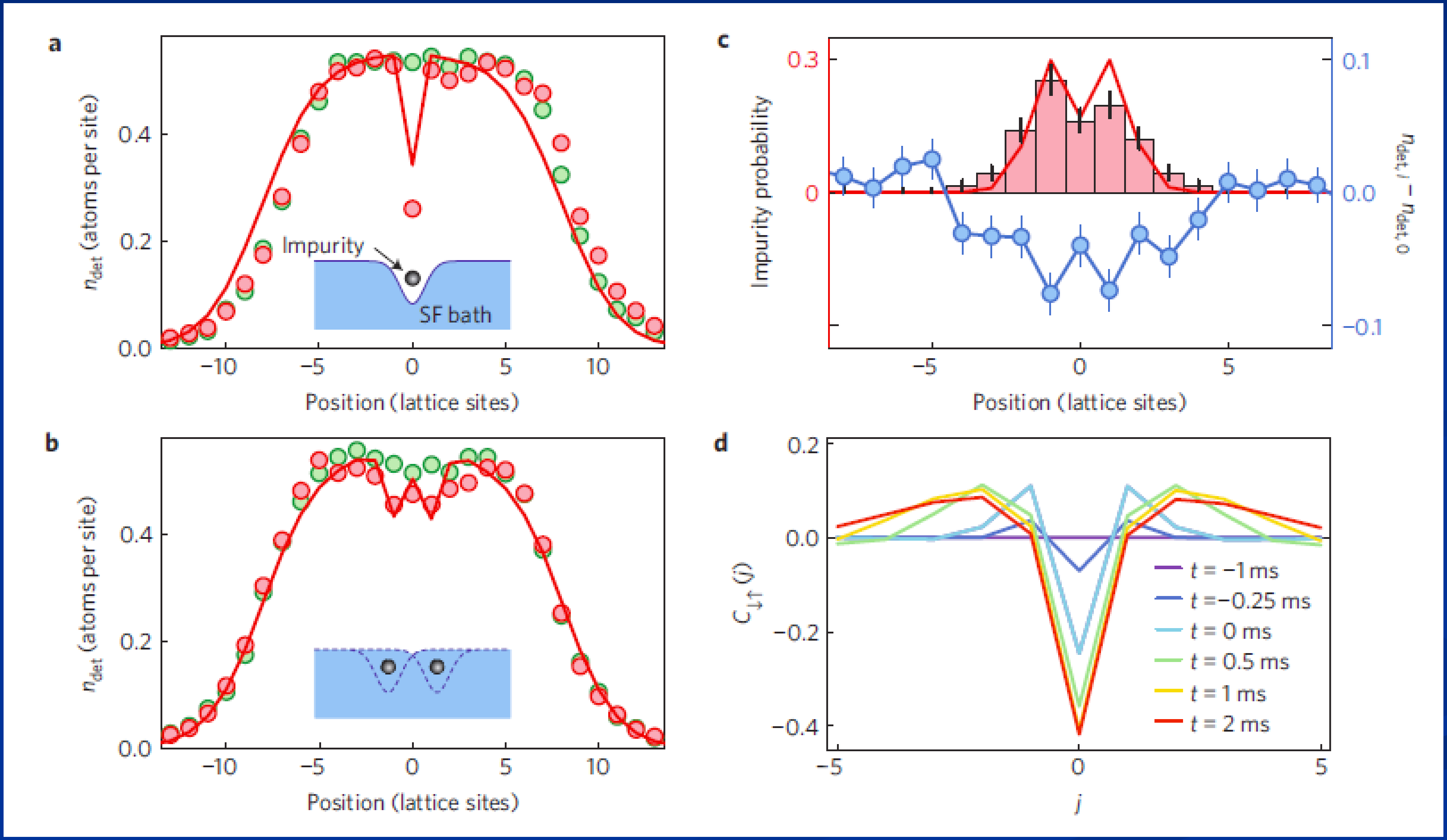


Heisenberg model : $H = J_{ex} \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$ $J_{ex} = \frac{4t^2}{U}$

Beyond the Mott insulator



Polaronic effect



Note

- Theory: two body correlations

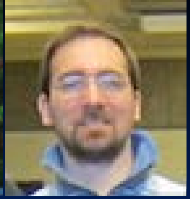
$$G_{\perp}(x,t) = \langle \Uparrow | s_{+}(x,t) s_{-}(0,0) | \Uparrow \rangle$$

- Experiment: more complex correlations

$$\langle GS | d_0 e^{iHt} d_x^{\dagger} d_x e^{-iHt} d_0^{\dagger} | GS \rangle$$

- Calculation ??

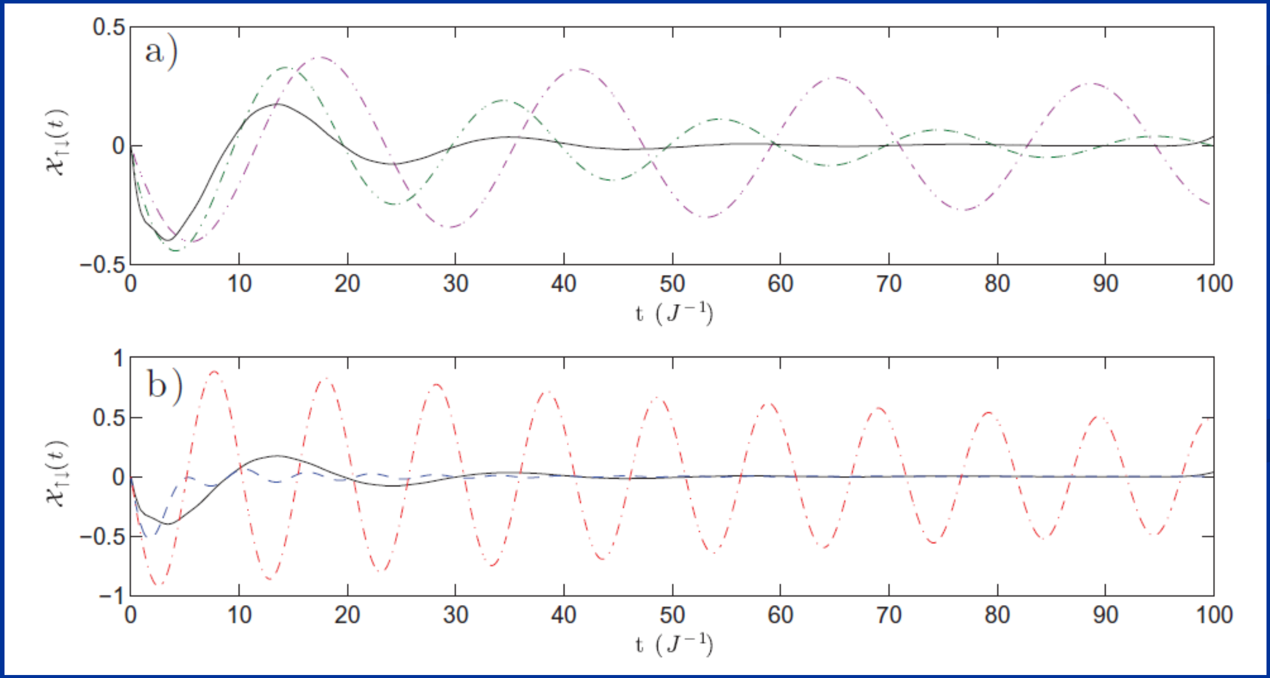
Many remaining problems/questions

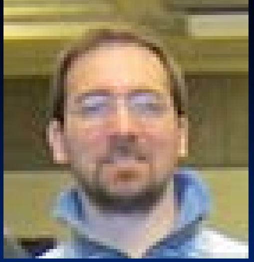



Kicked impurity


F. Massel, A. Kantian, A.J. Daley, TG, P. Torma, NJP 15 045018 (2013)

$$\mathcal{X}_{\uparrow\downarrow}(t) = \frac{\sum_i \left(i - \frac{L-1}{2}\right) \langle n_{i\uparrow} n_{i\downarrow} \rangle (t)}{\sum_i \langle n_{i\uparrow} n_{i\downarrow} \rangle (t)},$$





Two regimes: Infrared dominated vs polaronic

A. Kantian, U. Schollwoeck, TG, PRL 113 070601 (2014)

$$A(p,\omega) \propto \frac{\theta(\omega - \epsilon_p)}{(\omega - \epsilon_p)^{\Delta(p)}}$$

$$\Delta(p) \approx \Delta(0) + \beta p^2$$

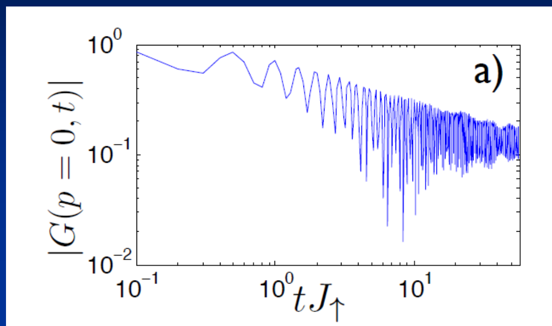
- Very efficient method: linked cluster expansion

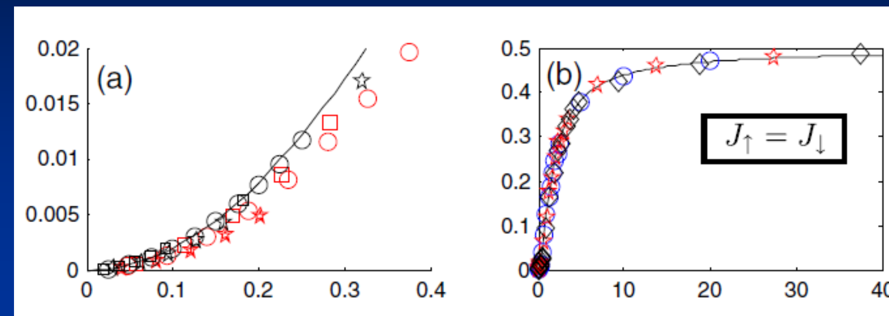
$$G_{\mathrm{LCE}}(p,t) = -ie^{-i\varepsilon_p t} e^{F_2(p,t)},$$

$$F_2(p,t) = \int du \frac{1 + itu - e^{itu}}{u^2} R(u),$$

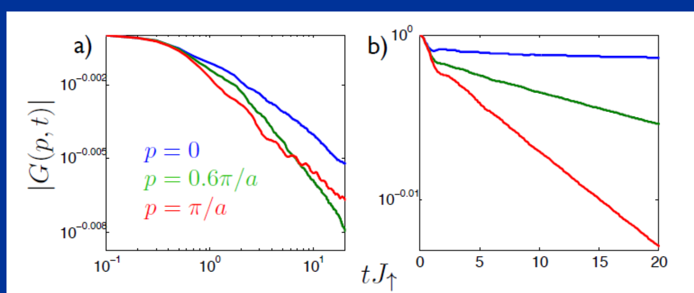
$$R(u) = \int dq V(q)^2 \delta(u + \varepsilon_p - \varepsilon_{p+q} - v|q|),$$

DMRG and LCE





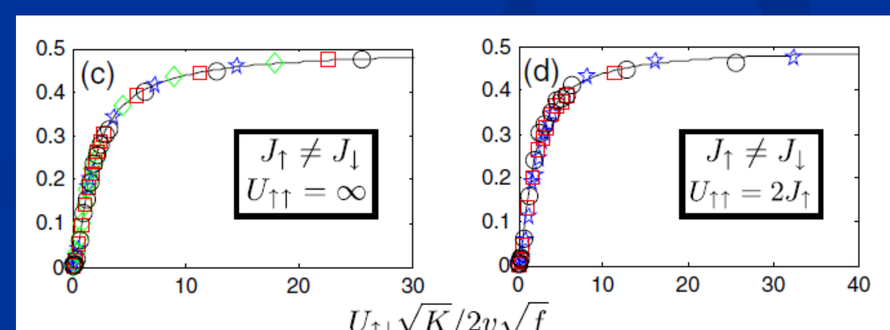
- Two regimes: quasiparticle and infrared dominated



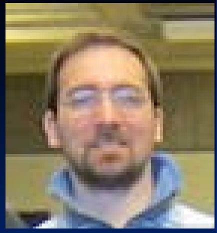
- Interpolating formula

$$\Delta(0) = -1 + \frac{2f}{\pi^2}\left[\arctan\left(\frac{2v}{U_{\uparrow\downarrow}}\sqrt{\frac{f}{K}}\right) - \frac{\pi}{2}\right]^2$$

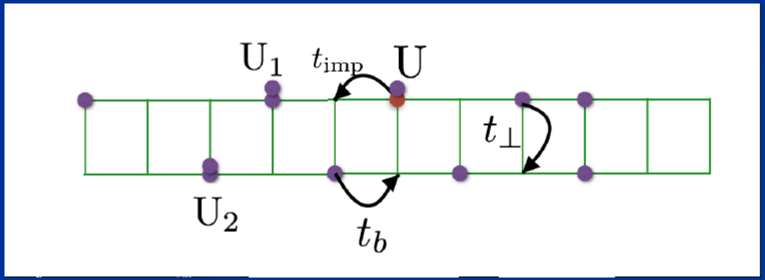
$$f = \frac{L}{K}\sum_{-d\leq x\leq d}\left(\frac{N_\uparrow}{L^2} - \langle \hat{n}_{x\uparrow}\hat{n}_{0\downarrow}\rangle_{N_\uparrow=N, N_\downarrow=1}\right)$$



Towards 2D: ladders



N.A. Kamar, A. Kantian, TG, PRA 100 023614 (2019)



$$H_s = \frac{1}{2\pi}\int dx\left[u_s K_s(\partial_x\theta_s)^2 + \frac{u_s}{K_s}(\partial_x\phi_s)^2\right],$$
$$H_a = \frac{1}{2\pi}\int dx\left[u_a K_a(\partial_x\theta_a)^2 + \frac{u_a}{K_a}(\partial_x\phi_a)^2\right]$$
$$-2\rho_0 t_\perp \int dx\, \cos[\sqrt{2}\theta_a(x)].$$

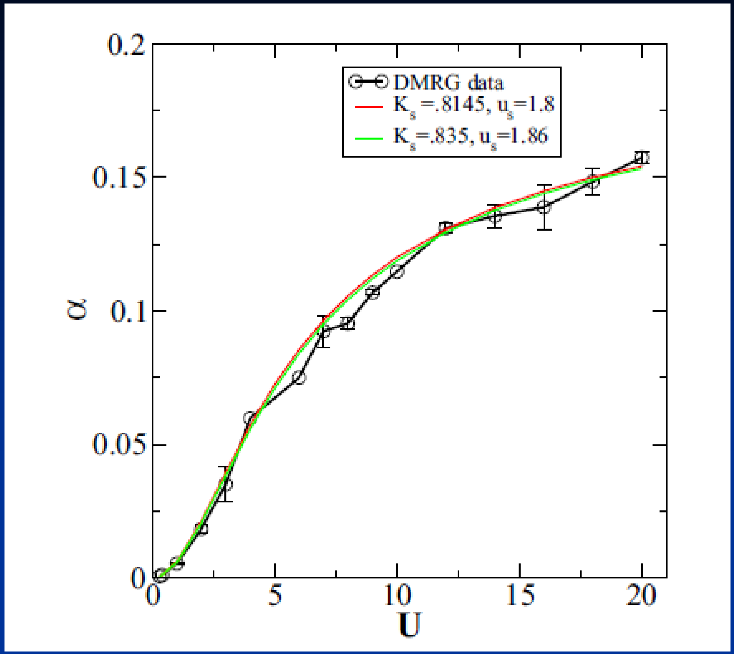
- Mixture of massive and massless modes

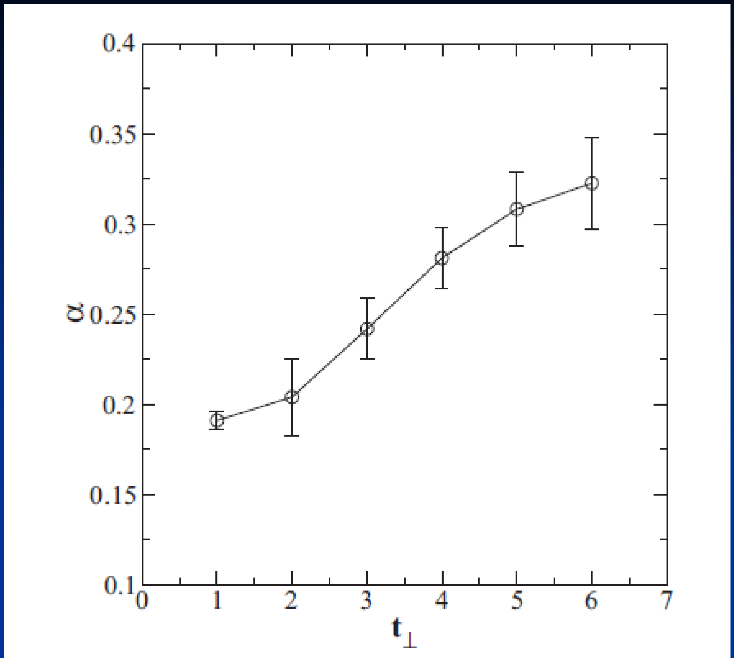
A. $U \ll \Delta_a$

B. $U \gg \Delta_a$

$$|G(p,t)| = e^{-\frac{K_s U^2}{4\pi^2 u_s^2}\left(1+\frac{12 t_{\text{imp}}^2 p^2}{u_s^2}\right)\ln(|t|)}.$$

$$= |t|^{-K_s/4\left(\frac{U_\phi}{\pi u_s}\right)^2}.$$





- Increase of the exponent even if Ks decreases
- Two regimes for impurity as for a single chain
- Hopping of the impurity between the legs: N.A. Kamard Phd thesis (2019) https://archive-ouverte.unige.ch/unige:128219

``Fermionic” type behavior (K <1)

Fermionic quantum bath

$$\rho(x,\tau) = \rho_0 - \frac{1}{\pi}\nabla\phi(x,\tau) + \rho_0 \cos(2\pi\rho_0 x - 2\phi(x,\tau))$$

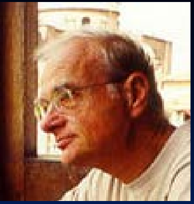
- Impurity coupled to N baths

$$S_{\rm imp} = \int d\tau 1/2 M_0 \dot{X}_\tau^2 - g_0 \sum_{i=1}^{N} \rho_i(X_\tau, \tau),$$

$$S_{\rm TLL} = \sum_{i=1}^{N} \int dx d\tau \frac{1}{2\pi K} [(\partial_\tau \phi_i)^2 + (\partial_x \phi_i)^2]$$

$$\cos[X_\tau - 2\phi_i(X_\tau, \tau)] \rightarrow \cos[X_\tau - 2\phi_i(0,\tau)].$$

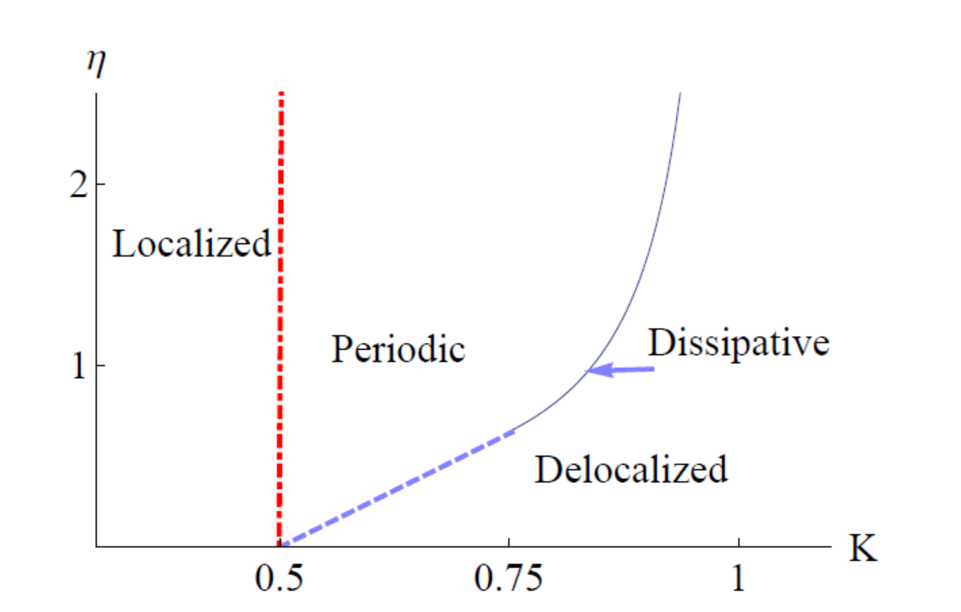
$$\int\int d\tau d\tau' \frac{1}{(\tau - \tau')^{2K}} \cos(X_\tau - X_{tau'})$$



RG analysis, N large

$$\langle \cos X_\tau \rangle \lesssim \langle \cos \tilde{X}_\tau \rangle \sim \left(\frac{4\pi^2 KM}{N\beta} \right)^{K/N} \overset{\beta\to\infty}{\to} 0.$$

B. Horovitz, TG, P. Le Doussal, PRL 111 115302 (2013)

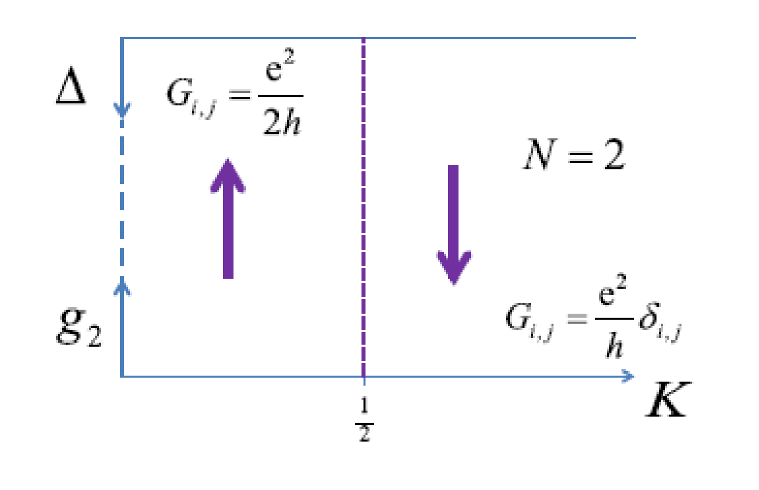


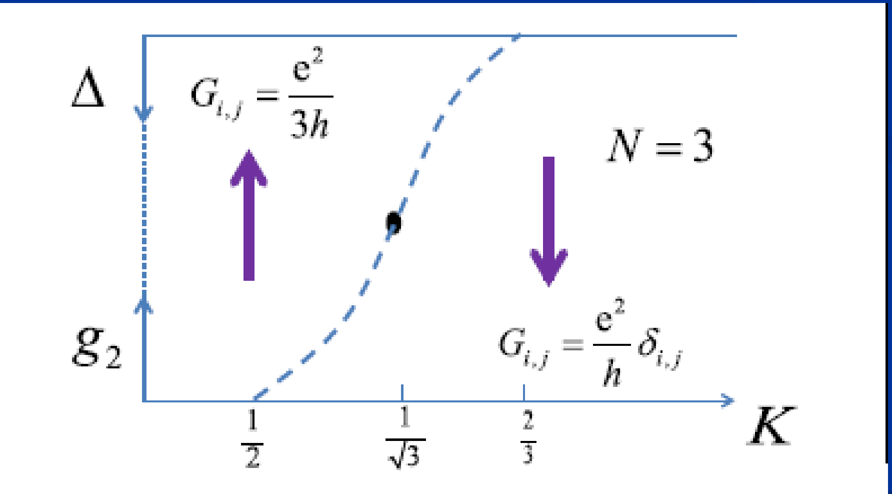
correlation	delocalized	dissipative	periodic	localized
$\langle \cos X_\tau \rangle$	0	0	constant	1
$\langle \cos X_\tau \cos X_0 \rangle$	$\sim \lvert\tau\rvert^{-2K}$	$\sim \lvert\tau\rvert^{-(2-2K)}$	constant	1
$\langle (X_\tau - X_0)^2 \rangle$	$\sim \lvert\tau\rvert$	$\sim \ln \lvert\tau\rvert$		0

N small

- g irrelevant, but coupling g2 between the modes generated

  B. Horovitz, TG, P. Le Doussal, PRL 121 166803 (2018)

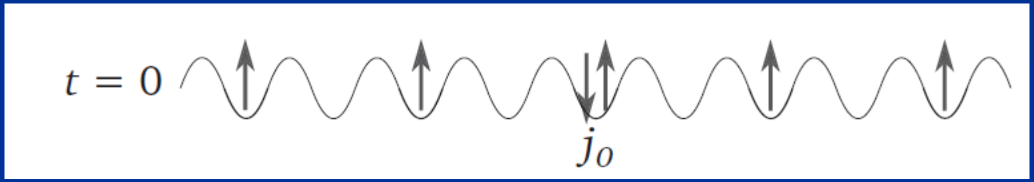


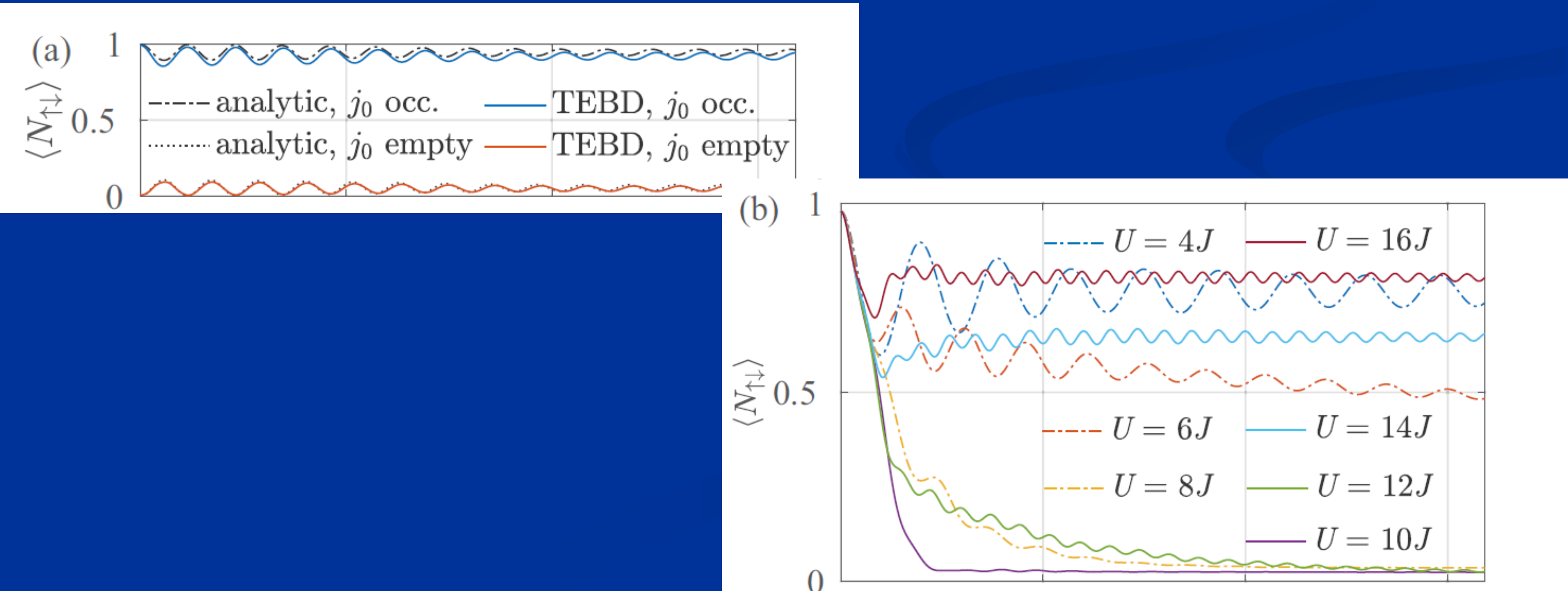


- Coupling between modes: sharp jump in transconductance

Bath with a ``structure" (K small)

A.M. Visuri, P. Torma, TG, PRB 93 125110 (16); PRA 95 063605 (17)





Conclusions and perspectives

- Fascinating problems for impurities in 1D correlated systems
- Many open questions !!
- We need experiments both on the bosonic and the fermionic side