

# Impurity particle driven through a quantum fluid by a constant force

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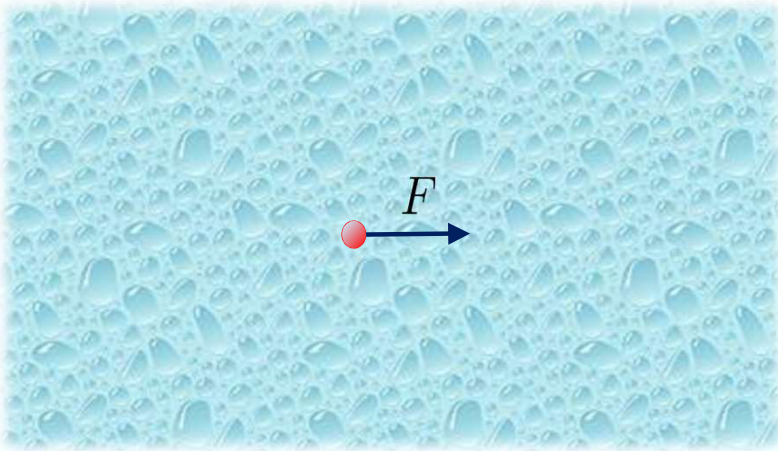
MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY

SEPT 07, 2021 [QUANTUM 2021, AUG 23 – SEPT 17, INSTITUTE PASCAL]

# Plan

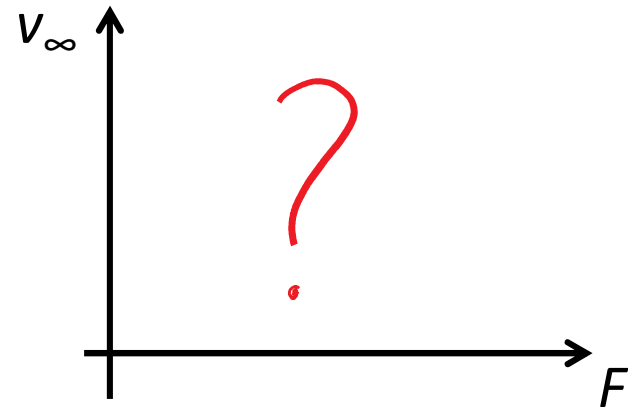
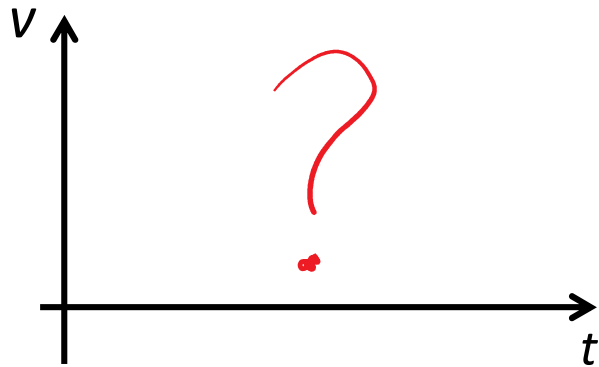
- Setup
- Depletion model
- Kinetic theory
- Comparison
- Cusp controversy
- Outlook

# Impurity particle dragged by a constant force through quantum fluid

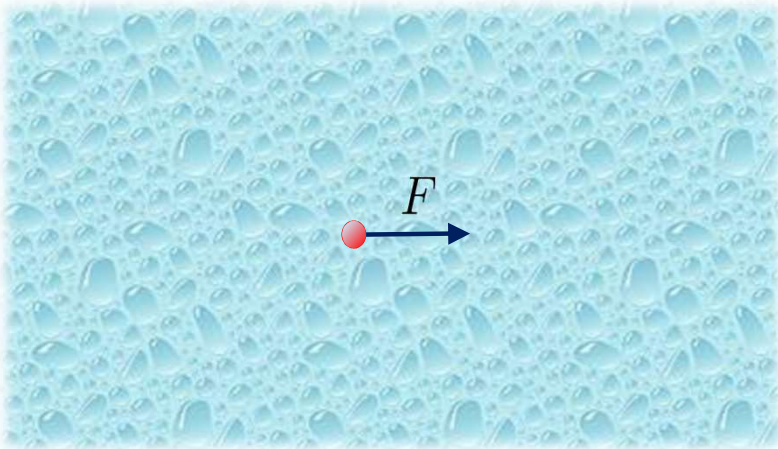


impurity particle of a finite mass  $m$

$$H = H_f + \frac{(P_{\text{imp}} + Ft)^2}{2m} + H_{\text{imp-f}}$$

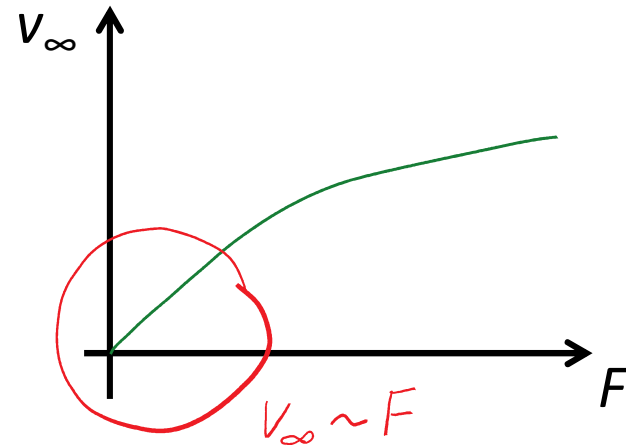
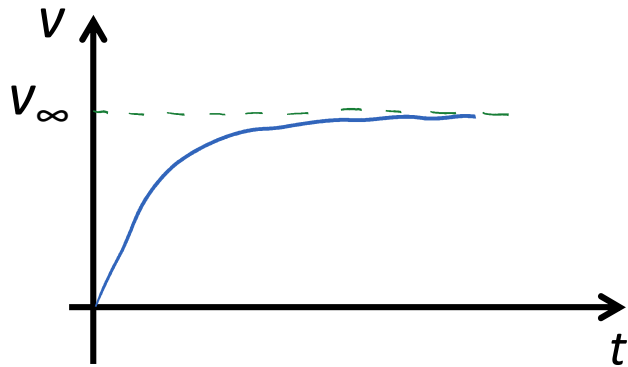


# Impurity particle dragged by a constant force through quantum fluid



impurity particle of a finite mass  $m$

classical liquid/gas, Fermi liquid:



# Fluids with nontrivial spectral edge

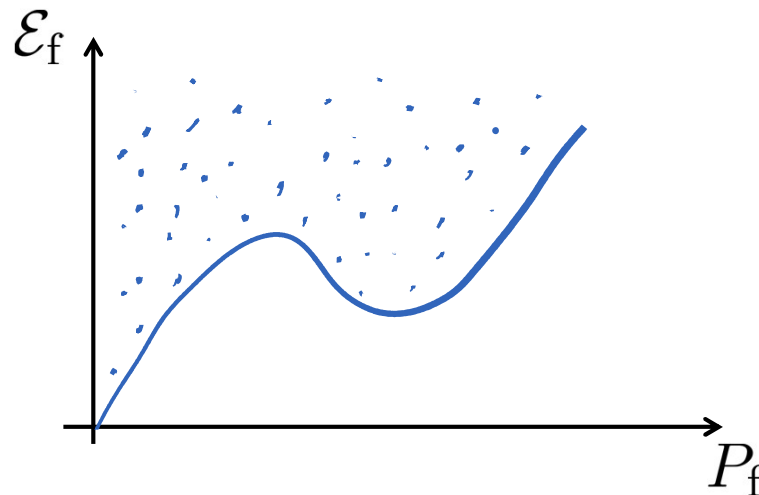
$\mathcal{E}_f$  - eigenenergy of  $H_f$

$$\mathcal{E}_f \geq \varepsilon(P_f)$$

spectral edge of the fluid (*aka* fluid dispersion )

nontrivial spectral edge:

$$\varepsilon(p) \neq \text{const}$$

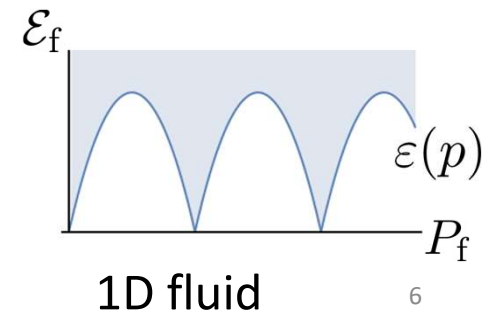
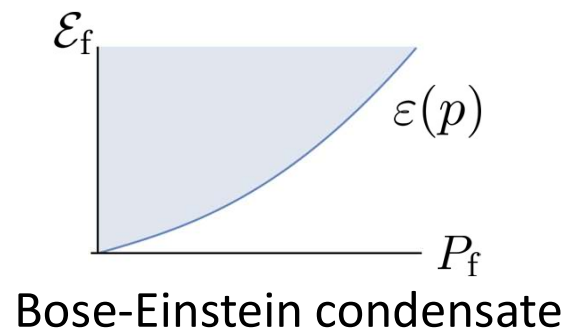
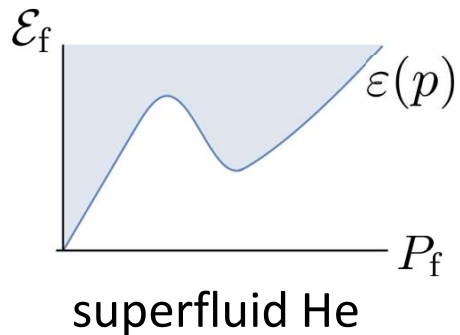


trivial spectral edge:

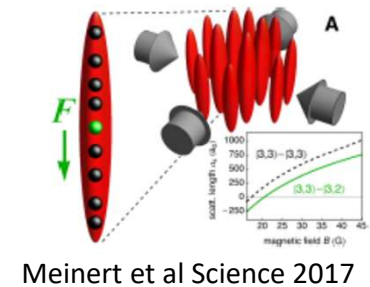
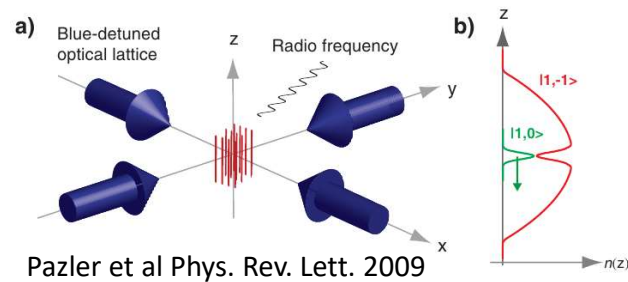
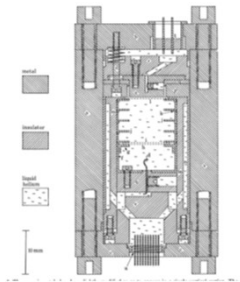
$$\varepsilon(p) = \text{const}$$

(classical gas and liquid,  
Fermi liquid, ...)

# Fluids with nontrivial spectral edge

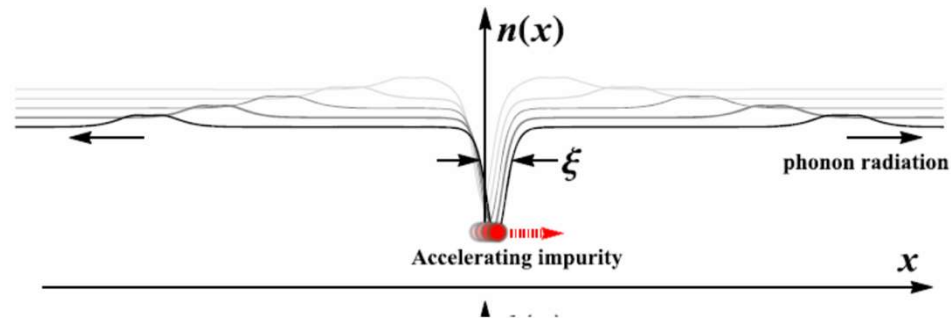


6



Allum, McClintock, Phillips  
Phil. Trans. R. Soc. Lond. A 1977

# Depletton model



D. M. Gangardt and A. Kamenev, Phys. Rev. Lett. **102**, 070402 (2009)

M. Schechter, D. Gangardt, and A. Kamenev, Ann. Phys. **327**, 639 (2012)

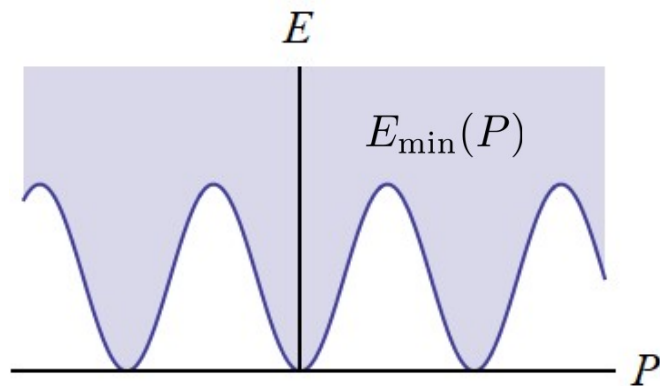
M. Schechter, A. Kamenev, D. M. Gangardt, and A. Lamacraft, Phys. Rev. Lett. **108**, 207001 (2012)

M. Schechter, D. Gangardt, and A. Kamenev, New J. Phys. **18**, 065002 (2016)

A. Campbell, D. Gangardt, SciPost Phys. **3**, 015 (2017)

# Bloch oscillations without a lattice

$E(P)$  – eigenenergy of  $H$  at a fixed  $P$



J. B. McGuire, J. Math. Phys. **6**, 432 (1965)

A. Lamacraft, Phys. Rev. B **79**, 241105(R) (2009)

• driving:  $P = Ft$

• (approximate) adiabatic following :

$$E(P(t)) \simeq E_{\min}(P(t))$$

• Hellmann-Feynman Theorem:

$$v(P) = \partial E / \partial P$$

**Bloch oscillations:**

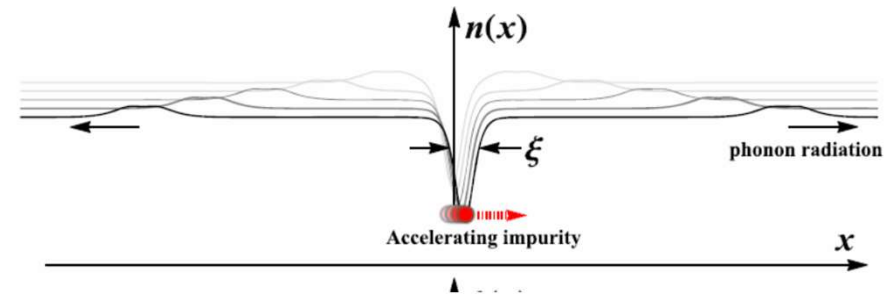
D. M. Gangardt and A. Kamenev, Phys. Rev. Lett. **102**, 070402 (2009)

$$v(t) \simeq \left. \partial E_{\min} / \partial P \right|_{P=Ft}$$



# Depleton model

$E_{\min}(P)$  - dispersion relation of a **depleton**



D. M. Gangardt and A. Kamenev, Phys. Rev. Lett. **102**, 070402 (2009)

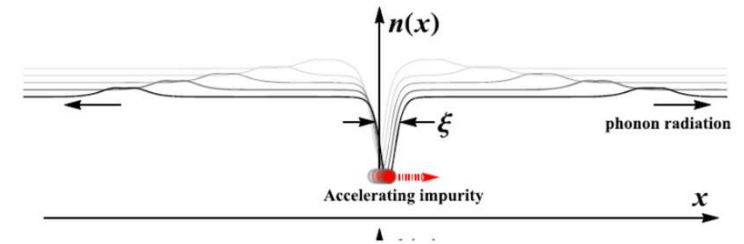
M. Schechter, D. Gangardt, and A. Kamenev, Ann. Phys. **327**, 639 (2012)

M. Schechter, A. Kamenev, D. M. Gangardt, and A. Lamacraft, Phys. Rev. Lett. **108**, 207001 (2012)

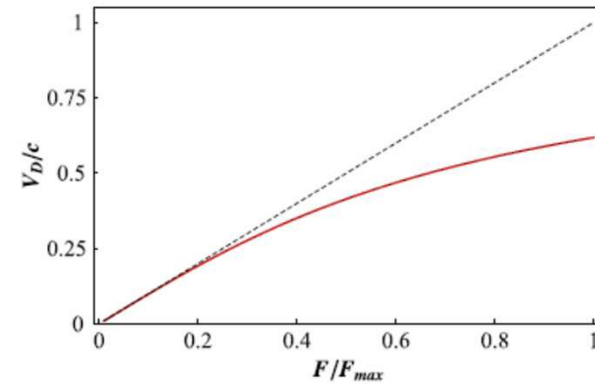
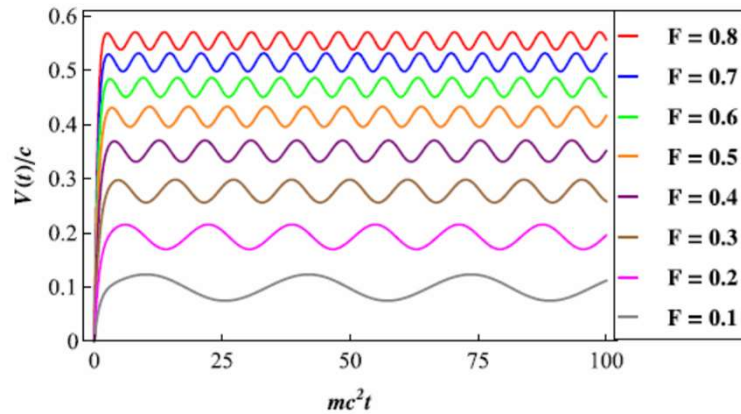
M. Schechter, D. Gangardt, and A. Kamenev, New J. Phys. **18**, 065002 (2016)

A. Campbell, D. Gangardt, SciPost Phys. **3**, 015 (2017)

# Bloch oscillations in depletion model at zero temperature



- Bloch oscillations are **universal** (any 1D fluid, any impurity mass and coupling), provided  $E_{\min}(P)$  is **smooth**
- Bloch oscillations are superimposed on drift velocity  $V_{\infty}$  linear in  $F$  (for small  $F$ )



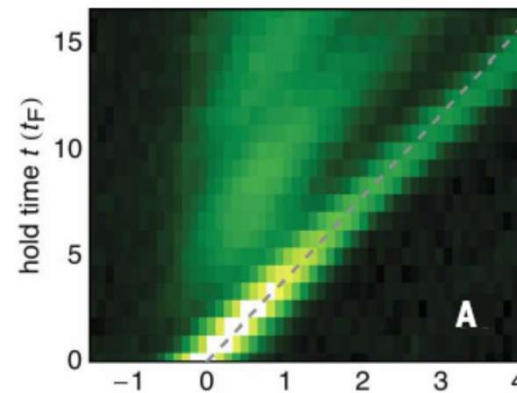
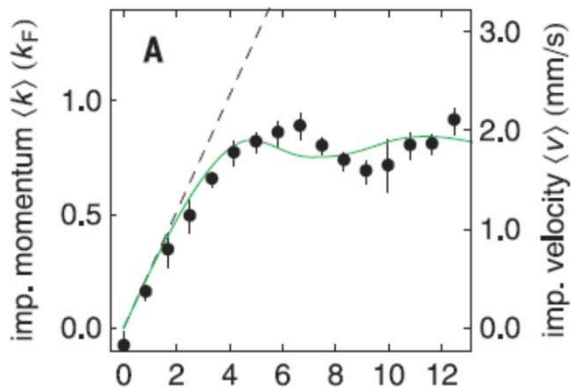
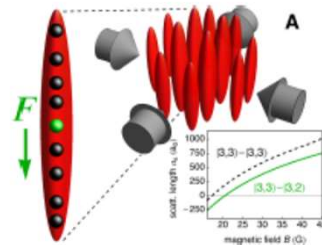
M. Schechter, D. Gangardt, and A. Kamenev, Ann. Phys. **327**, 639 (2012)

# Bloch oscillations without a lattice - observation

Meinert et al., Science 356, 945–948 (2017)

strongly interacting Bose gas

$$m_{imp} = m_{gas}$$



## REPORT

QUANTUM GASES

## Bloch oscillations in the absence of a lattice

Florian Meinert,<sup>1</sup> Michael Knap,<sup>2</sup> Emil Kirilov,<sup>1</sup> Katharina Jag-Lauber,<sup>1</sup> Mikhail B. Zvonarev,<sup>3</sup> Eugene Demler,<sup>4</sup> Hanns-Christoph Nägerl<sup>1\*</sup>

The interplay of strong quantum correlations and far-from-equilibrium conditions can give rise to striking dynamical phenomena. We experimentally investigated the quantum motion of an impurity atom immersed in a strongly interacting one-dimensional Bose liquid and subject to an external force. We found that the momentum distribution of the impurity exhibits characteristic Bragg reflections at the edge of an emergent Brillouin zone. Although Bragg reflections are typically associated with lattice structures, in our strongly correlated quantum liquid they result from the interplay of short-range crystalline order and kinematic constraints on the many-body scattering processes in the one-dimensional system. As a consequence, the impurity exhibits periodic dynamics, reminiscent of Bloch oscillations, although the quantum liquid is translationally invariant. Our observations are supported by large-scale numerical simulations.

# Kinetic theory of mobile impurity in a quantum fluid

GW Rayfield, Phys. Rev. Lett. **16**, 934 (1966)

R. Bowley and F. Sheard, in *Proceedings of the International Conference on Low Temperature Physics*, 1, 165 (1975)

R. Bowley and F. Sheard, Phys. Rev. B **16**, 244 (1977)

D. Allum, P. V. McClintock, and A. Phillips, Philos. Trans. R. Soc. London. Ser. A **284**, 179 (1977)

E. Burovski, V. Cheianov, O. Gamayun, and O. Lychkovskiy, Phys. Rev. A **89**, 041601 (R) (2014)

O. Gamayun, O. Lychkovskiy, and V. Cheianov, Phys. Rev. E **90**, 032132 (2014)

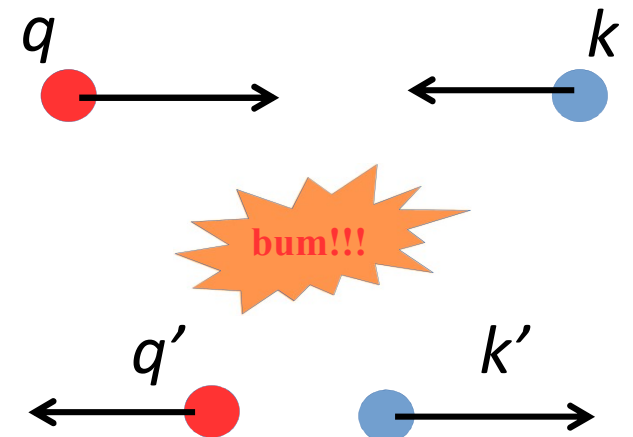
O. Gamayun, Phys. Rev. A **89**, 063627 (2014)

O. Lychkovskiy, Phys. Rev. A **89**, 033619 (2014)

O. Lychkovskiy, Phys. Rev. A **91**, 040101 (R) (2015)

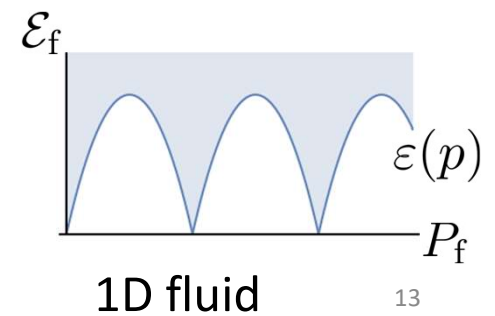
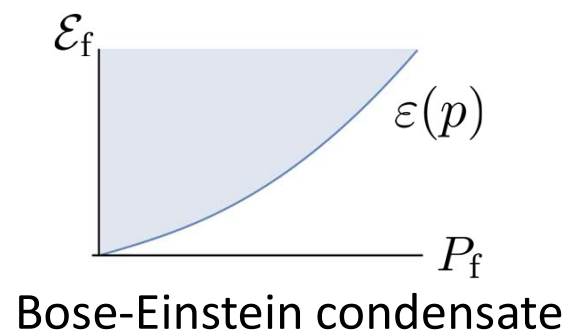
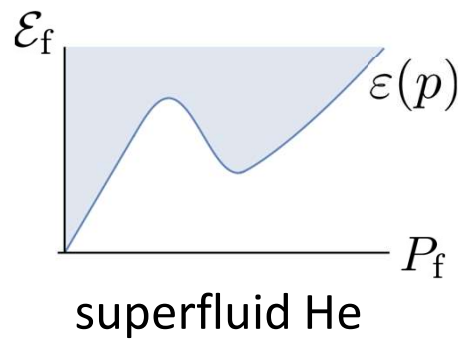
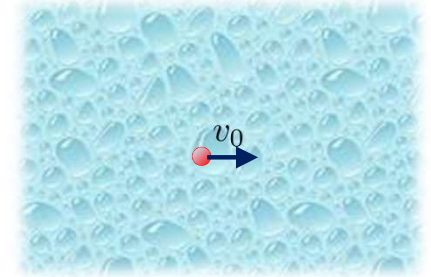
Y. Castin, I. Ferrier-Barbut, and C. Salomon, C. R. Phys. **16**, 241 (2015)

...



# Kinetic theory of mobile impurity in a quantum fluid

$$\begin{cases} m\mathbf{v}_0 = m\mathbf{v} + \mathbf{P}_f \\ \frac{m\mathbf{v}_0^2}{2} = \frac{m\mathbf{v}^2}{2} + \mathcal{E}_f \\ \mathcal{E}_f \geq \varepsilon(\mathbf{P}_f) \end{cases}$$

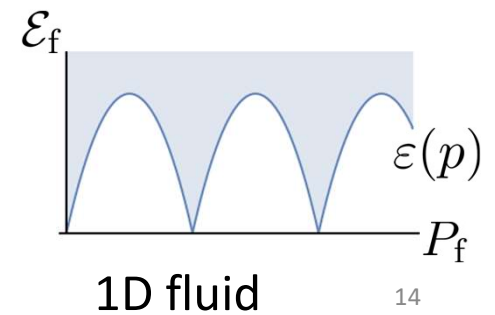
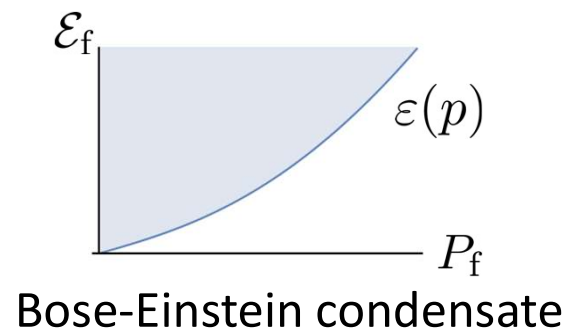
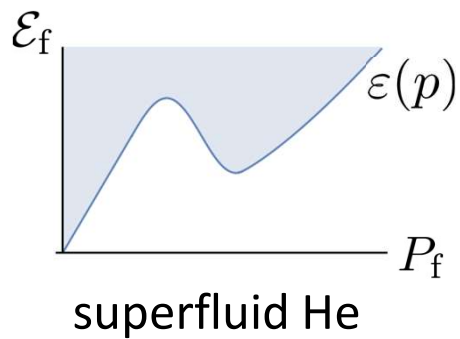
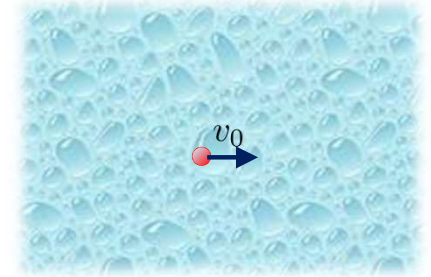


# Perpetual motion of impurity in a quantum fluid

$$\begin{cases} m\mathbf{v}_0 = m\mathbf{v} + \mathbf{P}_f \\ \frac{m\mathbf{v}_0^2}{2} = \frac{m\mathbf{v}^2}{2} + \mathcal{E}_f \end{cases}$$

$$\mathcal{E}_f \geq \varepsilon(\mathbf{P}_f)$$

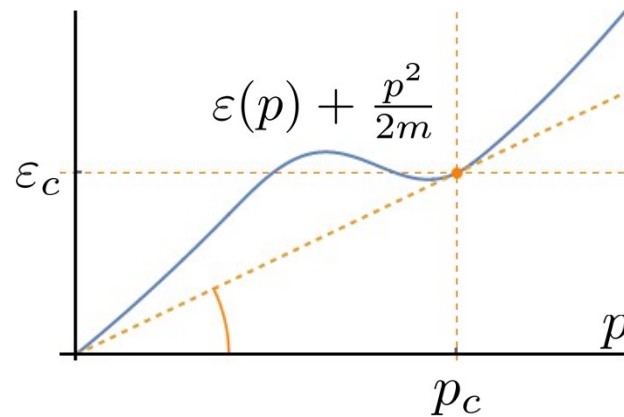
no solutions for  $v_0 = |\mathbf{v}_0| < v_c$   
 $v_c$  - generalized critical velocity



# Generalized critical velocity

$$v_c = \inf_p \frac{\varepsilon(p) + \frac{p^2}{2m}}{p}$$

GW Rayfield, Phys. Rev. Lett. **16**, 934 (1966)



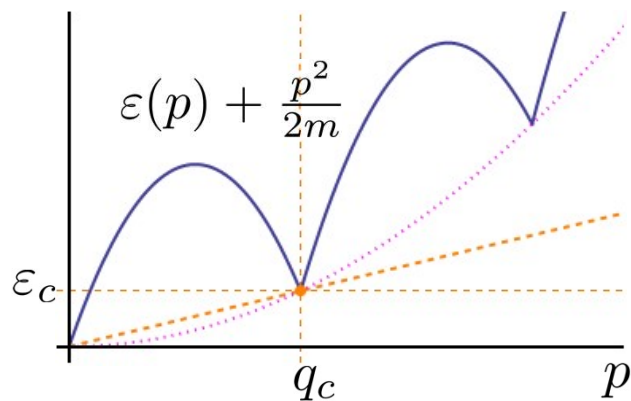
$$v_c = \varepsilon_c / p_c$$

Landau critical velocity (Landau, 1941):  $m=\infty$        $v_c > v_L$

# Critical velocity in one dimension

Generalized critical velocity:

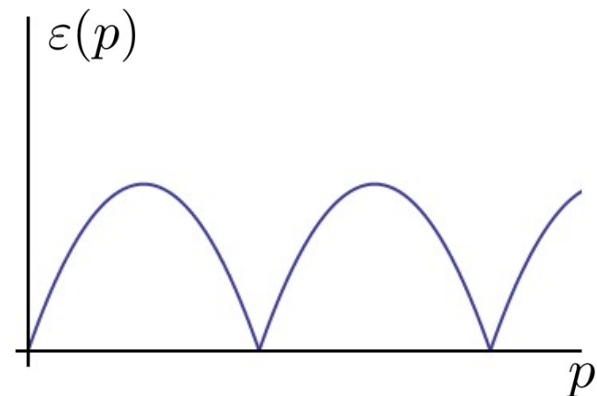
$$v_c = \inf_p \frac{\varepsilon(p) + \frac{p^2}{2m}}{p}$$



$$v_c \neq 0$$

Landau critical velocity:

$$v_L = \inf_p \frac{\varepsilon(p)}{p}$$

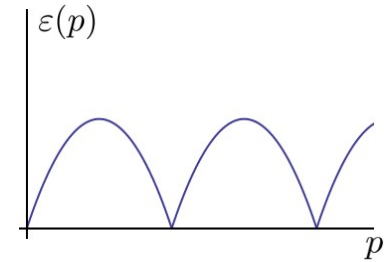


$$v_L = 0$$

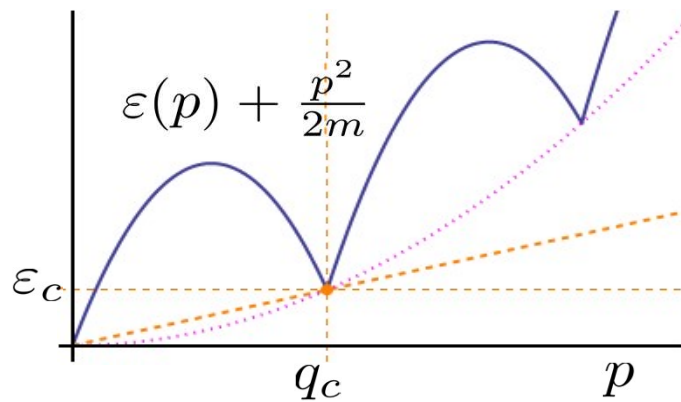


# Critical mass

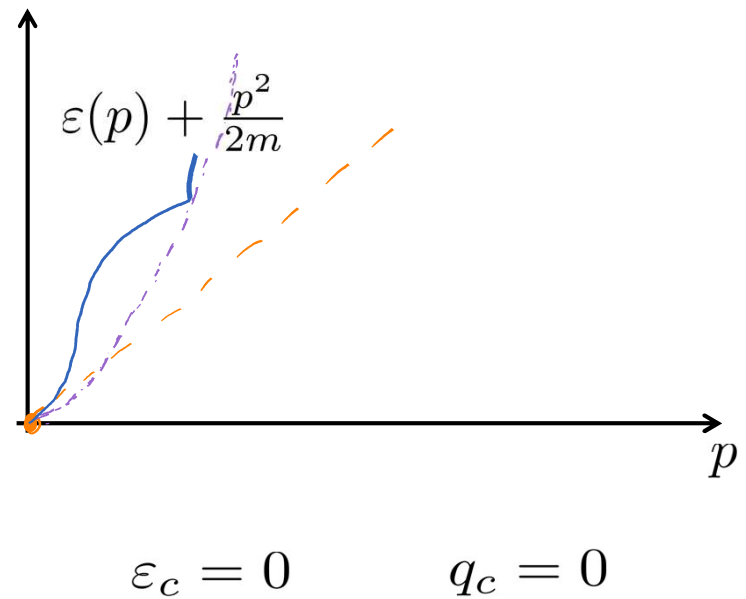
$$v_c = \inf_p \frac{\varepsilon(p) + \frac{p^2}{2m}}{p}$$



heavy impurity,  $m > m_c$

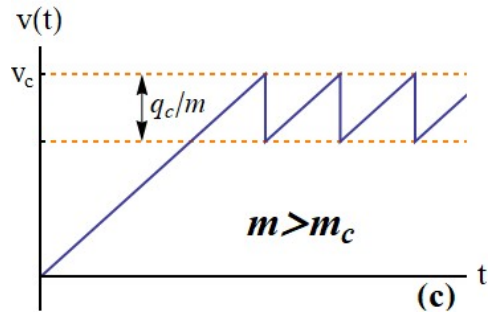
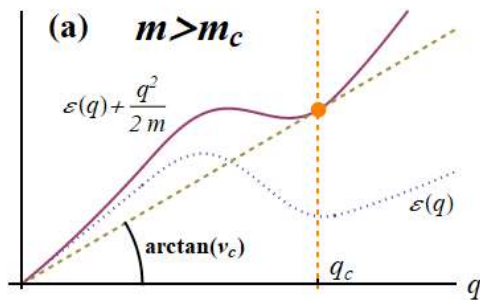


light impurity,  $m > m_c$



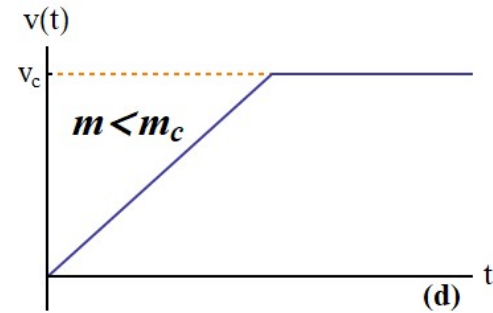
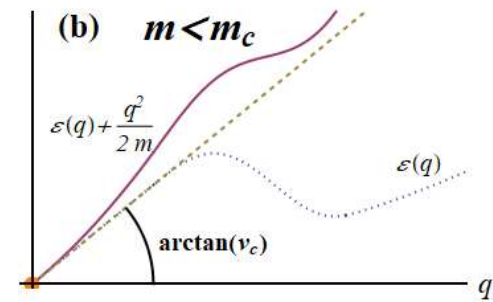
# Impurity dragged by a constant force: two regimes

heavy impurity



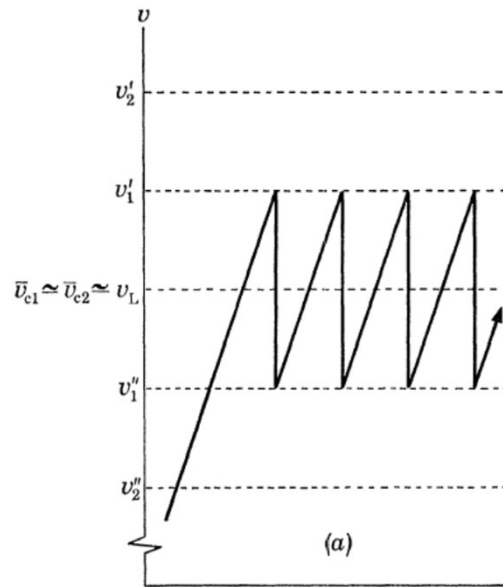
oscillations

light impurity

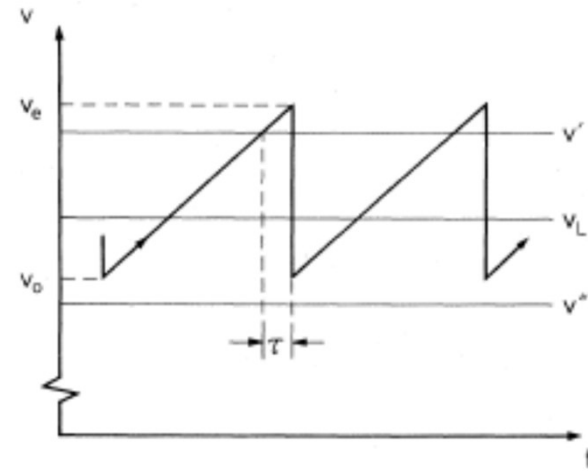


saturation without oscillations

# Ions accelerated by electric field in in superfluid He



D. Allum, P. V. McClintock, and A. Phillips, *Philos. Trans. R. Soc. London. Ser. A* **284**, 179 (1977).



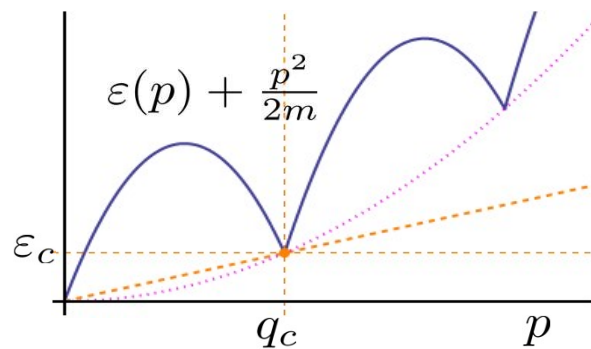
R. Bowley and F. Sheard, *Phys. Rev. B* **16**, 244 (1977).

# Impurity dragged through 1D gas of noninteracting fermions

$$H = \sum_{j=1}^N \frac{P_j^2}{2m_h} + \frac{P_{\text{imp}}^2}{2m} + \frac{\gamma\rho}{m} \sum_{j=1}^N \delta(x_j - x_{\text{imp}})$$

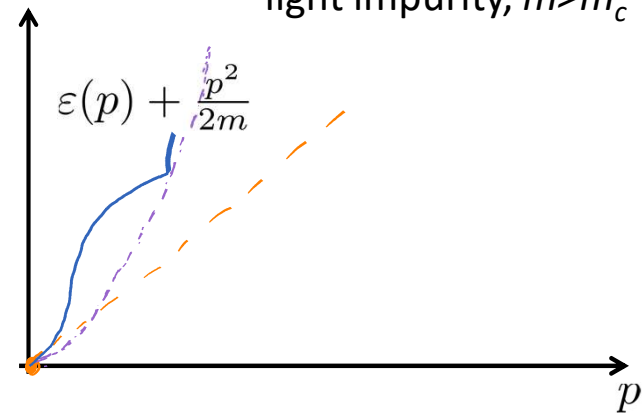
$$m_c = m_h$$

heavy impurity,  $m > m_c$



$$q_c = k_F$$

light impurity,  $m < m_c$

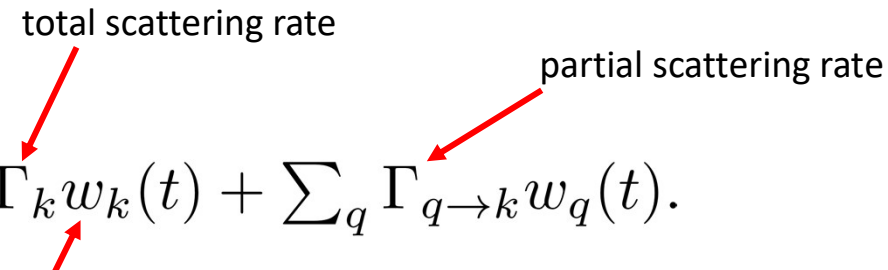


$$q_c = 0$$

Impurity dragged through 1D gas of noninteracting fermions

$$H = \sum_{j=1}^N \frac{P_j^2}{2m_h} + \frac{P_{\text{imp}}^2}{2m} + \frac{\gamma\rho}{m} \sum_{j=1}^N \delta(x_j - x_{\text{imp}})$$

Boltzmann equation:

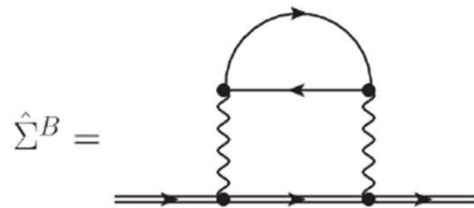
$$\frac{\partial w_k(t)}{\partial t} + F \frac{\partial w_k(t)}{\partial k} = -\Gamma_k w_k(t) + \sum_q \Gamma_{q \rightarrow k} w_q(t).$$


momentum distribution of impurity

# Quantum Boltzmann Equation (QBE)

Quantum kinetic equations  $[G]^{-1} = [G^{(0)}]^{-1} - \hat{\Sigma}$

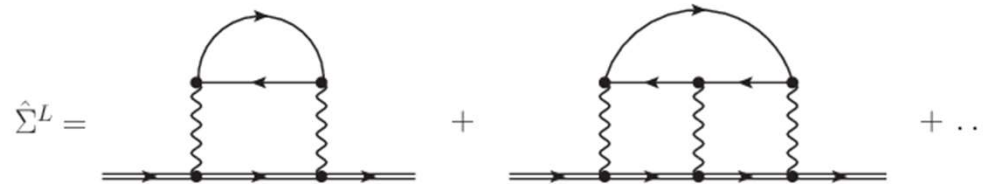
Pure Boltzmann



$$n_p^\infty = Z_{p_0} \frac{\theta(k_F - |p|)}{p_0 - p}, \quad p_0 > k_F.$$

$$p_\infty^B = p_0 - \theta(|p_0| - k_F) \frac{2k_F}{\ln \frac{p_0 + k_F}{p_0 - k_F}}$$

Multiple Scattering Events



$\gamma \rightarrow \frac{\gamma}{|\eta - 1|} \xrightarrow{\eta \rightarrow 1, \gamma \rightarrow 0}$

$$n_p^\infty = \tilde{Z}_{p_0} \frac{\theta(k_F - |p|)}{(p_0 - p)^2}, \quad p_0 > k_F.$$

$$p_\infty = p_0 - \theta(p_0^2 - k_F^2) \frac{p_0^2 - k_F^2}{2k_F} \ln \frac{p_0 + k_F}{p_0 - k_F}$$

O. Gamayun, Phys. Rev. A 89, 033619 (2014)

## Boltzmann equation – validity conditions

$\gamma \ll 1$                       weak coupling

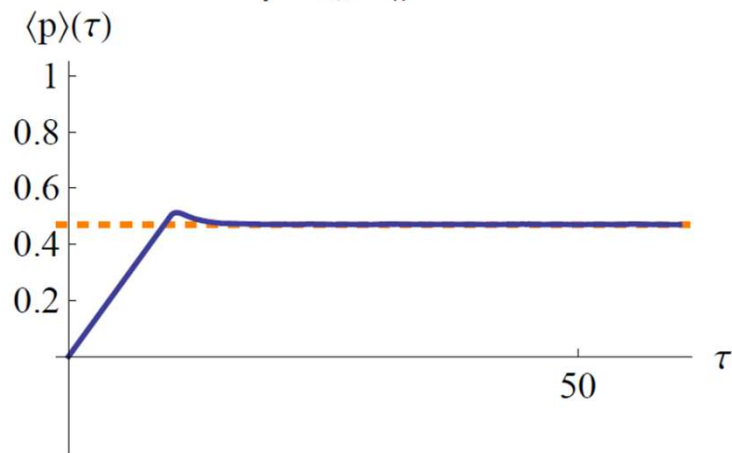
$$F \gg \left( \frac{\gamma^2 k_F^3}{2\pi^3 m_c} \right) e^{-\frac{\pi^4}{\gamma^2} \frac{m^2 - m_c^2}{m^2}}, \quad m > m_c$$

$$F \gg \left( \frac{\gamma^2 k_F^3}{2\pi^3 m_c} \right) \gamma^2 (m/m_c), \quad m < m_c$$

# Boltzmann equation: results

light impurity

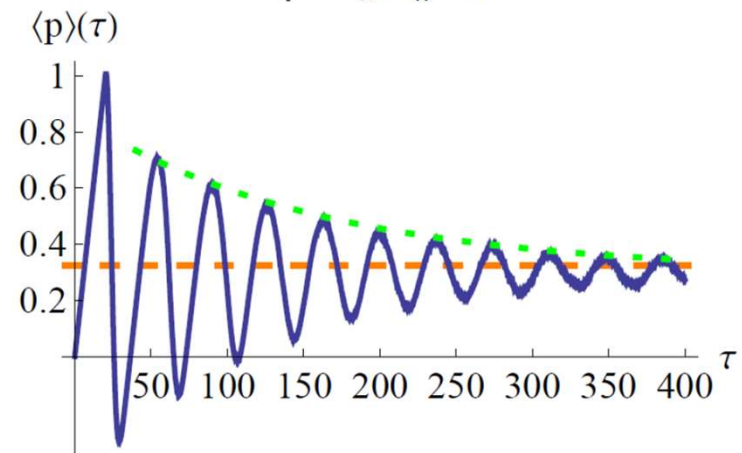
$$\eta = m_i/m_h = 0.5$$



saturation without oscillations

heavy impurity

$$\eta = m_i/m_h = 2$$

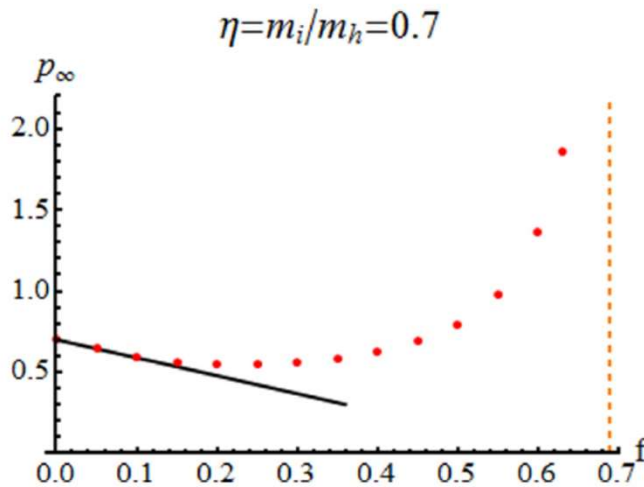


oscillations



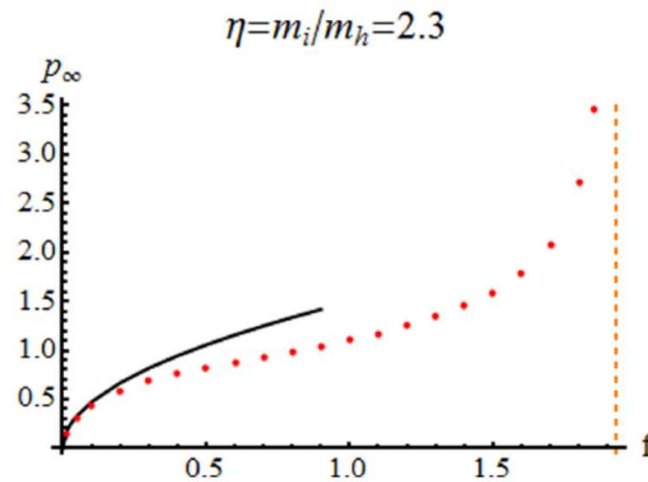
# Impurity dragged through 1D gas of noninteracting Fermions

light impurity



$$v_\infty = v_c - \zeta F$$

heavy impurity



$$v_\infty \sim \sqrt{F}$$

# Depleton model vs kinetic theory

## depleton model

universal oscillations:

any 1D fluid, any impurity mass and coupling

$$v_{\infty} \sim F$$

nonperturbative effective theory

## kinetic theory

$$m > m_c$$

oscillations

$$v_{\infty} \sim \sqrt{F}$$

perturbative

$$m < m_c$$

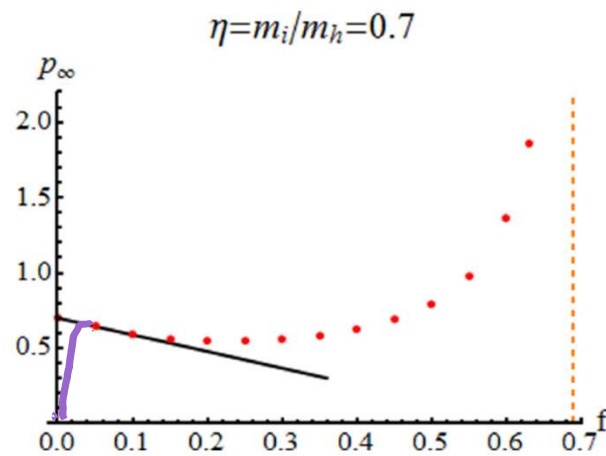
saturation without oscillations for

$$v_{\infty} = v_c - \zeta F$$

## Depletions model vs kinetic theory

Formally, two approaches can be reconciled if kinetic theory is grossly inadequate for  $F < F_c$

However, such reconciliation could be quite a bizarre, particularly for light impurities:



# Depleton model vs kinetic theory: nonperturbative effects

PHYSICAL REVIEW E 92, 016101 (2015)

Comment on “Kinetic theory for a mobile impurity in a degenerate Tonks-Girardeau gas”

Michael Schechter,<sup>1</sup> Dimitri M. Gangardt,<sup>2</sup> and Alex Kamenev<sup>1,3</sup>

perturbative treatment can miss binding  
between the hole and the impurity

$$\Delta \sim e^{-2\pi^2/\gamma}$$

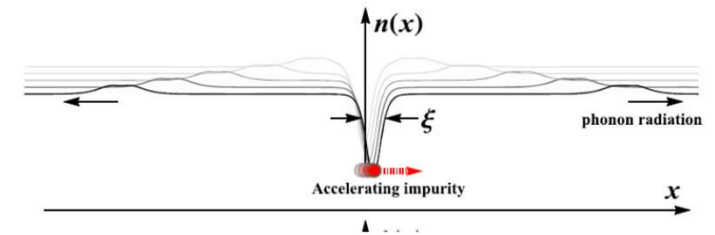
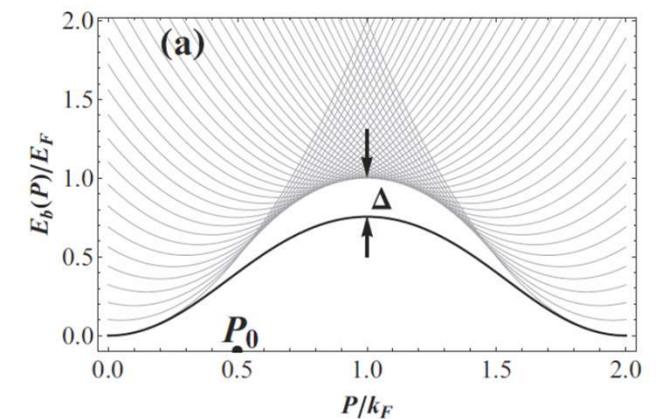
PHYSICAL REVIEW E 92, 016102 (2015)

Reply to “Comment on ‘Kinetic theory for a mobile impurity  
in a degenerate Tonks-Girardeau gas’”

O. Gamayun,<sup>1,2,3</sup> O. Lychkovskiy,<sup>4</sup> and V. Cheianov<sup>1,3</sup>

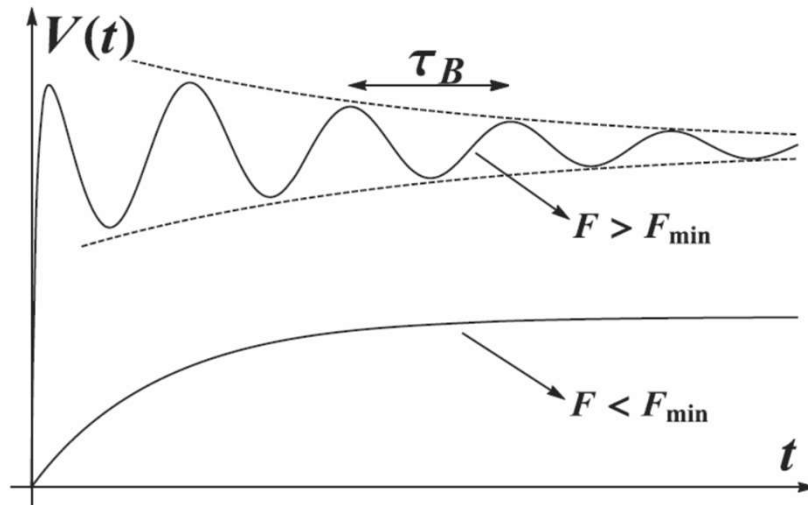
binding and related oscillations can be destroyed by  
quantum fluctuations

$$\delta E_4 = O(\gamma^4) \gg e^{-2\pi^2/\gamma}$$



## Depleton model: finite temperature

At least, **thermal** fluctuations do destroy the oscillations in the depletion model:

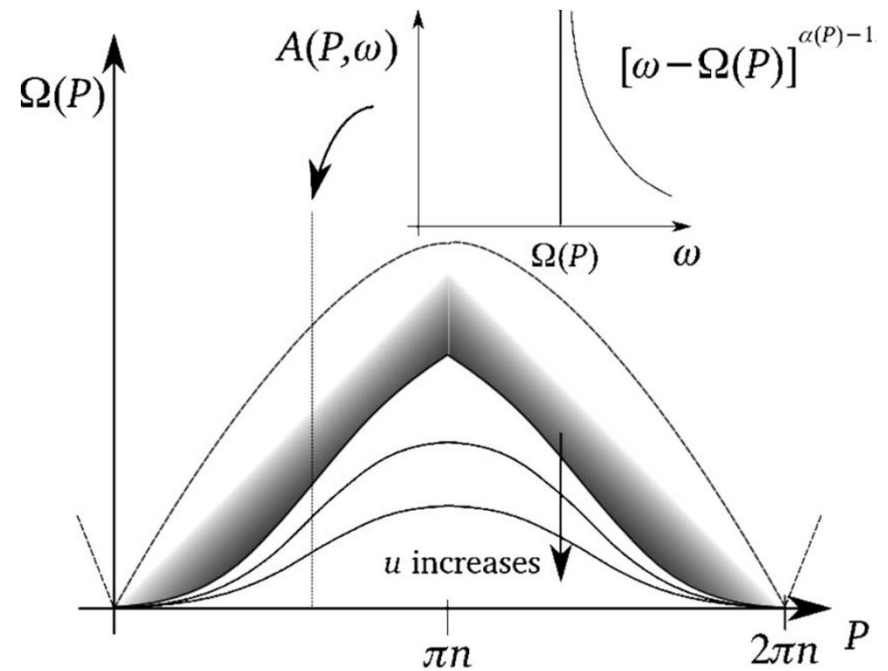


**Figure 4.** Schematic noise-averaged velocity as a function of time including the effects of fluctuations. For  $F < F_{\min}$  the impurity velocity saturates below the critical velocity and Bloch oscillations do not occur. For  $F_{\min} < F < F_{\max}$  Bloch oscillations occur, but are attenuated in time due to dephasing, see equation (24).

# Cusps in the impurity-fluid dispersion

A. Lamacraft, Phys. Rev. B **79**, 241105(R) (2009):

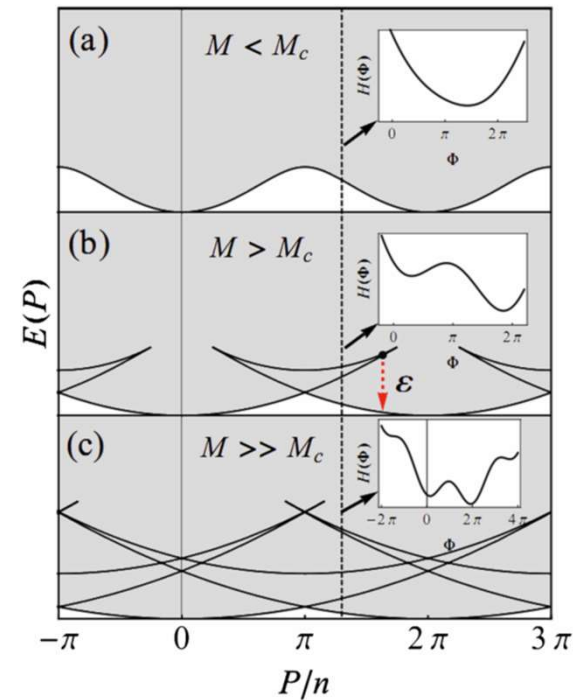
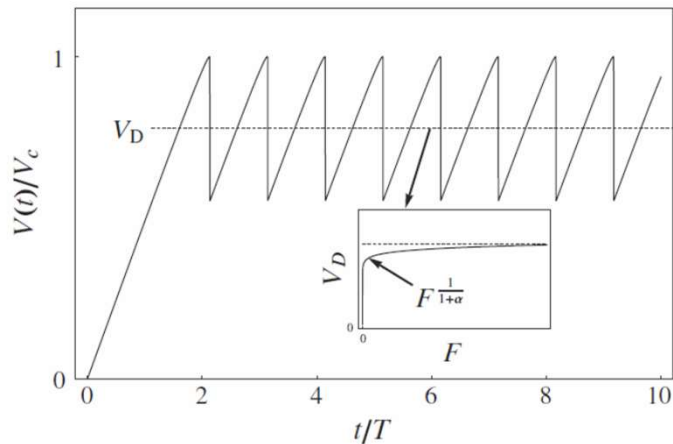
$E_{\min}(P)$  has a cusp at  $P=\pi n$   
for fluids with  $K < 1$  (bosons)  
and for  $\gamma < \gamma_c$   
impurity-fluid coupling



# Cusps in the impurity-fluid dispersion

impurity fails to follow  $E_{\min}(P)$  adiabatically

$V_{\infty}$  nonlinear in  $F$



M. Schechter, A. Kamenev, D. M. Gangardt, and A. Lamacraft, Phys. Rev. Lett. **108**, 207001 (2012)

## Cusps in the impurity-fluid dispersion: integrable model test

A. Lamacraft, Phys. Rev. B **79**, 241105(R) (2009):

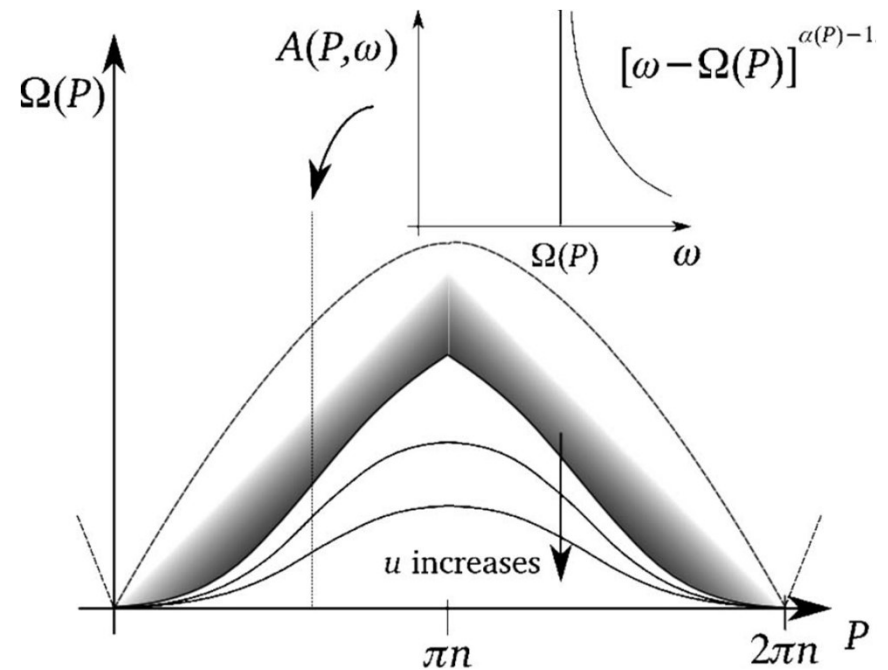
$E_{\min}(P)$  has a cusp at  $P=\pi n$

for fluids with  $K < 1$  (bosons)

and for  $\gamma < \gamma_c$

Integrable bosonic Yang-Gaudin model:

**no cusp for any  $\gamma$**





## Cusp in the McGuire model with attraction

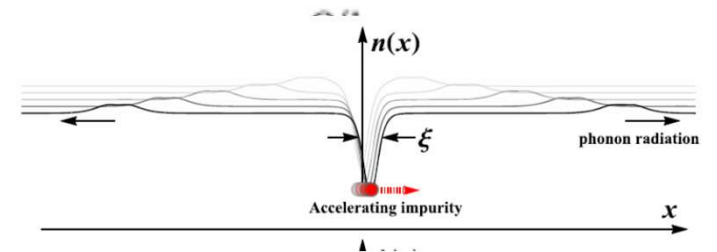
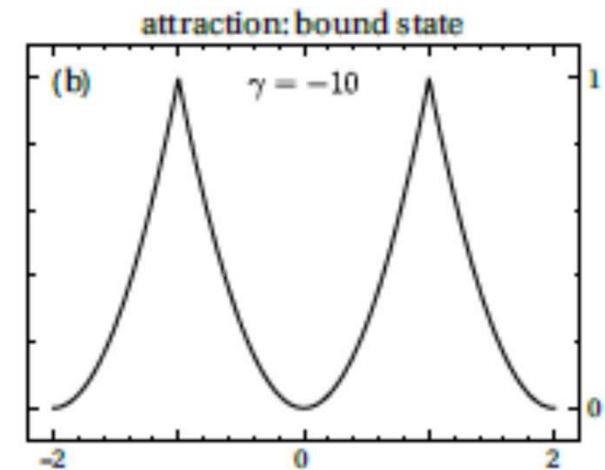
$$H = \sum_{j=1}^N \frac{P_j^2}{2m} + \frac{P_{\text{imp}}^2}{2m} + \frac{\gamma\rho}{m} \sum_{j=1}^N \delta(x_j - x_{\text{imp}})$$

$$\gamma < 0$$

Is this cusp there beyond integrability?

kinetic theory: not sensitive to the sign of  $\gamma$

depleton model: no impurity-hole binding any more



# Summary and outlook

- reconciling kinetic theory and depletion model is nontrivial
- more studies are welcome (numerics, adiabatic perturbation theory, ..., experiments?)
- non-linear-response theory is, in general, lacking
- attractive impurity-fluid interaction: little studied, potentially very interesting

Thank you for your attention!

# Funding

2016 -2017:

Quantum dynamics of a mobile impurity in a medium

Russian Foundation for Basic Research grant № 16-32-00669

2018-2020

Far-from-equilibrium dynamics in quantum impurity models

Russian Foundation for Basic Research grant № 18-32-20218

# Footnote: no many-body adiabaticity at finite $F$

necessary adiabatic condition:

$$F \sim \log L$$

O. Lychkovskiy, O. Gamayun, V. Cheianov  
PRL119, 200401 (2017);  
AIP Conf. Proc. 1936, 020024 (2018)

sufficient adiabatic condition:

$$F \sim L$$

(scaling of energy gap)

necessary and sufficient adiabatic condition:

$$F \sim L$$

O. Lychkovskiy, O. Gamayun, V. Cheianov  
Phys. Rev. B 98, 024307 (2018)

Thank you for your attention!

# Miscellaneous slides

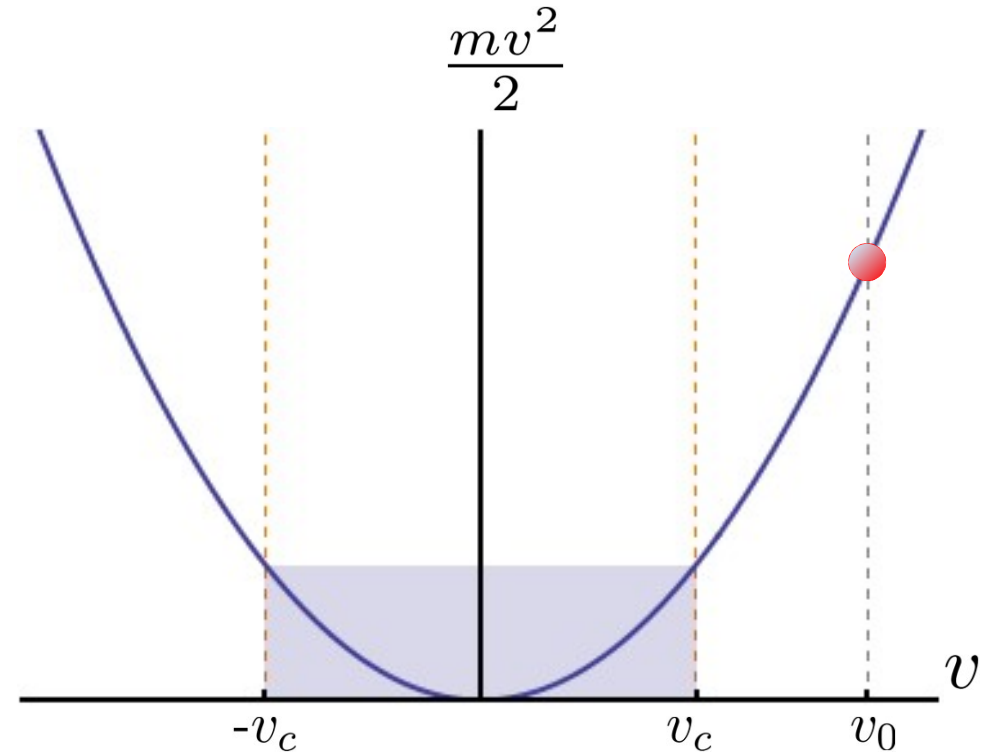
# Impurity dynamics for $v_0 > v_c$ : semiclassical picture

weak impurity-fluid coupling

$$\begin{cases} m\mathbf{v}_0 = m\mathbf{v} + \mathbf{P}_f \\ \frac{m\mathbf{v}_0^2}{2} = \frac{m\mathbf{v}^2}{2} + \mathcal{E}_f \\ \mathcal{E}_f \leq \varepsilon(\mathbf{P}_f) \end{cases}$$

$v_0 < v_c$ : scattering forbidden,  
 $v_\infty \simeq v_0$

$v_0 > v_c$ : scattering allowed





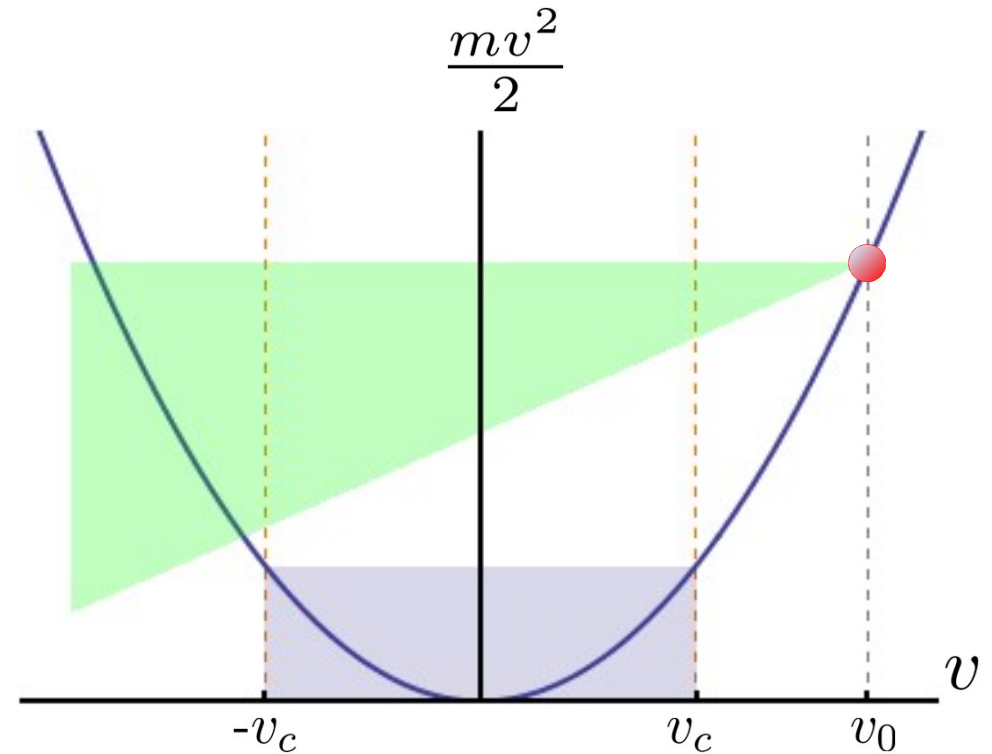
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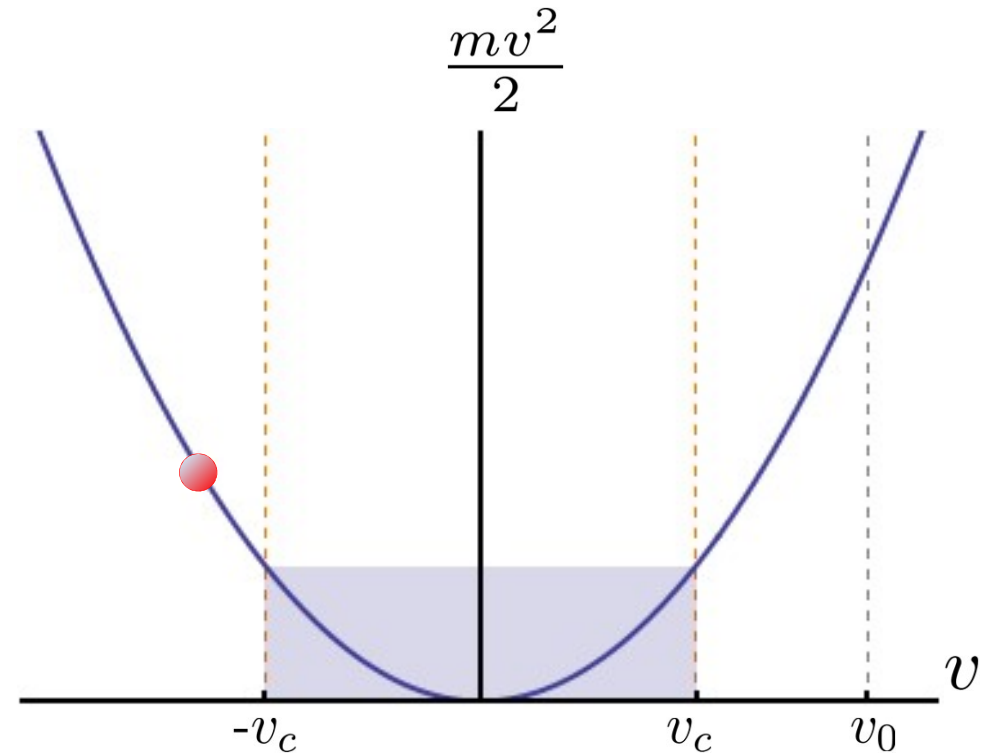
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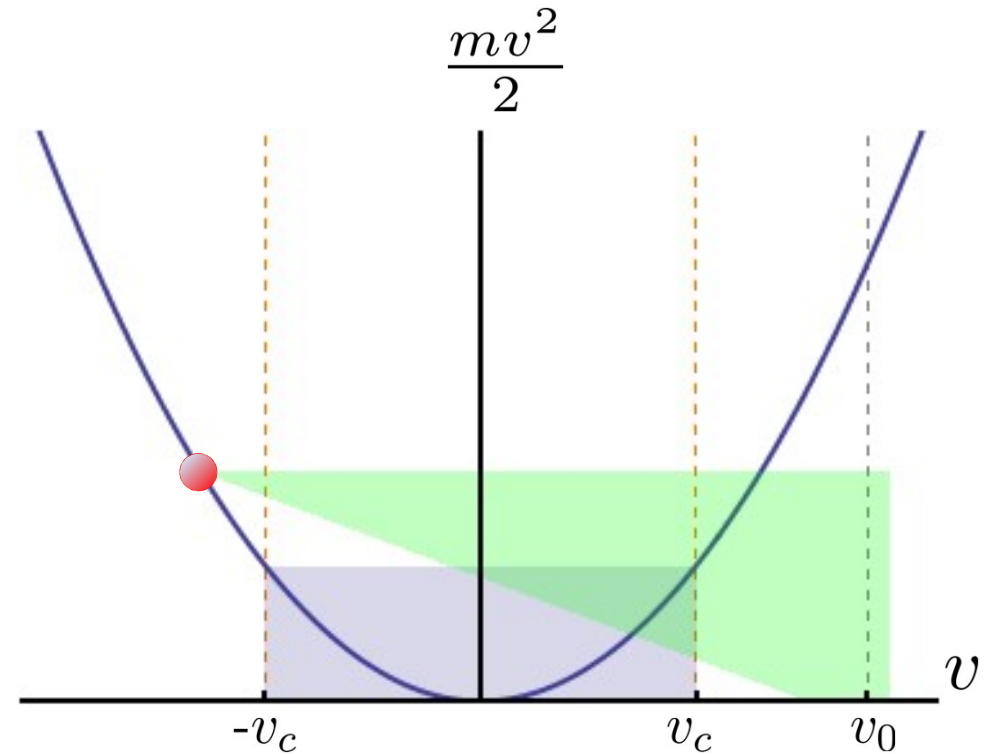
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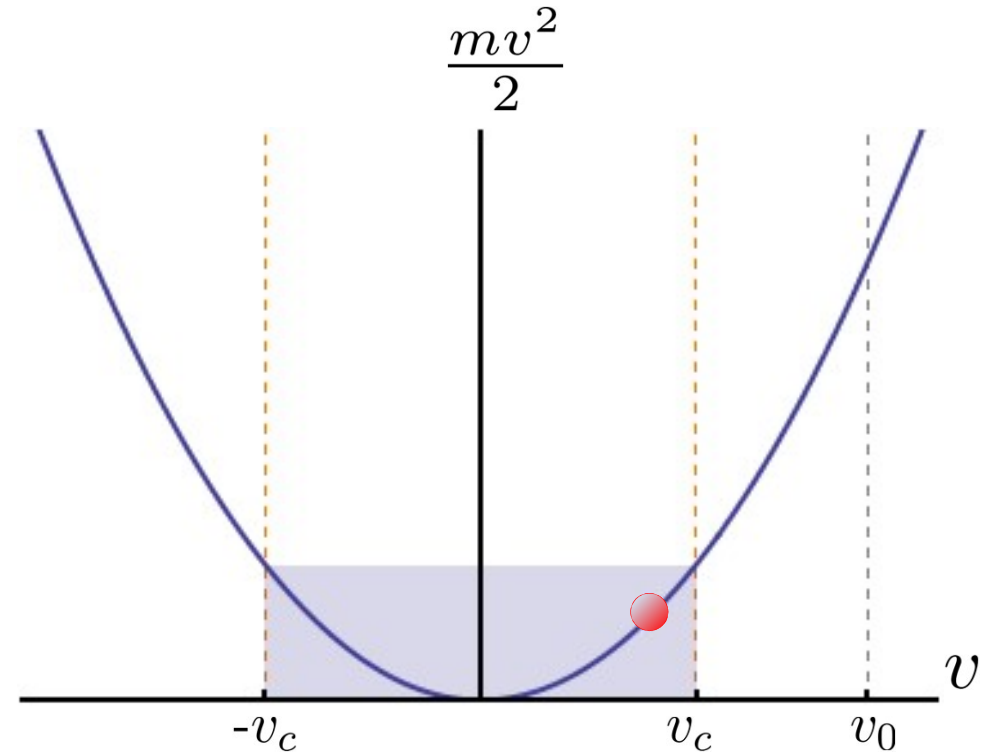
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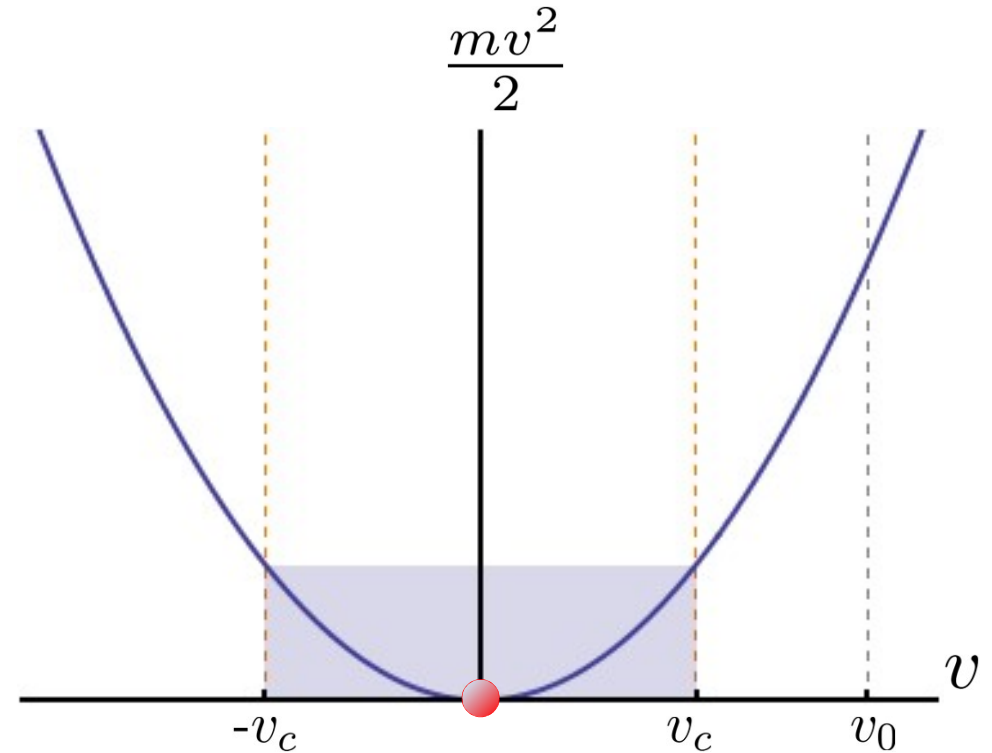
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# Impurity dynamics: weak coupling

Boltzmann equation:

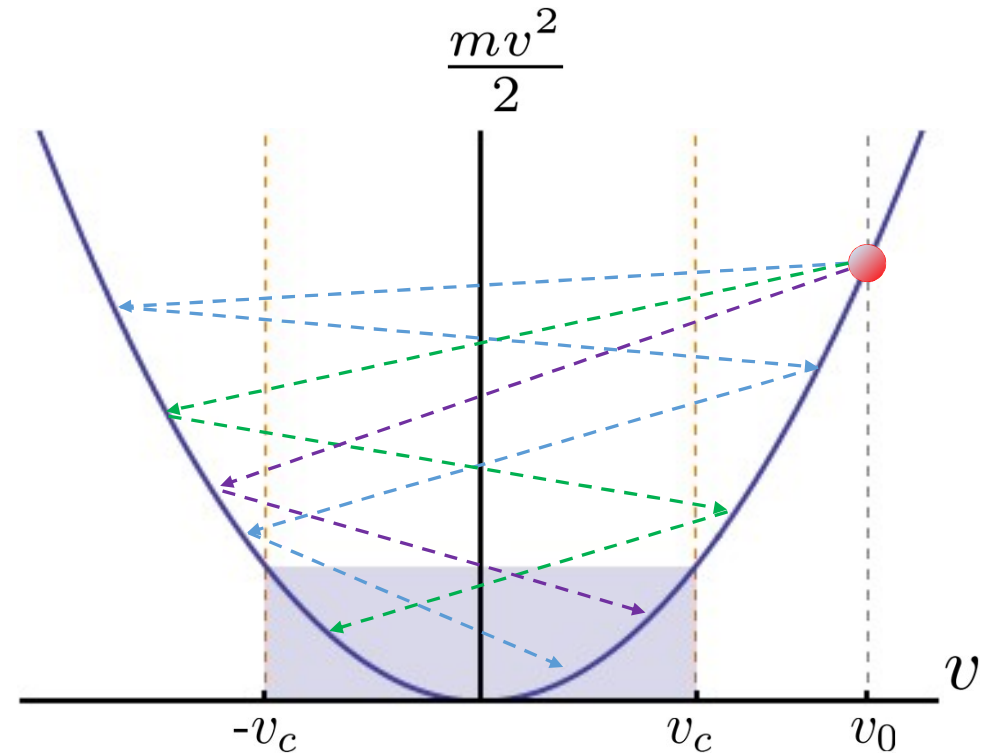
$$\frac{\partial n_k}{\partial t} = -n_k \sum_q \Gamma_{k \rightarrow q} + \sum_q n_q \Gamma_{q \rightarrow k}$$

$\Gamma_{k \rightarrow q}$  - scattering probability

works even in 1D, with reservations

E. Burovski, V. Cheianov, O. Gamayun and OL,  
Phys. Rev. A 89, 041601 (R) (2014)

O. Gamayun, Phys. Rev. A 89, 063627 (2014)



Footnote: generalized critical velocity beyond kinetic theory

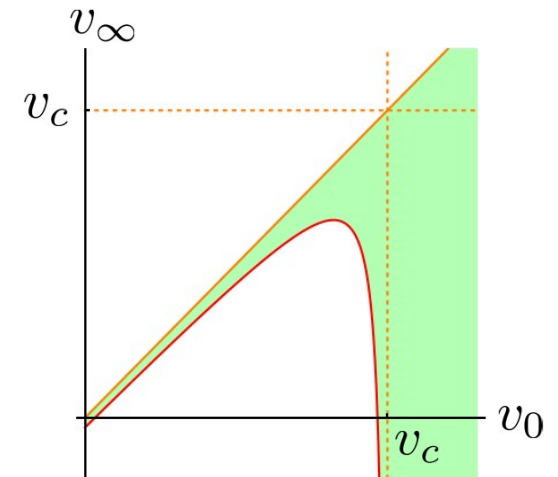
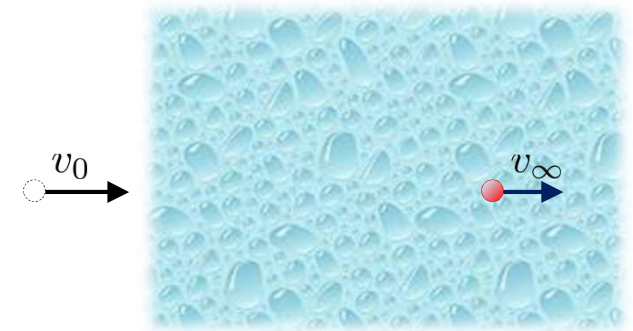
$$H = H_f + \frac{\mathbf{p}_{\text{imp}}^2}{2m} + \sum_{j=1}^N U(\mathbf{r}_j - \mathbf{r}_{\text{imp}}) \quad \text{impurity-fluid interaction}$$

rigorous bound:

$$|\mathbf{v}_0 - \mathbf{v}_\infty| \leq \frac{\bar{U}}{m(v_c - v_0)}$$

valid for  $v_0 < v_c$ ,  $U(\mathbf{r}) \geq 0$

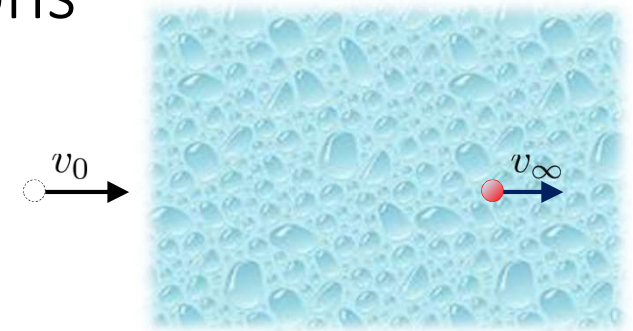
$$\bar{U} = \int d\mathbf{r} \rho U(\mathbf{r}), \quad \rho = N/V - \text{fluid density}$$



O. Lychkovskiy, Phys. Rev. A 91, 040101 (R) (2015)

O. Lychkovskiy, Phys. Rev. A 89, 033619 (2014)

# Accounting for impurity-fluid interactions



$$m\mathbf{v}_0 = m\mathbf{v} + \mathbf{p}$$

$$\frac{m\mathbf{v}_0^2}{2} = \frac{m\mathbf{v}^2}{2} + \varepsilon(\mathbf{p})$$

kinetic energy

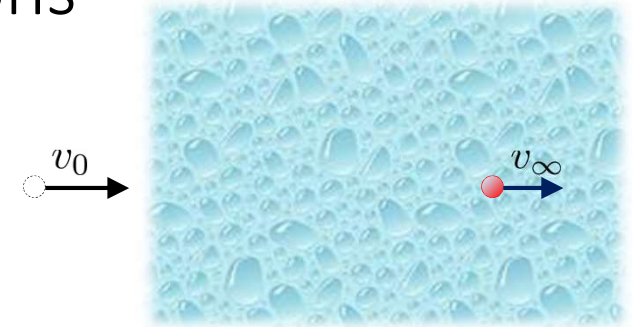
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impurity-fluid interaction



# Accounting for impurity-fluid interactions

$$H = H_f + \frac{\mathbf{p}_{\text{imp}}^2}{2m} + \sum_{j=1}^N U(\mathbf{r}_j - \mathbf{r}_{\text{imp}})$$

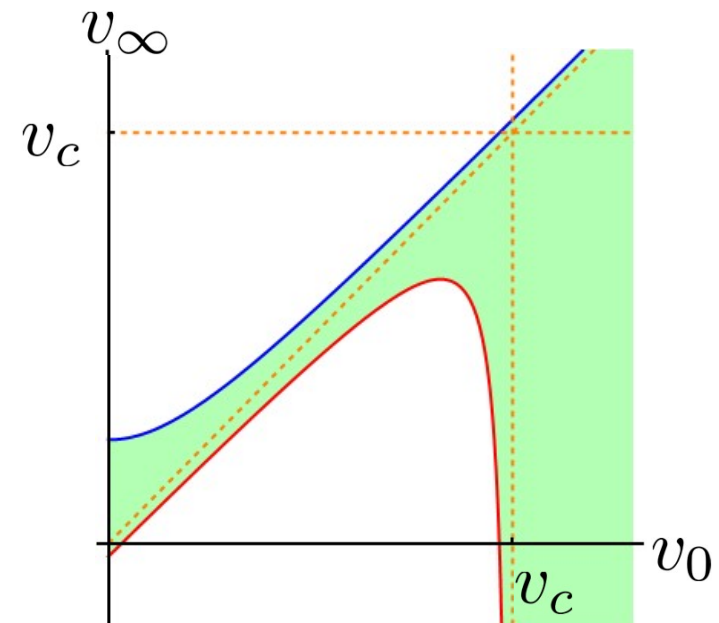


rigorous bound:

$$|\mathbf{v}_0 - \mathbf{v}_\infty| \leq \frac{\bar{U}}{m(v_c - v_0)}$$

valid for  $v_0 < v_c$ ,  $U(\mathbf{r}) \geq 0$

$\bar{U} = \int d\mathbf{r} \rho U(\mathbf{r})$ ,  $\rho = N/V$  – fluid density



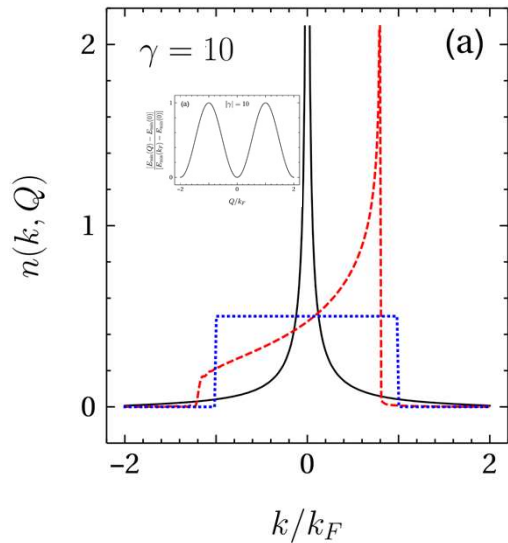
OL, Phys. Rev. A 89, 033619 (2014)

OL, Phys. Rev. A 91, 040101 (R) (2015)

# McGuire model: polarons and anyons

$$\varrho(y) = \det(\hat{I} + \hat{K} + \hat{W}) - \det(\hat{I} + \hat{K})$$

$$n(k, Q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dy e^{iky} \varrho(y)$$



$Q = 0$

$Q = 0.8 k_F$

$Q = k_F$

Anyons:

$$\psi(x_1)\psi^\dagger(x_2) = e^{-i\pi\kappa \text{sgn}(x_1-x_2)}\psi^\dagger(x_2)\psi(x_1) + \delta(x_1-x_2)$$

$\kappa = 0$  : bosons

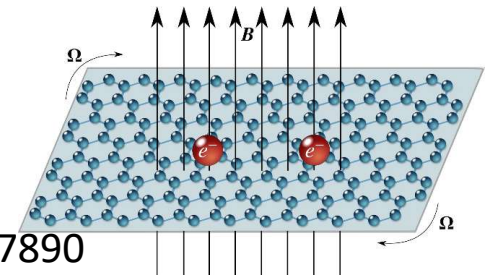
$\kappa = 1$  : fermions

$$\gamma \rightarrow \infty : \quad n(k, Q) = n_A(k) \quad \text{with} \quad \kappa = -Q/k_F$$

Patu, Korepin, Averin, 2008

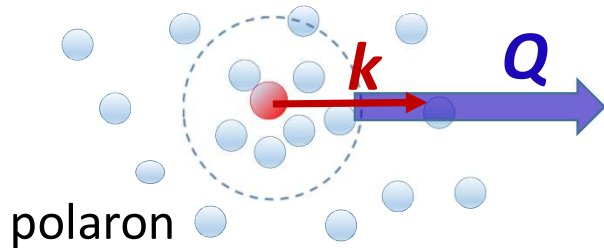
similar effect in 2D:

Yakoboylu et al, arXiv1912.07890



O. Gamayun, O. Lychkovskiy, M. Zvonarev  
arXiv 1909.07358

# McGuire model: polarons and anyons

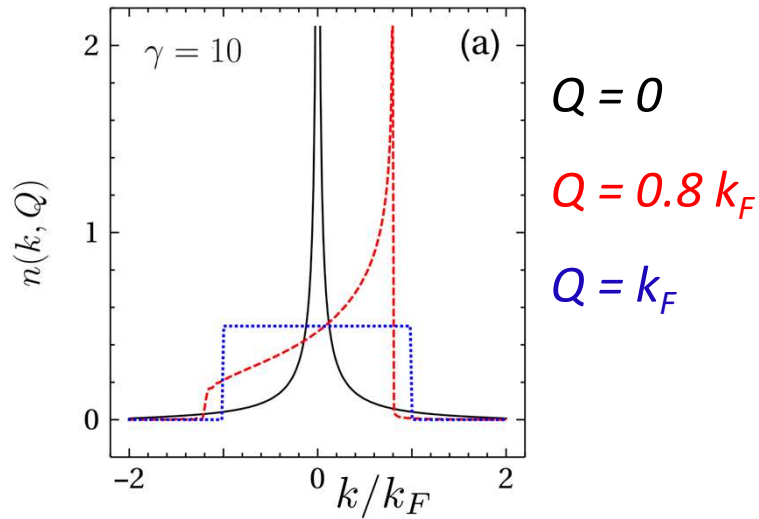


anyons:

$$\psi(x_1)\psi^\dagger(x_2) = e^{-i\pi\kappa \operatorname{sgn}(x_1-x_2)}\psi^\dagger(x_2)\psi(x_1) + \delta(x_1 - x_2)$$

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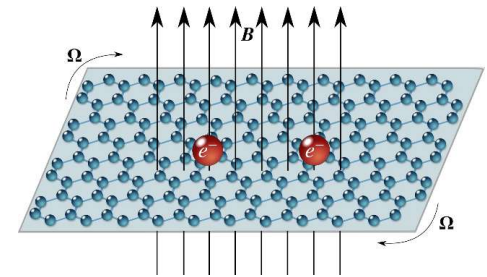
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