The 2N+1 body problem: an impurity immersed in a spin ½ fermionic superfluid

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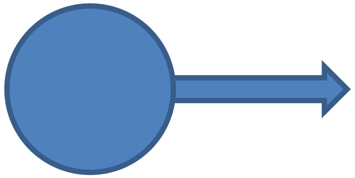
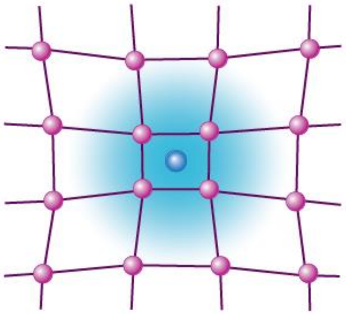
The polaron problem

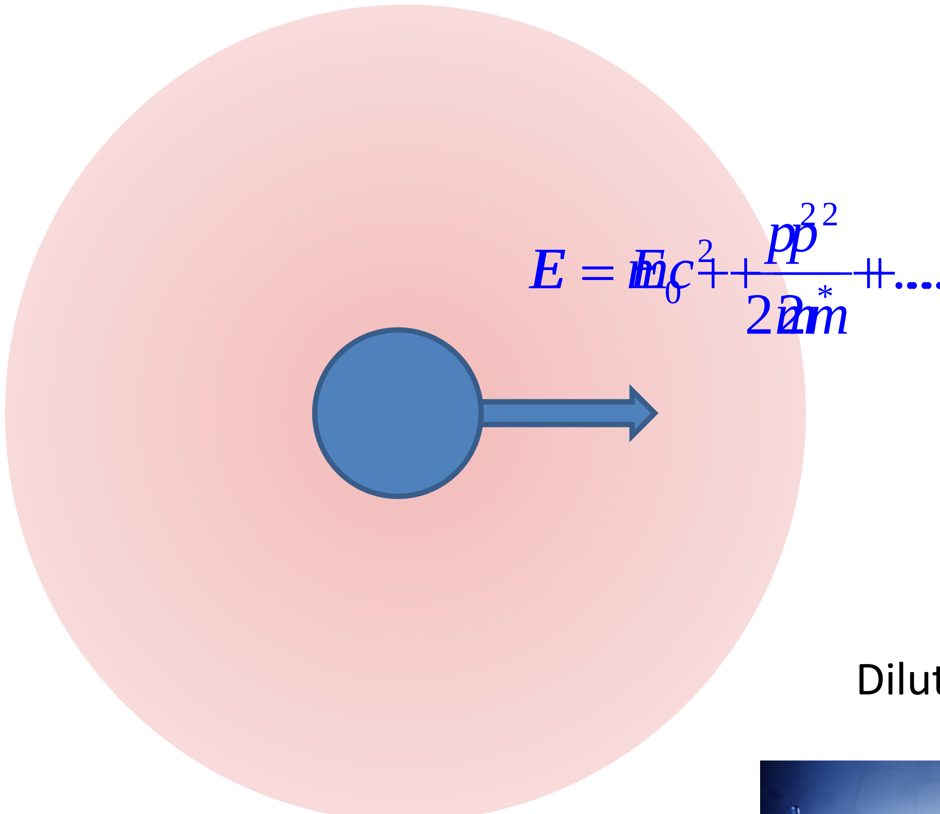
$$E = mc^2 + \frac{p^2}{2m} + \dots$$



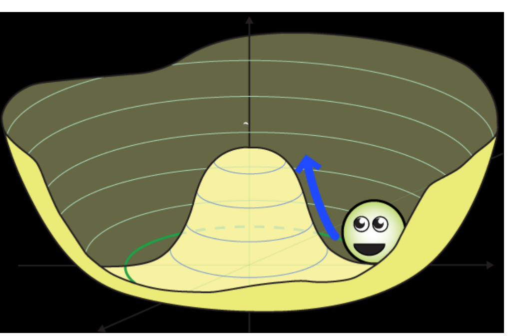
The polaron problem

Landau-Pekar-Frölich





Higgs mechanism

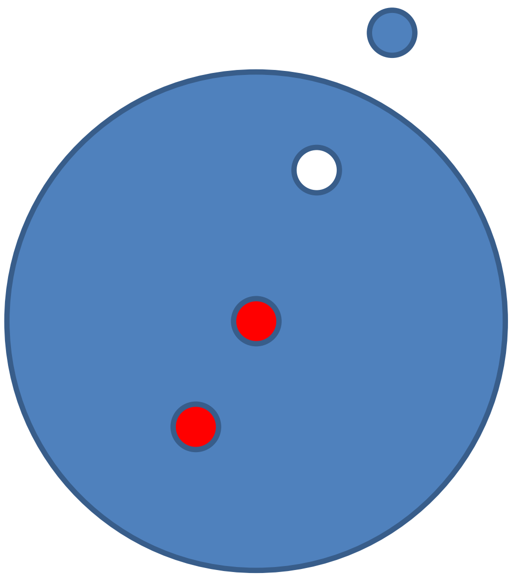


Dilute solutions

Ultracold atoms: Fermi and Bose polarons

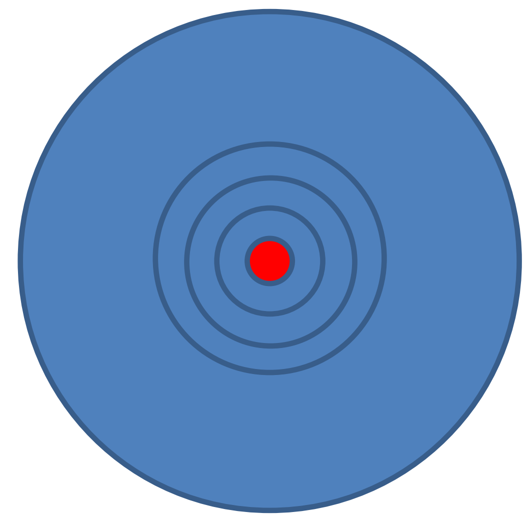
Ultracold regime: de Broglie wavelength >> range of the interatomic potential (**zero-range limit**)

Fermi polaron (ENS, MIT, Innsbruck, Technion…**)**: impurity immersed in an ideal gas of spin polarized fermions



Stable for any $k_F a$ (Moser & Seringer, 2016)

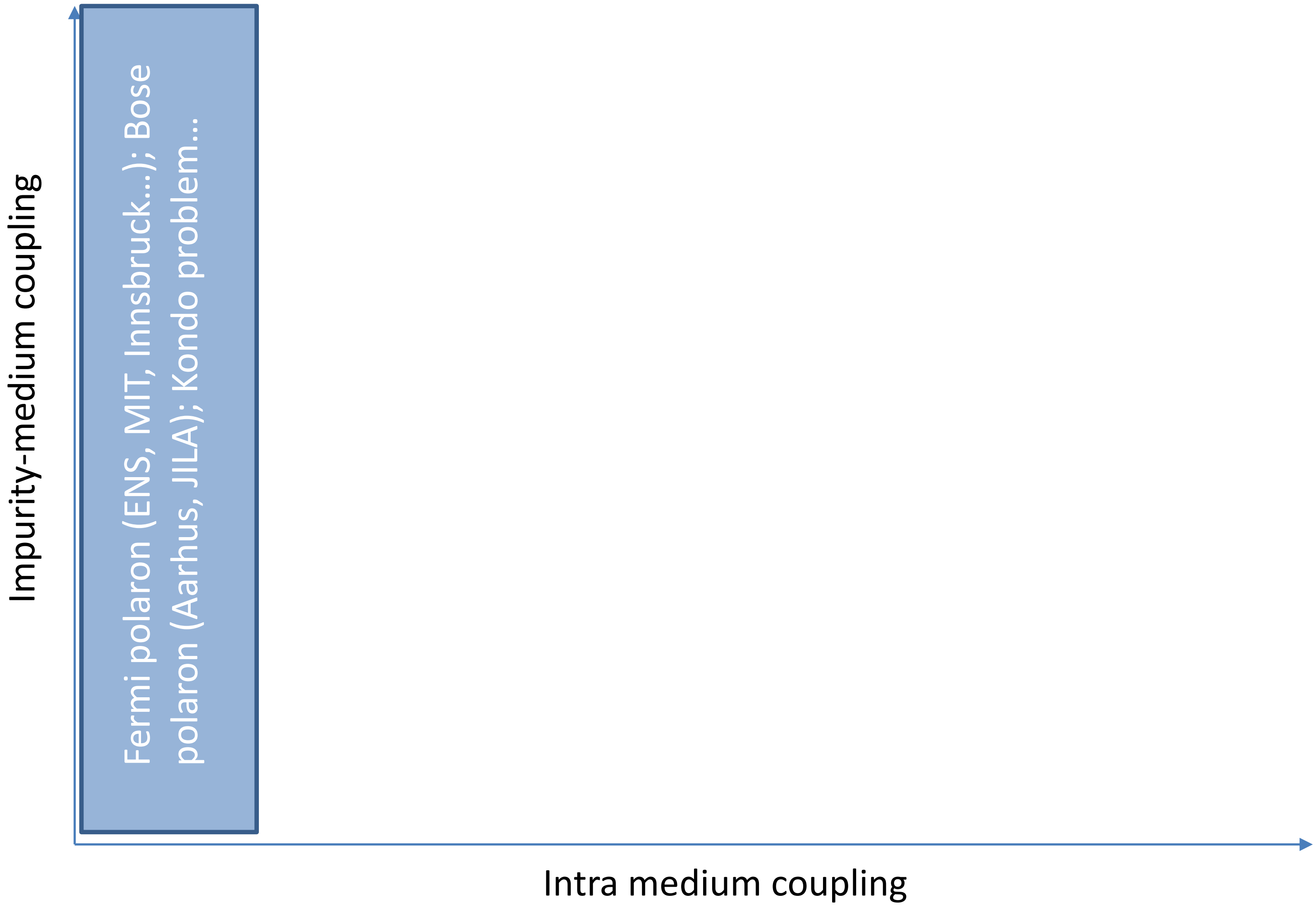
Bose polaron (Aarhus, JILA**)**: impurity immersed in an weakly interacting Bose-Einstein condensate



Efimov effect: polaron unstable for large na^3

Theory: see Landau, Frölich, Prokof'ev, Svistunov, Bruun, Parish, Levinsen, Massignan,Recati, Giorgini, Lobo…

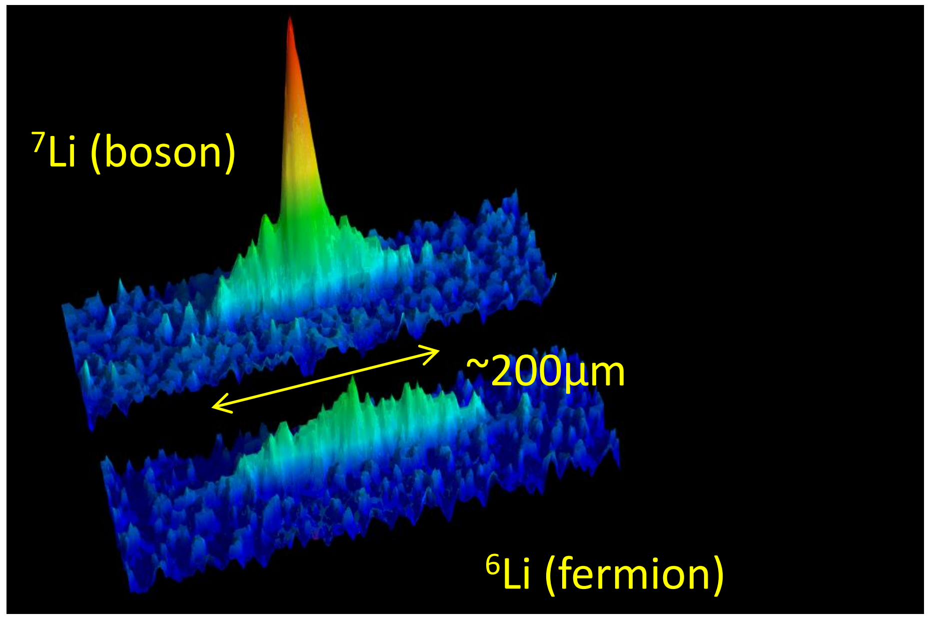
POLARON PHASE DIAGRAM



POLARON PHASE DIAGRAM

Impurity-medium coupling

Fermi polaron (ENS, MIT, Innsbruck…); Bose polaron (Aarhus, JILA); Kondo problem…



Impurity weakly coupled to a strongly correlated medium (ENS, Seattle, Tokyo, Shanghai…)

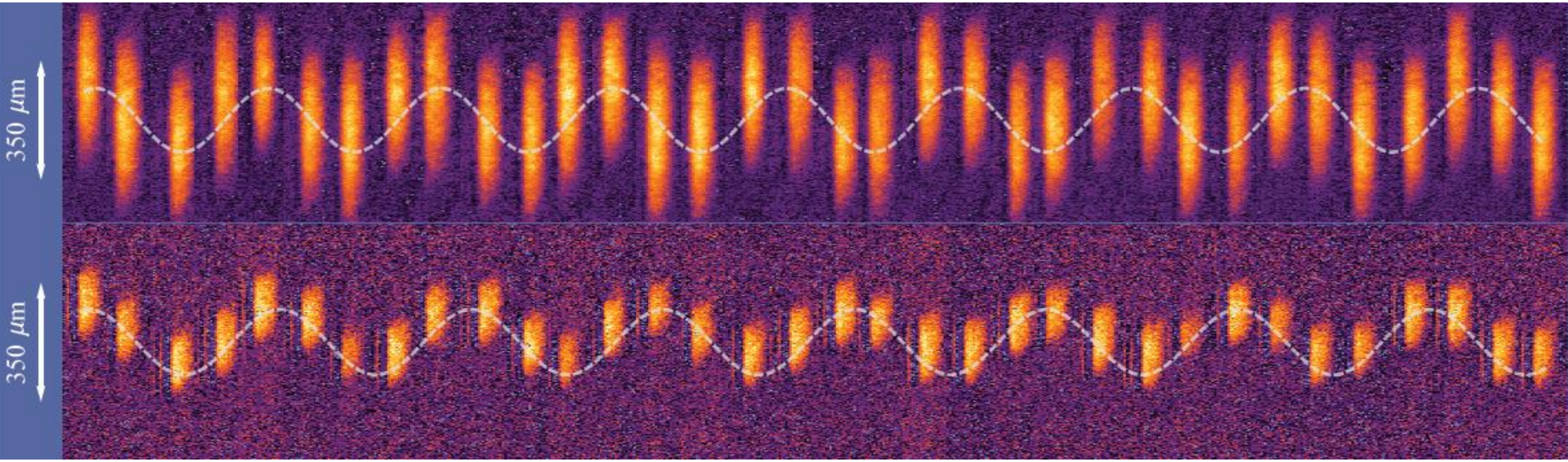
Intra medium coupling

Dynamics of the impurity

I. Ferrier-Barbut *et al.*, Science **345**, 1035 (2014)

See also C. Salomon's talk

Dynamics of the Mixtures



$$\omega_6 = 2\pi \times 16.80(2)\text{Hz} \qquad \tilde{\omega}_6 = 2\pi \times 16.80(1)\text{Hz}$$

$$\omega_7 = 2\pi \times 15.27(2)\text{Hz} \qquad \tilde{\omega}_7 = 2\pi \times 15.00(1)\text{Hz}$$

Single Superfluid
Ratio = $(7/6)^{1/2} = (m_7/m_6)^{1/2}$

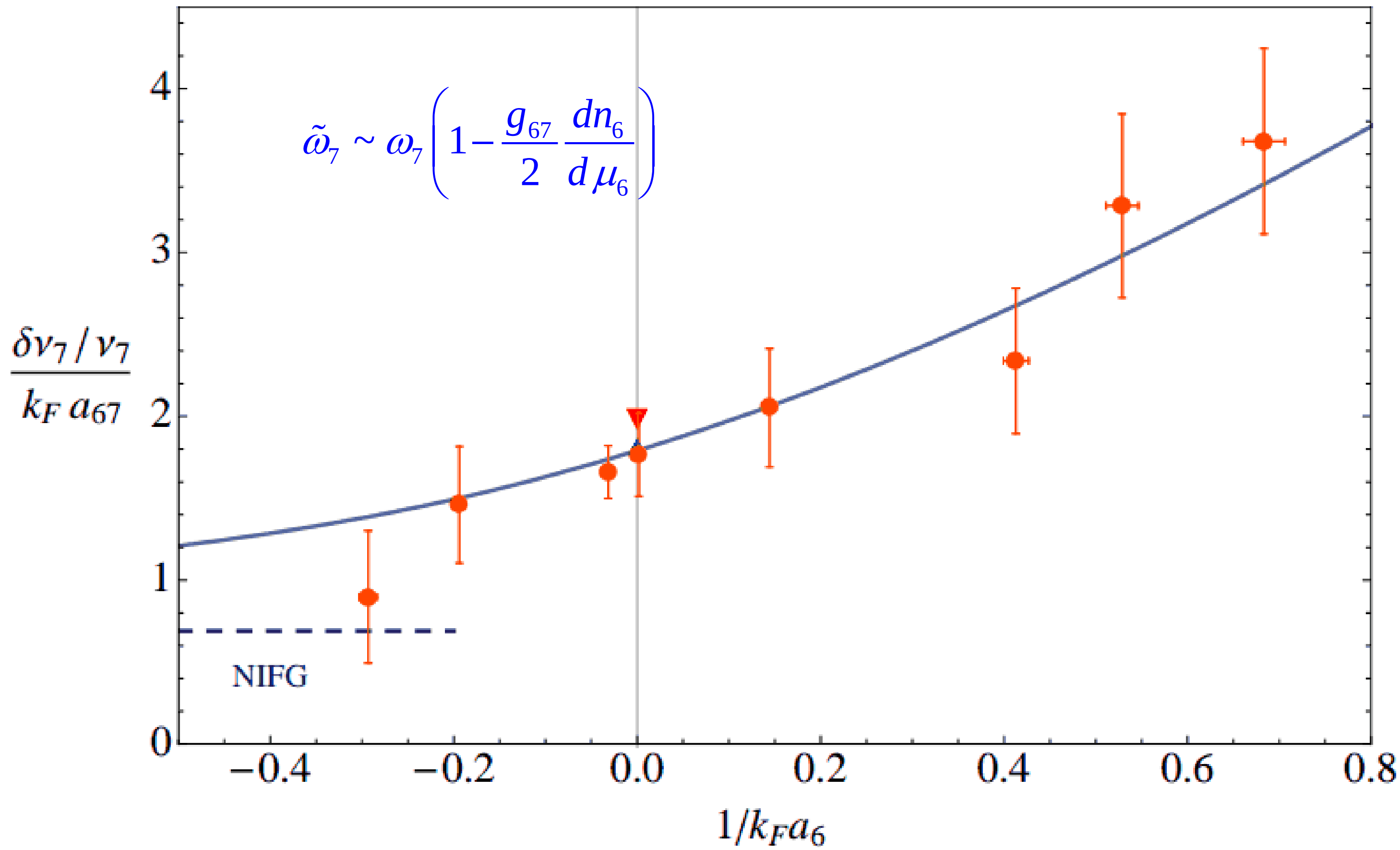
Coupled Superfluids

See also C. Hammer *et al* Phys. Rev. Lett. **106**, 065302 (2011) for boson-boson superfluid counterflow

FREQUENCY SHIFT

$$V_{\text{eff},7} = V(\mathbf{r}) + g_{67} n_6(\mu_6(\mathbf{r})) \qquad \mu_6(\mathbf{r}) = \mu_6^0 - V(\mathbf{r})$$ (Local Density Approximation)

$$\approx g_{67} n_6(\mu_6^0) + V(\mathbf{r}) \left(1 - g_{67} \frac{\partial n_6}{\partial \mu_6} \right)$$



Beyond weak coupling

M. Pierce *et al.* PRL **123**, 080403 (2019)

A. Bigué *et al., in preparation*

Energy of the polaron: beyond mean-field contributions

Contact interaction $V(r) = g'_{IF}\,\delta(\mathbf{r})$

$$g'_{IF}(\Lambda) = g_{IF} + \frac{g_{IF}^2}{\Omega}\sum_{q<\Lambda}\frac{1}{2\varepsilon_q} + \ldots$$

Second-order perturbation theory

$$\Delta E = g_{IF} n_F + \frac{g_{IF}^2 n_F}{\Omega}\sum_{q<\Lambda}\left[\frac{1}{2\varepsilon_q} - \chi(q,\varepsilon_q)\right]$$

UV (log) divergent!

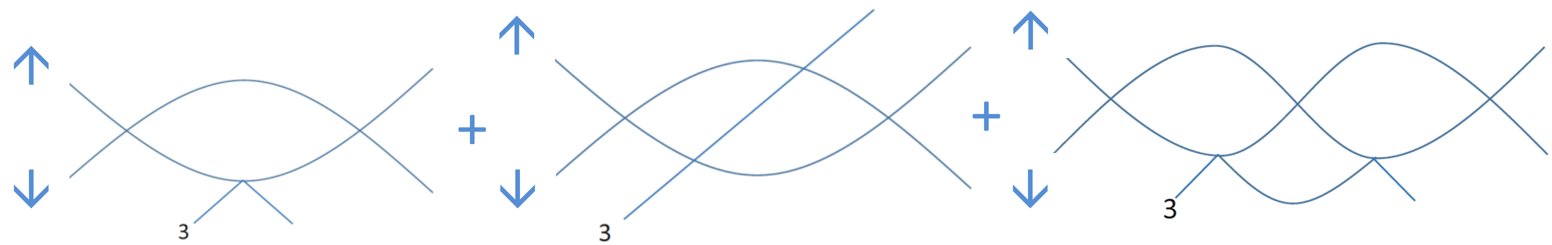
(see for instance BCS)

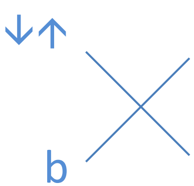
« Dynamical compressibility » of the superfluid

Back to the three-body problem

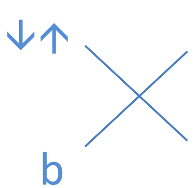
Same divergence in the 3-body scattering (≈Wu beyon mean-field correction for BECs)

$\Gamma =$





$= g'_{BF}$ (Born) $\Rightarrow$ log-divergent



$= \dfrac{g_{BF}}{1+ika_{BF}}$ (Faddeev) $\Rightarrow$ regular

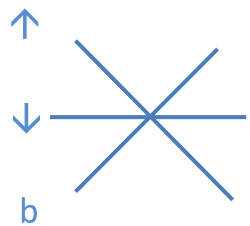
$$\Gamma_{\text{Born}} - \Gamma_{\text{Faddeev}} = g_{\text{BF}}^2 \kappa\left(\frac{m_{\text{B}}}{m_{\text{F}}}\right) \log(\Lambda R_{3\text{b}}) + o(1)$$

$$\kappa(7/6) = \frac{4}{9\sqrt{3}} + \frac{1}{3\pi} - \frac{\sqrt{3}}{2\pi^2}$$

$$R_{3\text{b}} \simeq 1.5 a_{\text{BF}} \;\; ({}^7\text{Li}/{}^6\text{Li})$$

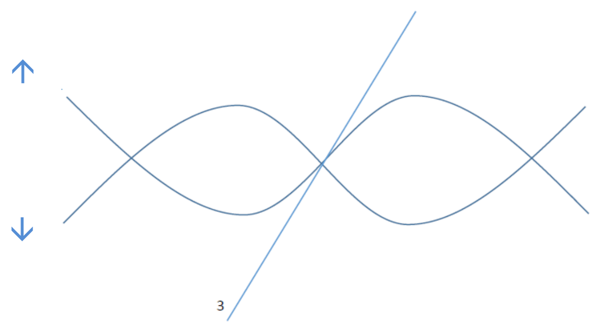
Curing the three-body problem

Add an explicit 3-body contact interaction to regularize the 3-body scattering amplitude (Braaten & Nieto ‘99)



$= g_3(\Lambda)$

Contribution to the 3-body amplitude



$= \Gamma_{3b}$

$$\Gamma_{\text{Faddeev}} = \Gamma_{\text{Born}} + \Gamma_{\text{3b}} \Rightarrow g_3(\Lambda)\left(\frac{1}{\Omega}\sum_{k<\Lambda}\frac{1}{2\varepsilon_k}\right)^2 = -g_{BF}^2\kappa\left(\frac{m_{\text{B}}}{m_{\text{F}}}\right)\log(\Lambda R_{3b}) + o(1)$$

Curing the polaron energy

Treat the three-body interaction perturbatively for the polaron

$$E = g_{\mathrm{IF}} n_{\mathrm{F}} \left[1 + k_{\mathrm{F}} a_{\mathrm{IF}} F\left(\frac{1}{k_{\mathrm{F}} a_{\mathrm{FF}}} \right) + \frac{a_{\mathrm{IF}} C_{\mathrm{F}}}{N} \kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}} \right) \ln\left(k_{\mathrm{F}} R_{3\mathrm{b}} \right) + \ldots \right]$$

$$F\left(\frac{1}{k_F a_{FF}} \right) = \frac{4\pi}{k_F} \left[\frac{\hbar^2}{\mu} \int_{q<\Lambda} \frac{d^3\mathbf{q}}{(2\pi)^3} \left(\frac{1}{2\varepsilon_q} - \chi(q, \varepsilon_q) \right) + \frac{C_{\mathrm{f}}}{N} \kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}} \right) \ln(\Lambda / k_{\mathrm{F}}) \right]$$

Expression ***does not make any assumption on the properties of the many-body background***

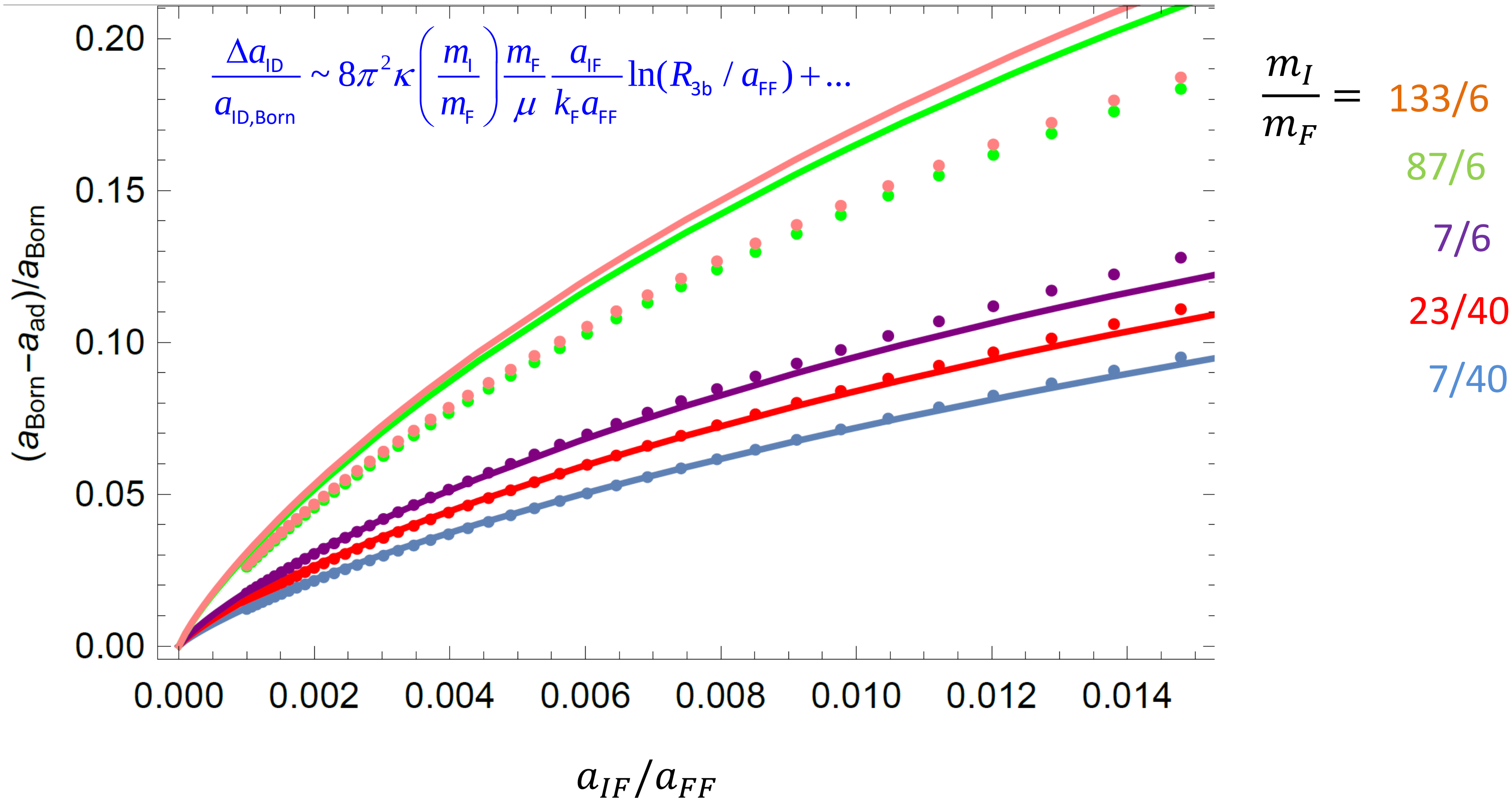
Asymptotic expressions

$a_{\mathrm{FF}} \to 0^- \Rightarrow$ Fermi polaron problem $\Rightarrow F(-\infty) = \dfrac{3}{4\pi}$

$a_{\mathrm{FF}} \to 0^+ \Rightarrow$ Bose polaron problem $\Rightarrow E = g_{\mathrm{ID}} n_{\mathrm{D}}$

$$\Rightarrow F(+\infty) \sim 8\pi^2 \kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}} \right) \frac{m_{\mathrm{F}}}{\mu} \frac{\ln(k_{\mathrm{F}} a_{\mathrm{FF}})}{k_{\mathrm{F}} a_{\mathrm{FF}}}$$

Cross-check: impurity-dimer scattering length



Zhang et al. (2014): Skornyakov Ter-Martirosyan

Calculation of F

$$F\left(\frac{1}{k_F a_{FF}}\right)=\frac{4\pi}{k_F}\left[\frac{\hbar^2}{\mu}\int_{q<\Lambda}\frac{d^3\mathbf{q}}{(2\pi)^3}\left(\frac{1}{2\varepsilon_q}-\chi(q,\varepsilon_q)\right)+\frac{C_{\mathrm{f}}}{N}\kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}}\right)\ln(\Lambda/k_{\mathrm{F}})\right]$$

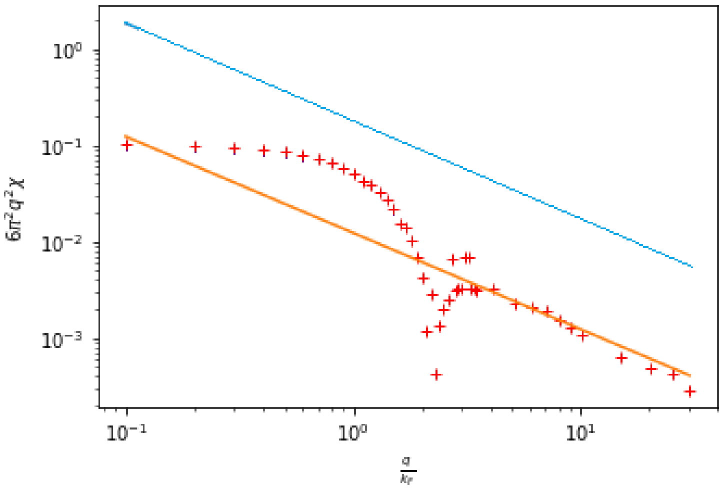
F depends only on the **properties of the medium**.

Use **BCS/Mean-field** theory to calculate χ.

- describes correctly the pair-breaking sector but misses collective modes (Bogoliubov Anderson modes)-> eg breakdown of the F-sum rule.

Regularization using 3-body interactions **fails within** BCS approximation

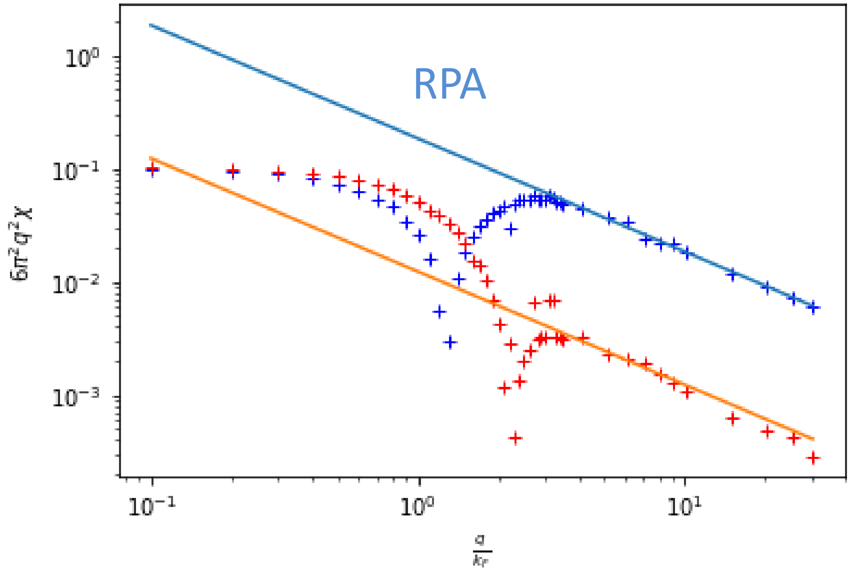
$$\chi(q,\varepsilon_q^{(r)})=\frac{1}{\varepsilon_q^{(r)}}\left[1-\pi^2\kappa(\eta)\frac{m_f}{m_r}\frac{C_f}{Nq}+\cdots\right]$$

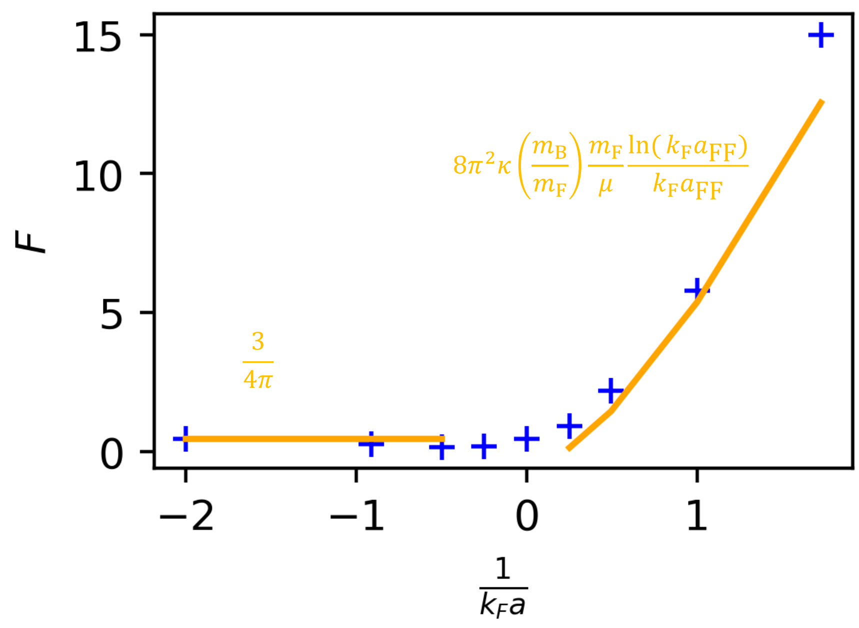


Random Phase Approximation

RPA: solve dynamic equations self-consistenly by allowing the gap to vary (Minguzzi *et al. 01* , Combescot *et al. 06,…*).

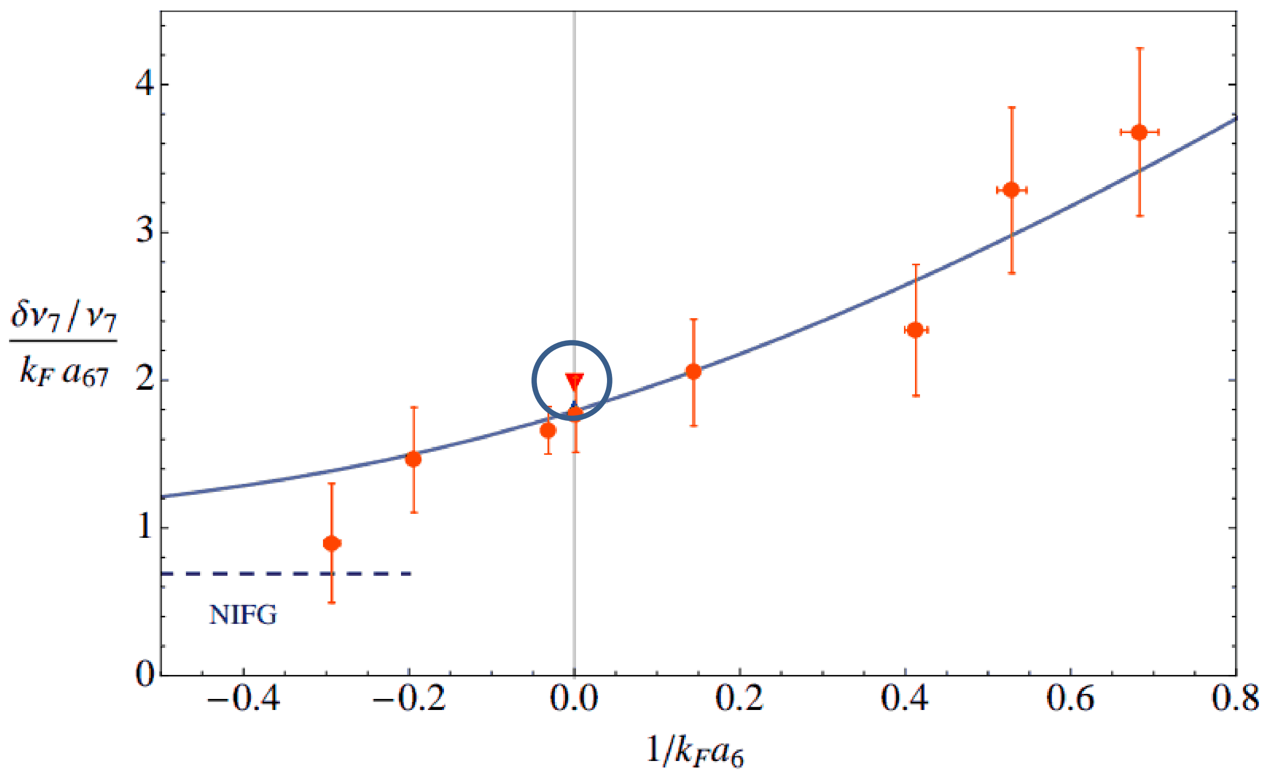
- Static properties still given by BCS theory but recovers the collective modes
- Correct scaling for the compressibility at large q.





The Next Steps

- **Challenge**: is it possible to observe the beyond mean-field corrections using high precision oscillation frequency measurements?



Conclusion and outlook

Beyond the weakly interacting regime, R. Alhyder *et al.* PRA **102**, 033322 (2020)

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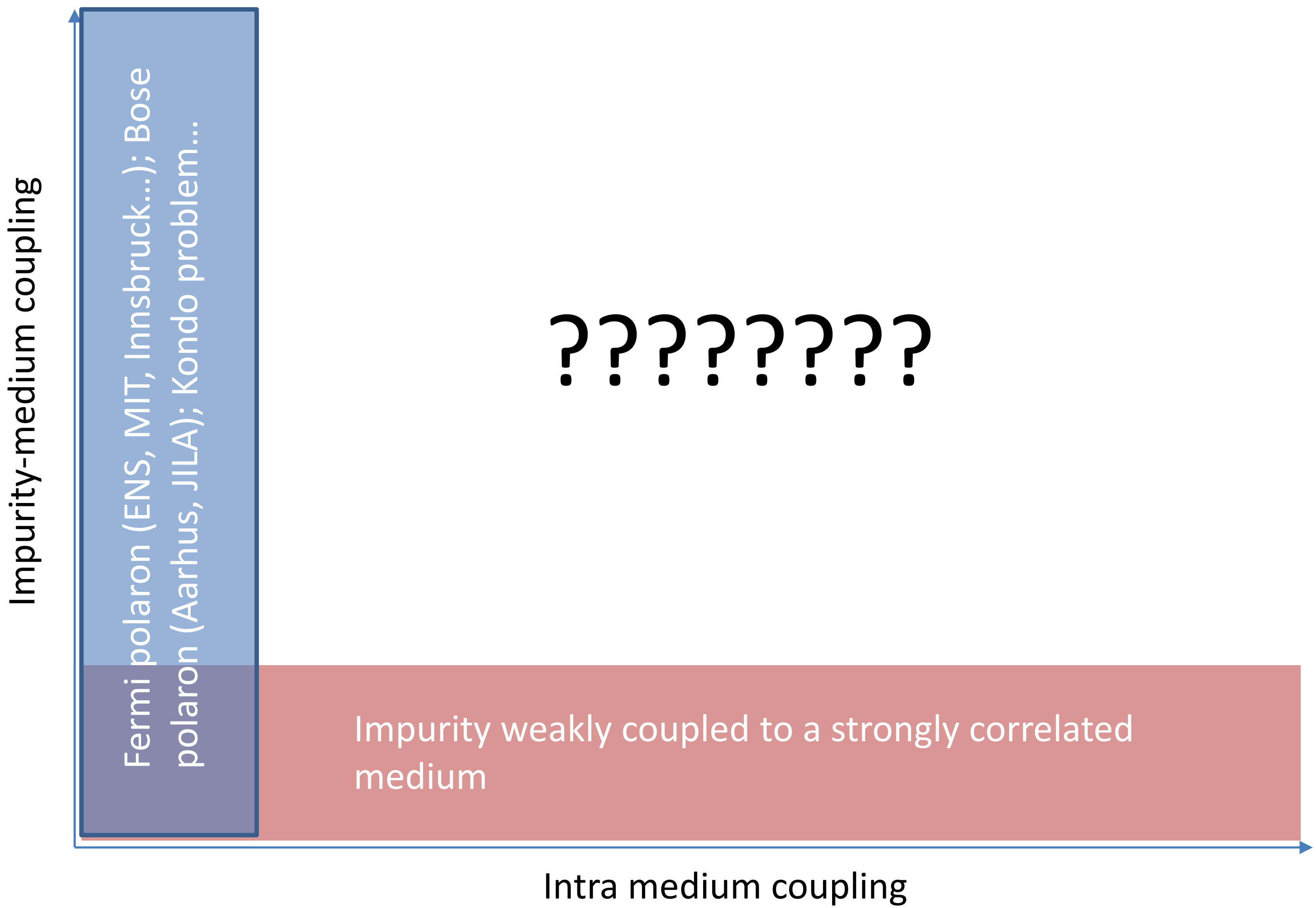
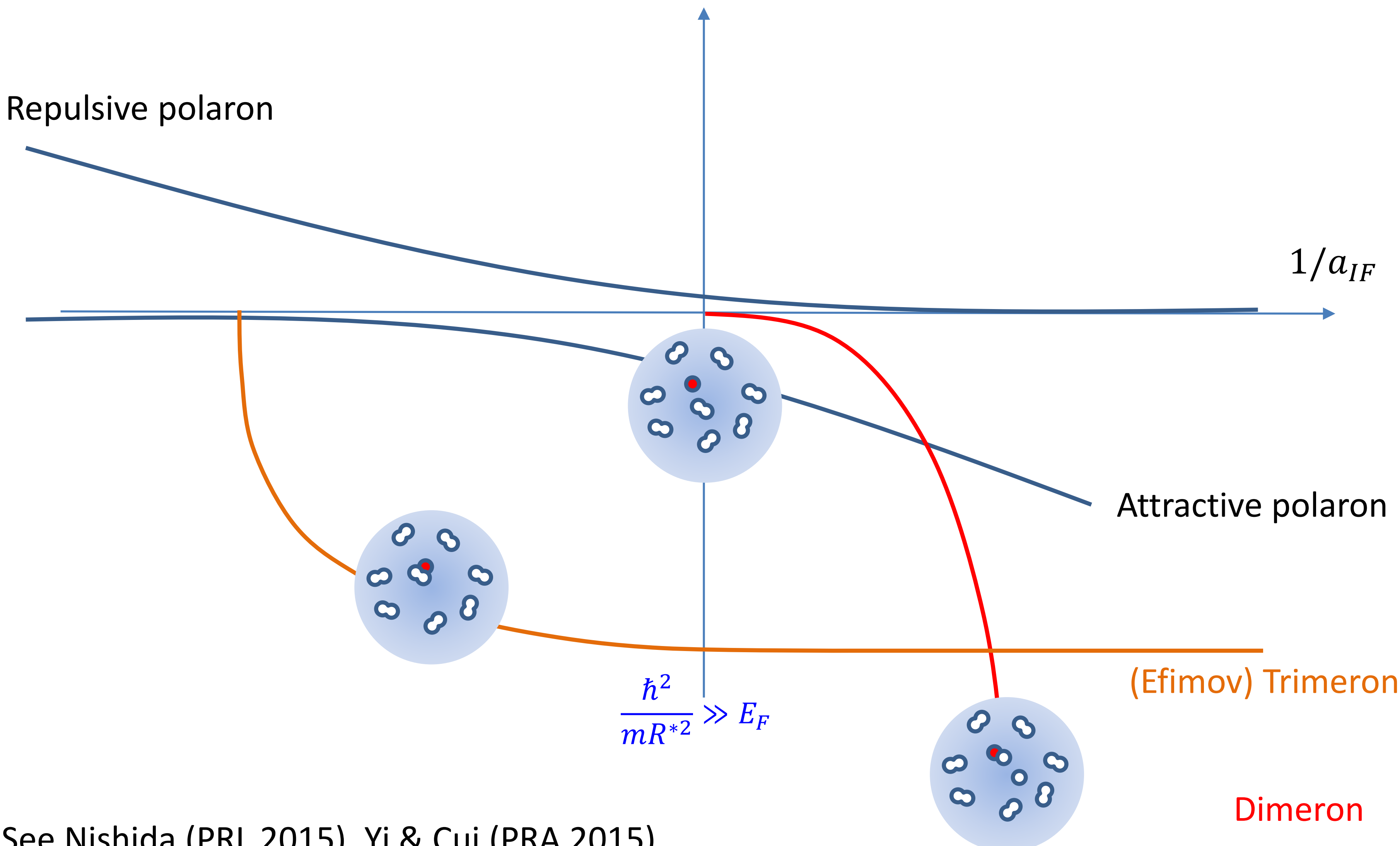
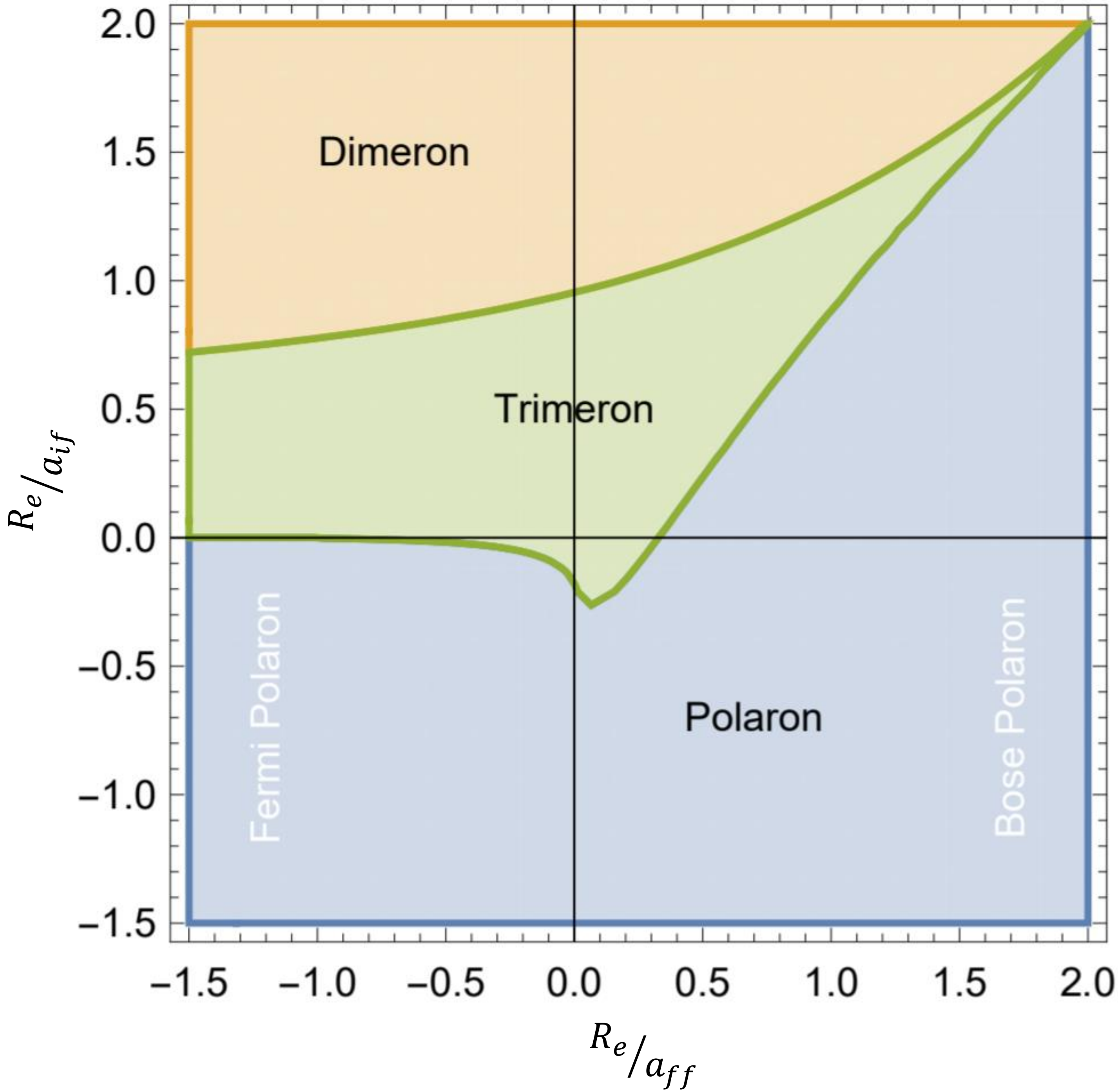


Polaron, dimeron, trimeron



See Nishida (PRL 2015), Yi & Cui (PRA 2015)



Dimeron
Trimeron
Polaron
Fermi Polaron
Bose Polaron
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
R_e/a_{if}
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
R_e/a_{ff}