Static and dynamic properties of a one-dimensional mobile impurity

Oleksandr Gamayun

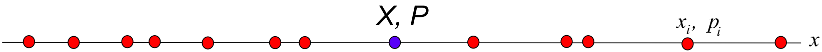
University of Amsterdam ⇒ Warsaw University

based on works in collaborations with:

O. Lychkovskiy, V. Cheianov, M. Zvonarev,
M. Malcolmson, E. Burovski, A. Pronko, J.S. Caux
Yu. Zhuravlev, N. Iorgov, M. Panfil, F. Taha Sant'Ana

Quantum 2021, IPa, 26/08/2021

Impurity propagation in 1D media



Blue: impurity

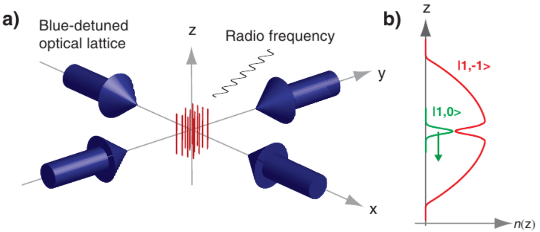
Red: host particles

Outline

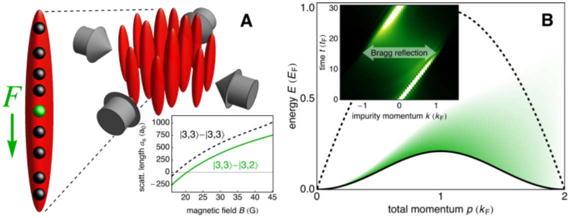
- McGuire model. Bethe Ansatz solution.
- Momentum distribution
- Quantum Flutter
- Log-diffusion

Physical motivation: Non-equilibrium dynamics in 1D

Quantum Transport through a Tonks-Girardeau Gas [PRL 103, 150601 (2009)]



Bloch oscillations in the absence of a lattice [Science 356, 945-948 (2017)]



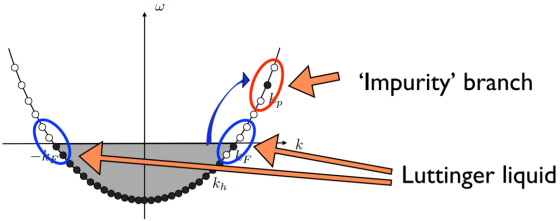
Motivation: Correlation functions in 1D

$$\langle \mathcal{O}(x,t)\mathcal{O}(0,0)\rangle = \sum_f |\langle \mathrm{vac}|\mathcal{O}(0,0)|f\rangle|^2 e^{-itE_f + iP_f x} = ?$$

Bosonization:

- Low-energy (soft-excitations), asymptotic $k_F x \gg 1$, $E_F t \gg 1$
- Linear Spectrum $S(\varphi) \sim \int d^2x(\partial\varphi)^2$
- $\mathcal{O} = P(\partial\varphi, \partial^2\varphi, e^{i\varphi})$

Non-linear model to capture high-energy behavior!!!

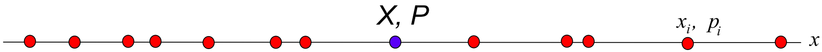


Glazman, Imambekov, Khodas, Kamenev, Cheianov, Pustilnik, Affleck, Pereira, Sirker, Caux . . .

Kitanine, Kozlowski, Maillet, Slavnov, Terras . . .

[Review: Adilet Imambekov, Thomas L. Schmidt, and Leonid I. Glazman, Rev. Mod. Phys. 84, 1253]

Mobile impurity [McGuire 1965]



$$H = \frac{P^2}{2m} + \sum_{i=1}^{N} \frac{p_i^2}{2m} + g \sum_{i=1}^{N} \delta(x_i - X)$$

- Model is integrable (specific sector of the fermionic Yang-Gaudin).
- Thermodynamic limit $N \to \infty$, $L \to \infty$, $\rho = N/L = \text{const}$
- Dimensionless coupling $\gamma = mg/\rho$, $m = 1$, $\rho = 1/\pi$, $k_F = 1$

$$\alpha = 2\pi/\gamma = 2/g$$

Impurity's reference frame

$$H = \frac{P^2}{2m} + \sum_{i=1}^{N} \frac{p_i^2}{2m} + g \sum_{i=1}^{N} \delta(x_i - X)$$

Total momentum

$$[H, \hat{Q}] = 0, \qquad \hat{Q} = -i \frac{\partial}{\partial X} - i \sum_{i=1}^{N} \frac{\partial}{\partial x_i}$$

Wave function transformation

$$\Psi(X, x_1, \ldots, x_N) = e^{iQX} \psi(y_1, \ldots y_N), \qquad y_i = x_i - X, \qquad y_i \sim y_i + L$$

$$H_Q = \frac{(Q - \sum_{i=1}^{n} p_i)^2}{2} + \sum_{i=1}^{N} \frac{p_i^2}{2} + g \sum_{i=1}^{N} \delta(y_i)$$

- $H_Q \psi(\mathbf{y}) = E \psi(\mathbf{y})$
- $\psi(\mathbf{y})$ is antisymmetric
- $\psi(\mathbf{y})$ is periodic and continuous $\psi(\mathbf{y})\Big|_{y_i=0} = \psi(\mathbf{y})\Big|_{y_i=L-0}$
- $\delta\psi_i \equiv \partial_{y_i} \psi(\mathbf{y})\Big|_{y_i=0}^{y_i=L-0} - g \frac{\psi(\mathbf{y})\Big|_{y_i=0} + \psi(\mathbf{y})\Big|_{y_i=L-0}}{2} = 0$

Coordinate Bethe Ansatz

$$\psi(\mathbf{y}) \sim \begin{vmatrix} e^{ik_1 y_1} & e^{ik_2 y_1} & \dots & e^{ik_{N+1} y_1} \\ e^{ik_1 y_2} & e^{ik_2 y_2} & \dots & e^{ik_{N+1} y_2} \\ \vdots & \vdots & \ddots & \vdots \\ e^{ik_1 y_N} & e^{ik_2 y_N} & \dots & e^{ik_{N+1} y_N} \\ A_1 & A_2 & \dots & A_{N+1} \end{vmatrix}$$

- ✓ $H_Q \psi(\mathbf{y}) = E\psi(\mathbf{y}), \qquad E = \sum_{i=1}^{N+1} k_i^2/2, \qquad Q = \sum_{i=1}^{N+1} k_i$
- ✓ $\psi(\mathbf{y})$ is antisymmetric
- $\psi(\mathbf{y})$ is periodic and continuous $\psi(\mathbf{y})\Big|_{y_i=0} = \psi(\mathbf{y})\Big|_{y_i=L-0}$
- $\delta\psi_i \equiv \partial_{y_i}\psi(\mathbf{y})\Big|_{y_i=0}^{y_i=L-0} - g\,\frac{\psi(\mathbf{y})\big|_{y_i=0} + \psi(\mathbf{y})\big|_{y_i=L-0}}{2} = 0$

Coordinate Bethe Ansatz

$$\psi(\mathbf{y})\Big|_{y_1=0}^{y_1=L-0} \sim \begin{vmatrix} e^{ik_1 L}-1 & e^{ik_2 L}-1 & \dots & e^{ik_{N+1} L}-1 \\ e^{ik_1 y_2} & e^{ik_2 y_2} & \dots & e^{ik_{N+1} y_2} \\ \vdots & \vdots & \ddots & \vdots \\ e^{ik_1 y_N} & e^{ik_2 y_N} & \dots & e^{ik_{N+1} y_N} \\ A_1 & A_2 & \dots & A_{N+1} \end{vmatrix}$$

- ✓ $H_Q\psi(\mathbf{y}) = E\psi(\mathbf{y}), \qquad E = \sum_{i=1}^{N+1} k_i^2/2, \qquad Q = \sum_{i=1}^{N+1} k_i$
- ✓ $\psi(\mathbf{y})$ is antisymmetric
- ✓ $\psi(\mathbf{y})$ is periodic and continuous $\psi(\mathbf{y})\Big|_{y_i=0} = \psi(\mathbf{y})\Big|_{y_i=L-0} \implies A_j = e^{ik_j L}-1$
- $\delta\psi_i \equiv \partial_{y_i}\psi(\mathbf{y})\Big|_{y_i=0}^{y_i=L-0} - g\,\frac{\psi(\mathbf{y})\Big|_{y_i=0}+\psi(\mathbf{y})\Big|_{y_i=L-0}}{2} = 0$

Coordinate Bethe Ansatz

$$\delta\psi_1 \sim \begin{vmatrix} ik_1(e^{ik_1L}-1)-\frac{g}{2}(e^{ik_1L}+1) & \dots & ik_{N+1}(e^{ik_{N+1}L}-1)-\frac{g}{2}(e^{ik_{N+1}L}+1) \\ e^{ik_1y_2} & \dots & e^{ik_{N+1}y_2} \\ \vdots & \ddots & \vdots \\ e^{ik_1y_N} & \dots & e^{ik_{N+1}y_N} \\ e^{ik_1L}-1 & \dots & e^{ik_{N+1}L}-1 \end{vmatrix}$$

- ✓ $H_Q\psi(\mathbf{y}) = E\psi(\mathbf{y}), \qquad E = \sum_{i=1}^{N+1} k_i^2/2, \qquad Q = \sum_{i=1}^{N+1} k_i$
- ✓ $\psi(\mathbf{y})$ is antisymmetric
- ✓ $\psi(\mathbf{y})$ is periodic and continuous $\psi(\mathbf{y})\Big|_{y_i=0} = \psi(\mathbf{y})\Big|_{y_i=L-0} \Longrightarrow A_j = e^{ik_jL}-1$
- ✓ $\delta\psi_i \equiv \partial_{y_i}\psi(\mathbf{y})\Big|_{y_i=0}^{y_i=L-0} - g\frac{\psi(\mathbf{y})\Big|_{y_i=0}+\psi(\mathbf{y})\Big|_{y_i=L-0}}{2} = 0 \Longrightarrow e^{ik_jL} = \frac{k_j-\Lambda+ig/2}{k_j-\Lambda-ig/2}$

Bethe Ansatz solution

- The eigenstate $|\mathbf{k}\rangle_Q \equiv |Q; k_1 \ldots, k_{N+1}\rangle$ at total momentum Q is given by the set of $N+1$ integers + phase shifts, via the "nested" set of Bethe equations
- $$E = \sum_{j=1}^{N+1} \frac{k_j^2}{2}, \qquad Q = \hat{P} + \sum_{i=1}^{N} \hat{p}_i = \sum_{i=1}^{N+1} k_i$$
$$e^{ik_j L} = \frac{k_j - \Lambda + ig/2}{k_j - \Lambda - ig/2}, \qquad k_j = \frac{2\pi}{L}\left(n_j - \frac{\delta_j}{\pi}\right)$$
- TD limit
$$\delta_j = \frac{\pi}{2} - \arctan(\Lambda - \alpha k_j) \approx \frac{\pi}{2} - \arctan\left(\Lambda - \alpha\frac{2\pi n_j}{L}\right), \quad \alpha = 2/g$$
- Fixed total momentum sector:
$$\boxed{Q = \frac{2\pi}{L}\sum_{j=1}^{N+1} n_j - 1 + \delta Q_\Lambda}$$
$$\delta Q_\Lambda = \frac{(\Lambda+\alpha)\arctan(\Lambda+\alpha) - (\Lambda-\alpha)\arctan(\Lambda-\alpha)}{\alpha\pi} + \frac{1}{2\alpha\pi}\ln\frac{1+(\alpha-\Lambda)^2}{1+(\alpha+\Lambda)^2}$$

Lieb-Liniger

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + c \sum_{i<j} \delta(x_i - x_j)$$

- The Bethe wave function

$$\boxed{\Psi(x_1, \ldots x_N) = \prod_{k>j} \left(\frac{\partial}{\partial x_k} - \frac{\partial}{\partial x_j} + c\,\mathrm{sgn}(x_k - x_j) \right) \det(e^{ik_j x_k})}$$

- Bethe Equations

$$\boxed{\exp(ik_j L) = \prod_{l \neq j} \frac{k_j - k_l + ic}{k_j - k_l - ic}}$$

- Dressed momentum (counting function)

$$e^{iL\mathcal{P}(q)} - 1 = 0 \qquad \mathcal{P}(q) = q + \int_{-q}^{q} \theta(q-k)\rho(k)dk$$

- Dressed phase shift

$$F(k_i) = \frac{q_i - k_i}{k_{i+1} - k_i} \Leftrightarrow F(k) - \frac{1}{2\pi} \int_{-q}^{q} K(k-q)F(q)dq = \frac{\pi + \theta(k-q)}{2\pi}$$

$$\theta(k) = 2 \arctan \frac{k}{c} \qquad K(k) = \partial_k \theta(k) = \frac{2c}{k^2 + c^2}$$

Form-Factors

- Direct computation from the coordinate Bethe Ansatz
- $|\mathbf{k}\rangle_Q \equiv |Q; k_1 \dots, k_{N+1}\rangle, \qquad k_j = \frac{2\pi}{L}\left(n_j - \frac{\delta_j}{\pi}\right)$
- $|\mathbf{q}\rangle \equiv |q_1 \dots, q_N\rangle, \qquad q_j = \frac{2\pi}{L} n_j$

$$|\langle \mathbf{q}|c_p|\mathbf{k}\rangle_Q|^2 = \delta_{p+\sum q_i, Q} \left(\frac{2}{L}\right)^N \frac{\prod_{j=1}^{N+1} \frac{\partial k_j}{\partial \Lambda}}{\sum_{j=1}^{N+1} \frac{\partial k_j}{\partial \Lambda}} \begin{vmatrix} \frac{1}{k_1 - q_1} & \dots & \frac{1}{k_{N+1} - q_1} \\ \vdots & \ddots & \vdots \\ \frac{1}{k_1 - q_N} & \dots & \frac{1}{k_{N+1} - q_N} \\ 1 & \dots & 1 \end{vmatrix}^2$$

$$\frac{\partial k_j}{\partial \Lambda} = \frac{2}{L} \frac{1}{1 + (\alpha k_j - \Lambda)^2 + 2\alpha/L}$$

Observables

- Momentum distribution

$$n_p = {}_Q\langle \mathbf{k}|c_p^+ c_p|\mathbf{k}\rangle_Q = \sum_{q_1,q_2,\ldots,q_N} |\langle \mathbf{q}|c_p|\mathbf{k}\rangle_Q|^2$$

- Dynamical and steady state momentum

$$\langle P(t)\rangle = \sum_{\mathbf{k},\mathbf{k}'} \langle \mathbf{q}_{\rm vac}|c_{p_0}|\mathbf{k}'\rangle\langle \mathbf{k}'|P|\mathbf{k}\rangle\langle \mathbf{k}|c_{p_0}^+|\mathbf{q}_{\rm vac}\rangle e^{-it(E_{\mathbf{k}'}-E_{\mathbf{k}})}$$

$$p_\infty = \sum_{\mathbf{k}} |\langle \mathbf{q}_{\rm vac}|c_{p_0}|\mathbf{k}\rangle|^2 \langle \mathbf{k}|P|\mathbf{k}\rangle$$

- Green's function

$$G(x,t) = \langle \psi_{\rm imp}(x,t)\psi_{\rm imp}^+(0,0)\rangle = \int \frac{dp_0}{2\pi} \sum_{\mathbf{k}} |\langle \mathbf{k}|c_{p_0}|\mathbf{q}_{\rm vac}\rangle|^2 e^{ip_0 x - itE_{\mathbf{k}}}$$

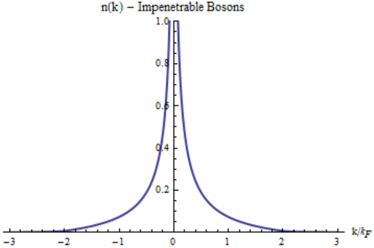
Momentum distribution

$$n_p = \sum_{q_1, q_2, \ldots, q_N} |\langle \mathbf{q} | c_p | \mathbf{k} \rangle_Q|^2 = \frac{1}{L} \int_0^L dx \, e^{ipx} \rho(x)$$

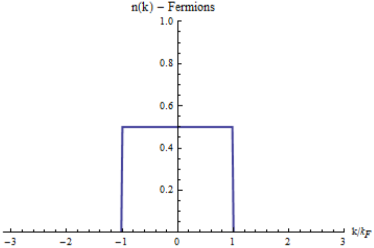
One-particle density matrix

$$\rho(x) = \int_0^L dx_2 \cdots \int_0^L dx_N \Psi(x, x_2, x_3, \ldots, x_N) \Psi^*(0, x_2, x_3, \ldots x_N)$$

$$\Psi(\ldots x_j, x_{j+1}, \ldots) = \Psi(\ldots x_{j+1}, x_j \ldots)$$



$$\Psi(\ldots x_j, x_{j+1}, \ldots) = -\Psi(\ldots x_{j+1}, x_j \ldots)$$



Summation

Summations over momenta $q_1, q_2, \dots q_N$ are independent!!!

$$\rho_N(y) = \det\left(\delta_{ij} + \frac{2\pi}{L} K_N(k_i, k_j) + \frac{2\pi}{L}\delta K_N(k_i, k_j)\right) - \det\left(\delta_{ij} + \frac{2\pi}{L} K_N(k_i, k_j)\right)$$

Thermodynamic limit

$$\rho(y) = \lim_{L\to\infty} \rho_N = \det(1 + \hat{K} + \delta\hat{K}) - \det(1 + \hat{K})$$

$\hat{K}$ and $\delta\hat{K}$ act on $[-1, 1]$ and are defined through the phase shift only

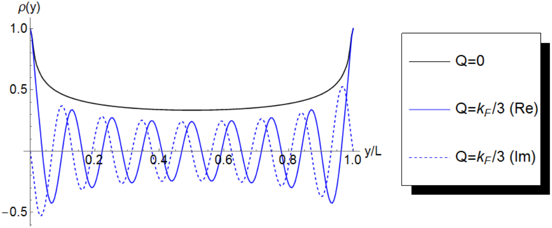
$$\boxed{F(k) \equiv \frac{\delta(\Lambda - \alpha k)}{\pi} = \frac{1}{2} - \frac{\arctan(\Lambda - \alpha k)}{\pi}}$$

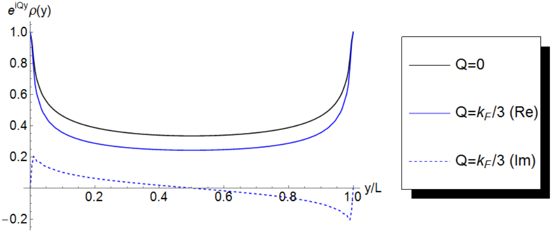
$$K(p, q) = \sin[\pi F(p)] \frac{e^{i(p-q)y/2}(\cot[\pi F(p)] + i) - e^{i(q-p)y/2}(\cot[\pi F(q)] + i)}{\pi(p - q)} \sin[\pi F(q)]$$

$$\delta K(p, q) = \frac{1}{\pi} \sin[\pi F(p)] e^{-i(p+q)y/2} \sin[\pi F(q)]$$

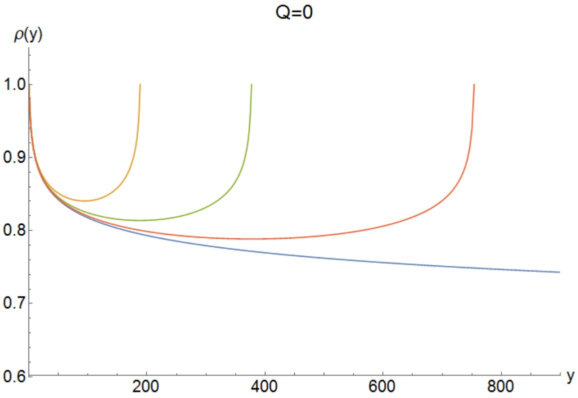
No need to solve Bethe equations!!!

One-particle density matrix





One-particle density matrix in TD limit

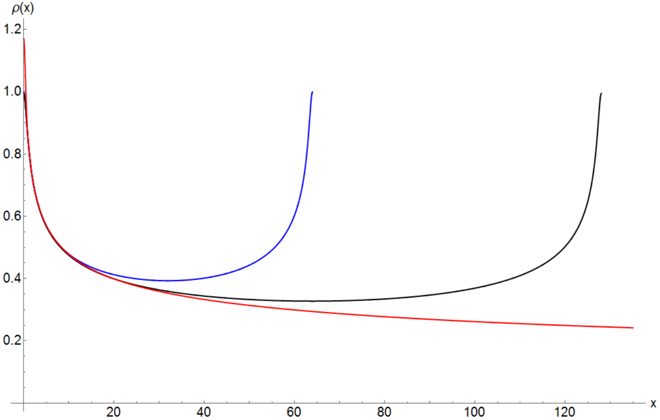


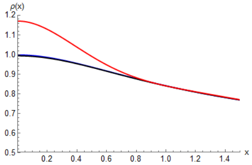
Lieb-Liniger model (ABACUS vs Fredholm; Preliminary)

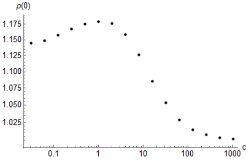
$$\rho(x) = \sum_{|\mu\rangle} e^{-ix(P_\lambda - P_\mu)} |\langle \mu | \hat{\Psi} | \lambda \rangle|^2 = \mathcal{R}\left(\det(1 + \hat{K} + \delta\hat{K}) - \det(1 + \hat{K})\right)$$

$$K(\lambda, \lambda') = \sin[\pi F(\lambda)] \sin[\pi F(\lambda')] \frac{e^{i(\mathcal{P}(\lambda) - \mathcal{P}(\lambda'))y/2}(\cot[\pi F(\lambda)] + i) - e^{i(\mathcal{P}(\lambda') - \mathcal{P}(\lambda))y/2}(\cot[\pi F(\lambda')] + i)}{\pi(\lambda - \lambda')}$$

$$Q = 0; \quad c = 4; \quad N/L = 1$$







Asymptotic = Microscopic bosonization

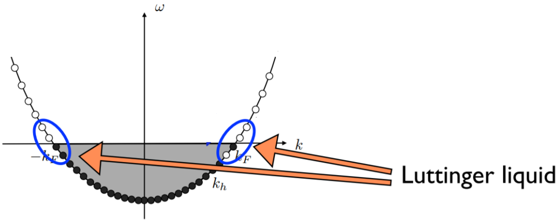
- Form-Factor summation

$$\rho(x) = \sum_{\mathbf{k}} |\langle \mathbf{q}|\mathcal{O}|\mathbf{k}\rangle|^2 e^{-ixP_{\mathbf{k}}} = \det(1+\hat{V})$$

- Orthogonality Catastrophe: $|\langle \mathbf{q}|\mathcal{O}|\mathbf{k}_{\mathrm{vac}}\rangle|^2 = \mathcal{A}/N^{2\beta}$
- Soft-mode summation

$$\rho(x) \sim \sum_{\mathrm{IR}} |\langle \mathbf{q}|\mathcal{O}|\mathbf{k}\rangle|^2 e^{-ixP_{\mathbf{k}}} = \sum_{k=0}^{\infty} \sum_{\substack{p_1,\dots p_k \\ h_1,\dots h_k}} \langle \Omega | e^{\sqrt{\beta}\varphi(x,t)} | \{p,h\}\rangle\langle\{p,h\}| e^{-\sqrt{\beta}\varphi(0,0)}|\Omega\rangle$$

$$= \mathcal{A} e^{-i(P_\Omega - P_{\mathrm{vac}})x} \langle e^{\sqrt{\beta}\varphi(x)} e^{-\sqrt{\beta}\varphi(0)}\rangle = \frac{\mathcal{A}}{x^{2\beta}} e^{-i(P_\Omega - P_{\mathrm{vac}})x}$$



Slavnov (1989); Slavnov and Korepin (1991); A. Shashi, L. I. Glazman, J.-S. Caux, and A. Imambekov (2011); N. Kitanine, K.K. Kozlowski, J.-M. Maillet, N.A. Slavnov, and V. Terras (2009-2012); K.K. Kozlowski, J.-M. Maillet (2015);

Asymptotic

$$\rho(y) \sim \frac{\mathcal{A} e^{-iQy}}{(2iy)^{F_-^2}(-2iy)^{(1-F_+)^2}}, \qquad y \to \infty$$

The exponents are defined by the phase shift

$$F_\pm = F(\pm 1)$$

$$\mathcal{A} = \alpha (2\pi)^{F_+ - F_- + 1} \frac{G^2(F_+) G^2(1 - F_-)}{F_+ - F_-} \exp(-C)$$

$$C = \frac{1}{2} \int_{-1}^{1} dq \int_{-1}^{1} dk \left(\frac{F(k) - F(q)}{k - q} \right)^2 + \int_{-1}^{1} dk \frac{F_-^2 - F^2(k)}{-1 - k} - \int_{-1}^{1} dk \frac{(1 - F_+)^2 - (1 - F(k))^2}{1 - k}$$

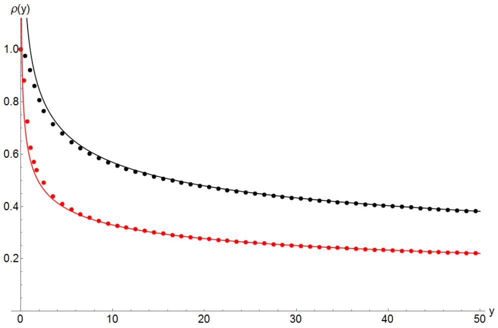
$$G(x+1) = \Gamma(x) G(x)$$

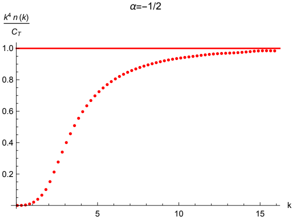
Impenetrable bosons [H. G. Vaidya and C. A. Tracy [Phys. Rev. Lett. 42, 3 (1979)]]

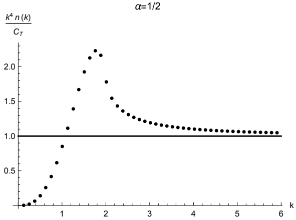
$$\rho(y) \sim \pi^2 \frac{G^2(1/4)}{\sqrt{2y}} = \frac{2^{-1/3} \pi \sqrt{e} A^{-6}}{\sqrt{y}}$$

$A = 1.2824271\ldots$ is Glaisher's constant.

Asymptotic $Q = 0$







[M. Olshanii and V. Dunjko (2003); O. I. Patu, A. Klumper (2017)]

Momentum dependent "statistics"

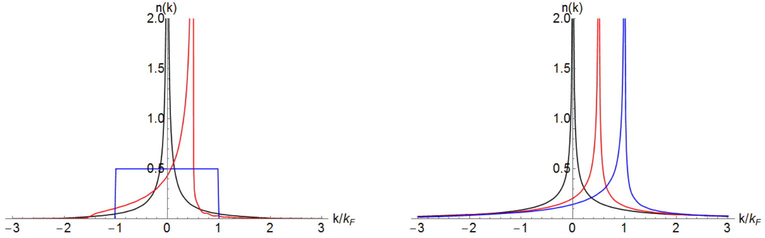


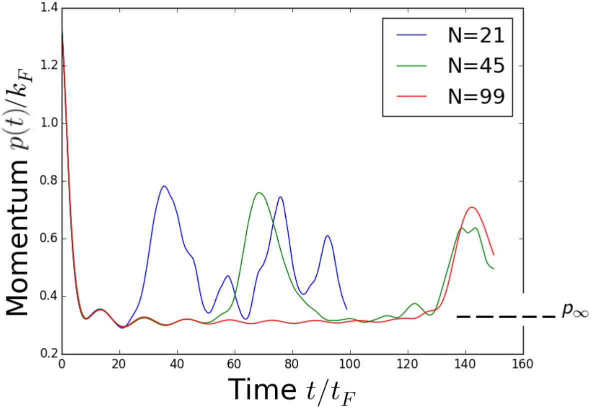
Figure: The momentum distribution for the different total momenta: $Q = 0$, $Q = k_F/2$, $Q = k_F$ - black, red and blue lines respectively. The left panel corresponds to $\alpha = 1/2$ and the right to $\alpha = -1/2$.

- External momentum as an anyonic parameter κ
- $\Psi^{\kappa}(\ldots, x_j, x_{j+1}, \ldots) = e^{-i\kappa\pi \mathrm{sgn}(x_{j+1}-x_j)}\Psi^{\kappa}(\ldots, x_{j+1}, x_j \ldots)$
- $\alpha = 2/g \to 0$ $\qquad$ $\boxed{\kappa = \frac{2}{\pi}\arctan\Lambda = Q/k_F \bmod 2}$

  [R. Santachiara, P. Calabrese (2008); O. I. Patu (2015)]

Time dependent momentum

$$\langle P(t)\rangle = \sum_{\mathbf{k},\mathbf{k}'} \langle \mathbf{q}_{\mathrm{vac}}|c_{p_0}|\mathbf{k}'\rangle\langle\mathbf{k}'|P|\mathbf{k}\rangle\langle\mathbf{k}|c^{+}_{p_0}|\mathbf{q}_{\mathrm{vac}}\rangle e^{-it(E_{\mathbf{k}'}-E_{\mathbf{k}})}$$

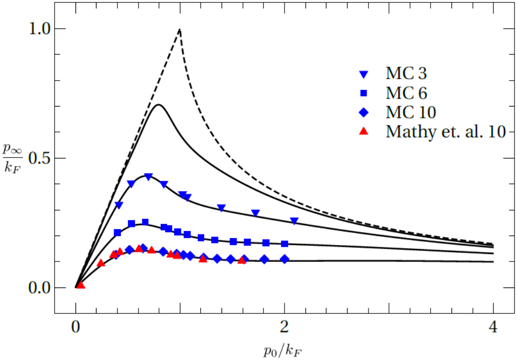


Steady state momentum

$$p_\infty = \sum_{\mathbf{k}} |\langle \mathbf{q}_{\text{vac}} | c_{p_0} | \mathbf{k} \rangle|^2 \langle \mathbf{k} | P | \mathbf{k} \rangle$$

Direct computation is numerically challenging for finite N. But as $N \to \infty$:

$$p_\infty = \int_{-\infty}^{\infty} \frac{d\Lambda}{2\pi} \mathcal{P}(\Lambda) \int_0^{\infty} dx \sin(p_0 x) [(h(x) - 1) \det(1 + V + \delta V) + \det(1 + V)]$$

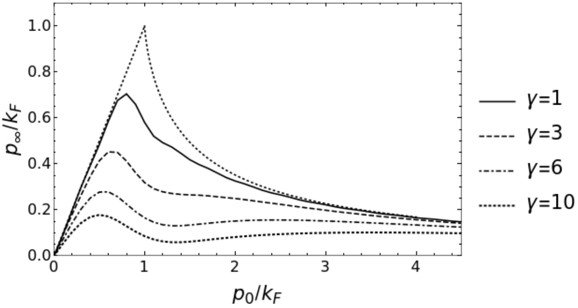


Kick Protocol (Preliminary)

Impurity is instantly accelerated to the additional momentum δP.

$$\Gamma_{\Lambda_q \to \Lambda_k} = \sum_{\mathbf{k};\Lambda_k} \left| \langle \mathbf{k} | e^{-i\hat{X}_{\mathrm{imp}} \delta P} | \mathbf{q} \rangle \right|^2 .$$

$$\boxed{\Gamma_{\Lambda_q \to \Lambda_k} = -\frac{1}{(\Lambda_q - \Lambda_k)^2 Q'(\Lambda_q)} \mathrm{Re} \int_0^\infty \frac{dx}{\pi} e^{-i\delta P x} \frac{d^2 \det(1 + \hat{K})}{dx^2} .}$$



Correlation function

- $$G(x,t) = \langle \psi_{\rm imp}(x,t)\psi^+_{\rm imp}(0,0)\rangle$$
- $$G(x,t) = \int \frac{dp_0}{2\pi}\sum_{\mathbf{k}} |\langle \mathbf{k}|c_{p_0}|\mathbf{q}_{\rm vac}\rangle|^2 e^{ip_0 x - itE_{\mathbf{k}}} = \int \frac{d\Lambda}{2\pi}\rho(x,t,\Lambda)$$
- Exact Result [O.G. A.G. Pronko, M.B. Zvonarev (Nucl. Phys. B 892; NJP 18, 045005)]

$$\rho(x,t,\Lambda) = (h-1)\det(1+\hat{V}) + \det(1+\hat{V}-\hat{W})$$

- "Jordan-Wigner" transformation to anyons

$$c_\lambda(x) = c(x)e^{i\lambda\int^x c^+ c(y)dy}$$

Averaging over the anyonic angle

$$G(x,t) = \int_0^{2\pi}\frac{d\lambda}{2\pi}\langle c^+_\lambda(x,t)c_\lambda(0,0)\rangle$$

Field theory treatment = Microscopic bosonization ($T = 0$)

- Form-Factor summation

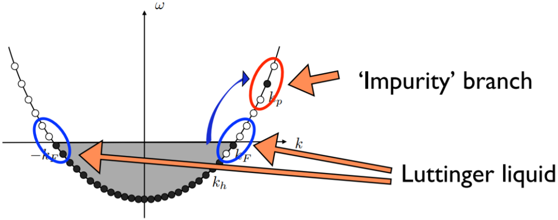
$$\tau(x,t) = \sum_{\mathbf{k}} |\langle \mathbf{q}|\mathcal{O}|\mathbf{k}\rangle|^2 e^{-ixP_{\mathbf{k}}+itE_{\mathbf{k}}} = \det(1+\hat{V})$$

- Orthogonality Catastrophe: $|\langle \mathbf{q}|\mathcal{O}|\mathbf{k}_{\rm vac}\rangle|^2 = \mathcal{A}/N^{2\beta}$
- Soft-mode summation

$$\tau(x,t) \sim \sum_{\rm IR} |\langle \mathbf{q}|\mathcal{O}|\mathbf{k}\rangle|^2 e^{-ixP_{\mathbf{k}}+itE_{\mathbf{k}}} = \langle e^{\sqrt{\beta}\varphi(x,t)} e^{-\sqrt{\beta}\varphi(0,0)}\rangle = \frac{\mathcal{A}e^{-ix\Delta P}}{(x-k_F t)^{\beta}(x+k_F t)^{\beta}}$$

- Nonlinear contributions

$$\tau(x,t) \sim \sum_{Q+\rm IR} |\langle \mathbf{q}|\mathcal{O}|\mathbf{k}\rangle|^2 e^{-ixP_{\mathbf{k}}+itE_{\mathbf{k}}+ix(Q-k_F)+it(Q^2-k_F^2)/2} = \frac{\mathcal{B}e^{-ix\Delta P+i\Delta Et}}{\sqrt{t}(x-k_F t)^{\tilde{\beta}}(x+k_F t)^{\beta}}$$



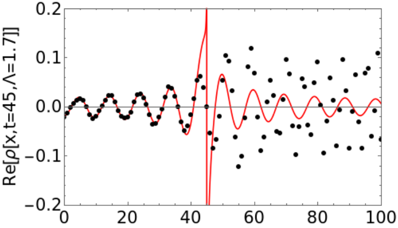
Slavnov (1989); Slavnov and Korepin (1991); A. Shashi, L. I. Glazman, J.-S. Caux, and A. Imambekov (2011); N. Kitanine, K.K. Kozlowski, J.-M. Maillet, N.A. Slavnov, and V. Terras (2009-2012); K.K. Kozlowski, J.-M. Maillet (2015);

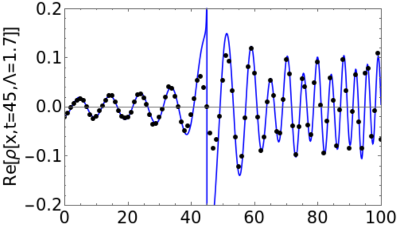
Asymptotics

$$\rho(x,t,\Lambda) = \frac{\mathcal{A}e^{-ixQ+it\Omega}}{(2i(t-x))^{(F_+-1)^2}(2i(t+x))^{F_-^2}} + \sqrt{\frac{2\pi}{it}}\frac{\mathcal{B}e^{-ixQ+it\Omega+i(t-x)^2/2t}}{(2i(t-x))^{F_+^2}(2i(t+x))^{F_-^2}}\theta(|x|>t)$$

$$Q = \frac{(\Lambda+\alpha)\arctan(\Lambda+\alpha)-(\Lambda-\alpha)\arctan(\Lambda-\alpha)}{\alpha\pi} + \frac{1}{2\alpha\pi}\log\frac{1+(\alpha-\Lambda)^2}{1+(\alpha+\Lambda)^2}$$

$$\Omega = \frac{2}{\alpha\pi} - \left(1+\alpha^{-2}(1-\Lambda^2)\right)\left(\frac{\arctan(\alpha-\Lambda)}{\pi}+\frac{\arctan(\alpha+\Lambda)}{\pi}\right) + \frac{\Lambda}{\alpha^2\pi}\log\frac{1+(\alpha-\Lambda)^2}{1+(\alpha+\Lambda)^2}$$



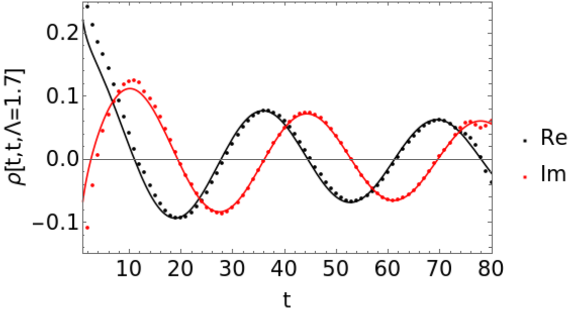


Diagonal Asymptotics

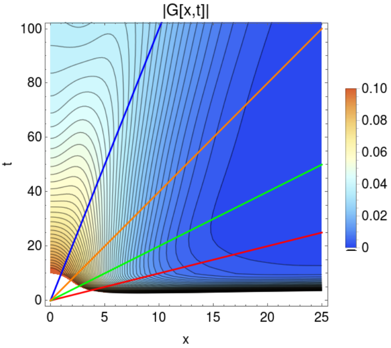
What about $\rho(t,t,\Lambda)$?

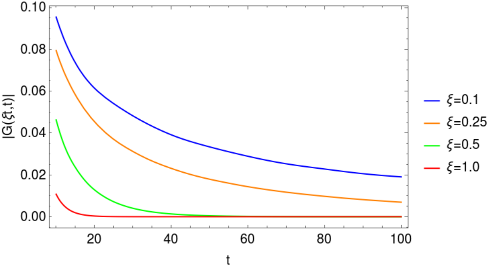
$$\rho(t,t,\Lambda) = \frac{\mathcal{A}e^{-ixQ+it\Omega}}{(2i(t-x))^{(F_+-1)^2}(2i(t+x))^{F_-^2}}\Bigg|_{x\to t} = \frac{\mathcal{A}e^{-itQ+it\Omega}}{(C\sqrt{t})^{(F_+-1)^2}(2i(t+t))^{F_-^2}}$$

$$C = \frac{4e^{i\pi/2}}{\pi}$$



Correlation function: different ray sections





$$G(x=\xi t,t)=\int\frac{d\Lambda}{2\pi}\frac{\mathcal{A}(\Lambda)e^{it(\Omega(\Lambda)-\xi Q(\Lambda))}}{(2i(t-x))^{(F_+(\Lambda)-1)^2}(2i(t+x))^{F_-(\Lambda)^2}}$$

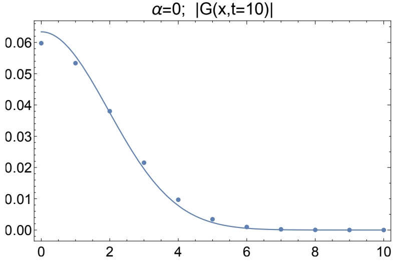
The integral is dominated by the saddle point Λ_s, which exists only for

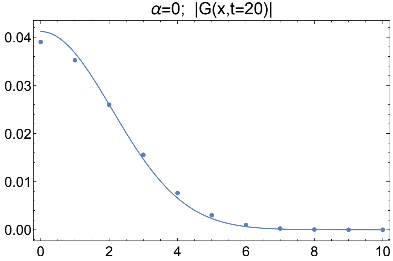
$$\xi<\xi_c=\max_\Lambda\frac{\partial\Omega(\Lambda)}{\partial Q(\Lambda)}$$

Correlation function; TG-limit

Asymptotic for $\rho(x, t, \Lambda)$ in the time-like regime $t \gg x$ leads to

$$G(x,t) \approx \frac{\pi^{3/2} G^4(1/2)}{2\sqrt{it}\sqrt{\log(2t) + \frac{i\pi}{2}}} \exp\left(-\frac{x^2}{2\left(\log(2t) + \frac{i\pi}{2}\right)} - \frac{it}{2} \right)$$

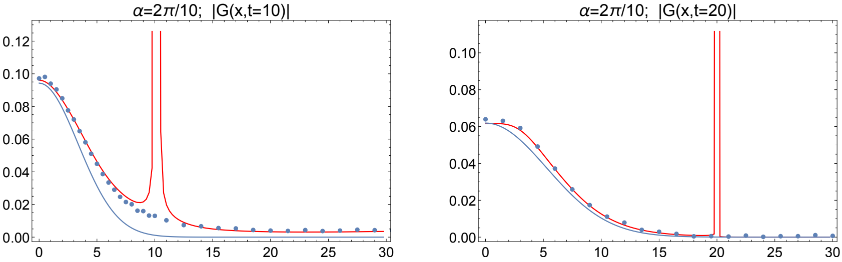




Correlation function; Finite coupling constant

$$G(x,t) = \frac{\pi^{3/2}(2\pi)^{F_+ - F_-} G^2(F_-) G^2(1-F_+) e^{-\tilde{C}[F]}}{(2it)^{2F_+^2}\sqrt{iT}} \exp\left(it\Omega(\alpha) + i\frac{x^2}{T}\left(\frac{\arctan\alpha}{\alpha}\right)^2 \right)$$

$$T = t\frac{\pi^2\Omega(\alpha)}{1+\alpha^2} - \frac{2i(1+2\pi\alpha F_+)\log(2t)}{(1+\alpha^2)^2}$$



Novel regimes? $G(p,t) = \int e^{ipx} G(x,t) = ?$

A. Kantian, U. Schollwöck, T. Giamarchi Phys. Rev. Lett. **113**, 070601 (2014)

Finite temperature

Distribution function

$$n_F(q) = \frac{1}{e^{\beta\epsilon(q)}+1}$$

The main change

$$\det{}_{[-k_F,k_F]}(1+\hat{V}) \to \det{}_{[-\infty,\infty]}(1+n_F\hat{V})$$

Static asymptotic

$$\det(1+K) \to \frac{\mathcal{A}e^{-ix\int\limits_{-k_F}^{k_F} F(k)dk}}{x^{F(k_F)^2+F(-k_F)^2}} \qquad \Rightarrow \qquad \det(1+n_F K) \to \mathcal{A}e^{-ix\int\nu(k)dk}$$

The dressed phase shift

$$\boxed{\nu(k) = \frac{1}{2\pi i}\log(1+(e^{2\pi i F(k)}-1)n_F(k))}$$

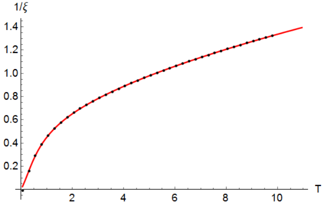
$$\mathcal{A} = \exp\left(-\frac{1}{2}\int dkdq\left(\frac{\nu(q)-\nu(k)}{k-q}\right)^2\right)$$

Finite temperature

$$\mathrm{Tr}(e^{-\beta H}\mathcal{O}(x,t)\dots)/\mathrm{Tr}e^{-\beta \mathrm{H}} = \langle \mathcal{O}(z=x+it)\dots\rangle_{\mathbb{S}^1\times\mathbb{R}^1} \overset{z\to z'=e^{2\pi z/\beta}}{=} \sim \langle \mathcal{O}(z')\dots\rangle_{\mathbb{R}^2}$$

CFT prediction for correlation length:

$$\tau(x)\Big|_{T=0} = \frac{\mathcal{A}}{x^{\nu^2}} \Longrightarrow \qquad \tau(x) = \frac{\mathcal{A}}{(\sinh(xT)/T)^{\nu^2}} \sim e^{-x/\xi} \Longrightarrow 1/\xi \sim T???$$



Finite temperature + Dynamics

$$\det(1+n_F V(\hat{x},t)) \to \frac{1}{t^{\nu(x/t)}} \exp\left(-i\int (x-qt)\nu(q)dq\right)$$

Summary and outlook

- Anyons $n_p = \sum_{\mathbf{q}} |\langle \mathbf{q}|c_p|\mathbf{k}\rangle|^2$ ✓
- Quantum flutter $p_\infty = \sum_{\mathbf{k}} |\langle \mathbf{q}|c_p|\mathbf{k}\rangle|^2 \langle \mathbf{k}|P_{\mathrm{imp}}|\mathbf{k}\rangle$ ✓
- Log-diffusion $G_p(t) = \sum_{\mathbf{k}} |\langle \mathbf{q}|c_p|\mathbf{k}\rangle|^2 e^{-itE_{\mathbf{k}}}$ ✓

To Do:

- Finite temperature
- "Kicked" profile (external force)
- $\langle P_{\mathrm{imp}}(t)\rangle$ as Fredholm determinant
- New formulation of universality? non CFT-like?

Extra Slides

Asymptotic without RHP = bosonization

- Form-Factor summation

$$\rho(y) = \sum_q |\langle \mathbf{q}|c_p|\mathbf{k}\rangle_Q|^2 e^{iy(P_{\mathbf{q}}-Q)} = \det(1+\hat{K}+\delta\hat{K}) - \det(1+\hat{K})$$

- Orthogonality Catastrophe

$$|\langle \mathrm{vac}|c_p|\mathbf{k}\rangle_Q|^2 = \frac{\mathcal{A}}{N^\alpha}$$

- Bosonization $\rho(y) \sim \dfrac{\mathcal{A}}{(-iy)^\alpha} e^{iy(P_{\mathrm{vac}}-Q)}$

Slavnov (1989); Slavnov and Korepin (1991); A. Shashi, L. I. Glazman, J.-S. Caux, and A. Imambekov (2011); N. Kitanine, K.K. Kozlowski, J.-M. Maillet, N.A. Slavnov, and V. Terras (2009-2012); K.K. Kozlowski, J.-M. Maillet (2015);

Soft mode summation

$$\frac{|\langle P_k, H_k | c_p | \mathbf{k} \rangle_Q|^2}{|\langle \mathrm{vac} | c_p | \mathbf{k} \rangle_Q|^2} = \left(\det_{1 \le i,j \le k} \frac{1}{p_i + h_j - 1} \right)^2 \left(\frac{\sin \pi F_+}{\pi} \right)^{2k} \prod_{j=1}^{k} \frac{\Gamma(p_j - F_+)^2}{\Gamma(p_j)^2} \prod_{j=1}^{k} \frac{\Gamma(h_j + F_+)^2}{\Gamma(h_j)^2}$$

$$P_{\mathbf{q}} - Q = \int_{-1}^{1} dq F(q) + \frac{2\pi}{L} \sum_{j=1}^{k} (p_j + h_j - 1)$$

$$e^{iy(P_{\mathbf{q}} - Q)} \frac{|\langle P_k, H_k | c_p | \mathbf{k} \rangle_Q|^2}{|\langle \mathrm{vac} | c_p | \mathbf{k} \rangle_Q|^2} = e^{iy\Delta P} \langle 0 | e^{F_+ \varphi(1)} | P_k, H_k \rangle \langle P_k, H_k | e^{F_+ \varphi(z)} | 0 \rangle$$

$$z = e^{2\pi i / L}$$

Soft mode summation

$$\{\psi_n, \psi_m^+\} = \delta_{nm}, \qquad \{\psi_n, \psi_m\} = \{\psi_n^+, \psi_m^+\} = 0.$$

$$\psi_n|0\rangle = 0, \quad \text{if} \quad n > 0, \qquad \psi_n^+|0\rangle = 0, \quad \text{if} \quad n \leq 0$$

$$|P_k, H_k\rangle = \psi_{p_1}^+ \dots \psi_{p_k}^+ \psi_{1-q_1} \dots \psi_{1-q_k}|0\rangle$$

Bosonization

$$\partial\varphi(z) =: \psi^+(z)\psi(z):$$

$$\sum_{P_k,H_k} e^{iy(P_{\mathbf{q}}-Q)}|\langle P_k, H_k|c_p|\mathbf{k}\rangle_Q|^2 = e^{iy\Delta P}|\langle \mathrm{vac}|c_p|\mathbf{k}\rangle_Q|^2 \sum_{P_k,H_k} \langle 0|e^{F_+\varphi(1)}|P_k, H_k\rangle\langle P_k, H_k|e^{F_+\varphi(z)}|0\rangle =$$

$$= |\langle \mathrm{vac}|c_p|\mathbf{k}\rangle_Q|^2 e^{iy\Delta P}\langle 0|e^{F_+\varphi(1)}e^{F_+\varphi(z)}|0\rangle = |\langle \mathrm{vac}|c_p|\mathbf{k}\rangle_Q|^2 \frac{e^{iy\Delta P}}{(1-z)^{F_+^2}} = \frac{\mathcal{A}}{N^{F_+^2}} \frac{e^{iy\Delta P}}{(1-e^{2\pi i/L})^{F_+^2}}$$

$$\boxed{\sum_{P_k,H_k} e^{iy(P_{\mathbf{q}}-Q)}|\langle P_k, H_k|c_p|\mathbf{k}\rangle_Q|^2 = \frac{\mathcal{A}e^{iy\Delta P}}{(-2k_F iy)^{F_+^2}}}$$

- Any soft modes excitations
- Non-linear bosonization