Quasiholes in the lowest Landau level pinned by impurities :

Detection of non-Abelian statistics by means of density profiles

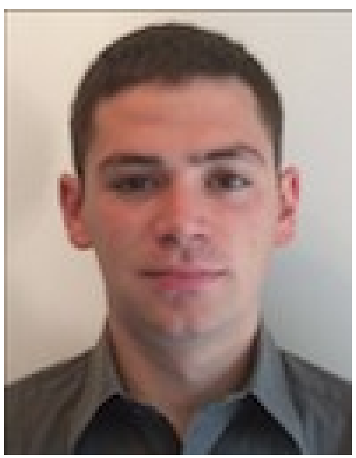
Leonardo Mazza

IPA, September 2021

Dynamics and control of impurities in complex quantum environments

LPTMS, Université Paris Saclay

Acknowledgements

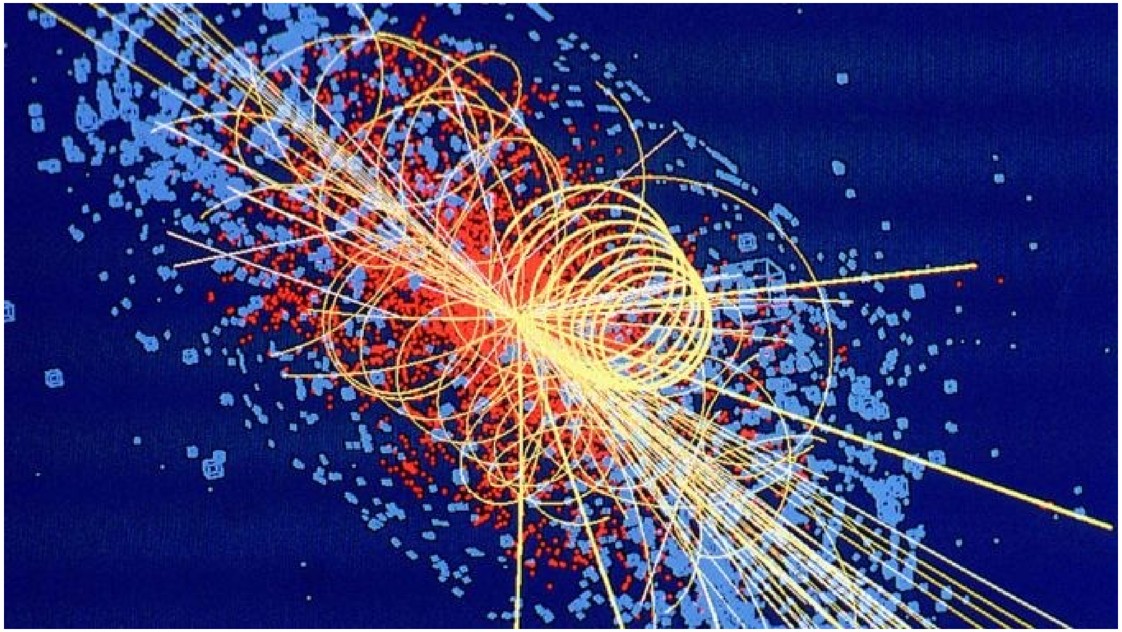


- Elia Macaluso (now private sector)
- Tommaso Comparin (now ENS Lyon)
- Iacopo Carusotto,

Trento, BEC center

E. Macaluso, T. Comparin, LM, I. Carusotto
Phys. Rev. Lett. **123**, 266801 (2019)

Quantum statistics



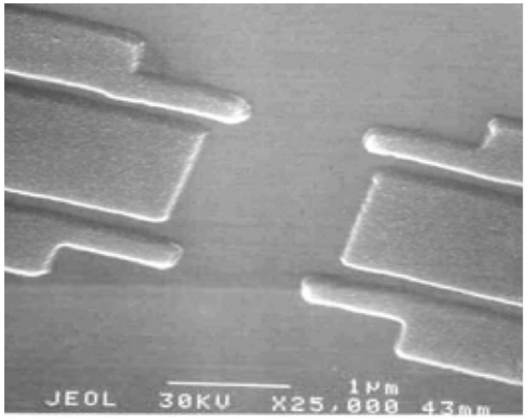
Spin-statistics theorem

Fundamental particles of the 4-dimensional space-time are:

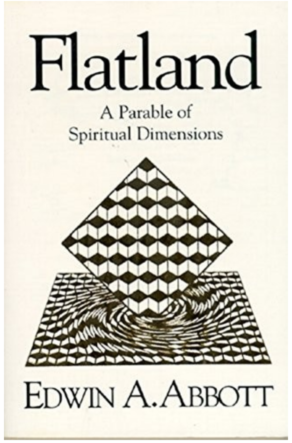
- **bosons** (integer spin, commuting field)
- **fermions** (half-integer spin, anti-commuting field)

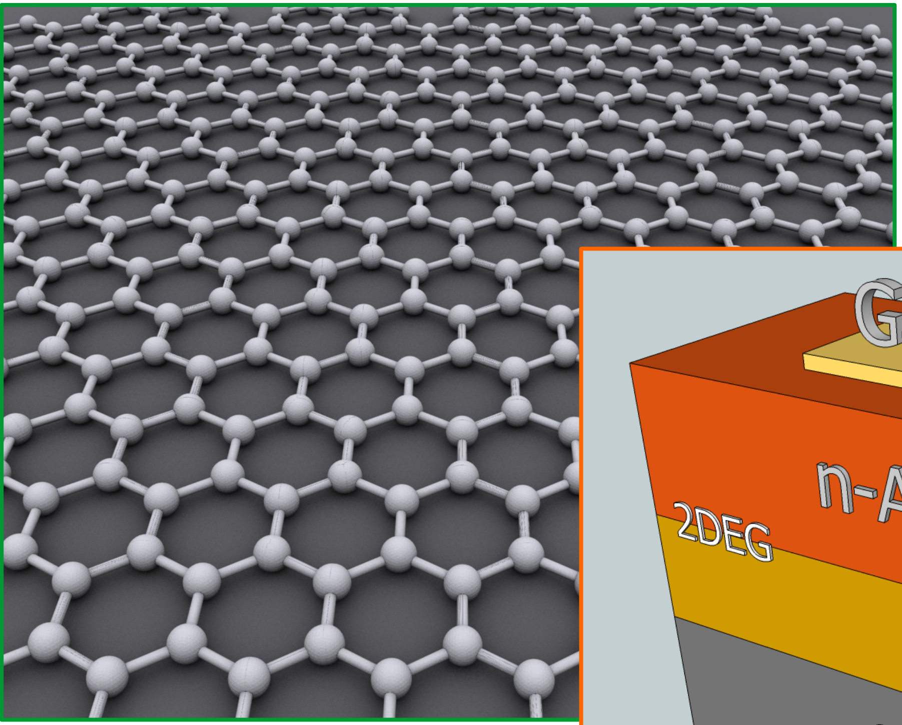
Anything else is an **anyon**

- No elementary particle in our universe
- Two spatial dimensions
- Quasi-particle excitation of a 2D many-body system

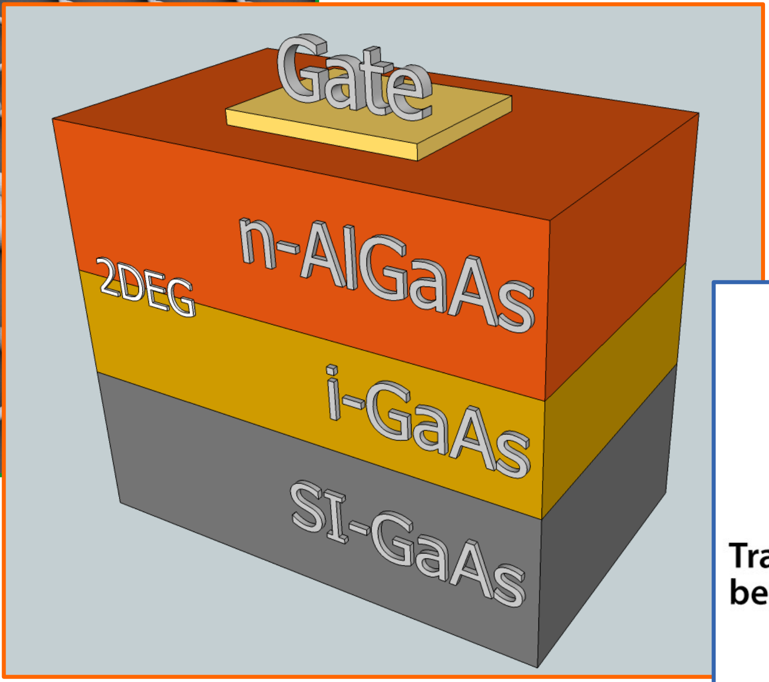


Quantum flatland



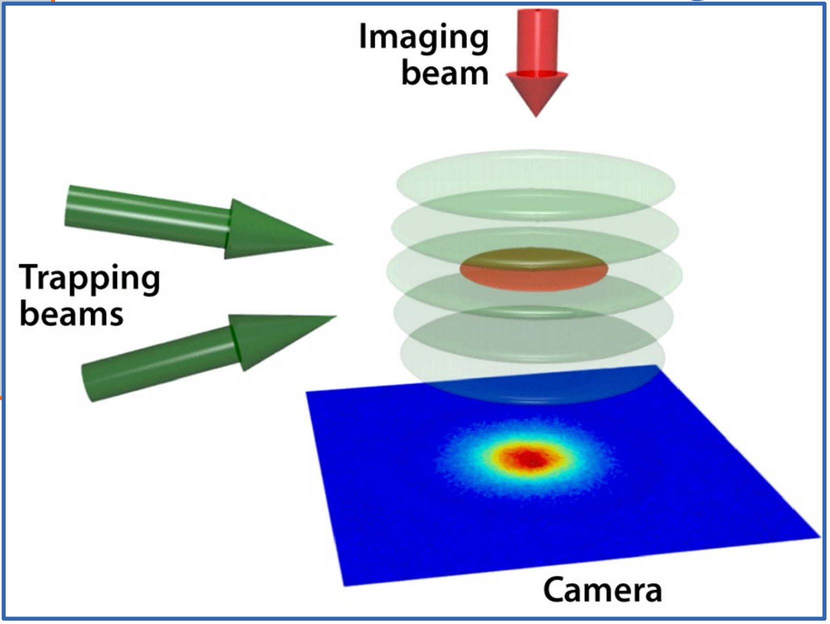


Graphene



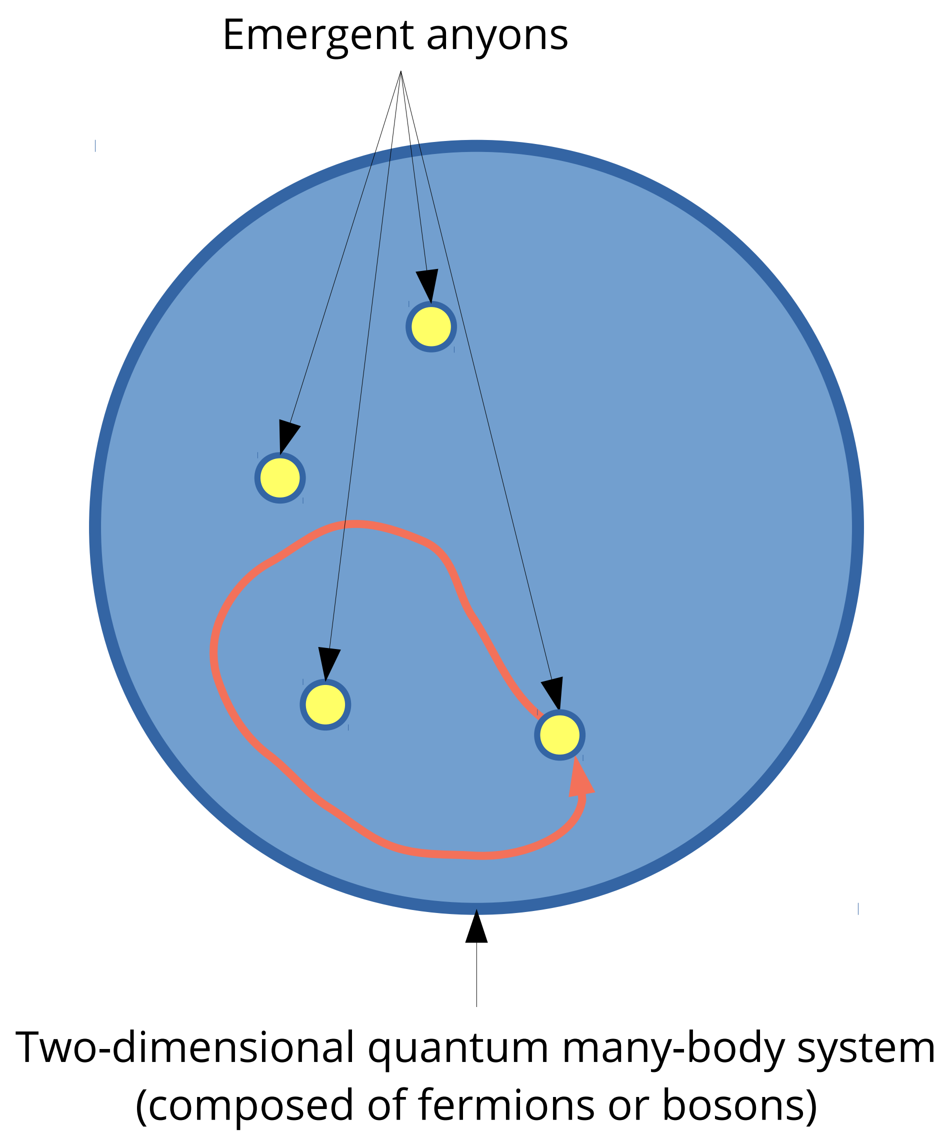
Two-dimensional
electron gases

Two-dimensional
atomic gases



- In these systems, we will (hopefully) find anyons

Anyonic statistics



No symmetrization postulate.

What is the geometric phase picked up by my quantum state after this loop process?

Topological contribution: presence/absence of an anyon in the loop

3D: contractible loops
2D: non trivial
1D: no loops possible

A. Kahre, Fractional Statistics and Quantum Theory (2005); Lerda, Anyons (1992)

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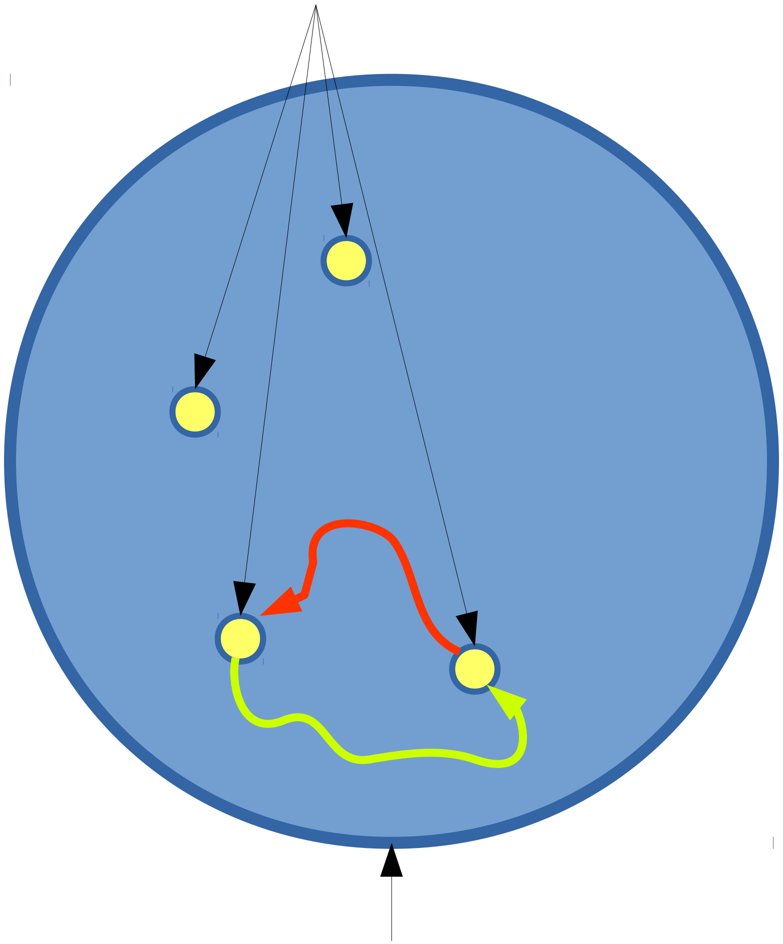
Anyonic statistics

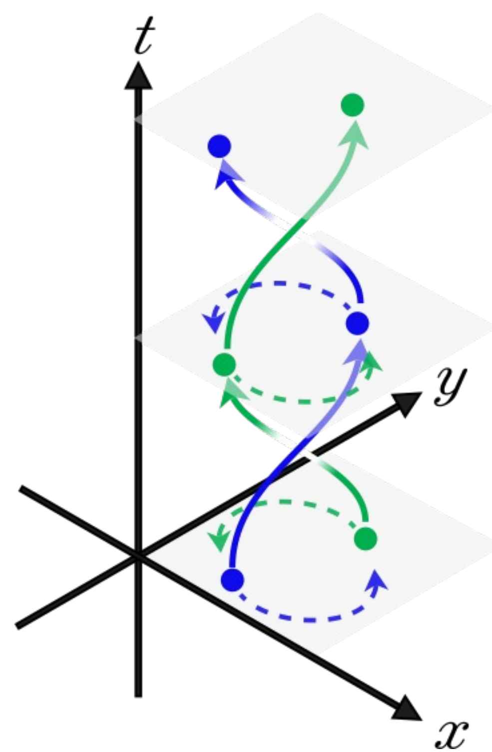
Emergent anyons



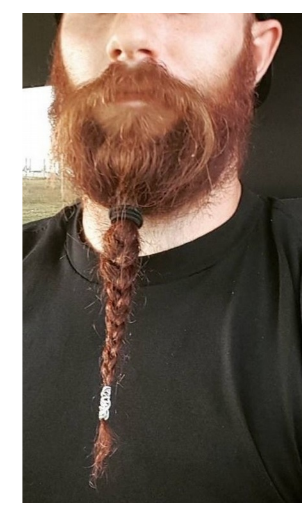
Two-dimensional quantum many-body system
(composed of fermions or bosons)

More precise:

Quantum statistics for identical particles
*from the notion of **exchange** of particles:*



One braiding
=
Two exchanges



A. Kahre, Fractional Statistics and Quantum Theory (2005); Lerda, Anyons (1992)

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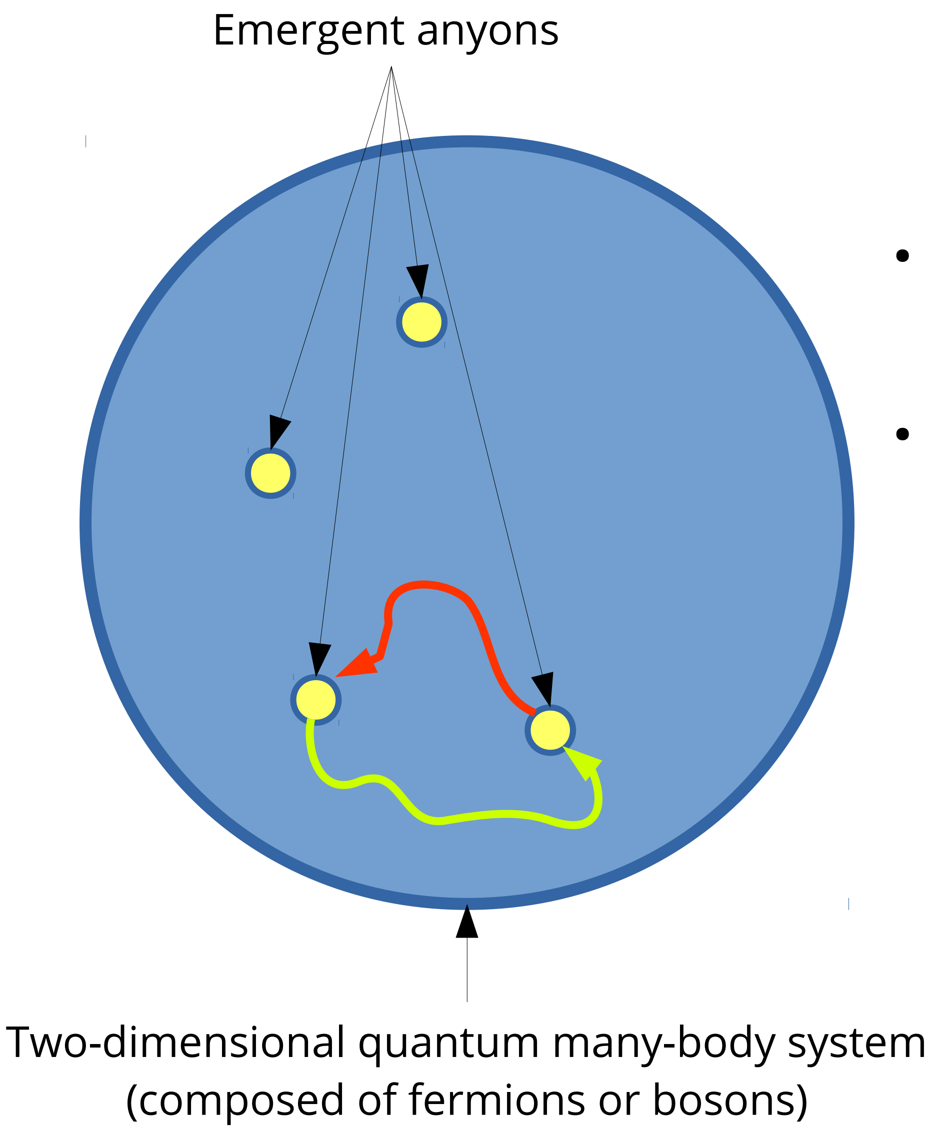
Anyonic statistics



Exchange phase = quantum statistics

- Bosons $\Psi(\vec{r}_1, \vec{r}_2) \to +\Psi(\vec{r}_2, \vec{r}_1)$
- Fermions $\Psi(\vec{r}_1, \vec{r}_2) \to -\Psi(\vec{r}_2, \vec{r}_1)$

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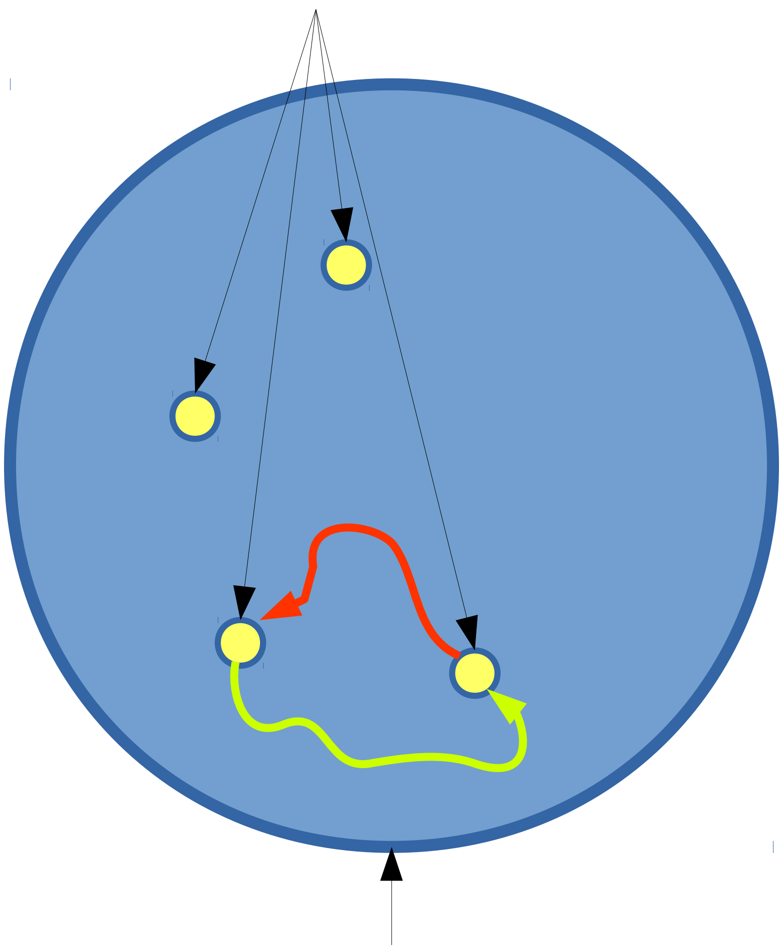
Anyonic statistics

Emergent anyons



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- Abelian anyons $\Psi(\vec{r}_1, \vec{r}_2) \to e^{i\varphi}\Psi(\vec{r}_2, \vec{r}_1)$
- Non-Abelian anyons $\vec{\Psi}(\vec{r}_1, \vec{r}_2) \to \mathcal{U}\vec{\Psi}(\vec{r}_2, \vec{r}_1)$

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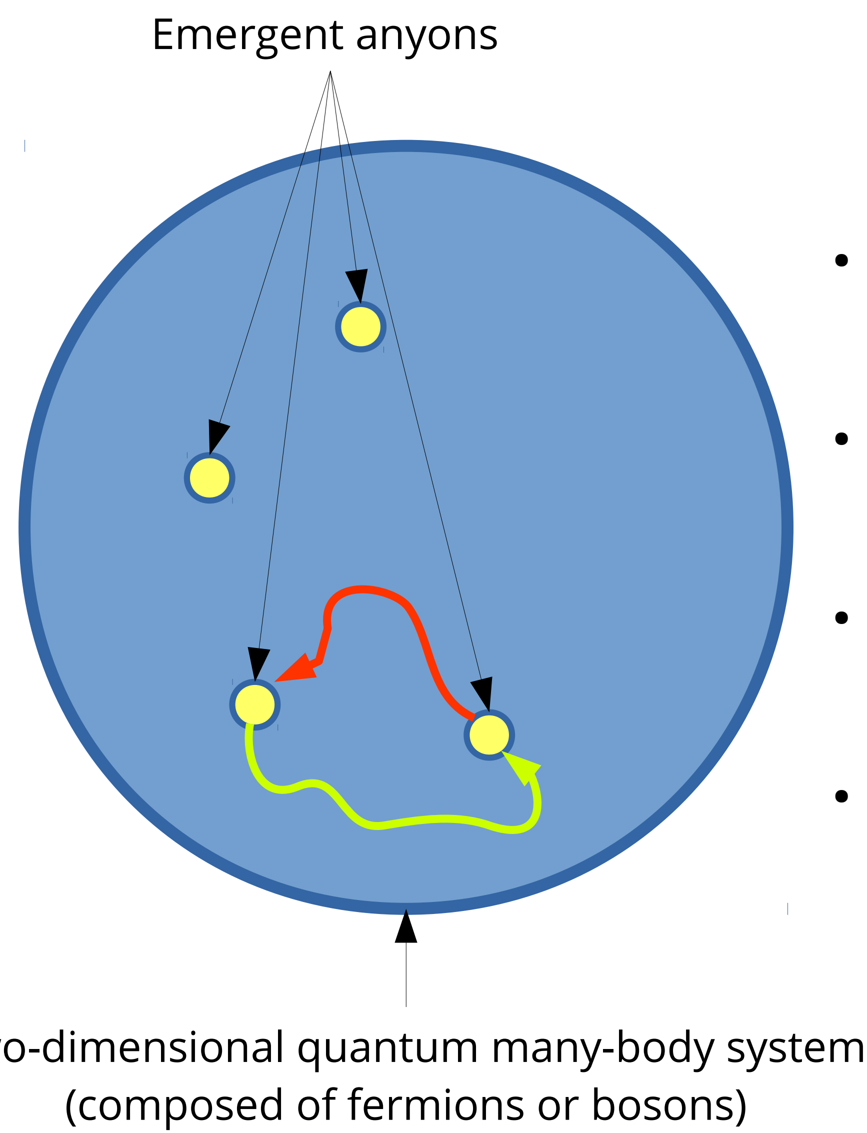

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Quantum gate for quantum computing?

A. Kahre, Fractional Statistics and Quantum Theory (2005); Lerda, Anyons (1992)

The problem

Anyons:

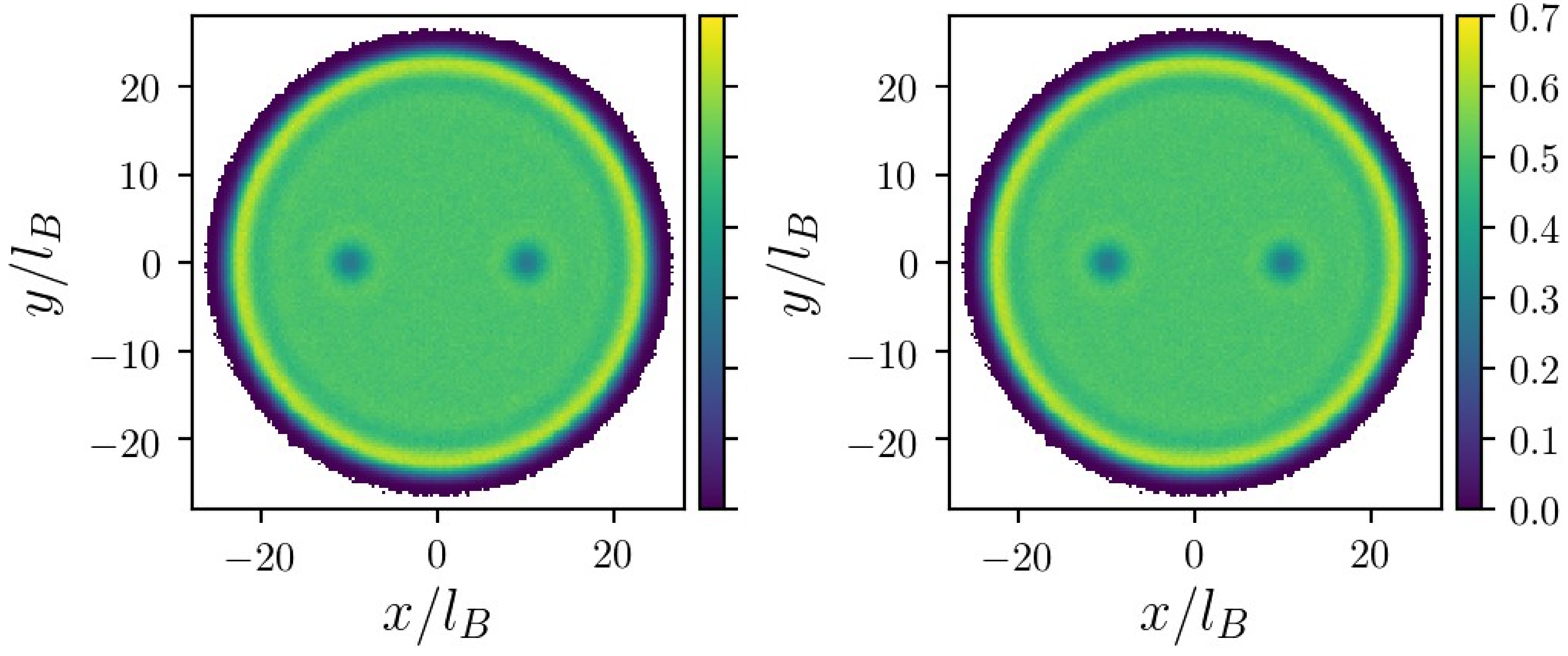
- One of the holy grails of the community working in cold atoms
  - Exceptional scientific interest
  - Possible technological relevance

Assuming we have created anyons,
how can we show that they are indeed anyons?

...the solid-state community is facing exactly this problem...

Today's talk

A novel characterisation method for non-Abelian anyons using **density profiles**



The comparison of these density profiles for anyons put in appropriate configurations gives info on their non-Abelian quantum statistics

Outline

- Introduction: Anyons
- Part #1: Detecting anyons with density measurements
- Part #2: The case of the Moore-Read wavefunction
- Conclusions

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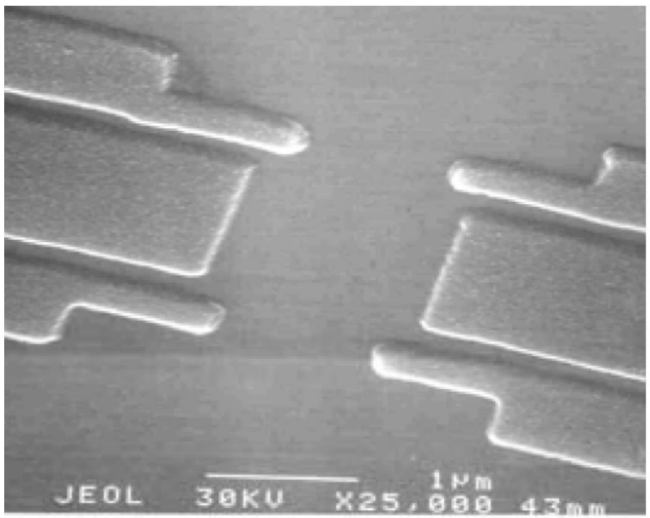
The curiosity of a theorist?

Anyons: a powerful concept for modeling/interpreting experiments

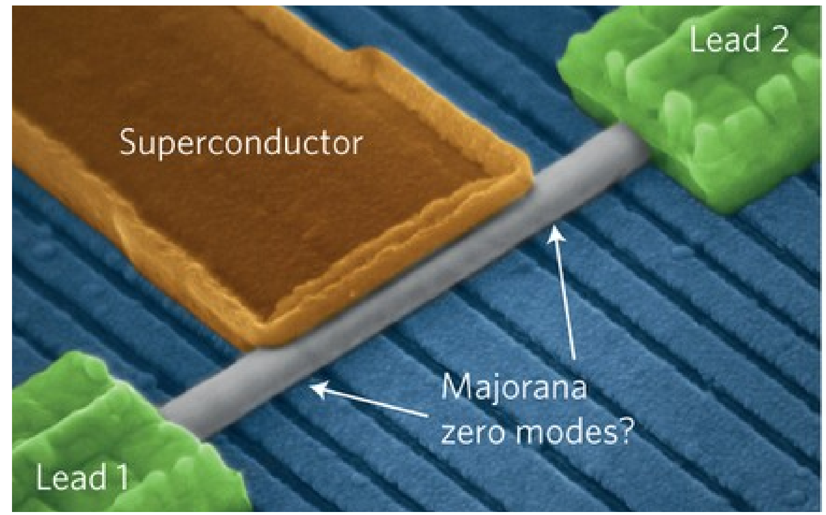


Two Hall bars
- 2DEG with perpendicular magnetic field

- should host anyons

- AlGaAs – GaAs heterostructures
- 25-100 mK / 10 Tesla

Willett, Pfeiffer, West, PNAS, 106, 8853 (2009)



One nanowire proximitized to a superconductor
- Effective Kitaev chain

- should host Majorana zero modes – non-Abelian anyons

- controversial

Kouwenhoven group, 2012

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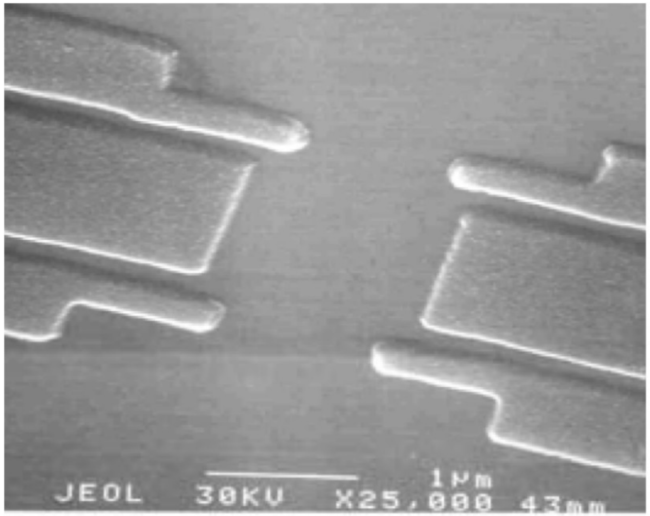

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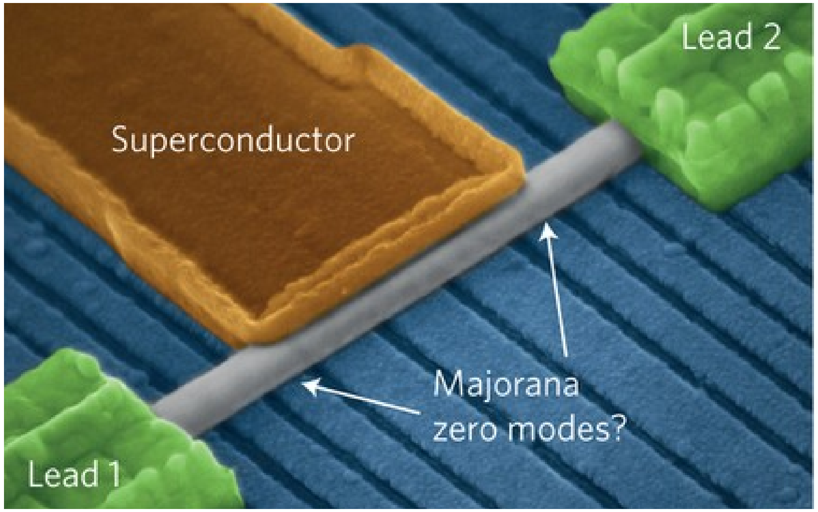

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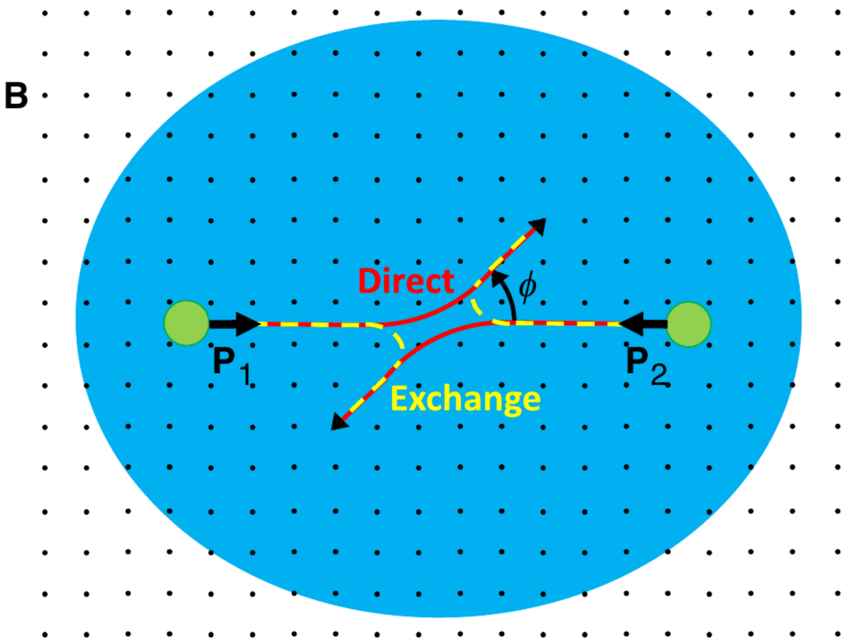
Anyons and impurities

Heavy impurities immersed in a fractional quantum Hall liquid

- Born-Oppenheimer approximation
- effective quasiparticles with anyonic statistics



See early works:
Zhang, Sreejith, Gemelke, Jain PRL 2014
Zhang, Sreejith, Jain PRB 2015
Loundholm, Rougerie PRL 2016
Grusdt, Yao, Abanin, Fleischhauer, Demler, Nat Comm 2016
Yakaboylu, Lemeshko PRB 2018
Yakaboylu, Ghazaryan, Lundholm, Rougerie, Lemeshko, Seiringer PRB 2020
Grass, Julia-Diaz, Baldelli, Lewenstein PRL 2020
Monuz de las Heras, Macaluso, Carusotto PRX 2020

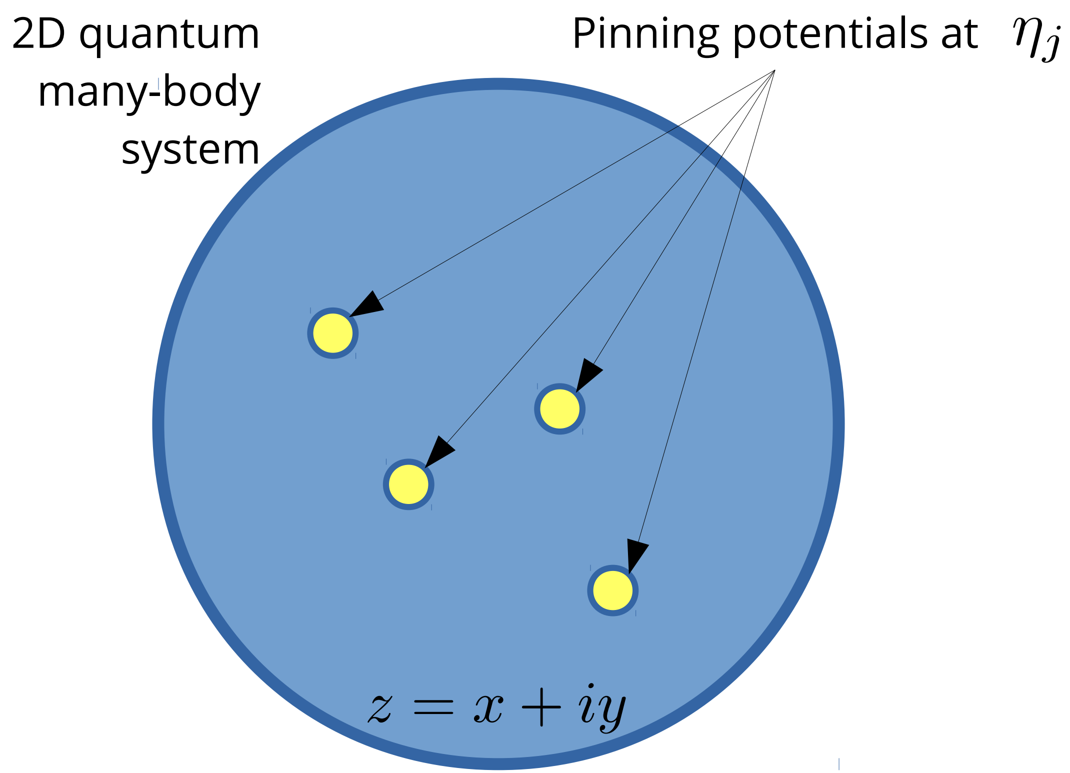
In this talk:

the simplest situation: external localised potentials pin the anyons

My goal:

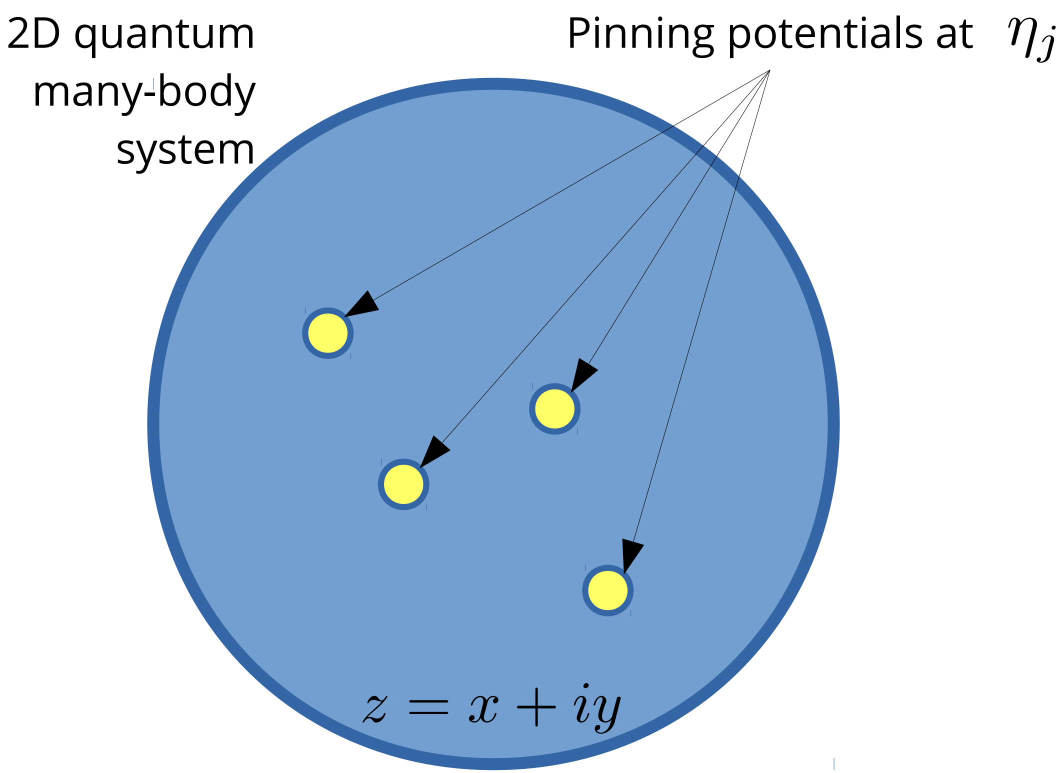
It is possible to measure their anyonic statistics
without interferometric schemes

Anyons in quantum many-body systems



$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j)$$

Anyons in quantum many-body systems



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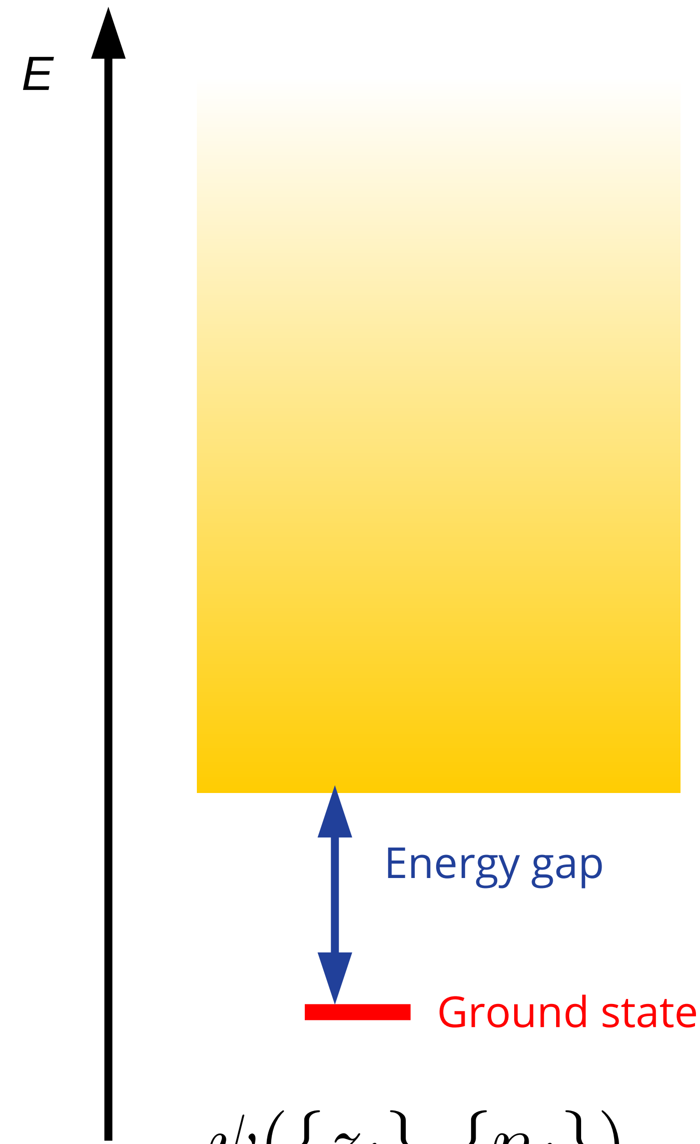
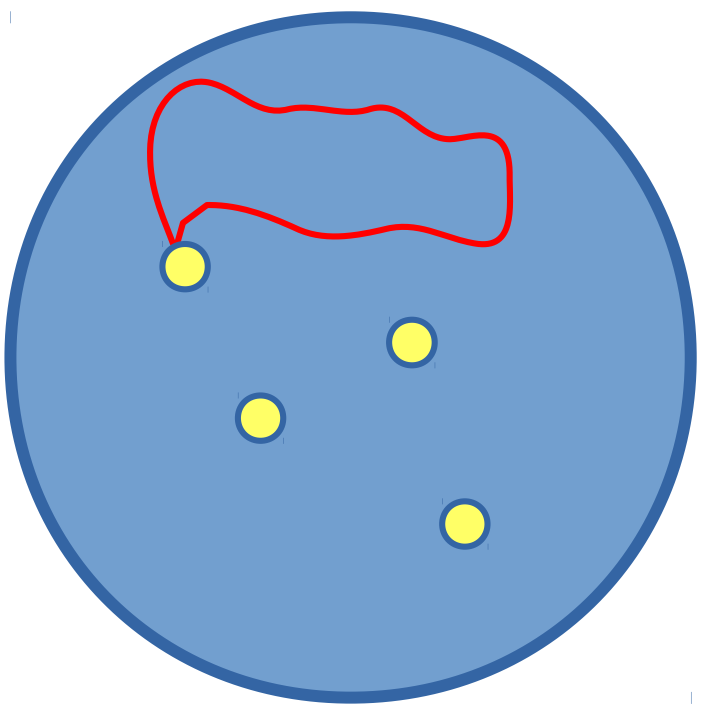


$$\psi(\{z_i\}, \{\eta_j\})$$

Exotic quantum statistics

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(t))$$



Quantum statistics for identical particles
from the notion of adiabatic exchange of particles

Adiabaticity:

- ensured by the many-body energy gap

$$T \gg h/E_{\text{gap}}$$

Old things, nicely reviewed here:
Bonderson, Gurarie and Nayak, PRB **83**, 075303 (2011)

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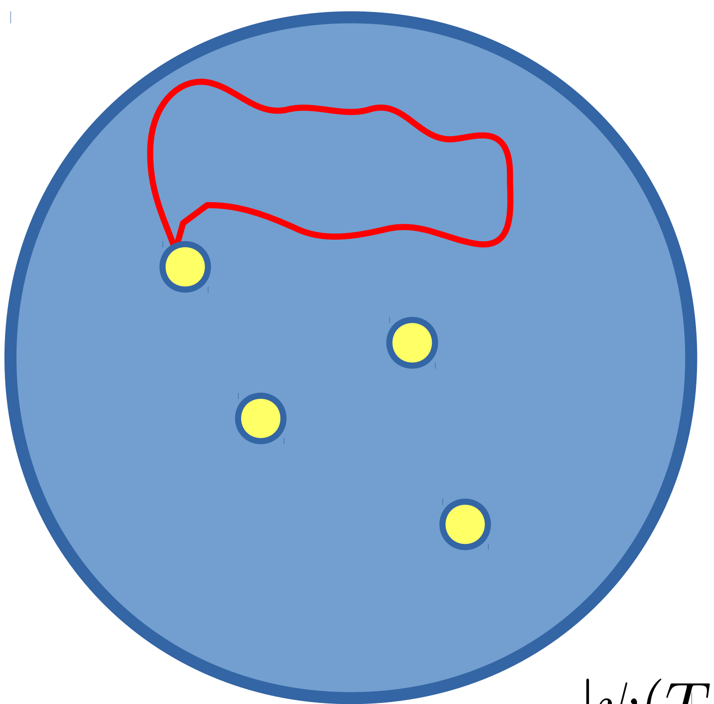
- geometric contribution to the time evolution (Berry phase)



$$|\psi(T)\rangle = e^{-\frac{i}{\hbar}E_{GS}T} \exp\left[i\int_0^T \mathcal{A}(t')\mathrm{d}t'\right] |\mathrm{GS}(\eta_j(T))\rangle$$

Dynamical phase

Geometric contribution

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Exotic quantum statistics

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(t))$$

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from the notion of adiabatic exchange of particles

Geometric contribution: Berry connection

$$\mathcal{A}(t) = i\langle \, \text{GS}(\eta_j(t)) \, | \frac{\text{d}}{\text{d}t} | \, \text{GS}(\eta_j(t)) \, \rangle$$

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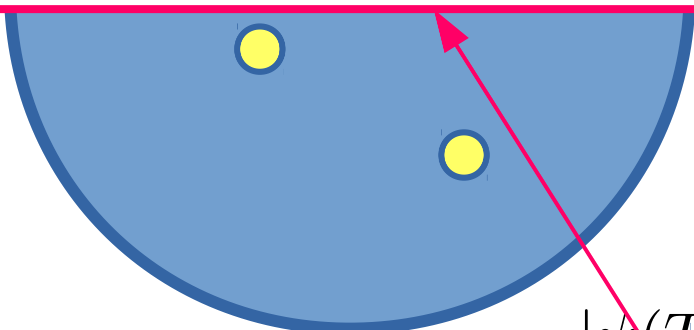

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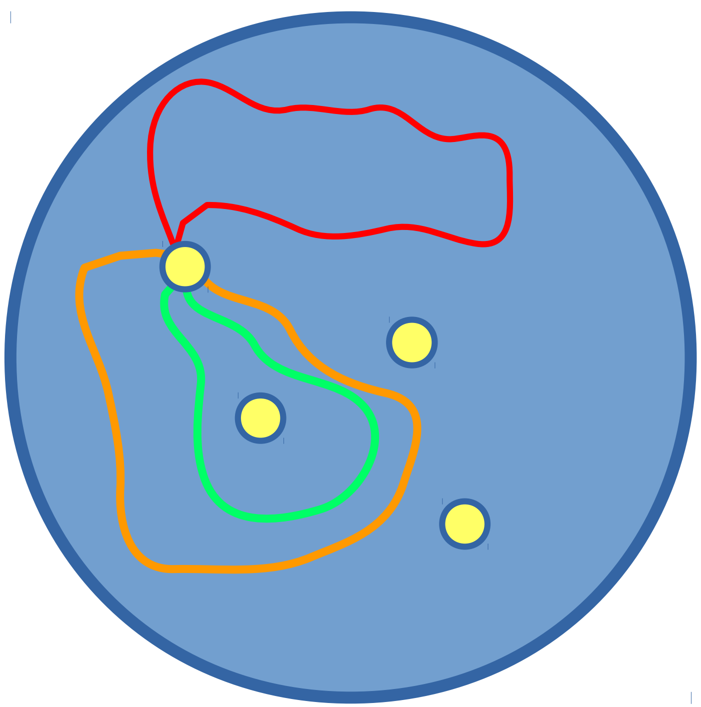

The geometric contribution:

- **Non-topological part**
  - Different in the three cases
- **Topological part**
  - Absent for the **red** path
  - Present and equal for the **orange** and **green** paths

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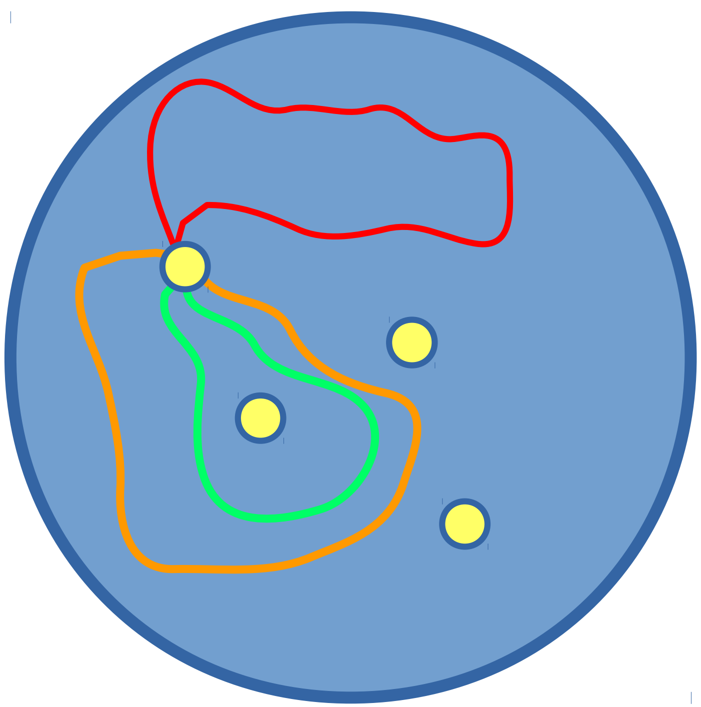

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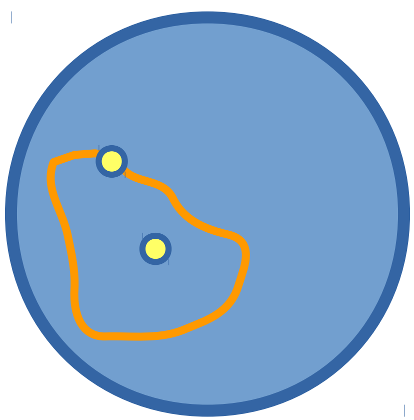
This encodes the quantum statistics
(and works only in 2D)

Exotic quantum statistics

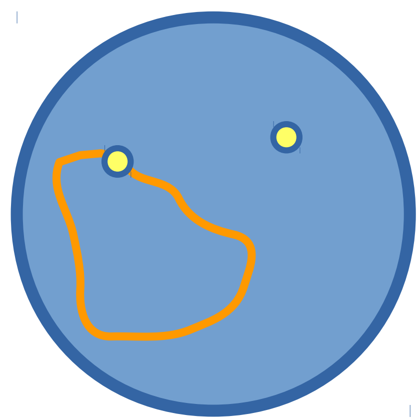
$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(t))$$

Quantum statistics for identical particles
from the notion of adiabatic exchange of particles

An idealised recipe for obtaining the geometric phase:



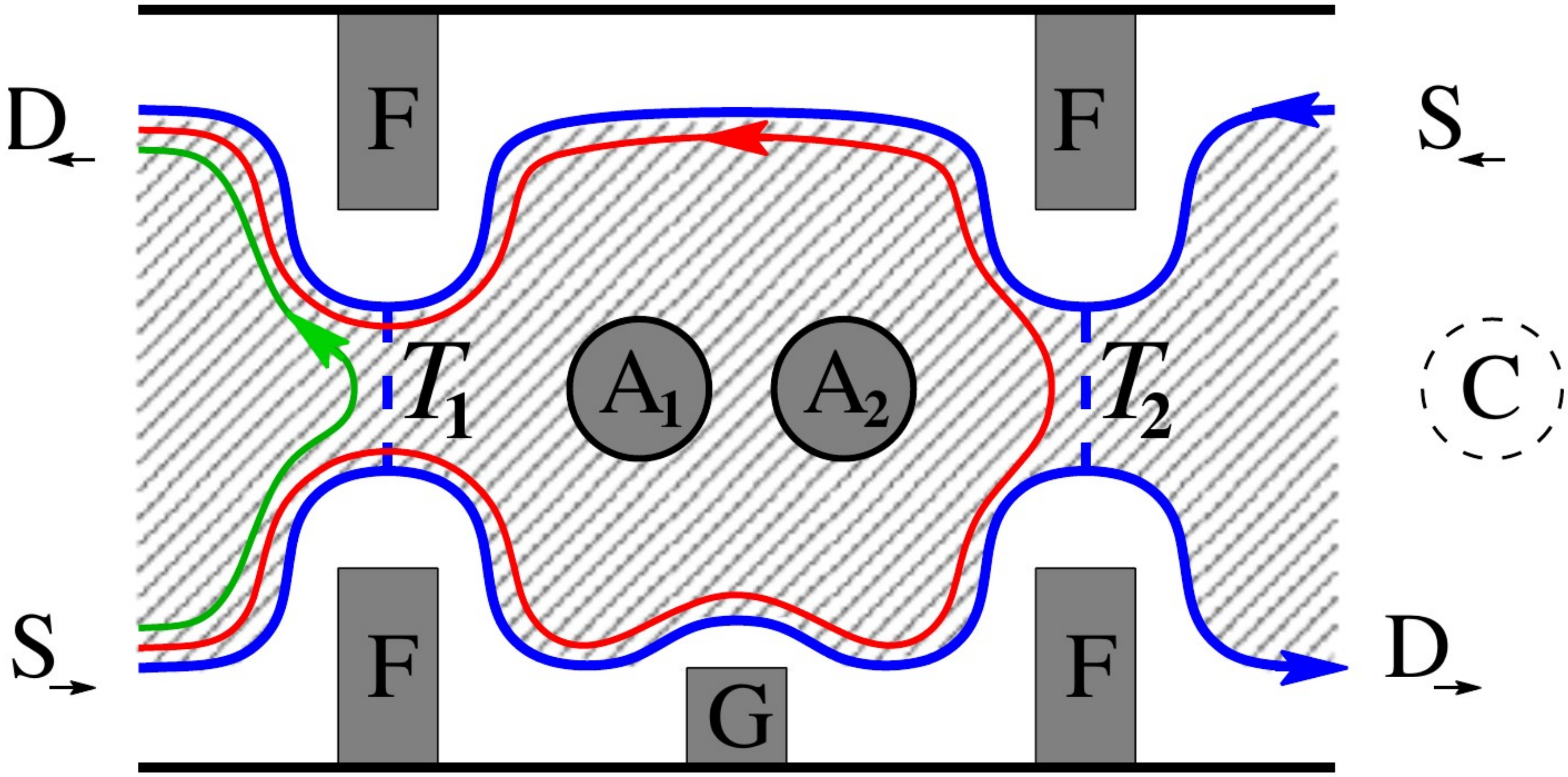
Φ_1



Φ_2

$$2\varphi_{br} = \Phi_1 - \Phi_2$$

Measuring anyonic statistics: Interferometry



Bonderson, Shtengel, Slingerland, Annals of Physics **323**, 2709 (2008)

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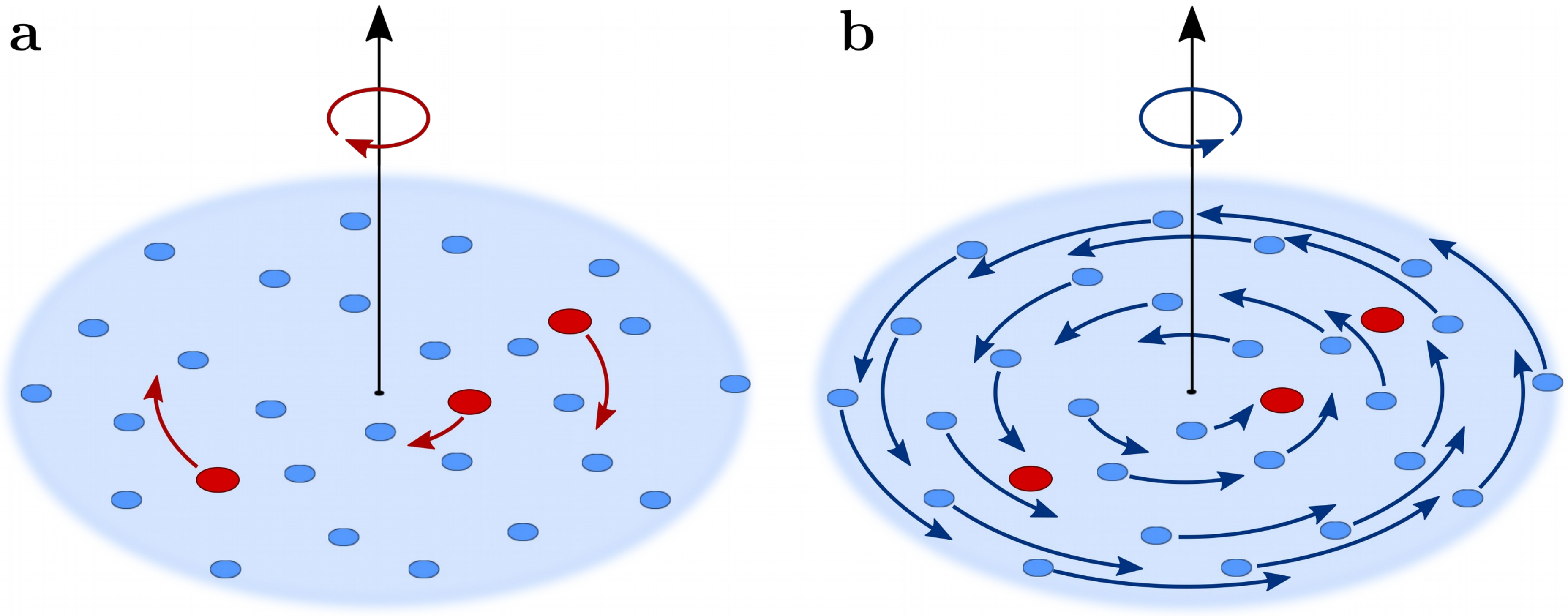
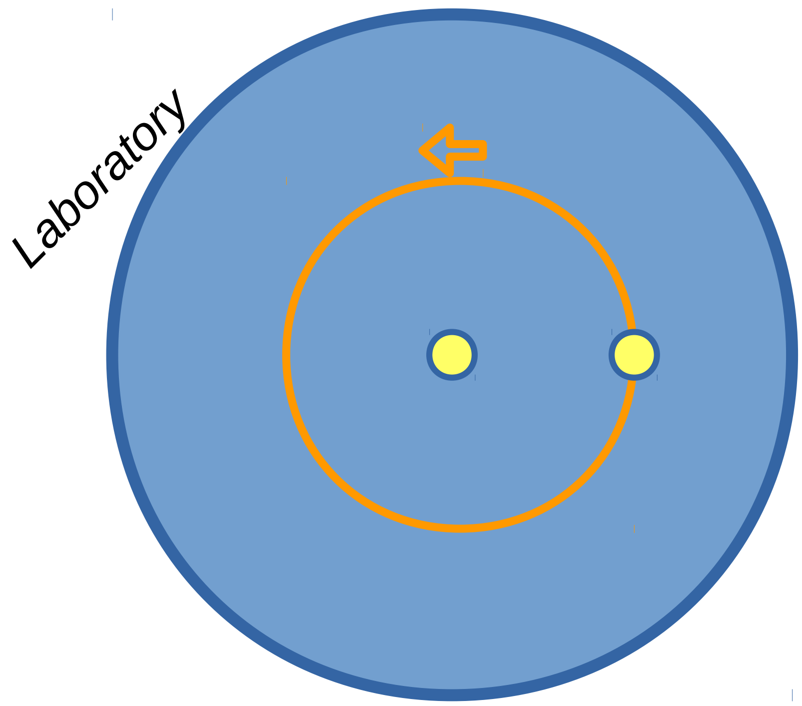
Semi-classically...



Instead of looking at anyons rotating around in the lab,

why don’t I look at them in the reference frame where they are at rest?

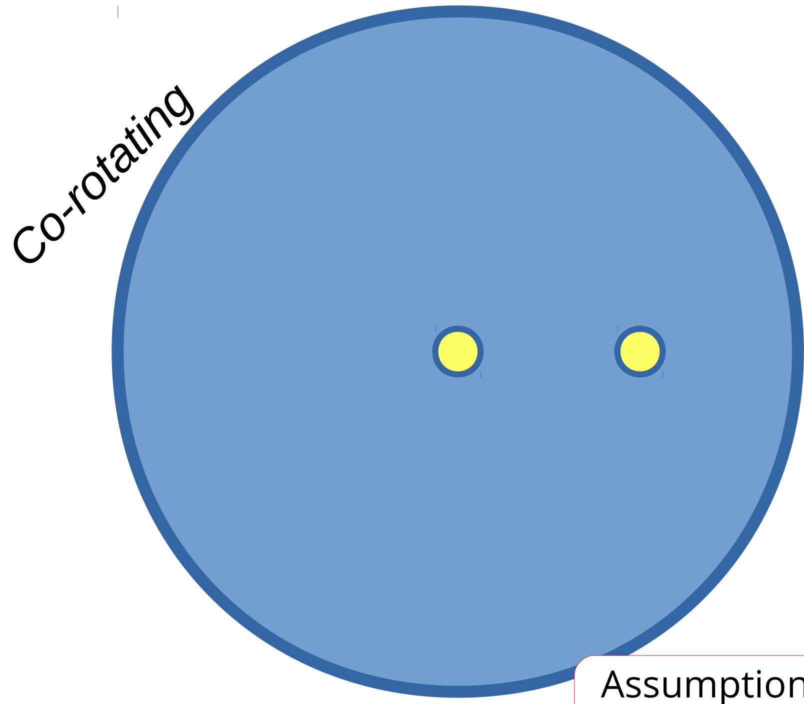
Co-rotating reference frame



$$\hat{H}(t) = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(t))$$

$$\eta_j(t) = \eta_j(0)e^{i\frac{\theta_f}{T}t} \quad \forall j$$

$$V(t) = e^{\frac{i}{\hbar}L_z\frac{t}{T}\theta_f}$$



Assumption: rotational invariance of $\hat{H}_0$

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(0)) - \frac{\theta_f}{T}\hat{L}_z$$

Co-moving Schroedinger equation

$$\hat{H} = \underbrace{\hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(0))}_{\text{Gapped many-body Hamiltonian}} \underbrace{- \frac{\theta_f}{T}\hat{L}_z}_{\text{Small perturbation}}$$

$t \in [0, T]$

In the infinite T limit, the system does not leave the ground space because the perturbation is small (adiabatic theorem)

Co-moving Schroedinger equation

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Gapped many-body Hamiltonian

Small perturbation

$t \in [0, T]$

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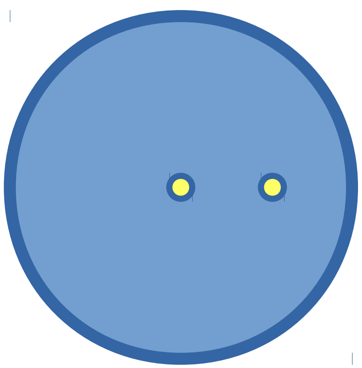

$|GS\rangle$

$$|\psi_2(t)\rangle = e^{-\frac{i}{\hbar}E_0 t}\gamma(t)|GS\rangle$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\gamma(t) = -\frac{\theta_f}{T}\langle GS|\hat{L}_z|GS\rangle\gamma(t)$$

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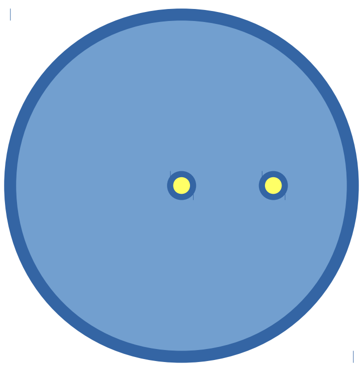

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In the lab reference frame

$$|\psi(T)\rangle = e^{-\frac{i}{\hbar}\hat{L}_z\theta_f}|\psi_2(T)\rangle$$

$$|\psi(T)\rangle = e^{-\frac{i}{\hbar}E_0 T}\; e^{-\frac{i}{\hbar}\hat{L}_z\theta_f}\; e^{\frac{i}{\hbar}\theta_f\langle\hat{L}_z\rangle}\;|GS\rangle$$

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Geometric phase:
we are interested in it

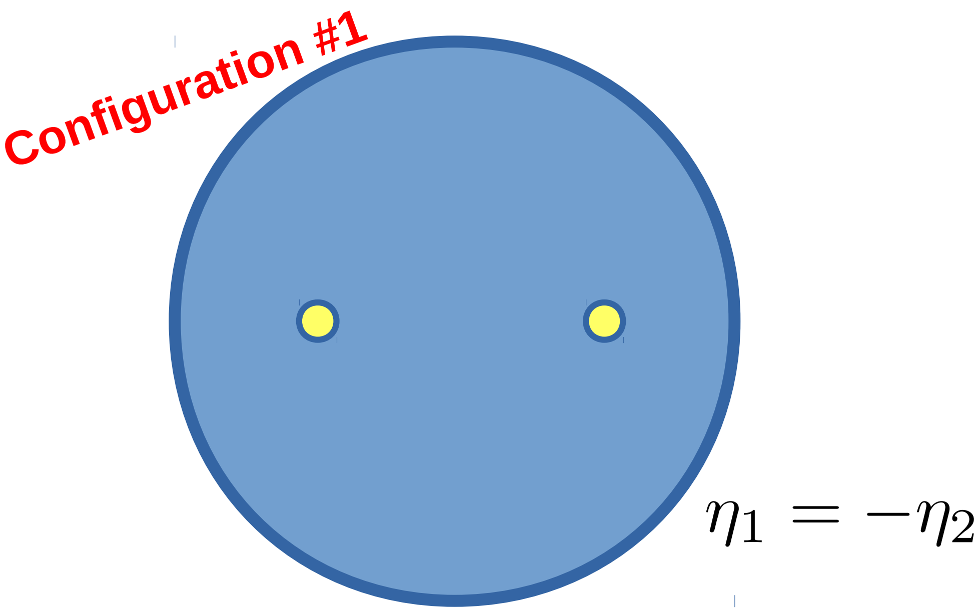
Dynamical phase:
we are not interested in it

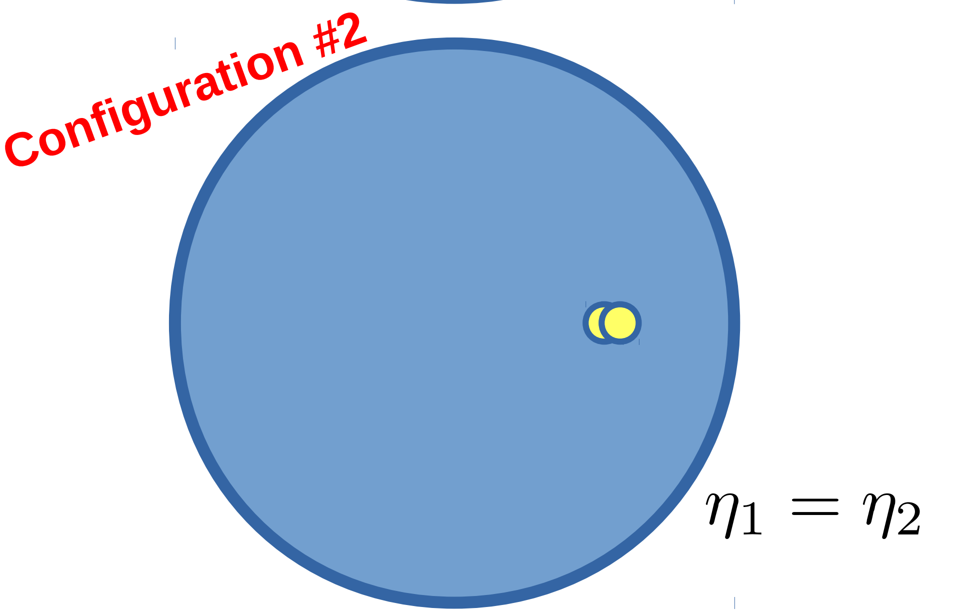
If we consider a rotation
of 360° this operator is
the identity

The geometric phase of 2pi rotations is simply: $e^{\frac{i}{\hbar}2\pi\langle\hat{L}_z\rangle}$

Can I use this formula to compute the non-Abelian statistics of anyons?

Topological contribution





360° rotation in the configuration #1
- Topological contribution
- Non-topological contribution

360° rotation in the configuration #2
- Non-topological contribution

$$\varphi_{br} = \frac{2\pi}{\hbar}\left[\langle \hat{L}_z \rangle_{\eta_1 = -\eta_2} - \langle \hat{L}_z \rangle_{\eta_1 = \eta_2}\right]$$

Lowest Landau level

Most of the known quantum states supporting anyons are defined in the lowest Landau level

$$\frac{1}{\hbar}\langle \hat{L}_z \rangle = \frac{\langle \hat{r}^2 \rangle}{2\ell_B^2} - N$$

$$\varphi_{br} = \frac{\pi}{\ell_B^2}\left[\langle \hat{r}^2 \rangle_{\eta_1=-\eta_2} - \langle \hat{r}^2 \rangle_{\eta_1=\eta_2}\right]$$

Can we use this relation to characterise non-Abelian anyons?

Interesting point: the density profile of the gas contains information about the anyonic statistics

Problem: difference of two thermodynamic quantities… phase is O(1)

Results on the Abelian anyons of the Laughlin state:

Umucalilar, Macaluso, Comparin and Carusotto, PRL **120** 230403 (2018)

Outline

- Introduction: Anyons
- Part #1: Detecting anyons with density measurements
- Part #2: The case of the Moore-Read wavefunction
- Conclusions

Moore-Read wavefunction

$$\Psi_{2qh}(\{z_j\},\eta_1,\eta_2) = \mathrm{Pf}\left(\frac{(\eta_1 - z_i)(\eta_2 - z_j) + i \leftrightarrow j}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^M e^{-\sum_j |z_j|^2/(4\ell_B^2)}$$

- Lowest Landau level wavefunction
- Defined for bosons (M=1) or fermions (M=2)
- Filling factor ν = 1/M
- Expected to explain the plateau at ν = 5/2

G. Moore and N. Read, Nuclear Physics B 360, 362 (1991)
Bonderson, Gurarie and Nayak, PRB **83**, 075303 (2011)

Moore-Read wavefunction

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- Lowest Landau level wavefunction
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- For two quasiholes, the wavefunction is not degenerate
- The quasiholes are not Abelian
- The fusion channel depends on the parity of the number of particles

$$\varphi_{br} = 2\pi\left[\frac{1}{4M} - \frac{1}{8} + \frac{P_N}{2}\right]$$

G. Moore and N. Read, Nuclear Physics B 360, 362 (1991)
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Moore-Read wavefunction

$$\Psi_{2qh} \; \ldots \; \mathrm{Pf}\left(\frac{(\eta_1 - z_i)(\eta_2 - z_j) + i \leftrightarrow j}{\ldots}\right) \prod_{i<j}(z_i - z_j)^M e^{-\sum_j |z_j|^2/(4\ell_B^2)}$$

- Measuring two different braiding phases is an unambiguous signature of non-Abelian statistics

Bosons:

N even → $\varphi_{br} = 0.125 * \pi$

N odd → $\varphi_{br} = 0.625 * \pi$

Fermions:

N even → $\varphi_{br} = 0.0$

N odd → $\varphi_{br} = 0.5 * \pi$

- For two quasiholes, the wavefunction is not degenerate
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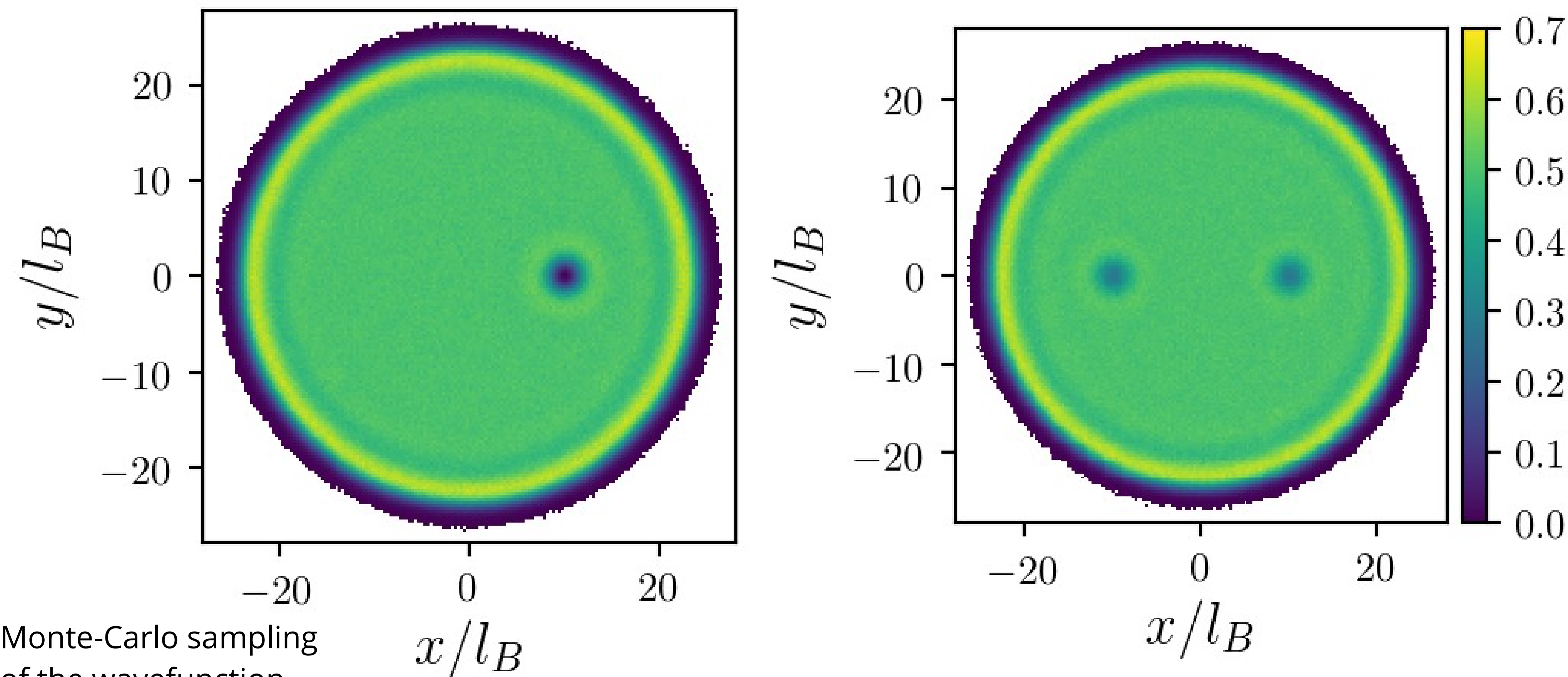
G. Moore and N. Read, Nuclear Physics B 360, 362 (1991)
Bonderson, Gurarie and Nayak, PRB **83**, 075303 (2011)

Braiding phase from density profile

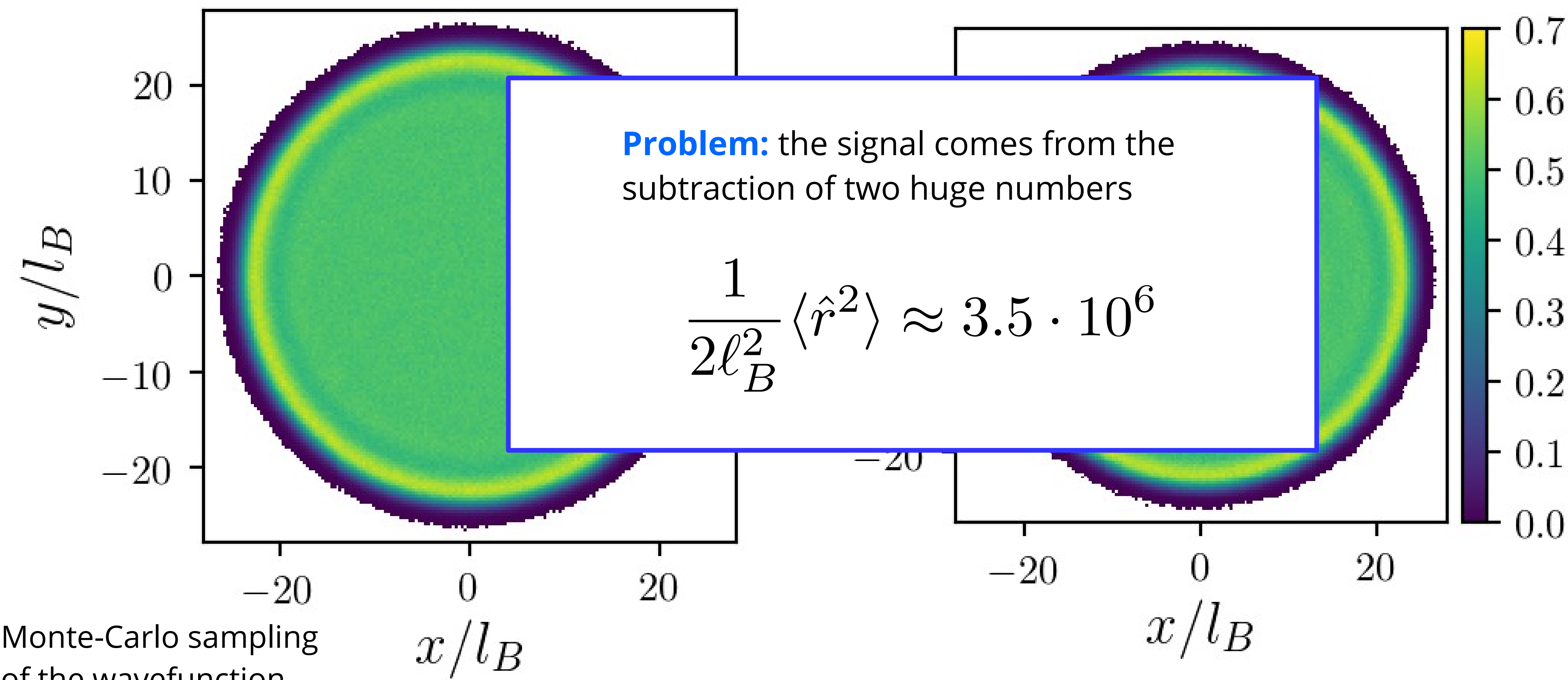
$$\varphi_{\rm br} = 2\pi \frac{1}{2\ell_B^2} \left[\langle \hat{r}^2 \rangle_{\eta_1=-\eta_2} - \langle \hat{r}^2 \rangle_{\eta_1=\eta_2} \right]$$



Monte-Carlo sampling
of the wavefunction
N=150; M=2

Braiding phase from density profile

$$\varphi_{\rm br} = 2\pi \frac{1}{2\ell_B^2} \left[\langle \hat{r}^2 \rangle_{\eta_1 = -\eta_2} - \langle \hat{r}^2 \rangle_{\eta_1 = \eta_2} \right]$$



Problem: the signal comes from the subtraction of two huge numbers

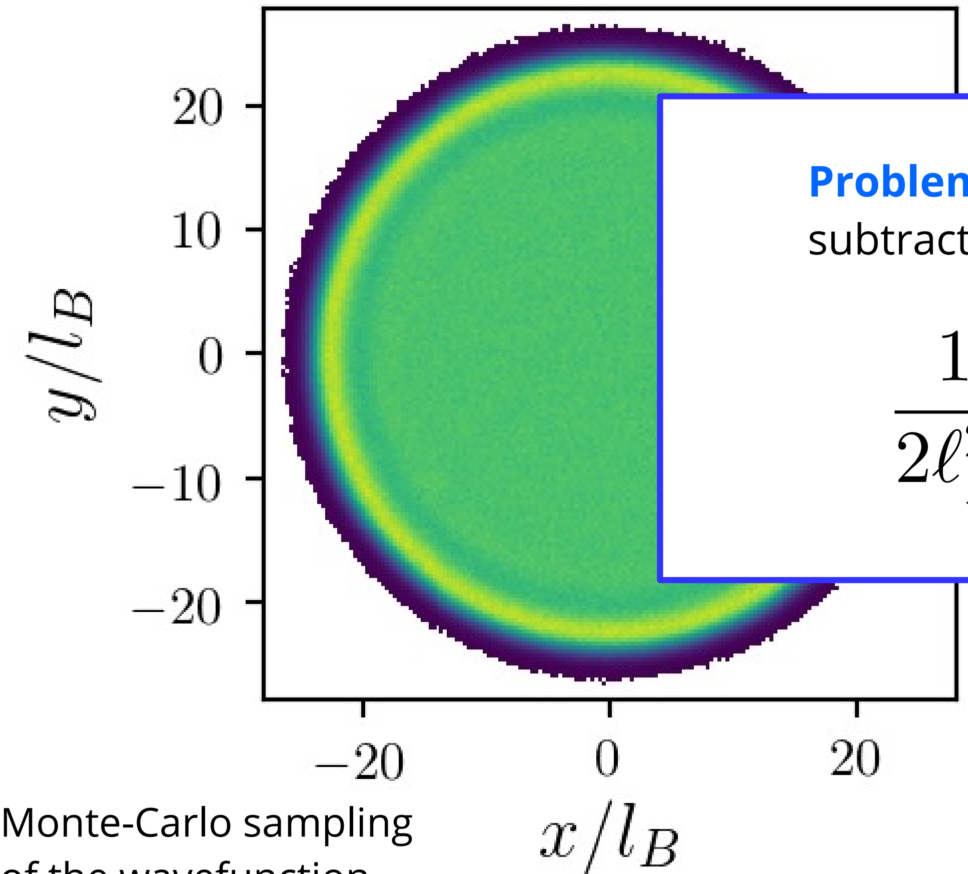
$$\frac{1}{2\ell_B^2} \langle \hat{r}^2 \rangle \approx 3.5 \cdot 10^6$$

Monte-Carlo sampling of the wavefunction
N=150; M=2

Braiding phase from density profile

Where is this difference significant?

$$\varphi_{\rm br} = 2\pi \frac{1}{2\ell_B^2} \left[\langle \hat{r}^2 \rangle_{\eta_1 = -\eta_2} - \langle \hat{r}^2 \rangle_{\eta_1 = \eta_2} \right]$$



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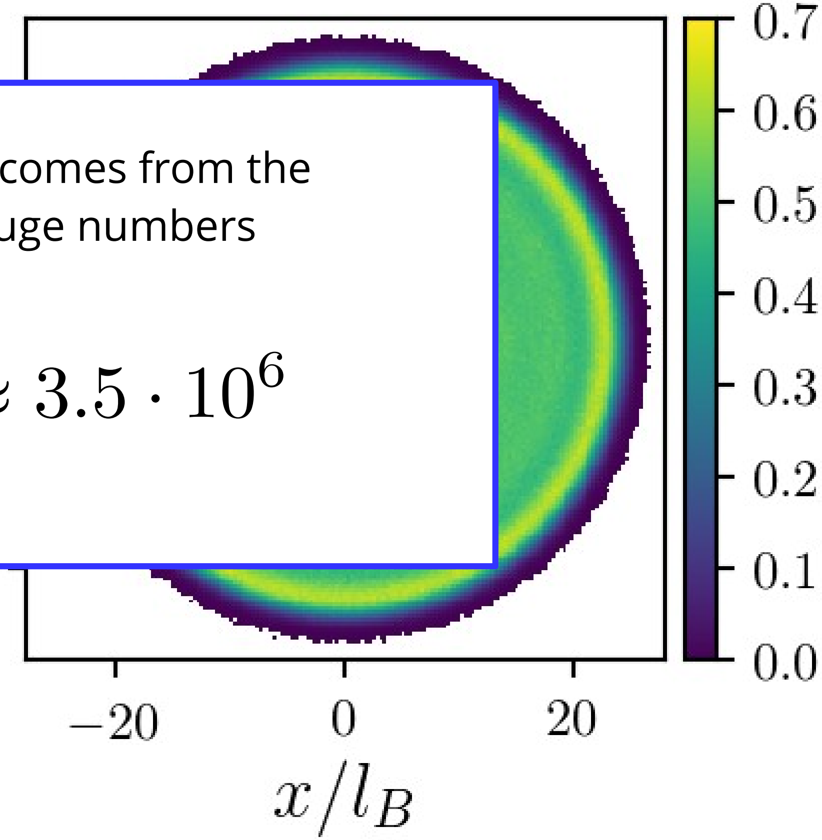
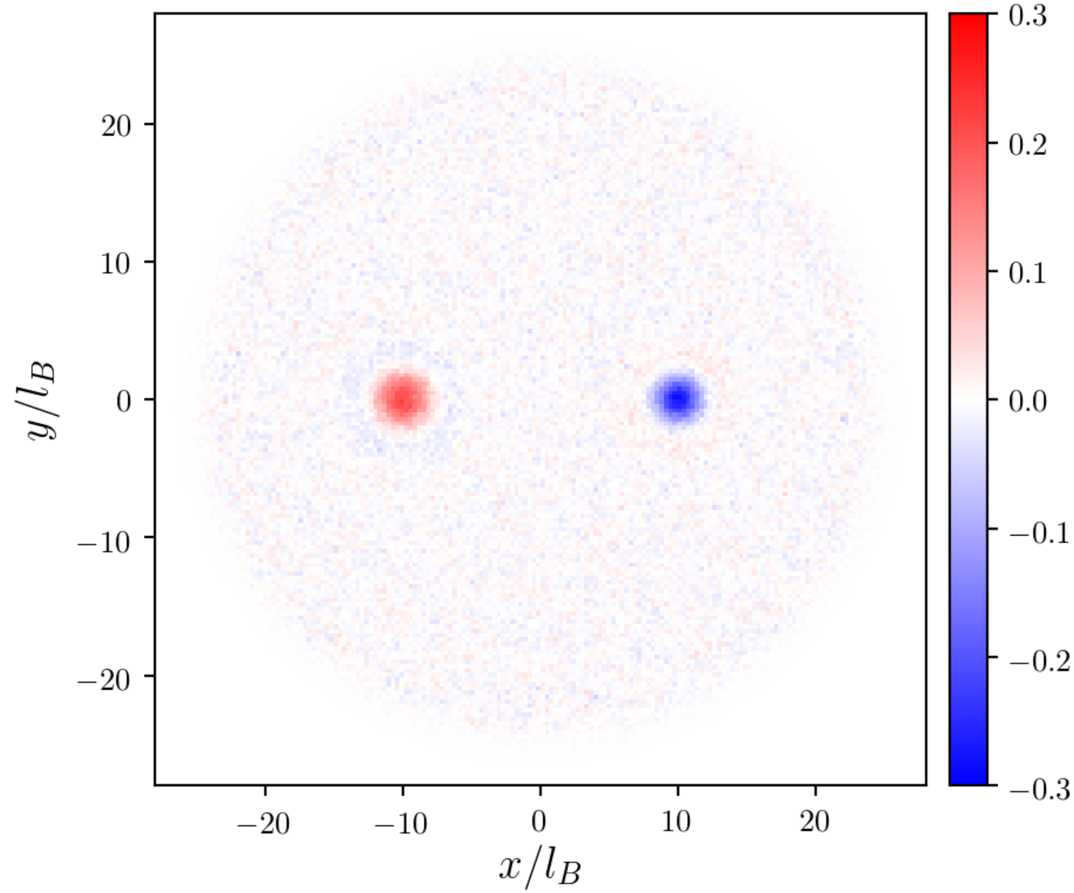


Monte-Carlo sampling of the wavefunction

N=150; M=2

Braiding phase from density profile

$$\varphi_{\rm br} = 2\pi \frac{1}{2\ell_B} \left[\langle \hat{r}^2 \rangle_{\eta_1 = -\eta_2} - \langle \hat{r}^2 \rangle_{\eta_1 = \eta_2} \right]$$



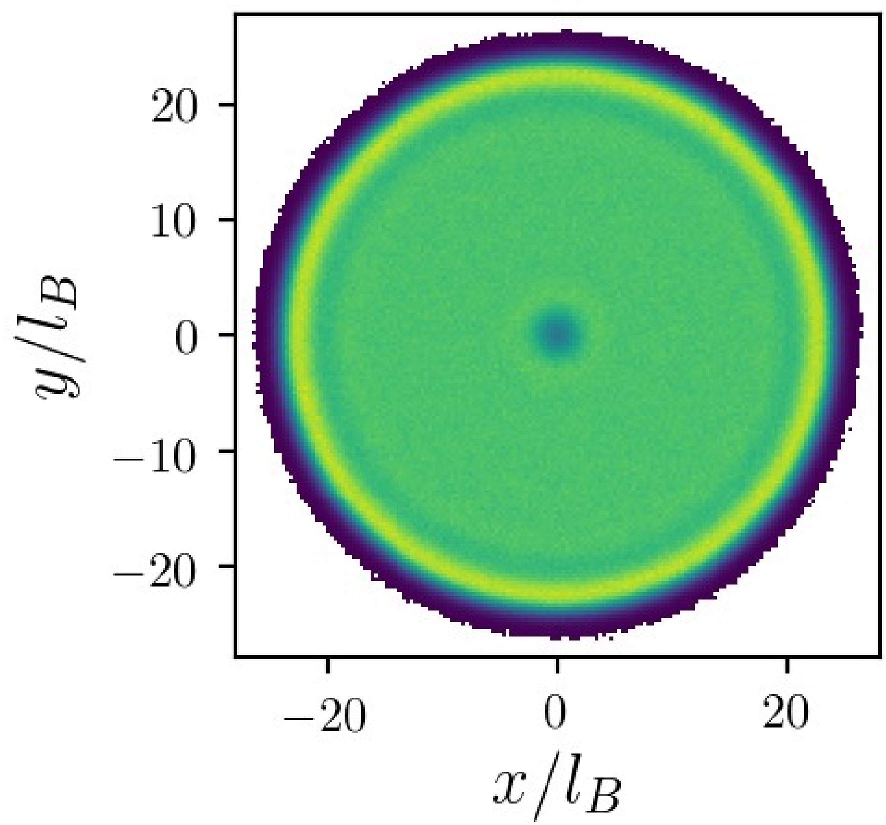
Difference of the two density profiles

This suggests that the braiding phase might be locally encoded around the quasiholes

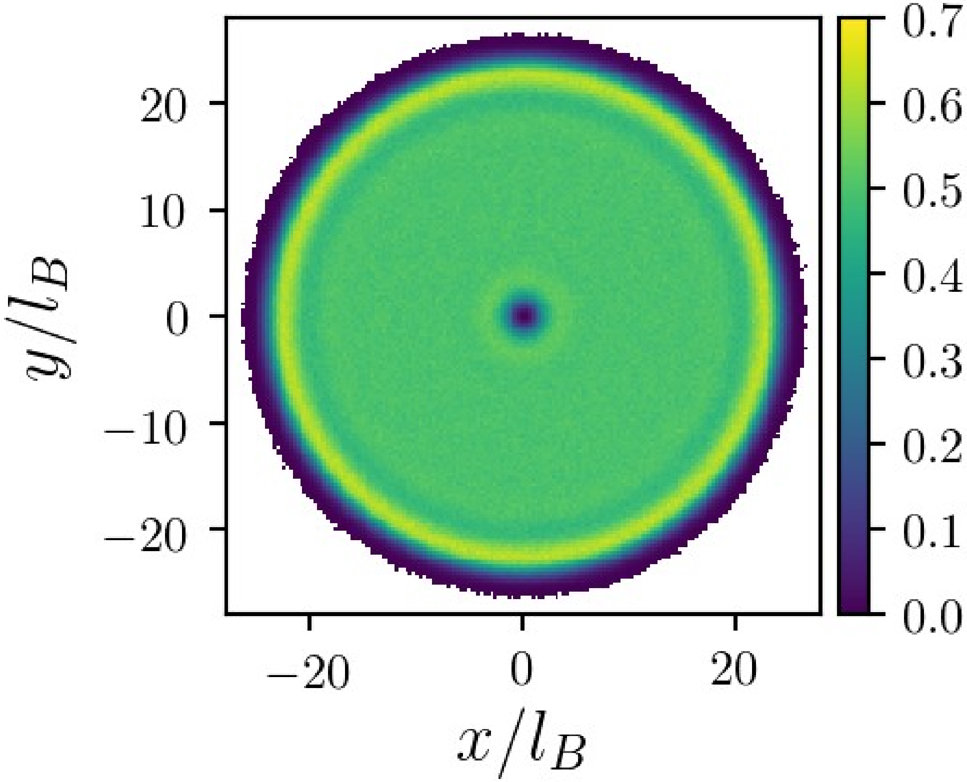
Braiding phase and depletion density

Depletion density: $d(r) = \bar{n} - n(r)$

1 quasihole in the origin

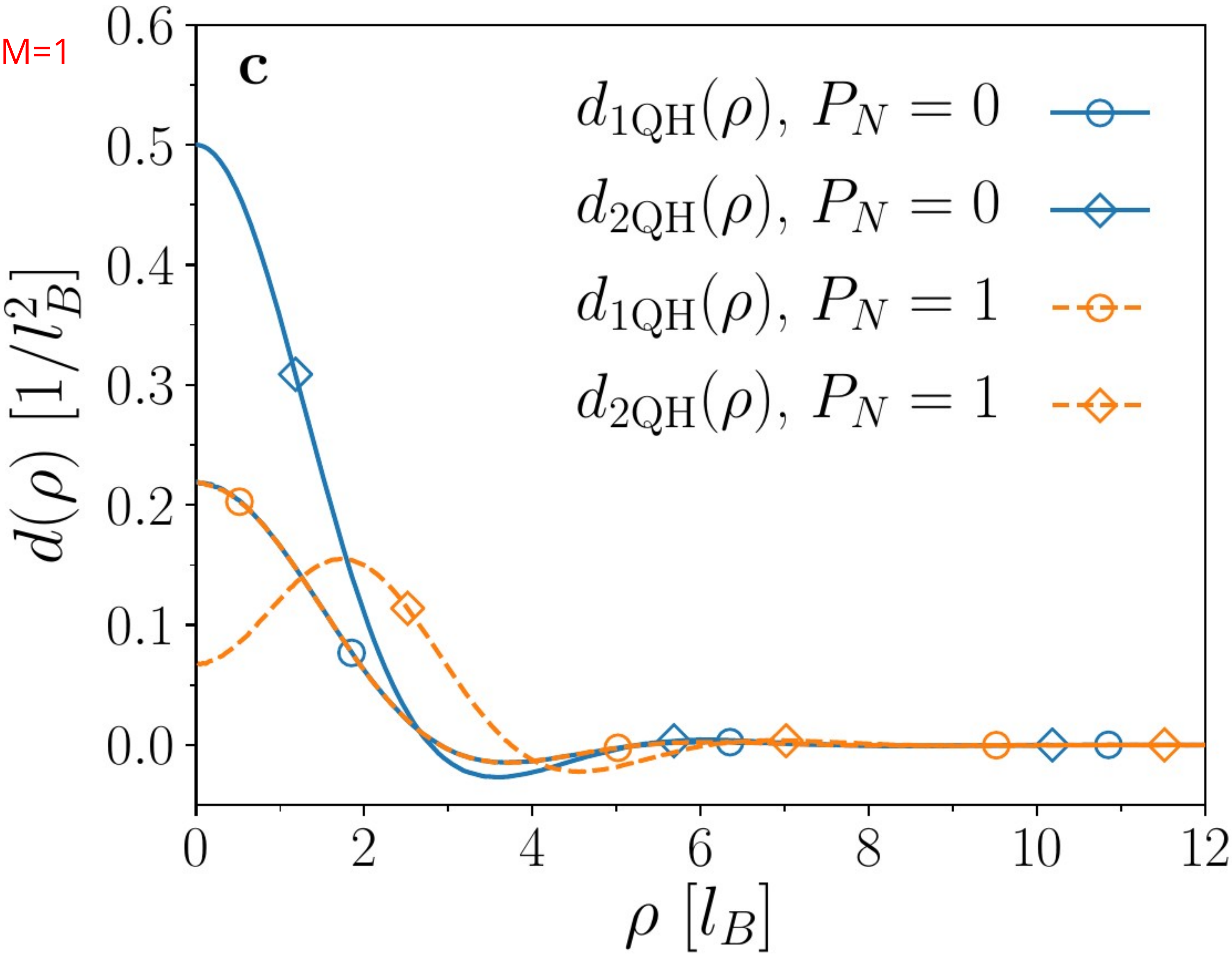


2 quasiholes in the origin



Results

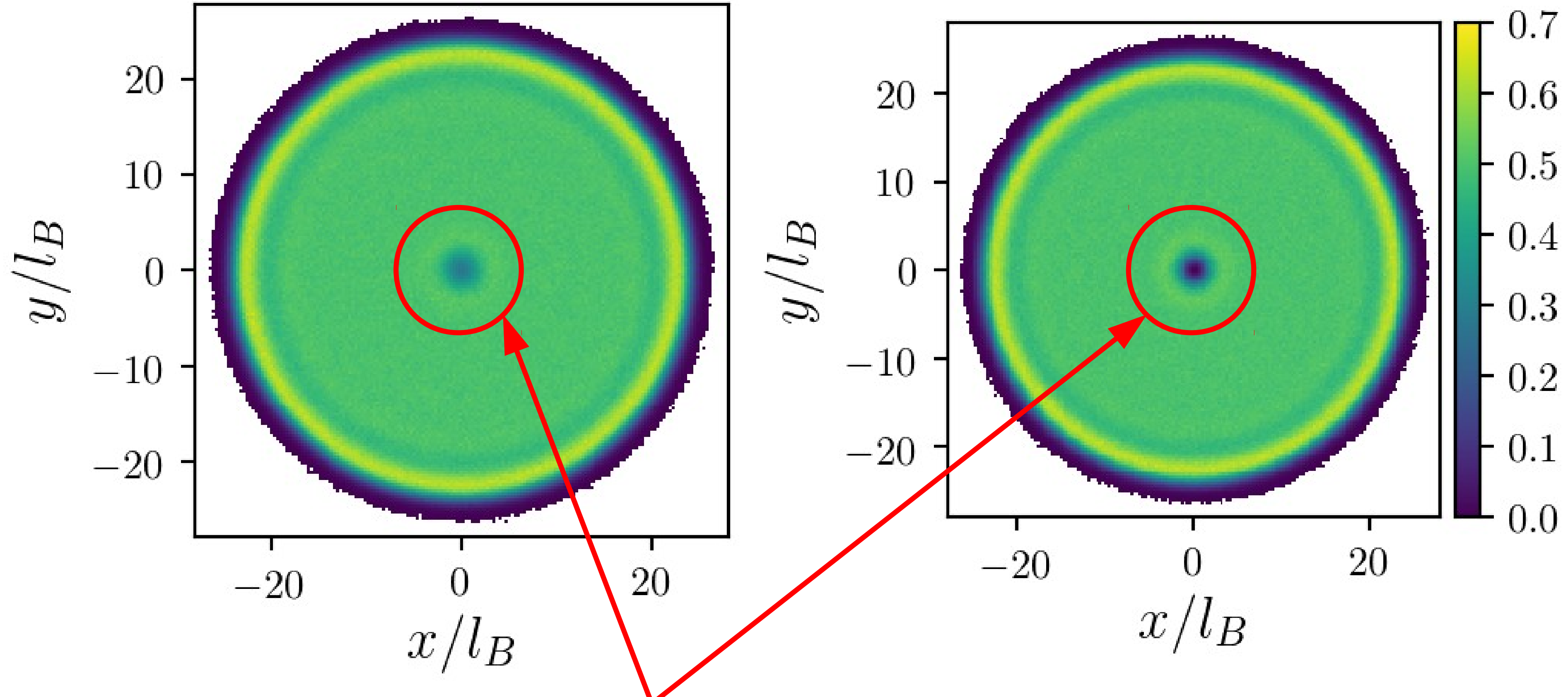
Bosons, M=1



Note that you can tell the difference between the two fusion channels only when the two quasi holes are close by!

We obtained similar results for fermions!

Braiding phase and depletion density

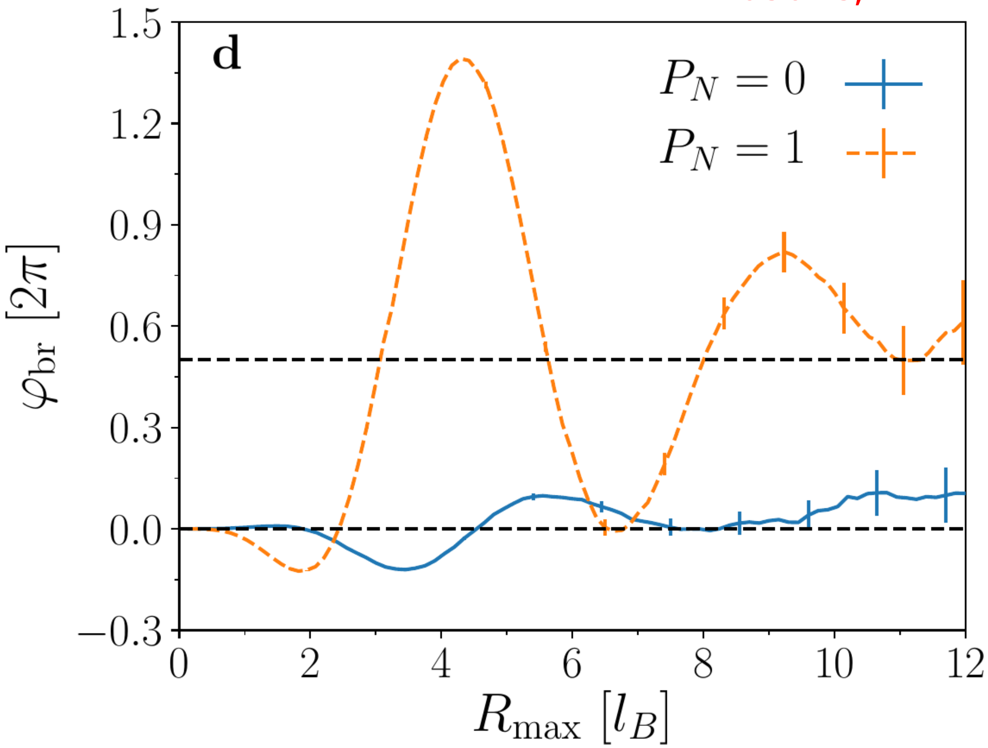


$$\varphi_{\mathrm{br}} = 2\pi \frac{2\pi}{2\ell_B^2} \int_0^{R} r^2 \big(d_{2qh}(r) - 2d_{1qh}(r)\big) r \,\mathrm{d}r$$

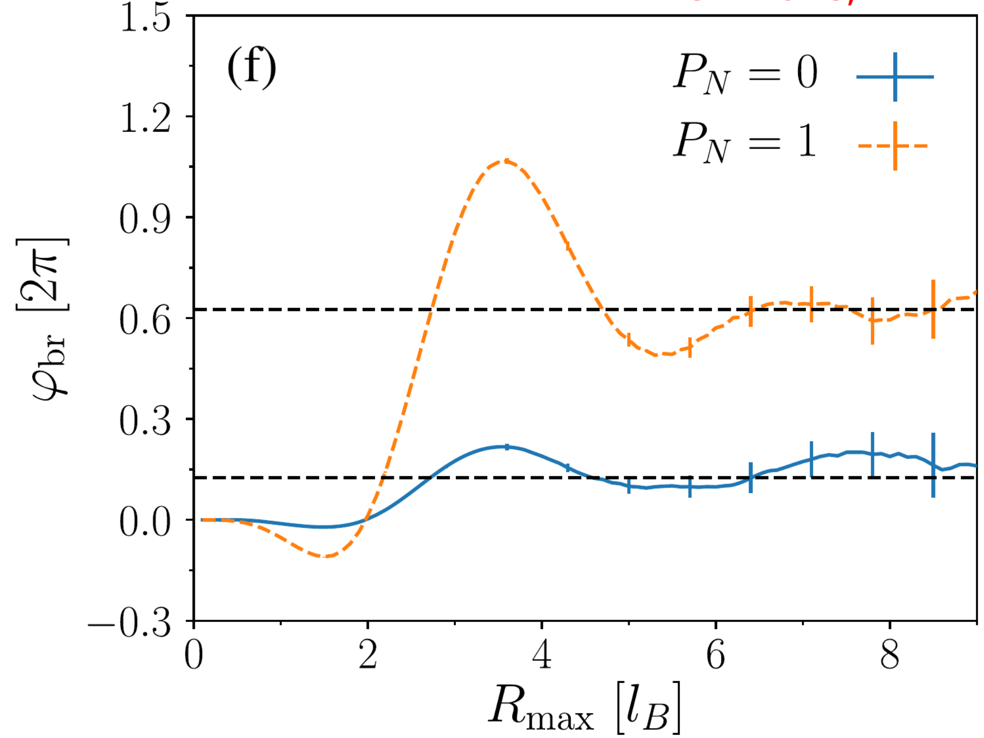
We obtained similar results for fermions!

Final result

Bosons, M=1



Fermions, M=2



How should the experiment work?

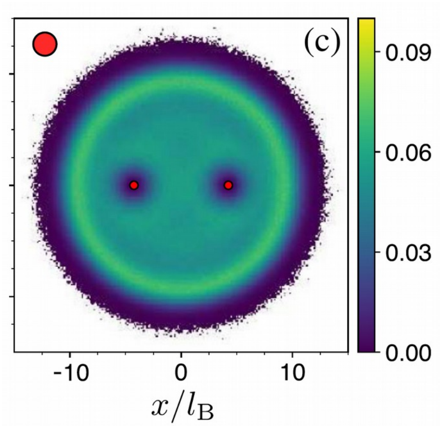
- **First:** create two separated quasiholes
  - Reconstruct their density profile
- **Second:** create two separated quasiholes
  - Bring them close by and reconstruct the density profile of a “double” quasi hole
  - Both fusion channels are possible
- **Third:** compute the statistical phase

Conclusions

My main message:

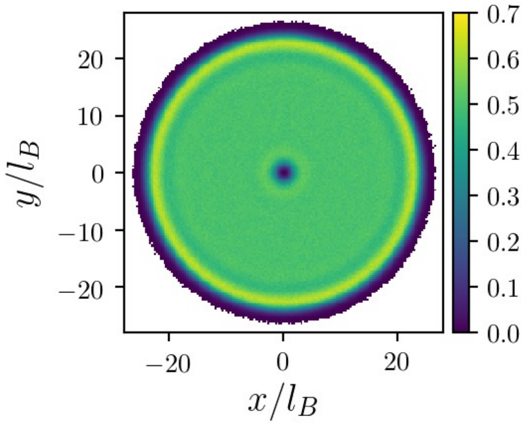
- Density profile of quasiholes contains quantitative information about their statistical phase

Laughlin wavefunction



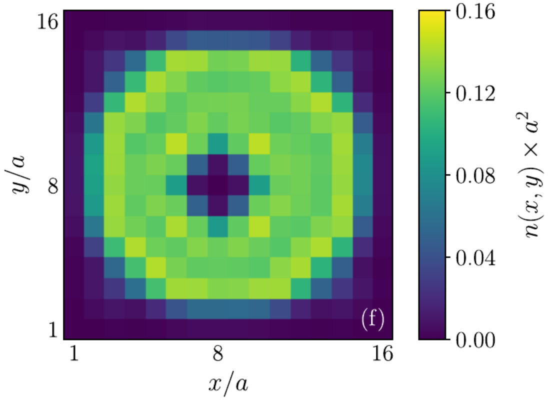
Umucalilar, Macaluso, Comparin and Carusotto, PRL **120** 230403 (2018)

Moore-Read wavefunction



Macaluso, Comparin, LM, Carusotto, PRL **123**, 266801 (2019)

Laughlin on a lattice
via Hamiltonian minimization



Macaluso, Comparin, Umucalilar, Gerster, Montangero, Rizzi Carusotto, PRR **2**, 013145 (2020)

Perspective

- Establish a connection with experimentalists to further tailor our technique to their laboratory possibilities
- Test our formula on more and more lowest-Landau-level wavefunctions with anyons (maybe in combination with novel MPS-quantumHall methods?)
- Extension of the formula to anyons that do not appear in the lowest Landau level
- A more in-depth understanding of the formula

$$\varphi_{\rm br} = 2\pi \frac{1}{\hbar} \left(\ell_{z,2} - 2\ell_{z,1} \right)$$

Difference in the rotational properties of two overlapping anyons versus 2 separated anyons

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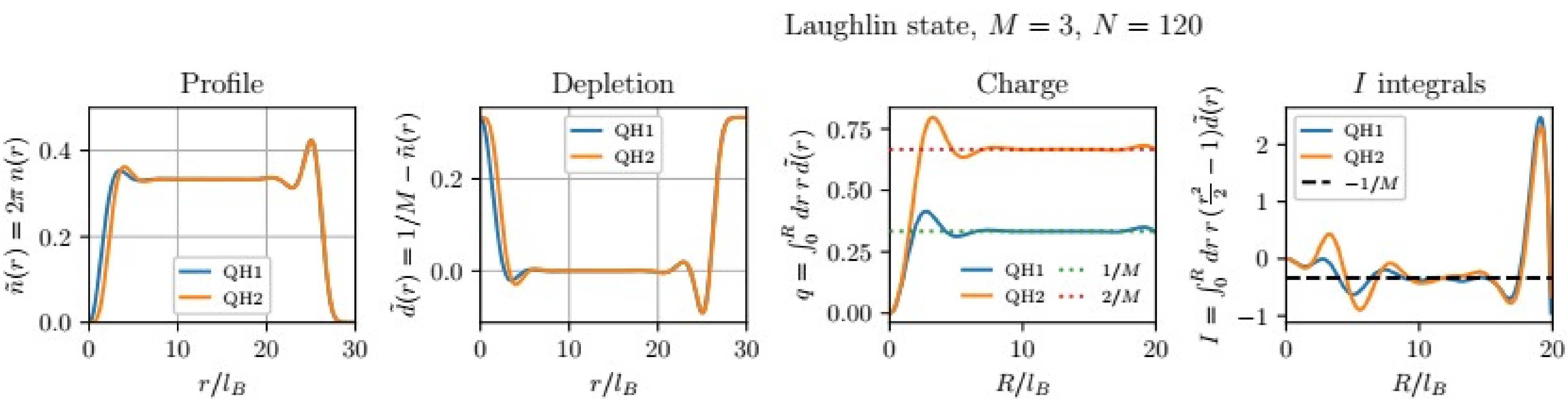
Rotations of anyons and statistics?

$$\varphi_{\mathrm{br}} = 2\pi \frac{1}{\hbar} \left(\ell_{z,2} - 2\ell_{z,1} \right)$$

Difference in the rotational properties of two overlapping anyons versus 2 separated anyons

$$\ell_{z,n} = 2\pi \int_0^R \left(\frac{r^2}{2\ell_B} - 1 \right) d_n(r) r dr$$



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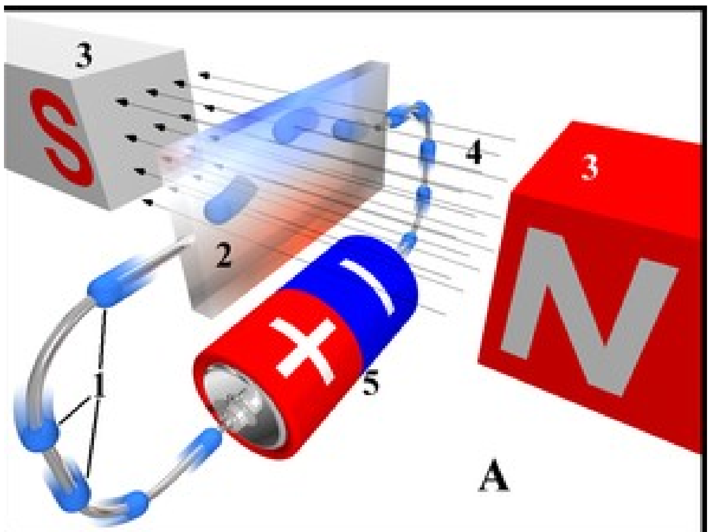
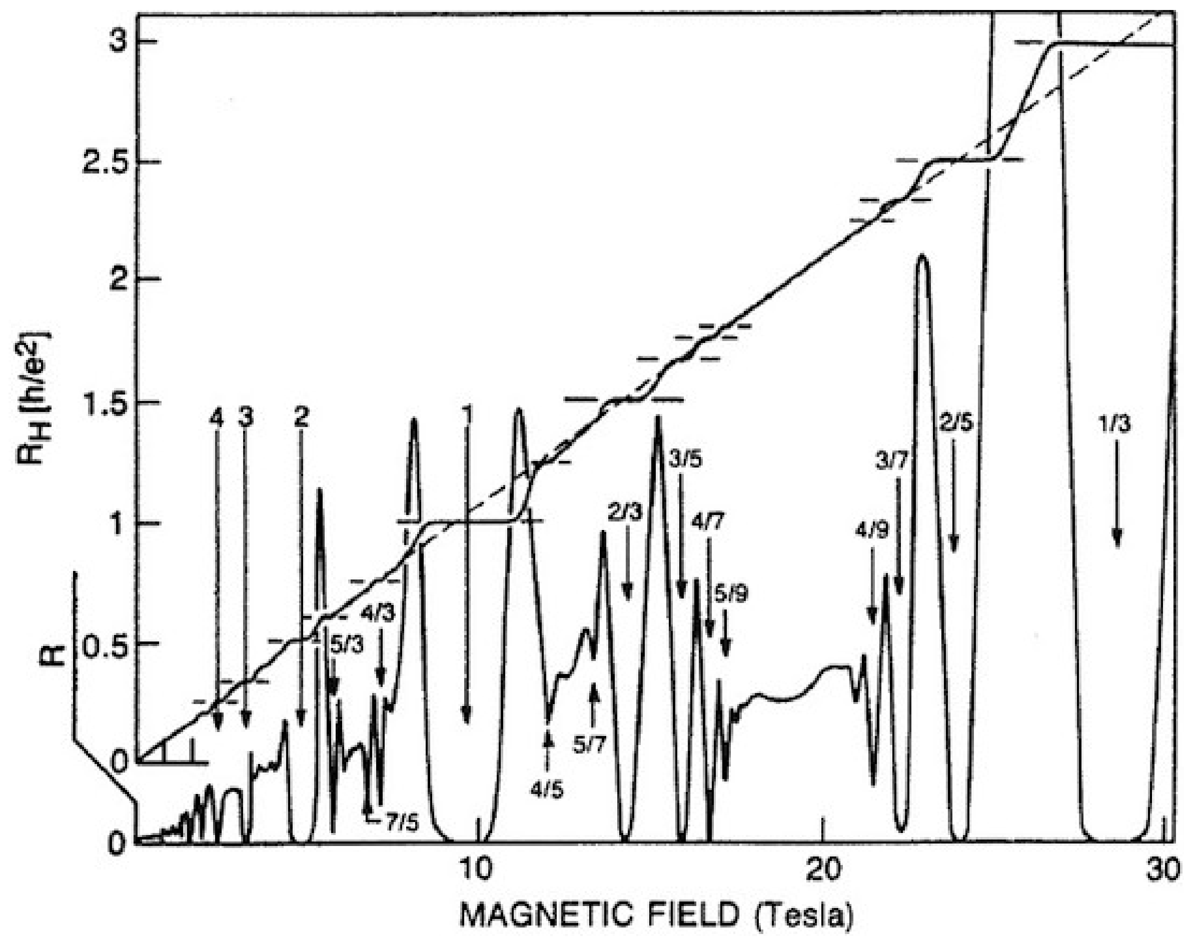
Thank you

Quantum Hall effect



2DEG in a strong magnetic field
- Voltage bias along x
- Current measured along y



Transverse resistance is quantised in fractional values
- Fractionally-charged quasi-particles
  - Quasi-hole and quasi-particle excitations
- Ballistic propagation along the boundaries
- The bulk of the sample is gapped
  - Fractional charge
  - Anyonic statistics

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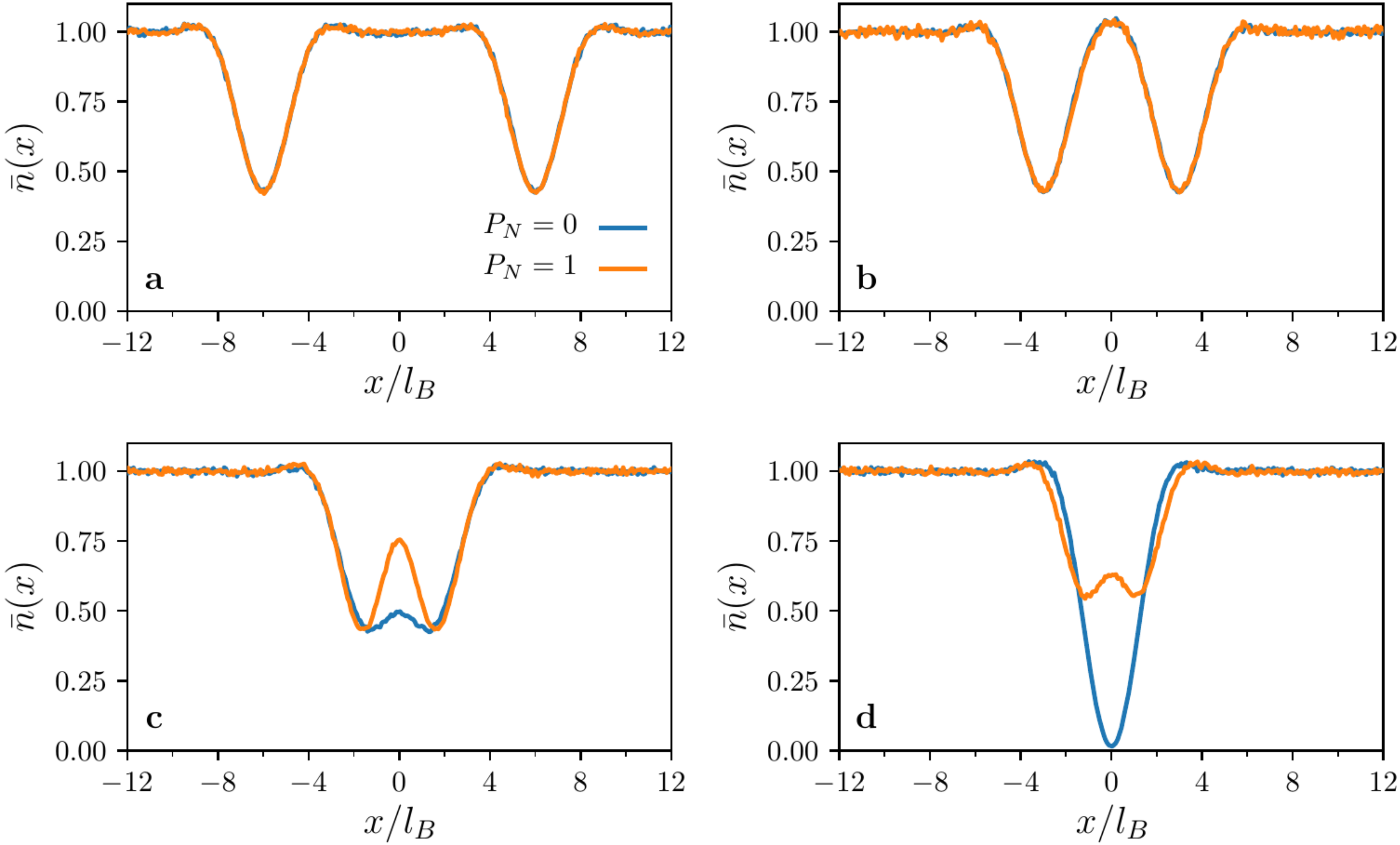
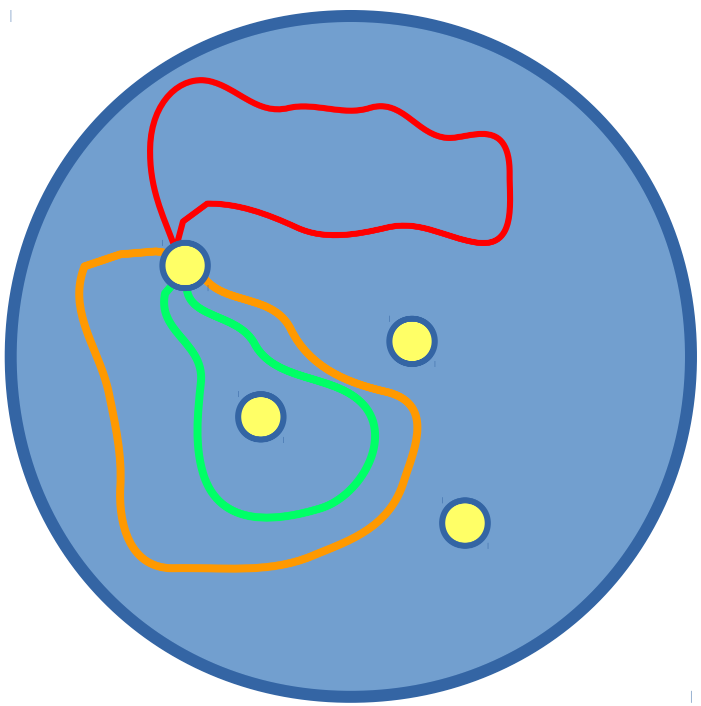
Again…



We obtained similar results for fermions!

Non-Abelian anyons

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(t))$$



The unitary Wilczek-Zee matrix can be diagonalised

$$\mathcal{U} = \begin{pmatrix} e^{i2\varphi_1} & 0 \\ 0 & e^{i2\varphi_2} \end{pmatrix}$$

The eigenstates represent
non-Abelian anyons
in different fusion channels

The existence of several fusion channels
is a definition of non-Abelian anyons

- Non-Abelian anyons
  - The ground state is degenerate $\{|\psi_1\rangle, |\psi_2\rangle\}$
  - Topological contribution: Matrix $\mathcal{U}$

Anyons in theory

- Explicit **analytical computation** of the topological contribution to the geometric term

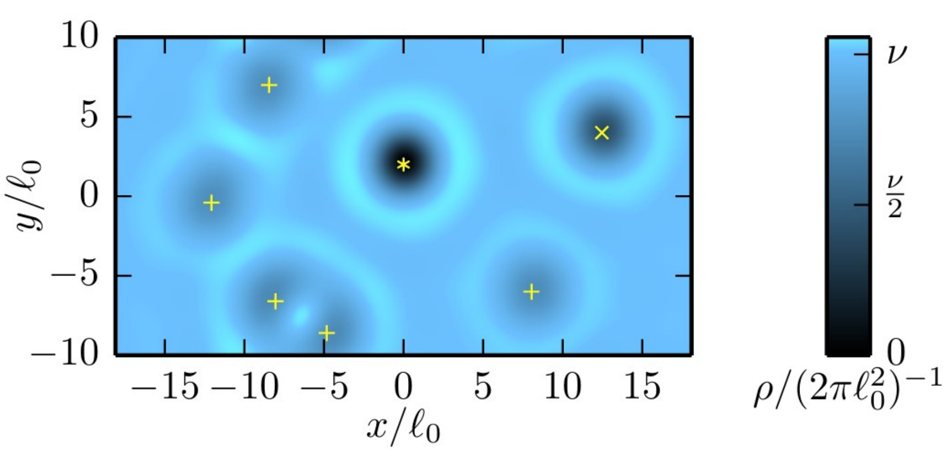
The Laughlin wavefunction:

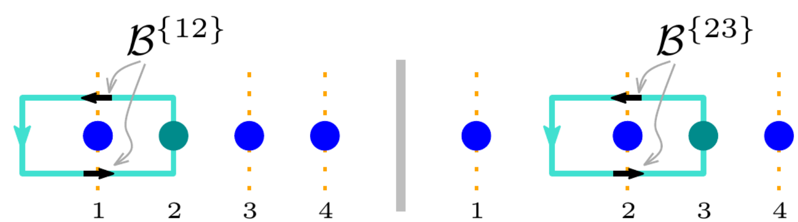
$$\Psi_{2qh}(\{z_j\},\eta_1,\eta_2) \sim \prod_j (z_j-\eta_1)(z_j-\eta_2)\prod_{i<j}(z_i-z_j)^3 e^{-\sum_j |z_j|^2/(4\ell_B^2)}$$

- **Numerical investigation**

Braiding non-Abelian quasiholes in fractional quantum Hall states

Yang-Le Wu,[1] B. Estienne,[2,3] N. Regnault,[1,4] and B. Andrei Bernevig[1]





$$\mathcal{A}_{ab}(\eta;\mathrm{d}\eta) \equiv e^{-i\mathrm{d}\eta A_{ab}(\eta)} \equiv \frac{\langle \Psi_a(\eta+\mathrm{d}\eta)|\Psi_b(\eta)\rangle}{\|\Psi_a(\eta+\mathrm{d}\eta)\|\cdot\|\Psi_b(\eta)\|}$$

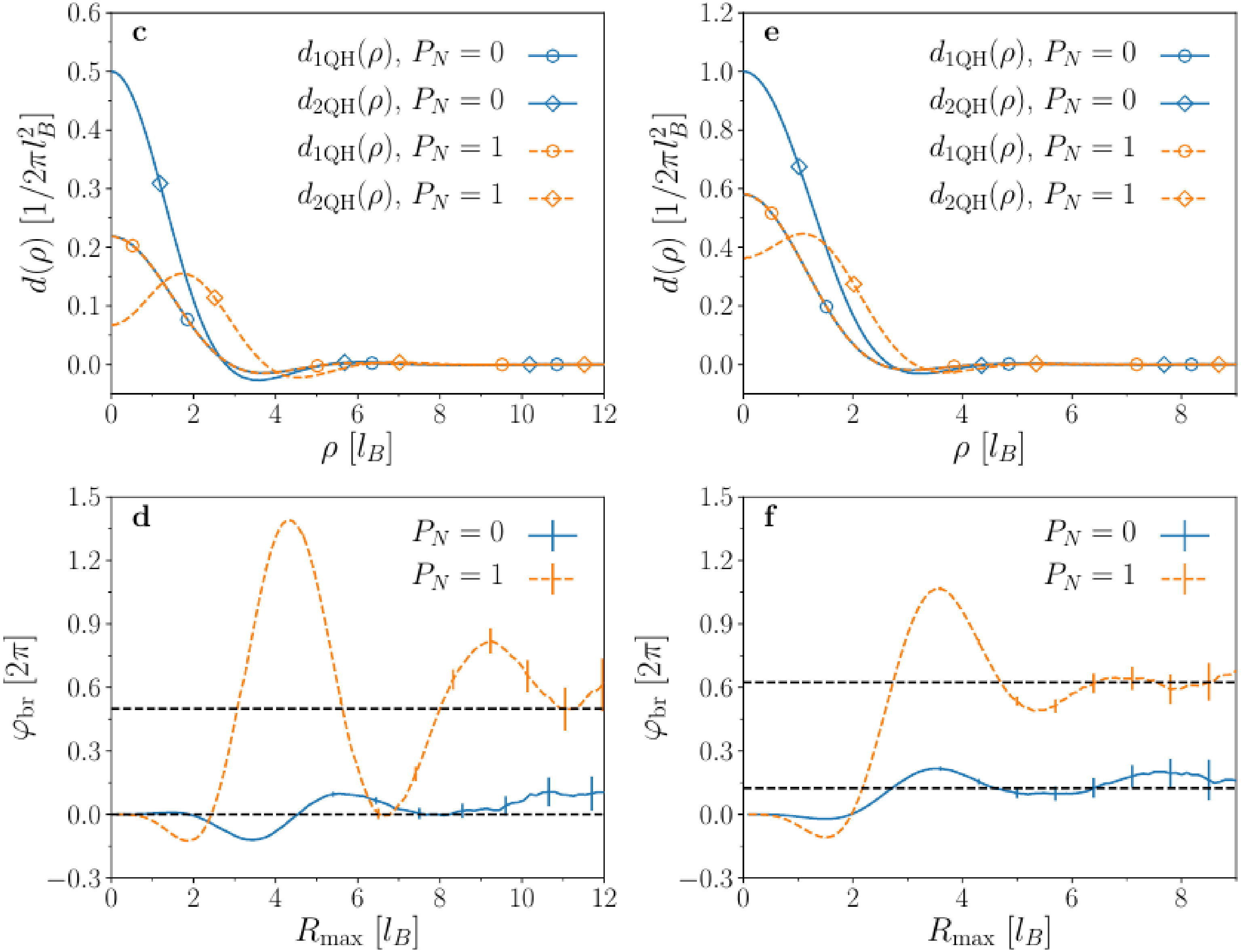
$$\mathcal{B}_{\mathrm{MR}}^{\{12\}} = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0\,i \end{bmatrix}$$

David Tong, The Quantum Hall Effect, Lecture notes. Bonderson, Gurarie and Nayak, PRB **83**, 075303 (2011)

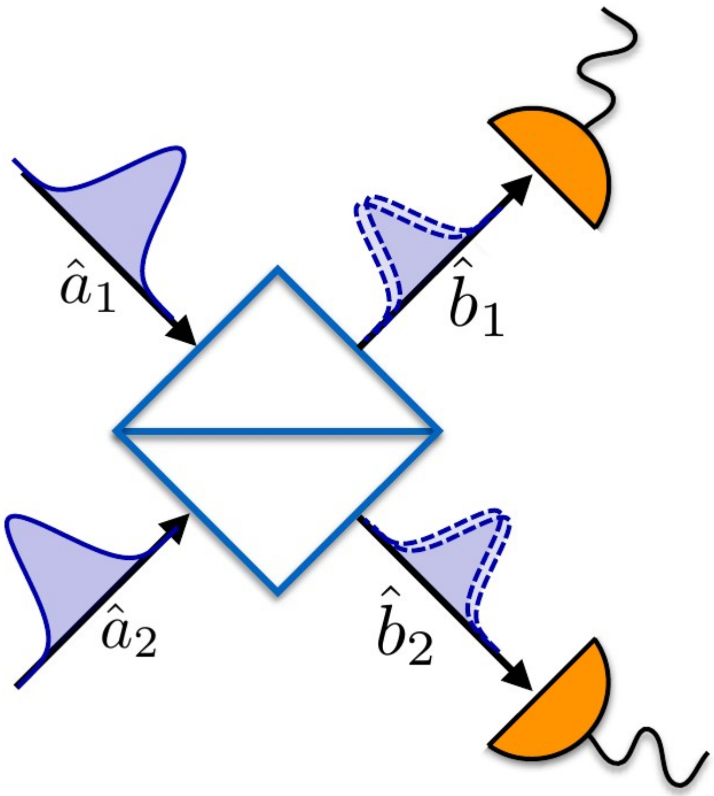
Results with fermions

Bosons, M=1

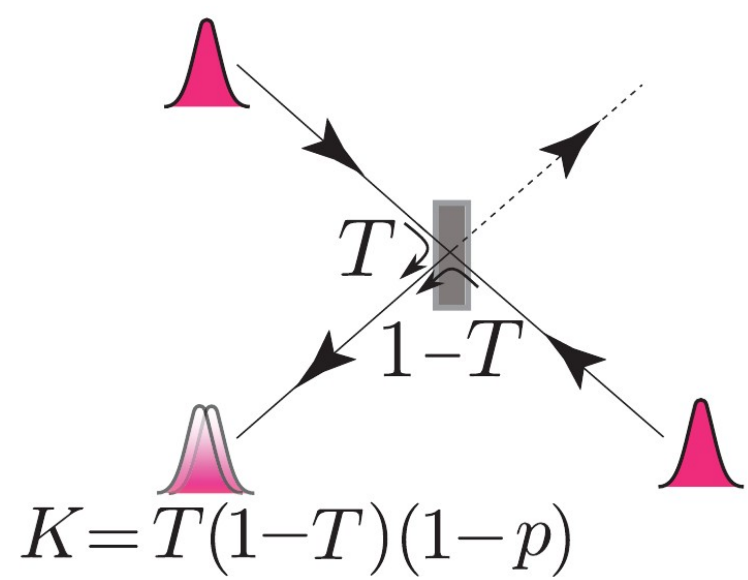
Fermions, M=2



Measuring anyonic statistics: Hong-Ou Mandel beam-splitter



What about anyons?



Two identical particles on two sides of a beamsplitter

- Bosons: bunching
- Fermions: antibunching

- Electrons: antibunching → K=0 → p=1
- Anyons: K > 0 → p ≠ 1

Bartolomei, Kumar, Bisognin, Marguerite, Berroir, Bocquillon, Plaçais, Cavanna, Dong, Gennser, Jin, Fève, Science (2020)

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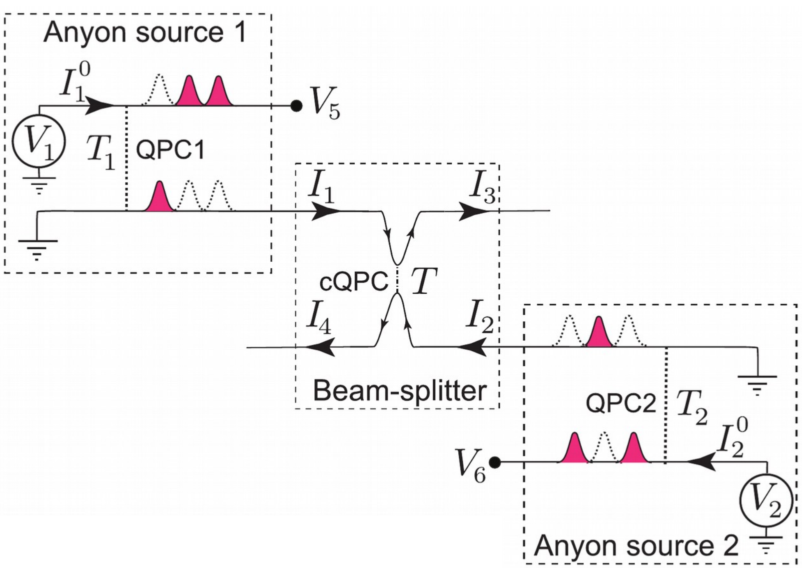
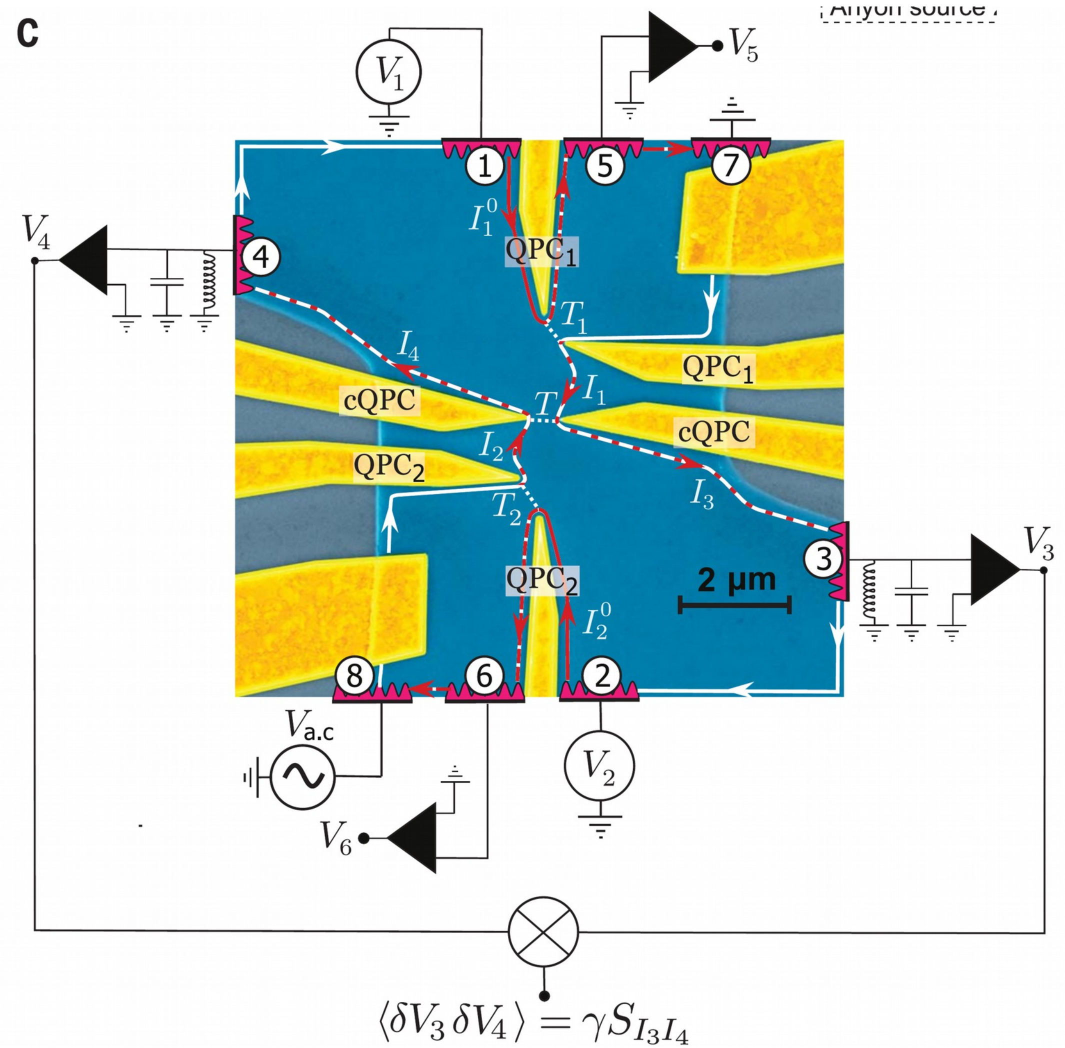
Measuring anyonic statistics: Hong-Ou Mandel beam-splitter





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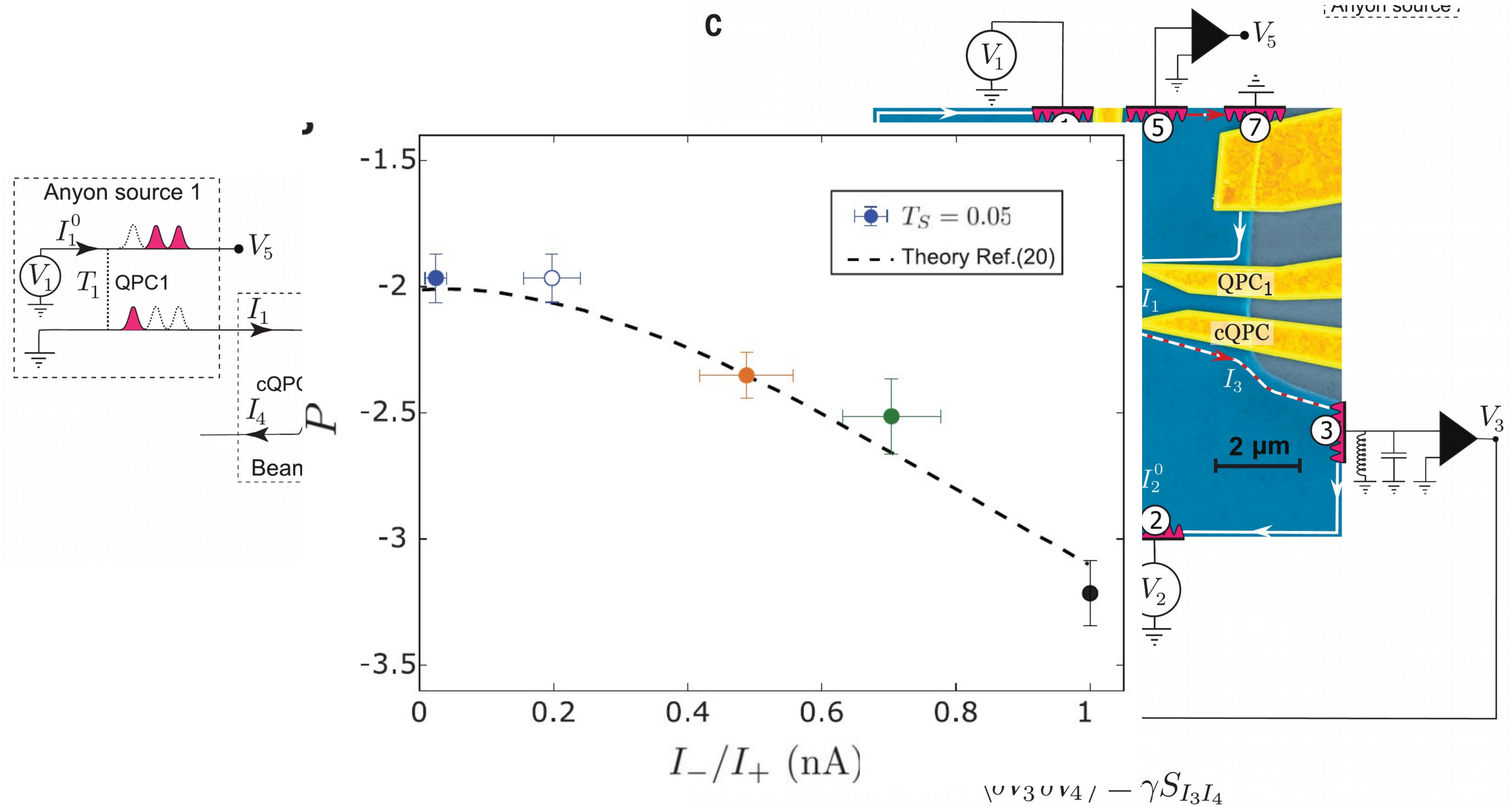

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