How the physics of quantum impurities became a central question in the relic neutrino detection

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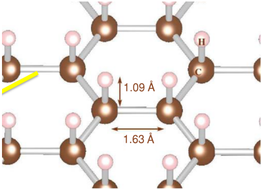
This talk will be about

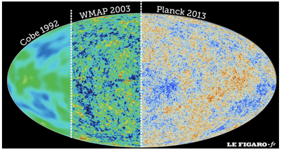
Quantum impurity as a central question of the **astrophysical experiment**

Quantum measurement devices with quantum impurities for particle physics and physics of the early universe

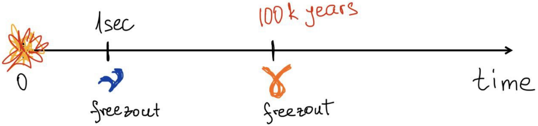
- New era in cosmology and particle physics through the experiments on quantum devices
- New pillar of the early universe picture comparable to CMB
- Devices with quantum impurities for the relic neutrino detection
- Zoo of condensed matter effects on small energy scales



CνB and CMB



- Similarly to how relic photons form **CMB**, relic neutrinos form **CνB**
- **As the Universe expands** at some moments it becomes *transparent* for ν/γ.
- *'' Frozen picture'' of the early Universe*.
  - The freezout of the neutrinos is much earlier than photons.



  - Right now, in your room, there are **411 relic photons** and **339 relic neutrinos** in every cm^3.
  - Most of them are **relic** neutrinos.

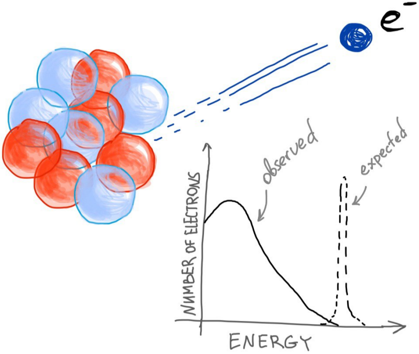
Observation of the cosmological neutrinos would then provide a window into the 1st second of creation

Why have we not discovered CνB yet?

Riminder: what is neutrino?

$$(A, Z) \rightarrow (A, Z+1) + e^{-} + ?$$

- Energy and angular momentum are not conserved in β decay processes?
- W. Pauli *predicted* in 1930 a new particle to ''save'' the conservation laws
- Pauli originally called his new particle "neutron" (neutral one)



- Chadwick discovered a massive nuclear particle in 1932, however it was *not* Pauli's particle.
- Fermi renamed Pauli's particle to *neutrino* ("little neutral one")

How was the neutrino detected?

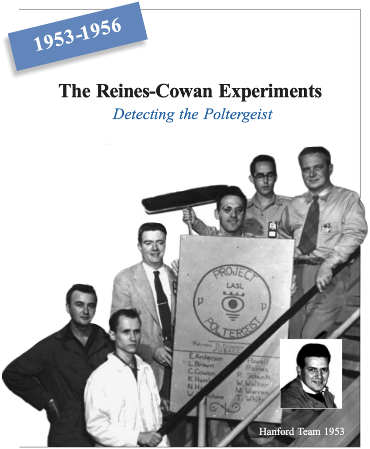
"I have done a terrible thing. I have postulated a particle that cannot be detected." (W. Pauli)

- Bethe and Peierls in 1934 estimated

$$\begin{aligned} \sigma &\sim 10^{-44}\ \mathrm{cm}^2 \\ \sigma_{\mathrm{Thomson}} &\sim 10^{-25}\ \mathrm{cm}^2 \\ \sigma_{\mathrm{nuclear}} &\sim 10^{-26}\ \mathrm{cm}^2 \end{aligned}$$

- Neutrino was first detected in 1956 by a group led by Clyde Cowan and Frederick Reines
- They used the enormous flux of antineutrinos from a nuclear reactor.

1995 - Nobel Prize



1953-1956

The Reines-Cowan Experiments

Detecting the Poltergeist

Hanford Team 1953

Fermi theory

$$n \to p + e^- + \bar{\nu}_e$$

Fermi: β decay is a decay of neutron inside the nucleus

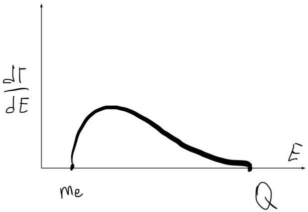
$$V_F(x_1, x_2, x_3, x_4) = G_F \delta(x_1 - x_2)\delta(x_2 - x_3)\delta(x_3 - x_4)$$

One can calculate the number of **transitions per unit time** into some range of final states $d\nu_f$ using **Fermi Golden Rule**

$$dw_{\text{if}} = 2\pi \left| \langle \psi_f^0 | \hat{V}_F | \psi_i^0 \rangle \right|^2 \delta(E_i - E_f) d\nu_f$$

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3}\sqrt{E_e^2 - m_e^2}\, E_e (Q - E_e)^2$$

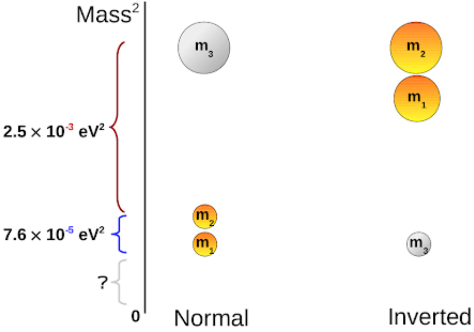
$$Q = m_N({}^A_Z X) - m_N({}^{A}_{Z+1} X') - m_e - m_{\bar{\nu}_e}$$



- Looks like neutrinos are **massless**
- Along with the β decay, Fermi theory predicts **neutrino capture**.

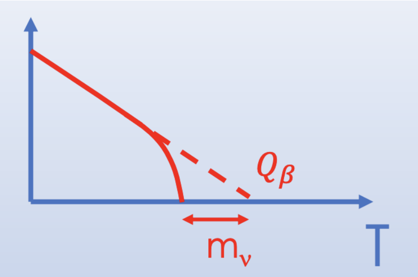
Neutrinos are massive

- In the last few decades neutrino flavor oscillations where convincingly observed, meaning that neutrinos are **massive**
- Neutrino oscillations can only measure Δm and hierarchy.

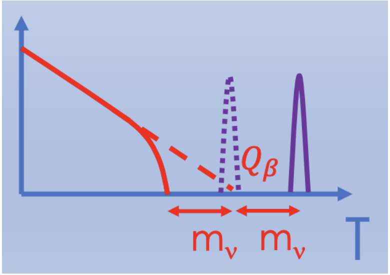


β-decay and neutrino capture

$(A, Z) \to (A, Z+1) + e^- + \bar{\nu}_e$



$(A, Z) + \nu_e \to (A, Z+1) + e^-$



- Neutrino capture is **threshold-less** – soft relic neutrino detection [Weinberg, 1962].
- The 2 parts of the spectrum are separated by $2m_\nu$ [1]
- Before the relic neutrino detection one would be able to measure the neutrino mass m_ν

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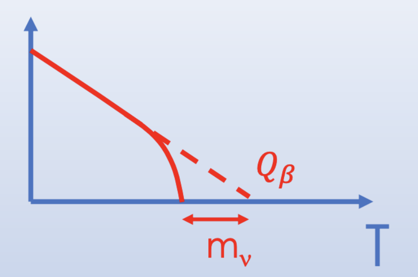
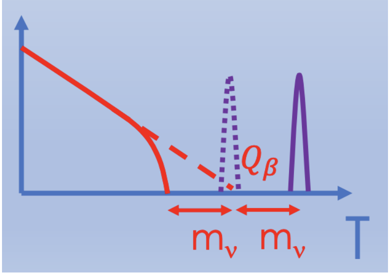
[1] Schematic picture that assumes only one neutrino flavour.

Relic neutrinos leave a signature in the spectrum of a radioactive atom

Is this goal technically achievable?

Challenges





- High energy precision (order of $m_\nu \sim \text{meV}$)
- Sufficient activity rate (several events per year)

Requirements to the experiment

High enough activity

Requirements to the experiment

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- Low emitter Q-value (Cocco et. al., 2007)

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$$(\sigma v)_\nu \propto \frac{1}{\tau Q^3}$$

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- Low emitter densities - *electron free path bigger than the system size*

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$$\lambda = (\sigma n)^{-1} = \left(R_{\text{atom}}^2 \frac{N}{L^3} \right)^{-1} > L$$

$$L > R_{\text{atom}} \sqrt{N} \sim 1\,\text{km}$$

Very naive estimate! In reality much bigger

Requirements to the experiment

High enough activity

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High enough precision

- Low emitter Q-value
- Low emitter densities - *electron free path bigger than the system size*
- Low volume

$$\Delta E \sim \frac{V_{\text{source}}}{V_{\text{detector}}}$$

Requirements to the experiment

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Requirements to the experiment

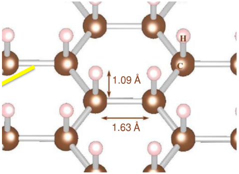
High enough activity

- Low emitter Q-value
- High number of emitters (order of 10^{25})
- Lifetime of emitter: small enough to have a high decay rate, but large enough not to decay instantly

- Radioactive material in gaseous form does not suit (0.93 eV resolution)
- Need in the solid-state based experiment

High enough precision

- Low emitter Q-value
- Low emitter densities - *electron free path bigger than the system size*
- Low volume



PTOLEMY[2] - state of the art

CνB detection experiment challemge:

- High energy resolution combined with sufficient number of events.

- **Tritium** as a β-decay emitter.
- Tritium is **deposed on graphene sheets** (vdW forces).
- ≈ 4 CνB **events per year**.
- Outstanding **energy resolution** of the apparatus $\approx 10\,\mathrm{meV}$.
- Strong collaboration funded by Simon's foundation
- Collaboration between Princeton (PI Chris Tully), Amsterdam, Milan, Rome

P on-
T ecorvo
O bservatory for
L ight,
E arly-universe,
M assive-neutrino
Y ield

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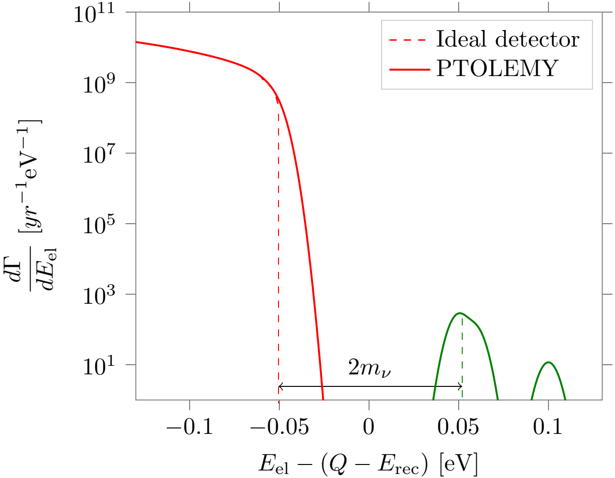
[2] *PTOLEMY collaboration "Neutrino physics with the PTOLEMY project", (2019)*

β-decay in Tritium [3]

$$\nu_e + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e$$

$$ {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e + \bar{\nu}_e$$



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[3] Mind the log scale

So that is it?

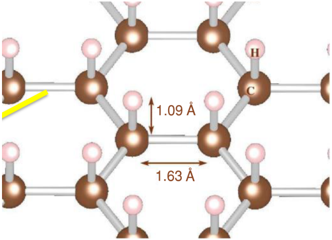
So that is it?

No

One needs to account for the intrinsic energy resolution

The **width of the peak** that serves as a signature of CνB is defined by

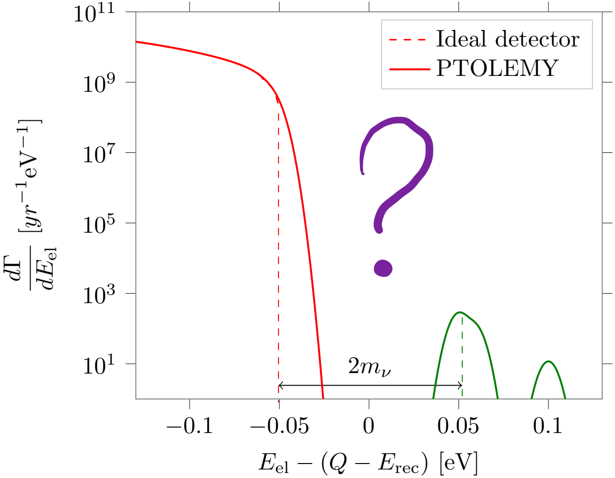
- *energy resolution* of the measurement
- physical *smearing* of the energies of individual electrons



- The presence of the substrate changes the intrinsic (before measurement) energy spectrum of the emitted electron.
- Introducing additional broadening of the electron spectrum.
- Which leads to intrinsic irreducible limitations on the energy resolution.

Real spectrum

Along with the finite energy resolution of the measurement device one has to account for the **intrinsic physical smearing** of the energies of individual electrons.

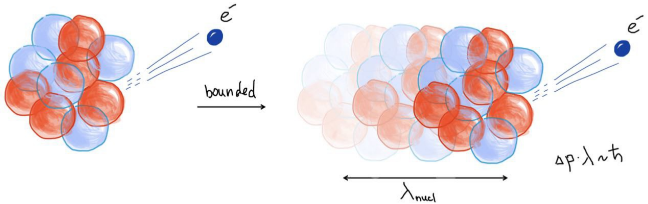


Mechanisms of the intrinsic energy broadening

- **Chemical bonding of the atom to the substrate.**
- **Impurity screening by charges in the substrate.**
- **X-ray edge singularity.**
- Lattice vibrations
- Emission of plasmons and surface polaritons
- Creation of shock wave emission due to the motion of the emitted electron at grazing angles at speeds exceeding the Fermi velocity
- Inhomogeneous broadening
- . . .

Chemical bonding of the atom to the substrate

General mechanism of the broadening



- For a bonded system, recoil energy of the nucleus is not fixed by the kinematics but has some distribution.
- Uncertainty[4] in the velocity of the centre of mass of the nucleus

$$\Delta u \approx \frac{\hbar}{m_{\text{nucl}} \lambda_{\text{nucl}}}.$$

- The energy of the electron is measured in the laboratory frame of reference, where it acquires an uncertainty[5]

$$\Delta E \approx m_e v_e \Delta u.$$

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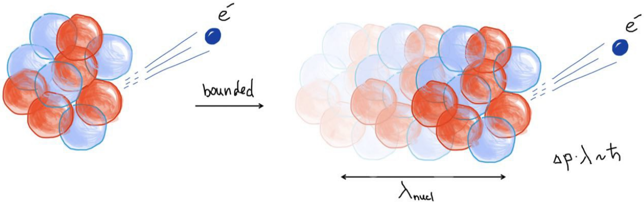
[4] from the Heisenberg uncertainty principle.

[5] ΔE has the same distribution as Δu.

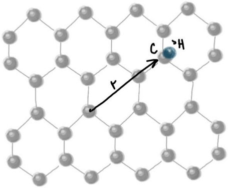
General mechanism of the broadening



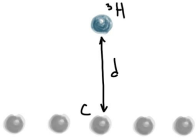
$$\Delta E \approx \hbar \frac{m_e v_e}{m_{nucl} \lambda_{nucl}},$$

- λ_{nucl} is the spread of the ground state of the nucleus that is defined by the bonding potential.

Bonding potential



in-plane



out-of-plane

For the heavy atom one can expand the potential near its minimum

$$U = \frac{1}{2}\varkappa_{i,j} r_i r_j + U_0$$

The energy uncertainty very weakly depends on the binding potential

$$\Delta E \propto \lambda_{\text{nucl}}^{-1} \propto \varkappa^{1/4}$$

Energy broadening for the β-decay of the Tritium on graphene

$$\frac{\Delta E}{\sqrt{\hbar m_e}} \approx \underbrace{\varkappa^{1/4}}_{\text{potential}} \underbrace{\sqrt{\frac{Q}{m_{\text{nucl}}^{3/2}}}}_{\text{nucleus}}$$

The uncertainty in the electron energy ΔE:

- Is of the order of $0.5\,\mathrm{eV}$.
- Is 2 orders of magnitude greater than the resolution needed to see the CνB signal.
- Weakly depends on the potential stiffness.
- For molecular tritium the estimate is of the same order.
- Strongly depends on the radioactive nucleus.
- Agrees with the the fully quantum calculation[6]

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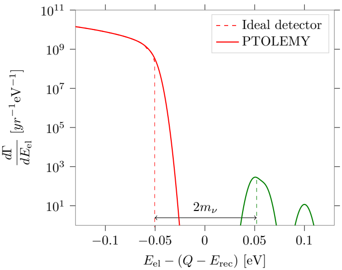
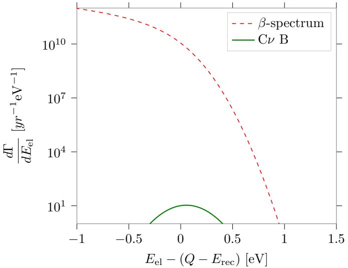
[6] Fermi Golden Rule.

Shape of the spectrum for the β-decay of the Tritium on graphene[7]



$$\xrightarrow{\text{smearing}}$$



$$\tilde{\mathcal{G}}(\mathsf{v}) = \int du\mathcal{F}(\mathsf{u})\mathcal{G}(\mathsf{v}+\mathsf{u}).$$

- $\mathcal{G}$ - distr. of the electron velocity in the centre of mass ref. frame.
- $\mathcal{F}$ - distr. of the velocity of the centre of mass.
- $\tilde{\mathcal{G}}$ - distr. of the electron velocity in the laboratory ref. frame.

---

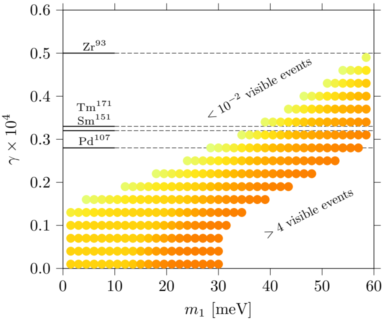
[7] $\Delta E \approx mu\Delta u$, therefore ΔE has the same distribution as Δu.

Solution [8]

$$\frac{\Delta E}{\sqrt{\hbar m_e}} \approx \varkappa^{1/4}\sqrt{\frac{Q}{m_{\mathrm{nucl}}^{3/2}}} \equiv \varkappa^{1/4}\gamma$$

Change the β-emitter to minimize γ

- Define the **visibility** as the number of CνB events that overlap with the continuous spectrum.
- $^{107}\mathrm{Pd}, {}^{151}\mathrm{Sm}, {}^{171}\mathrm{Tm}$ seem to work
- $^{107}\mathrm{Pd}$ has a very low activity
- $^{171}\mathrm{Tm}$ has ~ 10 times less events per year than $^{3}\mathrm{H}$
- $^{151}\mathrm{Sm}$ has $\sim 10^{3}$ times less events per year than $^{3}\mathrm{H}$



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[8] Mikulenko A., Cheipesh Y., Cheianov V., Boyarsky A., soon to appear

Is that it?

Is that it?

No, it is only beginning...

Mechanisms of the intrinsic energy broadening

- Chemical bonding of the atom to the substrate.
- **Impurity screening by charges in the substrate.**
- **X-ray edge singularity.**
- Lattice vibrations
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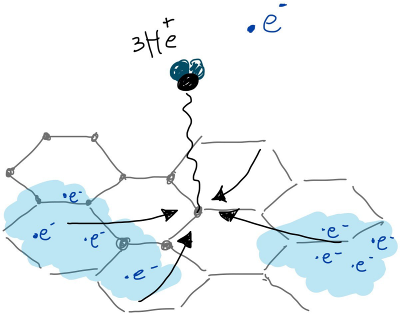
Charge screening

Once the electron is emitted, positive ion is formed. As a response to this, the **graphene will polarize** to screen the positive ion.



A screened weaker Couloumb potential **effectively increases the energy of the electron**.

Charge screening. Rough dimensional analysis

The only 2 **dimensionfull quantities** are

- $d \approx 3\,\text{Å}$ - the **distance of the tritium atom**.
- $v_F \approx 10\,\text{Å}\,\text{fs}^{-1}$ - **Fermi velocity** in graphene.

The rough estimate of the **relaxation time** is

$$\tau_{\text{relax}} = \frac{d}{v_f} \approx 0.3\,\text{fs} \tag{1}$$

The **velocity of the electron** near the edge of the spectrum is almost unchanged during the flight and is

$$\frac{v_0}{c} \approx 0.27\sqrt{\frac{E}{Q}} \tag{2}$$

Charge screening. Rough dimensional analysis

Such an electron will fly away from the atom on the distance

$$\lambda = v_e \tau_{\text{relax}} \approx \sqrt{\frac{E}{Q}} 0.27c \times 0.3\,\text{fs} \approx \sqrt{\frac{E}{Q}} \times 243\,\text{Å} \qquad (3)$$

The shift in the spectrum compared to unsreened case will be

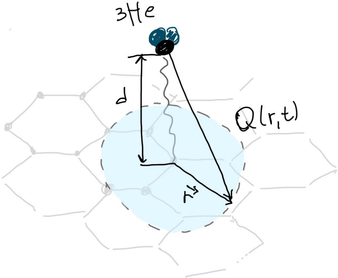
$$\Delta E(E) = k\frac{Ze^2}{\lambda} \approx \sqrt{\frac{E}{Q}} \times 59\,\text{meV} \qquad (4)$$

We see that the effect is significant and, most importantly, it is **energy-dependent**.

Charge screening. Quasiclassics

$$m_e\ddot{z} = -\nabla\varphi_{\text{eff}}(z,t) = -\nabla\left(\varphi_{\text{bare}}(z,t) - \varphi_{\text{polariz}}(z,t)\right)$$

$$\varphi_{\text{polariz}}(r,t) = \int d\vec{r}\,\frac{keQ(\vec{r},t)}{\sqrt{(z+d)^2+r^2}}$$



Where $Q(\vec{r},t)$ is the **renormalized** charge which is defined by the **dielectric permittivity** of graphene. For intrinsic graphene[a]

$$\varepsilon(q,t) = \left(\delta(t) + \frac{\alpha\pi}{4}qe^{-qd}J_0(v_F qt)\right)$$

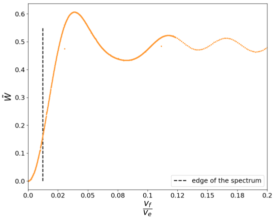
---

[a] Hwang, Das Sarma, PRB, 2007

Charge screening. Quasiclassics

The work performed on the electron by graphene

$$W = \underbrace{\frac{\pi\alpha^2}{16E_F d^2}}_{\approx 0.5\,\mathrm{eV}} \tilde{W}\left(\frac{v_F}{v_e}\right)$$



- For the electron at the end of the spectrum $W \approx 75\,\mathrm{meV}$
- Fermi velocity defines the relaxation time scale
- Near the edge of the spectrum $W(E)$ is almost linear.
- **The gap will not be "eaten" by this effect** as near the edge

$$|dW| = \frac{2\gamma}{E}\frac{dW}{d\gamma}dE \sim 10^{-6}dE$$

- Important to consider also **quantum fluctuations**

$$\mathrm{Var}(W) = \langle \hat{W}^2 \rangle - W^2$$

X-ray edge [9]

- Same Fermi Golden Rule but also taking into account graphene

$$
\begin{aligned}
|\psi\rangle_i^{(0)} &= |1\rangle_{\mathrm{H}} |0\rangle_{\mathrm{He}} |1\rangle_{\nu} |0\rangle_{e} |FS\rangle_{\mathrm{gr}} \\
|\psi\rangle_f^{(0)} &= |0\rangle_{\mathrm{H}} |1\rangle_{\mathrm{He}} |0\rangle_{\nu} |1\rangle_{e} |\lambda\rangle_{\mathrm{gr}} \\
dw_{\mathrm{if}} &= 2\pi \left| \langle\psi_f^0| \hat{V}_F |\psi_i^0\rangle \right|^2 \delta(E_i - E_f) d\nu_f
\end{aligned}
$$

- Ion of $^3\mathrm{He}^+$ is a scattering center (core level hole) in the X-ray edge singularity problem
- Energy transfer between the between the β emitters and the graphene system leading to the **smearing of the spectrum**
- The smeared beta decay spectrum is the convolution between the spectral density function of graphene $A(E)$ and the original beta decay spectrum $d\Gamma/dE$.

$$
\left.\frac{d\Gamma}{dE}\right|_{\mathrm{smeared}} = \left(\frac{d\Gamma}{dE} \star A\right)(E)
$$

---

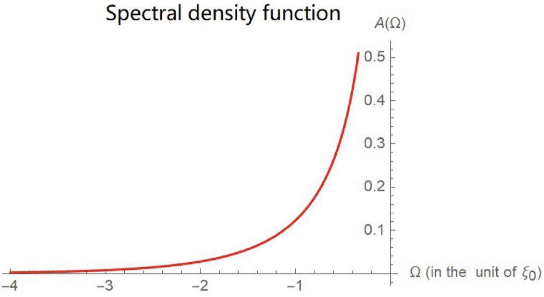
[9] Work done by T. Zhiyang and V. Cheianov

X-ray edge [10]

- Spectral density function has an X-ray edge

$$A(\Omega) = \Theta(-\Omega)\frac{\sin(\pi g)}{\pi}\Gamma(1-g)\frac{\exp(\Omega)}{\xi_0(-\Omega)^{1-g}}, \quad \Omega = \frac{E+E_i}{\xi_0},$$

where $g = e^4/2\varepsilon^2 v_F^2$ and $\xi = v_F/2d$ is the cut-off energy fixed by the distance d from the impurity to the graphene sheet.



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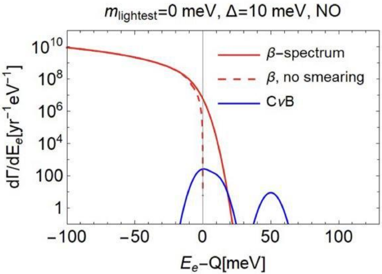
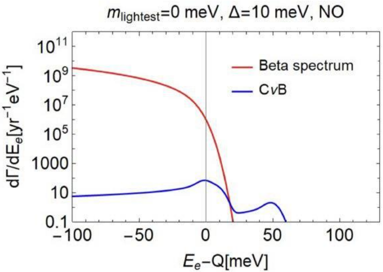
[10] Work done by T. Zhiyang and V. Cheianov

X-ray edge [11]



(a) *original beta decay spectrum when the lightest neutrino mass equals zero.*



(b) *smeared beta decay spectrum*

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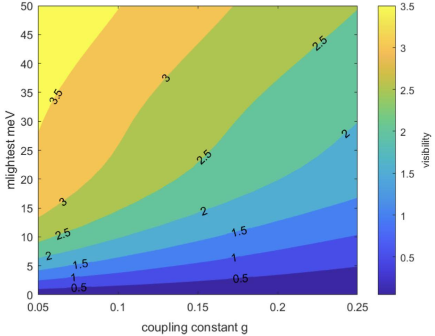
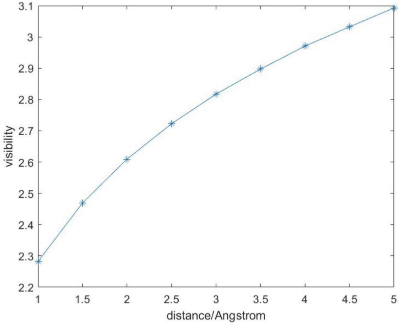
[11] Work done by T. Zhiyang and V. Cheianov

X-ray edge. Solutions [12]



Increase dielectric permittivity ε of the substrate



Increase the distance between the substrate and the β emitter

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[12] Work done by T. Zhiyang and V. Cheianov

Conclusions

- Quantum impurity problems play a key role in the new large-scale astro-particle experiment capable to open new era in Cosmology
- Fundamental questions of particle physics and the origin of our Universe could realistically be accessed only through the understanding of the condensed matter effects
- Limitations due to intrinsic effects in the solid state part of the measurement setup are crucial for the feasibility of this large-scale experiment.
- Before scaling to to a huge number of beta emitters, a program of theoretical and experimental study of small quantum devices with relevant impurities is needed.
- Zoo of effects appear in the $E \sim 10\,\mathrm{meV}$ which needs a much better understanding than usually in CM.
- Energy and time scales not accessible in the condensed matter experiments before
- More questions than answers [13]

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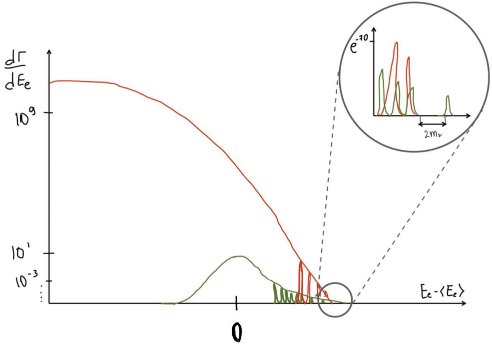
[13] gene.cheypesh@gmail.com, v.cheianov@gmail.com, boyarsky@lorentz.leidenuniv.nl, PTOLEMY collaboration

Backup slides

General shape of the spectrum

In case when the bonding potential is harmonic, the spectrum is

- Discrete near the edge.
- Continuous further from the edge.
- The envelope has a gaussian distribution.
- The distance between the discrete lines[14] is $\varepsilon = \hbar\sqrt{\frac{\varkappa}{m_{\text{nucl}}}}$.
- Biggest part of the CνB channel overlaps with the continuum.



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[14] 10 meV for the Tritium on graphene and 0.5 eV for the molecular tritium.

Comparison with molecular Tritium

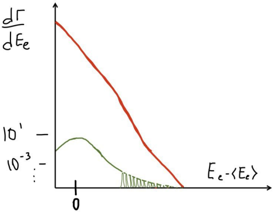
Similarities:

- Bonded by a harmonic potential ($\varkappa_{\text{graphene}} \approx 0.1, \varkappa_{\text{mol}} \approx 75$).
- Localized and therefore are subjects to Heisenberg's uncertainty principle $m\Delta v\Delta x \sim \hbar$.

Differences:

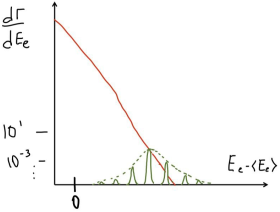
Atomic Tritium on graphene:

- All of the recoil energy goes to the harmonic modes.
- May break the bound after the recoil.

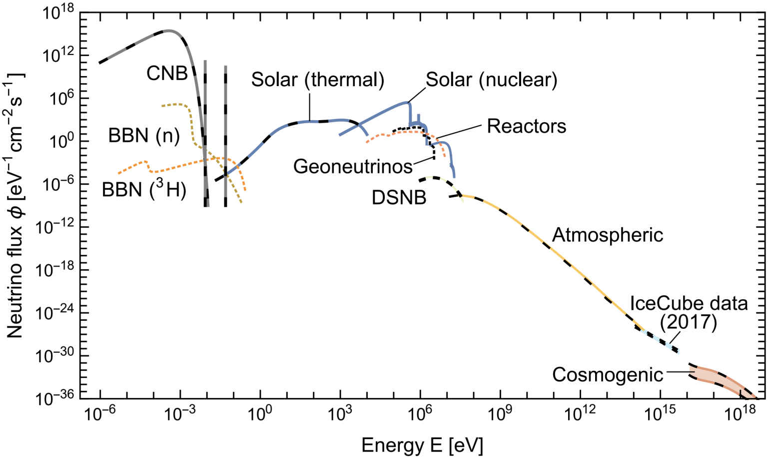


Gaseous molecular Tritium:

- Half of the recoil energy goes to the transnational motion.
- Remains bound after the recoil.



Neutrino flux[15]



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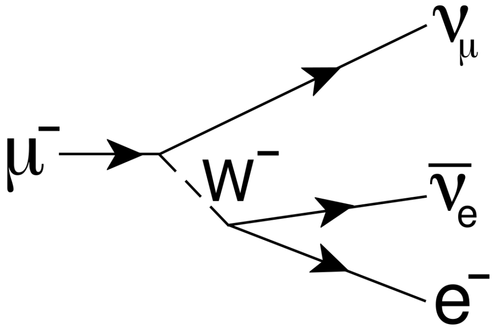
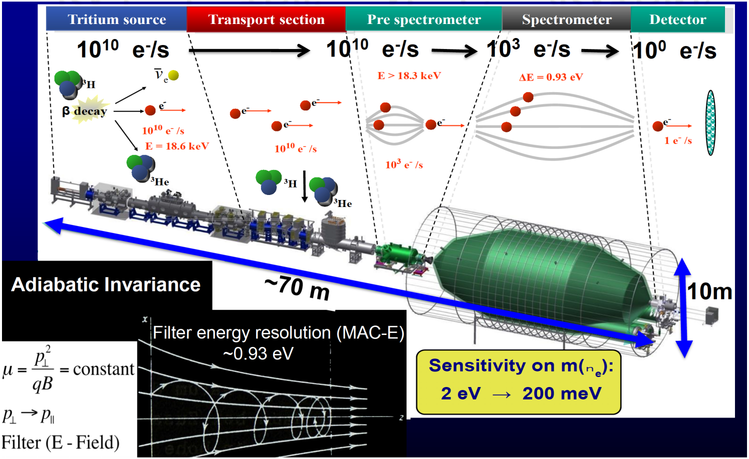
[15] *E. Vitagliano et.al. "Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components", (2020)*

Neutrino flavours



KATRIN

KArlsruhe TRItium Neutrino (KATRIN)



Adiabatic Invariance

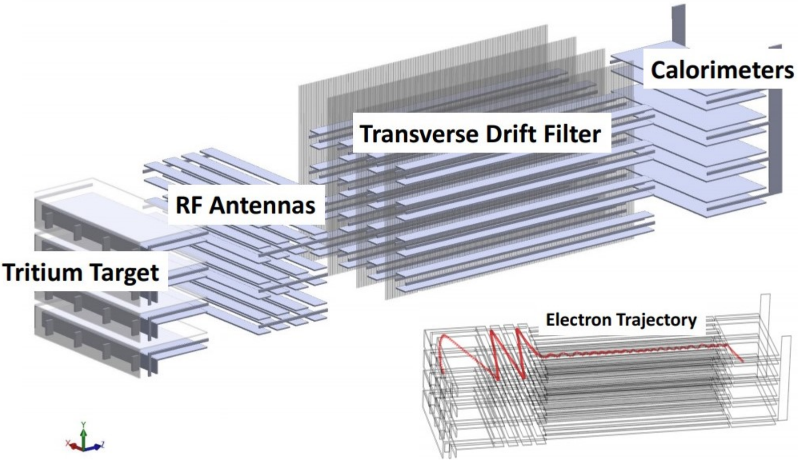
$$\mu = \frac{p_\perp^2}{qB} = \text{constant}$$

$$p_\perp \rightarrow p_\parallel$$

Filter (E - Field)

Sensitivity on $m(\nu_e)$:
2 eV → 200 meV

PTOLEMY


Calorimeters
Transverse Drift Filter
RF Antennas
Tritium Target
Electron Trajectory
Y
X
Z