Determination of CKM Angle $\phi_3$ at Belle and Belle II

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The CKM angle $\phi_3$

$$\phi_3/\gamma \equiv \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)$$

- Very precise theoretical prediction $\delta \phi_3/\phi_3 \sim 10^{-7}$
- Test physics beyond SM

The interference between color-favored and color-suppressed processes can be related:

$$\frac{A^{\text{suppr.}}[B^- \to \bar{D}^0 K^-]}{A^{\text{favor.}}[B^- \to \bar{D}^0 K^-]} = r_B e^{i(\delta_B - \phi_3)}$$

$r_B$ - the magnitude of the ratio of amplitudes $\sim 0.1$; $\delta_B$ - strong-phase difference

3 main methods to extract $\phi_3$:

- GLW method: CP eigenstates: $K^-K^+, \pi^-\pi^+, K_S^0\pi^0$
- ADS method: DCS modes: $K^+\pi^-, K^+\pi^-\pi^0$
- BPGGSZ method: self-conjugate multibody final states: $K_S^0\pi^+\pi^-, K_S^0K^+K^-, K_S^0\pi^+\pi^0\pi^0$

Foreseen precision of $\phi_3$ is expected to be $O(1^\circ)$ (current world-average $\delta \phi \sim 4^\circ$) with the full Belle II dataset of 50 $ab^{-1}$
BtoCharm Status

⇒ 200 fb⁻¹ Belle II data + Belle data
⇒ $B^- \rightarrow D_{CP} \left( \rightarrow K^- K^+ K_S^0 \pi^0 \right) h^- \rightarrow 01$ to measure $\phi_3$
⇒ Aim to publish 2022 summer

⇒ 128 fb⁻¹ Belle II data + Belle data
⇒ $B^- \rightarrow D \left( \rightarrow K_S^0 \pi^+ \pi^- \right) h^- \rightarrow$ to measure $\phi_3$

⇒ 62 fb⁻¹ Belle II data
⇒ Study of Cabibbo favored modes

arXiv:2104.03628
Study of $B \rightarrow D_{fav}^{(*)} h$ with 62.8 fb$^{-1}$ Belle II data

- The improved measurement of the color-favored hadronic two body decay of B meson helps to a better understanding of QCD effects.
- Decay ratio to be extracted:
  \[ R^{D_{}} = \frac{\Gamma(B \rightarrow D^{(*)} K)}{\Gamma(B \rightarrow D^{(*)} \pi)} \approx \tan^2 \theta_C \left( \frac{f_K}{f_\pi} \right)^2 \]
  which will eliminate some systematic uncertainties.
- Unbinned 1D simultaneous fit of $\Delta E$
  \[
  \begin{align*}
  N_{KID<0.6}^{D_{}} &= (1 - \kappa_{KID>0.6}) N_{Total}^{D_{}} \\
  N_{KID<0.6}^{D_{}} &= (1 - \epsilon_{KID>0.6}) R^{D_{}} N_{Total}^{D_{}} \\
  N_{KID>0.6}^{D_{}} &= \epsilon_{KID>0.6} R^{D_{}} N_{Total}^{D_{}} \\
  N_{KID>0.6}^{D_{}} &= \kappa_{KID<0.6} N_{Total}^{D_{}}
  \end{align*}
  \]
  $\kappa$ - pion fake rate (free); $\epsilon$-kaon efficiency (fixed from official PID table);
  KID - short for binary PID likelihood $L(K/\pi) = \frac{L(K)}{L(K)+L(\pi)}$.
Study of $B \to D^{(*)}_{fav} h$ done with 62.8 fb$^{-1}$ Belle II data

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$B^{-} \to D^{0}(K^{-}\pi^{+})h^{-}$</th>
<th>$B^{-} \to D^{0}(K^{0}_{S}\pi^{+}\pi^{-})h^{-}$</th>
<th>$B^{0} \to D^{-}h^{+}$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Belle II $R^{(*)+0}/($%$)$</td>
<td>7.66 ± 0.55±0.11−0.08</td>
<td>6.32 ± 0.81±0.09−0.11</td>
<td>9.22 ± 0.58 ± 0.09</td>
<td>6.80 ± 1.01 ± 0.07</td>
<td>5.99 ± 0.82±0.17−0.08</td>
</tr>
<tr>
<td>LHCb $R^{(*)+0}/($%$)$</td>
<td>7.77 ± 0.04 ± 0.07</td>
<td>7.77 ± 0.04 ± 0.07</td>
<td>8.22 ± 0.11 ± 0.25</td>
<td>7.93 ± 0.11 ± 0.56</td>
<td>7.76 ± 0.34 ± 0.26</td>
</tr>
</tbody>
</table>

We report measurements related to hadronic $B$ decays to final states that contain charm mesons. The analyses are performed on a 62.8 fb$^{-1}$ data set collected by the Belle II experiment at a center-of-mass energy corresponding to the mass of the $\Upsilon$(4S) resonance. The measurements reported are for the decay modes $B^{-} \to D^{0}h^{-}$, $B^{-} \to D^{0}h^{-}$, $B^{0} \to D^{-}h^{+}$ and $B^{0} \to D^{+}h^{+}$, where $h = \pi$ or $K$. These modes are either signal or control channels for measurements related to the unitarity triangle angle $\gamma$ in direct or time-dependent CP-violation measurements. The reported observables are the ratios between the $B \to D^{(*)}K$ and $B \to D^{(*)}\pi$ decay rates, which are found to be in agreement with previous measurements.
BPGGSZ Method Study of $B^- \rightarrow D(\rightarrow K_S^0 h^+ h^-)h^-$

First Belle + Belle II analysis Done By Niharika Rout, Jim Libby, Karim Trabelsi

- $B^- \rightarrow D(\rightarrow K_S^0 \pi^+ \pi^-)K$ is golden mode at Belle II
- Sensitivity to $\phi_3$ by comparing D Dalitz (fit with full amplitude model) distribution of $B^-$ and $B^+$:
  \[ A_B = \overline{A}(m_-, m_+) + r_B e^{i(\delta_B + \phi_3)} A(m_+^2, m_-^2) \]
- Model-dependent analysis have model uncertainty up to $3^\circ$ - $9^\circ$
- Using binned model-independent approach
  - Optimal binning of the D Dalitz plot which gives the maximum sensitivity to $\phi_3$
  - Observed yields in each bin can be related to physics parameters of interest and $D^0$ decay information

\[ N_{i} = h_{B} \pm \left[ F_{i} + r_{B}^{2} F_{i} + 2 \sqrt{F_{i} F_{i}^{\ast}} (c_{i} x_{i} + s_{i} y_{i}) \right]. \]

- $h_{B}$: Normalization constant
- Physics parameters of interest: $(x_{i}, y_{i}) = r_{B}(\cos(\phi_3 + \delta_B), \sin(\phi_3 + \delta_B))$
- Amplitude-averaged strong phase difference between $\overline{D}^0$ and $D^0$ over $i^{th}$ bin and are obtained from external charm factories like CLEO and BESIII
- Fraction of pure $D^0$ decay to bin $i$ taking into account the reconstruction and selection efficiency

https://arxiv.org/abs/2110.12125

$h = \pi, K$
BPGGSZ Method Study of $B^- \to D\left(\to K_S^0 h^+ h^-\right)h^-$

First Belle + Belle II analysis Done By Niharika Rout, Jim Libby, Karim Trabelsi

- Analysis with $711 fb^{-1}$ Belle data and $128 fb^{-1}$ Belle II data

- Unbinned 2D simultaneous fit of $\Delta E$ versus $C'$ (right plot) for $B^- \to D^0 \left(K_S^0 \pi^+ \pi^-\right)K^-$.

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<th>Component</th>
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<th>PDF (FBDT$_{trans}$)</th>
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<tr>
<td>Signal</td>
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<td>poly (1st)</td>
</tr>
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<td>$BB$ bkg</td>
<td>expo + (poly)</td>
<td>Chebychev poly-1st(2nd)</td>
</tr>
<tr>
<td>$q\bar{q}$ bkg</td>
<td>Chebychev poly (1st)</td>
<td>2 expo</td>
</tr>
<tr>
<td>$DK (D\pi)$ component</td>
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- Performed simultaneous fit in 160 categories; 80($16 \times 4 + 4 \times 4$) of Belle and 80 of Belle II

- Signal region:
  - $|\Delta E| < 0.05 \text{ GeV}$
  - $0.65 < C' < 1.0$

$(x_\pm, y_\pm) = r_B(\cos(\phi_3 + \delta_B), \sin(\phi_3 \pm \delta_B))$ are common to all the bins and are extracted from the fit

https://arxiv.org/abs/2110.12125

kaon enhanced $\mathcal{L}(K/\pi) > 0.6$

pion enhanced $\mathcal{L}(K/\pi) < 0.6$
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<td>Chebychev poly (1st)</td>
<td>2 expo</td>
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<tr>
<td>$D\bar{K}$ ($D\pi$) component</td>
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Uncertainty $\sim 14^\circ$ in earlier Belle measurement

Preliminary result:

$\delta_B(\circ) = 124.8 \pm 12.9 \text{ (stat.)} \pm 0.5 \text{ (syst.)} \pm 1.7 \text{ (ext. input)}$

$r_B^{DK} = 0.129 \pm 0.024 \text{ (stat.)} \pm 0.001 \text{ (syst.)} \pm 0.002 \text{ (ext. input)}$

$\phi_3(\circ) = 78.4 \pm 11.4 \text{ (stat.)} \pm 0.5 \text{ (syst.)} \pm 1.0 \text{ (ext. input)}$

https://arxiv.org/abs/2110.12125

$h = \pi, K$
GLW Method of $B^- \rightarrow D_{CP}h^-$, $h = \pi, K$

- Using CP eigenstates mode of $K^+K^-(CP^+)$, $K^0\pi^0(CP^-)$

- $D_{CP}$ modes interference => relate with $\phi_3$

$$A_{CP\pm} = \frac{\Gamma[B^- \rightarrow D_{CP}K^-] - \Gamma[B^+ \rightarrow D_{CP}K^+]}{\Gamma[B^- \rightarrow D_{CP}K^-] + \Gamma[B^+ \rightarrow D_{CP}K^+]} = \pm \frac{2r_B\sin\delta_B\sin\phi_3}{1 + r_B^2 \pm 2r_B\cos\delta_B\cos\phi_3}$$

$$R_{CP\pm} = \frac{R_{D_{CP}\pm}^{D^0}}{R_{D^0}} = 1 + r_B^2 \pm 2r_B\cos\delta_B\cos\phi_3$$

- $B^- \rightarrow D^0(K^-\pi^+)h^-$ as control modes to obtain $R_{D^0}$

- Related measurements listed on HFLAV

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$A_{CP^+}$</th>
<th>$A_{CP^-}$</th>
<th>$R_{CP^+}$</th>
<th>$R_{CP^-}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>0.25 ± 0.06 ± 0.02</td>
<td>-0.09 ± 0.07 ± 0.02</td>
<td>1.18 ± 0.09 ± 0.05</td>
<td>1.07 ± 0.08 ± 0.04</td>
<td>PRD 82 (2010) 072004</td>
</tr>
<tr>
<td>Belle (*)</td>
<td>0.06 ± 0.14 ± 0.05</td>
<td>-0.12 ± 0.14 ± 0.05</td>
<td>1.13 ± 0.16 ± 0.08</td>
<td>1.17 ± 0.14 ± 0.14</td>
<td>PRD 73 (2006) 051106</td>
</tr>
<tr>
<td>CDF</td>
<td>0.39 ± 0.17 ± 0.04</td>
<td>-</td>
<td>1.30 ± 0.24 ± 0.12</td>
<td>-</td>
<td>PRD 81 (2010) 031105(R)</td>
</tr>
<tr>
<td>LHCb</td>
<td>0.136 ± 0.009 ± 0.001</td>
<td>-</td>
<td>0.950 ± 0.009 ± 0.010</td>
<td>-</td>
<td>JHEP 2104 (2021) 081</td>
</tr>
</tbody>
</table>

Previous Belle measurement with 200 $fb^{-1}$ (~similar size as Belle II data today)
2D Simultaneous Fit of $B^- \rightarrow D_{CP} h^-$

- Signal enhanced fit: $-0.05<\Delta E <0.05$ & $C'>0.4$

- Concerning fit results (13 observables and 55 free fit parameters)

<table>
<thead>
<tr>
<th>Parameters (%)</th>
<th>$R^{D^0}$</th>
<th>$R^{D_{cp}^+}$</th>
<th>$R^{D_{cp}^-}$</th>
<th>$A^K_{D^0}$</th>
<th>$A^\pi_{D^0}$</th>
<th>$A^K_{D_{cp}^+}$</th>
<th>$A^\pi_{D_{cp}^+}$</th>
<th>$A^K_{D_{cp}^-}$</th>
<th>$A^\pi_{D_{cp}^-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC Input</td>
<td>8.264</td>
<td>8.264</td>
<td>8.264</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Stream 3</td>
<td>8.38$\pm$0.15</td>
<td>7.72$\pm$0.58</td>
<td>7.96$\pm$0.58</td>
<td>-2.40$\pm$1.72</td>
<td>-0.38$\pm$0.43</td>
<td>9.50$\pm$6.80</td>
<td>-0.73$\pm$1.55</td>
<td>0.53$\pm$6.02</td>
<td>0.17$\pm$1.36</td>
</tr>
</tbody>
</table>

$\rightarrow$ Uncertainty of $A_{CP}\sim 6\%$
$\phi_3$ Sensitivity

$B^- \to D(K_S^0\pi^+\pi^+)h^-$ Mode

$x_\pm = r_B \cos(\delta_B \pm \phi_3) \Rightarrow \text{Measurement}$

$y_\pm = r_B \sin(\delta_B \pm \phi_3)$

$\begin{array}{c|c}
\text{parameter} & \text{value} \\
\hline
x_+ & -0.1194\pm0.0346 \\
x_- & 0.0945\pm0.0360 \\
y_+ & -0.0634\pm0.0473 \\
y_- & 0.1192\pm0.0467 \\
\end{array}$

$\Rightarrow \phi_3$ Uncertainty $\sim10.7^\circ$

$\Rightarrow \text{Result from analysis: } \sim11.0^\circ$

$\Rightarrow \text{Estimation method is valid}$

GLW Method

$x_\pm = r_B \cos(\delta_B \pm \phi_3) = \frac{R_1(1 \mp A_1) - R_2(1 \mp A_2)}{4}$

$\begin{array}{c|c}
\text{parameter} & \text{Uncertainty from one stream MC} \\
\hline
R_1 & \pm0.0058 \\
A_1 & \pm0.0690 \\
R_2 & \pm0.0057 \\
A_2 & \pm0.0650 \\
\end{array}$

$\Rightarrow \phi_3$ Uncertainty $\sim13^\circ$

$\Rightarrow \text{Uncertainty } \sim9^\circ$ (will investigate further)

GLW $B^- \to D(K_S^0\pi^+\pi^+)K^-$ one stream Belle MC

$\Rightarrow \phi_3$ Uncertainty $\sim9^\circ$ (will investigate further)
Summary

- Determination of CKM angle \( \phi_3 \) is important and Belle II provides a good opportunity to extract it with high precision.

- Belle II analysis of \( B \to D^{(*)} h \) is done with Cabibbo favored modes to validate the performance of 62.8 fb\(^{-1} \) Belle II data. \[\text{arXiv:2104.03628}\]

- Performance of Belle II at this stage is similar to Belle.

- Determination of \( \phi_3 \) from the combined result of Belle + Belle II of \( B^- \to D^0 (K_S^0 h^+ h^-) K^- \) is submitted to JHEP and are available on arXiv.org. \[\text{https://arxiv.org/abs/2110.12125}\]

- GLW method with final states \( K^- K^+ , K_S^0 \pi^0 \) modes is being analyzed with Belle and Belle II data samples (summer 2022). Simultaneously fitting with 6 modes to extract the signal and concerning observables.

- We estimated the GLW+BPGGSZ sensitivity of \( \phi_3 \sim 9^\circ (0.711 \ ab^{-1}) , \sim 2^\circ (10 \ ab^{-1}) \)