QED corrections to $\bar{B} \to \bar{K}\ell^+\ell^-$

Flavour Physics Day

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Lepton Flavour Universality (LFU) predicted by SM.

One can thus define *lepton flavour universality* ratios, such as R_K:

$$R_K\left[q^2_{\mathrm{min}}, q^2_{\mathrm{max}}\right] = \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2 \frac{d\Gamma\left(B\to K\mu^+\mu^-\right)}{dq^2}}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} dq^2 \frac{d\Gamma(B\to Ke^+e^-)}{dq^2}},$$

where $q^2 = \left(\ell^+ + \ell^-\right)^2$.

SM predicts $R_K = 1$, whereas LHCb reports

$$R_K\left[1.1\mathrm{GeV}^2, 6\mathrm{GeV}^2\right] = 0.846^{+0.042+0.013}_{-0.039-0.012}$$

This represents a *3.1 σ deviation* from the SM.

$\Longrightarrow$ Hints to Physics beyond the SM.

Why are QED corrections to $\bar{B} \to \bar{K}\ell^+\ell^-$ important?

Expected to be small, since $\frac{\alpha}{\pi} \approx 2\cdot 10^{-3}$.

Due to kinematic effects, QED corrections are of $\mathcal{O}(\frac{\alpha}{\pi}) \ln \hat{m}_\ell$ [*Note:* $\hat{m}_\ell \equiv \frac{m_\ell}{m_B}$].

Moreover, R_K is a theoretically *clean observable*, since hadronic uncertainties cancel in the ratio.

Therefore, need to make sure QED corrections properly accounted for in experiments (PHOTOS).

Also, precise determination of CKM matrix elements.

Based on 2009:00929, with G. Isidori and R. Zwicky.

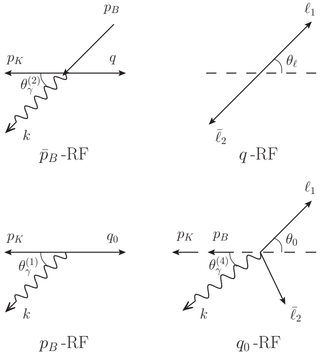
Introduction/Motivation

Bordone et al. (*1605.07633*) already performed a calculation to estimate QED corrections to R_K.

However, our work represents a more complete treatment since

- We work with the *full matrix elements* (real and virtual), starting from an EFT Lagrangian description. Hence, we can capture effects beyond collinear $\ln \hat{m}_\ell$ terms, such as $\ln \hat{m}_K$ which are not necessarily small.
- Results at the *double* differential level are given, and hence they can be used for angular analysis (moments).
- We present a *detailed discussion on IR divergences*, and demonstrate explicitly the conditions under which they cancel.

Differential Variables



$$\{q_a^2, c_a\} = \begin{cases} q_\ell^2 = (\ell_1+\ell_2)^2, \quad c_\ell = -\left(\frac{\vec{\ell_1}\cdot\vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q-\mathrm{RF}} & [\text{"Hadron collider"}]\ , \\ q_0^2 = (p_B - p_K)^2\ , \quad c_0 = -\left(\frac{\vec{\ell_1}\cdot\vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q_0-\mathrm{RF}} & [\text{"B-factory"}]\ , \end{cases}$$

where $q-\mathrm{RF}$ and $q_0-\mathrm{RF}$ denotes the rest frames of $q \equiv \ell_1 + \ell_2$ and $q_0 \equiv p_B - p_K = q + k$ respectively.

Differential variables and cut-off

Implement a *physical cut-off on the photon energy* (based on the visible kinematics),

$$\bar{p}_B^2 \;\geq\; m_B^2 \, (1 - \delta_{\rm ex}) \, ,$$

where

$$\bar{p}_B^2 \equiv m_{B_{\rm rec}}^2 = (p_B - k)^2 = (\ell_1 + \ell_2 + p_K)^2 .$$

Since there is *no photon-emission* in the non-radiative rate, there is no difference between the $\{q^2, c_\ell\}$- and $\{q_0^2, c_0\}$-variables.

IR Divergences

Split the differential rate as follows

$$\frac{d^2\Gamma_{\bar{B}\to\bar{K}\ell_1\bar{\ell}_2}(\delta_{\rm ex})}{dq_a^2 dc_a} = \frac{d^2\Gamma^{\rm LO}}{dq^2 dc_\ell} + \frac{\alpha}{\pi}\sum_{i,j}\hat{Q}_i\hat{Q}_j\left(\mathcal{H}_{ij} + \mathcal{F}_{ij}^{(a)}(\delta_{\rm ex})\right) + \mathcal{O}(e^4)\ ,$$

where $d^2\Gamma^{\rm LO}$ corresponds to the zeroth order differential rate and $\mathcal{H}$ and $\mathcal{F}$ stand for the virtual and real contributions respectively.

$$\begin{aligned}
\frac{\alpha}{\pi}\sum_{i,j}\hat{Q}_i\hat{Q}_j\mathcal{H}_{ij} &= \frac{1}{m_B}\rho_\ell|_{\bar{p}_B^2\to m_B^2} 2{\rm Re}[\mathcal{A}^{(2)*}\mathcal{A}^{(0)}]\ ,\\
\frac{\alpha}{\pi}\sum_{i,j}\hat{Q}_i\hat{Q}_j\mathcal{F}_{ij}^{(a)} &= \frac{1}{m_B}\int d\Phi_\gamma\,\rho_a\,|\mathcal{A}^{(1)}|^2\ ,
\end{aligned}$$

IR Divergences

The integrals are split into *IR sensitive parts* which can be done *analytically* and a necessarily regular part which is dealt with numerically.

$$\mathcal{H}_{ij} = \frac{d^2\Gamma^{\rm LO}}{dq^2 dc_\ell}\left(\tilde{\mathcal{H}}_{ij}^{(s)} + \tilde{\mathcal{H}}_{ij}^{(hc)}\right) + \Delta\mathcal{H}_{ij} \; ,$$

$$\mathcal{F}_{ij}^{(a)}(\delta_{\rm ex}) = \frac{d^2\Gamma^{\rm LO}}{dq^2 dc_\ell}\tilde{\mathcal{F}}_{ij}^{(s)}(\omega_s) + \tilde{\mathcal{F}}_{ij}^{(hc)(a)}(\underline{\delta}) + \Delta\mathcal{F}_{ij}^{(a)}(\underline{\delta}) \; ,$$

with $\tilde{\mathcal{H}}_{ij}^{(s)}$ ($\tilde{\mathcal{H}}_{ij}^{(hc)}$) and $\tilde{\mathcal{F}}_{ij}^{(s)}$ ($\tilde{\mathcal{F}}_{ij}^{(hc)(a)}$), containing all *soft* (*hard-collinear*) singularities, whereas $\Delta\mathcal{H}$ and $\Delta\mathcal{F}$ are regular.

We adopt the *phase space slicing method*, which requires the introduction of two auxiliary (unphysical) cut-offs $\omega_{s,c}$,

$$\underline{\delta} \equiv \{\delta_{\rm ex}, \omega_s, \omega_c\} \; , \quad \omega_s \ll 1 \; , \quad \frac{\omega_c}{\omega_s} \ll 1 \; .$$

IR Divergences and Cancellation

Phase Space slicing conditions

$$\bar{p}_B^2 \geq m_B^2\,(1-\omega_s) \iff E_\gamma^{p_B-\mathrm{RF}} \leq \frac{\omega_s m_B}{2},$$

$$k\cdot\ell_{1,2} \leq \omega_c m_B^2$$

All soft divergences cancel between real and virtual, independent of the choice of differential variables.

In the phase space slicing method, they are replaced by $\ln(\omega_s)$, which then cancel in the sum of the $\mathcal{F}$ terms.

IR Divergences: Hard Collinear Real

In the collinear limit $(k||\ell_1)$, the matrix element squared factorises:

$$|\mathcal{A}^{(1)}_{\ell_1||\gamma}|^2 = \frac{e^2}{(k\cdot\ell_1)}\hat{Q}^2_{\ell_1}\tilde{P}_{f\to f\gamma}(z)|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 + \mathcal{O}(m^2_{\ell_1}) \, ,$$

where $|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 = |\mathcal{A}^{(0)}_{\bar{B}\to\bar{K}\ell_{1\gamma}\bar{\ell}_2}|^2$ and $\tilde{P}_{f\to f\gamma}(z)$ is the collinear part of the splitting function for a fermion to a photon

$$\tilde{P}_{f\to f\gamma}(z) \equiv \left(\frac{1+z^2}{1-z}\right) \, .$$

z gives the momentum fraction of the photon and lepton.

$$\ell_1 = z\ell_{1\gamma}$$

$$k = (1-z)\,\ell_{1\gamma}$$

which then implies

$$q^2 = zq_0^2$$

IR Divergences: Cancellation of hc logs

In $\{q_0^2, c_0\}$ variables,

$$\frac{d^2\Gamma}{dq_0^2 dc_0}\bigg|_{\ln \hat{m}_{\ell_1}} = \frac{d^2\Gamma^{\,\mathrm{LO}}}{dq_0^2 dc_0}\left(\frac{\alpha}{\pi}\right)\hat{Q}_{\ell_1}^2 \ln \hat{m}_{\ell_1} \times C_{\ell_1}^{(0)} \;,$$

where

$$C_{\ell_1}^{(0)} = \left[\frac{3}{2} + 2\ln \bar{z}(\omega_s)\right]_{\tilde{\mathcal{F}}^{(hc)}} + \left[-1 - 2\ln \bar{z}(\omega_s)\right]_{\tilde{\mathcal{F}}^{(s)}} + \left[\frac{3}{2} - 2\right]_{\tilde{\mathcal{H}}} = 0$$

On the other hand, in $\{q^2, c_\ell\}$ variables,

$$\frac{d^2\Gamma}{dq^2 dc_\ell}\bigg|_{\mathrm{hc}} = \frac{\alpha}{\pi}\left(\hat{Q}_{\ell_1}^2 K_{\mathrm{hc}}(q^2, c_\ell)\ln \hat{m}_{\ell_1} + \hat{Q}_{\ell_2}^2 K_{\mathrm{hc}}(q^2, -c_\ell)\ln \hat{m}_{\ell_2}\right) ,$$

where $K_{\mathrm{hc}}(q^2, c_\ell)$ is a non-vanishing function.

After integration over q^2 and c_ℓ, the above vanishes.

However, with a cut-off δ_{ex}, collinear logs survive in both differential variables!

IR Divergences: Structure-dependent terms

Q: Do we miss any $\ln \hat{m}_\ell$ contributions due to structure dependence, by doing an EFT calculation?

A: No, gauge invariance ensures that there are no such additional contributions.

However, using the EFT analysis, we do *not* capture $\ln \hat{m}_K$ effects, which can be quite significant (*LCSR approach: ongoing work*).

Results

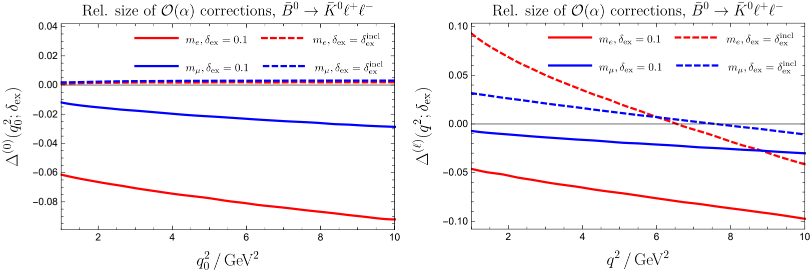
We consider *relative* corrections. For a single differential in $\frac{d}{dq_a^2}$,

$$\Delta^{(a)}(q_a^2; \delta_{\rm ex}) = \left(\frac{d\Gamma^{\rm LO}}{dq_a^2} \right)^{-1} \frac{d\Gamma(\delta_{\rm ex})}{dq_a^2}\Big|_{\alpha} ,$$

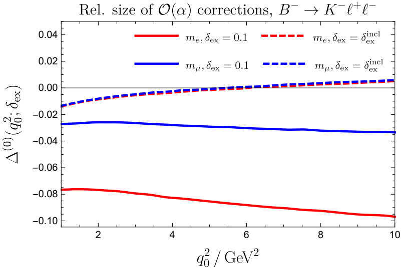
where the numerator and denominator are integrated separately over $\int_{-1}^{1} dc_a$ respectively.

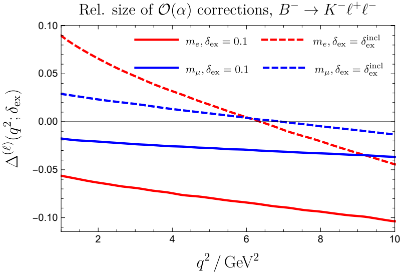
It is important to integrate the QED correction and the LO separately as this corresponds to the experimental situation.

Results: $\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ in q_a^2



- In photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$, dashed lines), all IR sensitive terms cancel in the q_0^2 variable locally.
- (Approximate) lepton universality on the plots on the left.
- Effects due to the photon energy cuts are sizeable since hard-collinear logs do not cancel in that case. More pronounced for electrons.

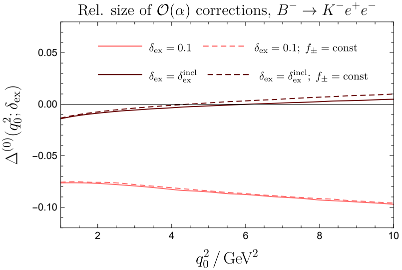


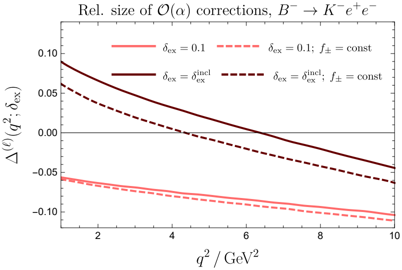


In the charged case, however, we see finite effects of the $\mathcal{O}(2\%)$ due to $\ln \hat{m}_K$ "collinear logs" which do not cancel.

Results: Distortion of the $\bar{B} \to \bar{K}\ell^+\ell^-$ spectrum

Distortion of the $\bar{B} \to \bar{K}\ell^+\ell^-$ spectrum due to γ-radiation





Effects are more prominent in the photon-inclusive case ($\delta_{\rm ex} = \delta_{\rm ex}^{\rm inc}$) since there is more phase space for the q^2- and q_0^2-variables to differ.

$\implies$ *Can be problematic for probing R_K in $q^2 \in [1.1, 6]$ GeV^2 range, due to charmonium resonances!*

Results: LFU and R_K

The net QED correction that should be applied to R_K according to our analysis amounts to

$$\Delta_{\mathrm{QED}} R_K \approx \left.\frac{\Delta\Gamma_{K\mu\mu}}{\Gamma_{K\mu\mu}}\right|_{q_0^2\in[1.1,6]\,\mathrm{GeV}^2}^{m_B^{\mathrm{rec}}=5.175\,\mathrm{GeV}} - \left.\frac{\Delta\Gamma_{Kee}}{\Gamma_{Kee}}\right|_{q_0^2\in[1.1,6]\,\mathrm{GeV}^2}^{m_B^{\mathrm{rec}}=4.88\,\mathrm{GeV}} \approx +1.7\%$$

Well below the current experimental error reported by LHCb.

However, effect of cuts can be significant. In Bordone et al. (*1605.07633*), in addition to the above energy cuts, a tight angle cut was also used, and a correction to R_K of

$$\Delta_{\mathrm{QED}} R_K \approx +3.0\% \quad ,$$

was reported.

$\Longrightarrow$ *Highlights the importance of building a MC to cross-check the experimental analysis (ongoing work)*

Future Work

- $\bar{B} \to \bar{K}\ell^+\ell^-$ differential distribution through Monte Carlo [ongoing].
- Fixing ambiguities in the UV counterterms, and structure-dependent corrections (including $\ln \hat{m}_K$ contributions) [ongoing].
- Analysis of moments of the angular distribution. Higher moments sensitive to QED corrections [ongoing].
- Calculation can be extended to other spin final states, such as K^*.
- Charged-current semileptonic decays ($\bar{B} \to D\ell\nu$). Unidentified neutrino in final state makes it hard to reconstruct B meson and to apply a cut-off on photon energy.

The END

BACKUP SLIDES

EFT

We use an *EFT*, for $\bar{B}(p_B) \to \bar{K}(p_K)\,\ell^+(\ell_2)\,\ell^-(\ell_1)$.

$$\mathcal{L}_{\rm int}^{\rm EFT} = g_{\rm eff}\, L^\mu V_\mu^{\rm EFT} + {\rm h.c.}\ ,$$

$$V_\mu^{\rm EFT} = \sum_{n\geq 0} \frac{f_\pm^{(n)}(0)}{n!}(-D^2)^n[(D_\mu B^\dagger)K \mp B^\dagger(D_\mu K)]\ ,$$

where D_μ is the covariant derivative and $f_\pm^{(n)}(0)$ denotes the $n^{\rm th}$ derivative of the $B \to K$ *form factor* $f_\pm(q^2)$.

$$\begin{aligned} H_0^\mu(q_0^2) &\equiv \langle \bar{K}|V_\mu|\bar{B}\rangle = f_+(q_0^2)(p_B+p_K)^\mu + f_-(q_0^2)(p_B-p_K)^\mu \\ &= \langle \bar{K}|V_\mu^{\rm EFT}|\bar{B}\rangle + \mathcal{O}(e), \end{aligned}$$

$$L_\mu \equiv \bar{\ell}_1\Gamma^\mu\ell_2\,, \quad V_\mu \equiv \bar{s}\gamma_\mu(1-\gamma_5)b\,,$$

$$g_{\rm eff} \equiv -\frac{G_F}{\sqrt{2}}\lambda_{\rm CKM}, \qquad \Gamma^\mu \equiv \gamma^\mu(C_V + C_A\gamma_5) \qquad C_{V(A)} = \alpha\frac{C_{9(10)}}{4\pi}$$

IR Divergences: Structure-dependent terms

The real amplitude can be decomposed,

$$\mathcal{A}^{(1)} = \hat{Q}_{\ell_1} a_{\ell_1}^{(1)} + \delta\mathcal{A}^{(1)} \,,$$

into a term $\hat{Q}_{\ell_1} a_{\ell_1}^{(1)}$ with all terms proportional to $\hat{Q}_{\ell_1}$, and the remainder $\delta\mathcal{A}^{(1)}$.

$$a_{\ell_1}^{(1)} = -e g_{\mathrm{eff}} \bar{u}(\ell_1) \left[\frac{2\epsilon^* \cdot \ell_1 + \not{\epsilon}^* \not{k}}{2k \cdot \ell_1} \Gamma \cdot H_0(q_0^2) \right] v(\ell_2) \,,$$

which contains all $1/(k \cdot \ell_1)$-terms.

The structure-dependence of this term is encoded in the form factor H_0.

IR Divergences: Structure-dependent terms

The amplitude square is given by

$$\sum_{\text{pol}} |\mathcal{A}^{(1)}|^2 = \sum_{\text{pol}} |\delta\mathcal{A}^{(1)}|^2 - \hat{Q}_{\ell_1}^2 \sum_{\text{pol}} |a_{\ell_1}^{(1)}|^2 + 2\hat{Q}_{\ell_1} \mathrm{Re}[\sum_{\text{pol}} \mathcal{A}^{(1)} a_{\ell_1}^{(1)*}] \,,$$

where it will be important that $\mathcal{A}^{(1)}$ is gauge invariant.

The *first term* is manifestly free from hard-collinear logs $\ln m_{\ell_1}$.

We use *gauge invariance* and set $\xi = 1$ under which the polarisation sum

$$\sum_{\text{pol}} \epsilon_\mu^* \epsilon_\nu = (-g_{\mu\nu} + (1-\xi) k_\mu k_\nu / k^2) \to -g_{\mu\nu}$$

collapses to the metric term only.

IR Divergences: Structure-dependent terms

The *second term* evaluates to

$$\int d\Phi_\gamma\, \hat{Q}_{\ell_1}^2 \sum_{\rm pol} |a_{\ell_1}^{(1)}|^2 = \int d\Phi_\gamma\, \hat{Q}_{\ell_1}^2 \frac{\mathcal{O}(m_{\ell_1}^2) + \mathcal{O}(k\cdot\ell_1)}{(k\cdot\ell_1)^2} = \mathcal{O}(1)\, \hat{Q}_{\ell_1}^2 \ln m_{\ell_1}$$

where we used $k - \ell_1 = \mathcal{O}(m_{\ell_1}^2)$, valid in the collinear region.

IR Divergences: Structure-dependent terms

We now turn to the *third term*.

Using anticommutation relations, $k - \ell_1 = \mathcal{O}(m_{\ell_1}^2)$ in the collinear limit, and the EoMs, we rewrite $a_{\ell_1}^{(1)}$ as

$$a_{\ell_1}^{(1)} = -e g_{\rm eff} \bar{u}(\ell_1) \left[\frac{4\epsilon^* \cdot \ell_1 + m_{\ell_1} \not{\epsilon}^*}{2k \cdot \ell_1} \Gamma \cdot H_0(q_0^2) \right] v(\ell_2) \; ,$$

Gauge invariance $k \cdot \mathcal{A}^{(1)} = 0$ implies $\ell_1 \cdot \mathcal{A}^{(1)} = \mathcal{O}(m_{\ell_1}^2)$ in the collinear region

IR Divergences: Structure-dependent terms

Therefore, the first part of $a_{\ell_1}^{(1)}$ contributes to

$$\hat{Q}_{\ell_1}\mathrm{Re}[\sum_{\mathrm{pol}}\mathcal{A}^{(1)}a_{\ell_1}^{(1)*}] \to c_1\hat{Q}_{\ell_1}^2\frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)^2} + c_2\hat{Q}_{\ell_1}\hat{Q}_X\frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)}$$

where $X \in \{\bar{B}, \bar{K}, \bar{\ell}_2\}$.

The second part of $a_{\ell_1}^{(1)}$ contributes to

$$\hat{Q}_{\ell_1}\mathrm{Re}[\sum_{\mathrm{pol}}\mathcal{A}^{(1)}a_{\ell_1}^{(1)*}] \to c_1'\hat{Q}_{\ell_1}^2\frac{\mathcal{O}(m_{\ell_1}^2)}{(k\cdot\ell_1)^2} + c_2'\hat{Q}_{\ell_1}\hat{Q}_X\frac{\mathcal{O}(m_{\ell_1})}{(k\cdot\ell_1)}$$

Thus, using gauge invariance, one concludes that $\delta\mathcal{A}^{(1)}$ (indicated by terms $\propto \hat{Q}_X$ in the above) does not lead to collinear logs.

Results: $\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ in c_a

We consider *relative* QED corrections. For a single differential in $\frac{d}{dq_a^2}$,

$$\Delta^{(a)}(q_a^2; \delta_{\rm ex}) = \left(\frac{d\Gamma^{\rm LO}}{dq_a^2} \right)^{-1} \frac{d\Gamma(\delta_{\rm ex})}{dq_a^2} \Big|_\alpha ,$$

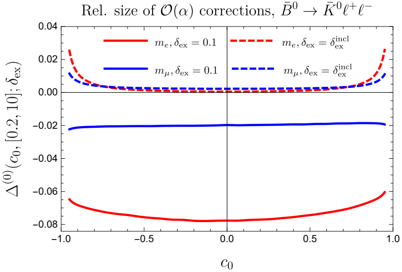
where the numerator and denominator are integrated separately over $\int_{-1}^{1} dc_a$ respectively. In addition, we define the single differential in $\frac{d}{dc_a}$

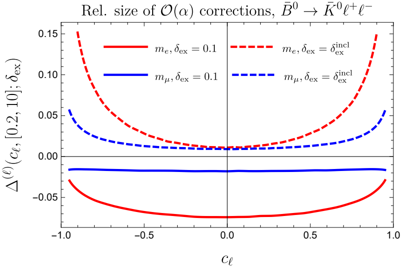
$$\Delta^{(a)}(c_a, [q_1^2, q_2^2]; \delta_{\rm ex}) = \left(\int_{q_1^2}^{q_2^2} \frac{d^2\Gamma^{\rm LO}}{dq_a^2 dc_a} dq_a^2 \right)^{-1} \int_{q_1^2}^{q_2^2} \frac{d^2\Gamma(\delta_{\rm ex})}{dq_a^2 dc_a} dq_a^2 \Big|_\alpha ,$$

where the non-angular variable is binned.

It is important to integrate the QED correction and the LO separately as this corresponds to the experimental situation.

Results: $\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ in c_a

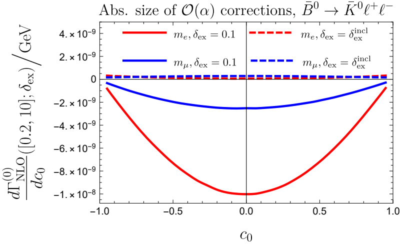


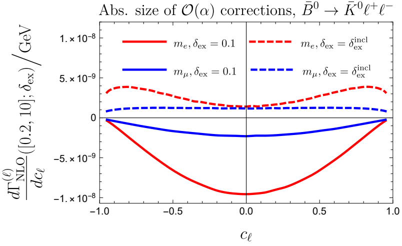


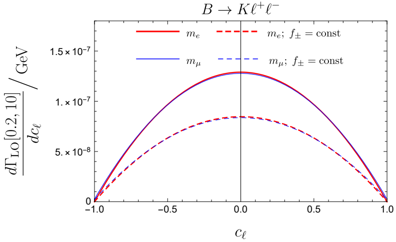
Enhanced effect towards the endpoints $\{-1, 1\}$ is partly due to the special behaviour of the LO differential rate which behaves like $\propto (1 - c_\ell^2) + \mathcal{O}(m_\ell^2)$ and explains why the effect is less pronounced for muons.

Even in c_ℓ. Almost even in c_0 (up to non-collinear effects).

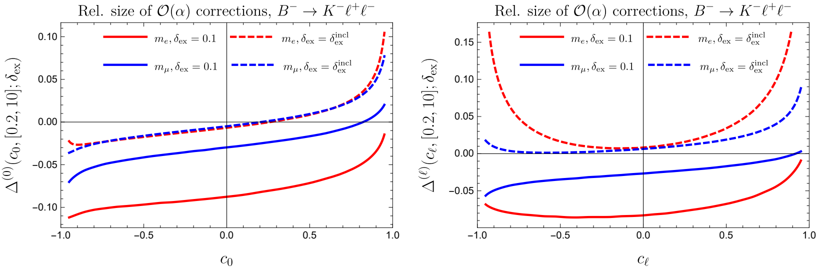
Results: $\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$ in c_a







Results: $B^- \to K^- \ell^+ \ell^-$ in c_a



- Same comments as before apply.
- More enhanced than the neutral meson case.
- 'Collinear' $\ln m_K$ odd in c_0/c_ℓ.

Results: Distortion of the $\bar{B} \to \bar{K}\ell^+\ell^-$ spectrum

To understand the distortion better, consider the following analysis in the collinear region:

$$|\mathcal{A}^{(0)}(q_0^2, c_0)|^2 \propto f_+(q_0^2)^2 = f_+(q^2/z)^2.$$

Since $z < 1$ in general, it is clear that momentum transfers of a higher range are probed.

Results: Distortion of the $\bar{B} \to \bar{K}\ell^+\ell^-$ spectrum

For example, when $c_\ell = -1$, maximising the effect, one gets

$$z_{\delta_{\rm ex}}(q^2)\Big|_{c_\ell=-1} = \frac{q^2}{q^2 + \delta_{\rm ex} m_B^2} \ , \quad (q_0^2)_{\rm max} = q^2 + \delta_{\rm ex} m_B^2 \ ,$$

For $\delta_{\rm ex} = 0.15$, $q^2 = 6\,\text{GeV}^2$ one has $(q_0^2)_{\rm max} = 10.18\,\text{GeV}^2$

$\Longrightarrow$ *Problematic for probing R_K in $q^2 \in [1.1, 6]$ GeV^2 range, due to charmonium resonances!*

Furthermore, in photon-inclusive case, the lower boundary for z becomes $z_{\rm inc}(c_\ell)|_{m_K \to 0} = \hat{q}^2$ such that $(q_0^2)_{\rm max} = m_B^2$.

$\Longrightarrow$ *Entire spectrum is probed for any fixed value of q^2*