HLbL to g-2 at large loop momenta

Antonio Rodríguez Sánchez

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In collaboration with:
Johan Bijnens (Lund)
Nils Hermansson-Truedsson (Bern)
Laetitia Laub (Bern)

Phys.Lett.B 798 134994
JHEP 10 (2020) 203
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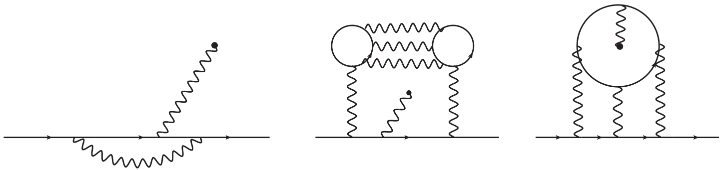
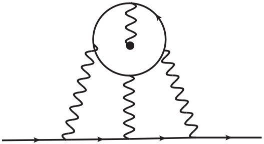
Muon g-2

- Muon response to weak magnetic field?
- Linear term in (soft) photon momentum of $\langle \mu^-_{p',\sigma'} | J^\mu_{\mathrm{EM}} e^{iS_{\mathrm{int}}} | \mu^-_{p,\sigma} \rangle$
- $S_{\mathrm{int}} \approx 0 \to g_\mu \approx 2$ $\qquad g_{\mu,\mathrm{exp}} \approx 2.002$
- $a_\mu^{\mathrm{exp}} \equiv (g_\mu^{\mathrm{exp}} - 2)/2 = 0.00116592061(41)$
  Phys.Rev.Lett. 126 (2021) 14, 141801



- $a_\mu^{\mathrm{SM}} = 0.00116591810(43)$ Phys.Rept. 887 (2020) 1-166

Data-driven HLbL: a multiscale problem

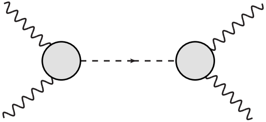


$$\Pi^q \sim \langle 0|\, T\left(\prod_j^4 \int dx_j e^{-iq_j x_j} J^q(x_j)\right) e^{iS_{\rm int}} |0\rangle$$

$$J_q^\mu = Q_q\, \bar{q}\gamma^\mu q$$

$$a_\mu^{\rm HLbL} \sim \int_0^\infty dQ_{1,2,3} \sum_i T_i' \,\overline{\Pi}_i$$

- Π^q? Weights T_i' enhance low-energy contributions
- A nonperturbative problem



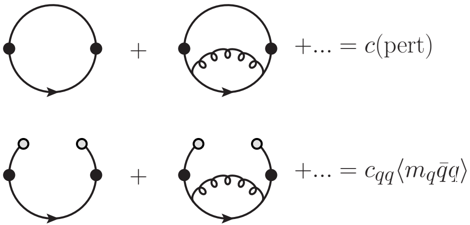
- From dispersion relations to resonance models and short-distance constraints
- $Q_1 \sim Q_2 \sim Q_3 \gg \Lambda_{\rm QCD}$

Asymptotic expansion of $(g-2)_\mu$ HLbL?

Operator Product Expansion (OPE)

Asymptotic behaviour of two-point correlation functions

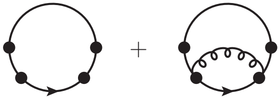
$$\Pi(q) = \int dx\, e^{-iqx} \langle 0| T(J_1(x) J_2(0)|0\rangle; \;\; J_i \sim \bar{q}\Gamma_i q$$



$$\Pi(Q) = \sum_{i,D} \frac{c_{i,D}(Q^2,\mu)\langle \mathcal{O}_{i,D}(\mu)\rangle}{Q^D}$$ Nucl.Phys.B 147 385-447

HLbL for $g-2$. Same procedure?

$$\Pi^{\mu_1\mu_2\mu_3\mu_4} = -i \int \frac{d^4 q_3}{(2\pi)^4} \left(\prod_i^4 \int d^4 x_i \, e^{-iq_i x_i} \right) \langle 0 | \, T \left(\prod_j^4 J^{\mu_j}(x_j) \right) | 0 \rangle$$



$+ \ldots = c(\mathrm{pert})$

$+ \ldots = c_{qq} \langle m_q \bar{q} q \rangle$

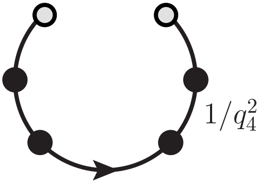
$$\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2, \mu) \langle \mathcal{O}_{i,D}(\mu) \rangle}{Q_1^{d_1} \ldots Q_4^{d_4}} \qquad \sum_i d_i = D$$

HLbL for g-2

- $\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle \mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1}\ldots Q_4^{d_4}}$ $\qquad \sum_i d_i = D$

- External photon: static limit $\rightarrow \lim_{q_4 \rightarrow 0} \frac{\partial \Pi^{\mu_1\mu_2\mu_3\nu_4}}{\partial q_{4,\,\mu_4}}$

- $\lim_{q_4 \rightarrow 0} \Pi^{\mathrm{OPE}}$?



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- OPE only supposed to work at large Euclidean Momenta

Rethinking the problem: soft static photon

$$\langle 0|e^{iS}|\gamma_1\gamma_2\gamma_3\gamma_4\rangle \to \Pi^{\mu_1\mu_2\mu_3\mu_4}$$

One step backwards

$$\Pi^{\mu_1\mu_2\mu_3} \sim \int \frac{d^4q_3}{(2\pi)^4}\left(\prod_i^3 \int d^4x_i\, e^{-iq_ix_i}\right)\langle 0|\, T\left(\prod_j^3 J^{\mu_j}(x_j)\right) e^{iS_{\rm int}}|\gamma_E(q_4)\rangle$$

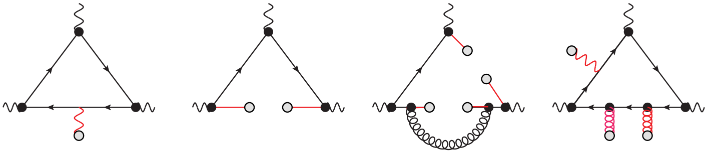
- $Q_{1,2,3} \gg \Lambda_{\rm QCD} \to$ OPE
- We are looking for a static (soft) photon contribution: $F^{\mu\nu}$
- From the OPE keep those operator contributions with the same quantum numbers as the static photon, $F^{\mu\nu}$

  Nucl.Phys.B 232 109-142, Phys.Lett.B 129 328-334, Phys.Rev.D 67 073006

OPE with background photon

$$S_{1,\,\mu\nu} \equiv e\, e_q F_{\mu\nu}, \quad S_{2,\,\mu\nu} \equiv \bar{q}\sigma_{\mu\nu} q, \ldots$$

$$\Pi^{\mu_1\mu_2\mu_3}(q_1, q_2) = \frac{1}{e} \vec{C}^{\,T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1, q_2) \langle \vec{S}_{\mu_4\nu_4} \rangle; \quad \langle S_{i,\mu\nu} \rangle = e e_q X_S^i \langle F_{\mu\nu} \rangle$$



- Leading order: massless quark loop
- Magnetic susceptibility $\sim \frac{m_q X}{Q^2}$
- First massless power corrections $\sim \frac{\Lambda_{\mathrm{QCD}}^4}{Q^4}$ JHEP 10 (2020) 203

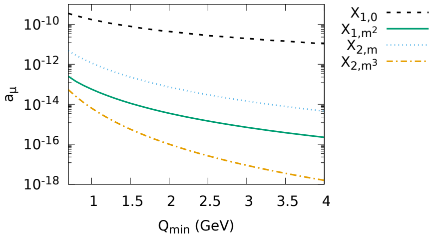
$g-2$ integral

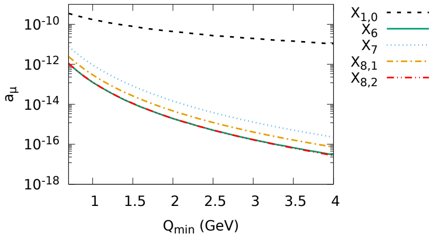
$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\,\vec{X}\,\langle e_q F_{\mu_4\nu_4}\rangle$$

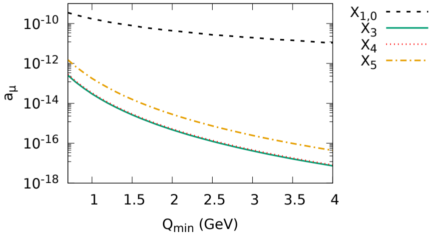
$$a_\mu^{\mathrm{HLbL}} \sim \int_0^\infty dQ_{1,2}\int_{-1}^{1} d\tau \sum_i T_i' \,\overline{\Pi}_i$$ JHEP 09 (2015) 074, JHEP 04 (2017) 161

1. Build general projectors P: $P_{\mu_1\mu_2\mu_3\mu_4\nu_4} C^{\mu_1\mu_2\mu_3\mu_4\nu_4} X \sim \overline{\Pi}$
2. Reduce scalar integrals KIRA, REDUZE
3. Perform the piece of the $g-2$ integral from some Q_{min} where the expansion is valid

Numerical results: quark loop vs power corrections

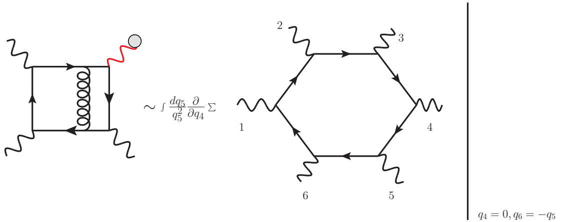




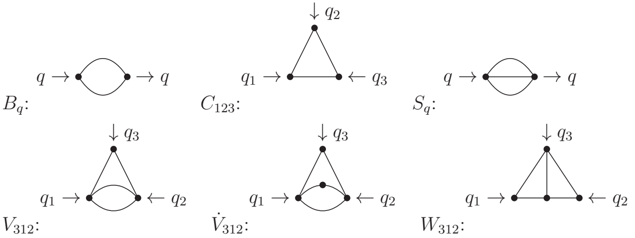


- Power corrections numerically very small above 1 GeV
- Gluonic corrections?...

The two loops: a symmetric sum of hexagons

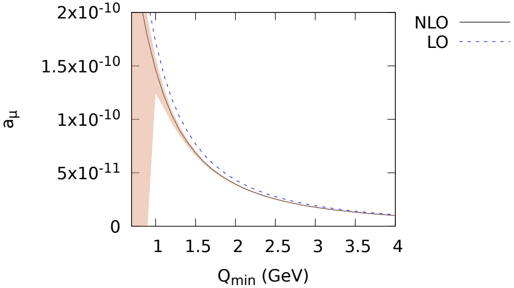


1. Build general projectors P: $P_{\mu_1\mu_2\mu_3\mu_4\nu_4} C^{\mu_1\mu_2\mu_3\mu_4\nu_4} = \overline{\Pi}$
2. Reduce $\sim \mathcal{O}(10^{3,4})$ scalar integrals (d dimensions) KIRA



Two loop: results

3. Master integrals known in terms of classical polylogs: analytic result for the HLbL tensor. Typically $\sim -\frac{\alpha_s}{\pi}$
4. Integrate from Q_{min}



Above $\sim 1-2\,\mathrm{GeV}$, gluonic corrections small and negative

Conclusions

- A systematic OPE with a background photon field can give a description of HLbL $g-2$ for large loop momenta
- The massless quark loop is the leading term
- Power corrections have been computed and found to be small
- Perturbative corrections are found small and negative
- Precise systematic expansion valid above $1-2\,\mathrm{GeV}$

Thank you