GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

LECTURE 2

COSMIC RAY PROPAGATION

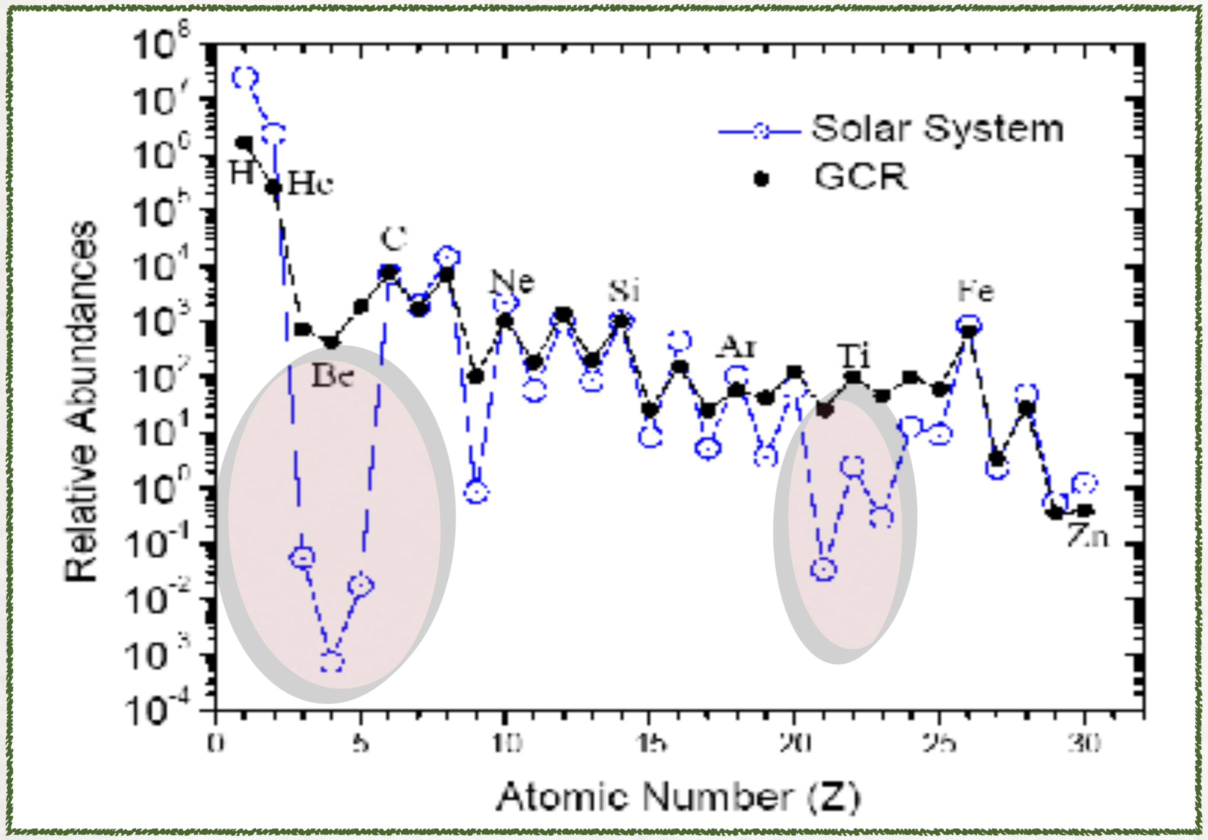
P. BLASI

GRAN SASSO SCIENCE INSTITUTE, CENTER FOR ADVANCED STUDIES

ISAPP school on "Astrophysical sources of cosmic rays", Paris Saclay, 28 March - 8 April 2022

BASIC INDICATORS OF DIFFUSIVE TRANSPORT

STABLE ELEMENTS

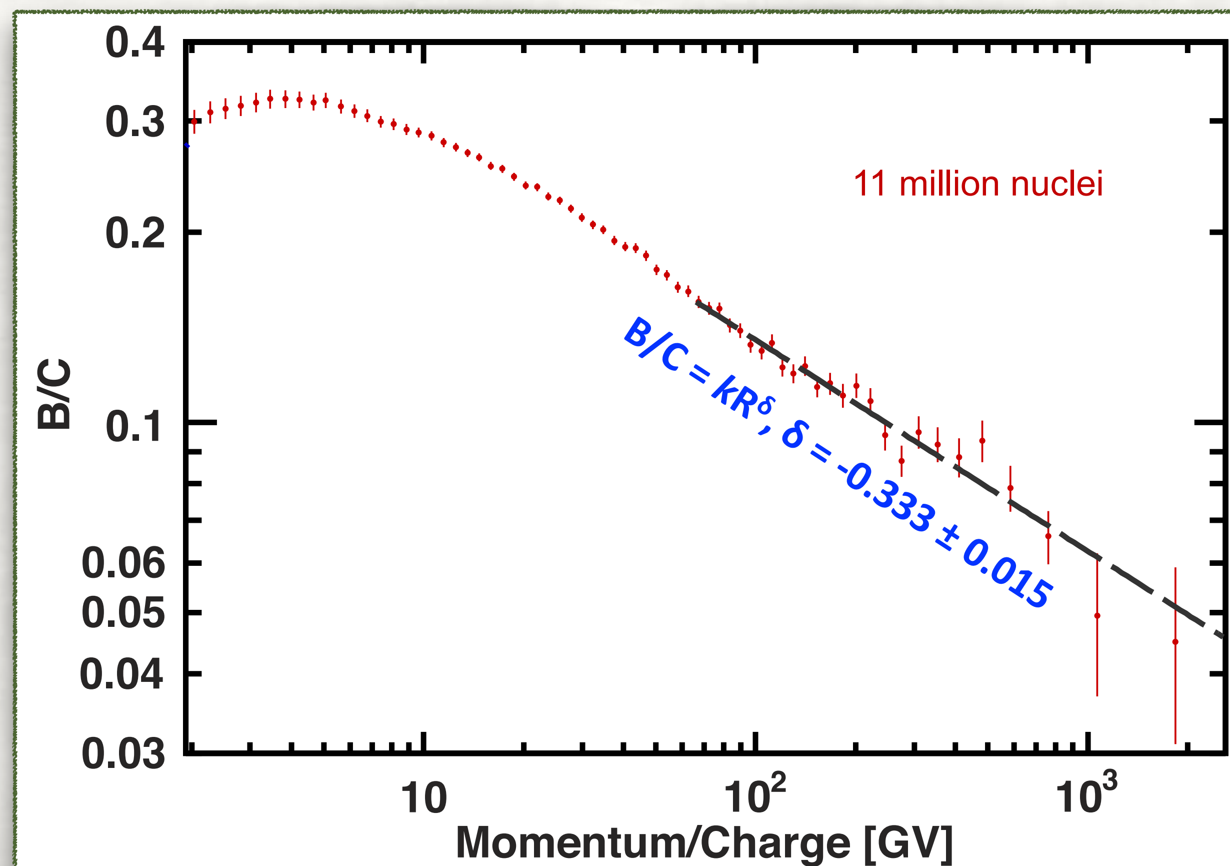


Measurements of the Boron and sub-iron elements in CRs show that CR live for tens of million years in the Galaxy

DIFFUSIVE TRANSPORT

$$\sigma_{sp}(A) \approx 45 A^{0.7} \text{ mb}$$

$$\tau_{sp} \approx [n_d (h/H) c \sigma_{sp}]^{-1} \approx 80 H_4 A_{12}^{-0.7} \text{ Myr}$$

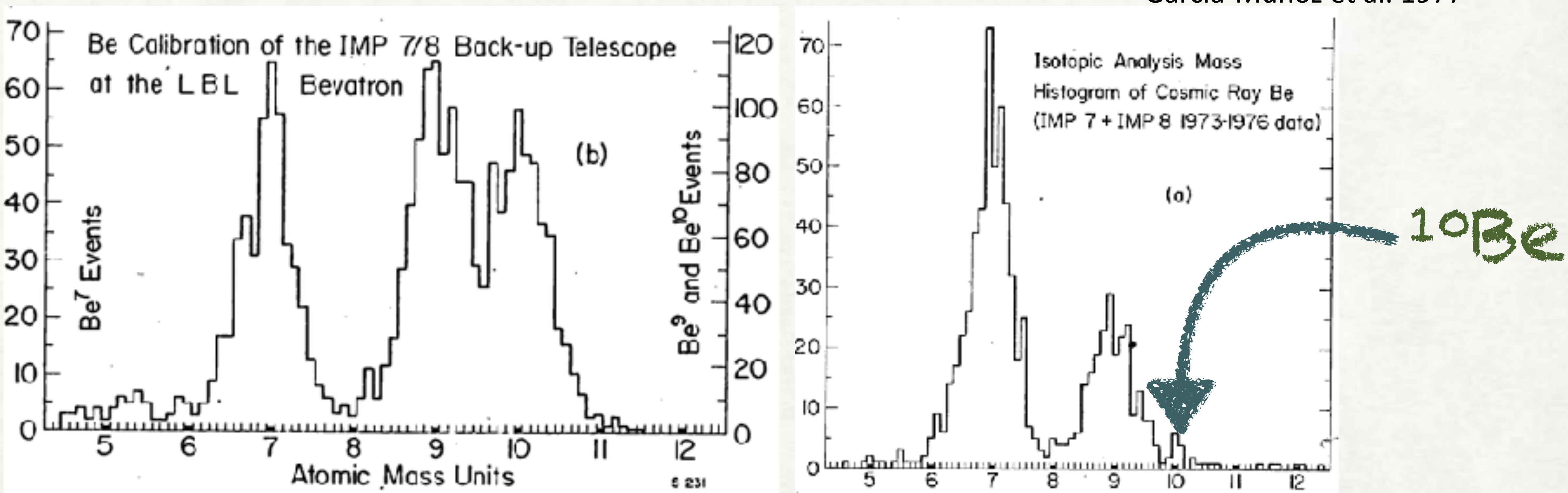


M. Aguilar *et al.*, Phys. Rev. Lett. **117**, 231101 (2016)

BASIC INDICATORS OF DIFFUSIVE TRANSPORT

UNSTABLE ELEMENTS

Garcia-Munoz et al. 1977



Be comes in three isotopes and ^{10}Be is unstable with a decay time of 1.4 Myr.

While in the Lab the three isotopes are roughly equally produced, in the CR we see the peak of ^{10}Be being much smaller —> information of decay vs production vs confinement

HOW DO COSMIC RAYS MOVE?

$$\tau_{DISC} = \frac{300\ \mathrm{pc}}{(1/3)c} \approx 3000\ \mathrm{years}$$

PROPAGATION TIME IN THE DISC OF THE GALAXY

$$\tau_{GAL} = \frac{15\ \mathrm{kpc}}{(1/3)c} \approx 150{,}000\ \mathrm{years}$$

PROPAGATION TIME ALONG THE ARMS OF THE GALAXY

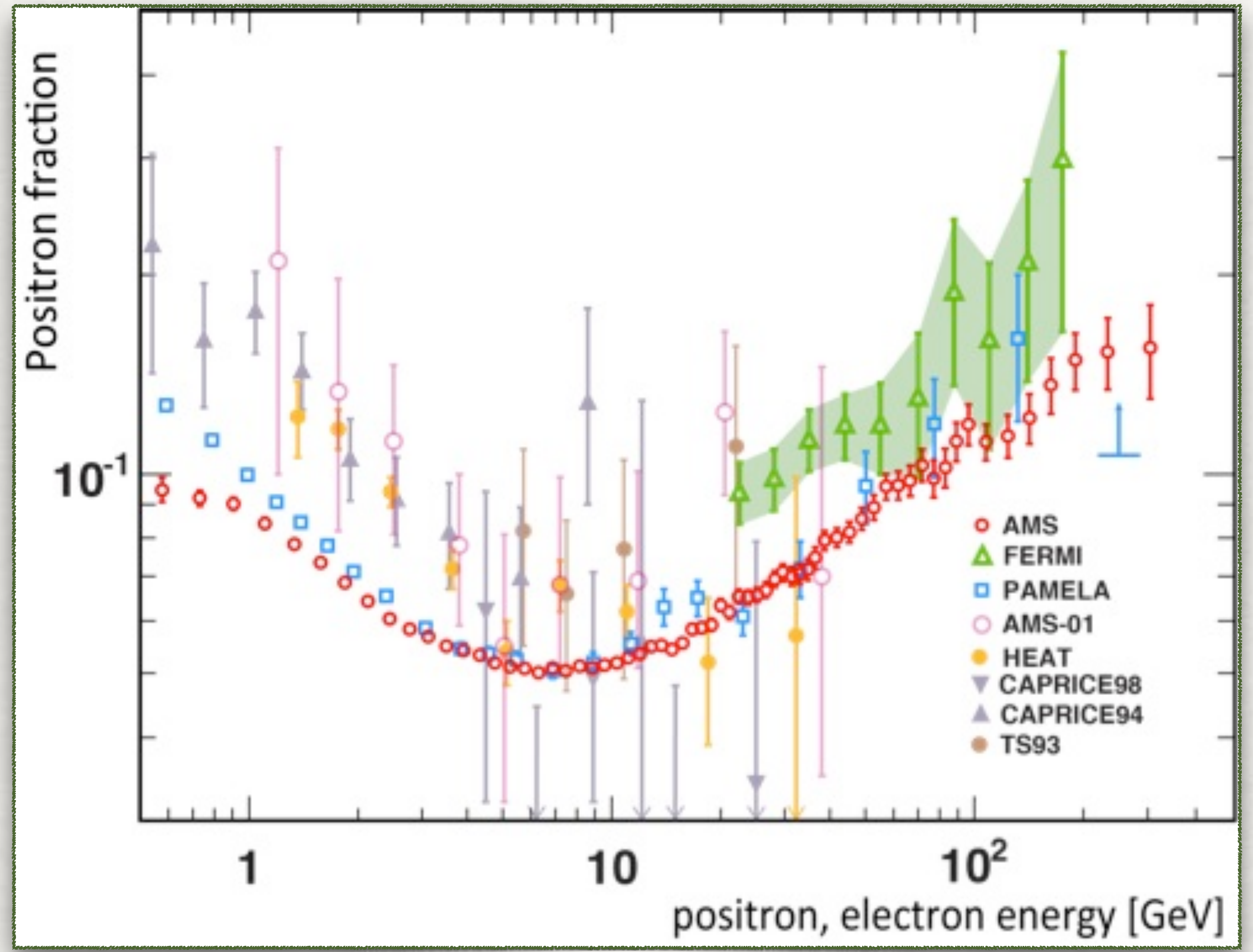
$$\tau_{HALO} = \frac{3\ \mathrm{kpc}}{(1/3)c} \approx 30{,}000\ \mathrm{years}$$

PROPAGATION TIME IN THE HALO

ALL THESE TIME SCALES ARE EXCEEDINGLY SHORT TO BE MADE COMPATIBLE WITH THE ABUNDANCE OF LIGHT ELEMENTS

PROPAGATION CANNOT BE BALLISTIC

SECONDARY/PRIMARY: POSITRON FRACTION



AMS-02 Coll. 2013

Reacceleration of secondary Pairs in old SNRs

PB 2009, PB & Serpico 2009; Mertsch & Sarkar 2009

Pulsar Wind Nebulae

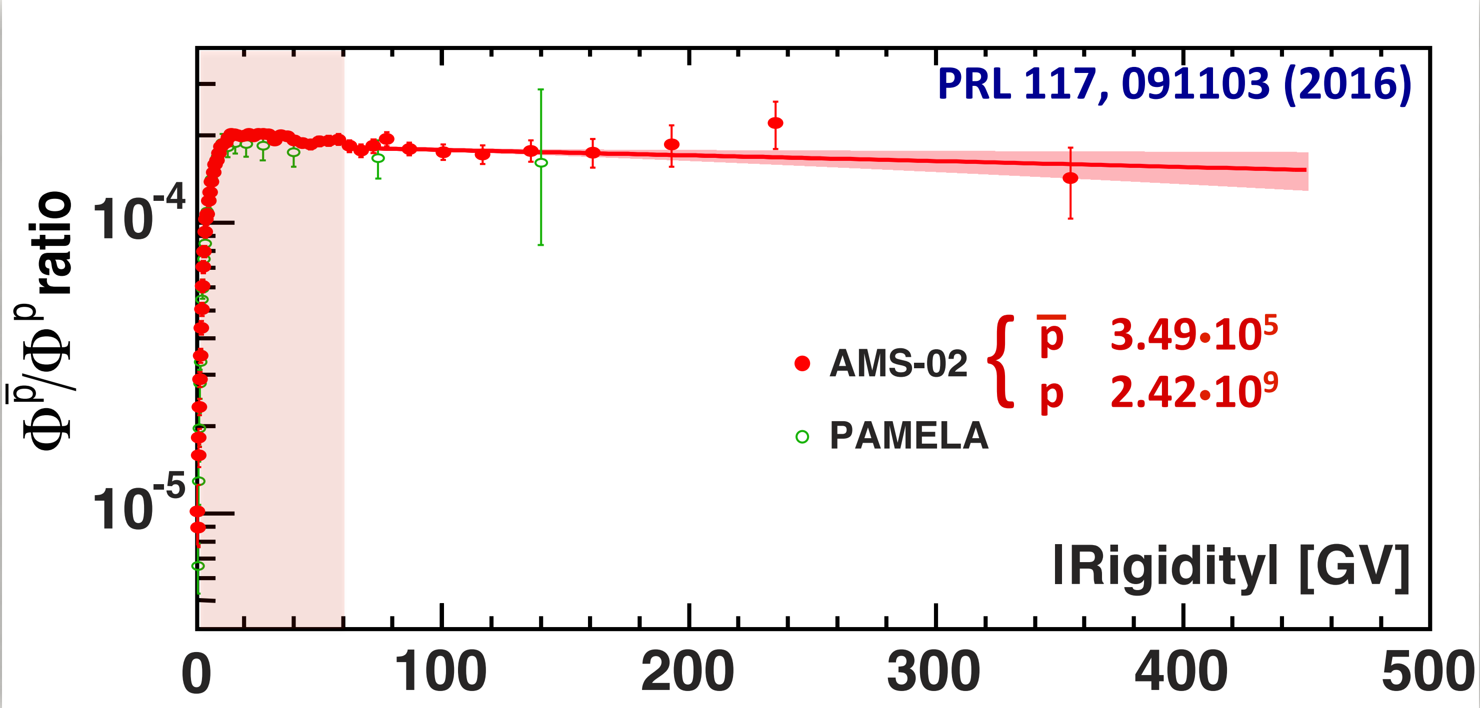
Hooper, PB & Serpico (2009); PB & Amato 2010

Dark Matter Annihilation

Difficult: high annihilation, Cross section, leptophilia, Boosting factor [Serpico (2012)]

SECONDARY/PRIMARY: ANTIPROTONS

- ANTIPROTONS ARE ALSO PRODUCED AS A RESULT OF CR INTERACTIONS
- THE IR SPECTRUM IS EXPECTED TO BE STEEPER THAN THAN OF PARENT PROTONS FOR THE SAME REASONS
- AMS-02 DATA SHOW A POSSIBLE ANOMALY, BUT NOT CLEAR AS YET (cross sections, astrophysics, ...)



Notice that $e^+/e^- \sim 0.1$

B/C~0.1

pbar/p~10^{-4}

DESCRIPTION OF TRANSPORT OF NUCLEI

For nuclei of mass A, it is customary to introduce the flux as a function of the kinetic energy per nucleon E_k: $I_\alpha(E_k)dE_k = p^2 F_\alpha(p)\, v(p)\, dp$ which implies: $I_\alpha(E_k) = Ap^2 F_\alpha(p)$

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(E_k)}{\partial z}\right] + 2h_d n_d v(E_k)\sigma_\alpha \delta(z) I_\alpha(E_k) =$$

$$= 2Ap^2 h_d q_{0,\alpha}(p)\delta(z) + \sum_{\alpha' > \alpha} 2h_d n_d\, v(E_k)\sigma_{\alpha' \to \alpha}\delta(z) I_{\alpha'}(E_k)$$

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DIFFUSION

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SPALLATION OF NUCLEI **α**

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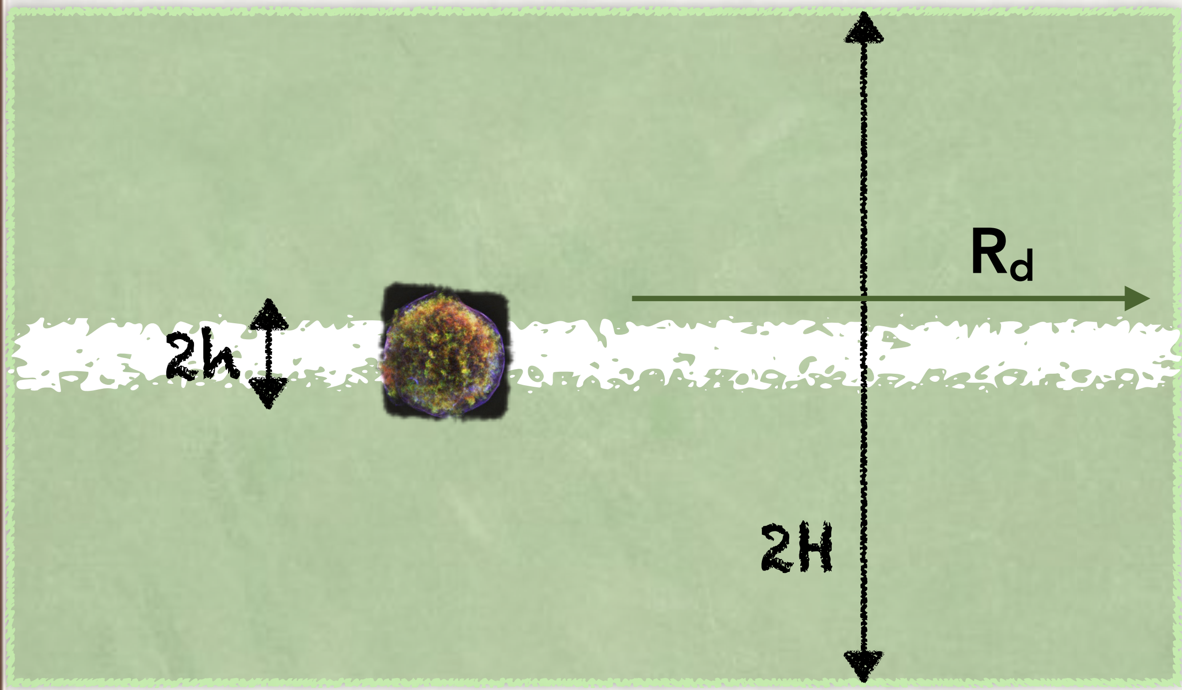
FOR SIMPLICITY THIS EQUATION DOES NOT CONTAIN SOME LOSS TERMS (IONIZATION), ADVECTION AND SECOND ORDER FERMI ACCELERATION IN ISM

ALL THESE EFFECTS MAY BECOME IMPORTANT AT E<10 GeV/nucleon

A TOY MODEL FOR PROTONS IN OUR GALAXY



HALO ~ several kpc

DISC ~ 300 pc

Assumptions of the model:

1. *CR are injected in an infinitely thin disc*
2. *CR diffuse in the whole volume*
3. *CR freely escape from a boundary*

1 $$Q(p,z) = \frac{Q_0(p)}{\pi R_d^2}\delta(z)$$

2 $$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] = Q(p,z)$$

3 $$f(z=H,p)=0$$

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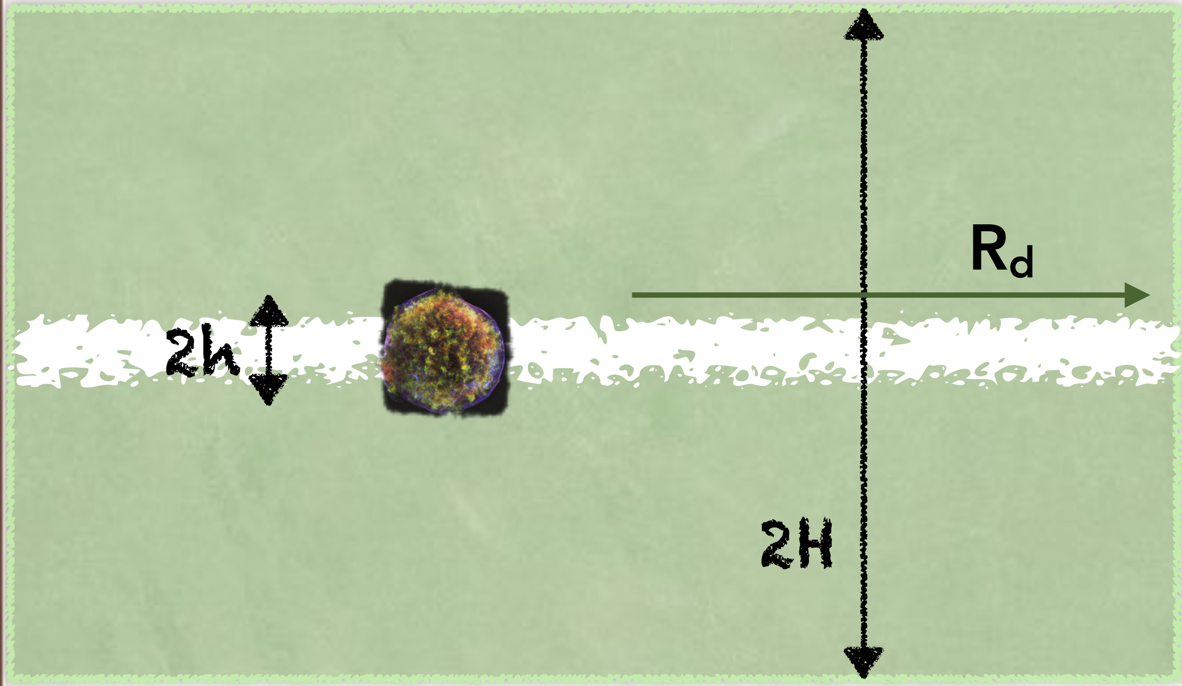

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For z≠0:

$$D\frac{\partial f}{\partial z} = Constant \rightarrow f(z) = f_0\left(1-\frac{z}{H}\right)$$

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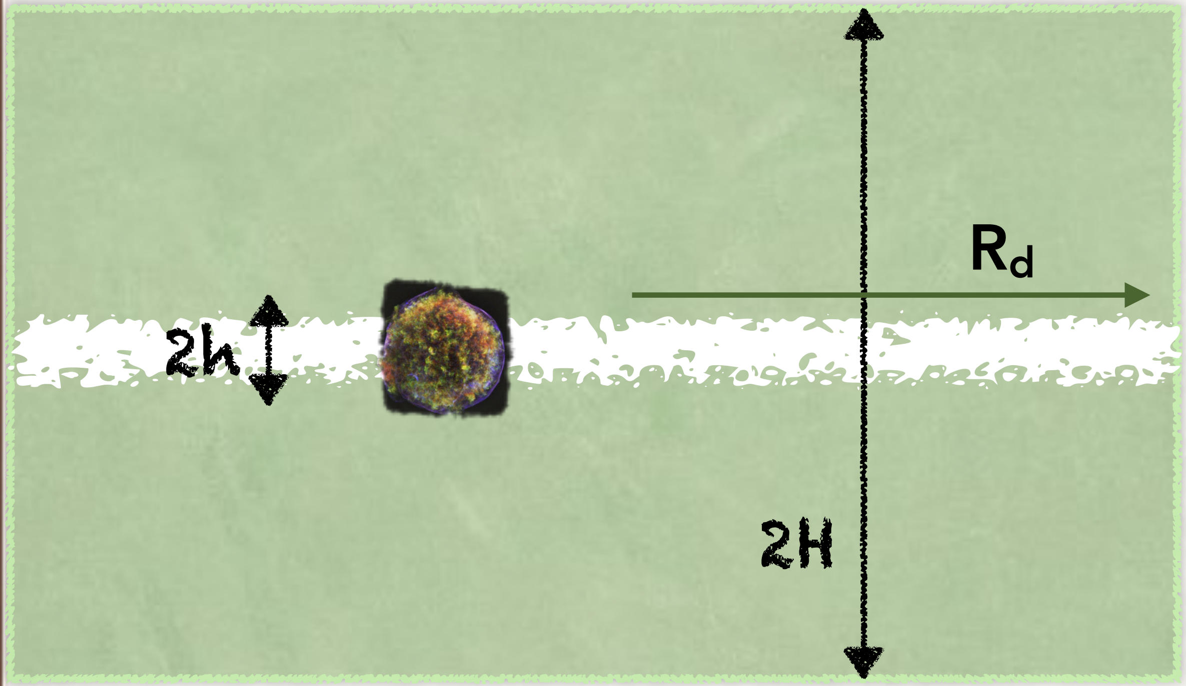

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$$D\frac{\partial f}{\partial z}|_{z=0^+} = -D\frac{f_0}{H}$$

A TOY MODEL FOR OUR GALAXY

Let us now integrate the diffusion equation around z=0

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] = \frac{Q_0(p)}{\pi R_d^2}\delta(z) \quad \Rightarrow \quad -2D\frac{\partial f}{\partial z}|_{z=0^+} = \frac{Q_0(p)}{\pi R_d^2}$$

and recalling that

$$D\frac{\partial f}{\partial z}|_{z=0^+} = -D\frac{f_0}{H} \quad \Rightarrow \quad f_0(p) = \frac{Q_0(p)}{2\pi R_d^2}\frac{H}{D} = \frac{Q_0(p)}{2\pi R_d^2 H}\quad\frac{H^2}{D}$$

Diffusion Time

Rate of injection per unit volume

Since $Q_0(p) \sim p^{-\gamma}$ and $D(p) \sim p^{\delta}$
$f_0(p) \sim p^{-\gamma-\delta}$

A TOY MODEL FOR OUR GALAXY: ESCAPE FLUX

WHICH CR FLUX WOULD BE MEASURED BY AN OBSERVER OUTSIDE OUR GALAXY?

WE ALREADY ESTABLISHED THAT

$$D\frac{\partial f}{\partial z} = constant$$

BUT THIS IS EXACTLY THE FLUX ACROSS A SURFACE IN DIFFUSIVE REGIME:

$$\Phi_{esc}(p) = -D\frac{\partial f}{\partial z}|_{z=H} = -D\frac{\partial f}{\partial z}|_{z=0^+} = \frac{Q_0(p)}{2\pi R_d^2}$$

THE SPECTRUM OF COSMIC RAYS OBSERVED BY AN OBSERVER OUTSIDE OUR GALAXY IS THE SAME AS INJECTED BY SOURCES, NOT THE SAME AS WE MEASURE AT THE EARTH!

MEANING OF FREE ESCAPE BOUNDARY?

The physics of CR transport is as much regulated by diffusion as it is by boundary conditions (this is true for toy models as well as it is for GALPROP)

What does "free escape" mean? $f(z=H,p)=0$

Conservation of flux at the boundary implies:

$$D\frac{\partial f}{\partial z}|_{z=H}=\frac{c}{3}f_{out}$$

$$D\frac{f_0}{H}=\frac{c}{3}f_{out}\rightarrow f_{out}=\frac{3D}{cH}f_0\approx\frac{\lambda(p)}{H}f_0\ll f_0$$

Beware that despite the great importance of this assumption we do not have any handle on what determines the halo size or weather the halo size 'felt' by particles depends on energy

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PRIMARY NUCLEI

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(E_k,z)}{\partial z}\right] + 2h_d n_d v(E_k)\sigma_\alpha \delta(z) I_\alpha(E_k) = 2Ap^2 h_d q_{0,\alpha}(p)\delta(z)$$

Formally similar to the equation for protons but with **spallation** taken into account

Technically the equation is solved in the same way:

1) *consider z>0 (or z<0) and then*
2) *integrate around z=0 between 0^- and 0^+*

1 $$D_\alpha \frac{\partial I_\alpha}{\partial z} = constant \to I_\alpha = I_{0,\alpha}\left(1 - \frac{z}{H}\right)$$ with free escape boundary condition

2 $$-D_\alpha \frac{\partial I_\alpha}{\partial z}|_{z=0} = -h_d n_d v \sigma_\alpha I_{0,\alpha} + Ap^2 h_d Q_{0,\alpha}(p)$$

PRIMARY NUCLEI

Injection X escape time

$$I_{0,\alpha}(E_k) = \frac{\frac{N_{inj,\alpha} R_{SN}}{2\pi R_{disc}^2 H} \frac{H^2}{D_\alpha}}{1 + \frac{X(E_k)}{X_\alpha}}$$

Flux of nuclei of type a

$$X(E_k) = n_d \left(\frac{h}{H}\right) m_p v \frac{H^2}{D_\alpha}$$

Grammage traversed by nuclei of type a

$$X_\alpha = \frac{m_p}{\sigma_\alpha}$$

Critical grammage for nuclei of type a

For $X \ll X_a$ (HIGH E) the equilibrium spectrum is the standard $E_k^{-\gamma-\delta}$

For $X \gg X_a$ (LOW E) the equilibrium spectrum same as injection spectrum $E_k^{-\gamma}$

SECONDARY NUCLEI - CASE OF B/C

SIMPLE CASE WITH CARBON AND OXYGEN AS PRIMARIES

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_B}{\partial z}\right] + 2h_d n_d v \sigma_B \delta(z) I_B = 2h_d n_d \, \sigma_{CB} v I_C \delta(z) + 2h_d n_d \, \sigma_{OxB} v I_{Ox} \delta(z)$$

Destruction of B

Production of B from carbon spallation

Production of B from oxygen spallation

Following the same strategy as in the previous cases one obtains easily:

$$I_{B,0}(E_k) = \frac{I_{C,0}(E_k)\frac{X(E_k)}{X_{cr,CB}}}{1+\frac{X(E_k)}{X_{cr,B}}} + \frac{I_{Ox,0}(E_k)\frac{X(E_k)}{X_{cr,OxB}}}{1+\frac{X(E_k)}{X_{cr,B}}}$$

which reflects in the following B/C ratio:

$$\frac{I_{B.0}(E_k)}{I_{C,0}(E_k)} = \frac{\frac{X(E_k)}{X_{cr,CB}}}{1+\frac{X(E_k)}{X_{cr,B}}} + \frac{I_{Ox,0}(E_k)}{I_{C,0}(E_k)} \frac{\frac{X(E_k)}{X_{cr,OxB}}}{1+\frac{X(E_k)}{X_{cr,B}}}$$

AT E>100 GeV THE B/C RATIO SCALES AS $X(E_K)$ NAMELY AS $1/D_\alpha$ NAMELY WE CAN MEASURE THE SLOPE OF D(E) FROM THE ENERGY DEPENDENCE OF B/C (in principle)

$$\frac{I_{B.0}(E_k)}{I_{C,0}(E_k)} = \frac{\frac{X(E_k)}{X_{cr,CB}}}{1+\frac{X(E_k)}{X_{cr,B}}} + \frac{I_{Ox,0}(E_k)}{I_{C,0}(E_k)} \frac{\frac{X(E_k)}{X_{cr,OxB}}}{1+\frac{X(E_k)}{X_{cr,B}}}$$

Since $X_{cr,\{B,C,O\}}$ are independent of energy and X(E) is a decreasing function of energy, the regime **$X(E) \gg X_{cr,\{B,C,O\}}$ (spallation dominated regime)** is at low energies, and the regime **$X(E) \ll X_{cr,\{B,C,O\}}$ (weak spallation)** is at high energies.

$$\frac{I_{B,0}}{I_{C,0}} \propto \text{constant}$$

Spallation dominated regime

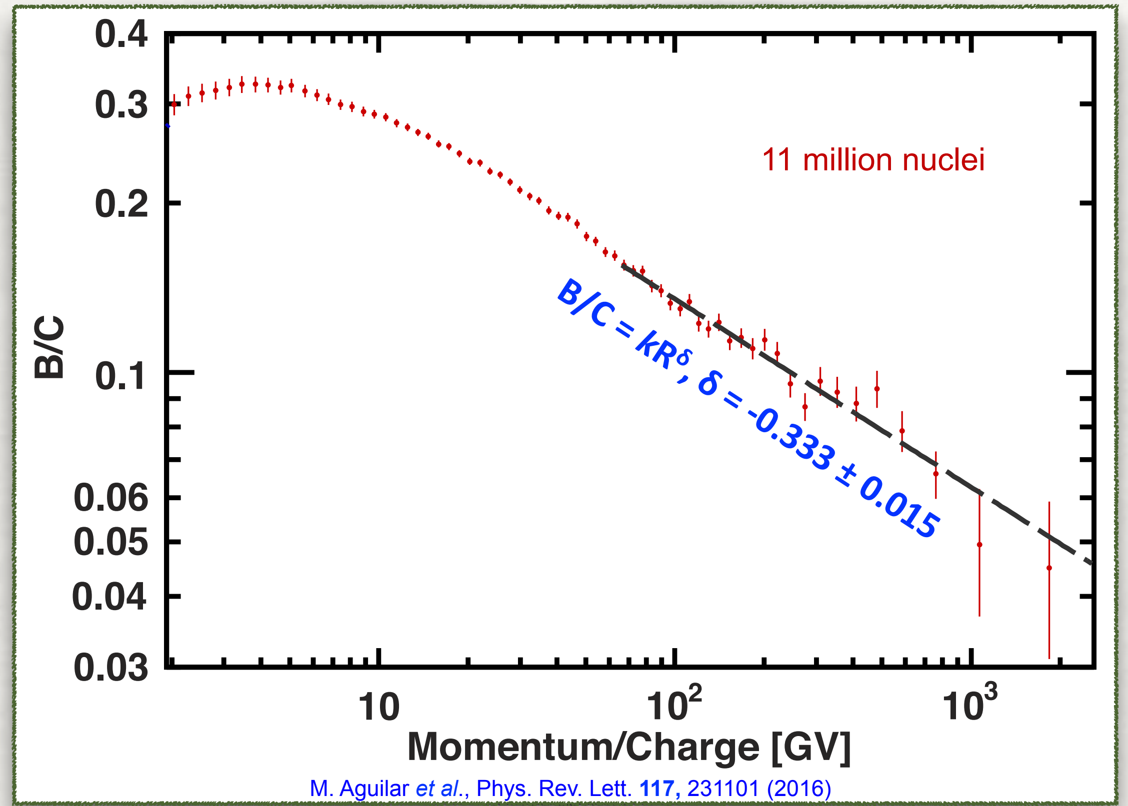
$$\frac{I_{B,0}}{I_{C,0}} \approx \frac{X(E_k)}{X_{cr,CB}} + \frac{I_{Ox,0}(E_k)}{I_{C,0}(E_k)} \frac{X(E_k)}{X_{cr,OxB}}$$

Weak spallation regime

AT HIGH ENERGIES THE RATIO IS SENSITIVE TO THE GRAMMAGE (~1/D)

SECONDARY/PRIMARY: B/C - SIMPLE RULE OF THUMB

Evidence for CR diffusive transport



primary equilibrium

$$n_{pr}(E/n) \propto Q(E/n)\tau_{diff}(E/n)$$

secondary injection

$$q_{sec}(E/n) \approx n_{pr}(E/n)\sigma v n_{gas}$$

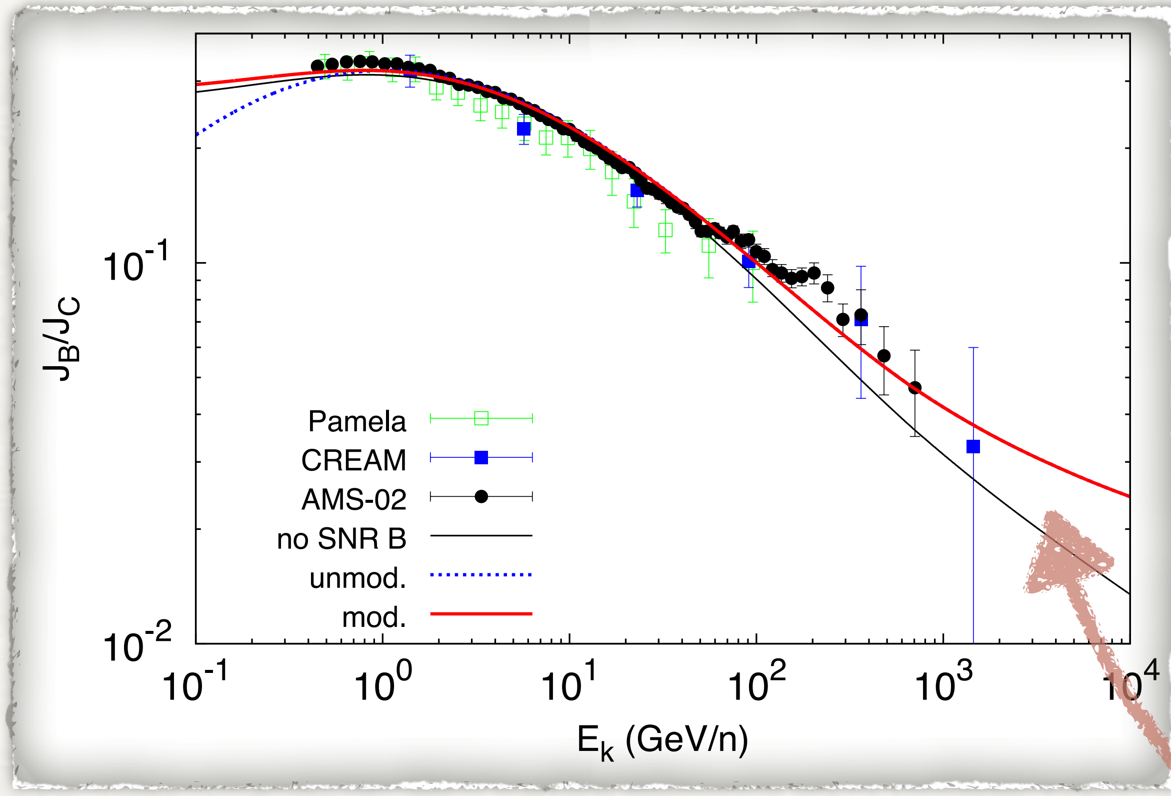
secondary equilibrium

$$n_{sec}(E/n) \approx q_{sec}(E/n)\tau_{diff}(E/n)$$

$$\frac{n_{sec}}{n_{pr}} \approx \frac{\sigma}{m_p}\left[v n_{gas} m_p \tau_{diff}\right]$$

GRAMMAGE: $X(E/n) \propto \tau_{diff}(E/n) \sim 1/D(E/n)$

BUT...CAVEATS AND COMMENTS



1) THIS CONCLUSION HOLDS ONLY AT ENERGIES WELL ABOVE 50-100 GeV/n BECAUSE AT LOW E_k ADDITIONAL PHYSICS (ADVECTION, 2nd ORDER FERMI IN ISM, SPALLATION, SOLAR MODULATION)

2) BUT... AT HIGH ENERGIES THERE IS: GRAMMAGE IN THE SOURCES, GRAMMAGE AROUND THE SOURCES, ACCELERATION OF SECONDARIES, ... **so be careful!!!**

A simple instance... the grammage accumulated by CR while trapped downstream of a supernova shock can be estimated as:

$$X_{\rm SNR} \approx 1.4 r_s m_p n_{\rm ISM} c T_{\rm SNR} \approx 0.17\,{\rm g\,cm^{-2}} \frac{n_{\rm ISM}}{\rm cm^{-3}} \frac{T_{\rm SNR}}{2\times 10^4 {\rm yr}}$$

REACCELERATION VS ACCELERATION

SHOCKS ARE BLIND TO THE NATURE OF THE CHARGED PARTICLES

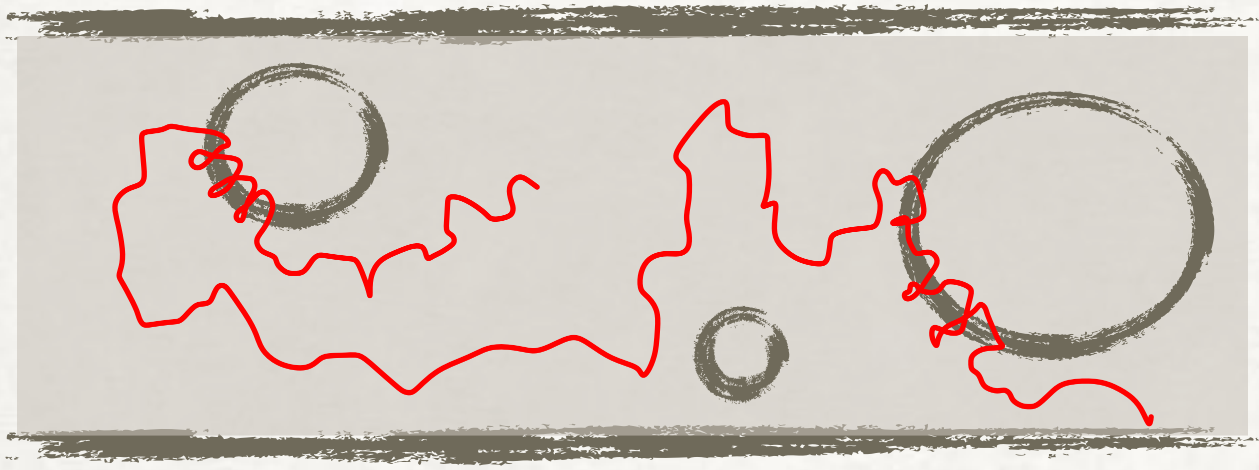
SEED CR, WHEN PRESENT, ARE ACCELERATED TOO

IF THEIR SPECTRUM IS STEEPER THAN THE ONE THAT IS ASSOCIATED WITH THE SHOCK MACH NUMBER —> THEIR SPECTRUM GETS HARDER

IF THEIR SPECTRUM IS HARDER THAN THE ONE THAT IS ASSOCIATED WITH THE SHOCK MACH NUMBER —> THEIR SPECTRUM REMAINS THE SAME

IN BOTH CASES ENERGY IS ADDED BUT THE TOTAL NUMBER OF PARTICLES IS CONSERVED

SHOCK ACCELERATION OF SECONDARY NUCLEI



SECONDARY NUCLEI (AS WELL AS PRIMARY) OCCASIONALLY ENCOUNTER A SN SHOCK AND GET ACCELERATED AT IT — SHOCK IS BLIND TO THE NATURE OF PARTICLES

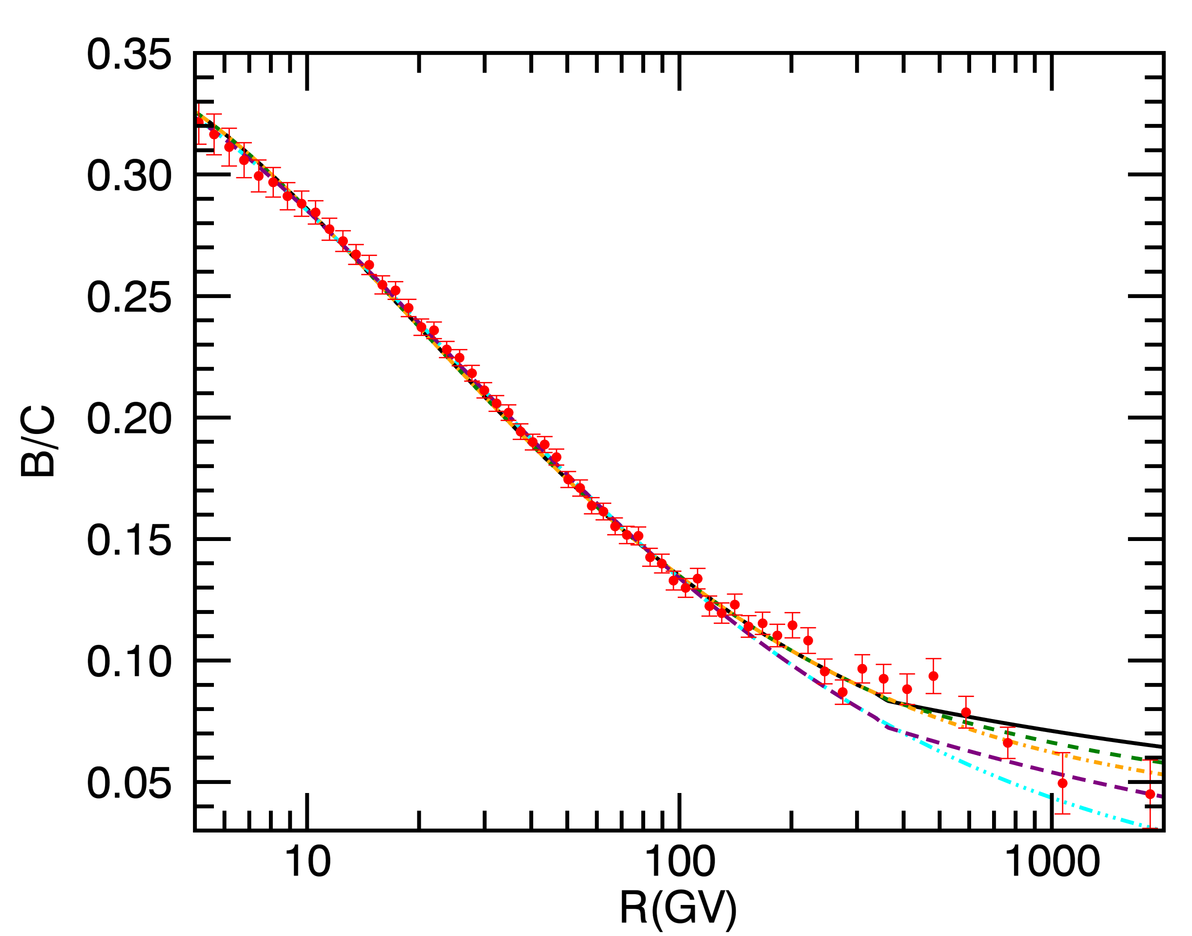
PRIMARY NUCLEI
thermal seeds —> $E^{-\gamma}$

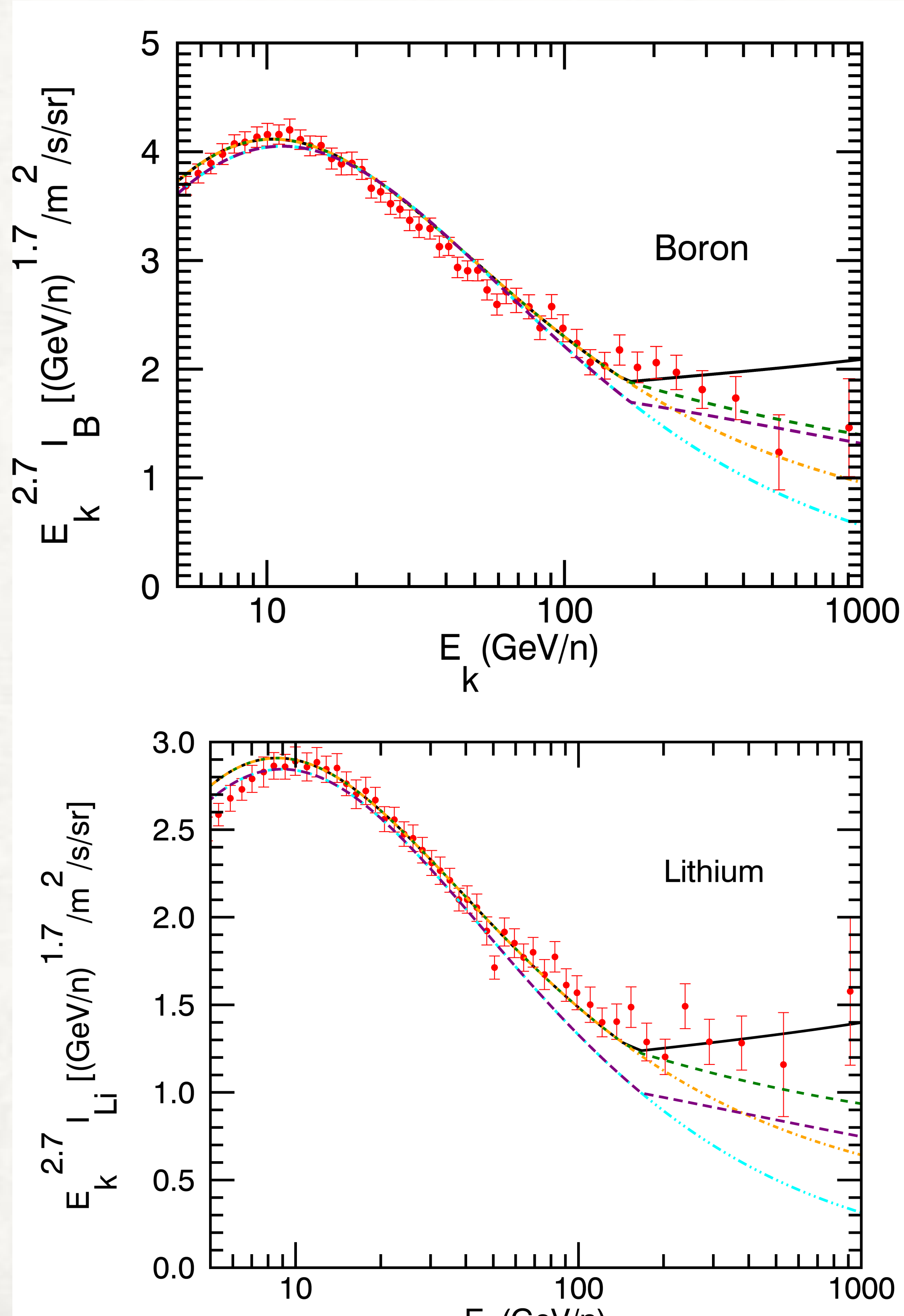
SECONDARY NUCLEI
$E^{-\gamma-\delta}$ —> $E^{-\gamma}$

IT IS CLEAR THAT THE OCCASIONAL ACCELERATION OF SECONDARY NUCLEI MUST BE THE MAIN CONTRIBUTION AT SUFFICIENTLY HIGH E, TYPICALLY ABOVE TeV (PB 2017)

SHOCK ACCELERATION OF SECONDARY NUCLEI

PB 2017, Bresci et al. 2019





CLEARLY NOT AN ATTEMPT TO MAKE A DETAILED PREDICTION, BUT RATHER DISCUSSION OF A NEW EFFECT

DECAY OF UNSTABLE ISOTOPES

THIS IS THE FIRST CASE WE MEET WHERE THE SOURCE OR LOSS TERM IS NOT IN THE FORM OF A DELTA FUNCTION (Z): THE DECAY OCCURS EVERYWHERE

$$-\frac{\partial}{\partial z}\left[D_a \frac{\partial f_a}{\partial z}\right] + v_A \frac{\partial f_a}{\partial z} - \frac{dv_A}{dz}\frac{p}{3}\frac{\partial f_a}{\partial p}$$

$$+\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \left(\frac{dp}{dt}\right)_{a,\mathrm{ion}} f_a\right] + \frac{\mu v(p)\sigma_a}{m}\delta(z) f_a + \frac{f_a}{\hat{\tau}_{d,a}}$$

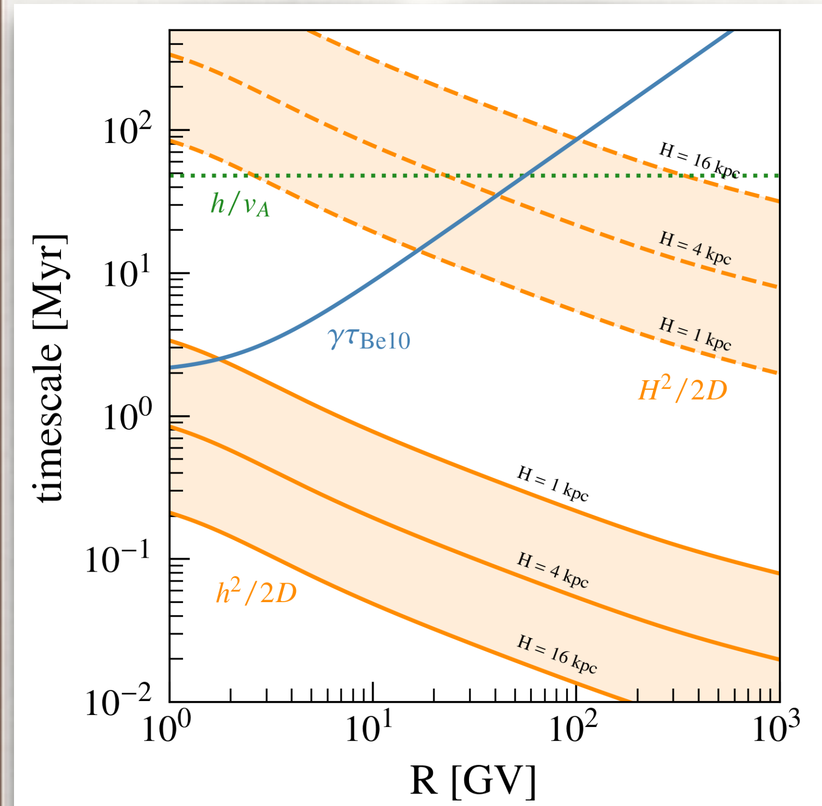
$$= 2h_d q_{0,a}(p)\delta(z) + \sum_{a'>a} \frac{\mu\, v(p)\sigma_{a'\to a}}{m}\delta(z) f_{a'} + \sum_{a'>a} \frac{f_{a'}}{\hat{\tau}_{d,a'}}$$

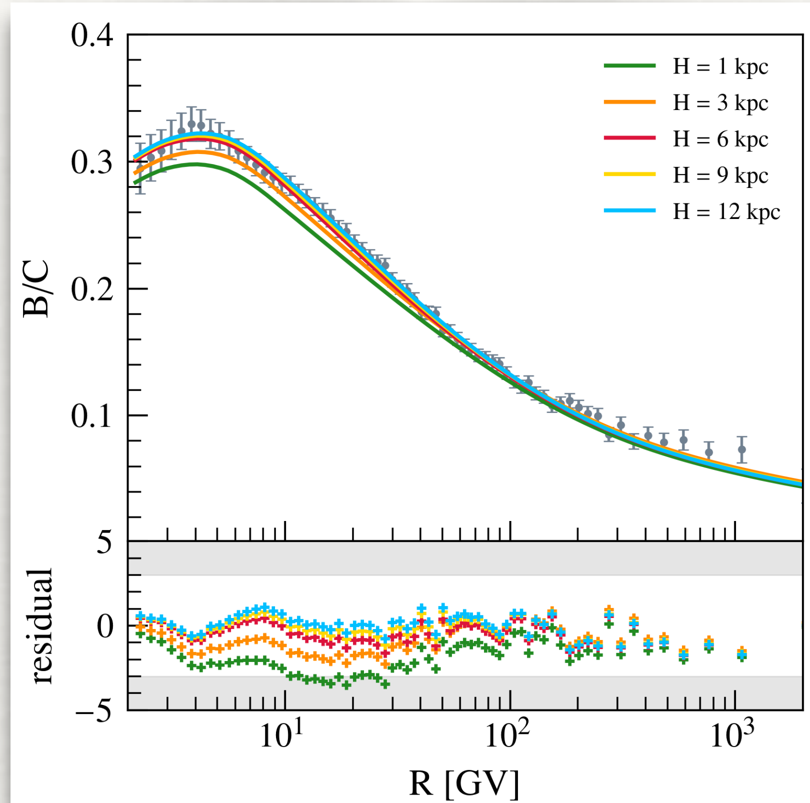
TWO MAJOR CHANGES:

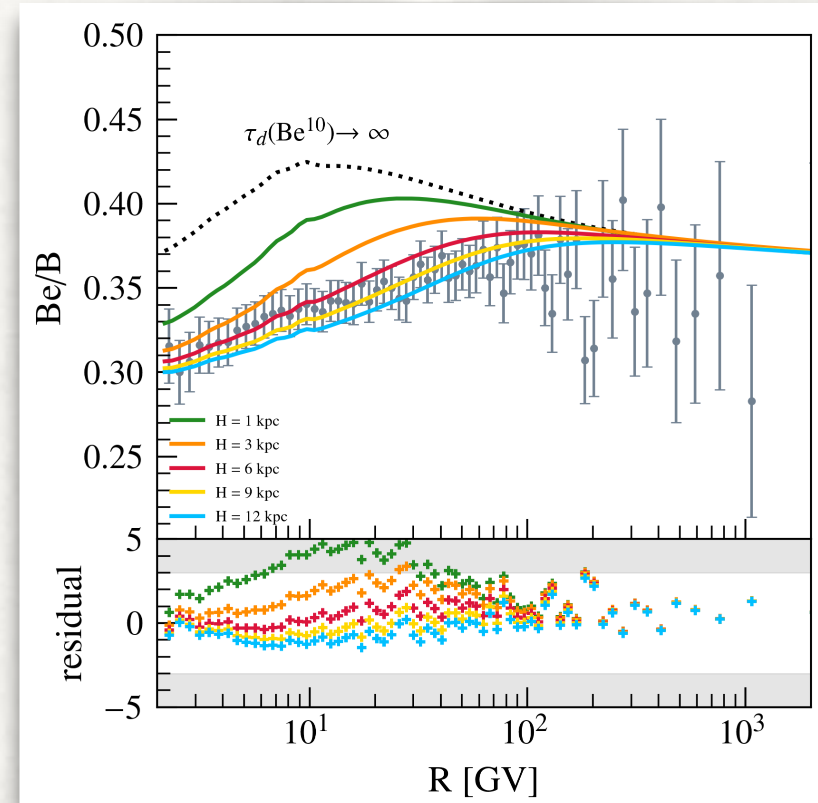
1) ^{10}Be decays on a time scale $\gamma\tau_d$ that at some high E becomes longer than $H^2/D(E)$

2) ^{10}Be decays mainly into ^{10}B so that it changes the abundance of stable elements

DECAY OF UNSTABLE ISOTOPES



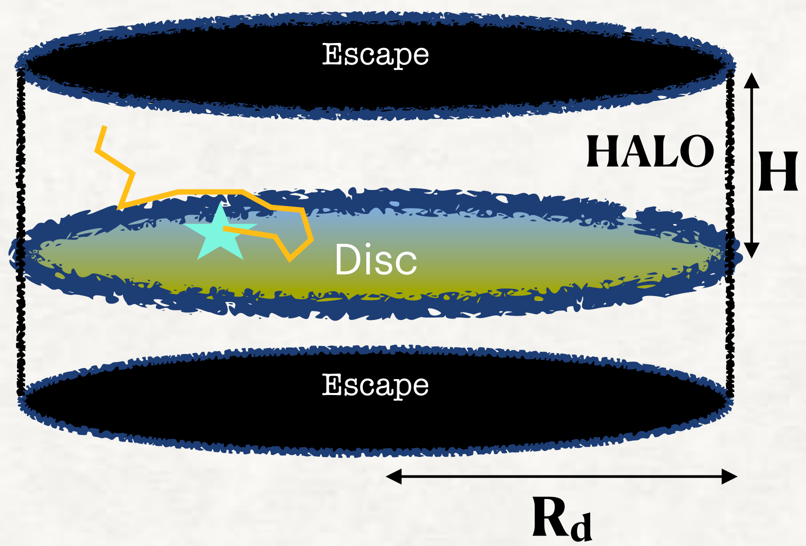




THE DECAY OF ^{10}Be SHOWS A PREFERENCE FOR RELATIVELY LARGE HALO SIZES H>5 kpc

POTENTIAL SOURCES OF GALACTIC CR

The energy density of Cosmic Rays at the position of the Earth is about 0.2 eV/cm3 at E>few GeV - how do we refill it?



$$\epsilon_{CR} = \frac{\dot{E}_{CR}}{2\pi R_d^2} \frac{H}{D(E)}$$

KNOWN FROM B/C

$$\dot{E}_{CR} \approx 2 \times 10^{40} \left(\frac{R_d}{15kpc}\right)^2 \text{ erg/s}$$

SN Type IA: Energetics 10^{51} erg Rate 1/100 years
Required Efficiency~6%

SN Type II: Energetics 10^{51} erg Rate 1/30 years
Required Efficiency~2%

SN Type II* (very luminous core collapse): Energetics 10^{52} erg Rate 1/10000 years
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Stellar Clusters: Typical luminosity 10^{37}-10^{38} erg/s - **if efficiency 10% about few thousand clusters required in the Galaxy**

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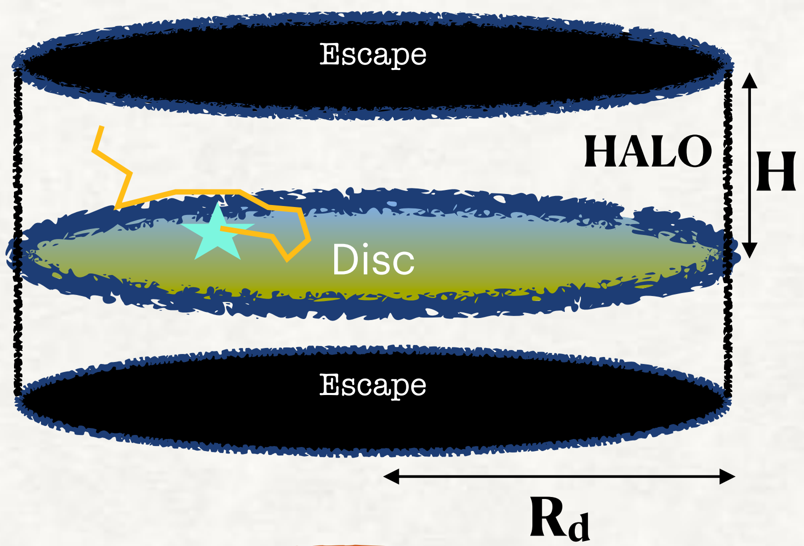

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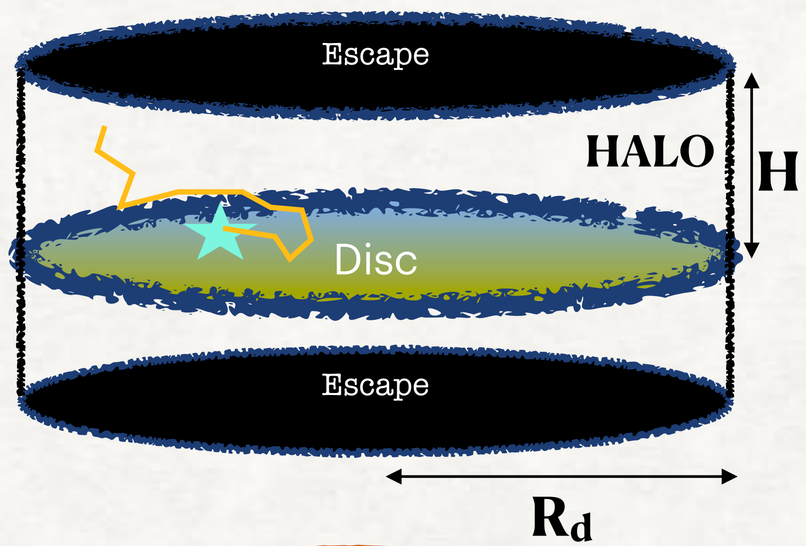

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WHAT DOES B/C TELL US?

Just to fix ideas, quantitatively one can say that $D_0/H=0.35$ in units of 10^{28} cm^2 s^{-1} kpc^{-1} where D_0 is approximately (not exactly) the diffusion coefficient at GV rigidity.

Since 10Be tells us that H~5 kpc, it follows that $D_0 \approx 1.8\times10^{28}$ cm^2 s^{-1}

Let us see what does it means in terms of what we learned in the previous lecture:

$$D(E) = \frac{1}{3} r_L(E) v \frac{1}{\mathcal{F}(k)} = \frac{1}{3} v \lambda_{Diff}(E) \qquad k = k_{res} = \frac{1}{r_L(E)}$$

Diffusive Pathlength

At 1 GV: $$\frac{1}{3} v \lambda_{Diff}(E) = 1.8\times 10^{28} cm^2/s \rightarrow \lambda_{Diff}(E=1GV) \approx 0.6 pc$$

This is how much a 1 GV CR has to travel before a deflection of 90 degrees

How much turbulent power is needed at 1GV?

$$r_L(1GV) \approx 10^{12} cm \qquad \mathcal{F}(k) \approx \frac{r_L c}{3 D_0} = 6\times 10^{-7} \equiv \left(\frac{\delta B}{B_0}\right)^2_{k_{res}}$$

A TINY PERTURBATION MAKES THE TRANSPORT TIME IN THE GALAXY GO FROM KYR TO 100S MILLION YEARS!!!

LIMITATIONS IN USING THIS METHOD

- The main limitation comes from the assumption that sources and gas are both in an infinitely thin disc
- This is fine as long as the interaction length is >h
- If this is not the case, one should also be aware that on scales ~h diffusion becomes sensitive to the structure of turbulence: be careful in trusting such results
- The other limitation comes from the 1D assumption on diffusion. This can be easily lifted by switching to a Green function formalism, as long as D does not vary strongly in space
- It is obvious that a more refined approach is needed to get morphological information (distribution of gas, light, etc)
- In principle one could adopt numerical approaches (GALPROP, DRAGON, USINE): these are able to account for complex spatial gas distribution and losses, but the D(E) is completely imposed by hand. This approach has many advantages but be always aware of its limitations as well

A QUICK LOOK AT TRANSPORT OF LEPTONS

The main difference with respect to what we already learned is that leptons are more prone to radiative energy losses (ICS, synchrotron, …)

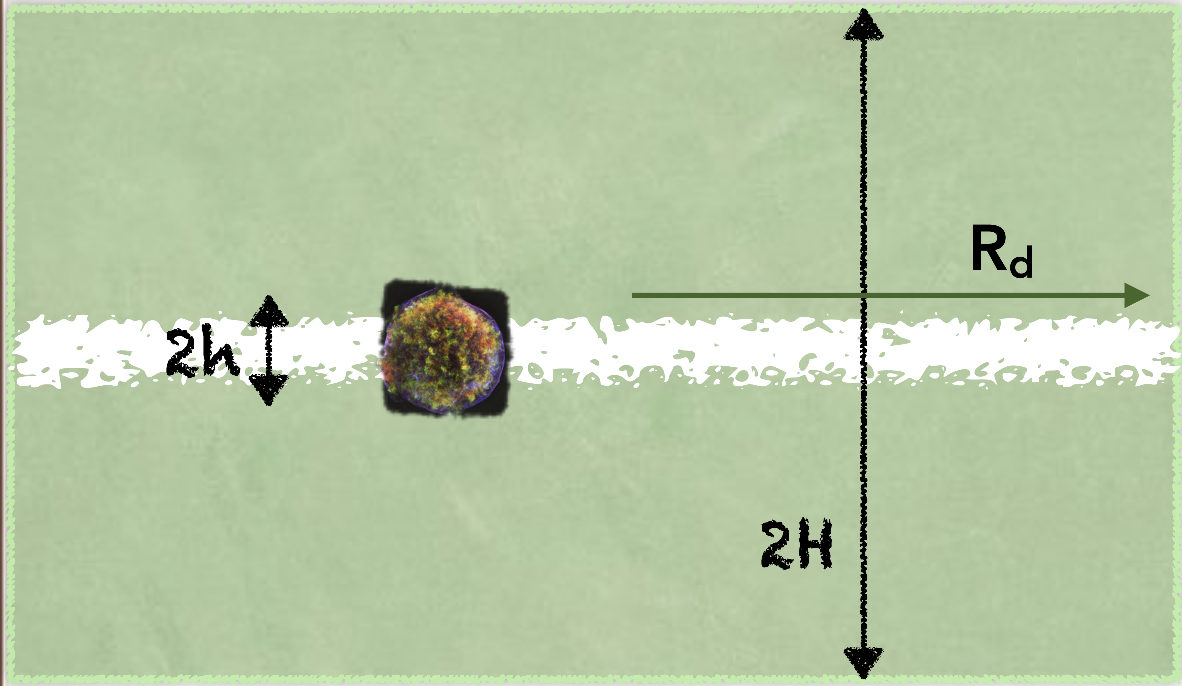
The transport equation to deal with should be

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} - \frac{1}{p^2}\frac{\partial}{\partial p}\left[\dot{p}p^2 f\right] + Q = 0$$

but if u—>0 and we approximate the loss terms as a catastrophic loss term:

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] = \frac{f}{\tau_{loss}} - Q_e \qquad Q_e(E) = \frac{N_e(E)\mathcal{R}_{SN}}{\pi R_d^2}\delta(z)$$

that can be solved in ways similar to the ones used above



Integrating the equation around the disc:

$$2D \left.\frac{\partial f}{\partial z}\right|_{0^+} = -\frac{N_e(E)\mathcal{R}_{SN}}{\pi R_d^2}$$

while for z>0:

$$D\frac{\partial^2 f}{\partial z^2} = \frac{f}{\tau_{loss}}$$

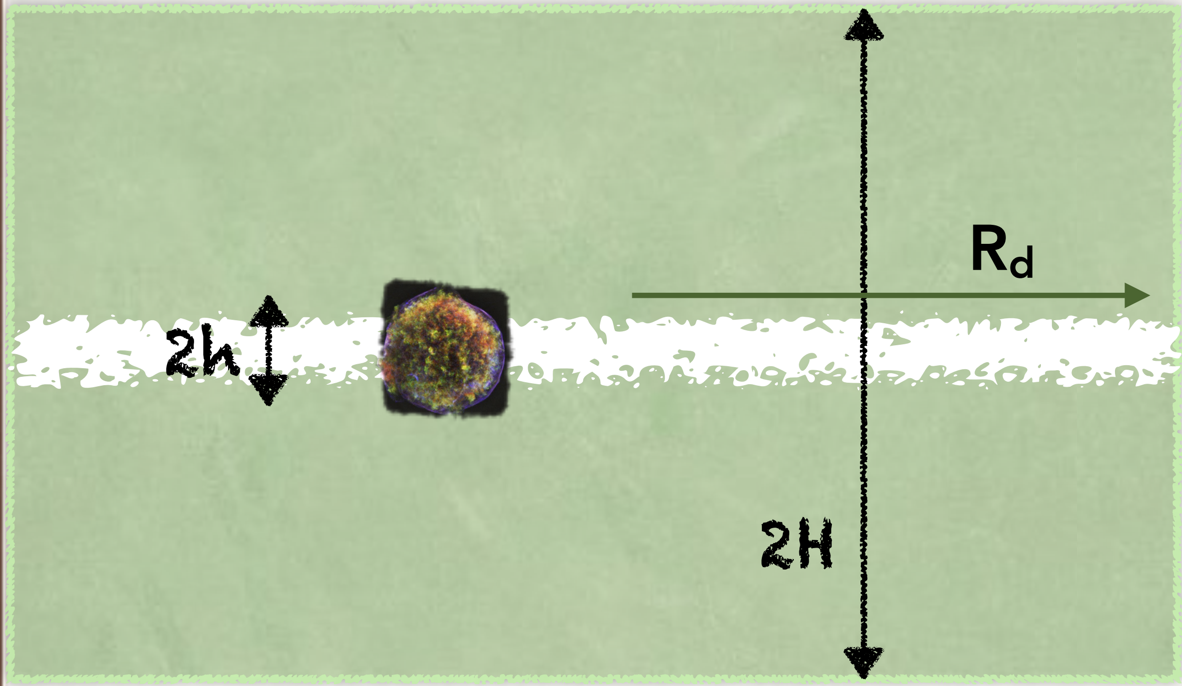
which has a general solution in the form:

$$f(z,p) = A\exp[-\beta z] + B\exp[\beta z]$$

It is easy to replace this in the original equation to get: $\beta^2 = \frac{1}{D\tau_{loss}}$

and requiring that $f(z=0)=f_0(E)$ and $f(z=H)=0$:

$$f = \frac{f_0 y^2}{y^2-1}\exp[-\beta z] - \frac{f_0}{y^2-1}\exp[\beta z] \qquad y=\exp(\beta H)$$



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Solving the equation: $$2D \left.\frac{\partial f}{\partial z}\right|_{0^+} = -\frac{N_e(E) R_{SN}}{\pi R_d^2}$$

we easily find:

$$f_0(E) = \frac{N_e(E) R_{SN}}{2\pi R_d^2} \sqrt{\frac{\tau_{loss}}{D}} \, \frac{y^2 - 1}{y^2 + 1}$$

$$y = \exp\left[\frac{H}{\sqrt{D \tau_{loss}}}\right]$$

IN THE LOW ENERGY LIMIT, WHERE LOSSES ARE WEAK:

$$\frac{H}{\sqrt{D\tau_{loss}}} = \left(\frac{H^2}{D\tau_{loss}}\right)^{1/2} \ll 1 \Rightarrow y \simeq 1 + \frac{H}{\sqrt{D\tau_{loss}}}$$

$$\longrightarrow \quad f_0(E) = \frac{N_e(E) R_{SN}}{2\pi R_d^2} \frac{H}{D}$$

IN THE OPPOSITE LIMIT, LOSSES DOMINATE TRANSPORT AND

$$\frac{H}{\sqrt{D\tau_{loss}}} \gg 1 \Rightarrow \frac{y^2 - 1}{y^2 + 1} \simeq 1$$

$$\longrightarrow \quad f_0(E) \simeq \frac{N_e(E) R_{SN} \tau_{loss}}{2\pi R_d^2 \sqrt{D\tau_{loss}}}$$

Rule of thumb: f(E)= Rate of Injection * Relevant Time/Relevant Volume

A SIMPLE APPLICATION TO THE POSITRON FRACTION

Secondary Positrons are produced through pp collisions and typically the energy of the secondary positrons is some fraction of the parent proton

$$E_e \simeq \xi E_p$$

The rate of production of positrons is then:

$$q_e(E)\,dE = n_p(E')\,dE'\, n_d\, \sigma_{pp}\, c\, 2h\, \delta(z) \Rightarrow \quad E' = \frac{E}{\xi}$$

You easily see that this injection terms has the same form as the one of primary electrons (delta function in z), hence one can simply apply what we already learned. As an exercise you can prove that:

$$f_{sec}(E) = \frac{N_p(E/\xi)\mathcal{R}_{SN}}{2\pi R_d^2}\frac{H}{D(E/\xi)}\frac{1}{\xi}n_d h \sigma_{pp} c \frac{H}{D(E)}$$

When losses are unimportant

$$f_{sec}(E) = \frac{N_p(E/\xi)\mathcal{R}_{SN}}{2\pi R_d^2}\frac{H}{D(E/\xi)}\frac{1}{\xi}n_d h \sigma_{pp} c \sqrt{\frac{\tau_{loss}}{D}}$$

When losses dominate

AS A CONSEQUENCE, IN BOTH CASES:

$$\frac{f_{sec}(E)}{f_e(E)} = \frac{N_p(E/\xi)}{N_e(E)}\frac{1}{\xi}\frac{X(E/\xi)}{X_{cr}} \propto E^{-\alpha_p+\alpha_e-\delta}$$

FUTURE DEVELOPMENTS

- **NATURE OF TURBULENCE**
  - Self-generated scattering
  - Pre-existing turbulence (anisotropic cascade, fast vs slow MS waves, etc)
- **ORIGIN OF THE HALO**
  - Why is there a magnetized halo, despite no sources of turbulence (aside from CRs)?
  - What sets the end of the halo and the free escape boundary?
  - Is there really a halo? or it's something more complex: Role of CR vs Gravity
- **TRANSITION FROM ACCELERATED PARTICLES TO COSMIC RAYS**
  - This is the contact point between acceleration and propagation, through the process of escape from sources
  - Much care needs to be taken, in that this stage may affect grammage and gamma ray observations

NON-LINEAR DIFFUSION

We have seen that on Galactic scale CR develop a gradient of order f_0/H

This implies a drift velocity of order $v_D = \frac{3D\nabla f}{f} \approx \frac{3D}{H}$

When $v_D > v_A$ streaming instability is excited at the rate that we found in the previous lecture

If this were the end of the story the self-generated D(E) would be the one for which $v_D = v_A$ at all energies

However, waves growth is limited by dampings, which introduce a non-trivial energy dependence in D(E) one-to-one connected with the spectrum of CR —> non linear transport

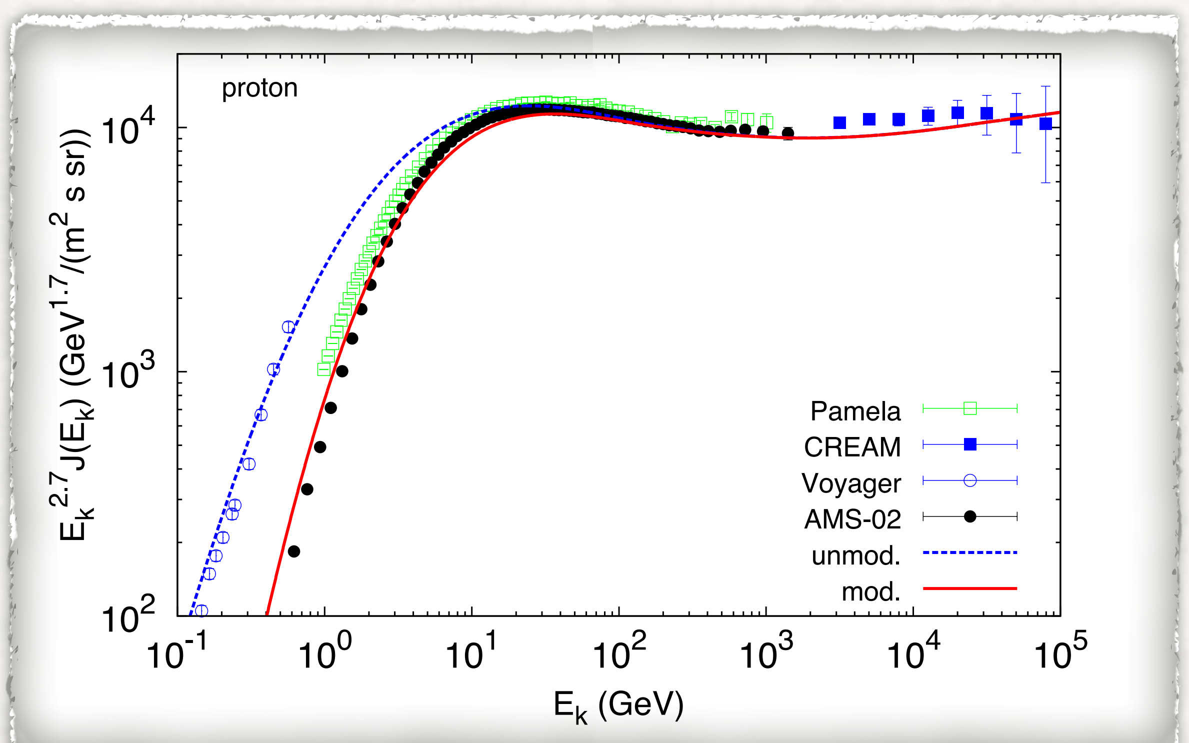
NON LINEAR GALACTIC TRANSPORT

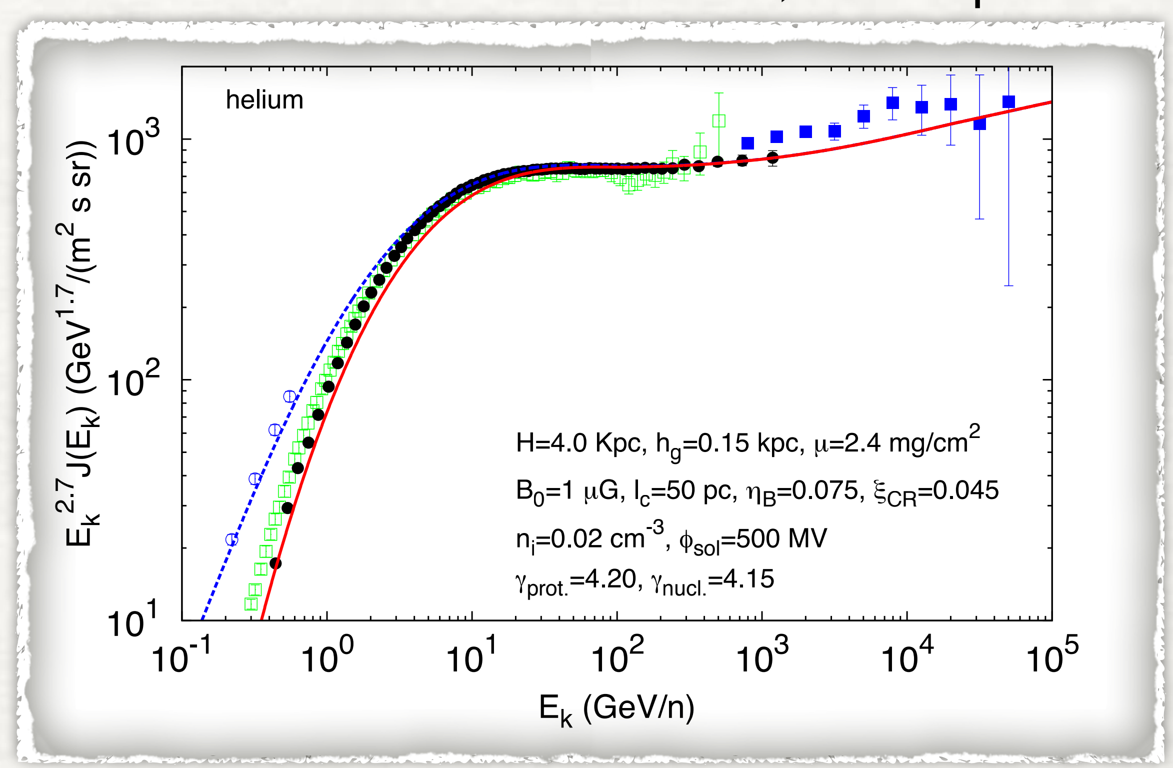
I. SELF-GENERATED WAVES VS PRE-EXISTING

Waves can cascade down from large scales (e.g. SNe) and be injected on resonant scales through streaming instability

The combination of the two phenomena leads to different energy scalings of D(p) and hence of anisotropy [*PB, Amato & Serpico 2012, Aloisio & PB 2014, Aloisio, PB & Serpico 2015*] — This phenomenon reflects in spectral breaks

Aloisio, PB & Serpico 2015

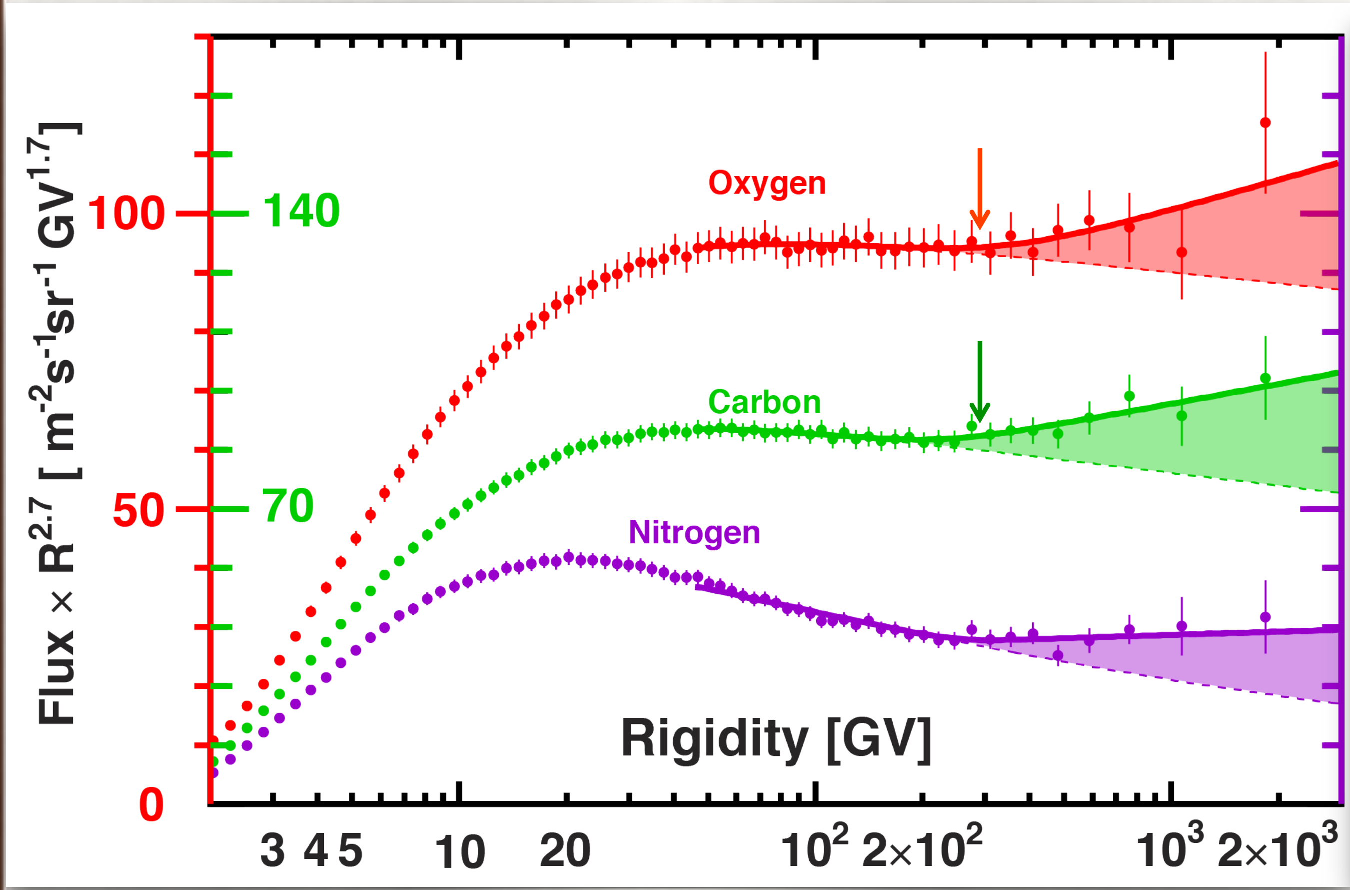


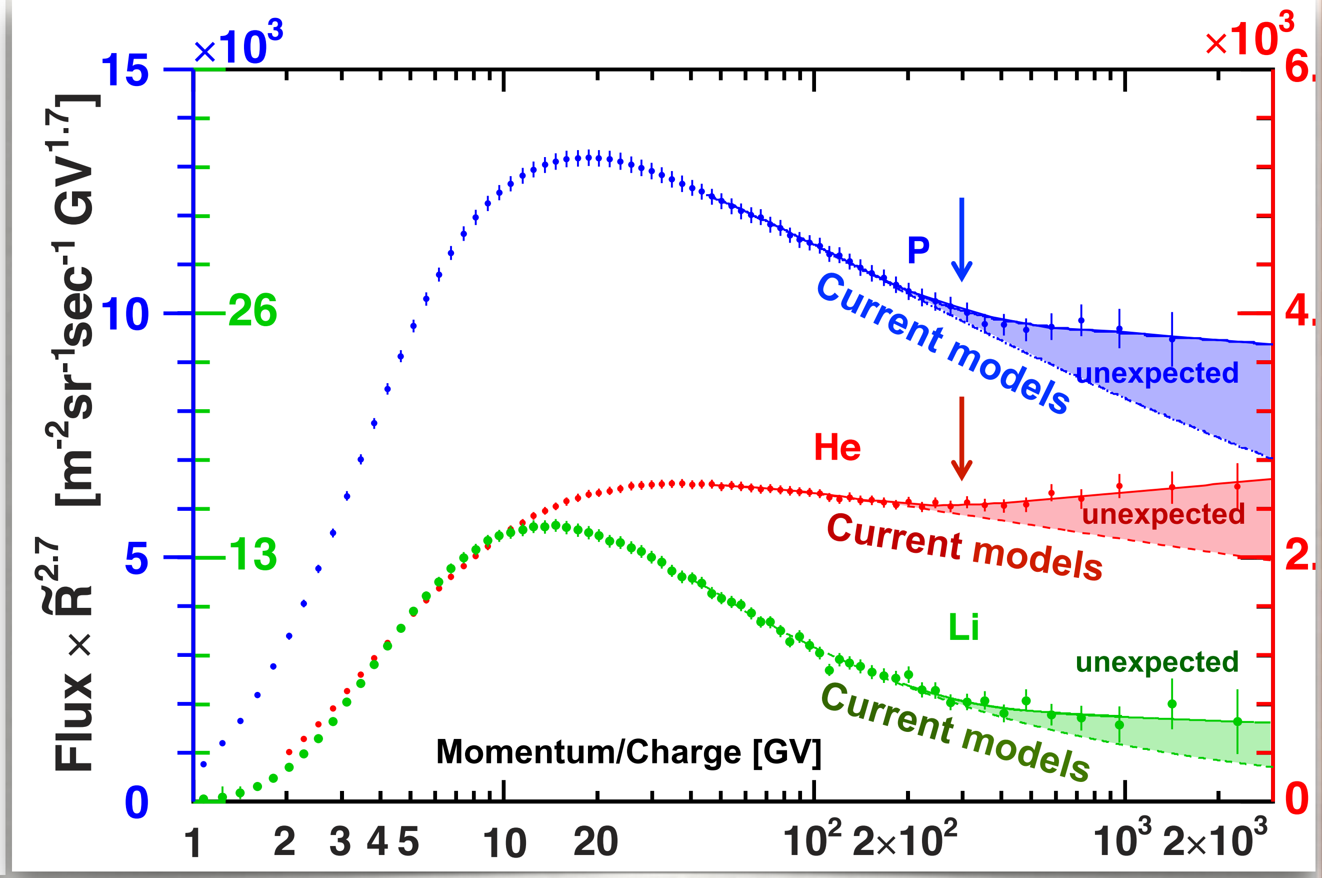


A GENERAL (PHENOMENOLOGICAL) TREND?

A change of slope around 300 GV seems to be visible in measurements of the fluxes of both primary and secondary nuclei

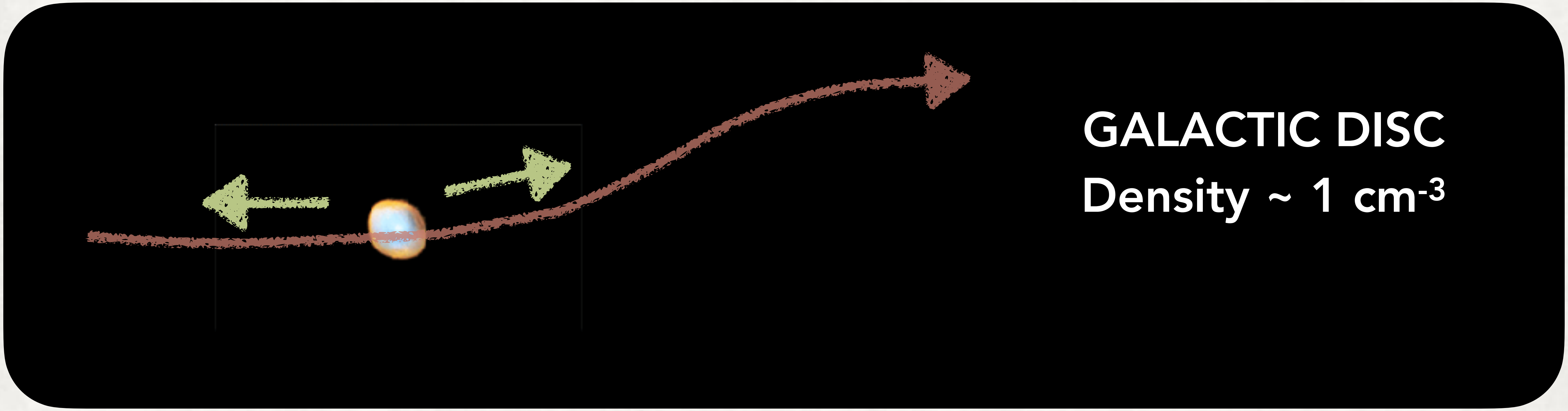
Whether this happens because of a break in the injection spectra or in the diffusion coefficient could be understood from quantitative assessment of the slopes below and above the break for primaries and secondaries





GRAMMAGE ACCUMULATED NEAR THE SOURCES

NEAR THE SOURCES THE DENSITY OF CR AND THE GRADIENTS ARE LARGE ENOUGH THAT INSTABILITIES ARE EXCITED AND MAY CONFINE CR CLOSE TO THE SOURCES FOR LONG TIMES



ACCOUNTING FOR THIS PROBLEM REQUIRES SOLVING A TIME DEPENDENT NON-LINEAR DIFFUSION PROBLEM

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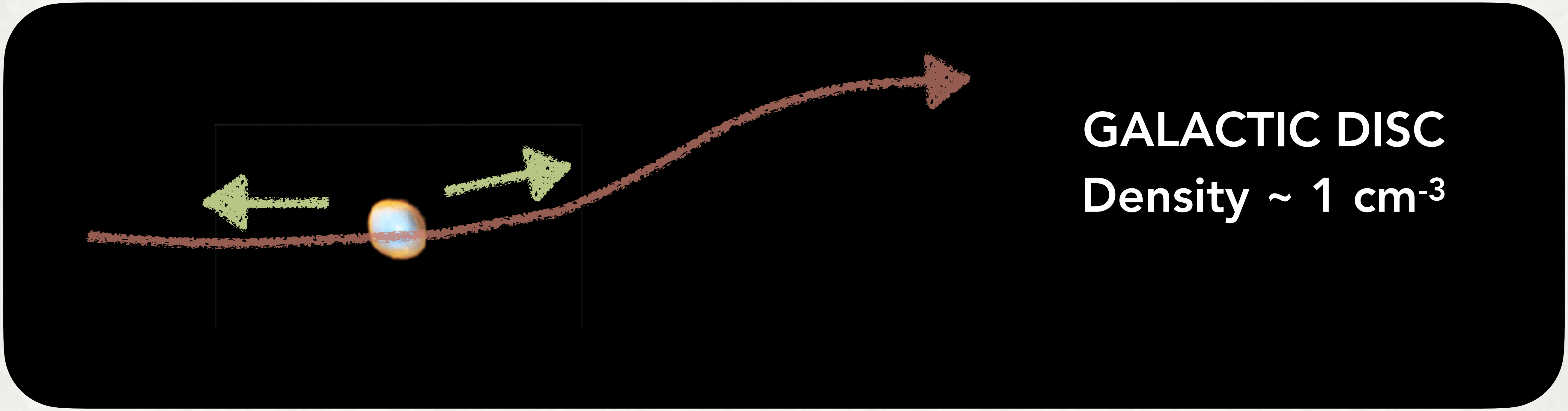

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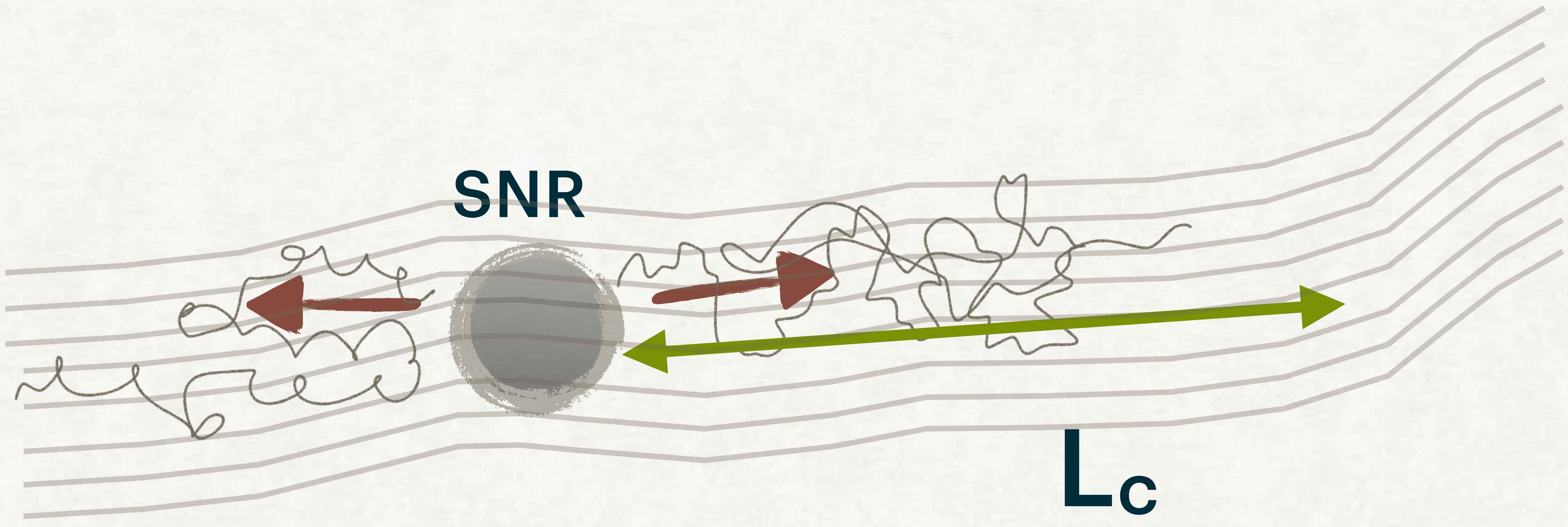
NEAR SOURCE TRANSPORT

The gradients in the particle distribution around a source are very large and can lead to excitation of fast streaming instability



In the absence of non-linear effects the CR density inside Lc remains > than the Galactic average for a time

$$t_s \sim 2 \times 10^4 E_{GeV}^{-1/3} \; yr$$

After that, propagation becomes 3D and the density drops rapidly

NEAR SOURCE TRANSPORT

In the presence of non-linear effects waves are excited

$$\Gamma_{\rm CR}(k) = \frac{16\pi^2}{3}\frac{v_{\rm A}}{\mathcal{F}B_0^2}\left[p^4 v(p)\frac{\partial f}{\partial z}\right]_{p=qB_0/kc}$$

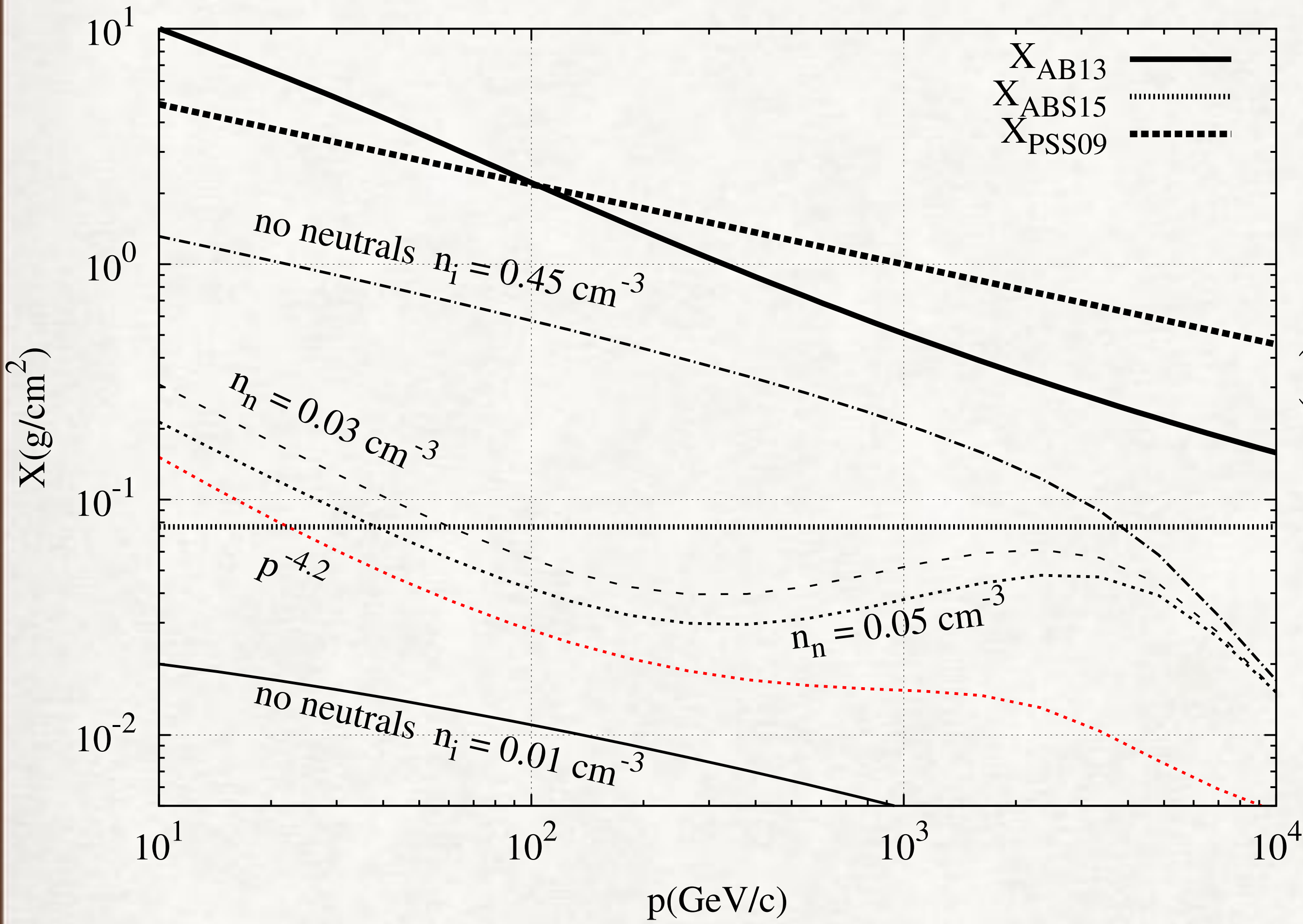
and damped:

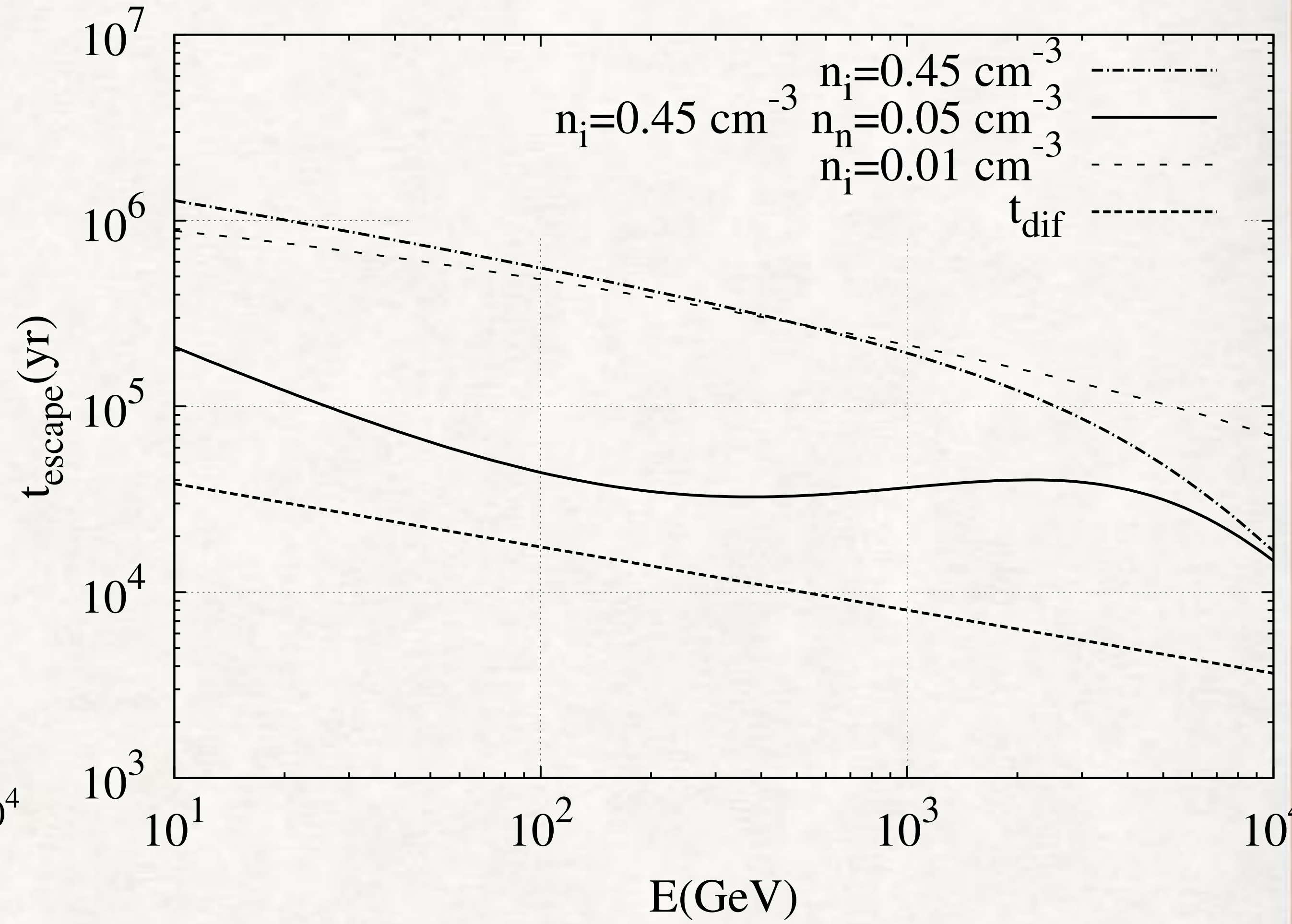
$$\Gamma_{NL} = (2c_K)^{-3/2} k v_A \mathcal{F}^{1/2}$$

The diffusion coefficient becomes a function of the CR density (NL transport)

$$\frac{\partial f}{\partial t} + v_A\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\left[D(p,z,t)\frac{\partial f}{\partial z}\right] = q_0(p)\delta(z)\Theta(T_{SN}-t)$$

GRAMMAGE ACCUMULATED NEAR THE SOURCES





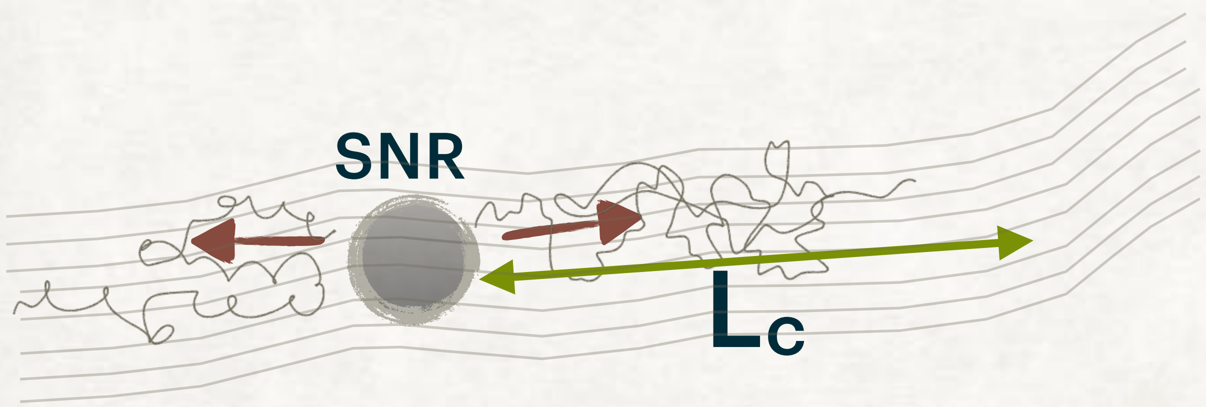
D'Angelo, PB & Amato 2017

THE NON LINEAR EFFECTS INDUCED BY CR CAN LEAD TO AN ENHANCED CONFINEMENT TIME CLOSE TO THE SOURCE IF MEDIUM IONIZED

IF NEUTRALS PRESENT, ION-NEUTRAL DAMPING LIMITS THIS PHENOMENON

HIGH ENERGY PARTICLES LEAVING A SNR

REMEMBER THE ESCAPE PROBLEM???



ADOPTING THE GALACTIC DIFFUSION COEFFICIENT AS A BENCHMARK

$$D(E) = \frac{1}{3} v\lambda \to \lambda \approx 1pc \left(\frac{E}{GeV}\right)^{1/2} \approx L_c \left(\frac{E}{2.5TeV}\right)^{1/2}$$

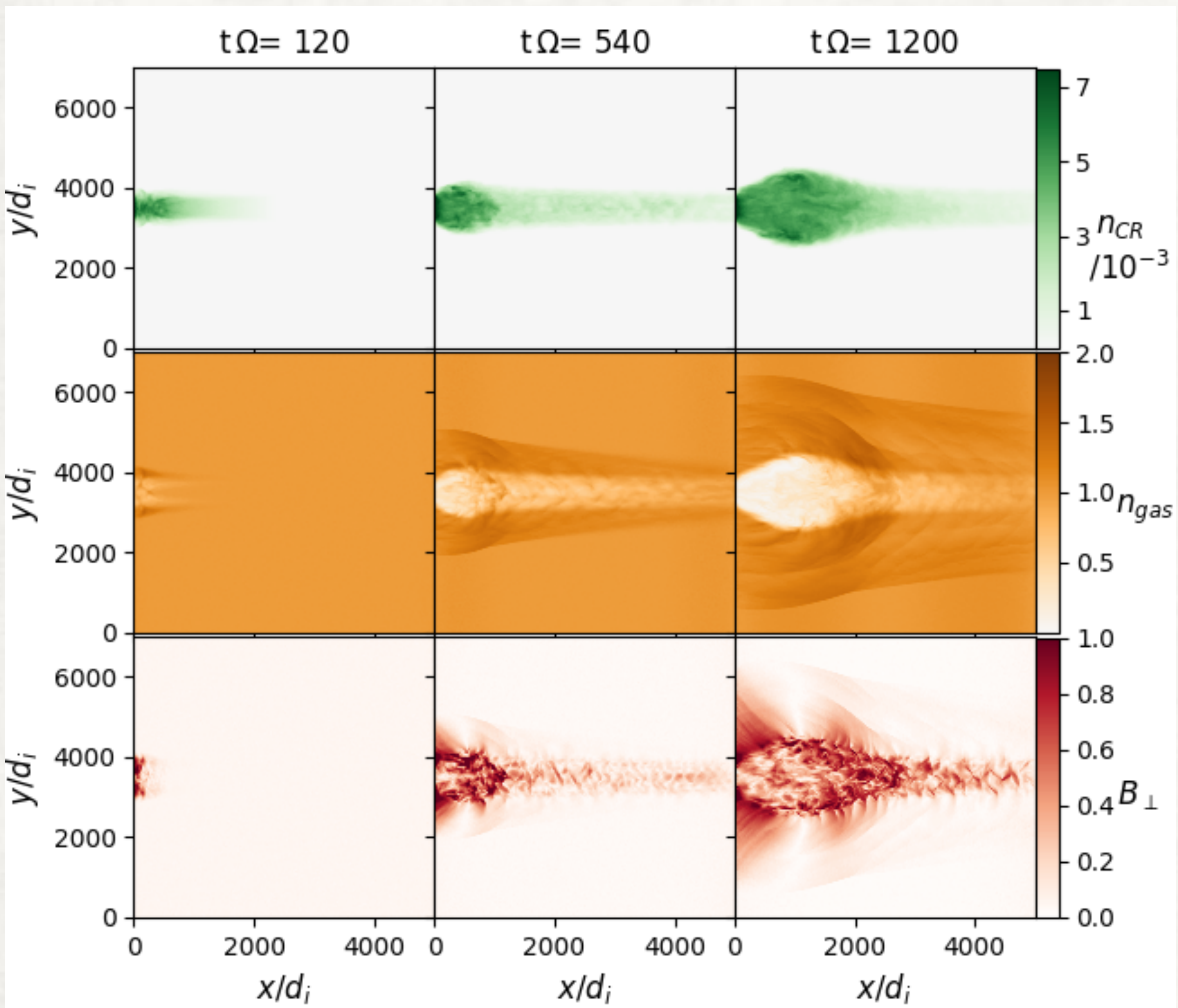
EVEN ASSUMING BALLISTIC MOTION IN SUCH REGION, FOR THE PARAMETERS OF A SNR:

$$n_{CR}(> E) \approx 5.4 \times 10^{-8} E_{GeV}^{-1} \text{ cm}^{-3} \to n_{CR}(> E)E > \frac{B_0^2}{4\pi}$$

THE NON RESONANT MODES 'a la Bell' ARE ALLOWED TO GROW ON A TYPICAL TIME SCALE:

$$\gamma_{max}^{-1} \approx 1.1(\mathrm{E}/2.5\mathrm{TeV})^{-1} \text{ years}$$

HYBRID SIMULATIONS



- THE EXCITATION OF THE INSTABILITY LEADS TO STRONG PARTICLE SCATTERING, WHICH IN TURN INCREASES CR DENSITY NEAR THE SOURCE
- THE PRESSURE GRADIENT THAT DEVELOPS CREATES A FORCE THAT LEADS TO THE INFLATION OF A BUBBLE AROUND THE SOURCE
- THE SAME FORCE EVACUATES THE BUBBLE OF MOST PLASMA
- THERE IS NO FIELD IN THE PERP DIRECTION TO START WITH, BUT CR CREATE IT AT LATER TIMES **(SUPPRESSED DIFFUSION, about 10 times Bohm)**

Schroer+, 2021 and 2022 *Dynamical effects of cosmic rays leaving their sources*

Grammage in the near source region

If the diffusion coefficient in the region surrounding the source gets suppressed enough and for long enough time, then CR can accumulate some grammage in the region

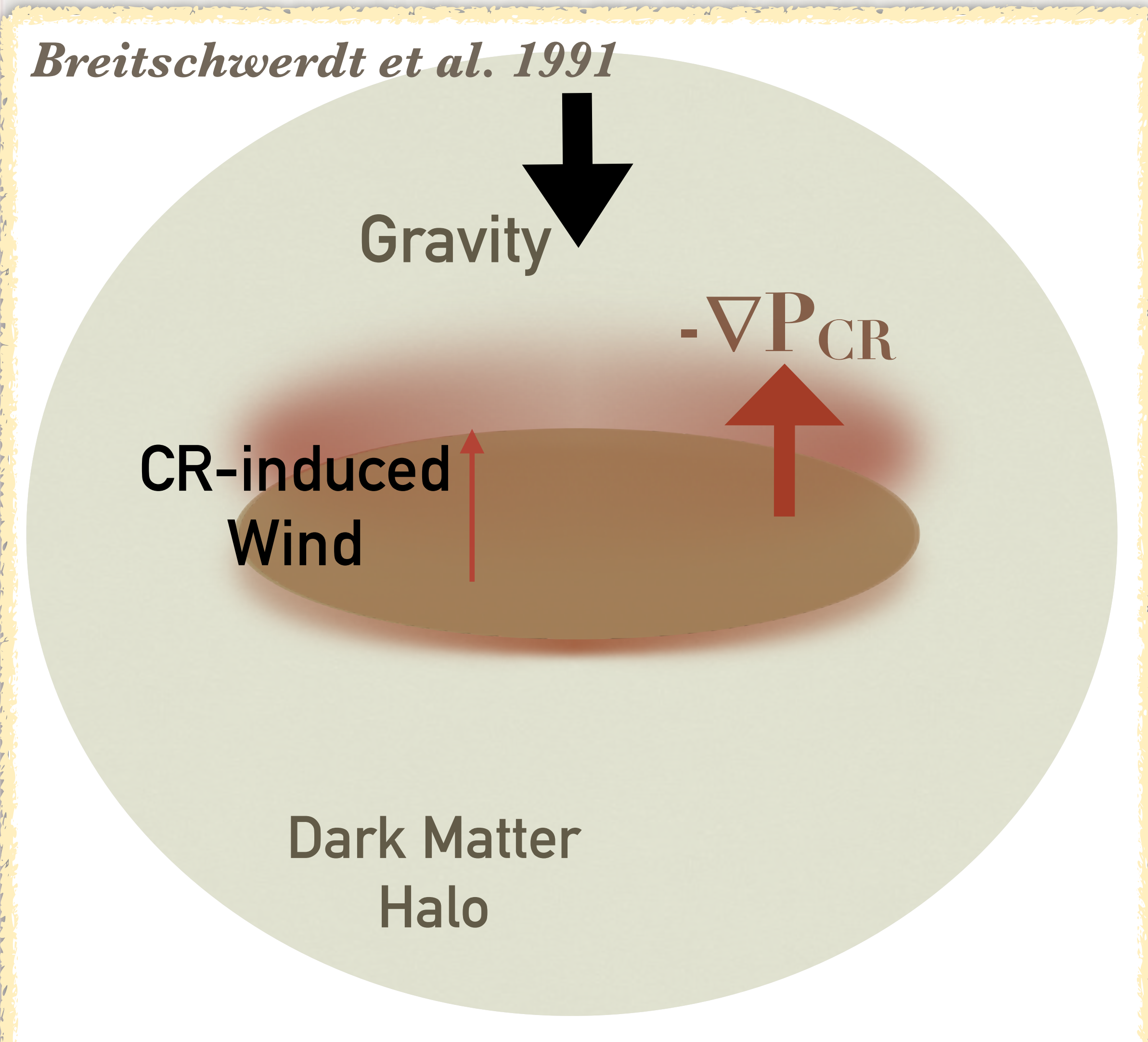
This near-source grammage depends on the D(E) suppression (ξ) and on the gas evacuation (η)

This grammage can be compared with that in the Galaxy. The near-source remains small if

$$\frac{\xi}{\eta} \lesssim \frac{L^2}{Hh} \approx 3 \times 10^{-3} \left(\frac{L}{50pc}\right)^2 \left(\frac{H}{5kpc}\right)^{-1} \left(\frac{h}{150pc}\right)^{-1}$$

Nevertheless the near-source grammage can significantly affect our modelling of the data as well as antiproton/proton and positron fraction

Cosmic Rays vs Gravity: Cosmic Ray Induced Galactic Winds



The force exerted by CR may win over gravity and a wind may be launched

$$\vec{\nabla}\cdot(\rho\vec{u}) = 0,$$

$$\rho(\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}(P_g + P_c) - \rho\vec{\nabla}\Phi,$$

$$\vec{u}\cdot\vec{\nabla}P_g = \frac{\gamma_g P_g}{\rho}\vec{u}\cdot\vec{\nabla}\rho - (\gamma_g - 1)\vec{v}_A\cdot\vec{\nabla}P_c,$$

$$\vec{\nabla}\cdot\left[\rho\vec{u}\left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1}\frac{P_g}{\rho} + \Phi\right)\right] = -(\vec{u}+\vec{v}_A)\cdot\vec{\nabla}P_c,$$

$$\vec{\nabla}\cdot\left[(\vec{u}+\vec{v}_A)\frac{\gamma_c P_c}{\gamma_c - 1} - \frac{\overline{D}\vec{\nabla}P_c}{\gamma_c - 1}\right] = (\vec{u}+\vec{v}_A)\cdot\vec{\nabla}P_c,$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\cdot\left[D\vec{\nabla}f\right] - (\vec{u}+\vec{v}_A)\cdot\vec{\nabla}f + \vec{\nabla}\cdot(\vec{u}+\vec{v}_A)\frac{1}{3}\frac{\partial f}{\partial \ln p} + Q = 0.$$

Diffusion determined by self-generation at CR gradients balanced by local damping of the same waves

No pre-established diffusion coefficient and no pre-fixed halo size

Cosmic Rays vs Gravity: CR driven winds

Aside from math, the Physics of the problem can be understood easily, though it turns out to be unrealistic: There is a critical distance above (and below) the disc (which depends on particle energy) where diffusion turns into advection:

$$\frac{z^2}{D(p)} \simeq \frac{z}{u(z)} \to z_*(p) \propto p^{\delta/2} \qquad D(p) \sim p^{\delta}$$

Ptuskin et al. 1997

No effective halo size H

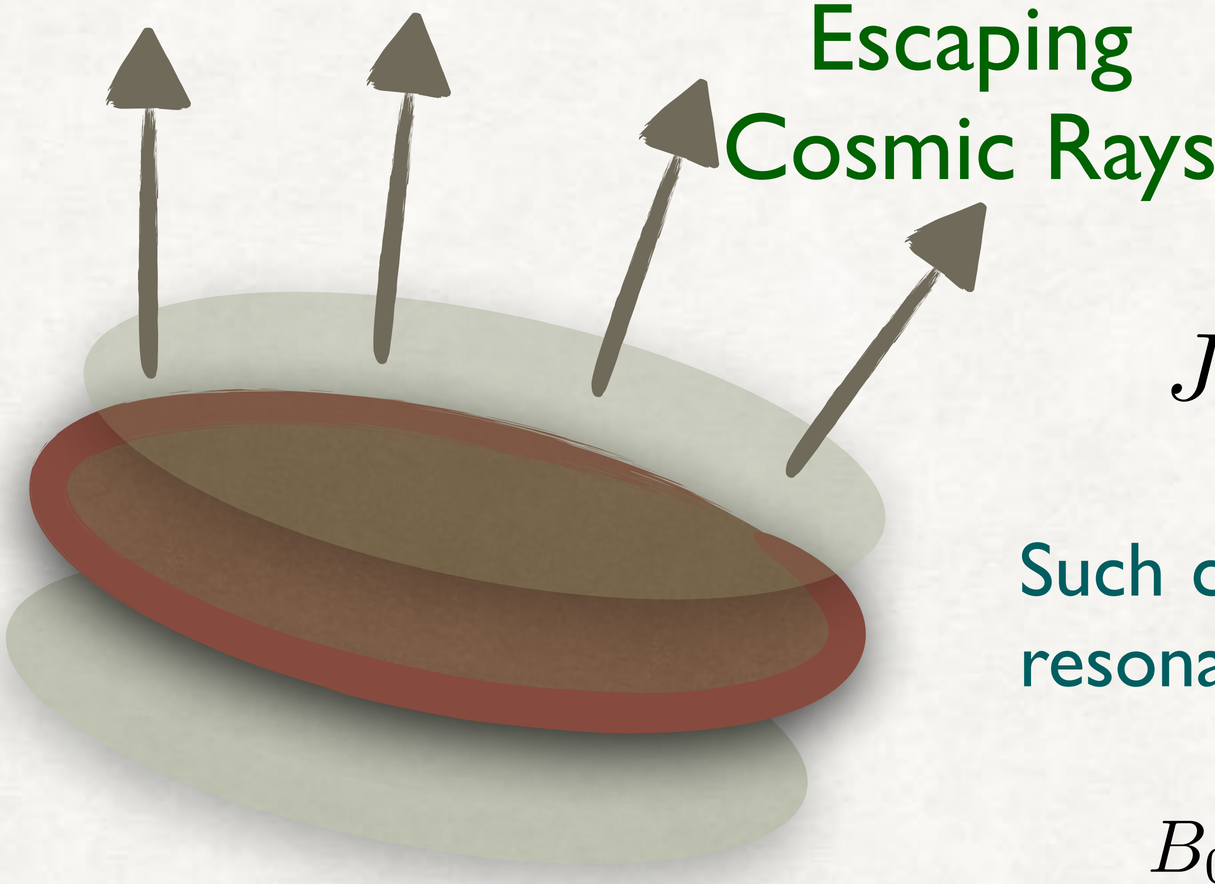
$$f_0(p) = \frac{Q(p)}{2A_{disc}} \frac{H}{D(p)} \sim E^{-\gamma-\delta} \qquad f_0(p) = \frac{Q(p)}{2A_{disc}} \frac{z_*(p)}{D(p)} \sim E^{-\gamma-\delta/2}$$

STANDARD CASE — CR-INDUCED WIND WITH SELF-GENERATION

At high energy, the critical scale becomes larger than the location where the geometry to he wind becomes spherical, and a steepening of the spectrum may be expected

ESCAPING THE GALAXY: A PHYSICAL PICTURE

PB&Amato 2019



As discussed above, the current of escaping CRs is very well known

$$J_{CR}(p) = eD\frac{\partial f}{\partial z}|_{z=H} = \frac{eQ_0(p)}{2\pi R_d^2}$$

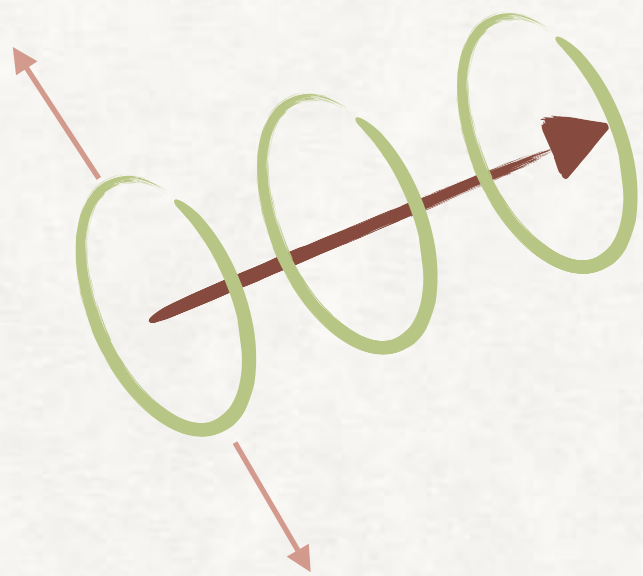
Such current in the typical IGM excites a non-resonant Bell-like instability provided:

$$B_0 \leq B_{sat} \approx 2.4 \times 10^{-8} L_{41}^{1/2} R_{10}^{-1} \text{ G}$$

At a wavenumber $k_{max} = \frac{4\pi}{cB_0} J_{CR}$

and with a growth rate: $\gamma_{max} = k_{max} v_A \approx 0.5 \text{ yr}^{-1} \ \delta_G^{-1/2} E_{\rm GeV}^{-1} L_{41} R_{10}^{-2}$

THE EASY WAY TO SATURATION



The current exerts a force of the background plasma

$$\rho \frac{dv}{dt} \sim \frac{1}{c} J_{CR} \delta B$$

which translates into a plasma displacement:

$$\Delta x \sim \frac{J_{CR}}{c\rho} \frac{\delta B(0)}{\gamma_{max}^2} exp(\gamma_{max} t)$$

which stretches the magnetic field line by the same amount…

The saturation takes place when the displacement equals the Larmor radius of the particles in the field **δ**B

$$\delta B \approx B_{\rm sat} \approx \sqrt{\frac{2L_{CR}}{c\ R_d^2 \Lambda}} \approx 2.4 \times 10^{-8} L_{41}^{1/2} R_{10}^{-1}\ \mathrm{G}$$

WHAT'S GOING ON?

We started from the assumption that CR escape freely and we got to the conclusion that perturbations are generated

I remind you that the escape spectrum is ~Q(p)~p^{-4} hence the spectrum of perturbations is scale invariant

One would be tempted to assume that CR would diffuse, but after a few γ_{max}^{-1} the pressure gradient built up because of scattering becomes sufficient to set the background plasma in motion with the speed

$$v_D \approx \frac{\delta B}{(4\pi\Omega_b \rho_{cr} \delta_G)^{1/2}} \sim 10 - 100 \text{ km/s}$$

local gas overdensity

PICTURE

When escaping CR reach a region where the field drops below ~10^{-8} G, they excite a non-resonant instability that sets the plasma in motion

Hence, their density is set by advection

$$n_{CR}(E) = \frac{\phi_{CR}}{\tilde{v}_A}$$

Instead of escaping at c they move at speed v_D so that their density is much higher around the Galaxy than in the case of free streaming

ASTROPHYSICAL NEUTRINOS (?)

COSMIC RAYS THAT ATTEMPTED ESCAPE FROM THE GALAXY ARE ACTUALLY TRAPPED IN A CIRCUMGALACTIC REGION, WHERE THE GAS DENSITY IS ABOUT ~200 $\Omega_b \rho_{cr}$

NEUTRINOS ARE PRODUCED THROUGH INELASTIC HADRONIC COLLISIONS

$$F_\nu(E_\nu)E_\nu^2 \approx \frac{L_{CR}}{2\pi R_d^2 \Lambda \tilde{v}_A} \frac{E_\nu^2}{E^2} \frac{dE}{dE_\nu} \frac{\delta_G \Omega_b \rho_{cr}}{m_p} \frac{c\sigma_{pp} R_d}{2\pi}$$