1. **The Neutrino Portal**
   (a) In the lecture, we argued that in the limit ($4\pi E/\Delta m^2_{41} \ll L \ll 4\pi E/\Delta m^2_{31}$, the three oscillation probabilities $P_{\nu_e\to\nu_e}$, $P_{\nu_\mu\to\nu_\mu}$, and $P_{\nu_\mu\to\nu_e}$ depend only on two parameters, $|U_{e4}|^2$ and $|U_{\mu 4}|^2$:

$$P_{\nu_e\to\nu_e} \simeq 1 - 2|U_{e4}|^2(1-|U_{e4}|^2) \quad (1)$$
$$P_{\nu_\mu\to\nu_\mu} \simeq 1 - 2|U_{\mu 4}|^2(1-|U_{\mu 4}|^2) \quad (2)$$
$$P_{\nu_\mu\to\nu_e} \simeq 2|U_{e4}||U_{\mu 4}|\,. \quad (3)$$

   Prove these relations.
   (b) For a while, early MiniBooNE suggested an excess in $\bar\nu_\mu \to \bar\nu_e$, but no corresponding excess in $\nu_\mu \to \nu_e$. How would one have needed to extend the $3+1$ scenario to accommodate such a result, had it been confirmed?

2. **Neutrino Magnetic Moments**
   (a) Why can Dirac neutrinos have flavour-diagonal magnetic moments, e.g.

$$\frac{1}{2}\mu_\nu^{ee}\bar\nu_L^e\sigma^{\mu\nu}\nu_R^e + h.c.\,, \quad (4)$$

   while Majorana neutrinos only admit transition magnetic moments, i.e. operators of the form

$$\frac{1}{2}\mu_\nu^{\alpha\beta}\bar\nu_L^\alpha\sigma^{\mu\nu}\nu_R^\beta + h.c. \quad (5)$$

   with $\alpha \neq \beta$?
   Hint: remember that for Majorana neutrinos $\nu_R \equiv \nu_L^c = -i\gamma^2\nu_L^*$ (in the chiral basis for the γ matrices) and use the properties of the γ matrices.
   (b) How far does a neutrino with a magnetic moment at the current limit $\mu_\nu \sim 10^{-12}\,\mu_B$ need to travel in the interstellar magnetic field ($B \sim \mathcal{O}(1\,\mu\mathrm{G})$) before its helicity is flipped? Discuss possible ways of detecting such magnetic moment-induced helicity flips of astrophysical neutrinos..

3. **Neutrinos in EFT**
   (a) Explain why the CC WEFT coefficient $\epsilon^{du,V}_{\alpha\beta}$ and the NC WEFT coefficient $\epsilon^{u,V}_{\alpha\beta}$ are not independent if we assume that SMEFT is the UV completion of WEFT.
   (b) Experiments searching for new physics in CC neutrino interactions are competing with strong constraints from searches for rare meson decays. We have seen in the lecture that neutrino experiments are particularly competitive in case of pseudoscalar dimension-6 operators. What is the reason for this?
   Hint: it is not neutrino processes that are enhanced, but SM meson decays that are suppressed.

4. **Neutrino Brain Teasers**
   (a) Imagine a world in which neutrinos are massive, but charged leptons are massless. Will neutrinos oscillate in such a world?
   (b) Do neutrinos produced in the decay $Z^0 \to \bar\nu\nu$ oscillate? If so, describe a gedankenexperiment in which these oscillations could be observed.

Cartoon by Jack Pairin, Harvard