Aspects of the modelling of GPDs at small Bjorken-x

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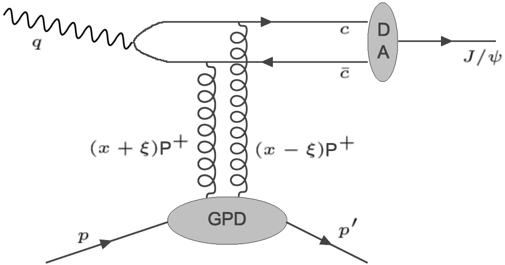
Vector meson production



LO depiction of J/ψ photoproduction. ξ is the **skewness** parameter measuring the transfer of plus-momentum to the hadron. x is the average plus-momentum of the active parton.

Transfer of four-momentum to the hadron $\rightarrow$ description in the framework of collinear factorization by **generalized parton distributions (GPDs)** and **non-relativistic QCD matrix element** for moderate or small photon virtuality $Q^2 = -q^2$. Hard scale provided by $m_V/2$ [Jones *et al*, 2015].

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2}, \quad t = (p' - p)^2$$

Vector meson production

- Vector meson production amplitude up to NLO [Ivanov *et al*, 2004]

$$\mathcal{F}(\xi, t) \propto \left(\frac{\langle O_1 \rangle_V}{m_V^3}\right)^{1/2} \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x \, T^a(x, \xi) \, F^a(x, \xi, t) \tag{1}$$

  where $\langle O_1 \rangle_V^{1/2}$ is the NR QCD matrix element, T a hard-scattering kernel and $F(x, \xi, t)$ is the GPD.
- The dominant region controlling the imaginary part of the amplitude is $x \gtrapprox \xi$.
- **Extracting the GPD from the convolution of Eq. (1) depends crucially on modelling choices on the GPD functional space $\rightarrow$ cf. deconvolution problem of DVCS [Bertone et al, 2021].**

Generalized parton distributions

Properties of GPDs [Müller *et al*, 1994], [Radyushkin, 1996] and [Ji, 1997]

- Depending on the helicity of the target and parton, several types of GPDs. Depending on the specific choice of observable, different sensitivity to various GPDs can be obtained. We focus on H in the following.
- The **forward limit** $t \to 0$ – and consequently $\xi \to 0$ – gives back the usual PDFs

$$H^q(x, \xi = 0, t = 0) = f^q(x) \tag{2}$$

$$H^g(x, \xi = 0, t = 0) = x f^g(x) \tag{3}$$

Since $\xi \sim 10^{-5}$ at LHCb, one could be tempted to write **GPD = PDF** at small ξ, such as

$$H^g(\xi, \xi = 10^{-5}) \approx H^g(\xi, 0) = \xi f^g(\xi) \tag{4}$$

But there is a problem...

Evolution of GPDs

GPD's dependence on scale is given by ξ **dependent renormalization group equations**. In the limit $\xi = 0$, we retrieve the usual DGLAP equation

$$\frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x,\mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu}\Bigg\{ \int_x^1 \mathrm{d}y\, \frac{f^{q+}(y,\mu) - f^{q+}(x,\mu)}{y-x}\left[1 + \frac{x^2}{y^2}\right] + f^{q+}(x,\mu)\left[\frac{1}{2} + x + \log\left(\frac{(1-x)^2}{x}\right)\right]\Bigg\} \tag{5}$$

But in the limit $x = \xi$, we obtain

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x,x,\mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu}\Bigg\{ \int_x^1 \mathrm{d}y\, \frac{H^{q+}(y,x,\mu) - H^{q+}(x,x,\mu)}{y-x} + H^{q+}(x,x,\mu)\left[\frac{3}{2} + \log\left(\frac{1-x}{2x}\right)\right]\Bigg\} \tag{6}$$

Assuming that GPD = PDF at small ξ and $x \approx \xi$ is incompatible with evolution, which generates an intrinsic ξ dependence!

Evolution of GPDs

- Conformal moments of GPDs are defined as

$$O_n^q(\xi,\mu) = \xi^n \int_{-1}^{1} \mathrm{d}x\, C_n^{3/2}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu) \tag{7}$$

  where $C_n^{3/2}$ are Gegenbauer polynomials of degree n. Under LO evolution, they evolve without mixing with one another (except for the quark - gluon mixing for n odd):

$$O_n^q(\xi,\mu) = O_n^q(\xi,\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/2\beta_0} \tag{8}$$

  where γ_n **is the same anomalous dimension as the one governing in the evolution of Mellin moments of the PDF**.
- The limit $\xi \to 0$ of GPD conformal moments give back PDF Mellin moments.

The Shuvaev transform

- The Shuvaev transform, noted by S in the following, (arXiv:9902410) allows to reconstruct a GPD from the knowledge of its conformal moments :

$$H^a(x, \xi, \mu) = S^a(x, \xi, n) \star O_n^a(\xi, \mu) \tag{9}$$

  Assume conformal moments are chosen to be independent of ξ at some scale. Then they will remain so under evolution since anomalous dimensions are independent of ξ. But the ξ dependence of the GPD varies under evolution. **So the ξ dependence of the Shuvaev transform contains an information on the ξ dependence introduced by evolution. How does it materialize?**

- Let's write the LO GPD evolution equation as

$$\frac{\mathrm{d}H^g(x, \xi, \mu)}{\mathrm{d}\mu} = \alpha_s(\mu) \sum_a \int_x^1 \mathrm{d}z \, K^{ga}(x, \xi, z) \, H^a(z, \xi, \mu) \equiv \sum_a K^{ga} \otimes H^a \tag{10}$$

The Shuvaev transform

- Then

$$H^g(x,\xi,\mu) = \sum_a \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{K^{ag}/2\beta_0} \otimes H^a(z,\xi,\mu_0) \tag{11}$$

$$\equiv \sum_a \Gamma^{ga}(x,\xi,z) \otimes H^a(z,\xi,\mu_0) \tag{12}$$

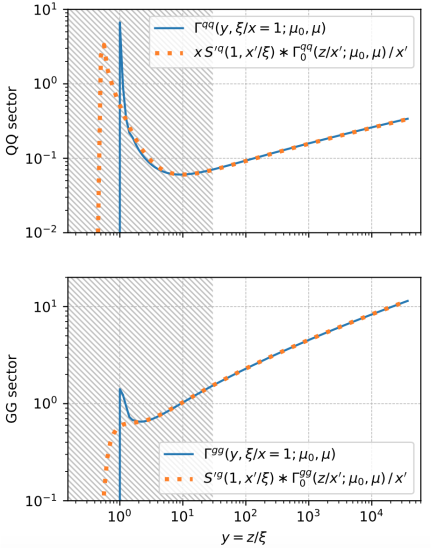
  where, since K is a distribution, α^K must be understood as $\exp(K\log(\alpha)) = \sum \log^n(\alpha)K^n/n!$ and K^n designates K convoluted with itself n times.
- **K is the splitting function appearing in the differential equation, and Γ the integrated evolution operator (essentially a convoluted exponential of K)**. It is remarkable that although K is a distribution, Γ is in practice an ordinary function. **It can be interpreted as a reweighting applied to the GPD at initial scale in order to form the GPD at final scale.**

The Shuvaev transform

- The GPD evolution code APFEL++ [Bertone *et al*, 2014, 2022] gives a direct access to the values of Γ, and allow to elucidate the link between the Shuvaev transform and the GPD evolution operator :

$$\Gamma^{ga}(x,\xi,z) \approx S^{g}(x,\xi,n) \star \mathcal{M}(n,y) \star \Gamma^{ga}(y,0,z) \tag{13}$$

The ξ dependent GPD evolution operator is approximately given by its limit for $\xi = 0$ (DGLAP evolution operator) to which are successively applied a Mellin transform and the Shuvaev transform. The approximation is excellent as soon as $z > 3\xi$.



The Shuvaev transform

- The reconstruction of ξ dependence introduced by evolution thanks to the Shuvaev transform is excellent when $z \gg \xi$. This is also the region where **safe to assume that the GPD and the PDF are equal at initial scale**. Combining the two hypothesis gives the actual Shuvaev modelling of small ξ GPD :

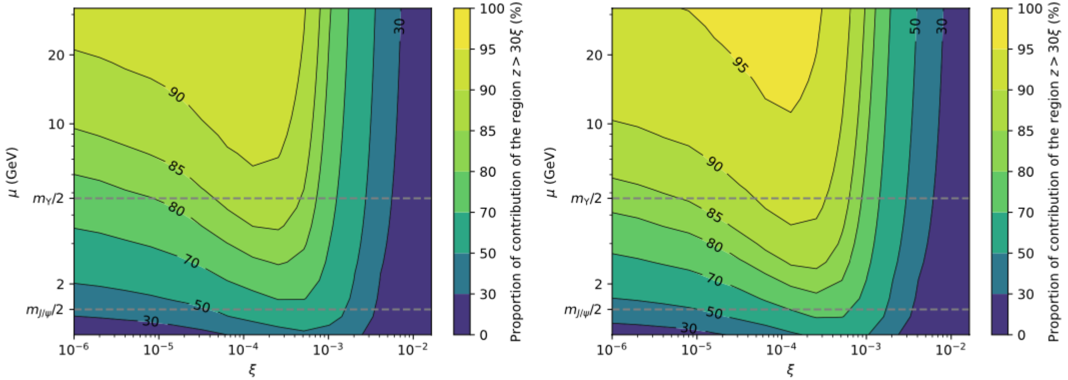
$$H^g(x, \xi, \mu) = S^g(x, \xi, n) \star \mathcal{M}(n, z) \star f^g(z, \mu) \tag{14}$$

  The GPD is constructed directly from the Shuvaev transform applied to the Mellin moments of the PDF, or in other words **the conformal moments of the GPD are assumed to be simply independent of** ξ.
- It now appears that this procedure is sound if it is possible to find an initial scale such that the region $z \gg \xi$ dominates in practice the evolution to the final scale $m_V/2$. **Then with an assumption of minimal regularity in ξ of the GPD at initial scale, we may assume that all the ξ dependence at final scale is purely generated by evolution**.

Uncertainty of the Shuvaev transform

- Any reference to this low lying scale has disappeared in the final formulation of the Shuvaev proposal, because it is equivalent to the assumption that conformal moments are ξ independent, which is a scale-independent statement (at LO)!
- But to characterize the uncertainty associated to the reconstruction procedure, it is useful to redefine this low lying scale μ_0 and measuring to what extent evolution from this scale to $m_V/2$ is indeed controlled by $z \gg \xi$.
- How to define μ_0? One should always take it as small as possible, as the larger the evolution range, the larger the dominance of the $z \gg \xi$ region. But the smaller μ_0, the larger the MHO corrections. All in all, choosing $\mu_0 = 1$ GeV seems a good compromise.

Uncertainty of the Shuvaev transform



Using the MMHT 2014 LO gluon PDF at $\mu_0 = 1$ GeV, and measuring for $x = \xi$ (left) and $x = 2\xi$ (right) the share of the region $z > 30\xi$ in the evolution of the GPD for various values of ξ and μ.

Uncertainty of the Shuvaev transform

- The higher the hard scale, the better!
- ξ has to be small enough for a region $z \gg \xi$ to even exist. When ξ gets very small, the GPD becomes steeper at small x, so the share of the region $z > 30\xi$ starts to weaken $\rightarrow$ **for the large momentum fraction at initial scale to effectively control evolution at small x at final scale, the GPD must no be too steep at small x!**
- Conservative estimate : this essentially assumes that at initial scale, the GPD at $\xi = 10^{-5}$ differs from the PDF by 100% for $z \in [\xi, 30\xi]$. In reality, we are probably closer to 10% – 20%.
- **Uncertainty for bottomonium of the order of a few percents, for charmonium of the order of 10%.**

An alternative procedure

How to use this rough estimate of systematic uncertainty in the extrapolation to $\xi = 0$?
Straightforward way : add the uncertainty to the PDF extraction performed thanks to the Shuvaev method.
More sophisticated way :

- Get a set of replicas representing your current knowledge of the PDF at μ_0,
- Compute your replicas to the actual evolution (not the Shuvaev approximate) at ξ and $m_V/2$,
- Reweight those replicas by assessing their compatibility with the experimental measurements (does not require a deconvolution procedure!). Take into account in the reweighting phase the systematic uncertainty computed before hand,
- Compute the effect of the reweighting on the PDF at scale $m_V/2$.

Conclusion

- The Shuvaev procedure to relate the GPD to the PDF relies on the fact that the region of large x at some initial scale μ_0 controls effectively the the region of small x at the hard scale $m_V/2$. This allows to consider that most of the ξ dependence at $m_V/2$ arises from the evolution.
- We propose to determine a systematic uncertainty associated to this procedure by measuring effectively the dominance of the large x region at scale 1 GeV using the best current knowledge of PDFs.
- Our alternative proposal does not require the technical implementation of the cumbersome Shuvaev integrals, and is adapted to assessing the impact of exclusive vector meson production on existing PDF fits.
- The method is also perfectly adaptable to higher orders, although Shuvaev's method is rooted in the LO properties of conformal moments, and by-passes the potential issue of deconvoluting the experimental