

HF2022: Heavy Flavours from small to large systems

Implications of exclusive quarkonia photoproduction in a tamed collinear factorisation to NLO

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HF2022

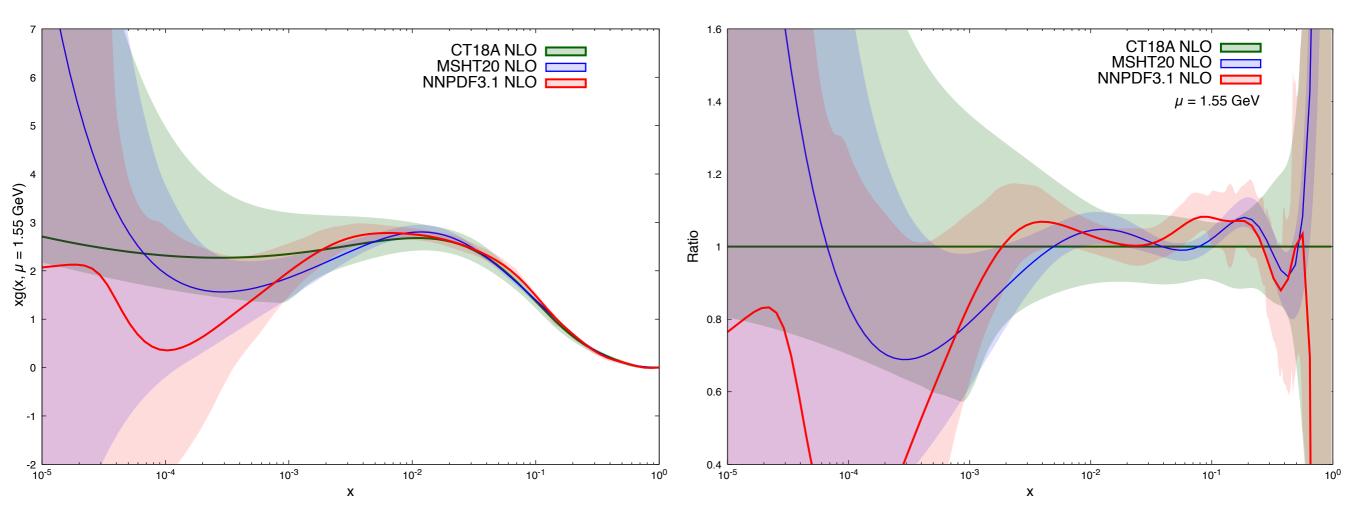




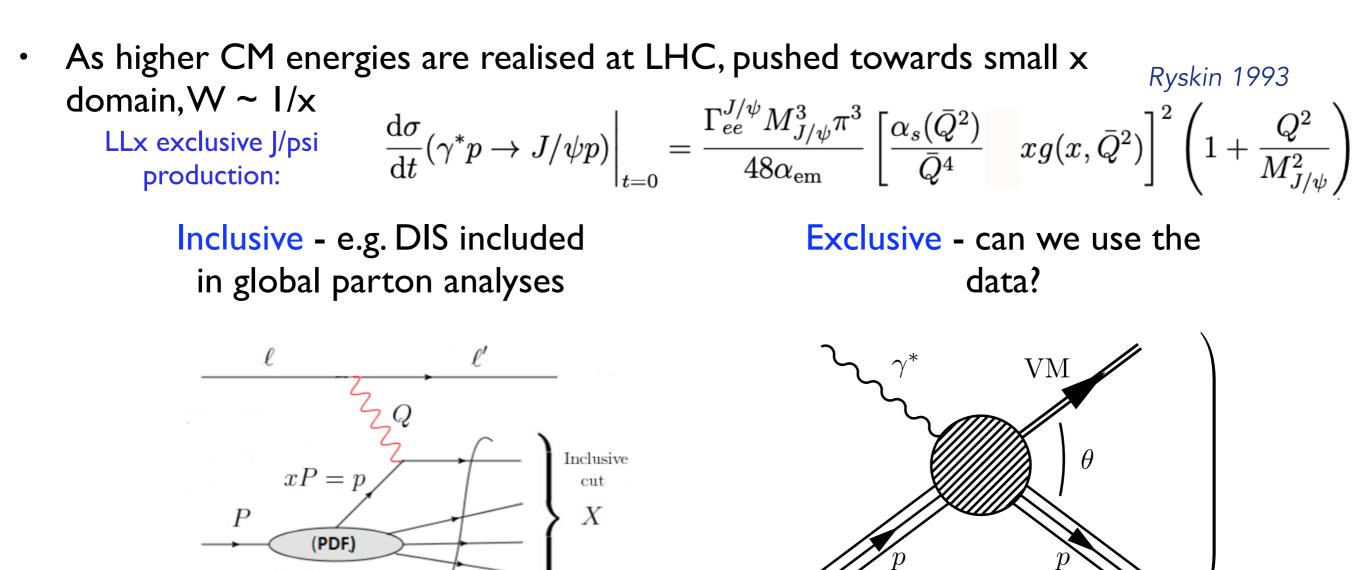


07/10/22

- Inclusive processes do not well constrain small x/Regge limit domain of PDFs
- Exclusive processes offer sensitive probe of this domain but as of yet not included in global analyses PDF determination - why?
 - I. Off forward kinematics imply sensitivity to GPD over conventional PDFs
 - 2. Scale dependence and stability of theoretical predictions

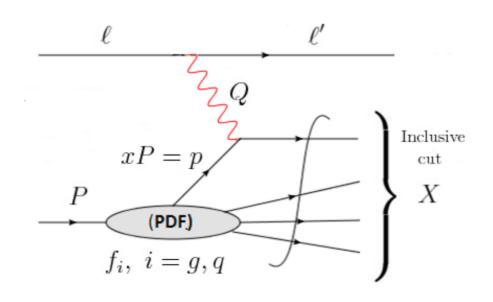


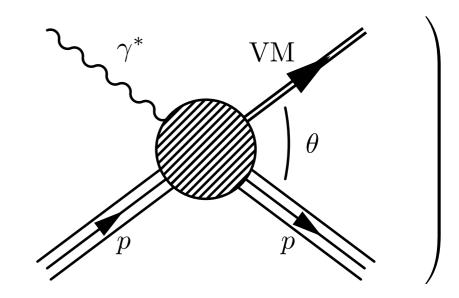
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 $f_i, i = g, q$

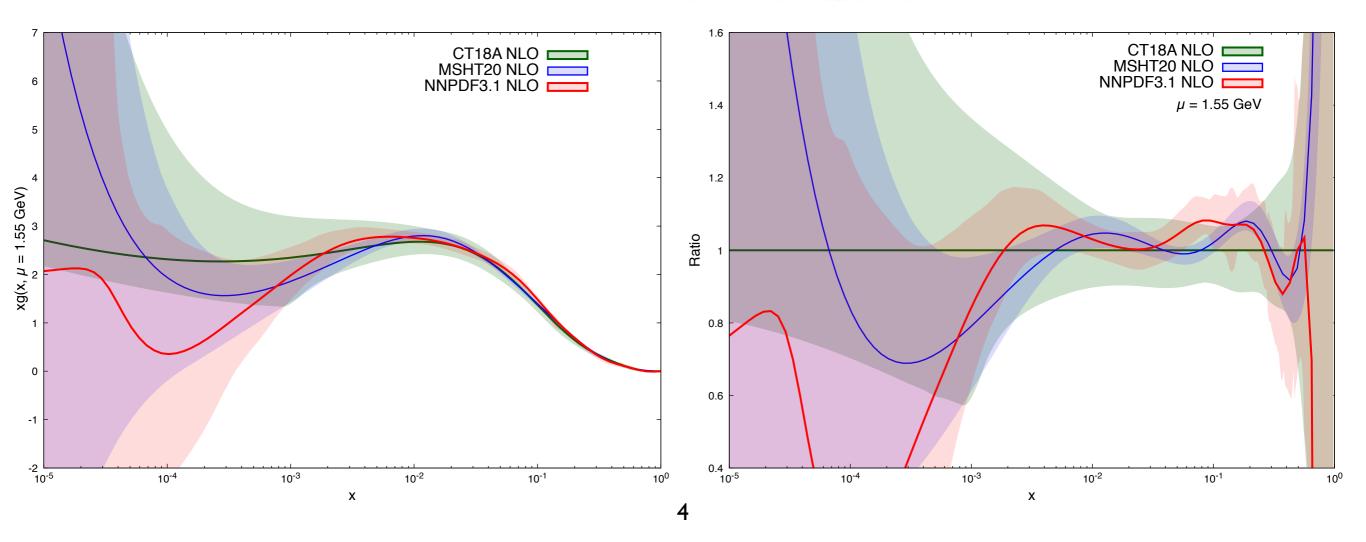
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- As higher CM energies are realised at LHC, pushed towards small x domain, W ~ I/x DLLA exclusive J/psi $\frac{d\sigma}{dt}(\gamma^* p \rightarrow J/\psi p)\Big|_{t=0} = \frac{\Gamma_{ee}^{J/\psi}M_{J/\psi}^3\pi^3}{48\alpha_{em}} \left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4}R_g xg(x,\bar{Q}^2)\right]^2 \left(1 + \frac{Q^2}{M_{J/\psi}^2}\right)$ Inclusive - e.g. DIS included in global parton analyses Exclusive - can we use the data?





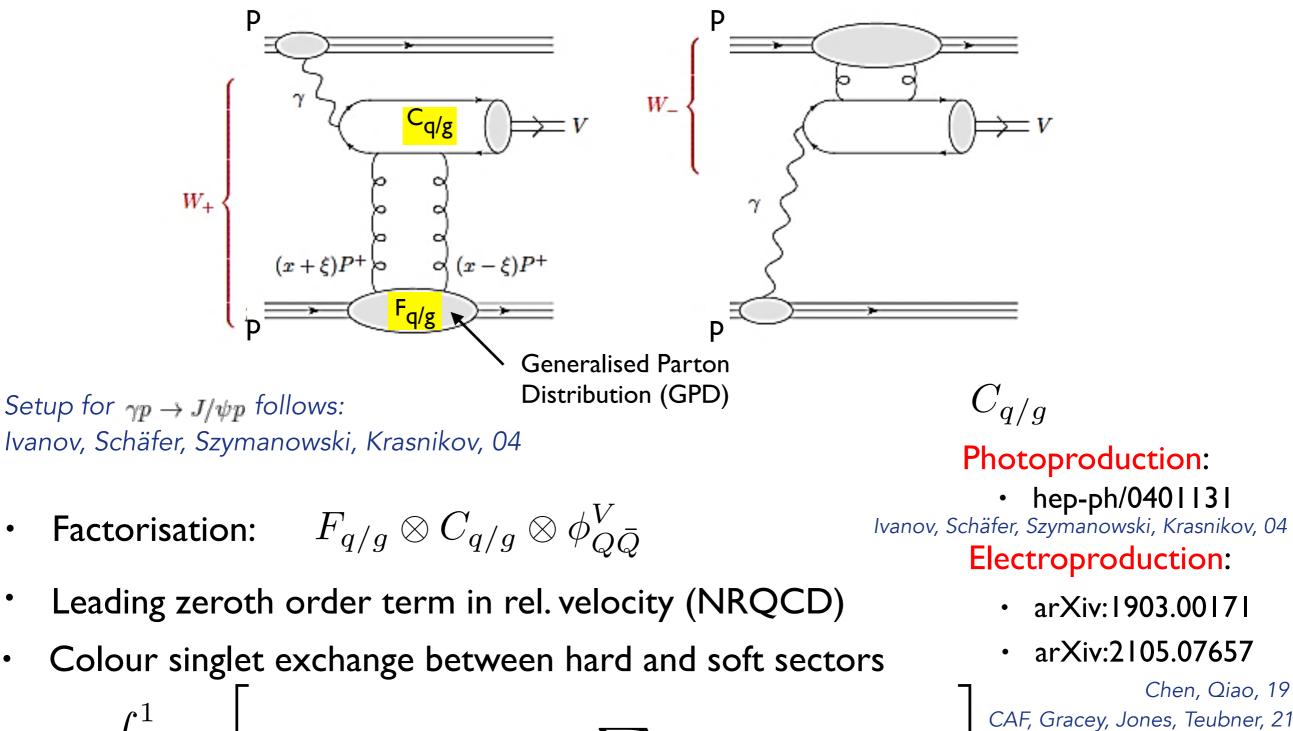
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This talk: how to counteract these problems and so allow exclusive J/psi data to probe gluon PDF down to $x\sim 3 imes 10^{-6}~\&~\mu=O(M_{J/\psi}/2)$



General Set up and Framework

Exclusive J/psi photoproduction in p+p UPC collisions in collinear factorisation



$$A \propto \int_{-1}^{1} \mathrm{d}x \left[C_g(x,\xi) F_g(x,\xi) + \sum_{q=u,d,s} C_q(x,\xi) F_q(x,\xi) \right]$$

5

General Set up and Framework

ccbar->J/psi:

• Effective field theory for production of heavy quarkonium [Bodwin et al. 1995]

$$\sigma_V = \sigma_{q\bar{q}} \cdot \langle O \rangle_V$$

• Relativistic corrections systematically computed by expanding matrix elements in powers of *r*:

$$\mathcal{M}[J/\psi] \propto (\mathcal{A}_{
ho} + \mathcal{B}_{
ho\sigma}r^{\sigma} + \mathcal{C}_{
ho\sigma\tau}r^{\sigma}r^{\tau} + \ldots)\epsilon^{
ho}_{J/\psi}$$

matrix elements $\epsilon^{
ho}_{J/\psi}$ - J/ψ polarization $r^{\mu} = q_1^{\mu} - q_2^{\mu}$

• We will compute to leading order in relative quark velocity v, for J/ψ :

$$\mathcal{M}[J/\psi] = \left(\frac{\langle O_1 \rangle_{J/\psi}}{2N_c m_C}\right)^{\frac{1}{2}} \mathcal{A}_{\rho} \epsilon^{\rho}_{J/\psi} \qquad \qquad \langle O_1 \rangle_{J/\psi} \equiv \langle O_1(^3S_1) \rangle_{J/\psi} \\ \mathcal{O}_1(^3S_1) = \psi^{\dagger} \boldsymbol{\sigma} \chi \cdot \chi^{\dagger} \boldsymbol{\sigma} \psi$$

• Extract
$$\langle O_1 \rangle_{J/\psi}$$
 from measurement of Γ_{ee}

 $\langle O_1 \rangle_V = \frac{N_c}{2\pi} |R_S(0)|^2 + \mathcal{O}(v^2)$

Leading zeroth order term in rel. velocity (NRQCD)

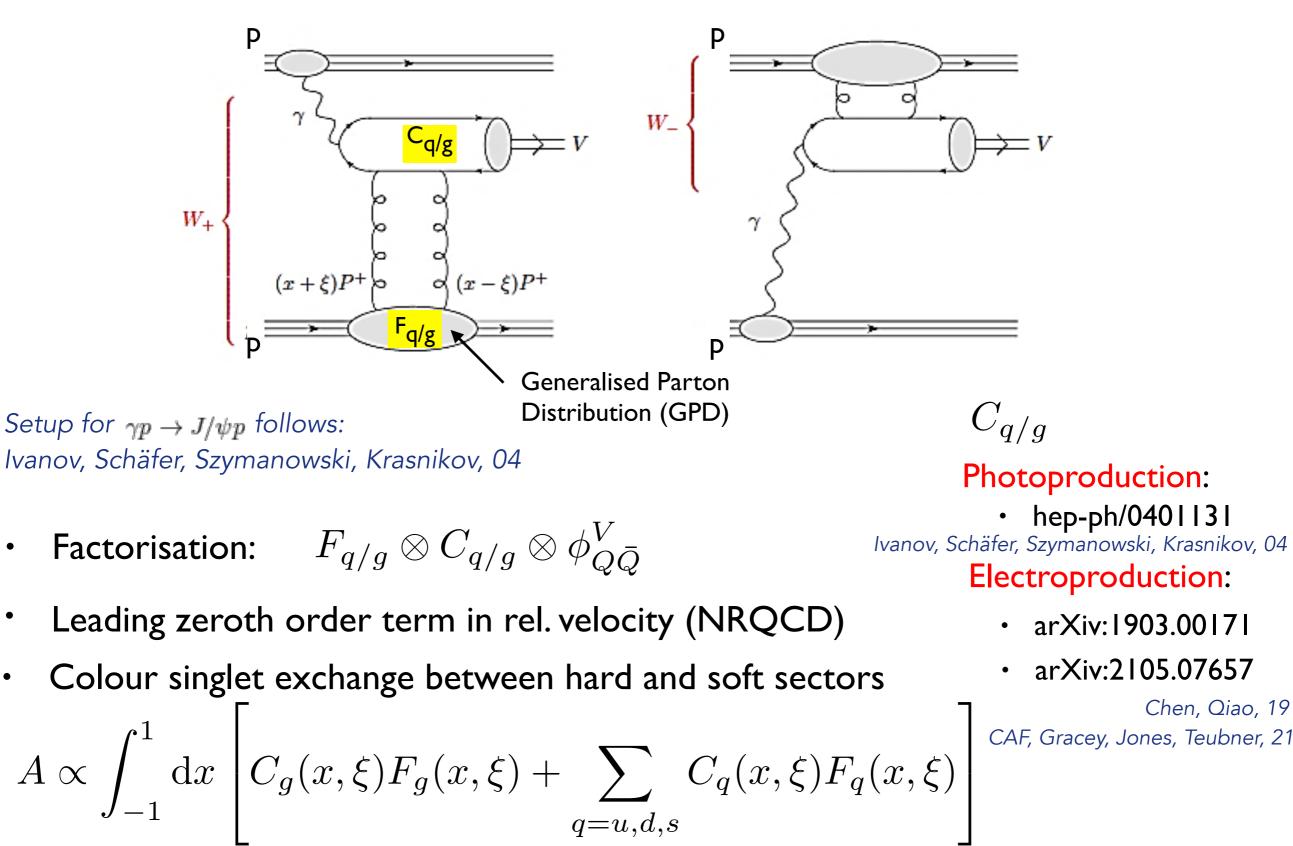
 $\mathcal{A}, \mathcal{B}, \mathcal{C}$ -

 First non-vanishing O(v^2) relativistic correction small AFTER additional ccbar+gg Fock state component considered for gauge invariance

O(6%) cross section correction factor proportional to derivative of square of J/psi w.f. at origin (and affecting normalisation only and not energy dependence)

General Set up and Framework

Exclusive J/psi photoproduction in p+p UPC collisions in collinear factorisation



GPDs and the Shuvaev transform

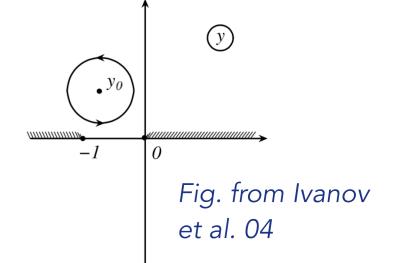
GPDs generalise PDFs: outgoing/incoming partons carry different momentum

fractions

 $\langle P' | \overline{\psi}_q(y) \mathcal{P}\{\} \psi_q(0) | P \rangle$

Müller 94; Radyushkin 97; Ji 97

 $x + \xi$ $\mathcal{H}_q(x, \xi, t)$ $x - \xi$ $\mathcal{H}_q(x, \xi, t)$ $x - \xi$ $\mathcal{H}_q(x, \xi, t)$ \mathcal{H} physically motivated assumptions c.f analyticity



Shuvaev 99 Martin et al. 09

Idea: Conformal moments of GPDs = Mellin moments of PDFs

(up to corrections of $O(xi^2)$ @ LO and O(xi) @ NLO)

- Construct GPD grids in multidimensional parameter space x, xi/x, qsq with forward • PDFs from LHAPDF
- Costly computationally due to slowly converging double integral transform
- Regge theory considerations => Shuvaev transform valid in space-like (DGLAP) region only. In time-like (ERBL) region imaginary part of coefficient function is zero

Shuvaev Transform

Full Transform:

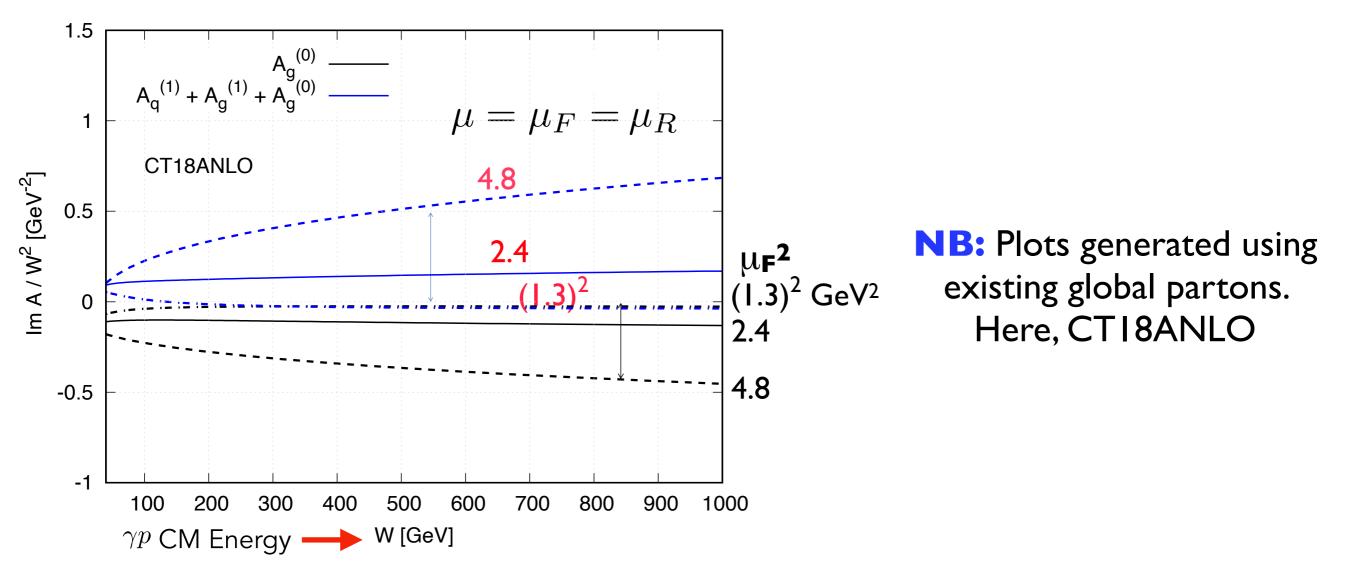
$$\begin{aligned} \mathcal{H}_{q}(x,\xi) &= \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi} \mathrm{Im} \int_{0}^{1} \frac{\mathrm{d}s}{y(s)\sqrt{1-y(s)x'}} \right] \frac{\mathrm{d}}{\mathrm{d}x'} \left(\frac{q(x')}{|x'|} \right), \\ \mathcal{H}_{g}(x,\xi) &= \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi} \mathrm{Im} \int_{0}^{1} \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\sqrt{1-y(s)x'}} \right] \frac{\mathrm{d}}{\mathrm{d}x'} \left(\frac{g(x')}{|x'|} \right), \\ y(s) &= \frac{4s(1-s)}{x+\xi(1-2s)}. \end{aligned}$$
[Shuvaev et. al 1999]

Stability of prediction I

opp. sign

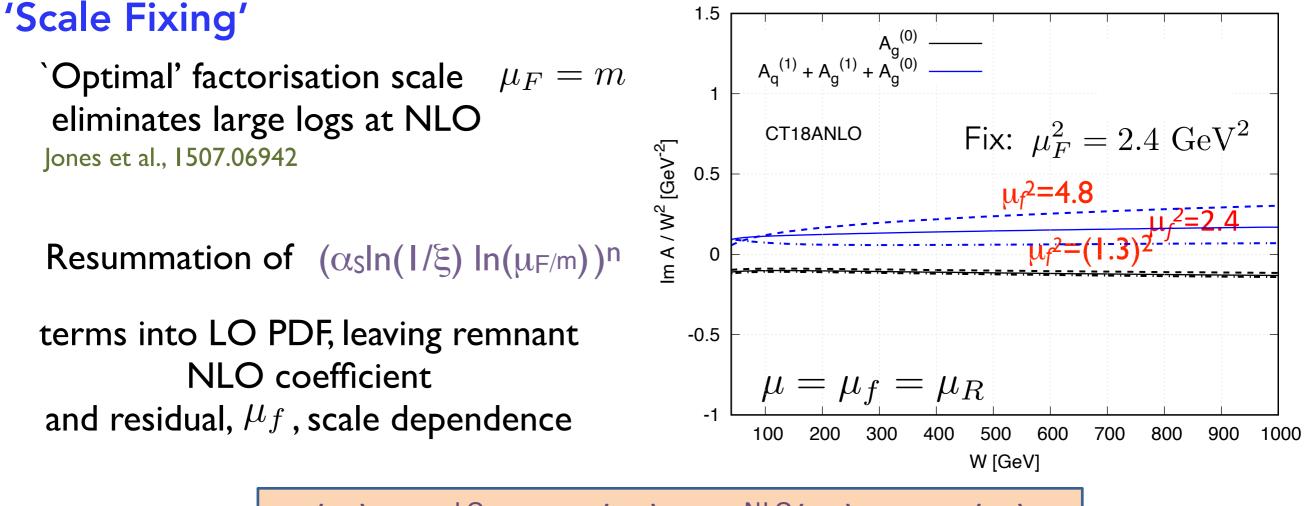
NLO in MSbar scheme D. Ivanov, et al., hep-ph/0401131

- A. Bad perturbative convergence |NLO_{correctn.}| > |LO| and
- B. Strong dependence on scale μ_F



Can do better... resummation of logarithmically enhanced contributions at low x!

Stability of prediction II



$$A(\mu_f) = C^{LO} \times GPD(\mu_F) + C^{NLO}(\mu_F) \times GPD(\mu_f)$$

Look for another sizeable correction that can reduce variations further -> implementation of a `Q0' cut

Stability of prediction II

'Scale Fixing'

`Optimal' factorisation scale $\mu_F = m$ eliminates large logs at NLO Jones et al., 1507.06942

Resummation of $(\alpha_{sln}(1/\xi) \ln(\mu_{F/m}))^n$

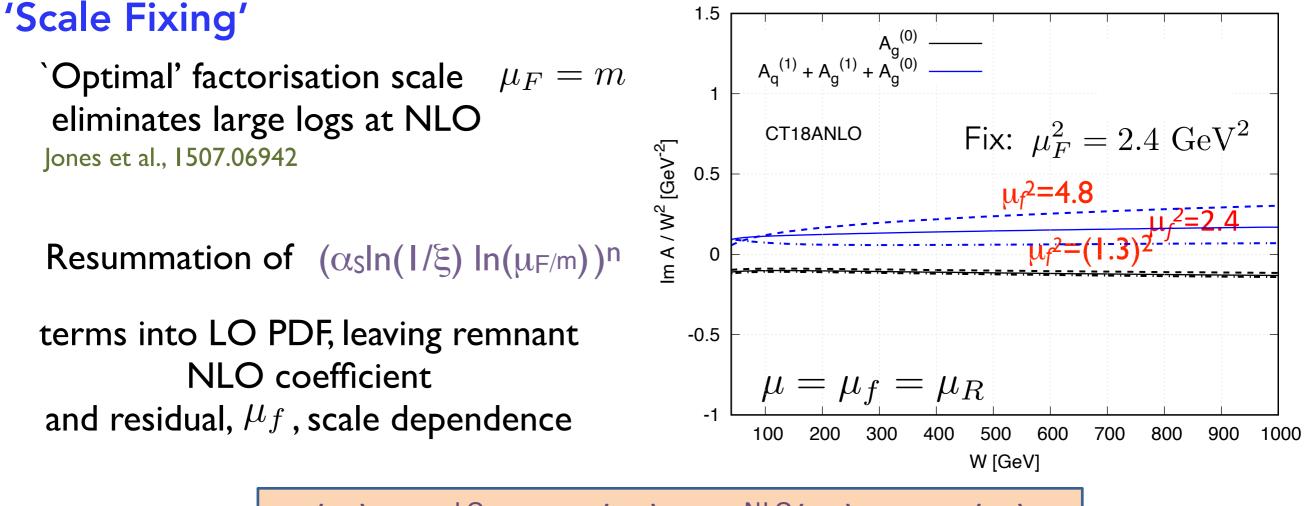
terms into LO PDF, leaving remnant NLO coefficient and residual, μ_f , scale dependence

$$\begin{split} & \mathsf{NLO High-energy limit:} \\ & \mathcal{M} \approx \frac{-4\,i\,\pi^2 \sqrt{4\pi\alpha}\,e_q(e_V^*e_\gamma)}{N_c\,\xi} \left(\frac{\langle O_1 \rangle_V}{m^3}\right)^{1/2} \times \\ & \times \left[\alpha_S(\mu_R)F^g(\xi,\xi,t) + \frac{\alpha_S^2(\mu_R)N_c}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_{\xi}^1 \frac{dx}{x}F^g(x,\xi,t) \right. \\ & \left. + \frac{\alpha_S^2(\mu_R)C_F}{\pi}\ln\left(\frac{m^2}{\mu_F^2}\right)\int_{\xi}^1 dx\left(F^{q,S}(x,\xi,t) - F^{q,S}(-x,\xi,t)\right) \right] \end{split}$$

 $A(\mu_f) = C^{LO} \times GPD(\mu_F) + C^{NLO}(\mu_F) \times GPD(\mu_f)$

Look for another sizeable correction that can reduce variations further -> implementation of a `Q0' cut

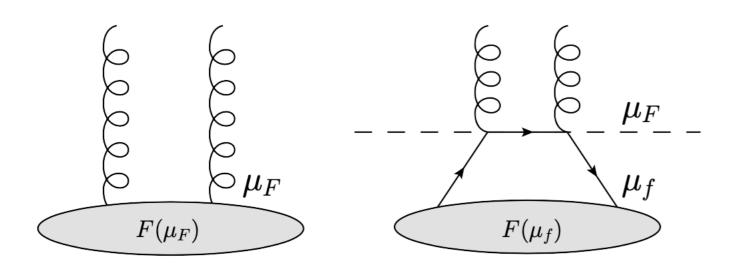
Stability of prediction II



$$A(\mu_f) = C^{LO} \times GPD(\mu_F) + C^{NLO}(\mu_F) \times GPD(\mu_f)$$

Look for another sizeable correction that can reduce variations further -> implementation of a `Q0' cut

Treatment of double logarithmic contribution



Ideology: Use scale shifting to find optimal scale that removes the largest contribution from the NLO correction *

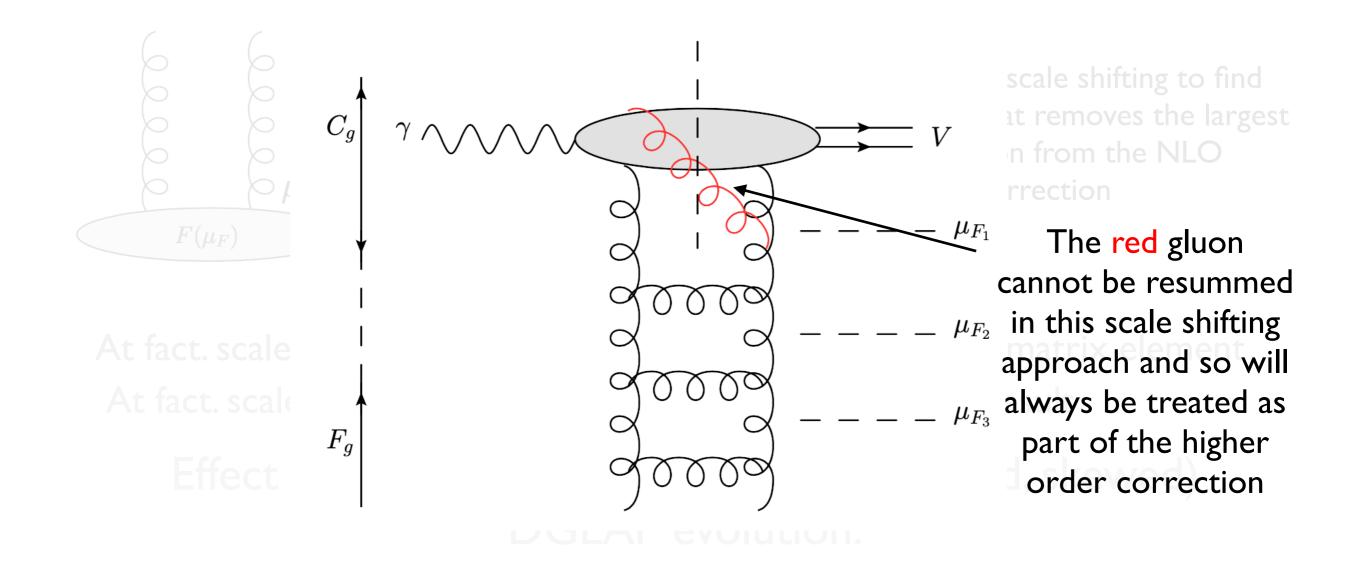
At fact. scale. μ_f , quark contribution is part of NLO hard matrix element At fact. scale μ_F , absorbed quark contribution into LO result

Effect of scale change driven by (generalised, skewed) DGLAP evolution:

$$A^{(0)}(\mu_f) = \left(C^{(0)} + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_f^2}{\mu_F^2}\right) C^{(0)} \otimes V\right) \otimes F(\mu_F)$$

* At small xi, this is the double logarithmic contribution $\sim \ln(1/x_i) \ln(m_e^2/m_e^2)$

Treatment of double logarithmic contribution

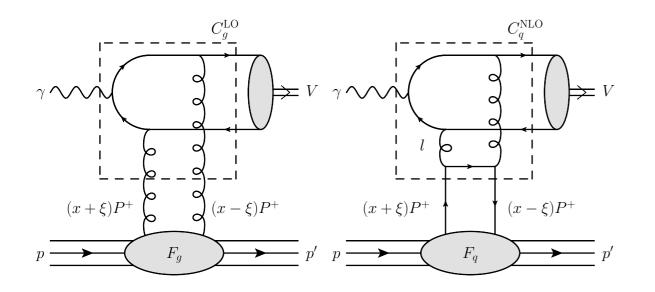


Choice muF = mc 'resums' the gluon ladder contributions, enhanced by this double logarithmic contribution. They are intrinsically resummed within the kt factorisation framework * and here by judicious choice of factorisation scale

^{*} But kt fact. framework treats only a subset of NLO corrections, those belonging to equivalence class of gluon-ladder diagrams

Stability of prediction III

Q0' cut Jones et al., 1610.02272



Fundamentally ubiquitous* and typically power suppressed, but sizeable here

$\mathcal{O}(Q_0^2/\mu_F^2)$

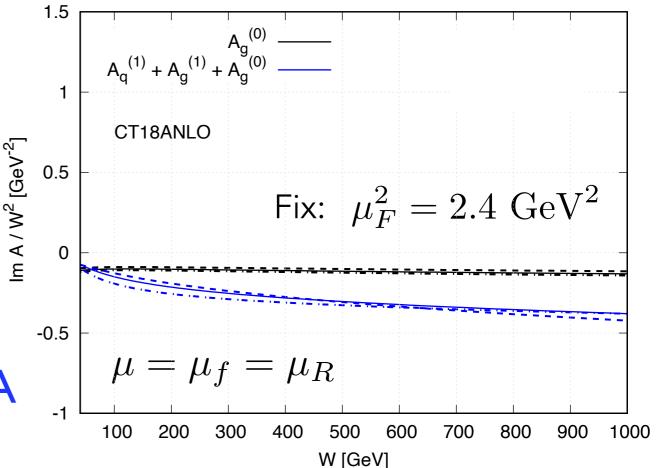
How do these predictions compare with the data at HERA and LHCb?

*see 1912.09304 for procedure applied to inclusive DIS and Drell-Yan production

Subtract DGLAP contribution

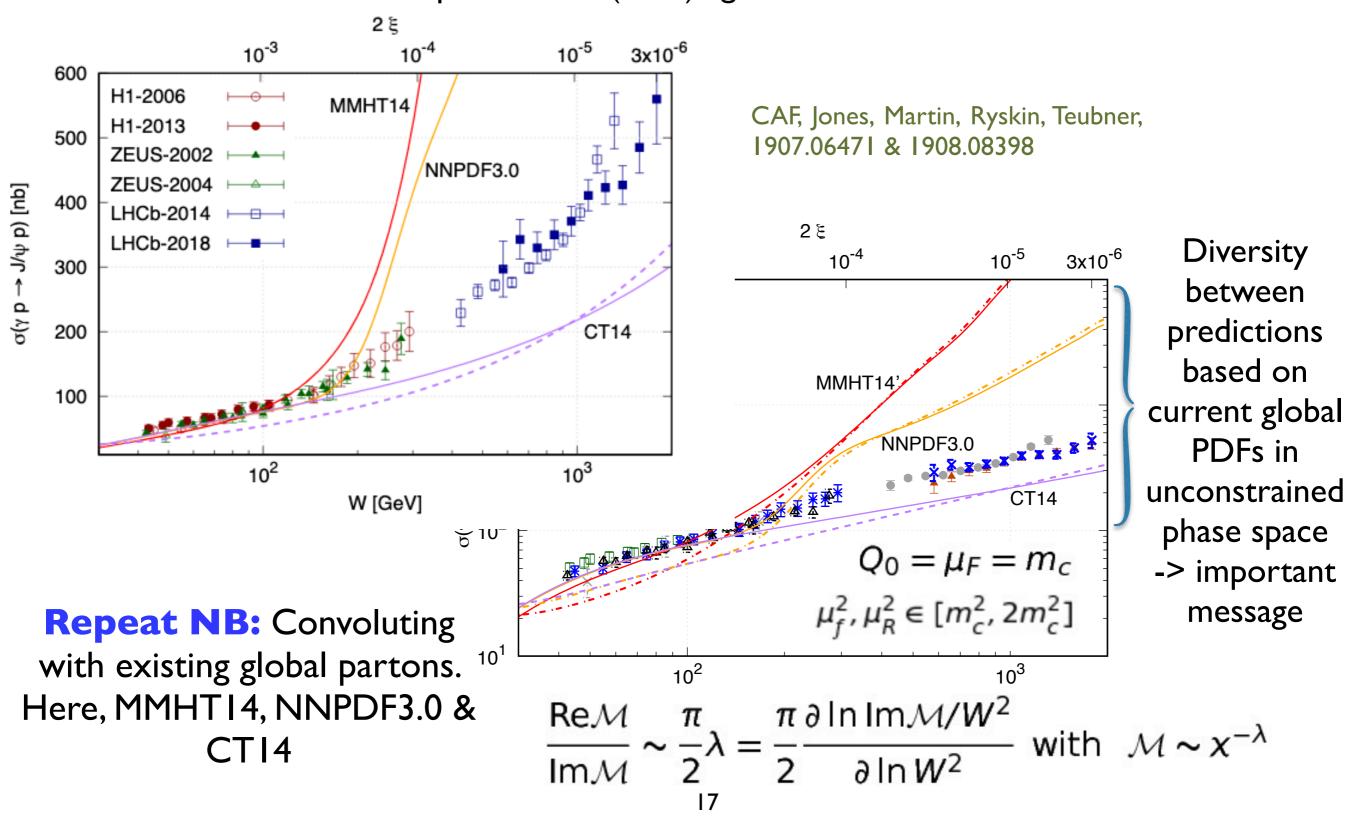
NLO ($|\ell^2| < Q_0^2$)

from known NLO MSbar coefficient function to avoid a double count with input GPD at Q_0 .



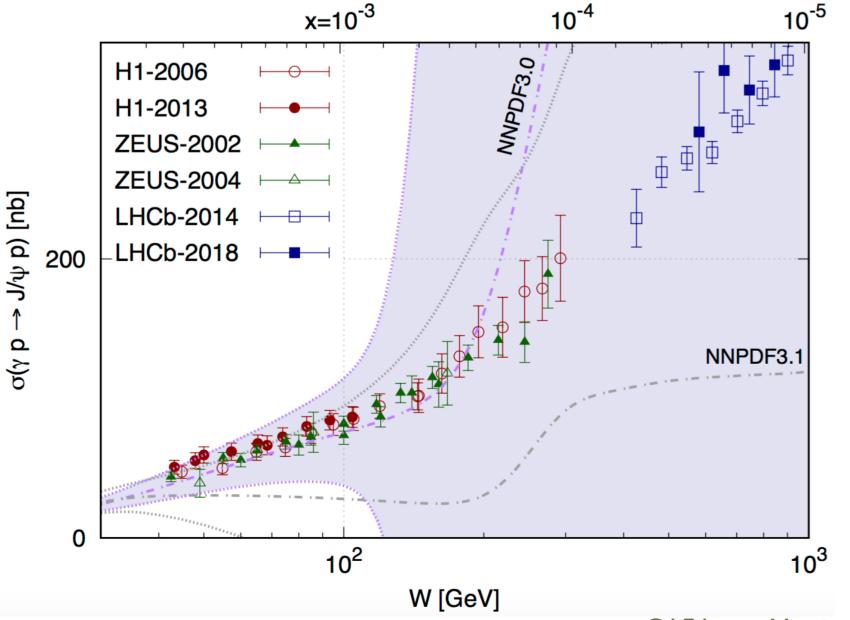
Towards the bigger picture

Plots demonstrates good scale stability of our NLO predictions in LHCb regime Predictions at optimal scale (solid) agree better with HERA data



Error budgets: errors due to parameter variations in global fits >> experimental uncertainty and scale variations in the theoretical result

..... exclusive data now in a position to readily improve global analyses



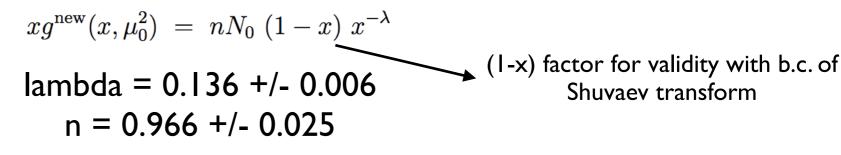
Exclusive LHCb data will

constrain small x growth whilst *exclusive* HERA data will improve determination of partons in regime with data constraints already from diffractive DIS HERA data

Extraction of low x gluon PDF via exclusive J/psi

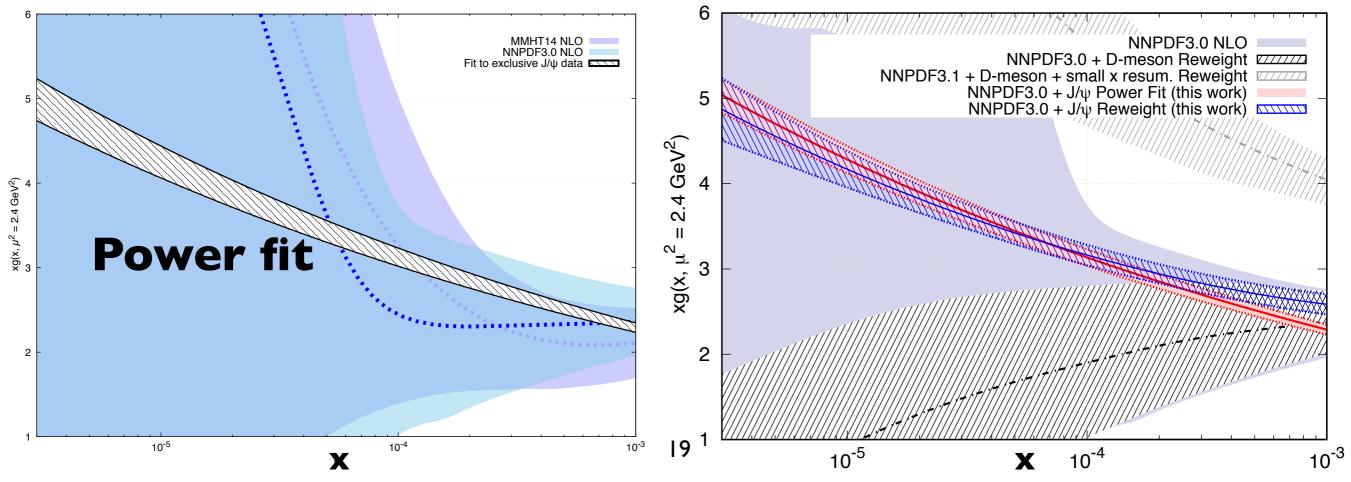
LeftApproach I:Fit a low x gluon PDF ansatz to the dataRightApproach 2:Bayesian reweight current global PDF analyses

	λ	n	$\chi^2_{ m min}$	$\chi^2_{ m min}/{ m d.o.f}$
NNPDF3.0	0.136	0.966	44.51	1.04
MMHT14	0.136	1.082	47.00	1.09
CT14	0.132	0.946	48.25	1.12



CAF, Martin, Ryskin, Teubner, 2006. I 3857





Extraction of low x gluon PDF via exclusive J/psi

Left

Reweighted gluon PDF extractions via exclusive J/psi data and inclusive D meson production differ:

Experimental inconsistencies in measurement of inclusive D meson production (?) (rapidity detection efficiency and self inconsistency with inclusive B meson detection),

Oliveira, Martin, Ryskin, 1712.06834

etac hadroproduction (conventional inclusive mode) favours harder gluon than that obtained from inclusive D meson production,

X

Lansberg, Ozcelik, 2012.00702

on PDF ansatz to the data

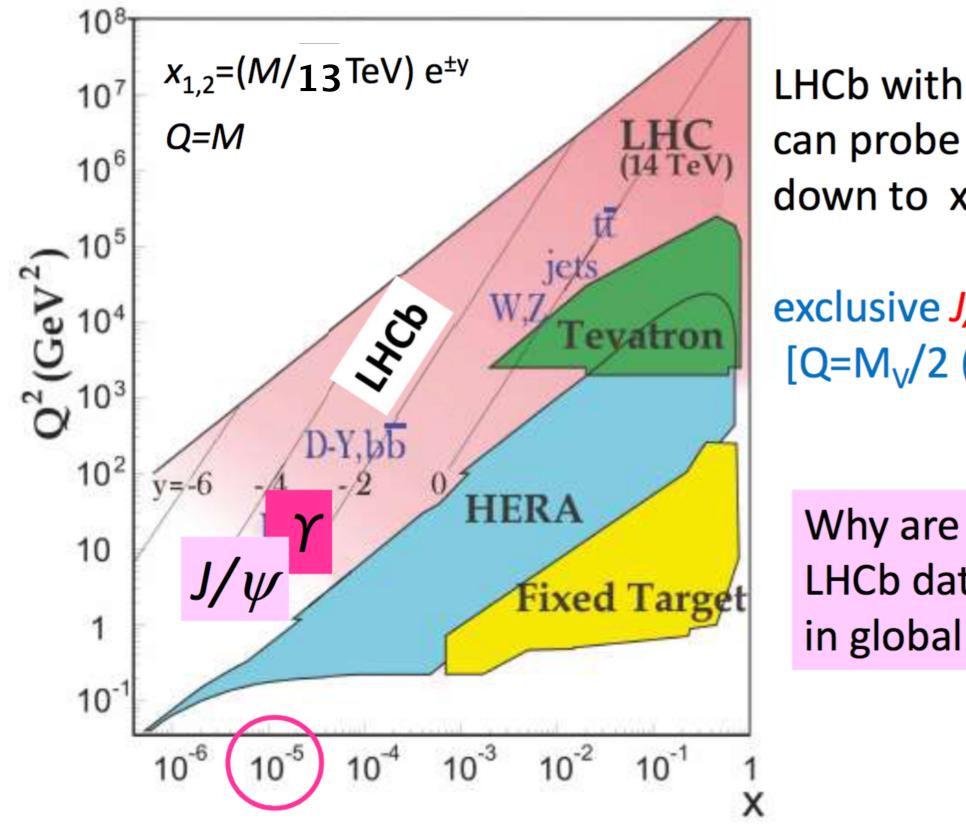
eight current global PDF analyses $= nN_0 (1-x) x^{-\lambda}$ = 0.136 + - 0.0060.966 + / - 0.025 $N_{\rm eff} \ll N_{\rm rep}$ 6 NNPDF3.0 NLO NNPDF3.0 + D-meson Reweight NNPDF3.1 + D-meson + small x resum. Reweight NNPDF3.0 + J/ ψ Power Fit (this work) 5 NNPDF3.0 + J/ $\dot{\psi}$ Reweight (this work) 2.4 GeV²) П xg(x, μ² : 3 2 20 10⁻³ 10⁻⁵ 10^{-4} Х

Summary

- Conventional MSbar NLO coll. fact. result unreliable and unstable
- Systematic taming via 'Q0' cut and resummation of large logarithmic contributions collectively reduce wild scale variations
- Predictions at cross section level have a good stability and central values in agreement of data within I sigma error bands
- Large difference between predictions based on global PDFs in LHCb regime
- Reconciliation at HERA energies -> motivated a low x and low scale gluon
 PDF extraction via two approaches and shown to be consistent
- Upshot: In a position to finally use exclusive J/psi data in a global fitter framework. Reweighting and profiling fit exercises in progress within the xFitter framework...

Thank you

Kinematic coverage

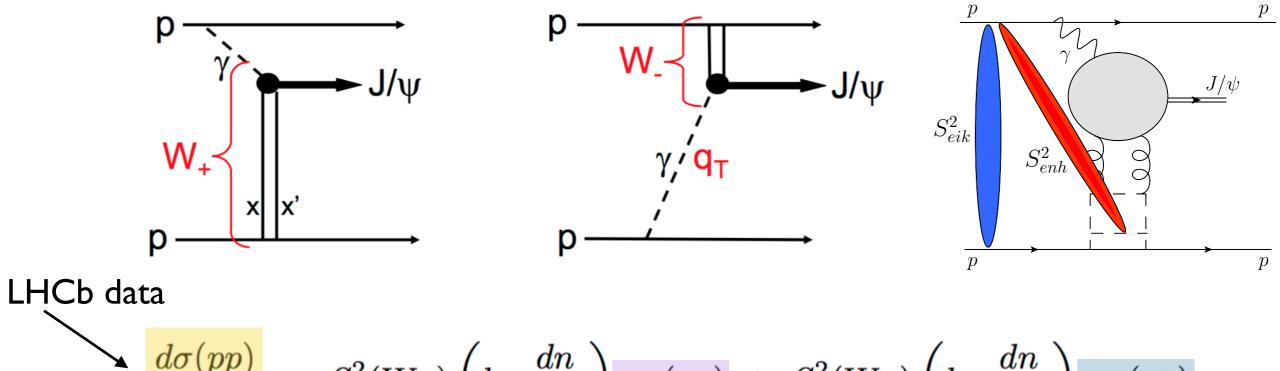


LHCb with *2* < *y* < *4.5* can probe gluon down to $x \sim 10^{-5}$

exclusive J/ψ , Y $[Q=M_{v}/2 (scale)]$

Why are these LHCb data not used in global PDF fits ??

General Set up and assumptions



$$\frac{d\sigma(pp)}{dy} = S^{2}(W_{+}) \left(k_{+} \frac{dn}{dk_{+}}\right) \sigma_{+}(\gamma p) + S^{2}(W_{-}) \left(k_{-} \frac{dn}{dk_{-}}\right) \sigma_{-}(\gamma p)$$
survival probability photon flux factors LHCb 'data' HERA gives W-

$$W_{\pm}^2 = M_{J/\psi} \sqrt{s} e^{\pm |y|} \Rightarrow x_{\pm} = \begin{cases} 10^{-5} \\ 0.02 \end{cases}$$
 at $y = 4, \sqrt{s} = 13$ TeV

Shuvaev Transform cont.

The conformal moments H_i^N of the GPDs are given by

$$H_{i}^{N} \equiv \int_{-1}^{1} \mathrm{d}x R_{N,i}(x_{1}, x_{2}) H_{i}(x, \xi),$$
 $i = q, g,$ Ohrndorf, 82

The conformal moments are polynomials in even powers of ξ ,

$$H_i^N = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} c_{k,i}^N \xi^{2k} = c_{0,i}^N + c_{1,i}^N \xi^2 + c_{2,i}^N \xi^4 + \dots, \quad , \ c_{0,i}^N = f_i^N$$

Leading term is Mellin moment of PDF

 Provided inverse exists then can relate GPDs to PDFs with suppression of order xi (i.e. good low x approx)

Shuvaev Transform cont.

Widely debated, certain conditions needing upheld, e.g lack of singularities in Re N > 1 plane e.g Diehl, Kugler, 08

Regge theory considerations => condition met Martin, Nockles, Ryskin, Teubner, 09

 Can check in physically motivated ansatz, e.g MSTW2008 global partons input parametrisation

Martin, Stirling,Thorne, Watt, 09

$$xg(x,Q_0^2) = A_g x^{\delta_g} (1-x)^{\eta_g} (1+\epsilon_g \sqrt{x} + \gamma_g x) + A_{g'} x^{\delta_{g'}} (1-x)^{\eta_{g'}}.$$
 We

Expand about x ~ 0

$$xg(x,Q_0^2) = A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}} + \dots,$$

Mellin transform: $xg^N(Q_0^2) = \int_0^1$

$$V(Q_0^2) = \int_0^1 \mathrm{d}x x^{N-1} (A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}}) + \dots$$

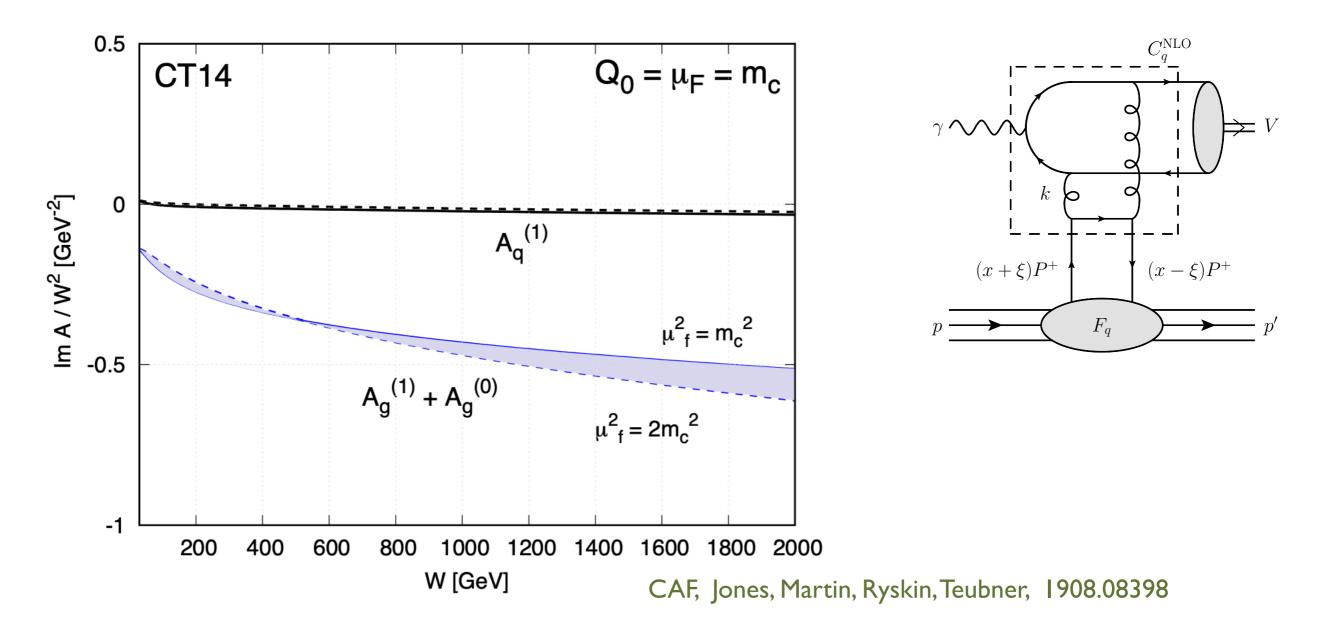
= $\frac{A_g}{N+\delta_g} + \frac{A_{g'}}{N+\delta_{g'}} + \dots$,

Fits to data (including 1sig. errors) suggest $\delta_g > -1$ and $\delta_{g'} > -1$

Shuvaev transform describes HVM and GDVCS data well

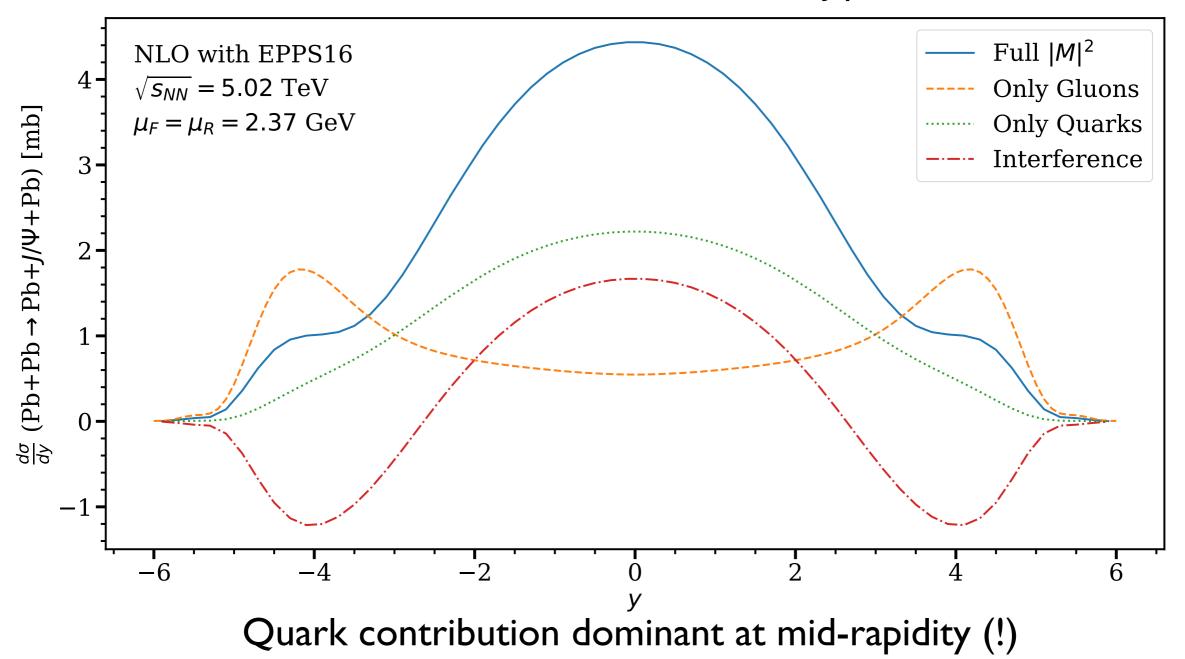
Kumericki, Muller, 10

After Q_O subtraction:



Interplay of quark and gluon at NLO

Conventional NLO Pb + Pb -> Pb + J/psi + Pb



Structure of amplitude detailed, interplaying between photoproduction cross section, photon flux, form factor and W_{\pm} components

Key: Cancellation of LO and NLO gluon amplitudes due to opp. signs

Eskola, CAF, Guzey, Löytäinen, Paukkunen 2203.11613

Interplay of quark and gluon at NLO

Conventional NLO Pb + Pb -> Pb + J/psi + Pb

Picture at NLO but (probably) a feature of NLO pQCD - superposition of higher-order quark terms will be present at e.g. NNLO so interesting to (ultimately!) assess the situation at successively higher orders

The gluons-only contribution in figure comes from the term

$$|\mathcal{M}_{G}^{\mathrm{LO}} + \mathcal{M}_{G}^{\mathrm{NLO}}|^{2} = [\operatorname{Re}(\mathcal{M}_{G}^{\mathrm{LO}}) + \operatorname{Re}(\mathcal{M}_{G}^{\mathrm{NLO}})]^{2} + [\operatorname{Im}(\mathcal{M}_{G}^{\mathrm{NLO}}) + \operatorname{Im}(\mathcal{M}_{G}^{\mathrm{NLO}})]^{2}$$

and the quarks-only contribution from

$$\mathcal{M}_Q^{\rm NLO}|^2 = [{\rm Re}(\mathcal{M}_Q^{\rm NLO})]^2 + [{\rm Im}(\mathcal{M}_Q^{\rm NLO})]^2,$$

from Eskola, CAF, Guzey, Löytäinen, Paukkunen 2203.11613

Quark contribution dominant at mid-rapidity (!)

Structure of amplitude detailed, interplaying between photoproduction cross section, photon flux, form factor and W_{\pm} components Key: Cancellation of LO and NLO gluon amplitudes due to opp. signs

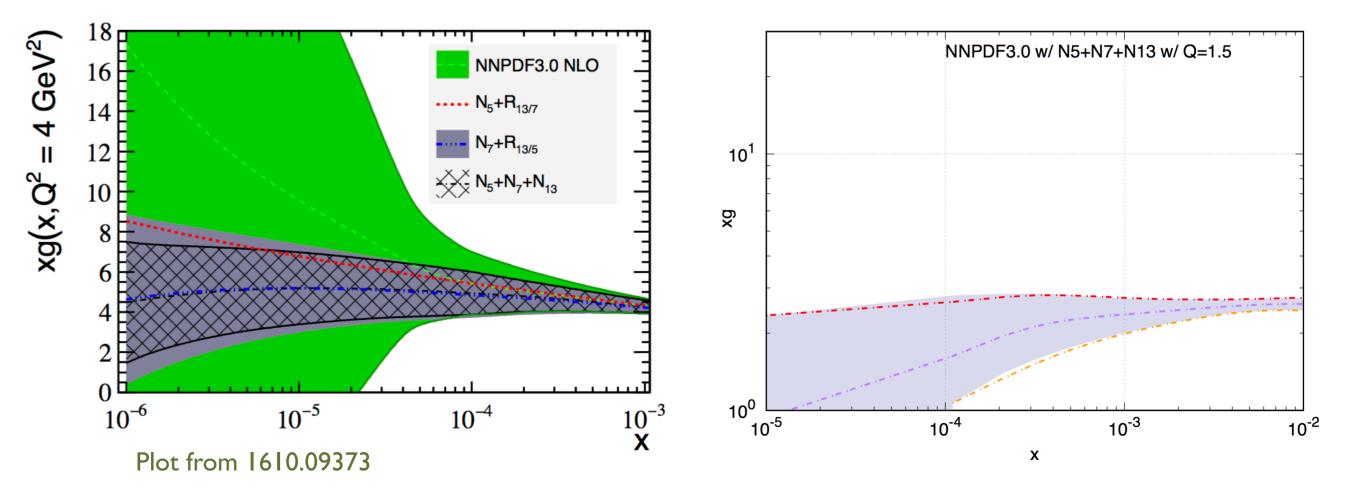
Constraints from inclusive D meson production data

Idea: Construct ratios of observables in y and p_t bins to combat various uncertainties

$$\begin{split} N_X^{ij} &= \frac{d^2 \sigma(\text{X TeV})}{dy_i^D d(p_T^D)_j} \middle/ \frac{d^2 \sigma(\text{X TeV})}{dy_{\text{ref}}^D d(p_T^D)_j} \\ R_{13/X}^{ij} &= \frac{d^2 \sigma(13 \text{ TeV})}{dy_i^D d(p_T^D)_j} \middle/ \frac{d^2 \sigma(\text{X TeV})}{dy_{\text{i}}^D d(p_T^D)_j} \end{split}$$

 \rightarrow

find decreasing gluon at the lowest x they may probe



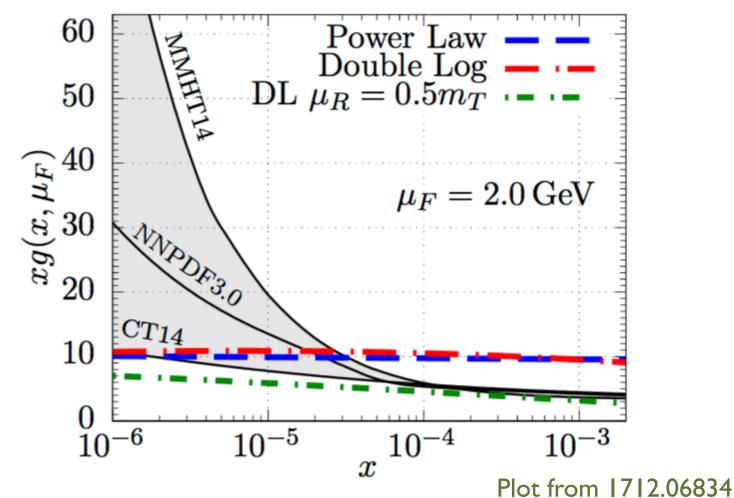
Tension with the J/psi data

We need a much harder gluon at low x to describe the exclusive J/psi LHCb data.

What's the reconciliation?

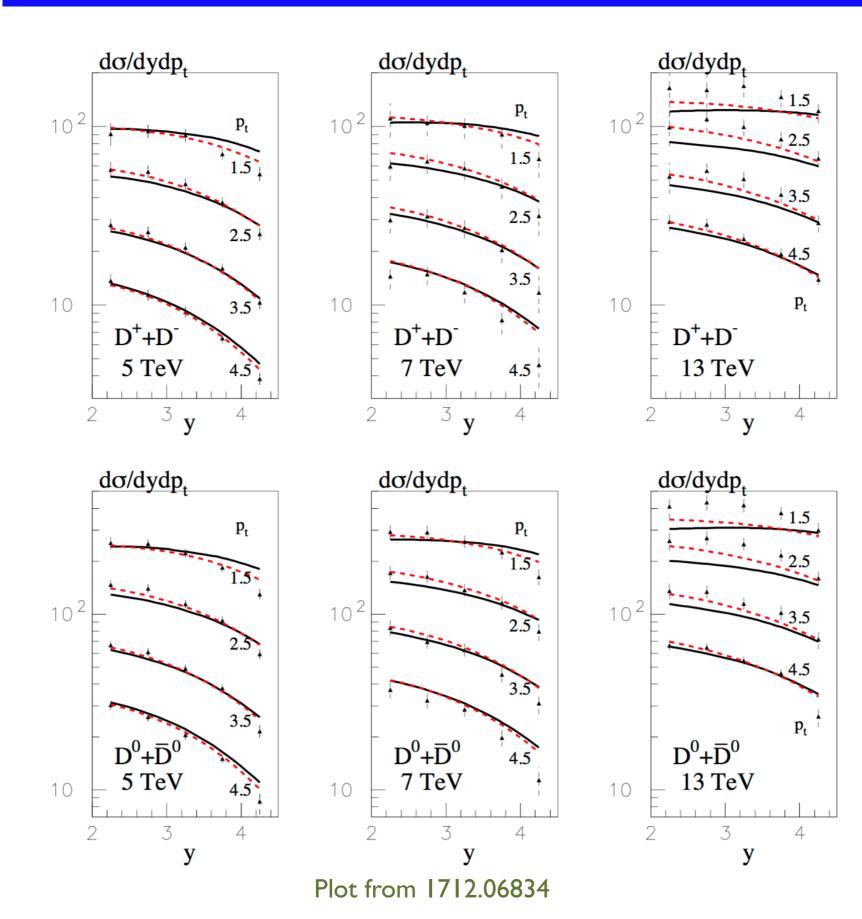
Indications of inconsistencies in the inclusive D experimental measurement

$$xg(x) = N\left(\frac{x}{x_0}\right)^{-\lambda}$$



$$xg(x,\mu^2) = N^{\text{DL}} \left(\frac{x}{x_0}\right)^{-a} \left(\frac{\mu^2}{Q_0^2}\right)^{b} \exp\left[\sqrt{16(N_c/\beta_0)\ln(1/x)\ln(G)}\right]$$

Rapidity and energy dependence of open charm cross section



- Need slower increasing gluon with decreasing x to describe rapidity dependence
- Need faster increasing gluon with decreasing x to describe energy dependence

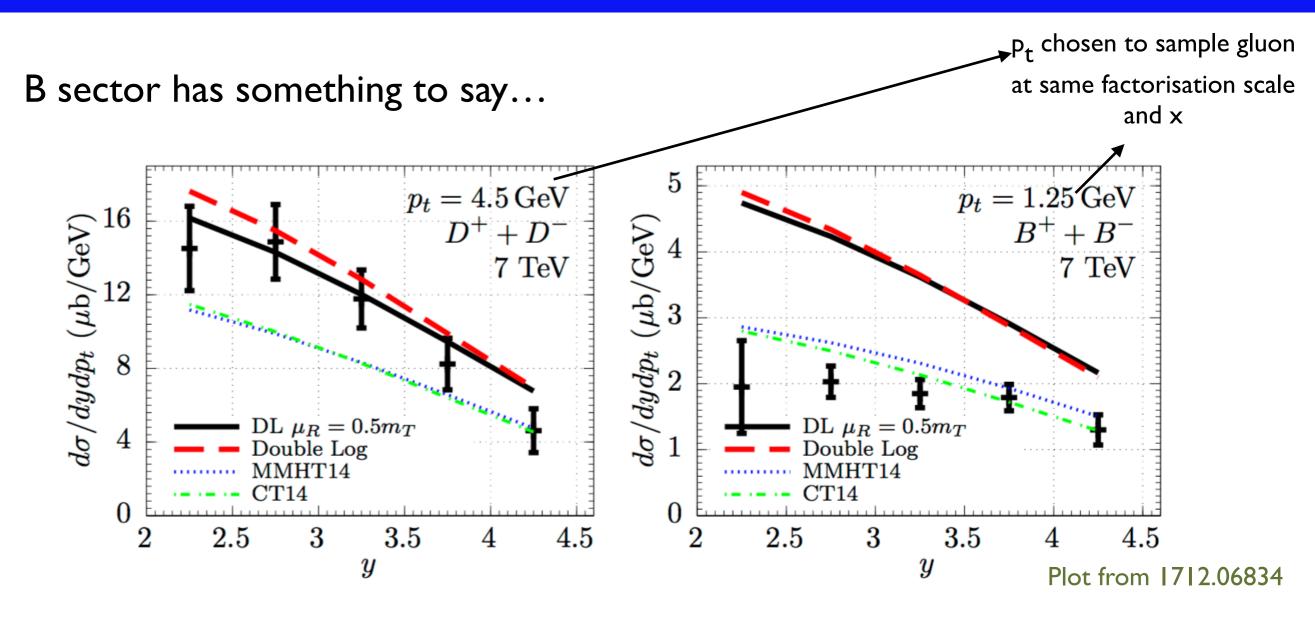
$$y \sim \ln(1/x) !!$$

solid

dash $Q_0=1$ GeV and $\mu_F=\mu_R=0.85m_T$

 $\mu_f=\mu_R=0.5m_T$ and $Q_0{=}0.5~{
m GeV}$

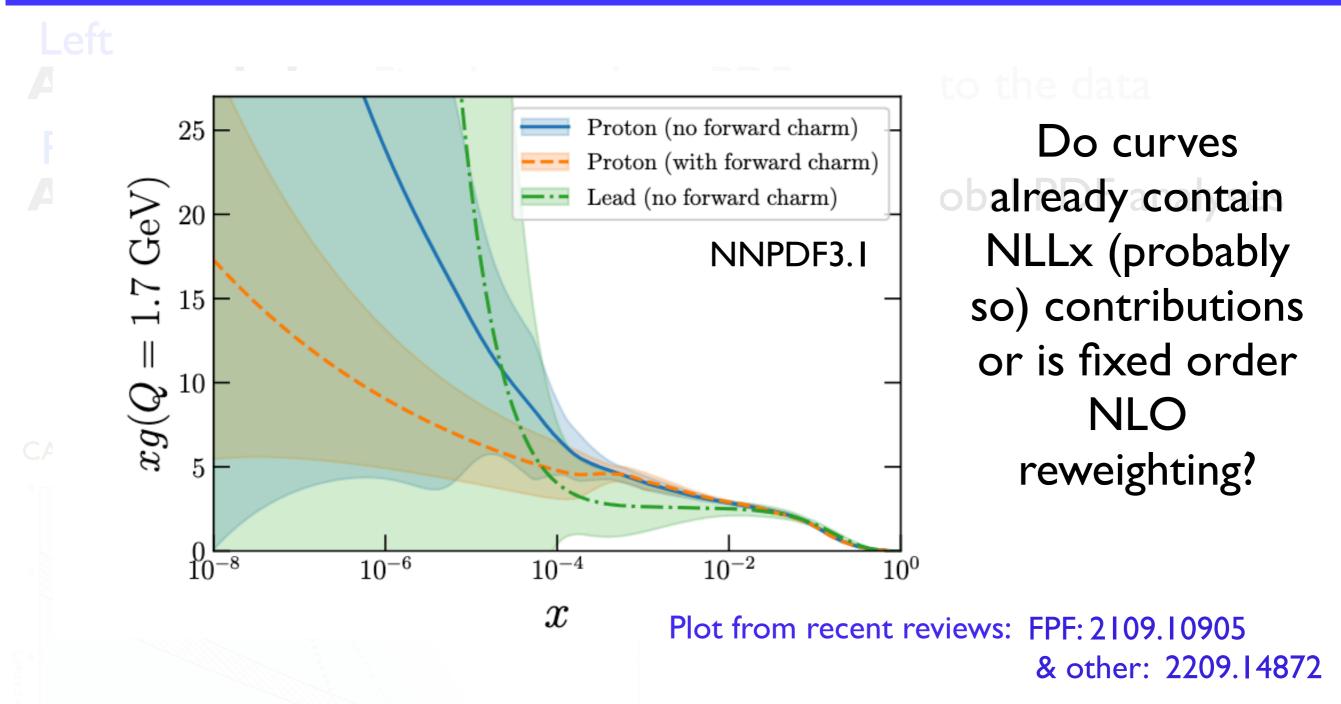
Open beauty results



Gluon found through fit to D meson data fails to describe the B meson distribution

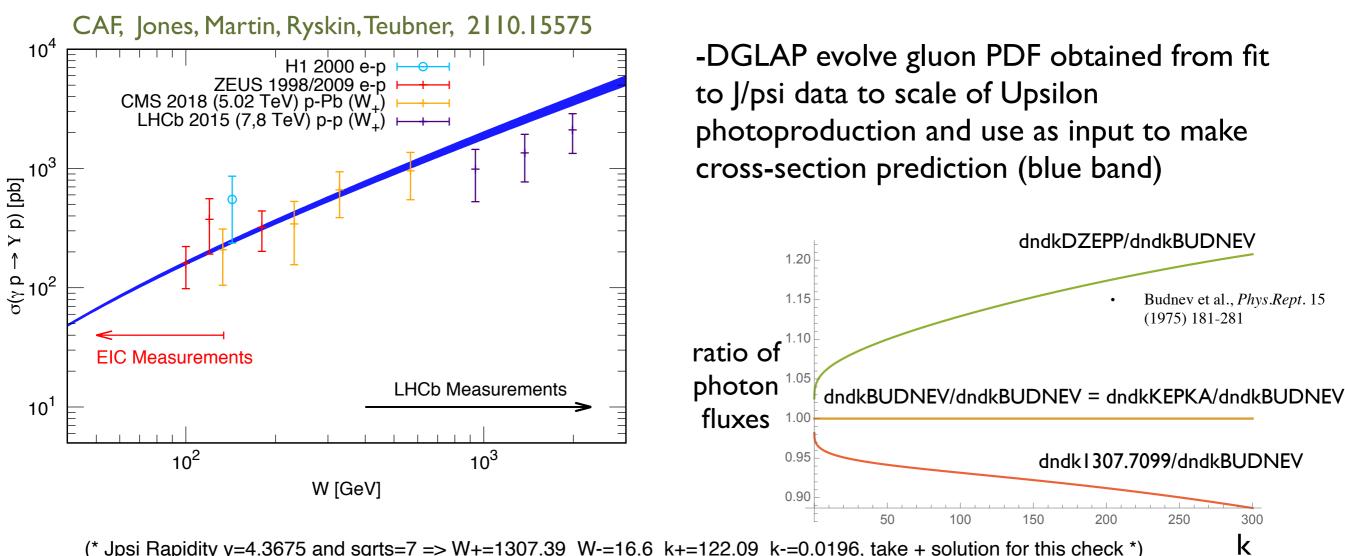
Should we really trust the decreasing nature of the low scale, low x gluon obtained via fit to LHCb open charm data?

Charm constraints from reviews



NNPDF3.1sx+charm given in 1808.02034

Other results in UPC: Upsilon in pp



(* Jpsi Rapidity y=4.3675 and sqrts=7 => W+=1307.39 W-=16.6 k+=122.09 k-=0.0196, take + solution for this check *) (ss1307.7099*flux1307.7099)/(ssBudnev*fluxBudnev)= 0.94901

So there is about a 5% effect that mostly comes from the flux and is partly (but not much) cancelled by the survival factor.

Now repeat exercise for a larger rapidity (beyond LHCb data/acceptance currently): (* Jpsi Rapidity y=5.125, sqrts=7 => W+=1909.38 W-=11.35 k+=260.41 k-=0.0092, , take + solution for this check *) (ss1307.7099*flux1307.7099)/(ssBudnev*fluxBudnev) = 1.24832

We see there is a 25% effect that mostly comes from the survival factors and is partly cancelled by the photon fluxes.

For Upsilon the k+ and w+ values will be larger and so the discrepancy between the fluxes will be larger and likely also between the survival factors.

At such large W+ the approximation used in 1307.7099 flux starts to break down and we should use the Budnev flux. The Upsilon mass is ~3x that of the Jpsi and k+ ~ M_v (whereas W+ ~ Sqrt[M_v]). The typical photon energy now is much larger than with Jpsi and we enter the region where the approximation breaks down at much lower rapidities and, importantly, within acceptance of LHCb => large W unfolded photoproduction LHCb data should be shifted upwards

Resummation

In pQCD, can write observables: $\sum_{n} \alpha_s^n c_n(x, Q^2)$

$$LL_QA: \sum_n \alpha_s^n \ln^n(Q^2) \left(\ln^n \left(\frac{1}{x}\right) + \ln^{n-1} \left(\frac{1}{x}\right) + \dots \right) \qquad c_n(x, Q^2) = \tilde{c}_n(x) \ln^n(Q^2),$$

$$LL_xA: \sum_n \alpha_s^n \ln^n \left(\frac{1}{x}\right) \left(\ln^n \left(Q^2\right) + \ln^{n-1} \left(Q^2\right) + \dots\right) \qquad c_n(x, Q^2) = \tilde{c}_n(Q^2) \ln^n \left(\frac{1}{x}\right)$$

Comments on the

- Consistency of use of resummed PDFs and fixed order pQCD coefficient functions
- Single In(1/x) resummation

Resummed PDFs were used previously and, strictly speaking, one should do resummation in coefficient function too but effect here anticipated to be small because Q_0 subtraction in the BFKL Kernel would make the effect of the alpha_s ln(1/x) terms weaker. In any case, LO BFKL is already known to be too large. Subtracting the k_T<Q_0 contribution we (A) diminish the BFKL term by factor $(1/2)^n$ at each order (qualitatively) and (B) remove enhanced alphas contribution at k_T < Q_0.

-> Status of other groups (c.f. discussion QAT22) ?

Sensitivity to the MSbar gluon PDF

- Remain in MSbar scheme with Q0 subtracted coefficient functions to NLO accuracy
- Subtraction does not affect IR or UV divergence renormalisation procedures
- Soft singularity at I=0 is removed after subtracting off the LO part of the NLO coefficient function before integral over loop momentum from 0 to Q0 is performed

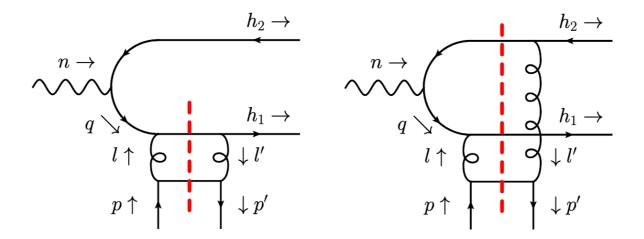
$$\Delta \text{Im}\mathcal{M}^{q} = \frac{\alpha_{s}^{2}}{2\pi} \int_{\xi}^{1} dx \left(F_{q}(x,\xi,m_{c}) - F_{q}(-x,\xi,m_{c}) \right) \left(\int_{0}^{Q_{0}^{2}} (M_{a}^{q} + M_{b}^{q}) \frac{2\pi m_{c}^{4}}{\hat{s}^{2}} dl^{2} \right)$$

Precisely this FINITE contribution that is subtracted from full MSbar coefficient functions to avoid double counting inherent within MSbar scheme (subtraction fundamentally ubiquitous but numerically relevant for low scale processes only*)

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 NLO diagrams for quark and gluon channel considered. Contain both LO and NLO contributions. Subtract off LO contribution (part given by LO (generalised) DGLAP evolution P_LO x C^0, see previous) before integration over I is performed, cancelling soft singularity dI^2/I^2.

Higher twist contributions

- Absorptive corrections, which provide the saturation, are described by higher-twist
 operators and formally not known within the collinear factorisation approach.
- The relative size of the contribution of the next twist absorptive correction is driven by parameter:

$$c = \alpha_s \frac{xg(x)}{R^2 \mu_0^2}$$

- Factor appearing in GLR equation (Phys. Rept. 100 (1983) 1–150) provides non-linear terms through computation of so-called 'fan' diagrams in pQCD that tame (linear) BFKL evolution
- Semi-quantitative estimate based on this scaling gives higher-twist term of O(few percent*). Details in 2006.13857.

^{*}If one takes into consideration the colour factor calculated assuming that the low x gluon is emitted by the valence quark in the proton, then there is an additional factor of 81/16 which enhances the estimate to ~6.5%. However, the point is that the higher-twist contribution may be relatively small and that, together with the additional factor of alphas from <v2> \sim alphas, all the parametric dependence is included in the GLR factor c.