

# Production of heavy hadrons via coalescence plus fragmentation in pp and AA collisions

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14/10/2022

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Laboratori Nazionali del Sud

# Outline

**Hadronization:**

- Fragmentation
- Coalescence model

**Heavy hadrons in AA collisions:**

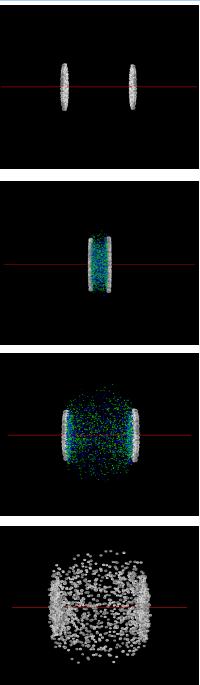
- $\Lambda_c$ , D spectra and ratio: RHIC and LHC

**Heavy hadrons in small systems (pp @ 5.02 TeV):**

- $\Lambda_c/D^0$
- $\Xi_c/D^0$  ,  $\Omega_c/D^0$

**Multicharm production**

# Quark Gluon Plasma in Ultra-Relativistic Heavy-Ion Collisions



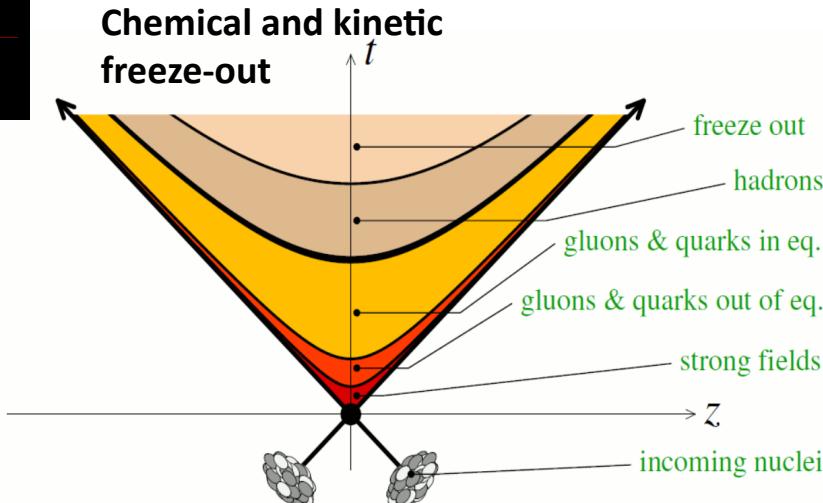
Initial Stage

Pre-equilibrium stage

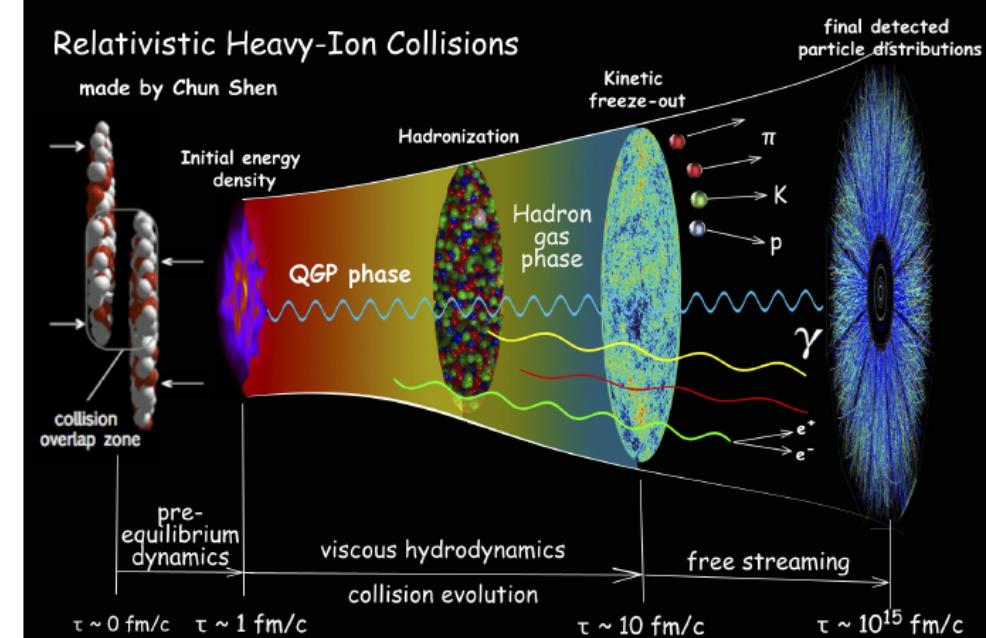
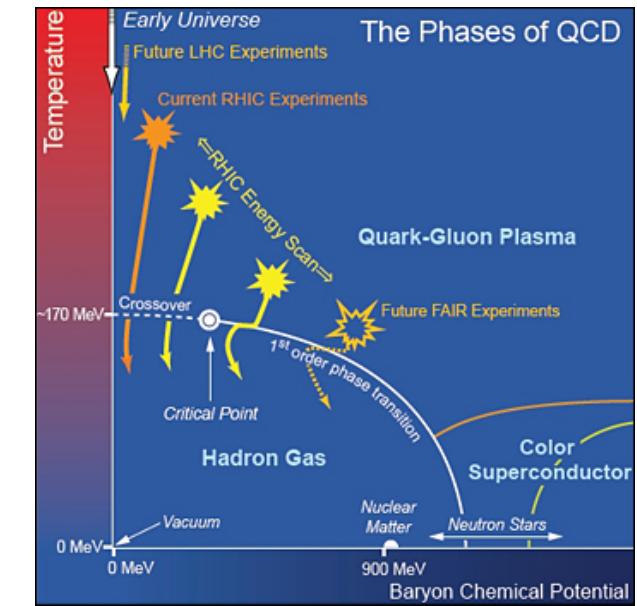
Expansion

QGP

**Hadronization**



- Nuclear matter: Critical Energy and Temperature in the transition between confined and deconfined phase  
 $\epsilon_c \approx 0.7 \text{ GeV/fm}^3$     $T_c \approx 165 \text{ MeV} \approx 10^{12} \text{ K}$
- If  $T > T_c$  colour charges are deconfined in a Quark Gluon Plasma (QGP)
- Different value of  $T$  and  $\rho$  for deconfinement → Phase Diagram



# Specific of Heavy Quarks

➤  $m_{c,b} \gg \Lambda_{\text{QCD}}$   
produced by pQCD process (out of equilibrium)

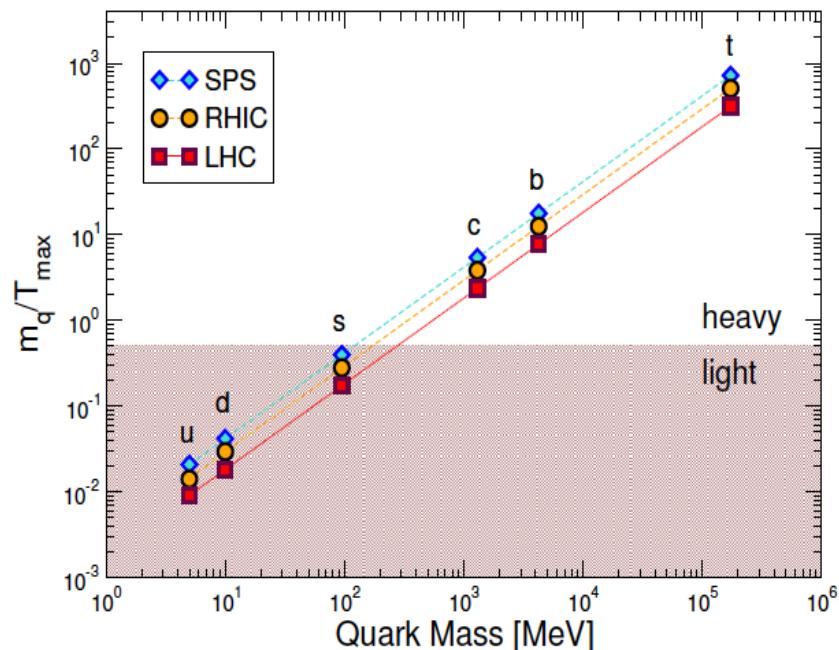
➤  $m_{c,b} \gg T_0$   
negligible thermal production

➤  $\tau_0 \ll \tau_{\text{QGP}}$

➤  $\tau_{\text{therm.}} \approx \tau_{\text{QGP}} \gg \tau_{g,q}$

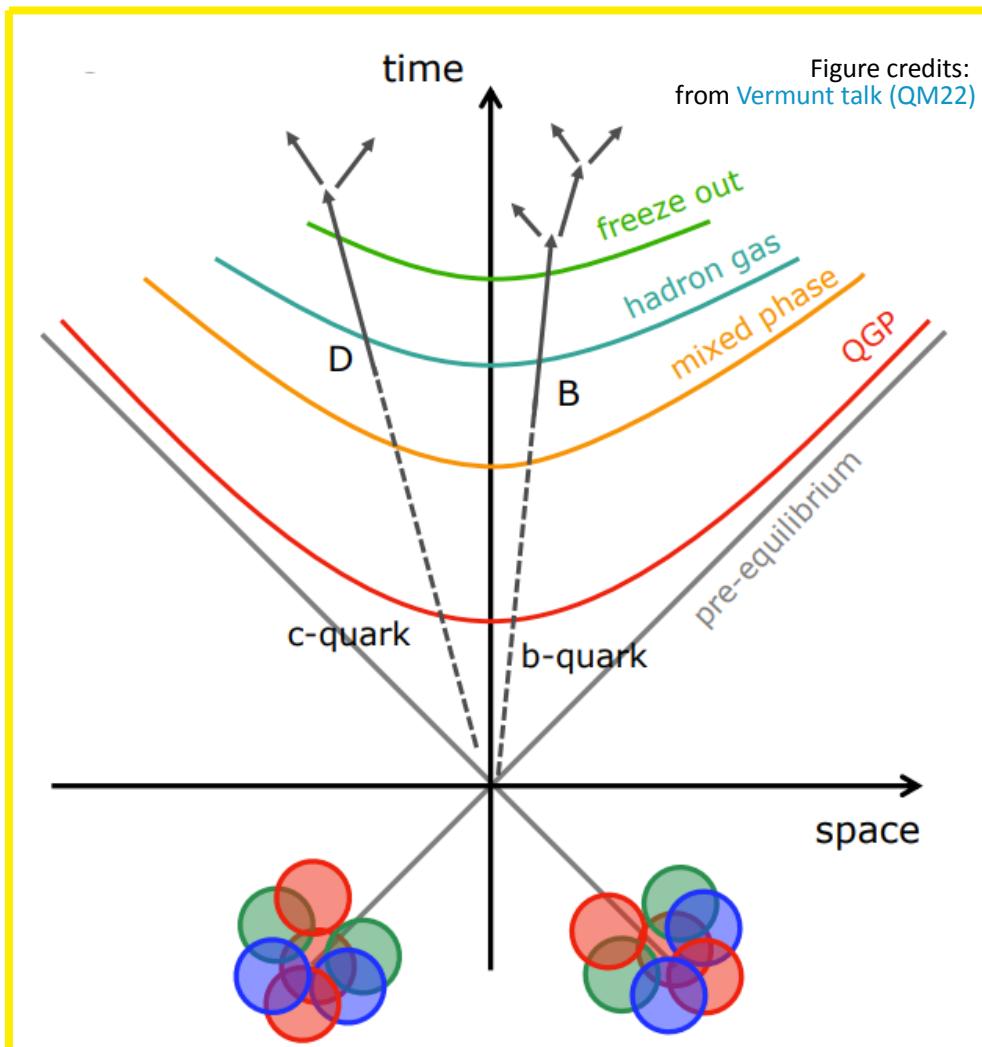
HQs experience the full QGP evolution

Carry informations about initial stages, more than light quarks



Recent reviews:

- 1) X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 (2019)
- 2) A.Andronic Eur.Phys.J.C 76 (2016) 3, 107
- 3) F.Prino, R.Rapp, J.Phys.G 43 (2016) 9, 093002



# Relativistic Boltzmann transport at finite $\eta/s$

## Bulk evolution

$$p^\mu \partial_\mu f_{q,g}(x, p) + M(x) \partial_\mu^x M(x) \partial_p^\mu f_{q,g}(x, p) = C_{22}[f_{q,g}]$$

free-streaming                                  field interaction  
 $\varepsilon - 3p \neq 0$                                   collisions  
 $\eta \neq 0$

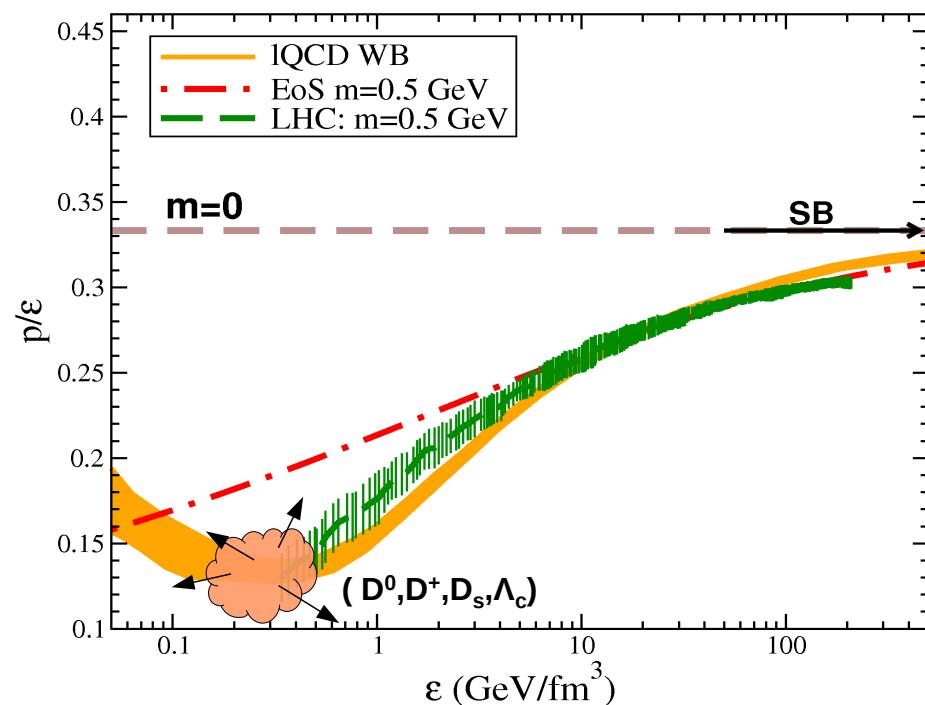
## Heavy quark evolution

$$p^\mu \partial_\mu f_Q(x, p) = C[f_q, f_g, f_Q]$$

• Describes the evolution of the one body distribution function  $f(x, p)$

• It is valid to study the evolution of both bulk and Heavy quarks

• Possible to include  $f(x, p)$  out of equilibrium



# Heavy flavour Hadronization

## Microscopic

### Fragmentation:

production from hard-scattering processes (PDF+pQCD).

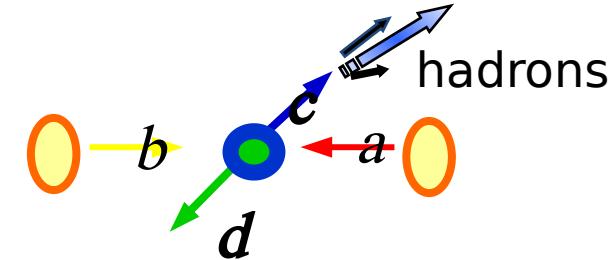
Fragmentation functions: data parametrization, assumed “universal”

$$\sigma_{pp \rightarrow h} = PDF(x_a, Q^2) PDF(x_b, Q^2) \otimes \sigma_{ab \rightarrow q\bar{q}} \otimes D_{q \rightarrow h}(z, Q^2)$$

### Parton shower: String fragmentation(Lund model – PYTHIA)

+colour reconnection(interaction from different scattering)

Cluster decay (HERWIG)



### Coalescence: recombination of partons in QGP close in phase space

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_w(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Have described first AA observations in light sector for the enhanced baryon/meson ratio and elliptic flow splitting

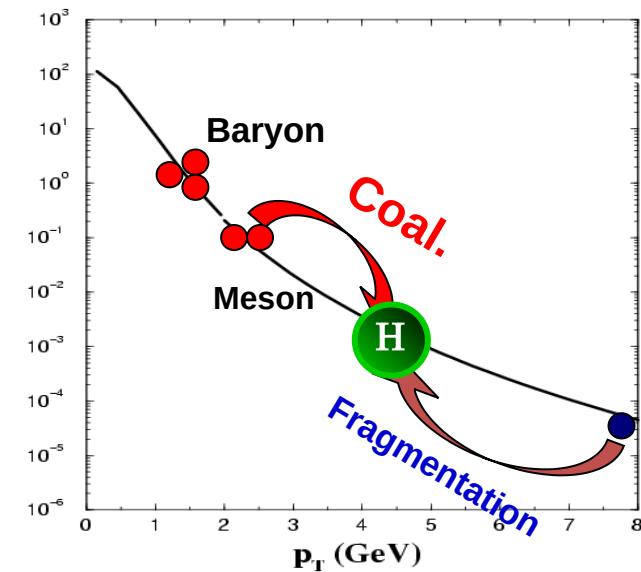
## Macroscopic

### Statistical hadronization:

Equilibrium + hadron-resonance gas + freeze-out temperature.

Production depends on hadron masses and degeneracy, and on system properties.

pQCD Charm production + total yield from charm cross section (not Temp.)  
charm hadrons according to thermal weights



# Catania Model: Coalescence + Fragmentation

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

LIGHT

Thermal+flow for u,d,s ( $p_T < 3$  GeV)

$$\frac{dN_{q,\bar{q}}}{d^2 p_T} \sim \exp\left(-\frac{\gamma_T - p_T \cdot \beta_T \mp \mu_q}{T}\right)$$

$$\beta(r) = \frac{r}{R} \beta_{max}$$

$$V = \pi R^2 \tau \cosh(y_z), R(\tau_f) = R_0 (1 + \beta_{max} \tau_f)$$

$$\text{PbPb@5ATeV(0-10%)}: \tau_f = 8.4 \frac{fm}{c} \rightarrow V|_{|y|<0.5} = 4500 fm^3$$

+quenched minijets for u,d,s ( $p_T > 3$  GeV)

Parton Distribution function

Hadron Wigner function

CHARM

In AA collisions charm distribution from the studies of  $R_{AA}$  and  $v_2$  of D-meson to determine the Space Diffusion coefficient:

parton simulations solving relativistic Boltzmann transport equation

In pp collisions the charm distribution are the FONLL distribution

Coalescence simulation in a fireball with radial flow for light quarks  $\rightarrow$  dimension set by experimental constraints

# Catania Model: Coalescence + Fragmentation

**Statistical factor  
colour-spin-isospin**      **Parton Distribution  
function**      **Hadron Wigner  
function**

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Wigner function – Wave function

Wigner function width fixed by root-mean-square charge radius from quark model

C.-W. Hwang, EPJ C23, 585 (2002)  
 C. Albertus et al., NPA 740, 333 (2004)

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \phi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \phi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$

$\phi_M(\mathbf{r})$  meson wave function

Assuming gaussian wave function

$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_w \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

only one width coming from  $\phi_M(\mathbf{r})$ ,  
 constraint  $\sigma_r \sigma_p = 1$

$$\langle r^2 \rangle_{ch} = \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 + \frac{3}{2} \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2$$

$$\sigma_r = 1/\sqrt(\mu_i \omega) \quad \mu_1 = \frac{m_1 m_2}{m_1 + m_2} \quad \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$

Meson	$\langle r^2 \rangle_{ch}$	$\sigma_{p1}$	$\sigma_{p2}$
$D^+ = [c\bar{d}]$	0.184	0.282	—
$D_s^+ = [\bar{s}c]$	0.083	0.404	—
Baryon	$\langle r^2 \rangle_{ch}$	$\sigma_{p1}$	$\sigma_{p2}$
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\Omega_c^0 = [ssc]$	-0.12	0.337	0.53

# Catania Model: Coalescence + Fragmentation

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Wigner function – Wave function

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \phi_M(\mathbf{r} + \frac{\mathbf{r}'}{2}) \phi_M^*(\mathbf{r} - \frac{\mathbf{r}'}{2})$$

$\phi_M(\mathbf{r})$  meson wavefunction

Assuming gaussian wave function

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only one width coming from  $\phi_M(\mathbf{r})$ ,  
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Wigner function width fixed by root-mean-square charge radius from quark model

C.-W. Hwang, EPJ C23, 585 (2002)  
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- Normalization of  $f_H(\dots)$  requiring that  $P_{coal}=1$  at  $p=0$
- The charm that does not coalesce undergo fragmentation

# Catania Model: Coalescence + Fragmentation

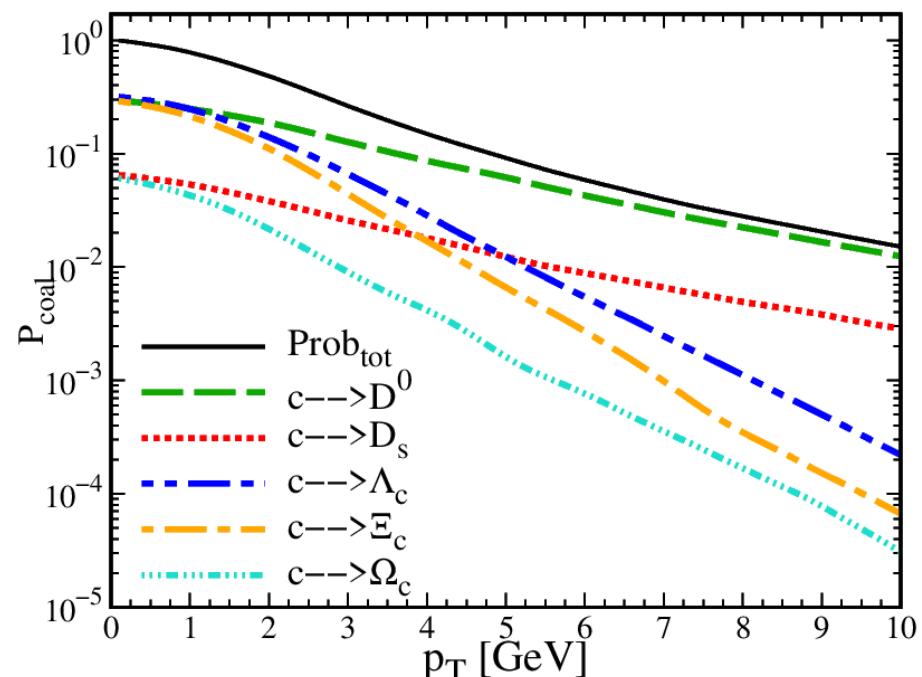
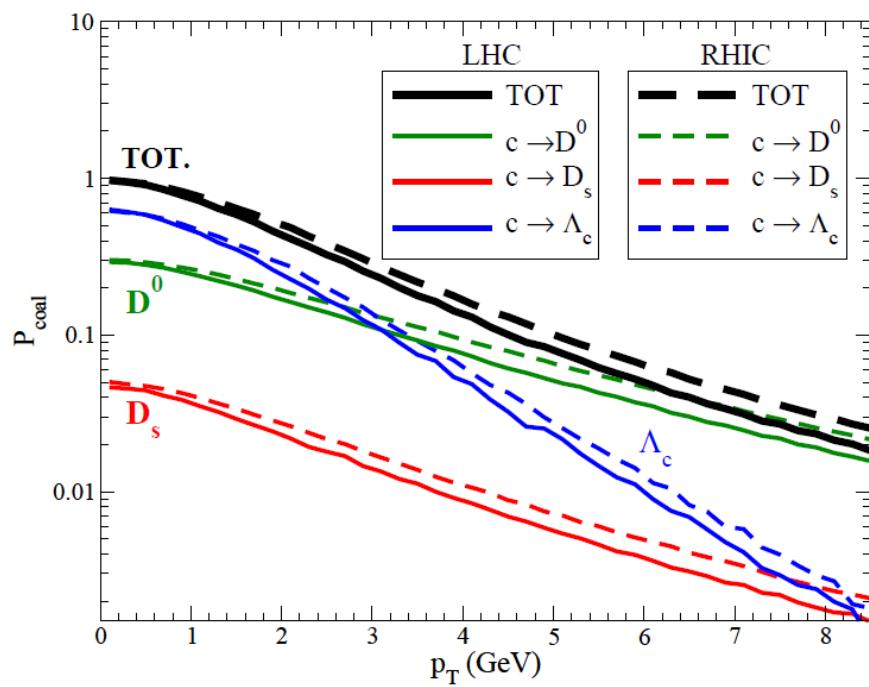
*Statistical factor  
colour-spin-isospin*

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*Parton Distribution  
function*

*Hadron Wigner  
function*

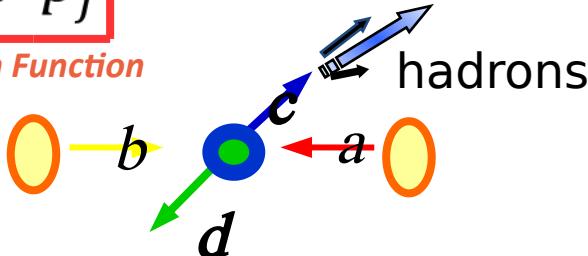
- Normalization of  $f_H(\dots)$  requiring that  $P_{coal}=1$  at  $p=0$
- The charm that does not coalesce undergo fragmentation



# Catania Model: Coalescence + Fragmentation

$$\frac{dN_h}{d^2 p_h} = \sum_f \int dz \frac{dN_f}{d^2 p_f} D_{f \rightarrow h}(z)$$

*Parton Distribution Function*

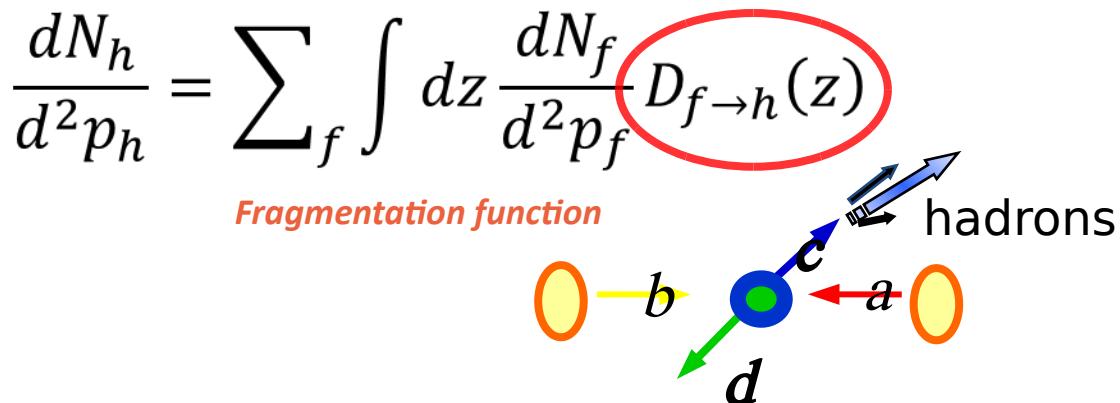


The distribution function is evaluated at the Fixed-Order plus Next-to-Leading-Log (FONLL)

M. Cacciari, P. Nason, R. Vogt, PRL 95 (2005) 122001

In AA: bulk+charm evolution with Relativistic Transport Boltzmann Equation

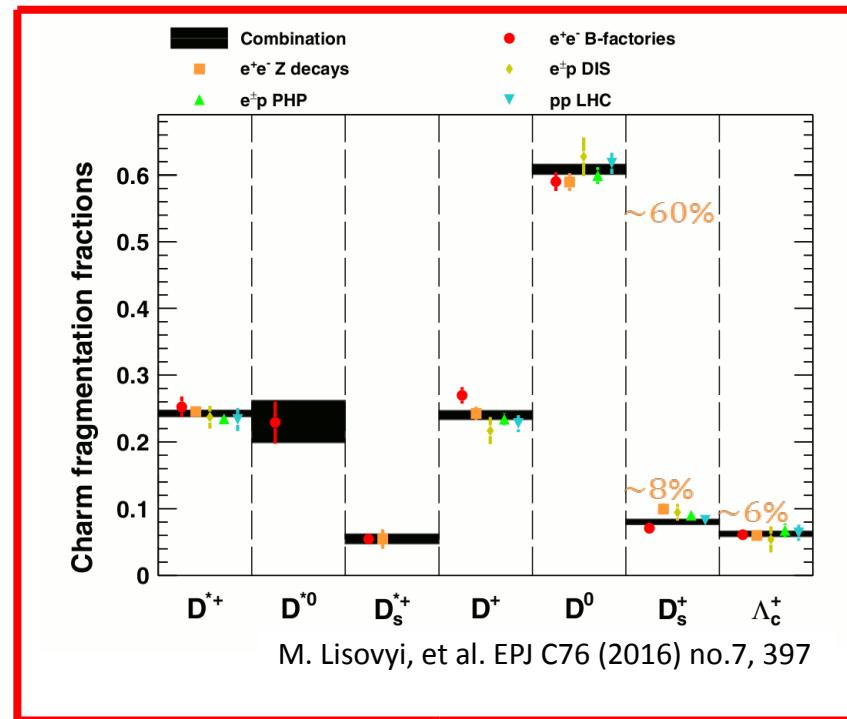
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In AA: bulk+charm evolution with Relativistic Transport Boltzmann Equation



We use the Peterson fragmentation function

C. Peterson, D. Schalatter, I. Schmitt, P.M. Zerwas PRD 27 (1983) 105

$$D_{f \rightarrow h}(z) \propto \frac{1}{z \left[ 1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right]^2}$$

Slightly modified to reproduce tail of the  $\Lambda_c/D^0$

**Charm Fragmentation Fraction ( $c \rightarrow h$ )**  
Measurement in  $e^\pm p$ ,  $e^+ e^-$  and old  $pp$  data

$$\left( \frac{\Lambda_c^+}{D^0} \right)_{pp} \underset{e^+ e^-}{\simeq} 0.1 \quad \left( \frac{D_s^+}{D^0} \right)_{pp} \underset{e^+ e^-}{\simeq} 0.13$$

# Heavy flavour: Resonance decay

Meson	Mass(MeV)	I (J)	Decay modes	B.R.
$D^+ = \bar{d}c$	1869	$\frac{1}{2}(0)$		
$D^0 = \bar{u}c$	1865	$\frac{1}{2}(0)$		
$D_s^+ = \bar{s}c$	2011	0(0)		
<b>Resonances</b>				
$D^{*+}$	2010	$\frac{1}{2}(1)$	$D^0\pi^+$ ; $D^+X$	68%,32%
$D^{*0}$	2007	$\frac{1}{2}(1)$	$D^0\pi^0$ ; $D^0\gamma$	62%,38%
$D_s^{*+}$	2112	0(1)	$D_s^+X$	100%
<b>Baryon</b>				
$\Lambda_c^+ = udc$	2286	0( $\frac{1}{2}$ )		
$\Xi_c^+ = usc$	2467	$\frac{1}{2}(\frac{1}{2})$		
$\Xi_c^0 = dsc$	2470	$\frac{1}{2}(\frac{1}{2})$		
$\Omega_c^0 = ssc$	2695	0( $\frac{1}{2}$ )		
<b>Resonances</b>				
$\Lambda_c^+$	2595	0( $\frac{1}{2}$ )	$\Lambda_c^+\pi^+\pi^-$	100%
$\Lambda_c^+$	2625	0( $\frac{3}{2}$ )	$\Lambda_c^+\pi^+\pi^-$	100%
$\Sigma_c^+$	2455	$1(\frac{1}{2})$	$\Lambda_c^+\pi$	100%
$\Sigma_c^+$	2520	$1(\frac{3}{2})$	$\Lambda_c^+\pi$	100%
$\Xi_c^{'+,0}$	2578	$\frac{1}{2}(\frac{1}{2})$	$\Xi_c^{+,0}\gamma$	100%
$\Xi_c^+$	2645	$\frac{1}{2}(\frac{3}{2})$	$\Xi_c^+\pi^-$ ,	100%
$\Xi_c^+$	2790	$\frac{1}{2}(\frac{1}{2})$	$\Xi_c'\pi$ ,	100%
$\Xi_c^+$	2815	$\frac{1}{2}(\frac{3}{2})$	$\Xi_c'\pi$ ,	100%
$\Omega_c^0$	2770	0( $\frac{3}{2}$ )	$\Omega_c^0\gamma$ ,	100%

In our calculations we take into account hadronic channels including the ground states + first excited states

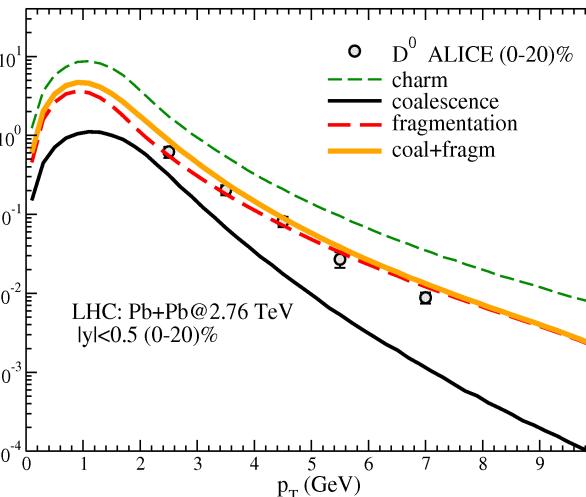
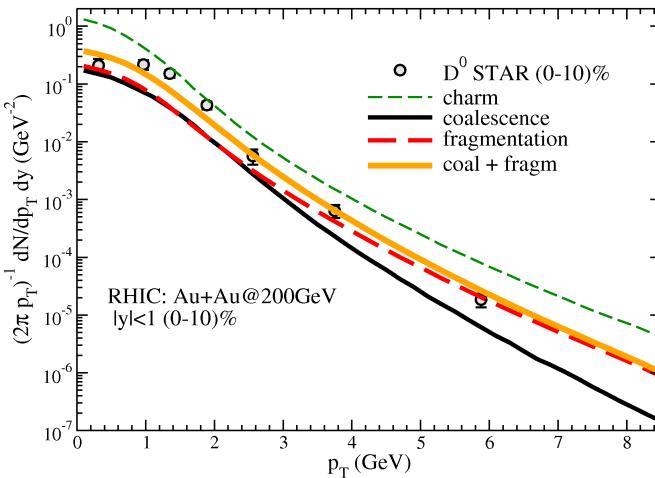
**Statistical factor suppression for resonances**

$$\frac{[(2J+1)(2I+1)]_{H^*}}{[(2J+1)(2I+1)]_H} \left( \frac{m_{H^*}}{m_H} \right)^{3/2} e^{-(m_{H^*} - m_H)/T}$$

# AA @ RHIC & LHC

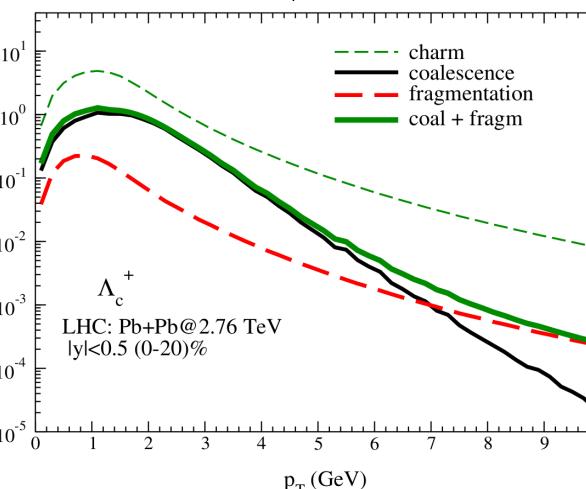
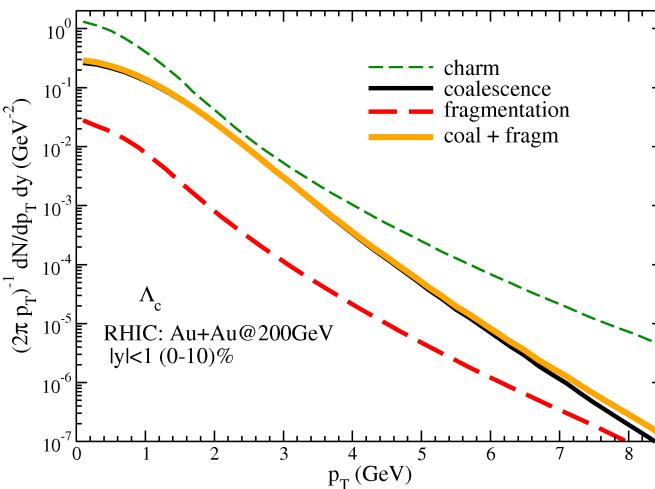
wave function widths  $\sigma_p$  of baryon and mesons are the same at RHIC and LHC!

Data from: STAR Coll. PRL 113, 142301 (2014), ALICE Coll. JHEP 09 (2012) 112



Coalescence lower at LHC than at RHIC

main contribution:  
Fragmentation



Coalescence lower at LHC than at RHIC

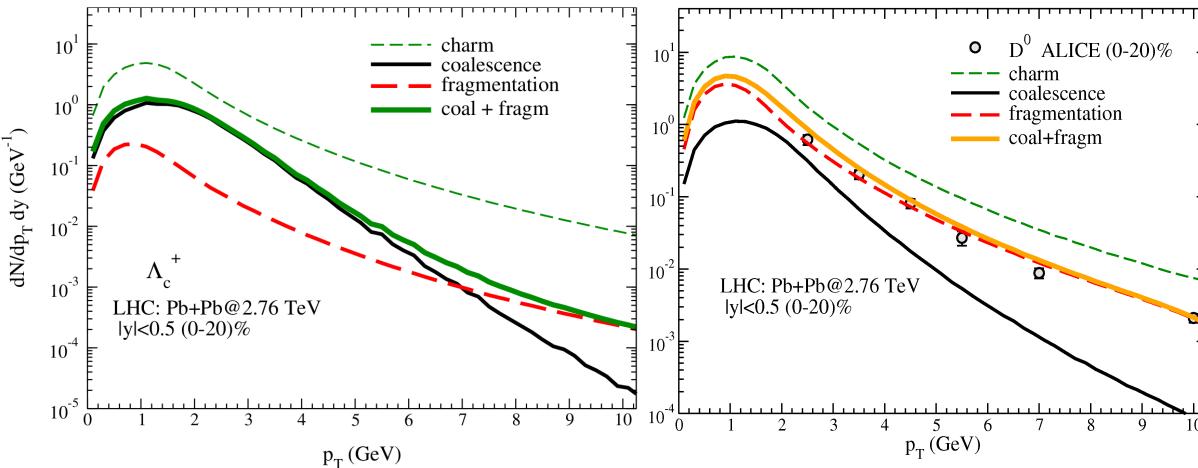
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RHIC

LHC

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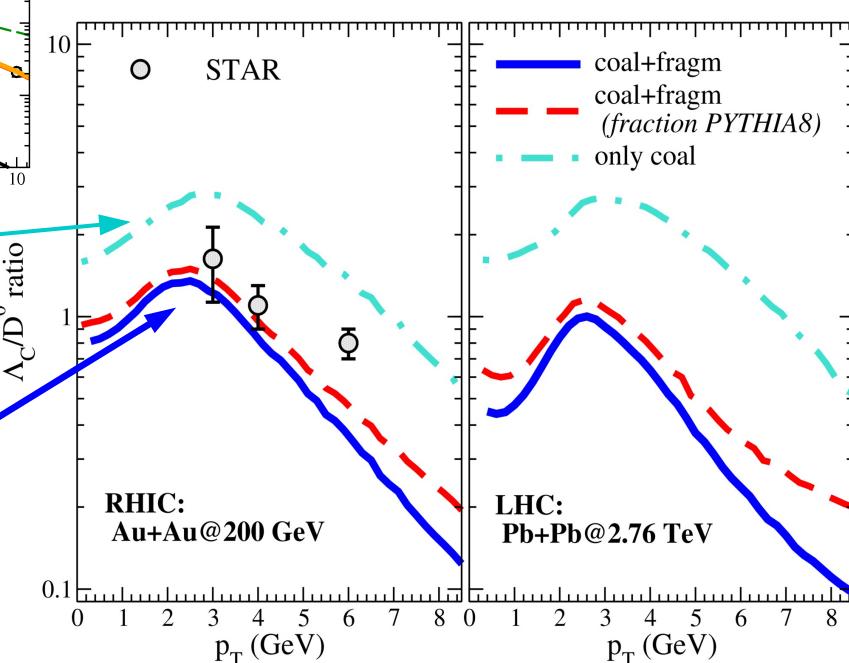
Only Coalescence ratio is similar at both energies.

Fragmentation  $\sim 0.1$  at both energies.

the **combined ratio is different** because the coalescence over fragmentation ratio at LHC is smaller than at RHIC

Therefore at LHC the larger contribution in particle production from fragmentation leads to a final ratio that is smaller than at RHIC.

Coalescence lower at LHC than at RHIC



STAR Coll., Phys.Rev.Lett. 124 (2020) 17, 172301

S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348

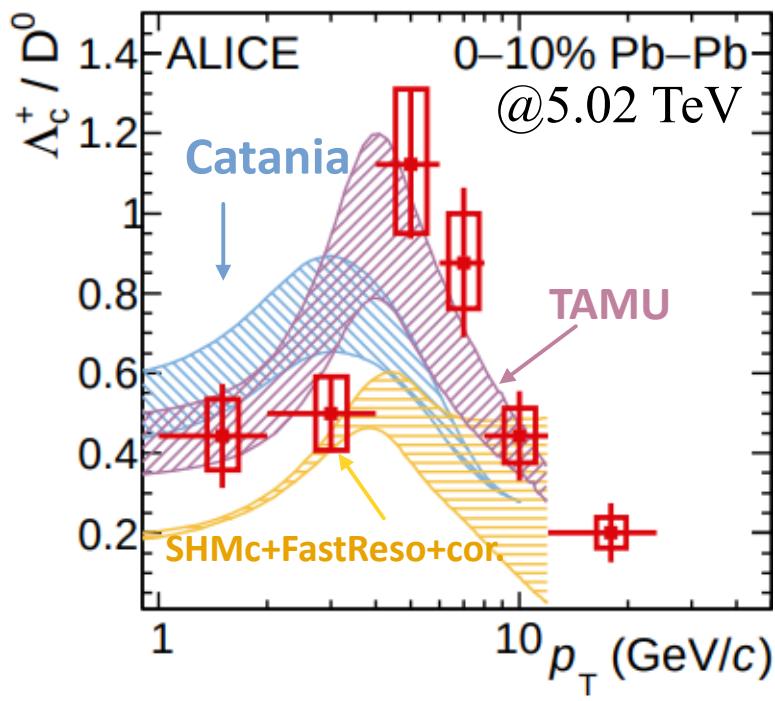
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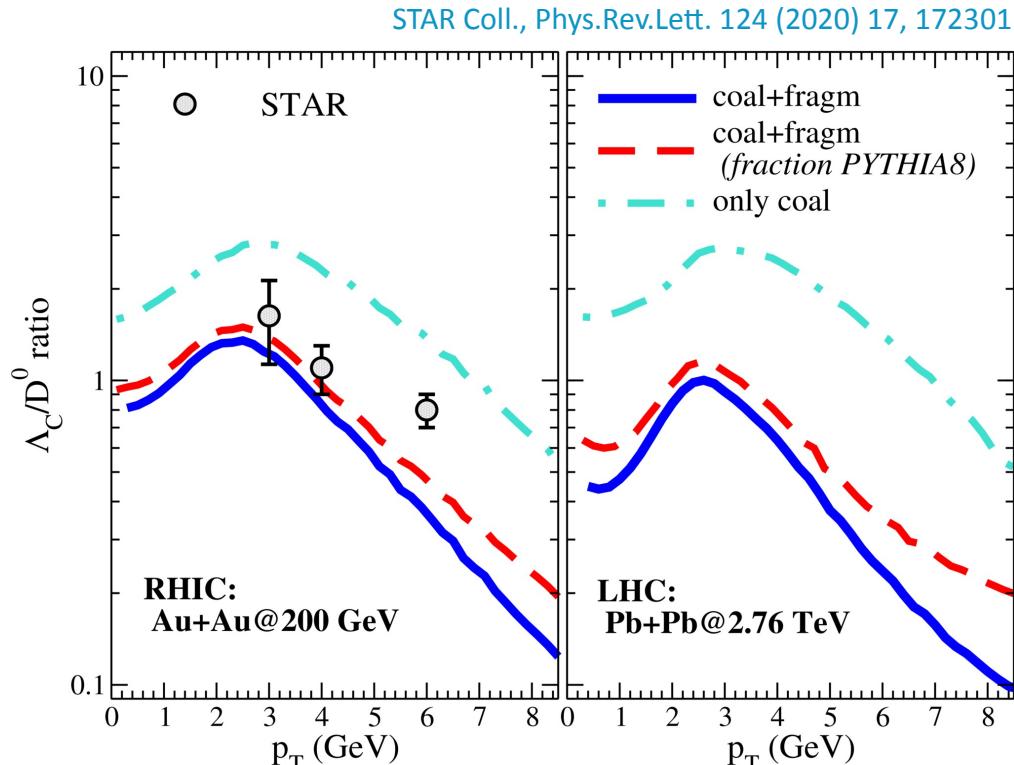
Results for 0-10% in PbPb @5.02TeV:

Consistent with the trend shown at RHIC and LHC @2.76TeV

Available data at low  $p_T$  → differences recombination vs SHM



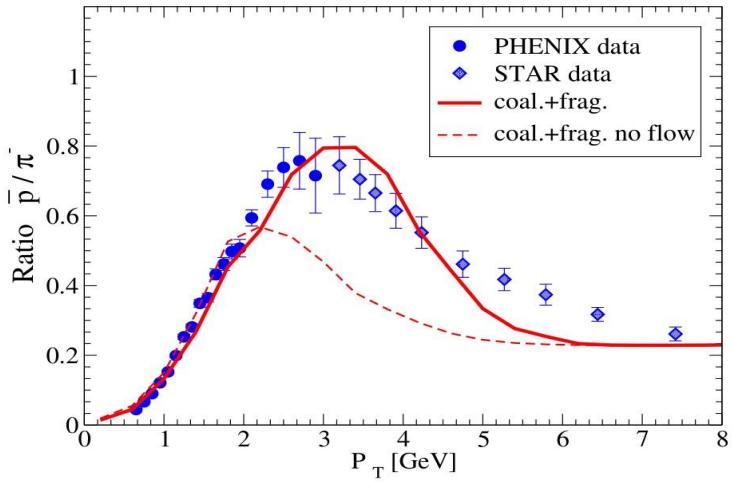
ALICE Coll. arXiv:2112.08156v1



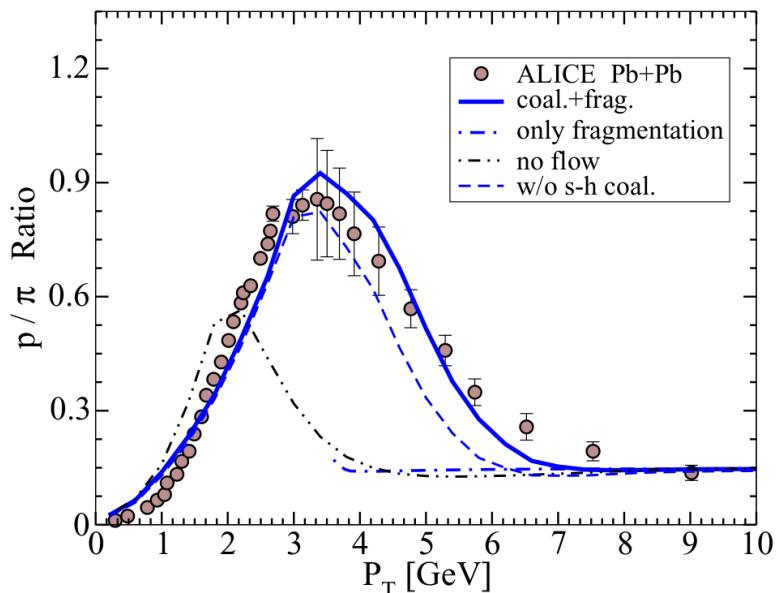
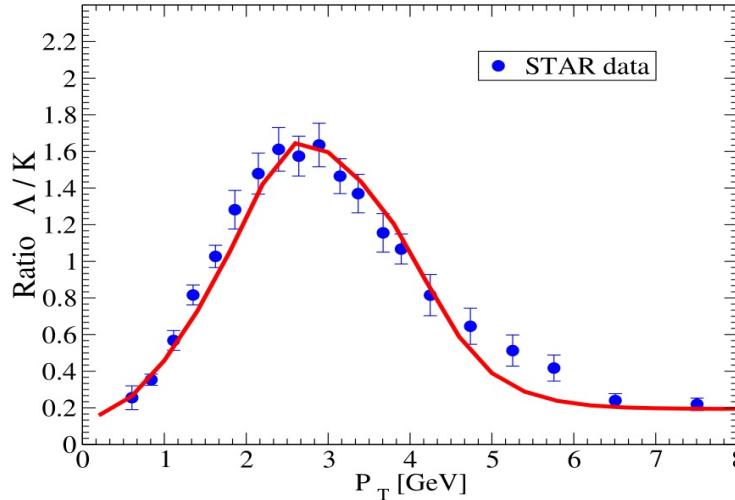
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# Baryon to meson ratio at RHIC & LHC

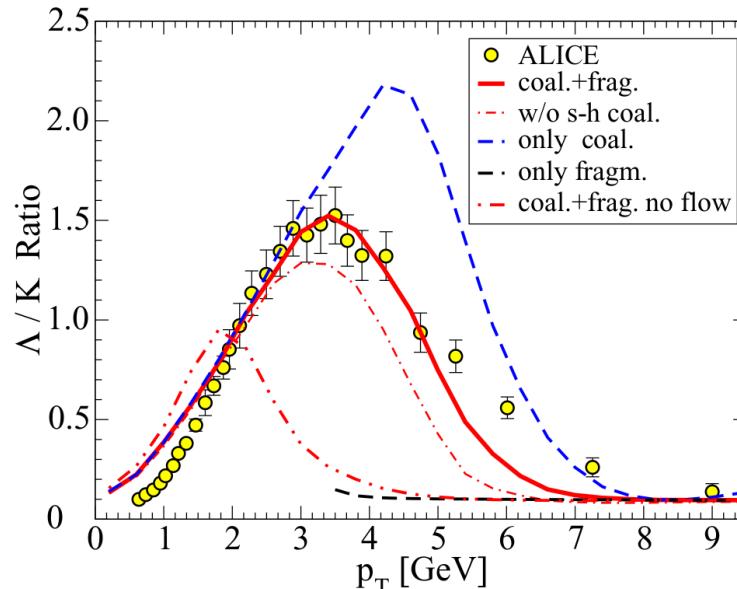
Minissale, Scardina, Greco, Phys.Rev. C 92 (2015) 5,054904



RHIC



LHC



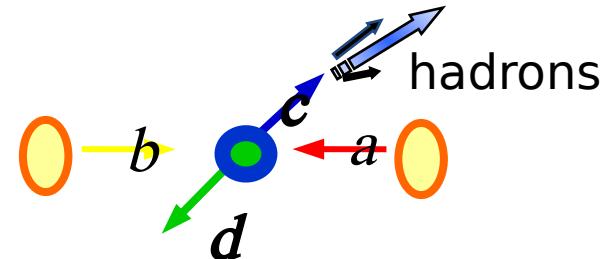
- coalescence naturally predict a baryon/meson enhancement in the region  $p_T \approx 2-4$  GeV with respect to  $pp$  collisions
- Lack of baryon yield in the region  $p_T \approx 5-7$  GeV

# Heavy flavour Hadronization

**Fragmentation:** production from hard-scattering processes (PDF+pQCD).

Fragmentation functions: data parametrization, assumed “universal”

$$\sigma_{pp \rightarrow h} = PDF(x_a, Q^2) PDF(x_b, Q^2) \otimes \sigma_{aa \rightarrow q\bar{q}} \otimes D_{q \rightarrow h}(z, Q^2)$$

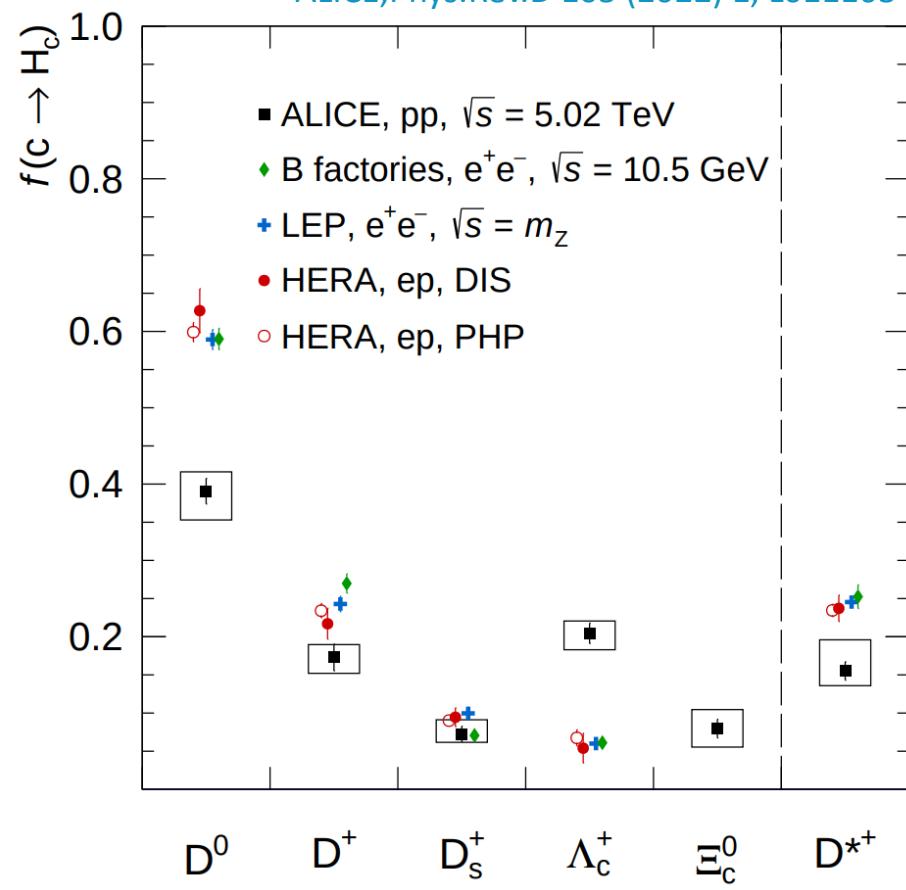


ALICE, Phys. Rev. D 105 (2022) 1, L011103

Things get more complicated after experimental evidence in pp@5TeV:

Fragmentation fractions ( $c \rightarrow h$ ) depends on collision system...and QGP presence?

No more Universality?

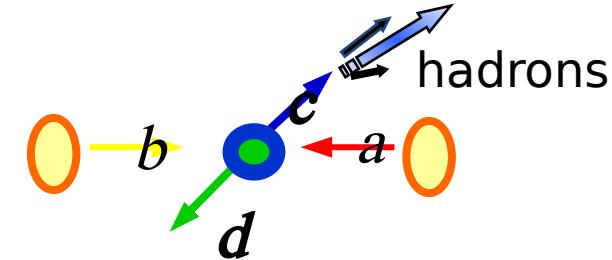


# Heavy flavour Hadronization

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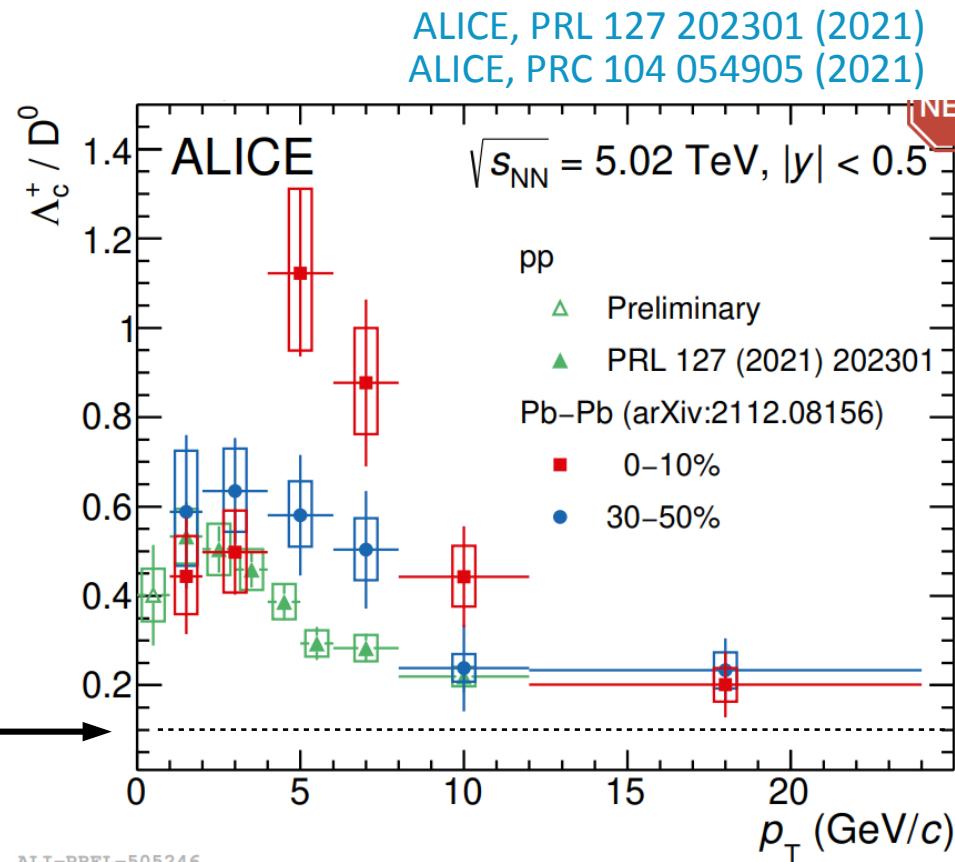
Things get more complicated after experimental evidence in pp@5TeV:

Fragmentation fractions ( $c \rightarrow h$ ) depends on collision system...and QGP presence?

No more Universality?

Baryon/meson ratio is underestimated, and no  $p_T$  dependence

$$\left( \frac{\Lambda_c^+}{D^0} \right)_{e^+ e^-} \simeq 0.1$$



# Small systems: Coalescence in pp?

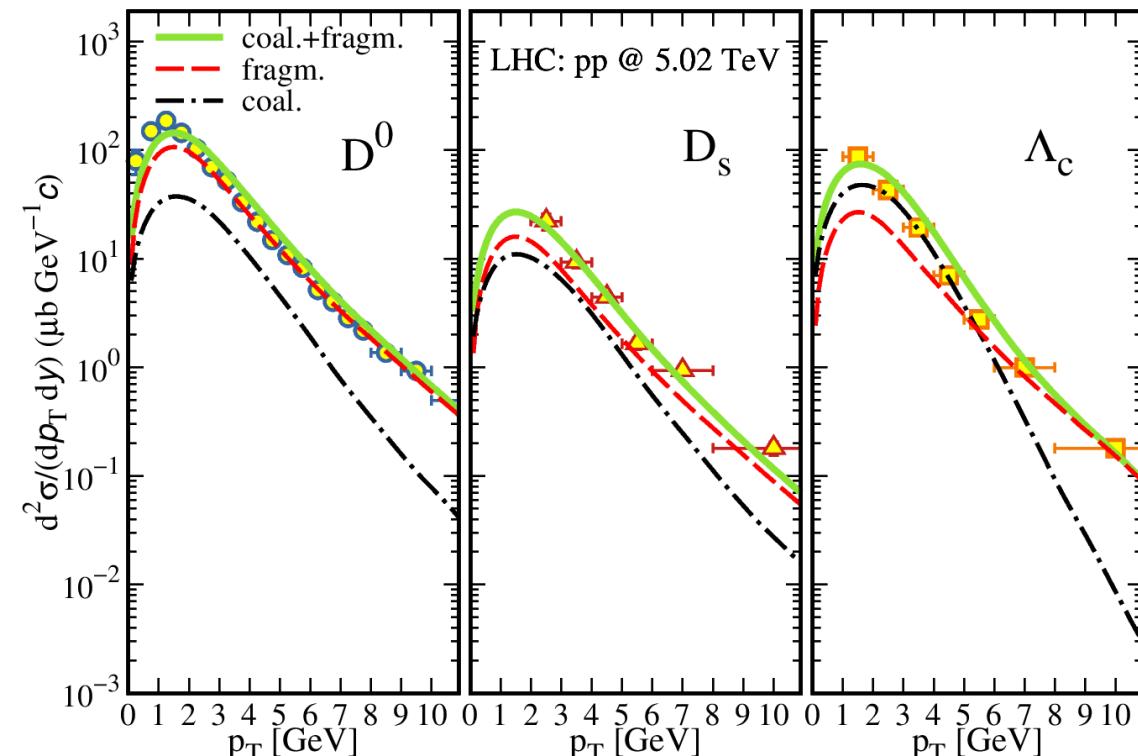
Common consensus of possible presence of QGP in smaller system.

If we assume in p+pp @ 5 TeV a medium similar to the one simulated in hydro:

What if:

- Assuming QGP formation also in pp?
- What coalescence+fragmentation predicts in this case?

Data from:  
S. Acharya et al. (ALICE), Eur. Phys. J. C 79, 388 (2019)  
ALICE Coll., Phys.Rev.Lett. 127 (2021) 20, 202301 - Phys.Rev.C 104 (2021) 5, 054905



p+p @ 5 TeV

- $\tau_{pp}=2 \text{ fm}/c$
- $\beta_0=0.4$
- $R=2.5 \text{ fm}$
- $V \sim 30 \text{ fm}^3$

■ Thermal Distribution ( $p_T < 2 \text{ GeV}$ )

$$\frac{dN_q}{d^2 r_T d^2 p_T} = \frac{g_g \tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right)$$

■ Minijet Distribution ( $p_T > 2 \text{ GeV}$ )  
NO QUENCHING

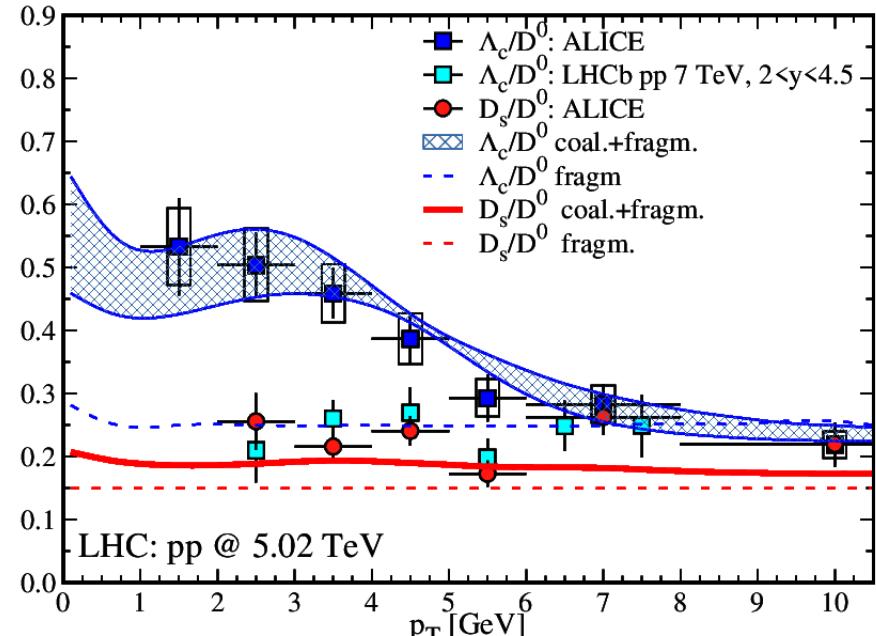
CHARM

FONLL Distribution

wave function widths  $\sigma_p$  of baryon and mesons kept the same from AA to pp

# Small systems: Coalescence in pp?

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622

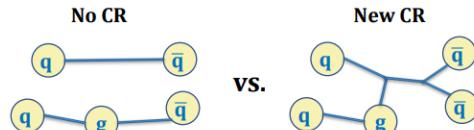


Error band correspond to  $\langle r^2 \rangle$  uncertainty in quark model

Other models:

He-Rapp, Phys.Lett.B 795 (2019) 117-121: Increase  $\approx 2$  to  $\Lambda_c$  production: SHM with resonance not present in PDG

**PYTHIA8 + color reconnection**  
CR with SU(3) weights and string length minimization

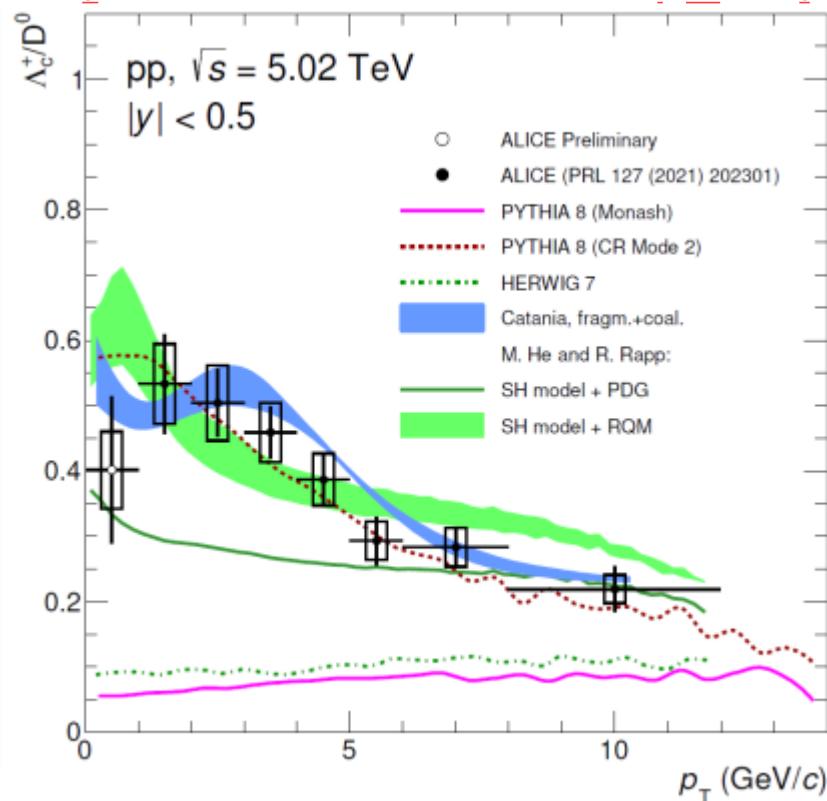


21

Reduction of rise-and-fall behaviour in  $\Lambda_c / D^0$  ratio:

- Confronting with AA: Coal. contribution smaller w.r.t. Fragm.
- FONLL distribution flatter w/o evolution trough QGP
- Volume size effect

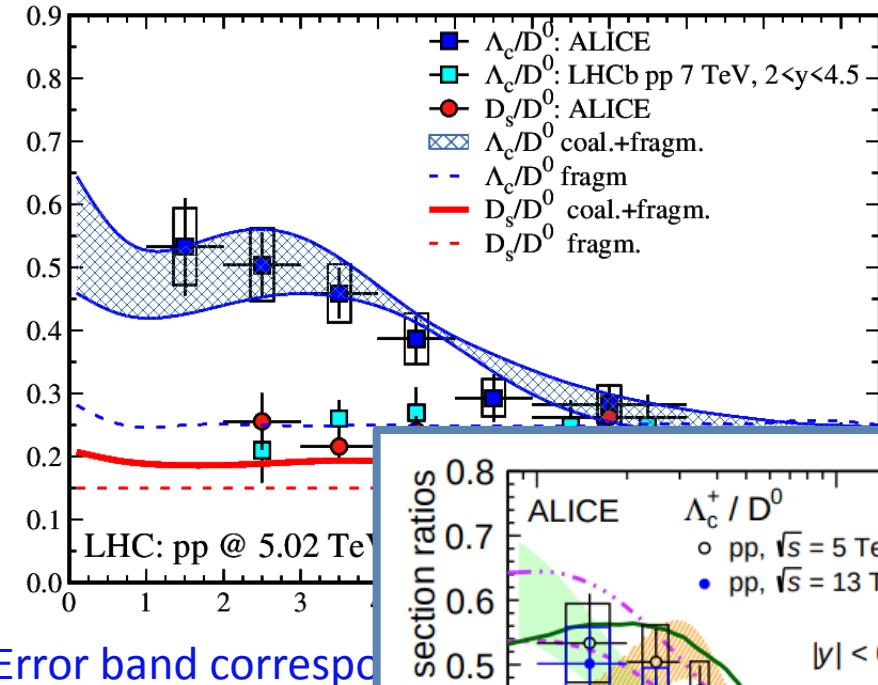
The increase of  $\Lambda_c$  production in pp have effect on  $R_{AA}$  of  $\Lambda_c$



ALICE, Phys.Rev.Lett. 127 (2021) 20, 202301

# Small systems: Coalescence in pp?

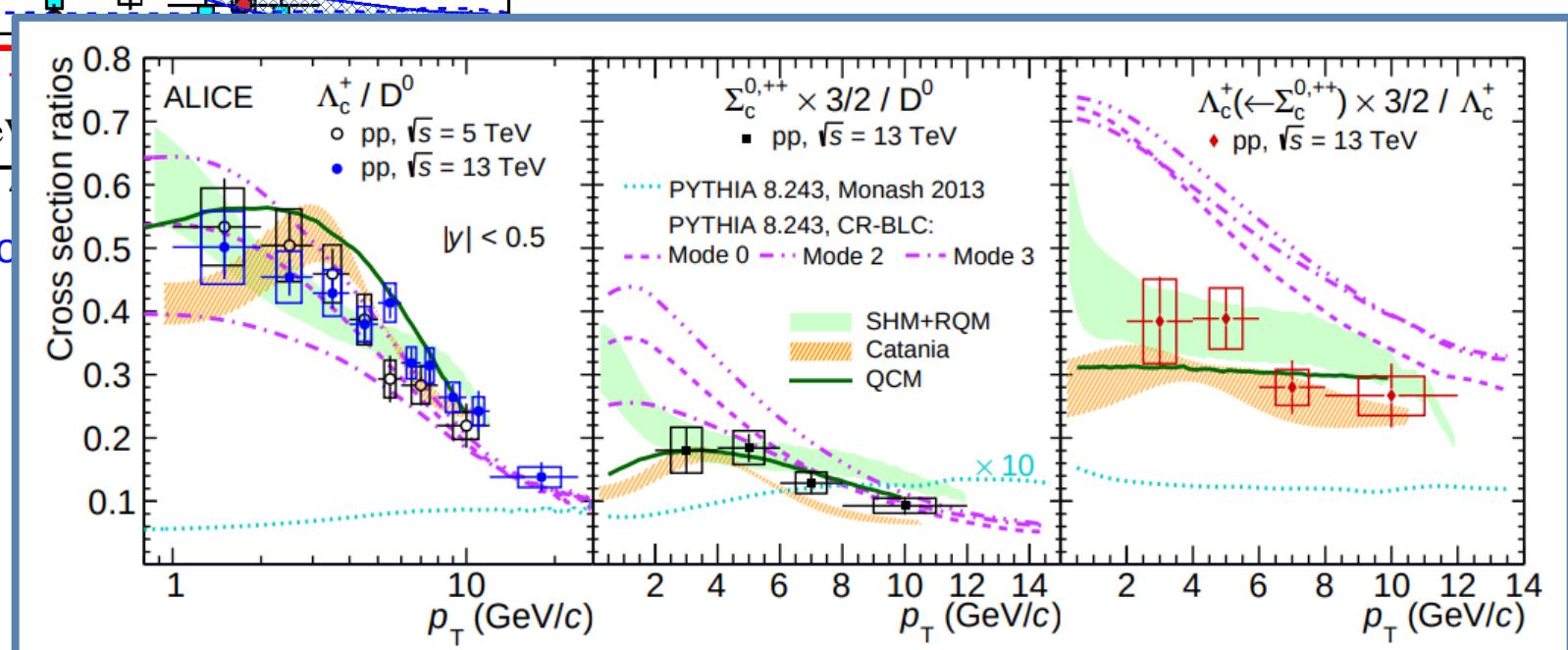
V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622



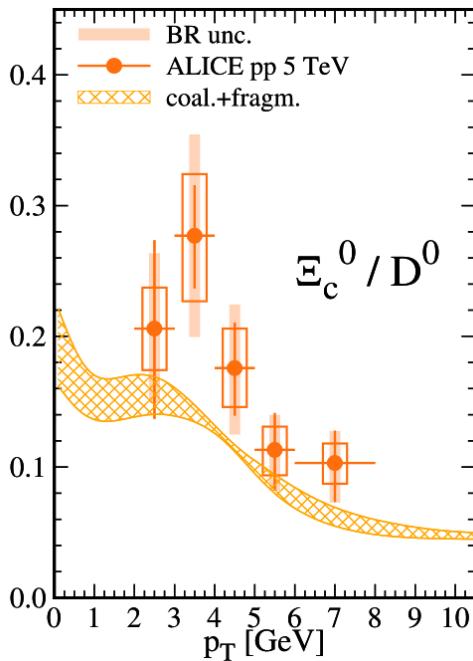
Reduction of rise-and-fall behaviour in  $\Lambda_c / D^0$  ratio:

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- Volume size effect

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# Small systems: Coalescence in pp?



## New measurements of heavy hadrons at ALICE:

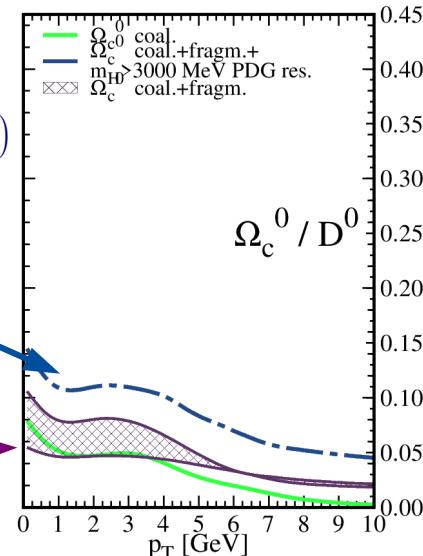
- $\Xi_c/D^0$  ratio, same order of  $\Lambda_c/D^0$ : coalescence gives enhancement
- very large  $\Omega_c/D^0$  ratio, our model does not get the big enhancement

Assuming additional PDG resonances with  
 $J=3/2$  and decay to  $\Omega_c$  additional to  $\Omega_c^0(2770)$

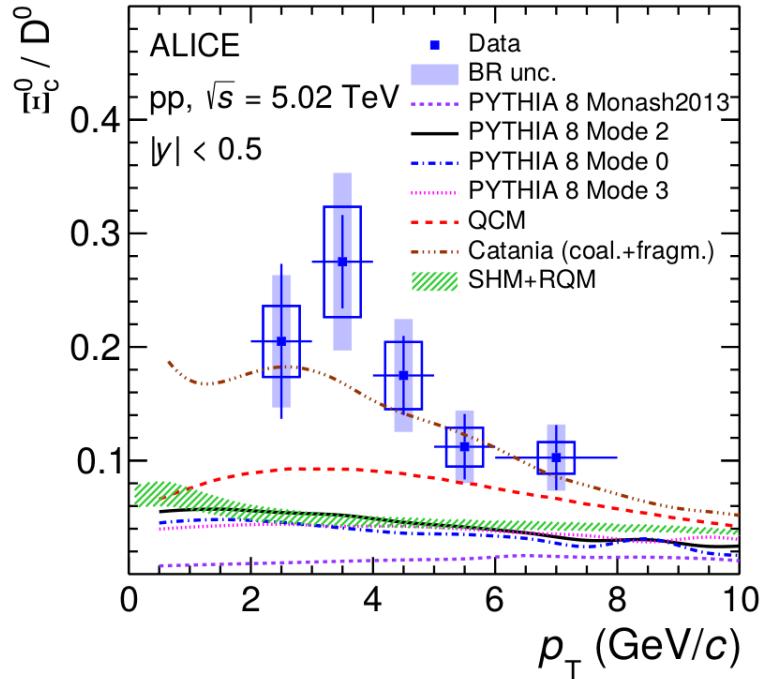
$\Omega_c^0(3000), \Omega_c^0(3005), \Omega_c^0(3065), \Omega_c^0(3090), \Omega_c^0(3120)$

supply an idea of how these states may affect  
the ratio

Error band correspond to  $\langle r^2 \rangle$  uncertainty in  
quark model



# Small systems: Coalescence in pp?



Assuming additional PDG resonances with  $J=3/2$  and decay to  $\Omega_c^0$  additional to  $\Omega_c^0(2770)$

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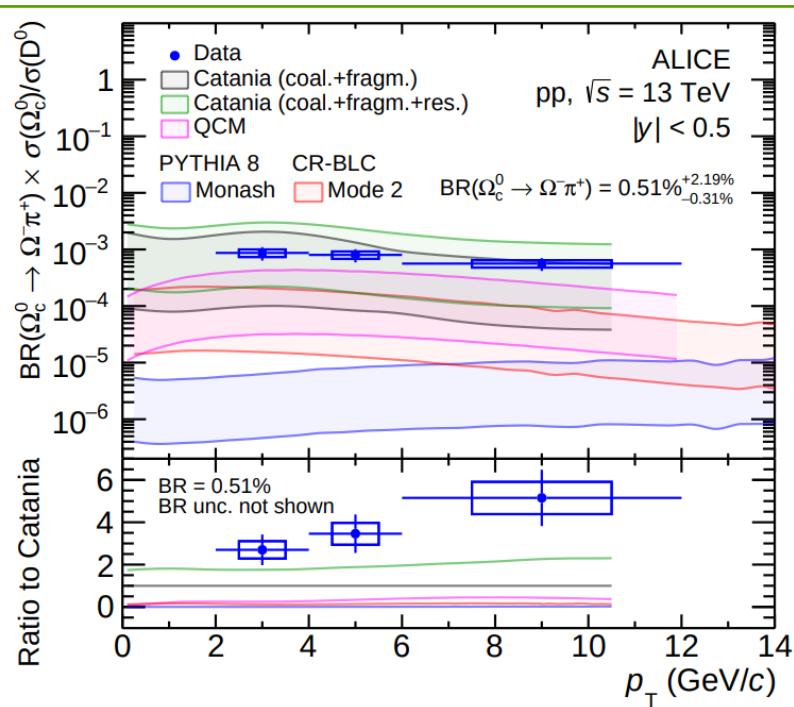
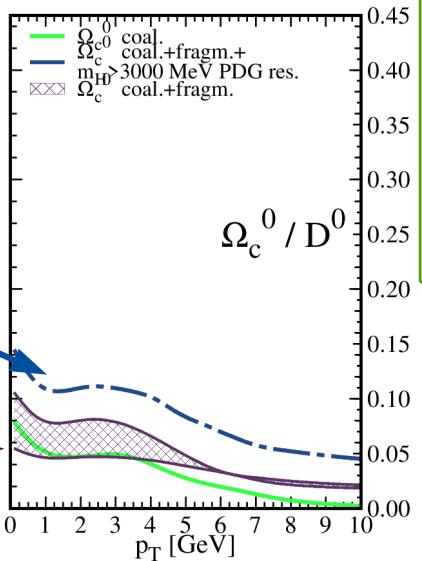
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- very large  $\Omega_c/D^0$  ratio, our model does not get the big enhancement

Uncertainties bands coming from the Branching Ratio error



ALICE Coll. JHEP 10 (2021) 159  
ALICE Coll. arXiv:2205.13993

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 136622

# Multicharm production Pb-Pb, Kr-Kr, Ar-Ar, O-O

## Baryon

$\Xi_{cc}^{+,++} = dcc, ucc$	3621	$\frac{1}{2} (\frac{1}{2})$
$\Omega_{scc}^+ = scc$	3679	$0 (\frac{1}{2})$
$\Omega_{ccc}^{++} = ccc$	4761	$0 (\frac{3}{2})$

## Resonances

$\Xi_{cc}^*$	3648	$\frac{1}{2} (\frac{3}{2})$	$1.71 \times g.s$
$\Omega_{scc}^*$	3765	$0 (\frac{3}{2})$	$1.23 \times g.s$

like S.Cho and S.H. Lee, PRC101 (2020)  
from R.A. Briceno et al., PRD 86(2012)

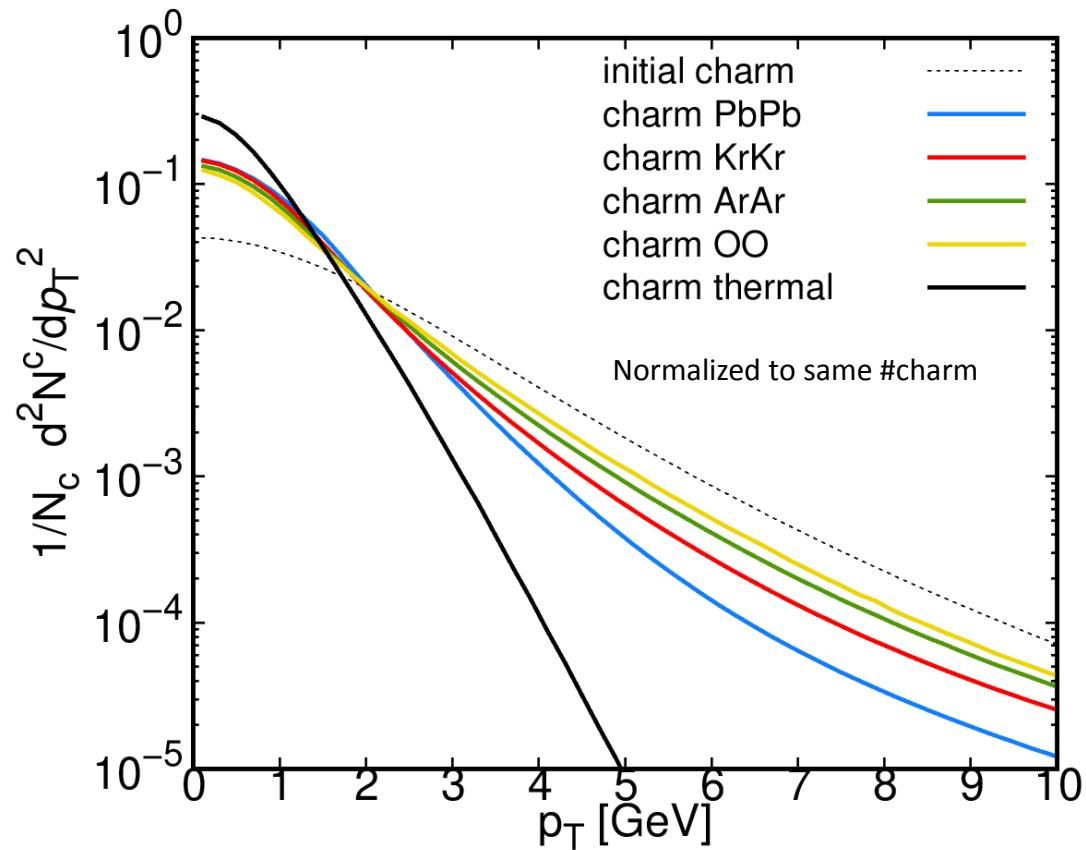
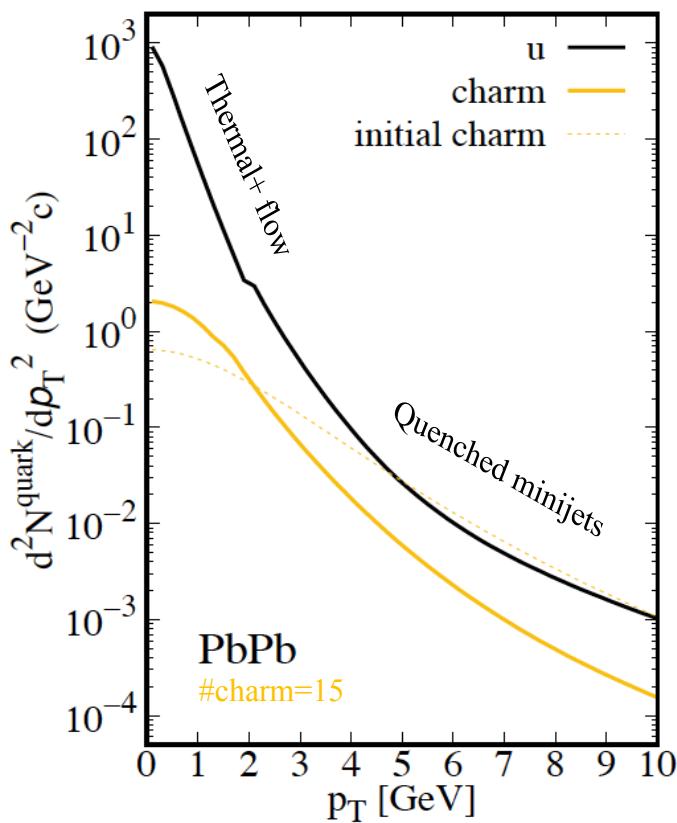
## Strengths of the approach:

- Does not rely on distribution in equilibrium for charm  
→ useful for small AA down to pp collisions and at  $p_T > 3\text{-}4 \text{ GeV}$
- Provide a  $p_T$  dependence of spectra and their ratios vs  $p_T$

Widths from harmonic oscillator  
rescaling and from  $\langle r \rangle$  of  
Tsingua approach

	$\sigma_{p_1}(\text{GeV})$	$\sigma_{p_2}(\text{GeV})$	$\sigma_{r_1}(fm)$	$\sigma_{r_2}(fm)$
$\Xi_c$	0.262	0.438	0.751	0.450
$\Omega_c$	0.345	0.557	0.572	0.354
$\Xi_{cc}^\omega$	0.317	0.573	0.622	0.344
$\Omega_{ccc}^{\sigma_r \sigma_p = 3/2}$	0.522	0.522	0.566	0.566

# Charm distribution in PbPb-KrKr-ArAr-OO from transport approach

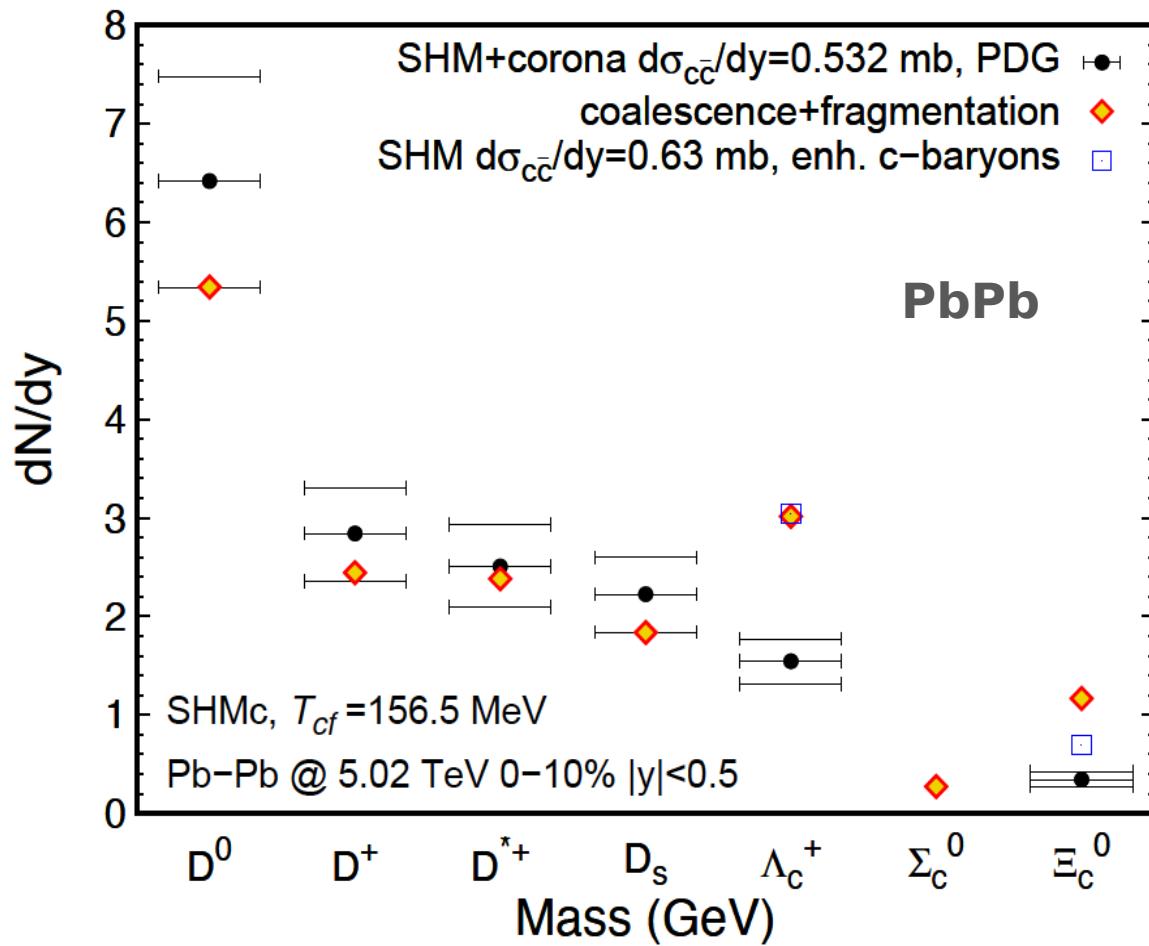


	OO	ArAr	KrKr	PbPb
$R_0(f m)$	2.76	3.75	4.9	6.5
$R_{\max}(f m)$	5.2	7.65	10.1	14.1
$\tau(f m)$	4	5	6.2	8
$\beta_{\max}$	0.55	0.6	0.64	0.7
$V_{ y <0.5}(f m^3)$	345	920	2000	5000

Volume scales with  $A$ , now we employ the same value of SHM  
 A. Andronic et al., JHEP (2021) 035

Shadowing on charm included as a  $K = 0.65$  factor [no  $p_T$  dependence]  
 $\# \text{charm} = 15$  (PbPb), 4.35 (KrKr), 1.5 (ArAr), 0.4 (OO)

# Yields in PbPb from coalescence vs SHM



We have performed a small readjustment of the widths and  $\langle r^2 \rangle$  to reduce  $\Lambda_c$  to get similar yields as SHM with enhanced set of c-baryons with  $\sigma_{cc} = 0.63 \text{ mb}$

using SHM parameters:

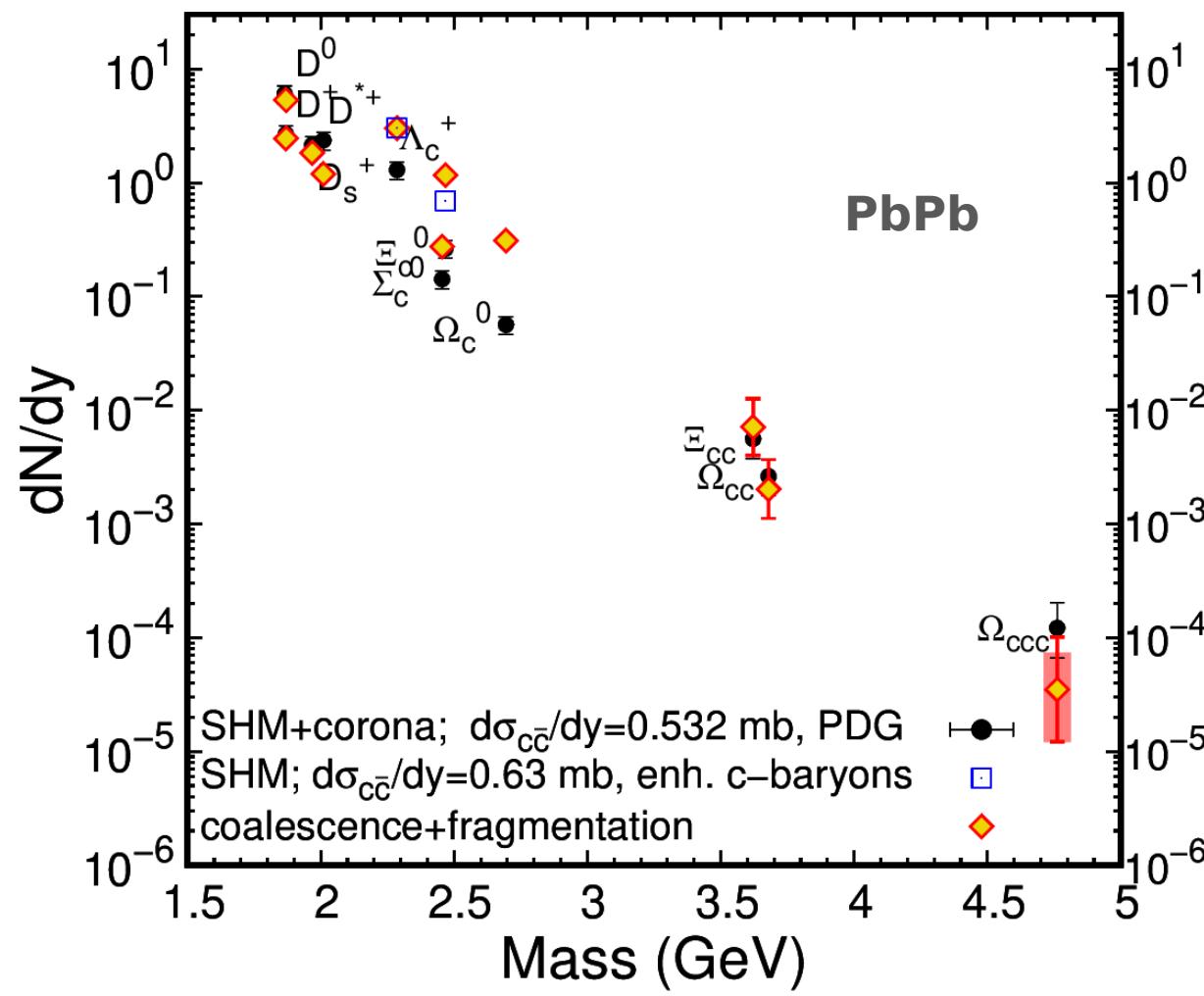
Volume =  $5000 \text{ fm}^3$

Temperature =  $0.155 \text{ MeV}$

$N_c \approx 15$  with  $\sigma_{cc} = 0.63 \text{ mb}$

$\Xi_c^0, \Omega_c^0$  (next slide), have a larger difference wrt SHM

# Yields in PbPb from coalescence vs SHM



according to Tsinghua PLB746 (2015) [Solution of Schoedinger eq. under  $V(r)$ ]

Obtained starting for D from the  $\langle r^2 \rangle$  of the quark model, but for  $\Lambda_c$  we have reduce it by 20%.

$\Sigma_c^0, \Xi_c^0, \Omega_c^0$ , from quark model

$\Xi_{cc}, \Omega_{cc}$  obtained rescaling the width according to the harmonic oscillator relations

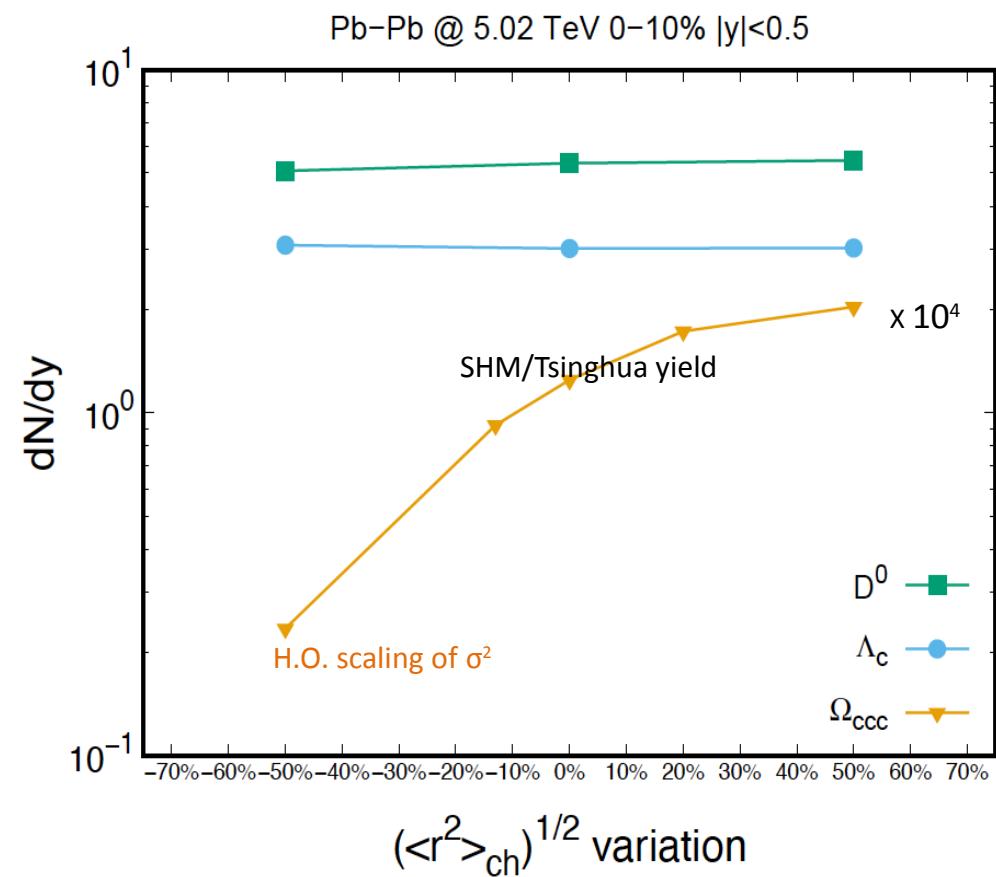
$$\sigma_{ri} = \frac{1}{\sqrt{\mu_i \omega}}$$

$$\mu_1 = \frac{m_1 m_2}{m_1 + m_2}; \mu_2 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}$$

$\Omega_{ccc}$

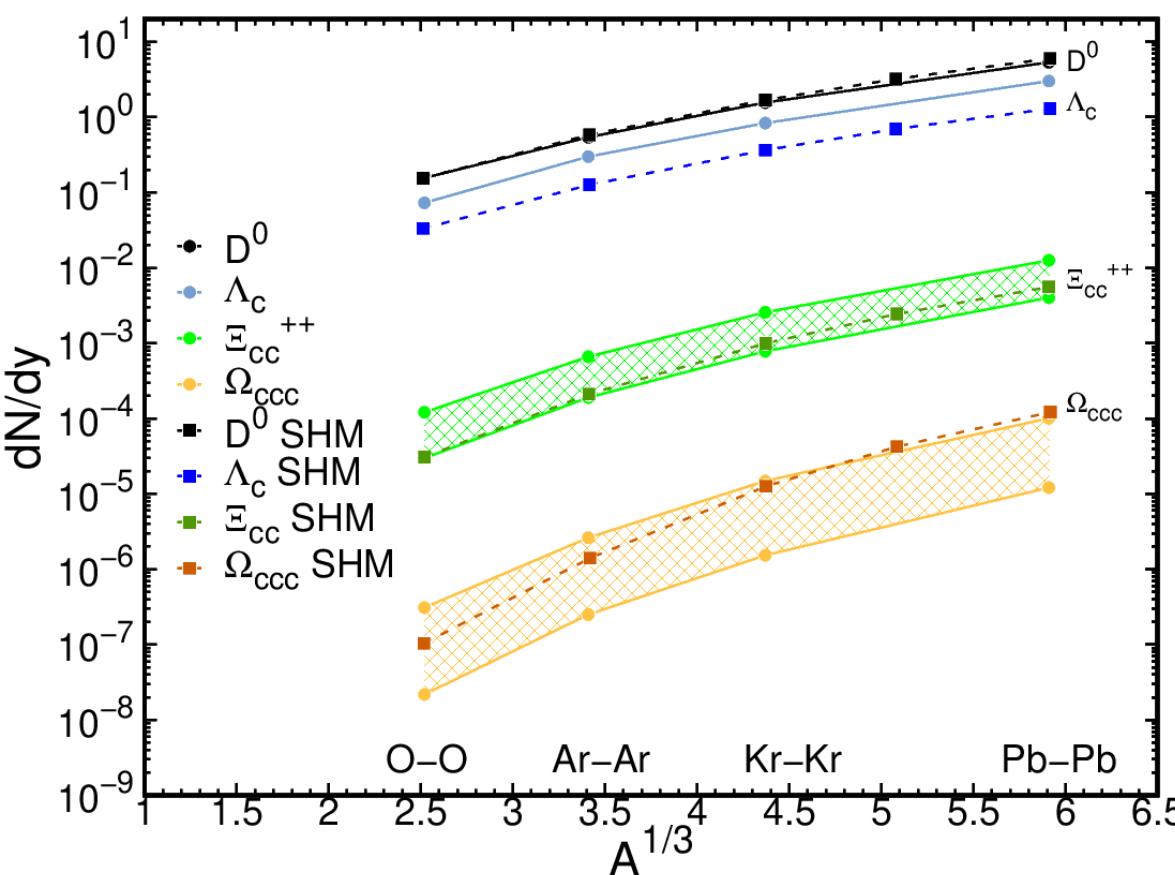
- upper limit: thermal distribution
- lower limit: PbPb distribution with widths scaled as standard Harm. Oscill. ( $\omega$  from  $\Omega_c^0$ )
- box upper limit:  $\sigma_r \cdot \sigma_p \approx 1.5 + \langle r \rangle = 0.5$  fm

# Microscopic details effect on $\Omega_{ccc}$ production



- $D^0$  and  $\Lambda_c$  determine the majority of the yield, the radius variation is compensated by the constraint on the charm hadronization
- A  $\pm 50\%$  in the radius of  $\Omega_{ccc}$  induces **a change in the yield by about 1 order of magnitude**
- Here 0% corresponds to  $\langle r \rangle = 0.5$  fm that gives a yield nearly equal to SHM, a simple harmonic oscillator rescaling of  $\sigma^2$  would give a value similar to -50%

# Multi-charm production vs A-A: Yields



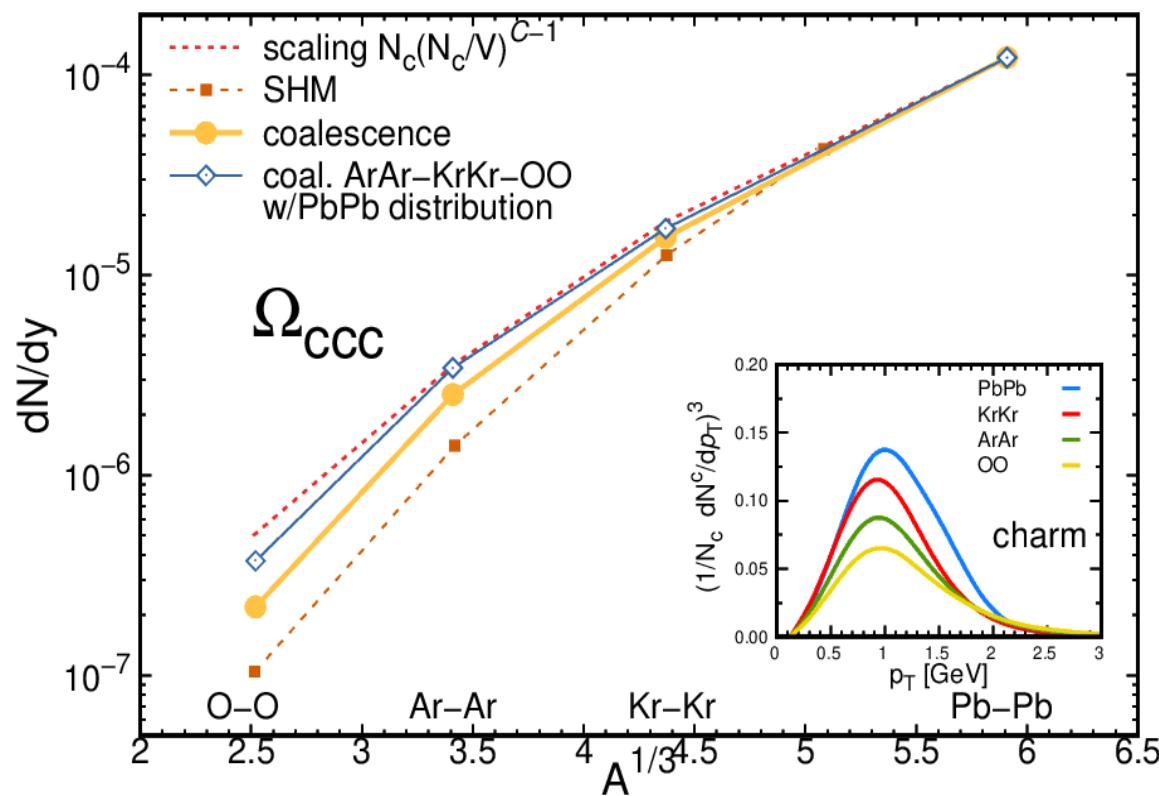
$\Xi_{cc}$      $\Omega_{ccc}$

→ upper limit: thermal distribution

→ lower limit: PbPb distribution with widths scaled as standard Harm. Oscill.  
( $\omega$  from  $\Xi_c, \Omega_c^0$ )

- Compatible yields within the two scenarios w.r.t. SHM
- Different trends with  $A^{1/3}$  increasing the number of constituent charm quarks  
Lack of canonical suppression but...

# Yields scaling with A



## Scaling of SHM (for A>40)

$$\frac{dN^{AA}}{dy}(h^i) = \frac{dN^{PbPb}}{dy}(h^i) \left(\frac{A}{208}\right)^{(\alpha+3)/3} \frac{f_{can}(\alpha, A)}{f_{can}(\alpha, Pb)}$$

For coalescence, in an homogeneous density background in equilibrium at fixed T, discarding flow and wave functions effects the expected scaling is:

$$V \left(\frac{N_c}{V}\right)^c = N_c \left(\frac{N_c}{V}\right)^{C-1}$$

with  $N_c \propto A^{4/3}$  and  $V \propto A$   
 → the scaling corresponds to  $\frac{dN}{dy} \propto A^{\frac{C+3}{3}}$

like in SHM w/o canonical suppression

- If the  $p_T$ -distribution does not change we obtain the scaling expected
- There is an effect due to different charm distributions. In Ar-Ar it reduces  $\Omega_{ccc}$  by  $\approx 1.3$  factor, in O-O it is  $\approx 1.7$
- the cube of the distribution gives an idea of this difference, but Wigner function mitigate the effect

A larger production of coalescence w.r.t. SHM for small systems:

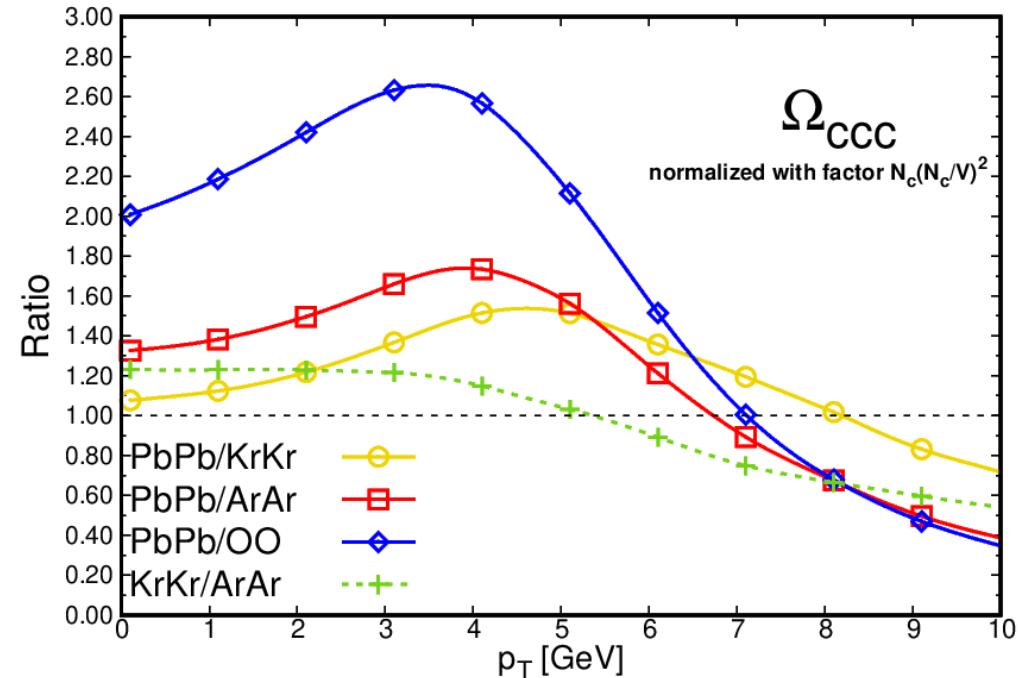
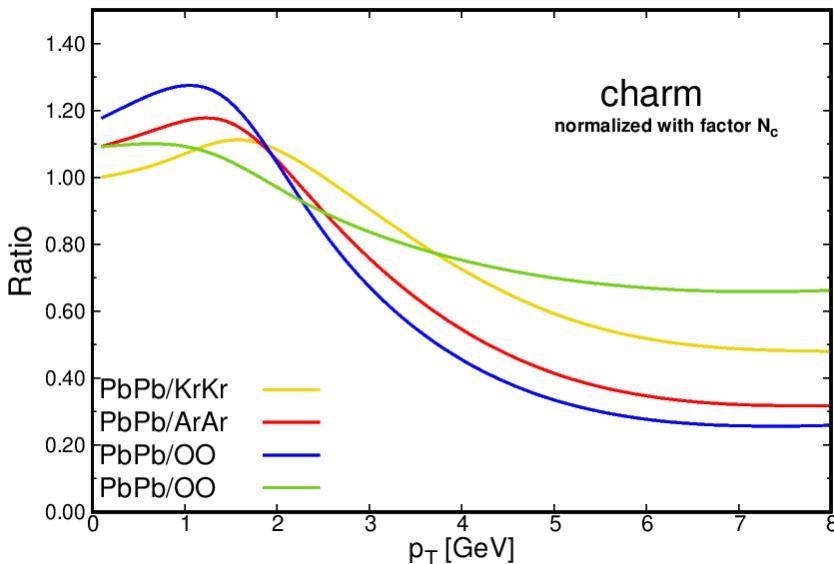
**- Lack of canonical suppression, but e-b-e fluctuations can enhance production?**

$$\langle N^3 \rangle > \langle N \rangle^3$$

# Ratios of $p_T$ distribution of $\Omega_{ccc}$ in PbPb/KrKr/ArAr

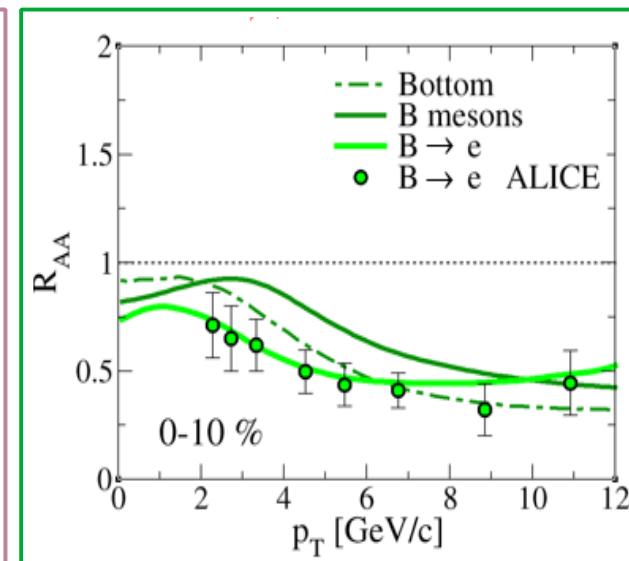
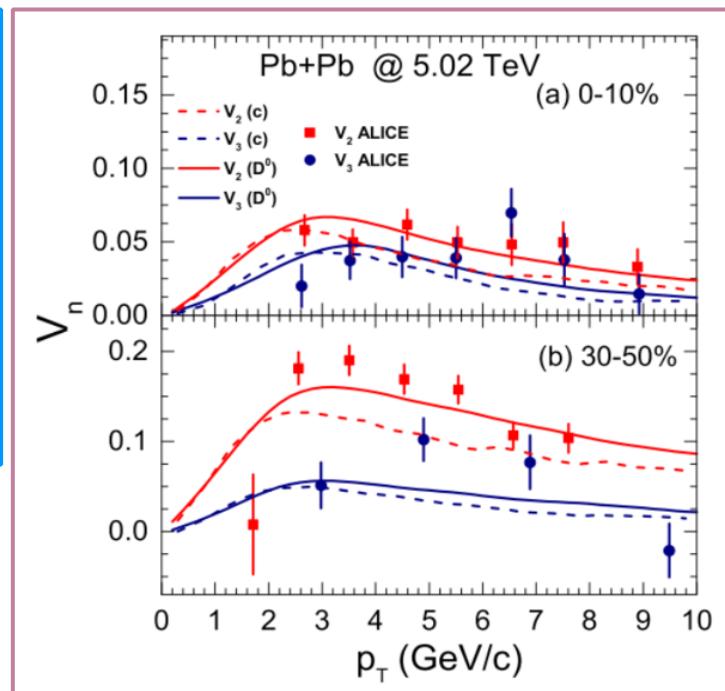
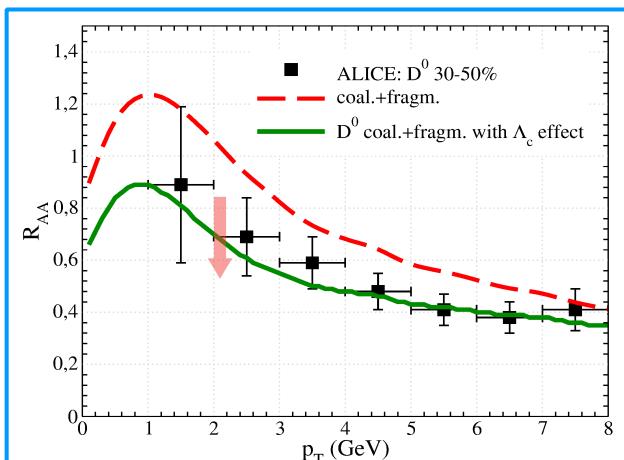
caveat: in O-O no  $N_c$  and V scaling with fixed distribution (multipl. factor)

The  $\Omega_{ccc}$   $p_T$  distribution, with only coalescence, in the intermediate region decreases faster in larger systems.



- It can be a meter of non-equilibrium. Translation of feature of charm spectra at low  $p_T$  into higher momentum region.
- More sensitive for multicharm respect to D mesons and  $\Lambda_c$ . Both effects of light quarks and fragmentation

## Implications and developments:



The large  $\Lambda_c$  production has effects on the  $R_{AA}$  of  $D^0$ , because of the charm conservation

Coalescence give an enhancement to the  $v_n(p_T)$  of final hadrons compared to the charm  $v_n(p_T)$ .  
 Sambataro,Sun,Minissale,Plumari,Greco,  
 Eur.Phys.J.C 82 (2022) 9, 833

Electrons from semileptonic  $B$  meson decay with a coal + fragm model for  $B$  meson production  
 Sambataro, Minissale et al.(in preparation)

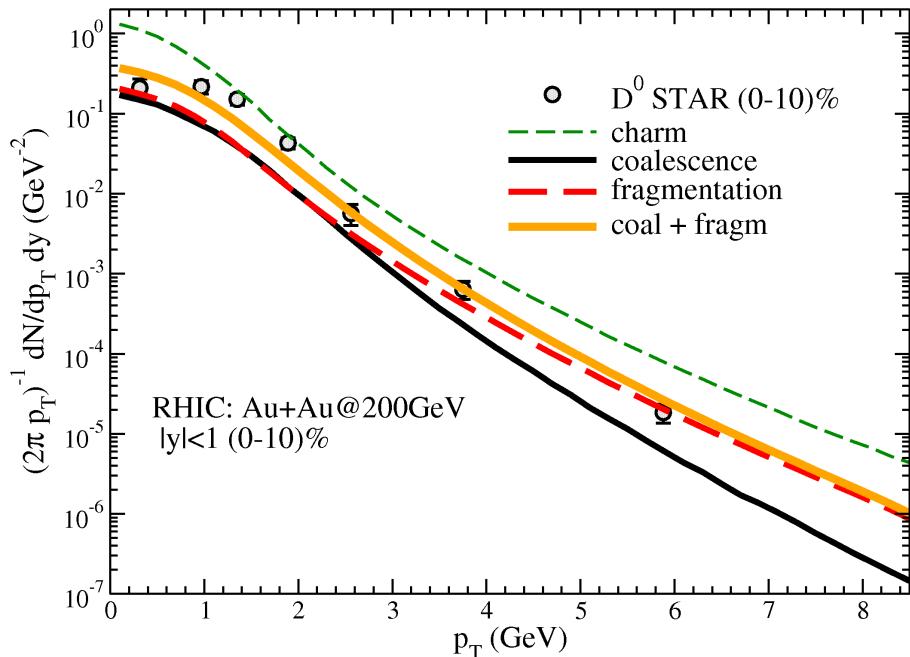
# Conclusions

- *Good agreement with experimental data of hadrons spectra in AA collisions from RHIC to LHC*
- *Extension to pp: description of D mesons and  $\Lambda_c$  spectra*
- *Coalescence plus fragmentation gives peculiar enhancement in baryon/ meson ratio for all heavy hadrons  $\Lambda_c, \Xi_c, \Omega_c$*
- *Outlook: multicharm hadrons production*

# Backup Slides

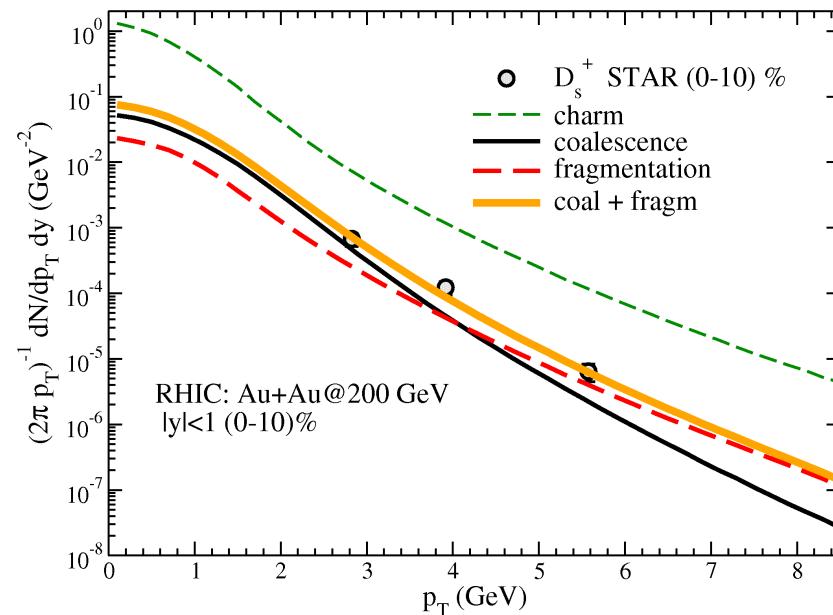
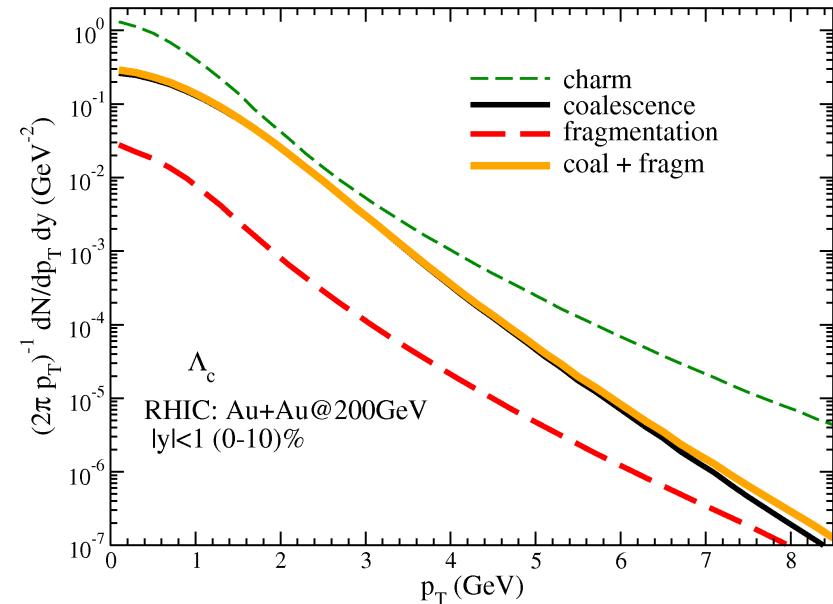
# RHIC: results

S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348



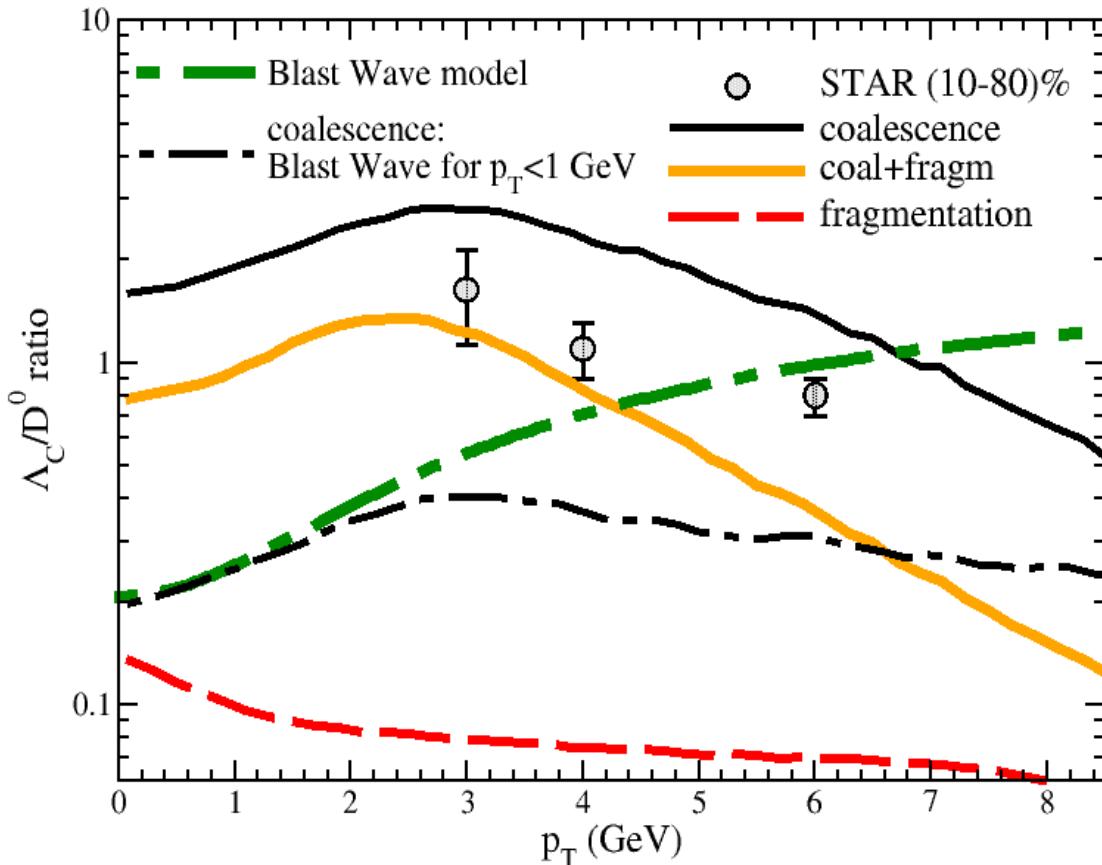
Data from STAR Coll. PRL 113 (2014) no.14, 142301

- For  $D^0$  coalescence and fragmentation comparable at 2 GeV
- fragmentation fraction for  $D_s^+$  are small and less than about 8% of produced total heavy hadrons
- $\Lambda_c^+$  fragmentation is even more smaller, coalescence gives the dominant contribution



# RHIC: Baryon/meson

STAR, Phys.Rev.Lett. 124 (2020) 17, 172301



Compared to light baryon/meson ratio  
the  $\Lambda_c/D^0$  ratio has a larger width  
(flatter)

More flatter  $\rightarrow$  should coalescence  
extend to higher  $p_T$ ? Indication also in  
light sector

V. Minissale, F. Scardina, V. Greco PRC 92,054904 (2015)  
Cho, Sun, Ko et al., PRC 101 (2020) 2, 024909

Needed data at low  $p_T$

# Elliptic Flow – Quark Number Scaling

Fourier expansion of the azimuthal distribution

$$f(\varphi, p_T) = 1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos n\varphi$$

momentum anisotropy in the transverse plane

$n=2$  Elliptic flow

coalescence brings to

$$\begin{aligned} v_{2,M}(p_T) &\approx 2v_{2,q}(p_T/2) \\ v_{2,B}(p_T) &\approx 3v_{2,q}(p_T/3) \end{aligned}$$

Partonic  
elliptic flow

Hadronic  
elliptic flow

Assumption

- one dimensional
- Dirac delta for Wigner function
- isotropic radial flow
- not including resonance effect

# Transport approaches

## Fokker-Planck ( $T \ll m_b$ soft scattering)

$$\frac{\partial}{\partial t} f_Q = \gamma \frac{\partial}{\partial p_i} [p_i f_Q] + D_p \nabla_p^2 [f_Q]$$

Drag coeff.                      Momentum diffusion coeff.

Background: Hydro/transport expanding bulk

### -Fluctuation dissipation theorem

### -Spatial diffusion coefficient

a measure of thermalization time

$$\langle x^2 \rangle - \langle x \rangle = 6 D_s t$$

$$D_p = E T \gamma$$

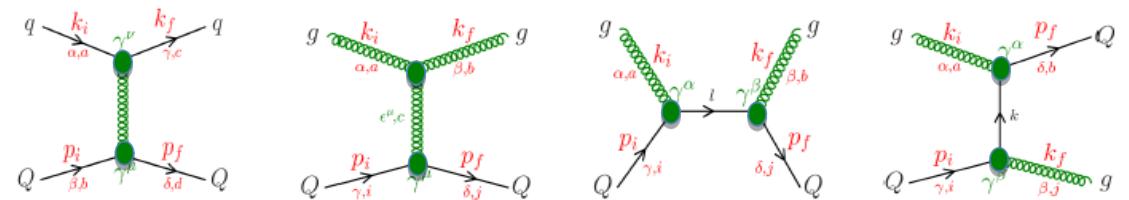
$$D_s = \frac{T}{M \gamma} = \frac{T^2}{D_p} = \frac{T}{M} \tau_{th}$$

## Boltzmann kinetic transport

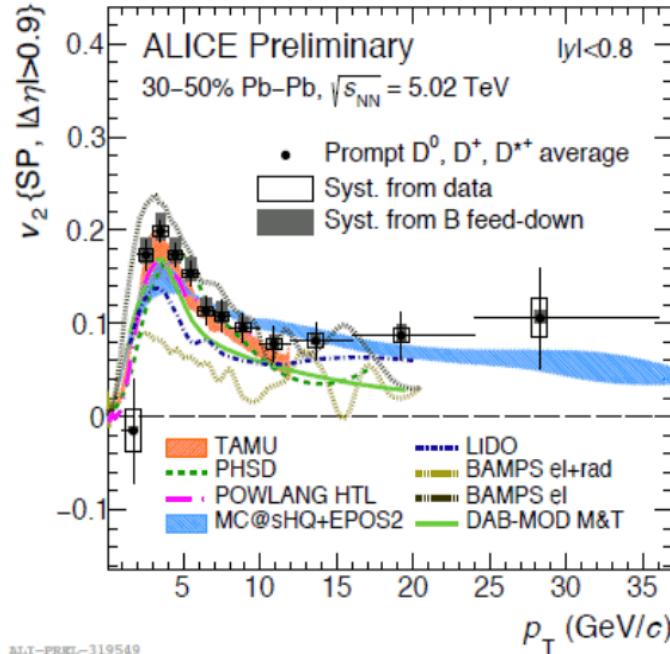
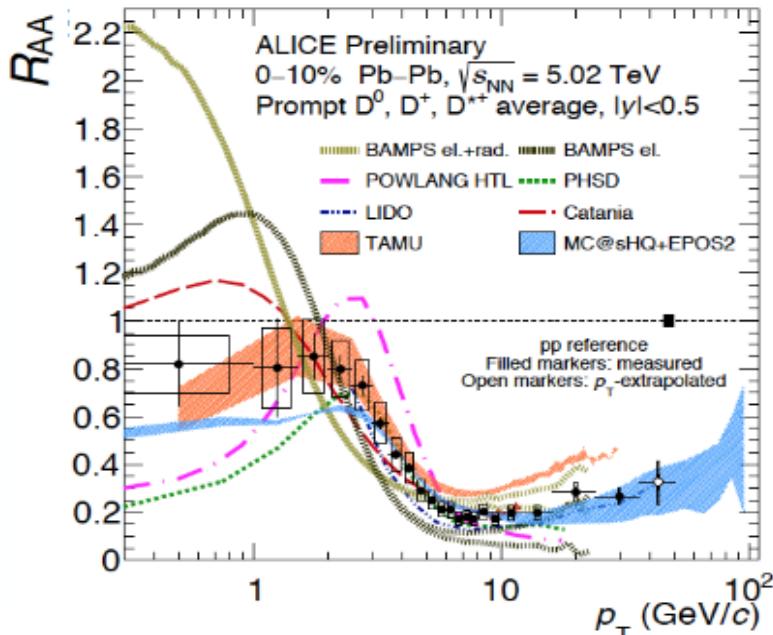
$$p^\mu \partial_\mu f_Q(x, p) = C[f_q, f_g, f_Q]$$

### Collision integral

$$C[f_q, f_g, f_Q] = \frac{1}{2E_1} \int \frac{d^3 p_2}{2E_2(2\pi)^3} \int \frac{d^3 p_1'}{2E_1'(2\pi)^3} [f_Q(p_1') f_{q,g}(p_2') - f_Q(p_1) f_{q,g}(p_2)] \times |M_{(q,g) \rightarrow Q}(p_1 p_2 \rightarrow p_1' p_2')| (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')$$



# Transport approaches



Models not really tested at  $p \rightarrow 0$

The new data  $\rightarrow$  determine  $D_s(T)$  more properly,

i.e.  $p \rightarrow 0$  where it is defined and computed in IQCD

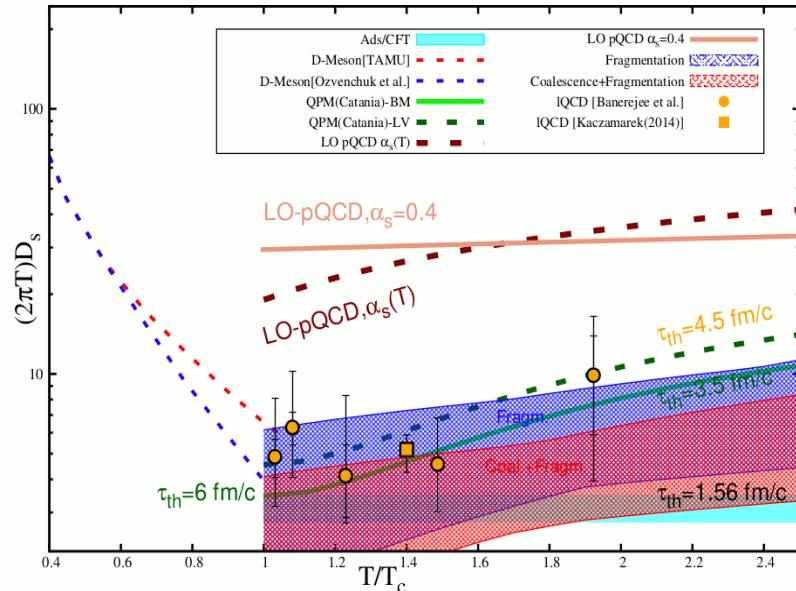
2018-2019

Several Collab. in joint activities:

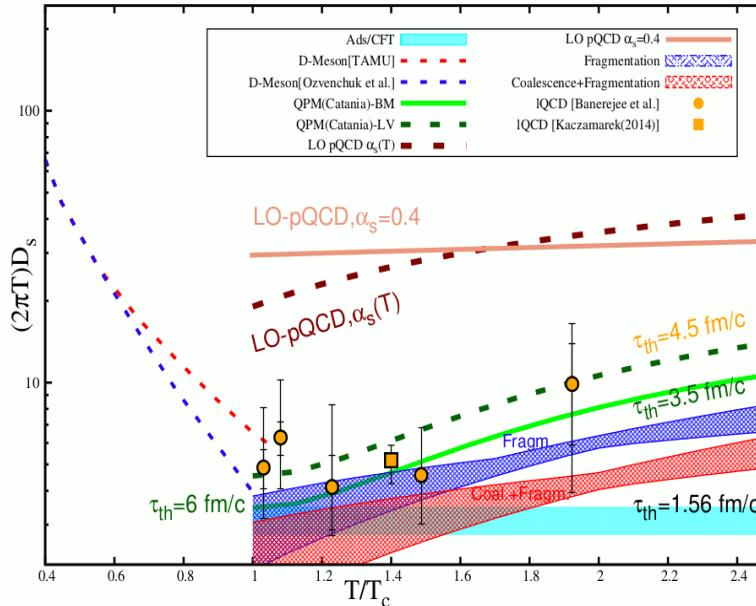
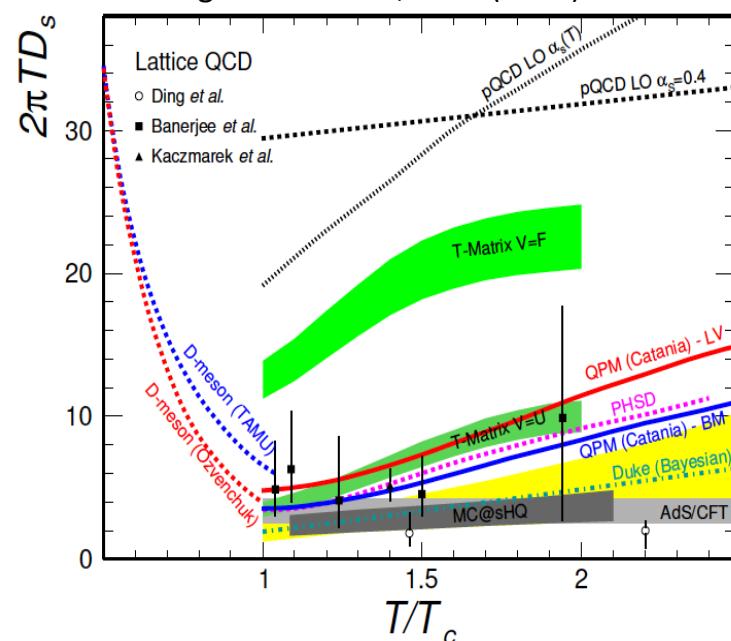
- EMMI-RRTF:  
R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS:  
S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

# Transport coefficient

Z. Citron et al., CERN Yellow Rep. Monogr. 7 (2019) 1159



X. Dong and V. Greco, PPNP(2019)



Different hadronization models can affect the extraction of the charm quark diffusion coefficient

**2018-2019**

**Several Collab. in joint activities:**

- EMMI-RRTF: R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS: S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)