Studying gluon TMDs via double J/ψ production in proton-proton collisions

Based on work of Florent SCARPA and Alice COLPANI SERRI

Jelle BOR

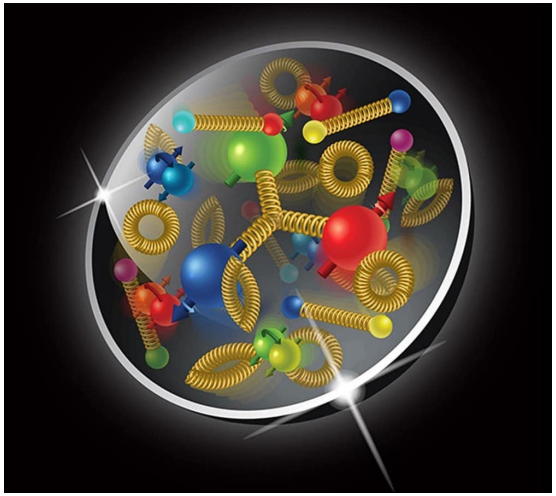
Heavy flavours from small to large systems, Institut Pascal (Orsay, France); recent results and perspectives in hadron physics

October 17, 2022

Introduction

Inclusive production of J/ψ pairs in pp collisions (via gluon fusion) gives azimuthal asymmetries

$\hookrightarrow$ understanding the internal structure of nucleons
$\hookrightarrow$ gluon dynamics poorly known



Results $\Longleftrightarrow$ (future) measurements at LHC (fixed-target) experiments $\hookrightarrow$ Transverse Momentum Dependent PDFs (TMDs)

Transverse Momentum Dependent PDFs (1)

PDFs → great precision
Collinear QCD phenomenology

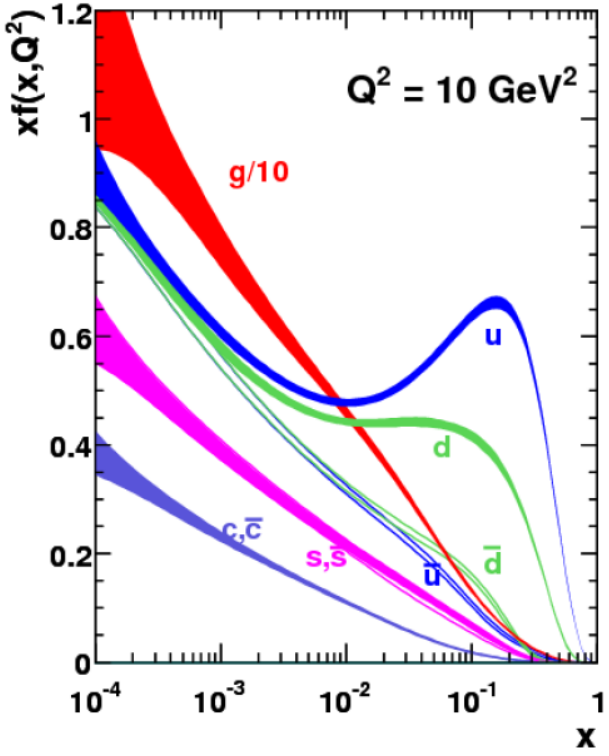
↪ only 1D information
↪ x dependence

⇓

3D structure of the nucleon
Beyond collinear factorisation

⇓

Transverse dynamics



▷ Nucleon structure in terms of **TMDs**

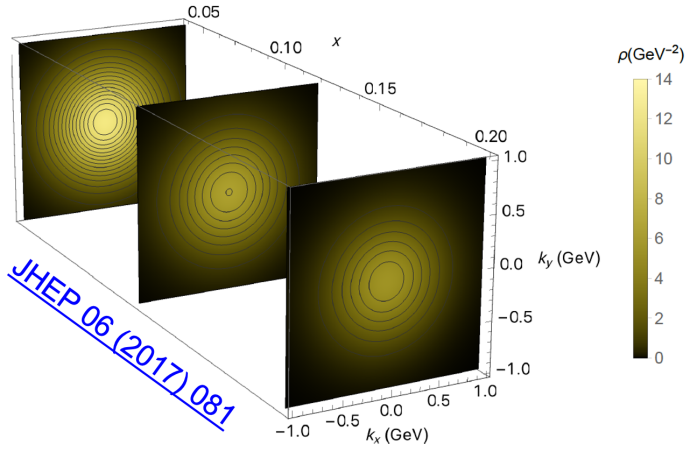
Transverse Momentum Dependent PDFs (2)

TMDs $\rightarrow$ 3D structure of the nucleon
Correlations between k_T and the polarisation of the nucleon/parton

2 components ▷ collinear (x)
▷ transversal ($\vec{k_\perp}$) $\rightarrow$ **generate** q_T (final-state)

Quark TMDs extracted from data
$\hookrightarrow$ SIDIS, DY processes

Gluon TMDs
$\hookrightarrow$ Extremely poorly known
$\hookrightarrow$ How to measure them?
Inclusive **quarkonium** production



JHEP 06 (2017) 081

A. Bacchetta et al. (JHEP 08 (2008) 023)

Quarkonium production processes

Experimental point of view:

- Quarkonium production observed in different experiments
- J/ψ: easy to produce and detect
  $\hookrightarrow$ plenty of experimental data

Theoretical point of view:

- Not clear how to treat quarkonium production in general
- 3 common models $\rightarrow$ Colour Singlet Model (CSM)
  $\rightarrow$ Colour Octet Mechanism (COM)
  $\rightarrow$ Colour Evaporation Model (CEM)
- Not complete agreement with experimental data
- However, for J/ψ-pair production: **CSM** is the best!

TMD factorisation

Study of gluon TMDs $\rightarrow$ TMD factorisation ($q_T \ll Q$)

General factorised cross section

$\hookrightarrow$ partonic scattering amplitude (*perturbative*)

$\hookrightarrow$ k_T-dependent correlators (*non-perturbative*)

$$d\sigma = \int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T)$$
$$\times \Phi_g^{\mu\nu}(x_1, \vec{k}_{T1}) \Phi_g^{\rho\sigma}(x_2, \vec{k}_{T2}) \left[\hat{\mathcal{M}}_{\mu\rho} \hat{\mathcal{M}}^*_{\nu\sigma} \right]_{\Big|_{k_2 = x_2 P_2}^{k_1 = x_1 P_1}} + \mathcal{O}\left(\frac{q_T^2}{Q^2} \right)$$

- In order to stay in TMD regime: $\max(q_T) = Q/2$

Gluon TMDs and correlators

TMD correlator parametrisation for an unpolarised proton

- ▷ unpolarised: $f_1^g \longrightarrow$
- ▷ linearly polarised: $h_1^{\perp g} \longrightarrow$

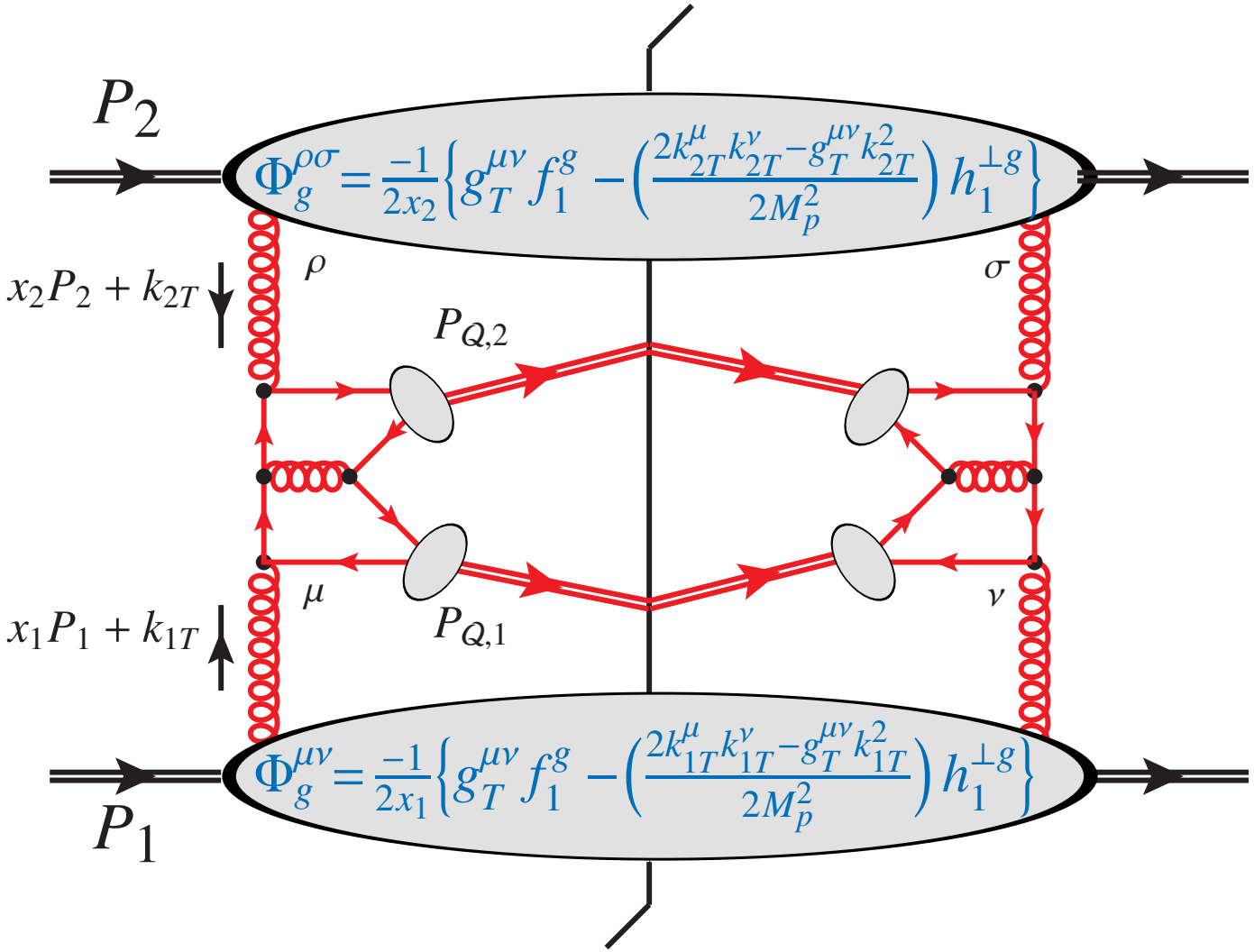
		Gluon		
		U	C	L
Nucleon	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

$$\Phi_g^{\mu\nu}(x,\vec{k}_T) = -\frac{1}{2x}\left[g_T^{\mu\nu} f_1^g(x,\vec{k}_T^2) - \left(\frac{k_T^\mu k_T^\nu}{M_H^2} + g_T^{\mu\nu}\frac{\vec{k}_T^2}{2M_H^2}\right) h_1^{\perp g}(x,\vec{k}_T^2)\right]$$

$\hookrightarrow$ Second term goes to 0 if if $k_T = 0$

P.J. Mulders and J. Rodrigues (Phys.Rev.D 63 (2001) 094021)

LO Feynman diagram for $p(P_1) + p(P_2) \to \mathcal{Q}(P_{Q,1}) + \mathcal{Q}(P_{Q,1}) + X$



Why di-J/ψ production?

- Single J/ψ production (CSM): a lot of data at low q_T ✓
  $\hookrightarrow$ but gluon in the final state $\rightarrow$ presence of soft gluons (non-perturbative) between Initial State Interactions (ISIs) and Final State Interactions (FSIs) can be problematic
  $\hookrightarrow$ **no** TMD factorisation ✗
- Single η_c production: no gluon in the final state ✓
  $\hookrightarrow$ but **no** data at low q_T ✗
- Double J/ψ production:
  ▷ data at low q_T ✓
  ▷ no gluon in the final state ✓
  $\hookrightarrow$ gluon fusion: ISI can be encapsulated in the TMDs ✓
  $\hookrightarrow$ consider CSM: no FSIs ✓
  $\rightarrow$ **Safe TMD factorisation**

PhD Thesis F. Scarpa (10.33612/diss.128346301)

Hadronic cross section

The general formula for the cross section of gluon fusion is:

$$
\begin{aligned}
d\sigma_{UU}^{gg} \propto\ & F_1 \times \mathcal{C}[f_1^g f_1^g] \\
& + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] \\
& + (F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3' \times \mathcal{C}[w_3' h_1^{\perp g} f_1^g]) \cos(2\Phi_{CS}) \\
& + (F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]) \cos(4\Phi_{CS})
\end{aligned}
$$

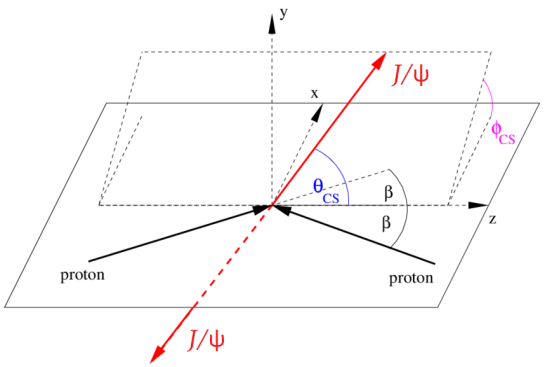
- First two members: azimuthally independent
- Third member: $\cos(2\Phi_{CS})$-asymmetry
- Fourth member: $\cos(4\Phi_{CS})$-asymmetry

Computation of azimuthal asymmetries (average)

The corresponding expressions for $\cos(2\Phi_{CS})$ and $\cos(4\Phi_{CS})$:

$$\langle\cos(2\phi_{CS})\rangle = \frac{1}{2}\frac{F_3\mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3'\mathcal{C}[w_3' h_1^{\perp g} f_1^g]}{F_1\mathcal{C}[f_1^g f_1^g] + F_2\mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$

$$\langle\cos(4\phi_{CS})\rangle = \frac{1}{2}\frac{F_4\mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{F_1\mathcal{C}[f_1^g f_1^g] + F_2\mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$



- The hard-scattering coefficients (F_1, F_2, F_3, F_3', F_4) give the explicit dependence on $M_{\psi\psi}$ and θ_{CS} (given in backup slides)
- Modulations due to $h_1^{\perp g}$
- Set hard scale $Q \equiv M_{\psi\psi}$
- TMD evolution applied within the convolutions

Introduction Evolution (1)

- Beyond tree level, the TMDs and hard factors F become scale dependent J. Collins (ISBN: 9781107645257)
- Implementing evolution is more easily done in impact parameter space (b_T), where convolutions become simple products:

$$d\sigma_{UU}^{gg} \propto \int d^2\mathrm{b}_T\, e^{-i\mathrm{b}_T\cdot\mathrm{q}_T}\, \hat{W}(\mathrm{b}_T, Q) + \mathcal{O}(\mathrm{q}_T^2/Q^2)$$

$$\hat{W}(\mathrm{b}_T, Q) = \hat{f}(x_1, \mathrm{b}_T; \zeta_f, \mu)\, \hat{g}(x_2, \mathrm{b}_T; \zeta_g, \mu)\, \mathcal{H}(Q; \mu)\,.$$

- The convolutions are rewritten by Fourier transforming:

$$\mathcal{C}[w\, f\, g](x_1, x_2, \vec{q}_T) = \int d^2\vec{k}_{T1} \int d^2\vec{k}_{T2}\, \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T)$$
$$\times\, w_{n,m}(\vec{k}_{T1}, \vec{k}_{T2})\, f(x_1, \vec{k}_{T1})\, g(x_2, \vec{k}_{T2})$$
$$\Rightarrow \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T)\, \hat{f}(x_1, \mathrm{b}_T)\, \hat{g}(x_2, \mathrm{b}_T)$$

Introduction Evolution (2)

$$\mathcal{C}[w\, f\, g](x_1, x_2, \vec{q}_T; Q) = \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T) \times e^{-S_A(b_T; Q^2, Q)}\, \hat{f}(x_1, \mathrm{b}_T; \mu_b^2, \mu_b)\, \hat{g}(x_2, \mathrm{b}_T; \mu_b^2, \mu_b)$$

- S_A contains $\ln Qb_T$
- Expressions (based on pQCD) are valid when: $b_0/Q \leq b_T \leq b_{T,\mathrm{max}}$
- At lower limit $\mu_b = b_0/b_T$ becomes larger than Q, i.e. evolution should stop ($S_A = 0$)
- At upper limit perturbation theory starts to fail, which is not exactly known. Common to take $b_{T,\mathrm{max}} = 0.5\,\mathrm{GeV}^{-1}$ or $b_{T,\mathrm{max}} = 1.5\,\mathrm{GeV}^{-1}$.
- This effectively boils down to a different resummation: $\mu_b(b_T)/Q \to \mu_b(b_T^*)/Q$

Introduction Evolution (3)

- We need to add a component that takes over as $b_T > b_{T,\mathrm{max}}$:

$$\hat{W}(b_T, Q) \equiv \hat{W}(b_T^*, Q) e^{-S_{NP}(b_T, Q)}$$

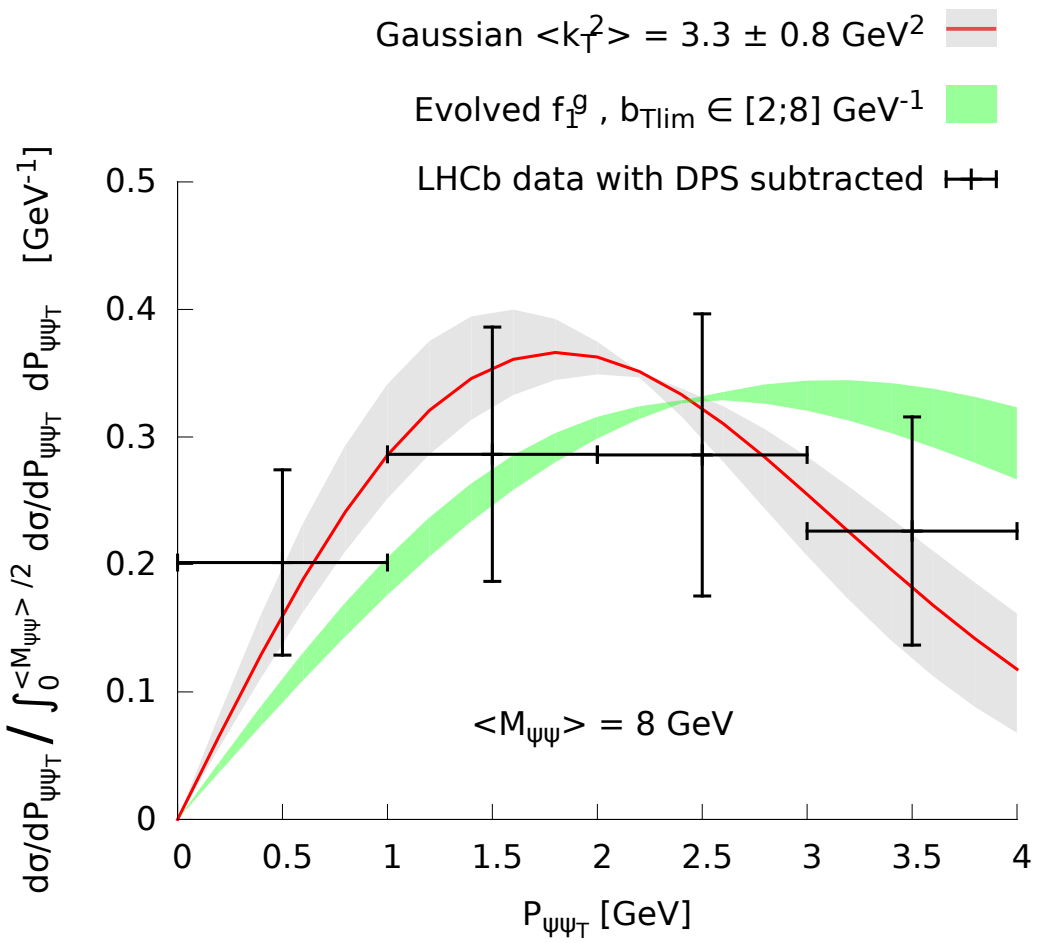
- There are different parameterizations for S_{NP} in the literature, but typically it is chosen to be a Gaussian:

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1\,\mathrm{GeV}$$

- We obtain the following expression for the convolutions:

$$\mathcal{C}[w\, f\, g](x_1, x_2, \vec{q}_T; Q) = \int_0^\infty \frac{db_T}{2\pi}\, b_T^n\, J_m(b_T\, q_T)\, e^{-S_A(b_T^*; Q^2, Q)}\, e^{-S_{NP}(b_T; Q)} \times \hat{f}(x_1, \mathrm{b}_T^*; \mu_b^2, \mu_b)\, \hat{g}(x_2, \mathrm{b}_T^*; \mu_b^2, \mu_b)$$

Results for $x_1 = x_2$: Florent



F. Scarpa et al. (Eur.Phys.J. C 80 no.2, (2020) 87)
R. Aaij et al. (JHEP06(2017)047)

$x_1 \neq x_2$; Master project Alice; co-supervised by me

Goal: phenomenological study of the azimuthal asymmetries for J/ψ pair production in pp collisions with $\rightarrow x_1 \neq x_2$

Implementation: **ex-novo code** in Python; faster and we implement use of LHAPDF package for PDFs (for perturbative tails TMDs)

$\Downarrow$

Code validation: reproduced published results ($x_1 = x_2$)

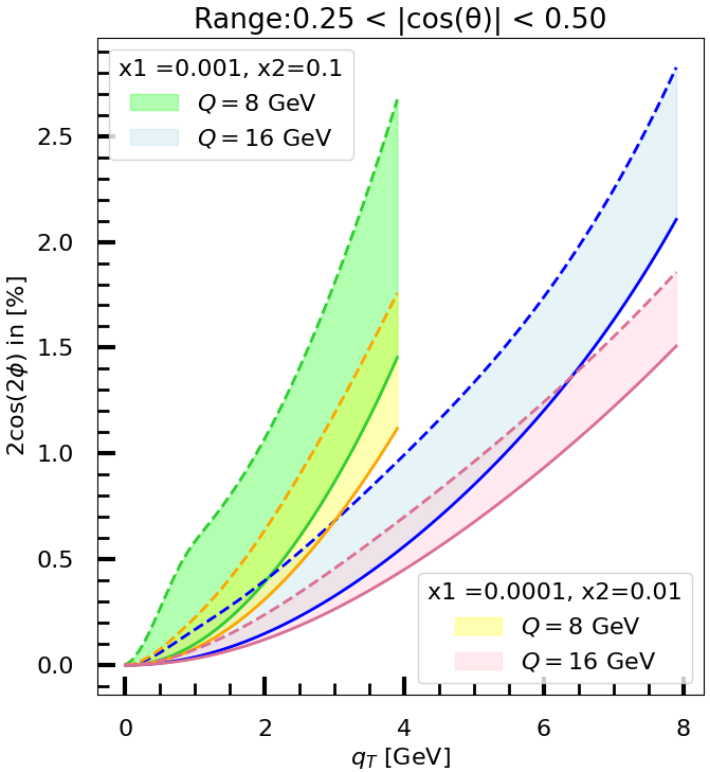
$\Downarrow$

NEW: **first studies** with $x_1 \neq x_2$
(two sets of x_1, x_2 but same rapidity $y = \frac{1}{2} \ln \frac{x_1}{x_2}$)

Preliminary: predictions for $\cos(2\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: [0.25; 0.50], $Q = M_{\psi\psi} = 8, 16$ GeV
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ contribution up to 3%

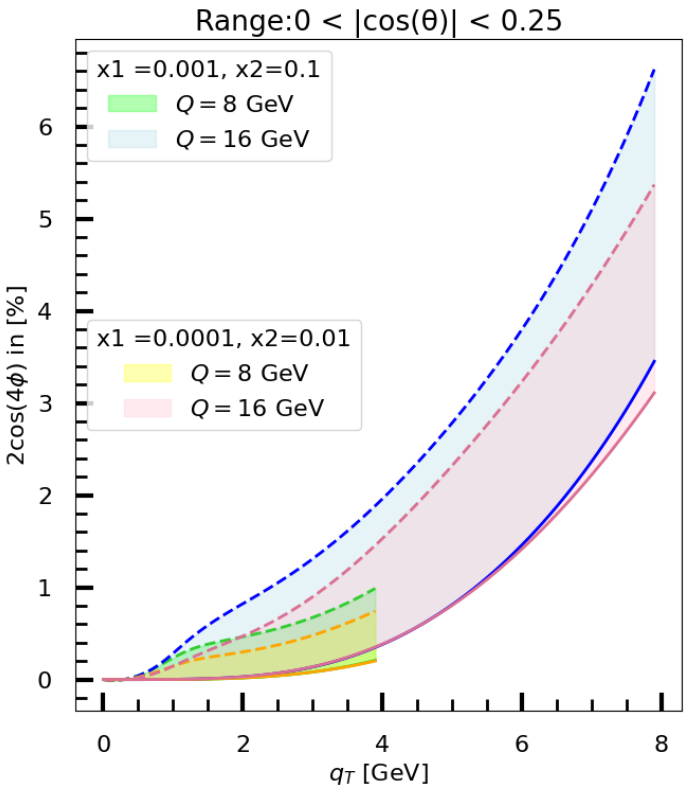
▷ big overlap in the low q_T region, not for large q_T

▷ $\sim$ same magnitude for low and high Q

Preliminary: predictions for $\cos(4\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: $[0; 0.25]$, $Q = 8, 16$ GeV
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ max contribution $5 - 6\%$

▷ overlap $\forall q_T$

▷ much higher amplitude for high Q (at high q_T)

Summary

- Double J/ψ production is a very promising process to investigate gluon TMDs
- Quarkonium q_T spectrum probes gluon transverse momenta
- Azimuthal asymmetries arise due to linear polarization of gluons inside unpolarized hadrons
- $x_1 \approx x_2$ seems to be favoured: lower azimuthal asymmetries for $\frac{x_1}{x_2} \neq 1$
- Further studies can be made considering polarised protons $\rightarrow$ access to more gluon TMDs

- For $pp \rightarrow \eta_{c,b} X$:

A. Bacchetta et al. (arXiv:2208.06252) and proceedings

- For $ep \rightarrow J/\psi X$:

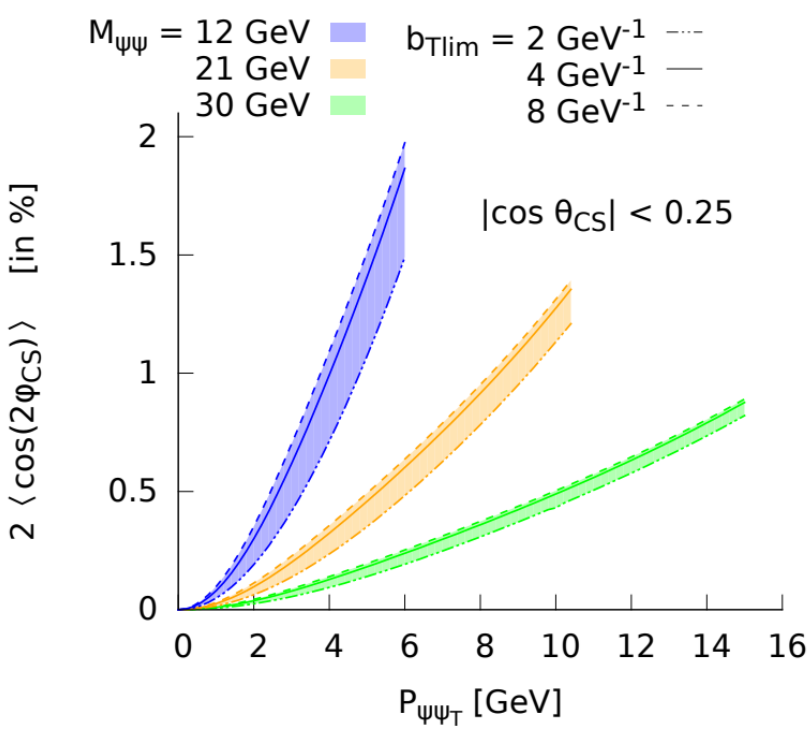
J. Bor and D. Boer (Phys.Rev.D 106 (2022) 1)

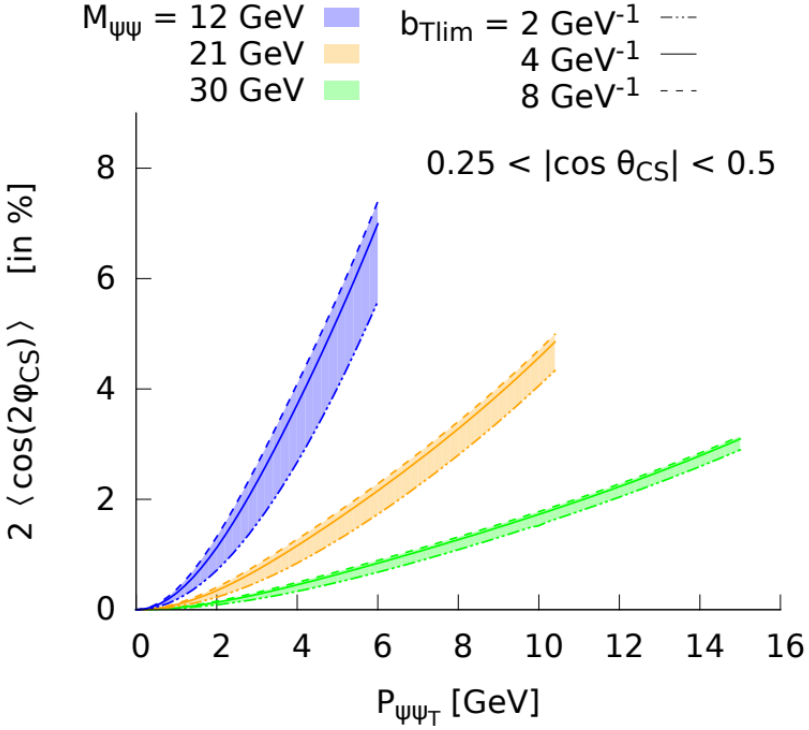
Backup slides

Results for $x_1 = x_2$: $\cos(2\Phi_{CS})$

Plots considering:

- Two ranges of $\cos(\theta_{CS})$: [0; 0.25] and [0.25; 0.50]
- Three values for the invariant mass: 12, 21, 30 GeV; **x1=x2**





Results for $x_1 = x_2$: $\cos(4\Phi_{CS})$

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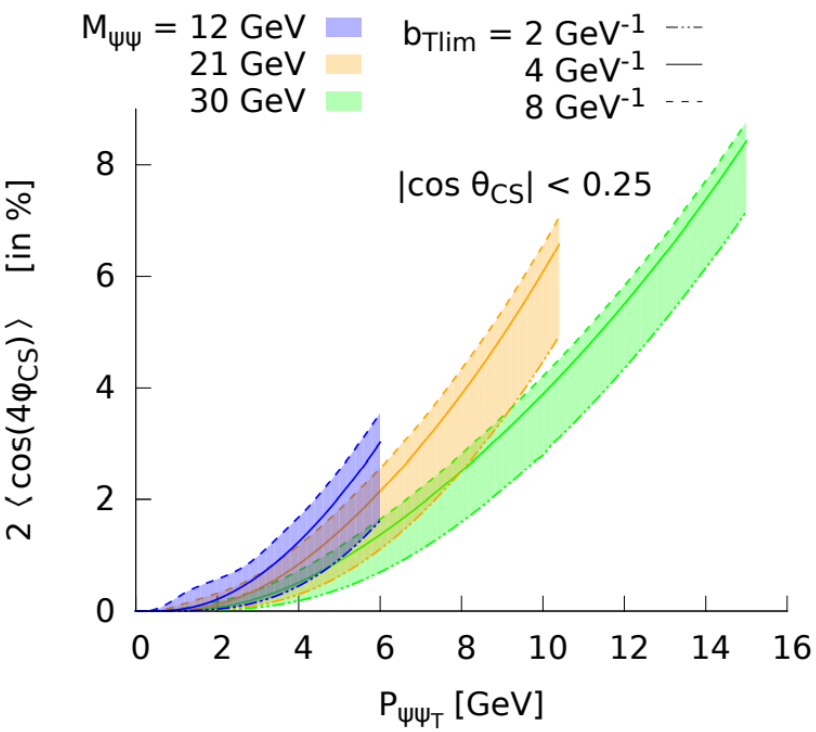
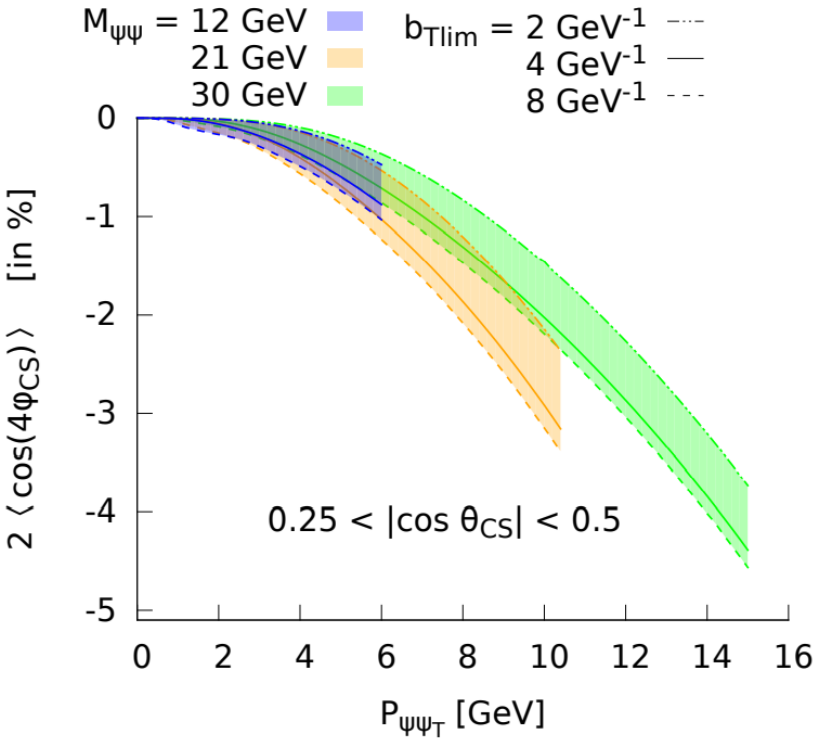
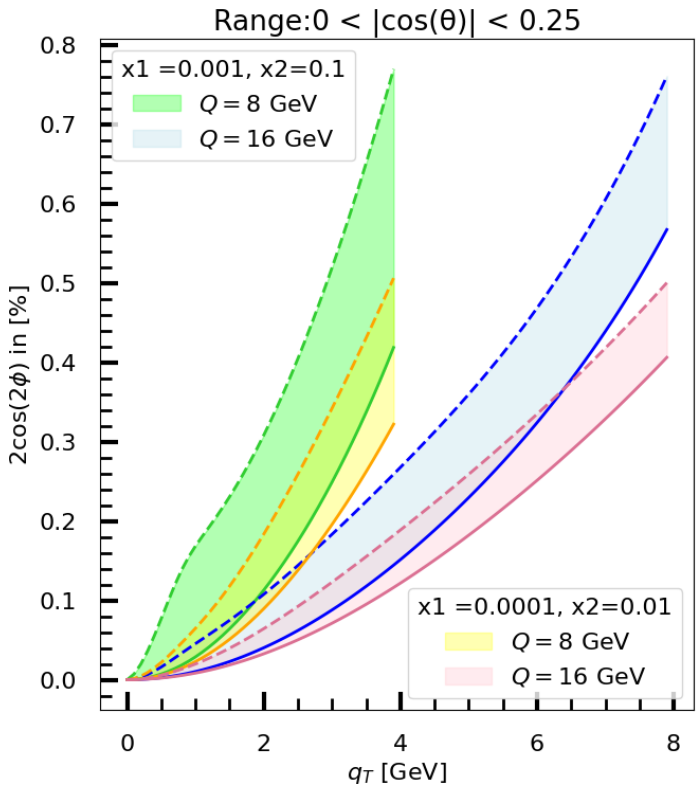




Results in for $x_1 \neq x_2$: $\cos(2\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: [0; 0.25], $Q = M_{\psi\psi}$
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ contribution below 1%

▷ big overlap in the low q_T region, not for large q_T

▷ $\sim$ same magnitude for low and high Q

Results in for $x_1 \neq x_2$: $\cos(4\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: [0.25; 0.50], $Q = M_{\psi\psi}$
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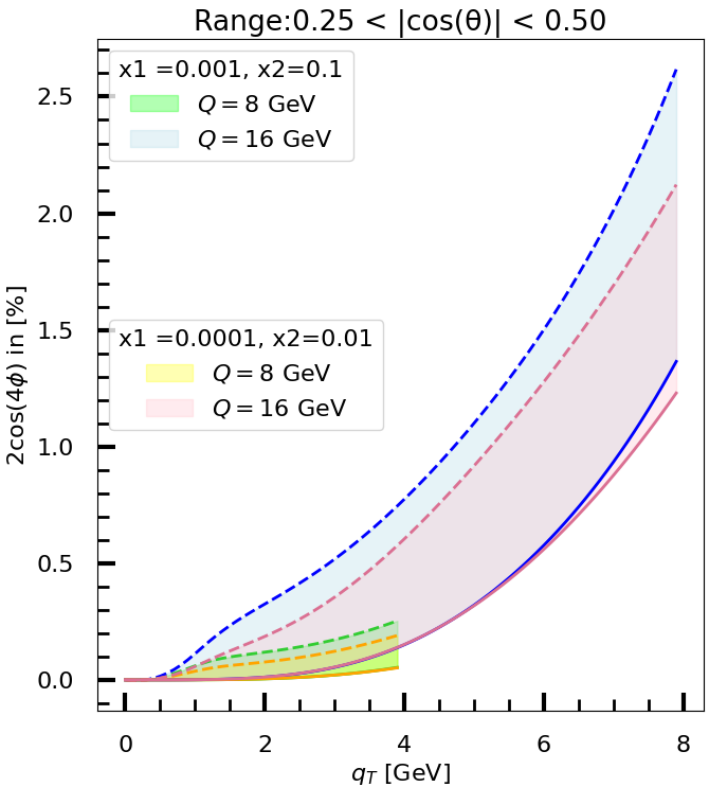

▷ contribution up to 3%

▷ higher amplitude for high Q (low Q negligible)

Hard scattering coefficients

$$F_1 = \frac{\mathcal{N}}{\mathcal{D} M_\Psi^2} \sum_{n=0}^{6} f_{1,n} (\cos\theta_{CS})^{2n} \qquad F_2 = \frac{2^4 3 M_\Psi^2 \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^4} \sum_{n=0}^{4} f_{2,n} (\cos\theta_{CS})^{2n}$$

$$F_3' = F_3 = \frac{-2^3 (1-\alpha^2)\mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^{5} f_{3,n} (\cos\theta_{CS})^{2n}$$

$$F_4 = \frac{(1-\alpha^2)^2 \mathcal{N}}{\mathcal{D} M_{\Psi\Psi}^2} \sum_{n=0}^{6} f_{4,n} (\cos\theta_{CS})^{2n}$$

with: $\alpha = \frac{2M_\Psi}{M_{\Psi\Psi}}$, $\mathcal{N} = 2^{11} 3^{-4} (N_c^2 - 1)^{-2} \pi^2 \alpha_s^4 |R_\Psi(0)|^4$, $\mathcal{D} = M_{\Psi\Psi}^4 (1 - (1-\alpha^2)\cos\theta_{CS}^2)^4$ and $R_\Psi(0)$ is the J/ψ radial wave function at the origin and $N_c = 3$.

The Sudakov Factor and Scales

- The solution of the evolution equations results in:

$$\hat{f}_1^g(x_1, \mathrm{b}_T; \zeta, \mu) = e^{-\frac{1}{2} S_A(b_T;\zeta,\mu)} \hat{f}_1^g(x_1, \mathrm{b}_T; \mu_b^2, \mu_b)$$
$$\hat{h}_1^{\perp g}(x_1, \mathrm{b}_T; \zeta, \mu) = e^{-\frac{1}{2} S_A(b_T;\zeta,\mu)} \hat{h}_1^{\perp g}(x_1, \mathrm{b}_T; \mu_b^2, \mu_b)$$

- $\mu \sim Q$ avoids large logarithms in $\mathcal{H}$
- TMDs should be evaluated at their natural scale: $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- $\Rightarrow$ take $\sqrt{\zeta_0} \sim \mu_0 \sim \mu_b \equiv b_0/b_T$ (with $b_0 = 2e^{-\gamma_E}$), in order to minimize both logarithms of μb_T and ζb_T^2 in S_A, and then evolved up to $\sqrt{\zeta} \sim \mu \sim Q$

Perturbative tails

- The large transverse momentum perturbative tail of the TMDs can be written as:

$$\hat{f}_1^g(x, b_T; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$$

$$\hat{h}_1^{\perp g}(x, b_T; \mu_b^2, \mu_b) = -\frac{\alpha_s(\mu_b)}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1 \right) \left\{ C_A \, f_{g/P}(x'; \mu_b) + C_F \sum_{i=q,\bar{q}} f_{i/P}(x'; \mu_b) \right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$$

P. Sun et al. (Phys.Rev.D 84 (2011) 094005)

b_T-Domains

- To ensure $b_0/Q \leq b_T$ we take:

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}$$

- For $b_T \leq b_{T,\text{max}}$:

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T,\mathbf{max}}}\right)^2}}$$

J. Collins et al. (Phys.Rev.D 94 (2016) 3, 034014)

The Non-perturbative Sudakov Factor

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1\,\text{GeV}$$

$b_{T,\text{lim}}$ (GeV^{-1})	r (fm $\sim 1/(0.2$ GeV$)$)	A (GeV^2)
2	0.2	0.64
4	0.4	0.16
8	0.8	0.04

Table 1: Values of the parameter A for $b_{T,\text{lim}}$ and r determined at $Q = 12$ GeV. A is defined at which $\exp(-S_{NP})$ becomes negligible ($\sim 10^{-3}$). To estimate the uncertainty associated with the S_{NP} we vary $b_{T,\text{lim}}$ spanning roughly from $b_{T,\text{max}} = 1.5$ GeV^{-1} to the charge radius of the proton. r is the range over which the interactions occur from the centre of the proton.

Abstract

In this talk I will explain why double J/ψ production in proton-proton collisions is a promising process to study gluon TMDs. I will touch upon the azimuthal asymmetries that arise in this process and the TMD evolution that is included in the computations. To conclude I will present some numerical results from F. Scarpa and A. Colpani and refer to some other interesting studies.