Discussion section:

Heavy flavor hadronization mechanism in Heavy ion collisions

13/10/2022

Propositions in Tuesday:

Guiseppe:
take the standard hadronization procedure of your code (with the from us provided hypersurface) and calculate dn/dpt of D- mesons produced by a c-quark of 3 GeV/c and 10 GeV/c

Elena/Pol:
for all codes which use Wigner densities: do the calculation with the standard hadronization procedure with our hypersurface and c-quark distribution after fixing the parameters, which enter the Wignerdensity, to a common value.

Ivan:
v_2 calibration by comparing the light meson v_2 with experiment

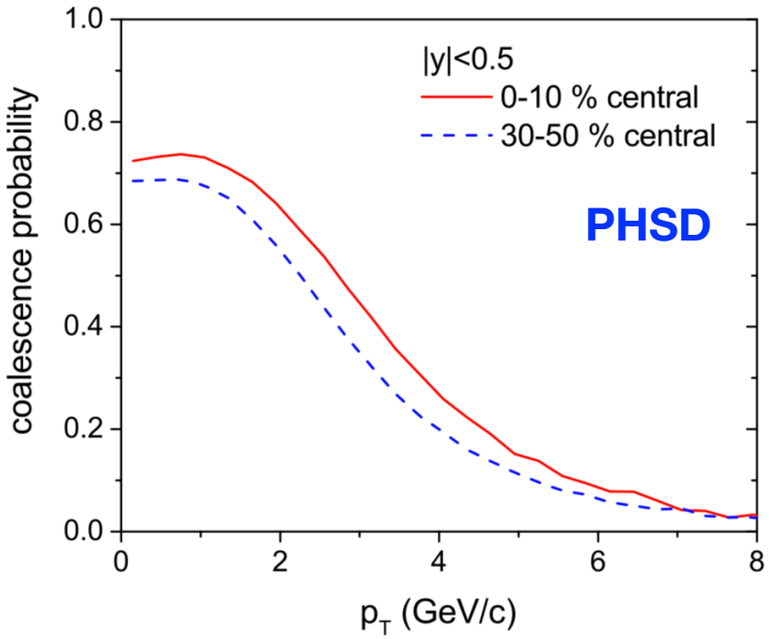
Pol:
test of momentum space- space correlations of the c-qaurks by providing such a correlated c-quark distribution

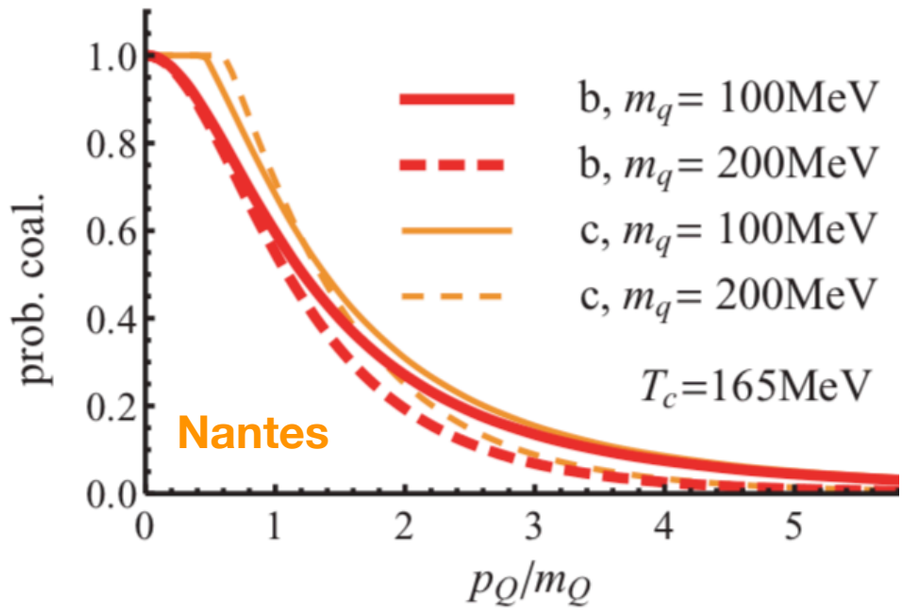
Model comparison for hadronization

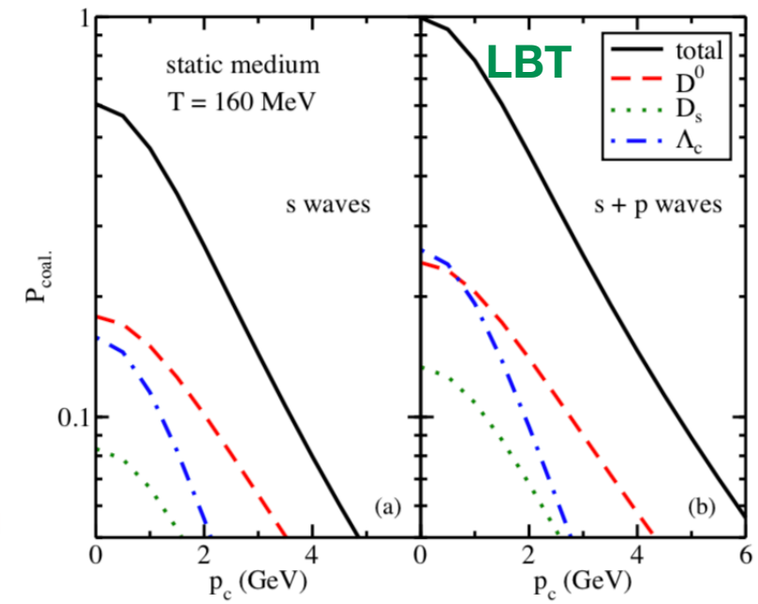
Descriptions:

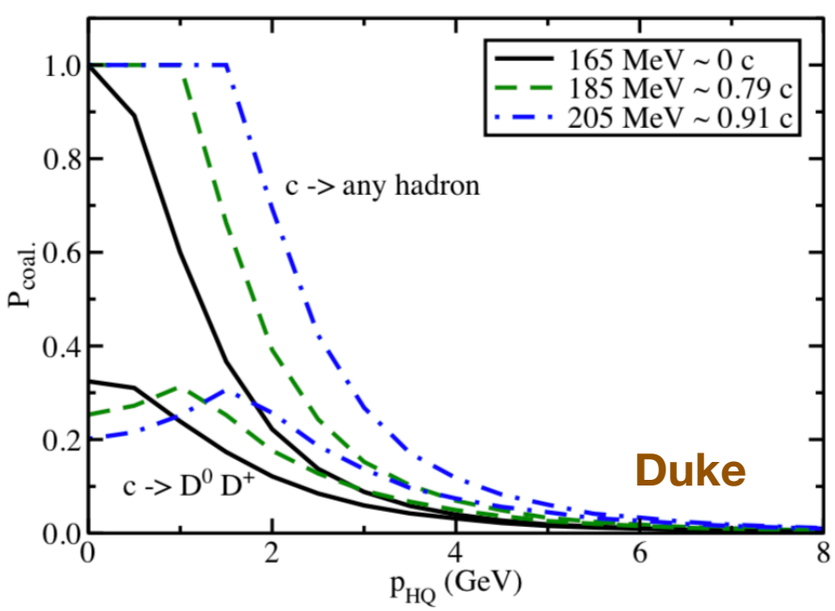
	Frag.	Recom.	NOTE
Catania	Peterson	Wigner function	S-wave, D+,D0,Ds, D*+,D*0,D*s,\Lambda_c,\Sigma_c
Duke	Pythia 6.4	Wigner function	S-wave,D,D*,\Lambda_c,\Sigma_c,\Xi_c,\Omega_c
LBT	Pythia 6.4	Wigner function	S-wave,P-wave,D,Ds,D*,\Lambda_c
Nantes	Extracted from e^+e^- data	Wigner function	S-wave, D+,D0
Nantes(new)	Extracted from e^+e^- data	Wigner function	S-wave, D+,D0
PHSD	Peterson	Wigner function	S-wave, P-wave(S=1,2) D+,D0,Ds, D*+,D*0,D*s
TAMU	thermal density correlated HQET	RRM	D+,D0,Ds, D*+missing baryons
Turin	Pythia 6.4	Invariant mass	S-wave, D+,D0,Ds,\Lambda_c,\Sigma_c,\Omega_c
Vitev	HQET with energy loss	–	S-/P-wave, D+,D0,Ds, c-baryons

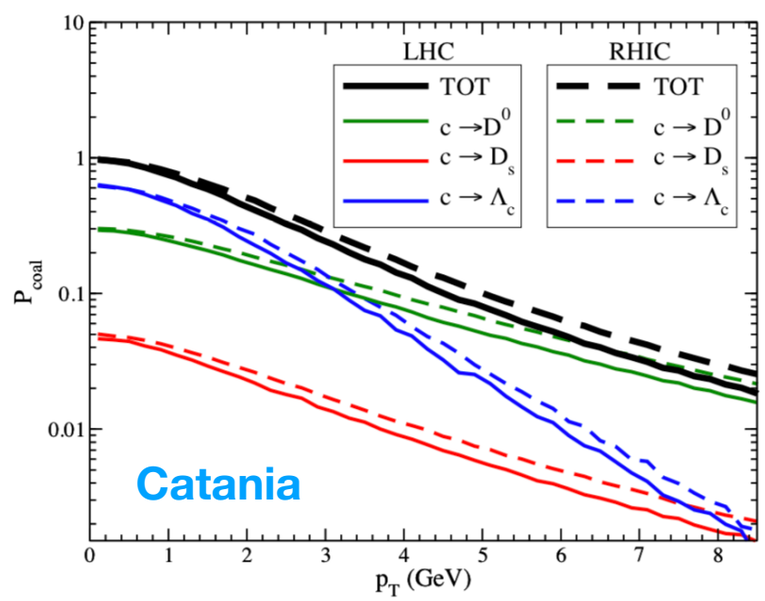
Recombination/coalescence probability











Low pT heavy flavor hadronize via the recombination, while high pT through the fragmentation!

Wigner function used in each model

Catania

$$W(x,p) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2\sigma_{ri}^2\right).$$

Duke

$$W(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$$

LBT

$$\begin{aligned} W_s(p) &= g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2}, \\ W_p(p) &= g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 p^2 e^{-\sigma^2 p^2}. \end{aligned}$$

Nantes

$$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q)\cdot u_Q]^2}{2R_c^2}\right) \exp\left(-\alpha_d^2(u_Q \cdot u_q - 1)\right),$$

PHSD

$$\begin{aligned} W_s(r,p) &= \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2}, \\ W_p(r,p) &= \frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2} + \frac{16}{3}\sigma^2 p^2 - 8\right) e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2}, \end{aligned}$$

Some groups use momentum space Wigner function, some use phase space Wigner function

Wigner function used in each model

Catania

$$W(x,p) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2\sigma_{ri}^2\right).$$

$$r = |\mathbf{r}_1 - \mathbf{r}_2|, \qquad p = \frac{|m_2\mathbf{p}_1 - m_1\mathbf{p}_2|}{m_1+m_2}.$$

Duke

$$W(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$$

$$r = |\mathbf{r}_1 - \mathbf{r}_2| \qquad p = \frac{|E_2\mathbf{p}_1 - E_1\mathbf{p}_2|}{E_1+E_2},$$

LBT

$$W_s(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$$

$$W_p(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 p^2 e^{-\sigma^2 p^2}.$$

$$r = |\mathbf{r}_1 - \mathbf{r}_2| \qquad p = \frac{|E_2\mathbf{p}_1 - E_1\mathbf{p}_2|}{E_1+E_2},$$

Nantes

$$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q)\cdot u_Q]^2}{2R_c^2}\right) \exp\left(-\alpha_d^2(u_Q \cdot u_q - 1)\right),$$

PHSD

$$W_s(r,p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$$

$$W_p(r,p) = \frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2} + \frac{16}{3}\sigma^2 p^2 - 8\right) e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$$

$$r = |\mathbf{r}_1 - \mathbf{r}_2|, \qquad p = \frac{|m_2\mathbf{p}_1 - m_1\mathbf{p}_2|}{m_1+m_2}.$$

The definition of relative momentum is also different.

Wigner function used in each model

Catania	$W(x,p) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2\sigma_{ri}^2\right).$	**Different σ for D, D_s, Λ_c**
Duke	$W(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$	**Same σ for all charmed hadrons**
LBT	$W_s(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 p^2},$ $W_p(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 p^2 e^{-\sigma^2 p^2}.$	**Same σ for all charmed hadrons**
Nantes	$W(x_Q, x_q, p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q)\cdot u_Q]^2}{2R_c^2}\right) \exp\left(-\alpha_d^2(u_Q \cdot u_q - 1)\right),$	**$\alpha_d = 0.51$, R_c is fixed by α_d . Different for D and D_s**
PHSD	$W_s(r,p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$ $W_p(r,p) = \frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2} + \frac{16}{3}\sigma^2 p^2 - 8\right) e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$	**Same mean radius, different σ for different states**

The choice of the sigma in the Wigner function is different.

Light quark distribution and masses used in each model

Catania	$f_q = \frac{g\tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right),$	Uniform distribution in coordinate
Duke	$f_q = \frac{V}{(2\pi)^3} \frac{g_i}{e^{E_i/T}+1}.$	No coordinate information
LBT	$f_q = \frac{V}{(2\pi)^3} \frac{g_i}{e^{E_i/T}+1}.$	No coordinate information
Nantes	$f_q = g_i \exp(-\sqrt{m^2+p^2}/T).$	Uniform distribution in coordinate
PHSD	$f_q = \frac{g_i}{e^{E_i/T}+1}.$	Uniform distribution in coordinate (for comparison)

Momentum distribution is almost same for different groups.

Light quark distribution and masses used in each model

Catania	$f_q = \frac{g\tau m_T}{(2\pi)^3} \exp\left(-\frac{\gamma_T(m_T - p_T \cdot \beta_T)}{T}\right),$	$m_{u,d} = 0.3\text{GeV},\ m_s = 0.38\text{GeV},\ m_c = 1.5\text{GeV}.$
Duke	$f_q = \frac{V}{(2\pi)^3} \frac{g_i}{e^{E_i/T} + 1}.$	$m_{u,d} = 0.3\text{GeV},\ m_s = 0.475\text{GeV},\ m_c = 1.27\text{GeV}.$
LBT	$f_q = \frac{V}{(2\pi)^3} \frac{g_i}{e^{E_i/T} + 1}.$	$m_{u,d} = 0.3\text{GeV},\ m_s = 0.4\text{GeV},\ m_c = 1.8\text{GeV}.$
Nantes	$f_q = g_i \exp(-\sqrt{m^2 + p^2}/T).$	$m_{u,d} = 0.1\ \text{GeV},\ m_c = 1.5\ \text{GeV}.$
PHSD	$f_q = \frac{g_i}{e^{E_i/T} + 1}.$	**Light quark is off-shell and sampled with a given distribution,** $m_c = 1.5GeV$

Different light quark mass in each group.

Fragmentation process

Fragmentation function

Catania	Peterson
Duke	Pythia 6.4
LBT	Pythia 6.4
Nantes	Extracted from e^+e^- data
PHSD	Peterson

Different value even in the same fragmentation function.

Backup

Model comparison for hadronization

Fix the freeze-out hypersurface and charm distribution at freeze-out hypersurface.

We prepared four tasks:

1. *Description of the hadronization scheme in your model*

2. *Final yield of charm hadrons with given charm distribution at hadronization hypersurface.*

   - *For pure fragmentation (assuming all c quarks proceed through fragmentation)*
   - *For pure coalescence / recombination*
   - *For your genuine hadronization model*

   For each cases, calculate the H_{AA} *of D* ($D^+ + D^0$), D_s *and* Λ_c.

$$H_{AA} = \frac{dN_D/dp_T}{dN_c/dp_T}$$

3. *Same as the second one but in p+p collisions.*

4. *Elliptic flow* v_2 *with and without the c-quark flow.*

$$\frac{dN}{d^2\mathbf{p}_T dy} = \frac{1}{\pi dp_T^2 dy}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos\left(n\left(\phi - \Psi_n\right)\right)\right]$$

Model comparison for hadronization

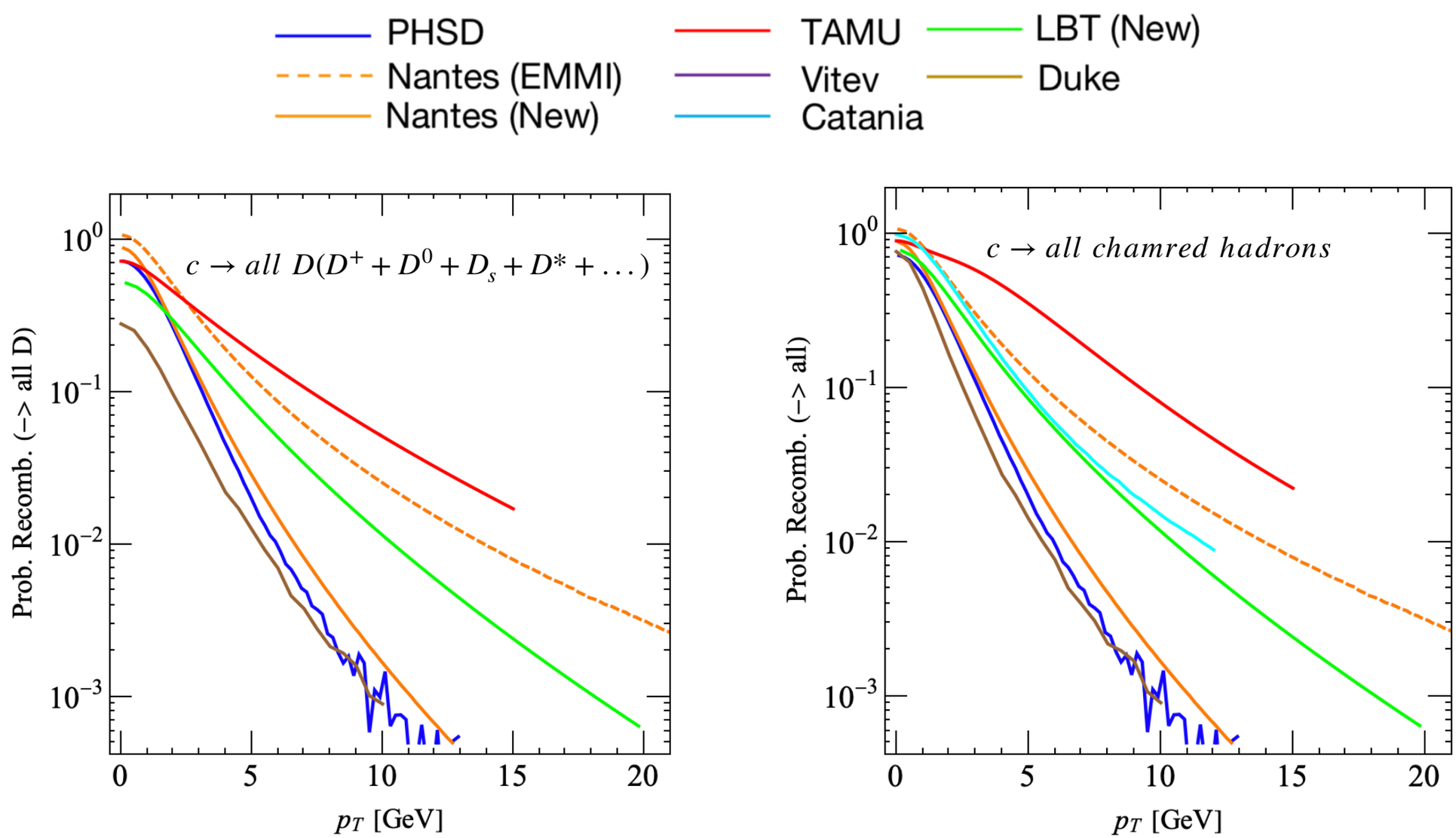
We prepared four tasks, but so far only two tasks are finished :

2. *Yield (H_{AA}) via fragmentation, recombination, and both with given c-quark distribution in HIC.*

4. *Elliptic flow v_2 with and without the c-quark flow.*

	Frag.	Recom.	Mix	$D(D^0 + D^+)$	D_s	Λ_c
Catania	✓	✓	✓	✓	✓	✓
Duke	✓	✓	✓	✓	✗	✗
LBT	✓	✗	✓	✓	✓	✓
Nantes	✓	✓	✓	✓	✗	✗
Nantes(new)	✓	✓	✓	✓	✗	✗
PHSD	✓	✓	✓	✓	✓	✗
TAMU	✓	✓	✓	✓	✓	✓
Turin	✗	✗	✓	✓	✓	✓
Vitev	✓	✗	✗	✓	✓	c-baryons

Model comparison — Recombination probability

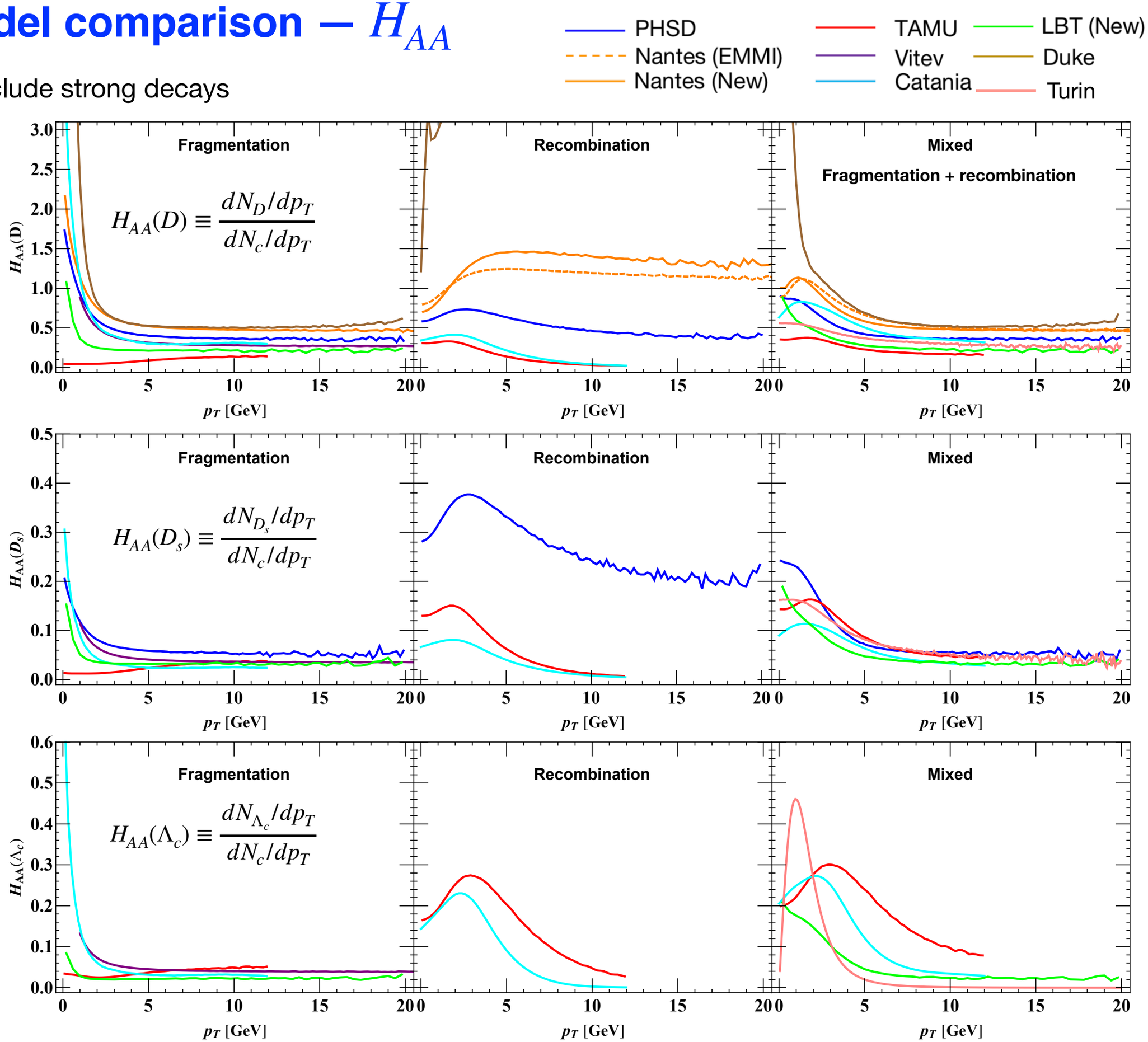


! TAMU: pT of charm quark in the local rest frame; Others in the Lab frame.

- *Huge difference at high pT region (pT>3 GeV)*
- *Total recombination probability ~1.0 at zero pT required by all charm quarks hadronized via recombination at pT~0.*
- $P_{frag.}(p_T) = 1 - P_{coal.}(p_T)$

Model comparison — H_{AA}

Include strong decays



Model comparison — H_{AA}

What we learned:

- *Nothing but the sequence* $H_{AA}(D) > H_{AA}(D_s) > H_{AA}(\Lambda_c)$

The large difference may come from the branching ratios between various charmed-hadrons

$$R = \left(\int \frac{dN_c}{dp_T} \times H_{AA} dp_T \right) \Big/ \left(\int \frac{dN_c}{dp_T} dp_T \right)$$

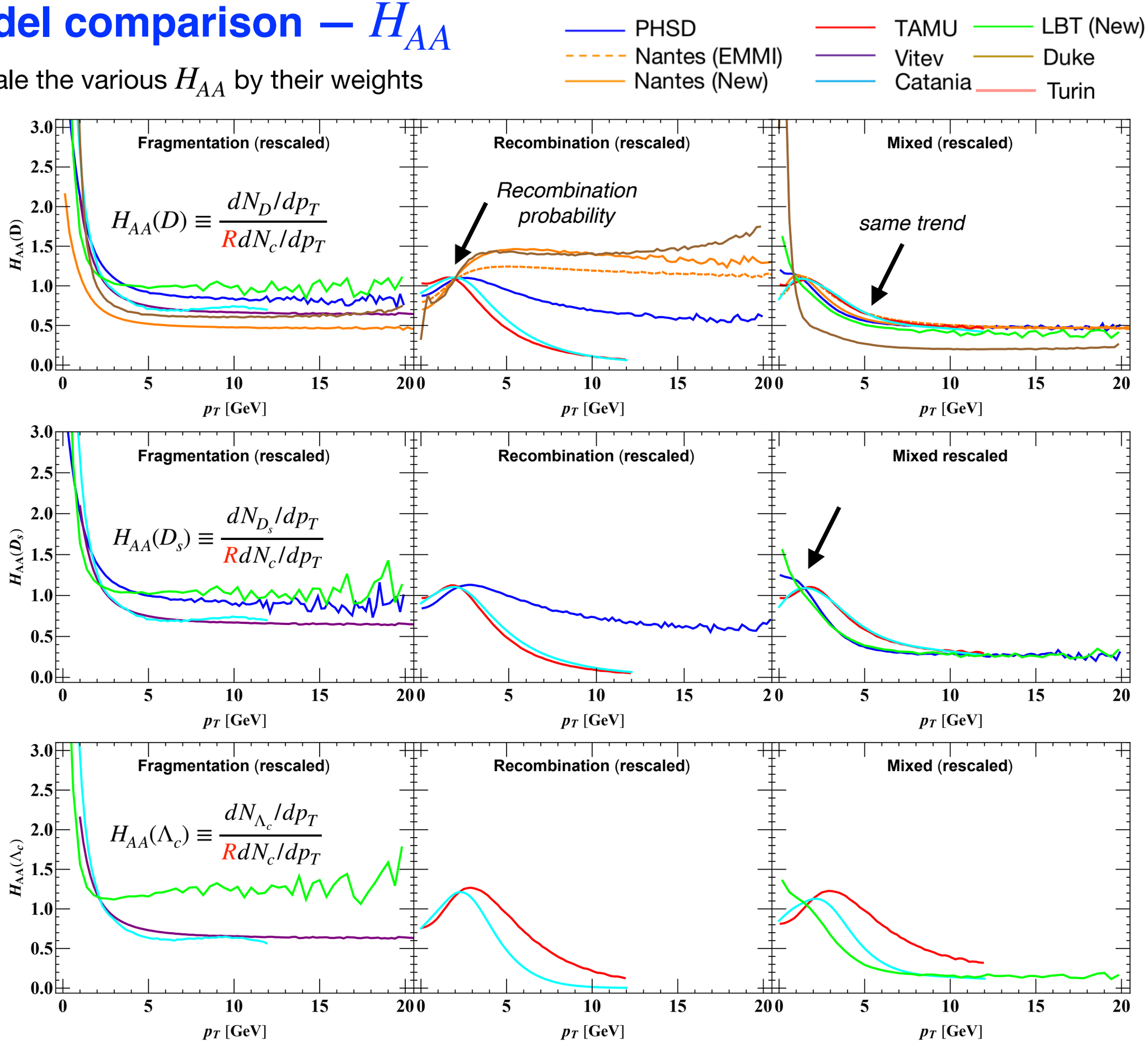
Frag.	D	D_s	Λ_c
Catania	99%	8%	13%
Duke	277%	-	-
LBT	37.8%	5.4%	3%
Nantes	100%	-	-
Nantes (new)	100%	-	-
PHSD	81%	10%	-
TAMU	5.1%	1.4%	2.9%
Vitev	42.1%	5.5%	6%

Recom.	D	D_s	Λ_c
Catania	37.5%	7.3%	19%
Duke	367%	-	-
LBT	-	-	-
Nantes	100%	-	-
Nantes (new)	100%	-	-
PHSD	67%	33%	-
TAMU	30%	13%	22%
Vitev	-	-	-

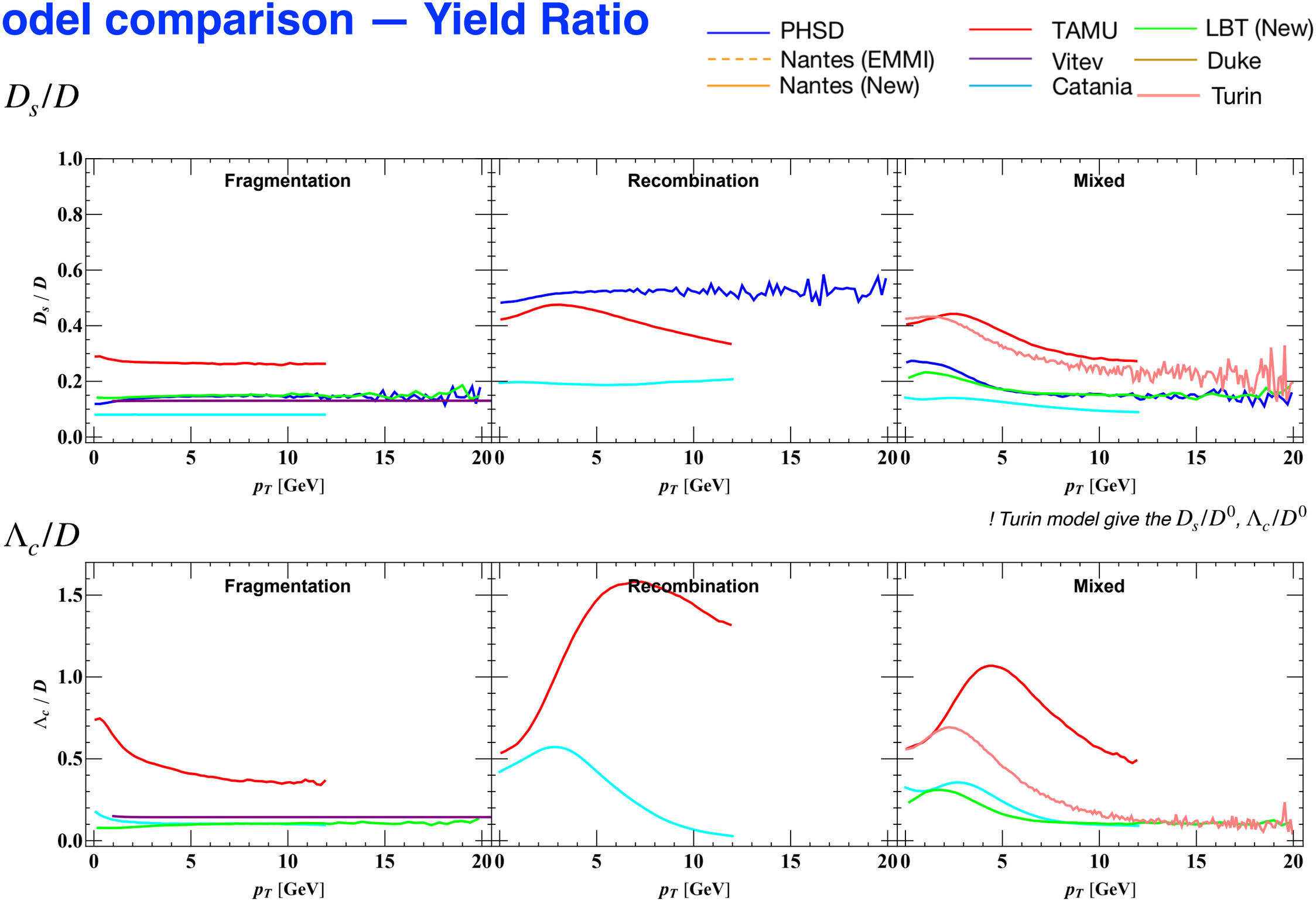
Mixed.	D	D_s	Λ_c
Catania	76.1%	10.4%	24.2%
Duke	257%	-	-
LBT	54.7%	12.1%	15.3%
Nantes	100%	-	-
Nantes (new)	100%	-	-
PHSD	75%	20%	-
TAMU	35%	15%	25%
Vitev	-	-	-

Model comparison — H_{AA}

Rescale the various H_{AA} by their weights



Model comparison — Yield Ratio



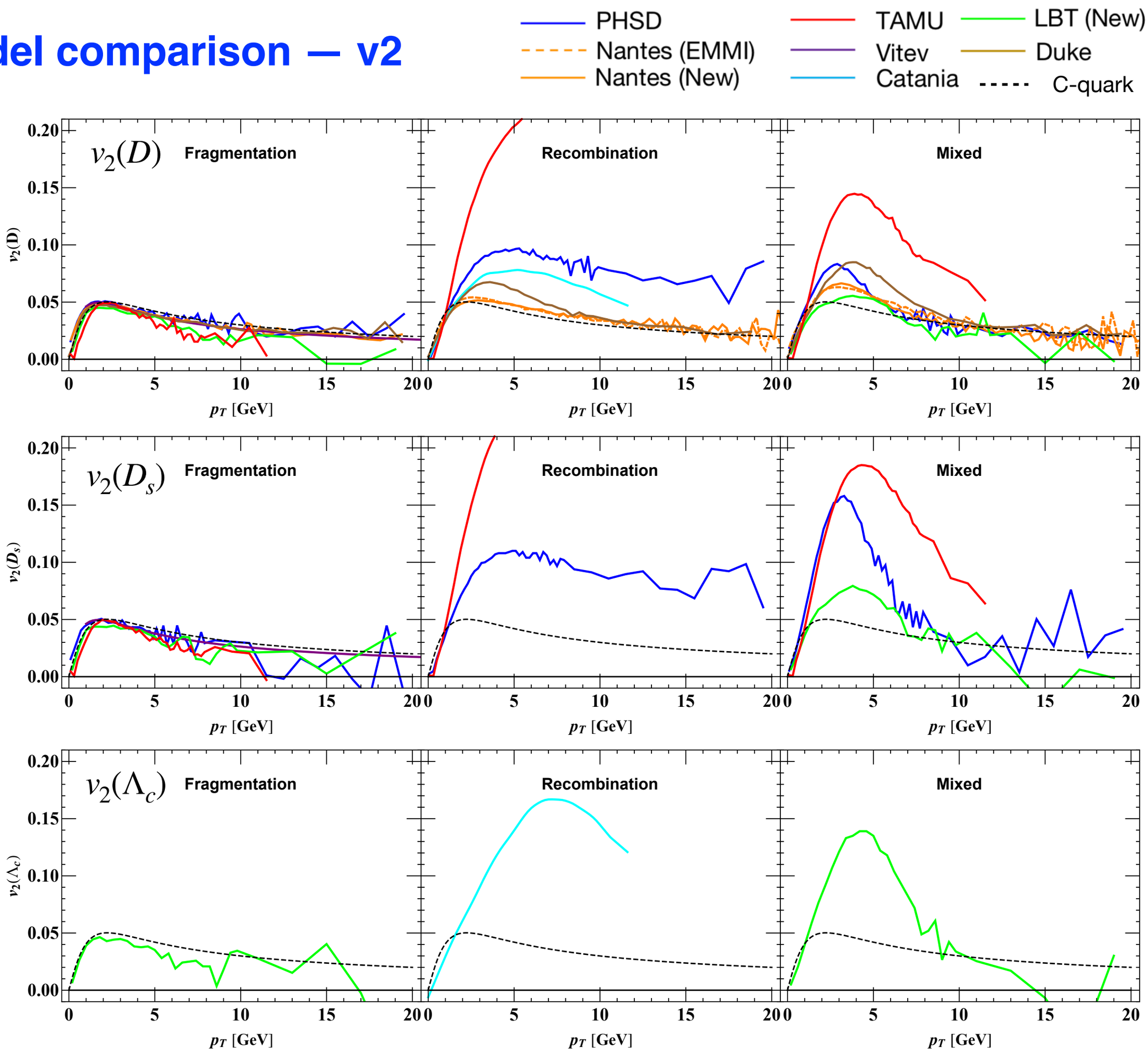
! Turin model give the D_s/D^0, Λ_c/D^0

TAMU model gives a larger Λ_c/D^0 ratio than others; may cause by "missing" baryons

Similar D_s/D, Λ_c/D ratio from fragmentation (except TAMU)

Λ_c/D ratio from recombination is much larger than fragmentation———baryon/meson enhanced

Model comparison — v2

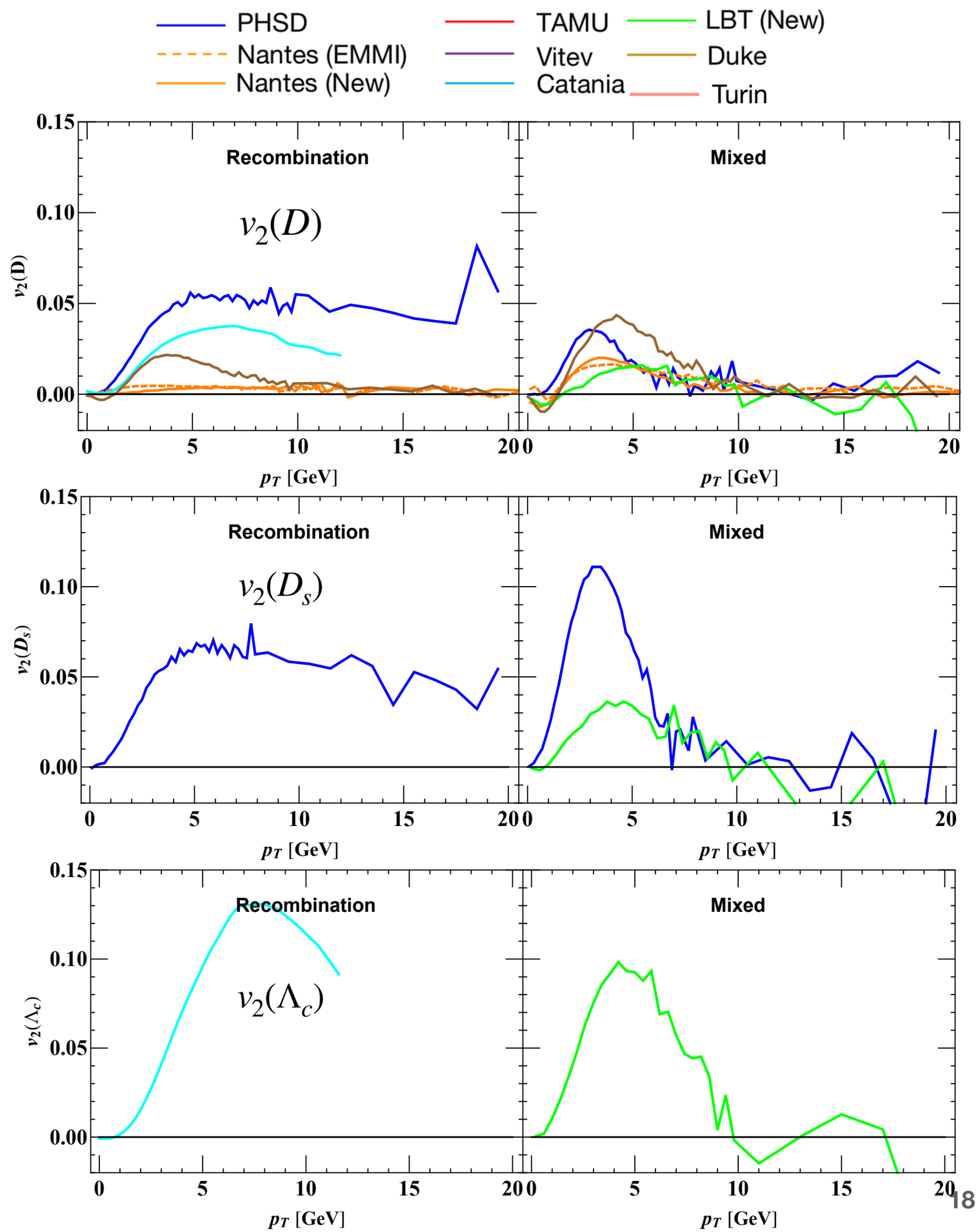


Model comparison — v2

We take an isotropic c-quark distribution (eliminate the elliptic flow)

$$\frac{dN_c}{dp_T} \times [1 + 2v_2(p_T)\cos(2\phi)]$$

- *v2 from the fragmentation is zero. Elliptic flow fully comes from the recombined light quark/quarks.*



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Thanks for your attention!