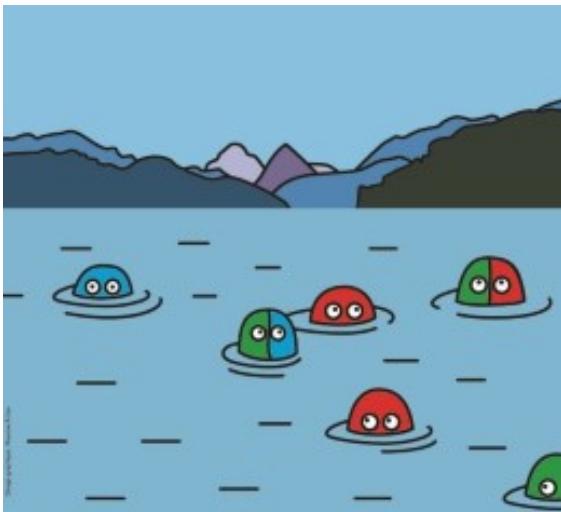


# Heavy quark hadronization: Coalescence and fragmentation

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INFN-LNS



UNIVERSITÀ  
degli STUDI  
di CATANIA



DIPARTIMENTO DI  
FISICA E  
ASTRONOMIA  
“ETTORE MAJORANA”



Istituto Nazionale di Fisica Nucleare

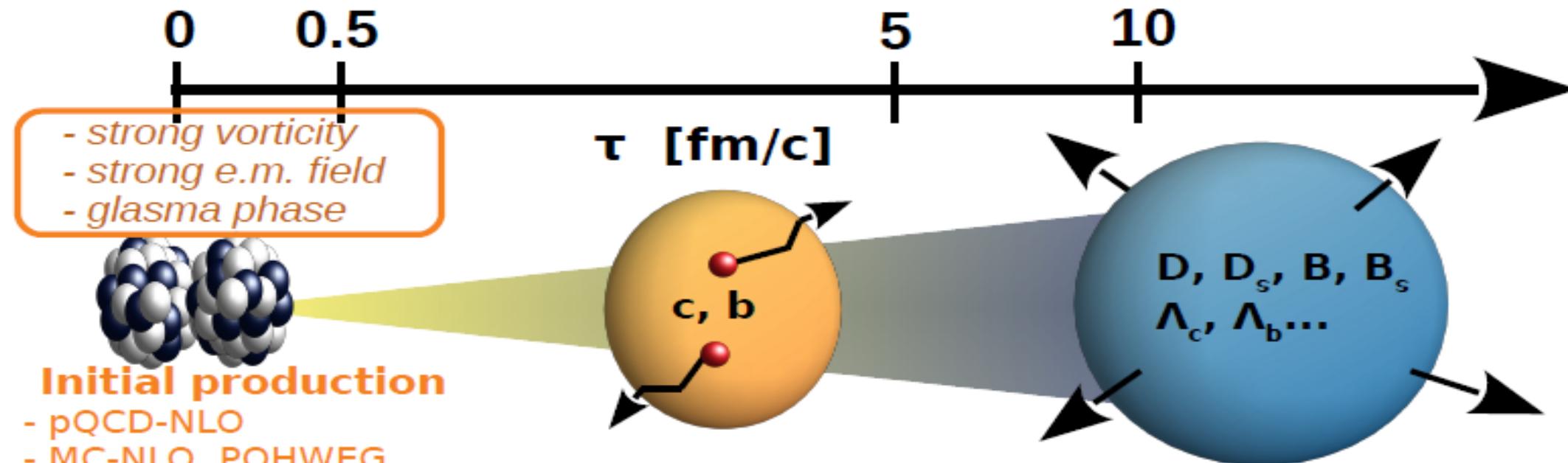
Thanks to:

V. Minissale, A. Arena, S. K. Das, Y. Sun, M.L. Sambataro, V. Greco

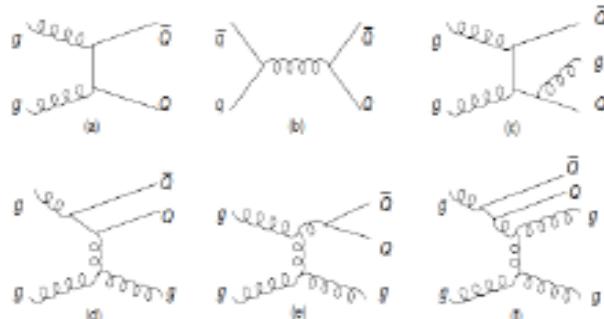
HF2022: Heavy Flavours from small to large systems

Oct 3 – 21, 2022  
Institut Pascal

# Heavy quarks in uRHIC



$$\sigma_{pp \rightarrow cc} = \int_0^1 dx_1 dx_2 \sum_{i,j} f_i(x_1, Q^2) f_j(x_2, Q^2) \sigma_{ij \rightarrow cc}(x_1, x_2, Q^2),$$



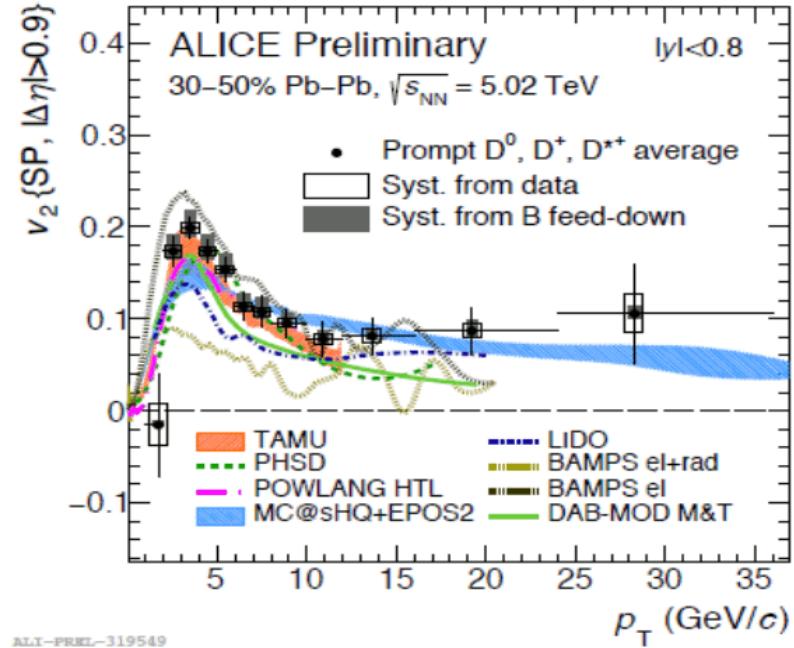
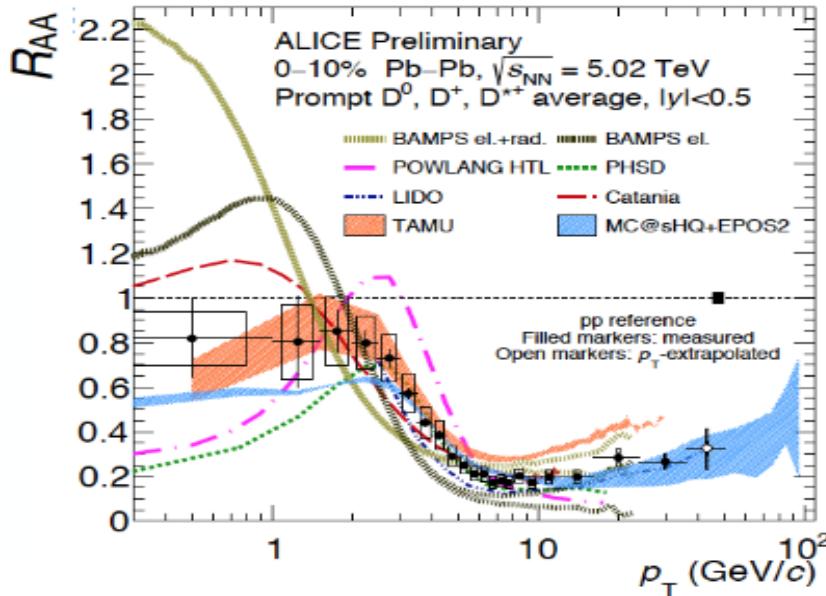
## Dynamics in QGP

- Transport approaches:  
Boltzmann/Fokker-Planck
- Thermalization
- Transp. Coeff. of QCD matter  $D_s(T)$
- Jet Quenching

## Hadronization

- SHM/coalescence and/or fragm.  
 $D, D_s, B, B_s, \Lambda_c, \Lambda_b, \Xi_c, \Omega_c\dots$
- $\Lambda_c/D$  in pp,pA,AA
- $R_{AA}$ , collective flow harmonics

# Transport coefficient



Models not really tested at  $p \rightarrow 0$

The new data  $\rightarrow$  determine  $D_s(T)$  more properly,  
i.e.  $p \rightarrow 0$  where it is defined and computed in IQCD

|                         | Catania      | Duke         | Frankfurt(PHSD)        | LBL          | Nantes   | TAMU           |
|-------------------------|--------------|--------------|------------------------|--------------|----------|----------------|
| Initial HQ (p)          | FONLL        | FONLL        | pQCD                   | pQCD         | FONLL    |                |
| Initial HQ (x)          | binary coll. | binary coll. | binary coll.           | binary coll. |          | binary coll.   |
| Initial QGP             | Glauber      | Trento       | Lund                   |              | EPOS     |                |
| QGP                     | Boltzm.      | Vishnu       | Boltzm.                | Vishnu       | EPOS     | 2d ideal hydro |
| partons                 | mass         | m=0          | m(T)                   | m=0          | m=0      | m=0            |
| formation time QGP      | 0.3 fm/c     | 0.6 fm/c     | 0.6 fm/c (early coll.) | 0.6 fm/c     | 0.3 fm/c | 0.4 fm/c       |
| interactions in between | HQ-glasma    | no           | HQ-preformed plasma    | no           |          | no             |

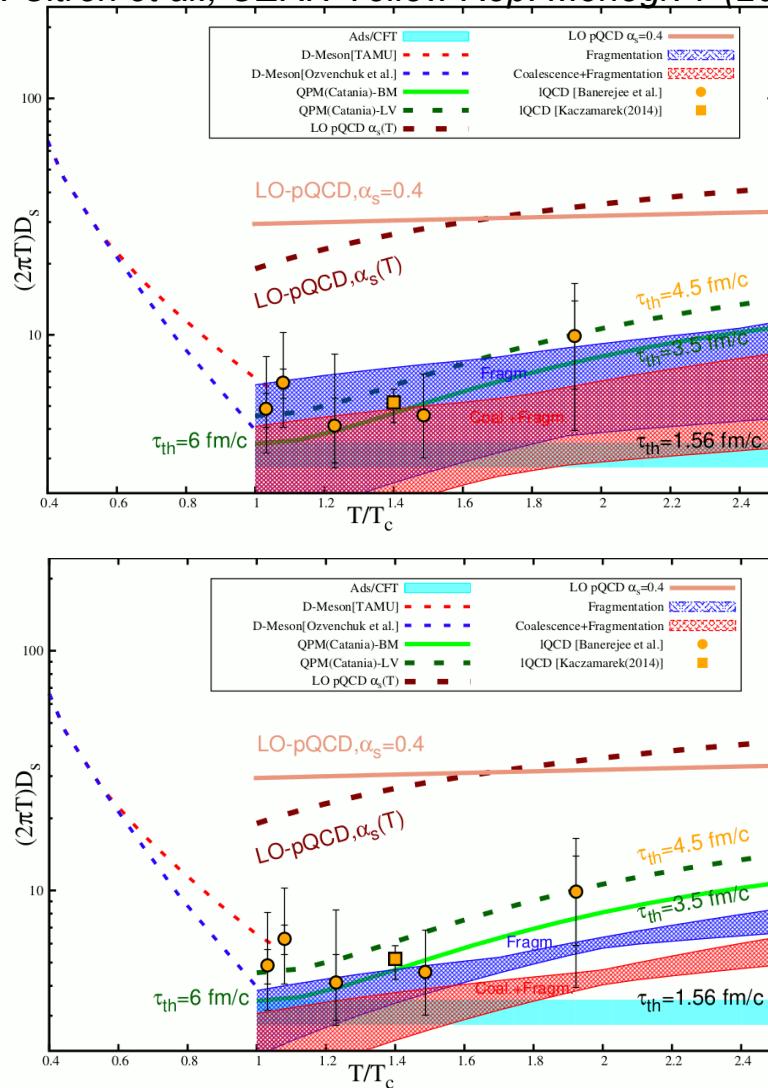
**2018-2019**

**Several Collab. in joint activities:**

- EMMI-RRTF:  
R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS:  
S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

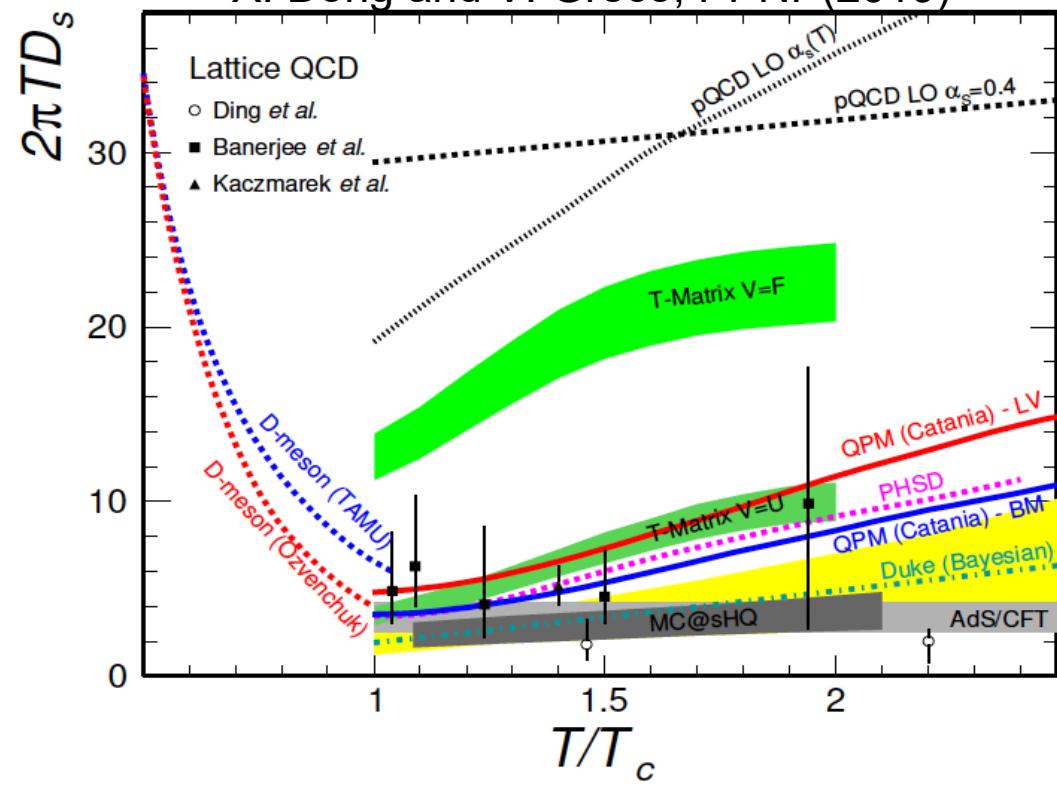
# Transport coefficient

Z. Citron et al., CERN Yellow Rep. Monogr. 7 (2019) 1159



Different hadronization models can affect  
the extraction of the charm quark diffusion coefficient  
**New joint activity is starting**

X. Dong and V. Greco, PPNP(2019)



**2018-2019**  
**Several Collab. in joint activities:**

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R. Rapp et al., Nucl. Phys. A 979 (2018)
- HQ-JETS:  
S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

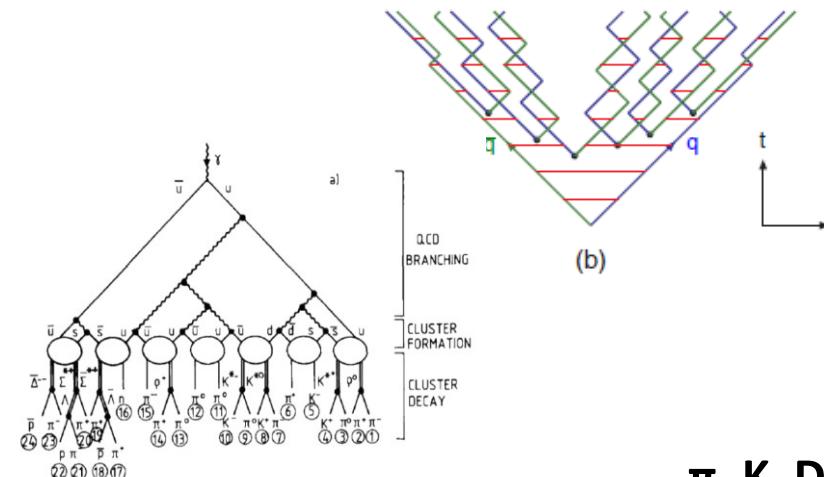
# HF Hadronization schemes

- Independent fragmentation

$$q \rightarrow \pi, K, p, \Lambda \dots$$

$$c \rightarrow D, D_s, \Lambda_c, \dots$$

- String fragmentation (PYTHIA)



- In medium hadronization with Cluster decay

A. Beraudo et al., arXiv:2202.08732v1 [hep-ph]

- Coalescence/recombination

P.B. Gossiaux, R. Bierkandt and J. Aichelin, PRC 79 (2009) 044906.

S. Plumari, V. Minissale et al, Eur. Phys. J. C78 no. 4, (2018) 348

S. Cao et al. , Phys. Lett. B 807 (2020) 135561

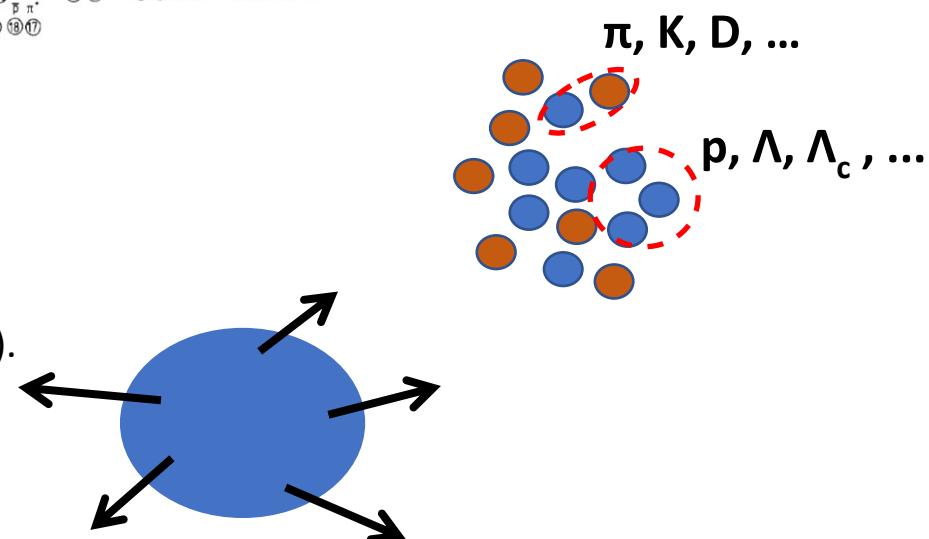
## Resonance Recombination model

L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).

L. Ravagli, H. van Hees and R. Rapp, Phys. Rev. C 79, 064902 (2009).

- Statistical hadronization model (SHM)

A. Andronic et al, JHEP 07 (2021) 035



# Indipendent fragmentation

Inclusive hadron production from hard-scattering processes (large  $Q^2$ ):

Factorization of: PDFs, partonic cross section (pQCD),  
fragmentation function

$$\frac{dN_h}{d^2 p_h} = \sum_f \int dz \frac{dN_f}{d^2 p_f} D_{f \rightarrow h}(z)$$

*Fragmentation function*

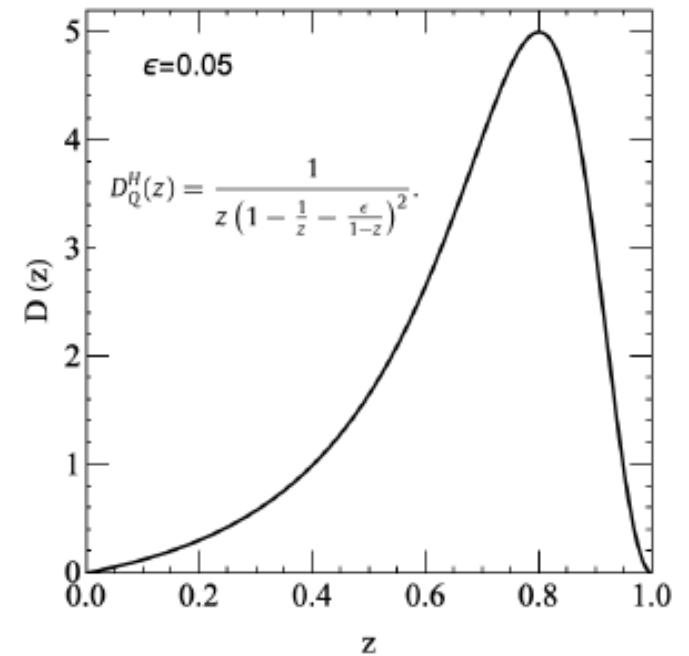
$q \rightarrow \pi, K, p, \Lambda \dots$   
 $c \rightarrow D, D_s, \Lambda_c, \dots$

Fragmentation functions  $D_{f \rightarrow h}$  are phenomenological functions

to parameterize the *non-perturbative parton-to-hadron transition*

$z$  = fraction of the parton momentum taken by the hadron  $h$

Fragmentation functions assumed **universal** among energy  
and collision systems and constrained from  $e^+e^-$  and  $e\mu$



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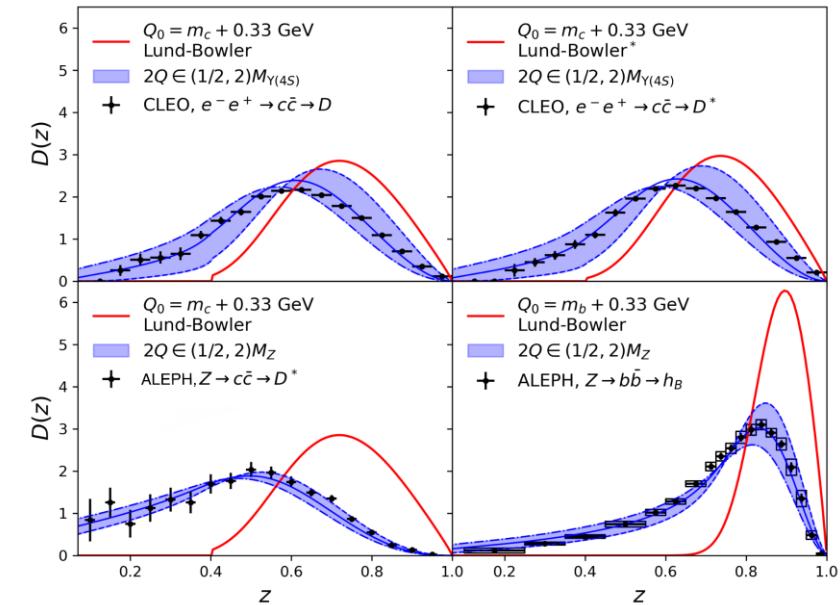
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Fragmentation functions assumed **universal** among energy  
and collision systems and constrained from  $e^+e^-$  and  $e p$

W. Ke and I. Vitev, arXiv:2204.00634 [hep-ph]



Evolution softens HQ fragmentation functions

# Hadronization: fragmentation and coalescence

## Proton to pion ratio Enhancement:

In vacuum from fragmentation functions  
the ratio is small

$$\frac{D_{q \rightarrow p}(z)}{D_{q \rightarrow \pi}(z)} < 0.25$$

## Elliptic flow splitting:

For  $p_T > 2$  GeV Both hydro and fragmentation predicts similar  $v_2$  for pions and protons

## Another hadronization mechanism is by coalescence:

Formalism originally developed for light-nuclei production from coalescence of nucleons on a freeze-out hypersurface.

Extended to describe meson and baryon formation in AA collisions from the quarks of QGP through  $2 \rightarrow 1$  and  $3 \rightarrow 1$  processes

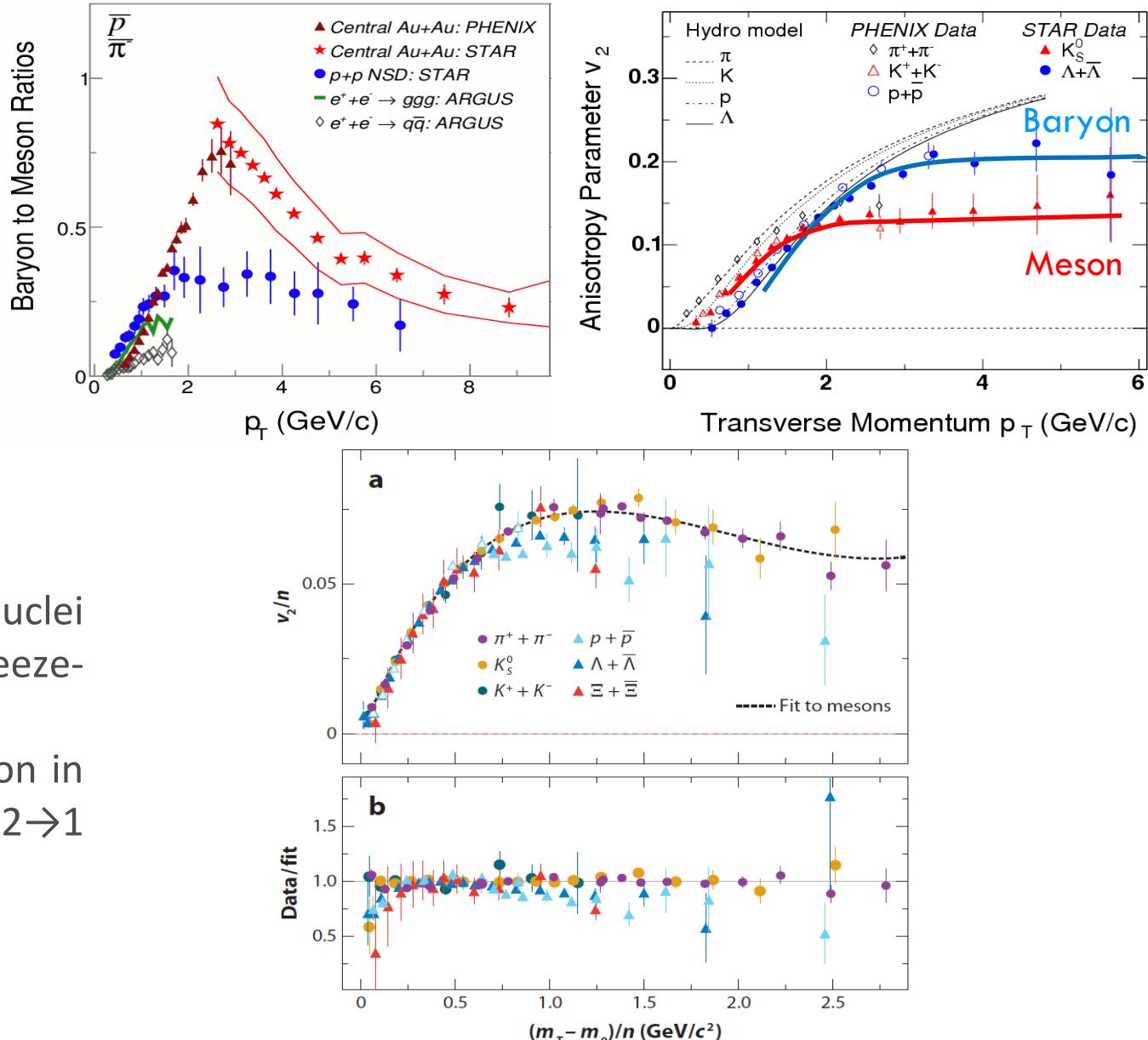
V. Greco, C.M. Ko, P. Levai PRL 90, 202302 (2003).

V. Greco, C.M. Ko, P. Levai PRC 68, 034904 (2003).

R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRL 90, 202303 (2003).

R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRC 68, 044902 (2003).

R. J. Fries, V. Greco, P. Sorensen Ann.Rev.Nucl.Part.Sci. 58 (2008) 177



## Coalescence model

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

# Coalescence model

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Meson



$$H = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{K}{2} (r_1 - r_2)^2 \rightarrow H = \frac{P_{CM}^2}{2M} + \frac{P_{x1}^2}{2\mu_1} + \frac{1}{2} \mu_1 \omega^2 {x_1}^2$$

$$W(r, p) = \int d^4y e^{-ipy} \psi(r + \frac{y}{2}) \psi(r - \frac{y}{2}) \rightarrow \frac{8}{(2\pi\hbar c)^3} e^{-\frac{2x_1^2}{\sigma_r^2}} e^{-\frac{p^2}{2\sigma_p^2}}$$

# Coalescence model

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) W(x_1, \dots, x_n; p_1, \dots, p_n) \delta\left(p_T - \sum_i p_{iT}\right)$$

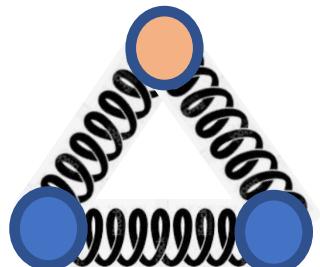
## Meson



$$H = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{K}{2} (r_1 - r_2)^2 \rightarrow H = \frac{P_{CM}^2}{2M} + \frac{P_{x1}^2}{2\mu_1} + \frac{1}{2} \mu_1 \omega^2 {x_1}^2$$

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## Baryon



$$H = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{P_3^2}{2m_3} + \frac{K'}{2} (r_1 - r_2)^2 + \frac{K'}{2} (r_2 - r_3)^2 + \frac{K''}{2} (r_3 - r_1)^2$$

$$W(r, p) = \int d^4y e^{-ipy} \psi(r + \frac{y}{2}) \psi(r - \frac{y}{2}) \rightarrow \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - \frac{p_{ri}^2}{2\sigma_{ri}^2}\right)$$

# Coalescence in AA: Catania

S. Plumari, V. Minissale, S.K. Das, G. Coci, and V. Greco, EPJC 78 (2018) 4, 348 , V. Minissale, S. Plumari, V. Greco, PLB 821 (2021) 136622.

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Wigner function <-> Wave function

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$

$\varphi_M(\mathbf{r})$  meson wave function

Assuming gaussian wave function

$$f_M(x_1, x_2; p_1, p_2) = A_W \exp\left(-\frac{x_{r1}^2}{\sigma_r^2} - p_{r1}^2 \sigma_r^2\right)$$

For baryon  $N_q=3$

$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

Normalization  $f_H(\dots)$  fixed by requiring  $P_{coal}(p>0)=1$  which fixes  $A_w$ , additional assumption wrt standard coalescence which does not have confinement

Wigner function width fixed by root-mean-square charge radius from quark model

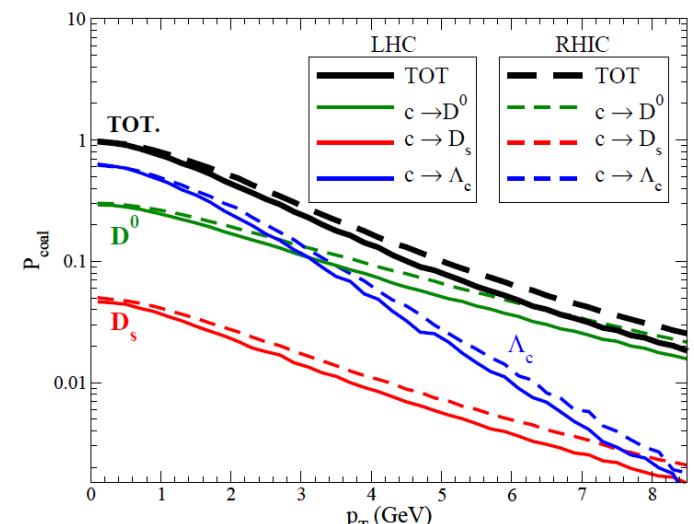
C.-W. Hwang, EPJ C23, 585 (2002);  
C. Albertus et al., NPA 740, 333 (2004)

$$\begin{aligned} \langle r^2 \rangle_{ch} &= \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 \\ &+ \frac{3}{2} \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2 \end{aligned} \quad (8)$$

$\sigma_{ri} = 1/\sqrt{\mu_i \omega}$  Harmonic oscillator relation

$$\mu_1 = \frac{m_1 m_2}{m_1 + m_2}, \quad \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}.$$

| Meson                 | $\langle r^2 \rangle_{ch}$ | $\sigma_{p1}$ | $\sigma_{p2}$ |
|-----------------------|----------------------------|---------------|---------------|
| $D^+ = [c\bar{d}]$    | 0.184                      | 0.282         | —             |
| $D_s^+ = [\bar{s}c]$  | 0.083                      | 0.404         | —             |
| Baryon                | $\langle r^2 \rangle_{ch}$ | $\sigma_{p1}$ | $\sigma_{p2}$ |
| $\Lambda_c^+ = [udc]$ | 0.15                       | 0.251         | 0.424         |
| $\Xi_c^+ = [usc]$     | 0.2                        | 0.242         | 0.406         |
| $\Omega_c^0 = [ssc]$  | -0.12                      | 0.337         | 0.53          |



# Coalescence in AA: Catania

S. Plumari, V. Minissale, S.K. Das, G. Coci, and V. Greco, EPJC 78 (2018) 4, 348 , V. Minissale, S. Plumari, V. Greco, PLB 821 (2021) 136622.

$$\frac{dN_{Hadron}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$

Wigner function <-> Wave function

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$

$\varphi_M(\mathbf{r})$  meson wave function

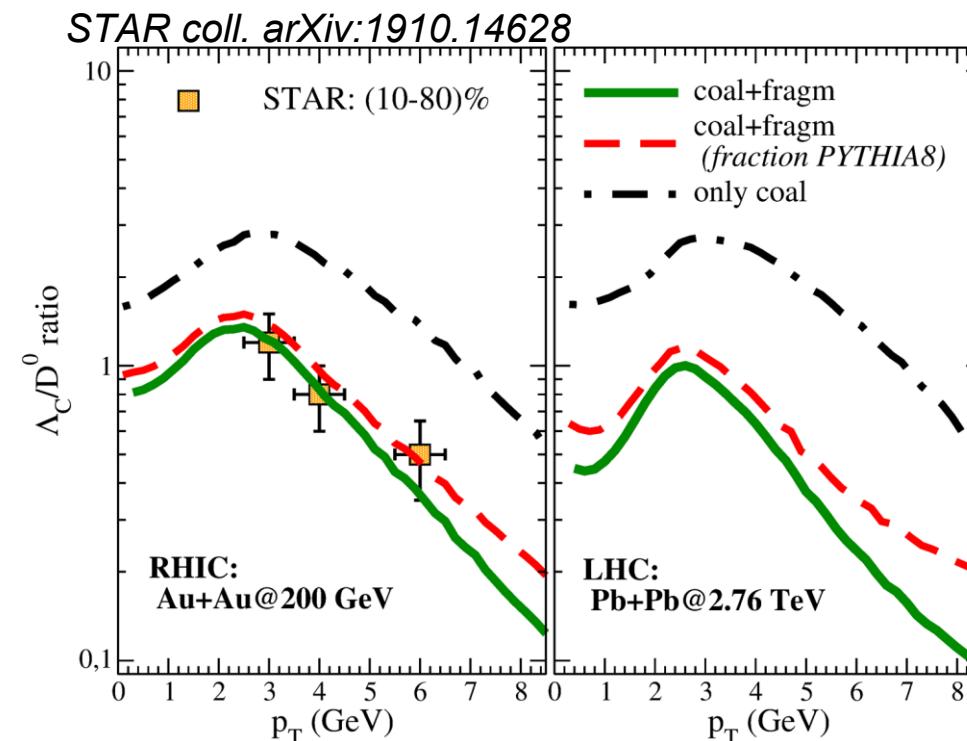
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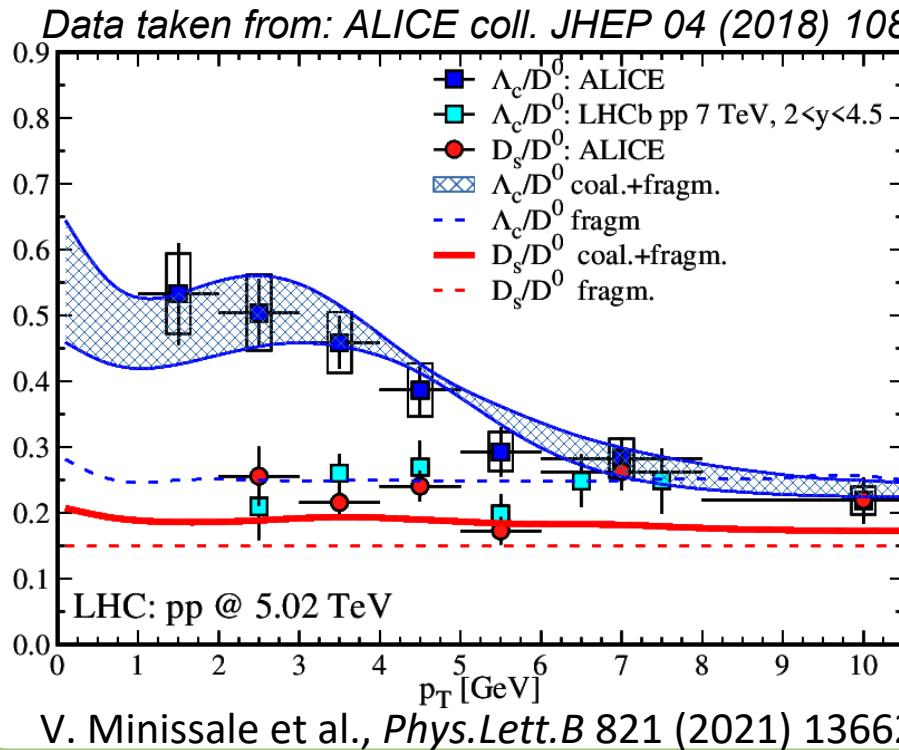
$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

Normalization  $f_H(\dots)$  fixed by requiring  $P_{coal}(p>0)=1$   
which fixes  $A_w$ , additional assumption wrt standard  
coalescence which does not have confinement



The  $\Lambda_c/D^0$  ratio is smaller at LHC energies:  
fragmentation play a role at intermediate  $p_T$

# Coalescence in pp: Catania



Error band correspond to  $\langle r^2 \rangle$  uncertainty in quark model

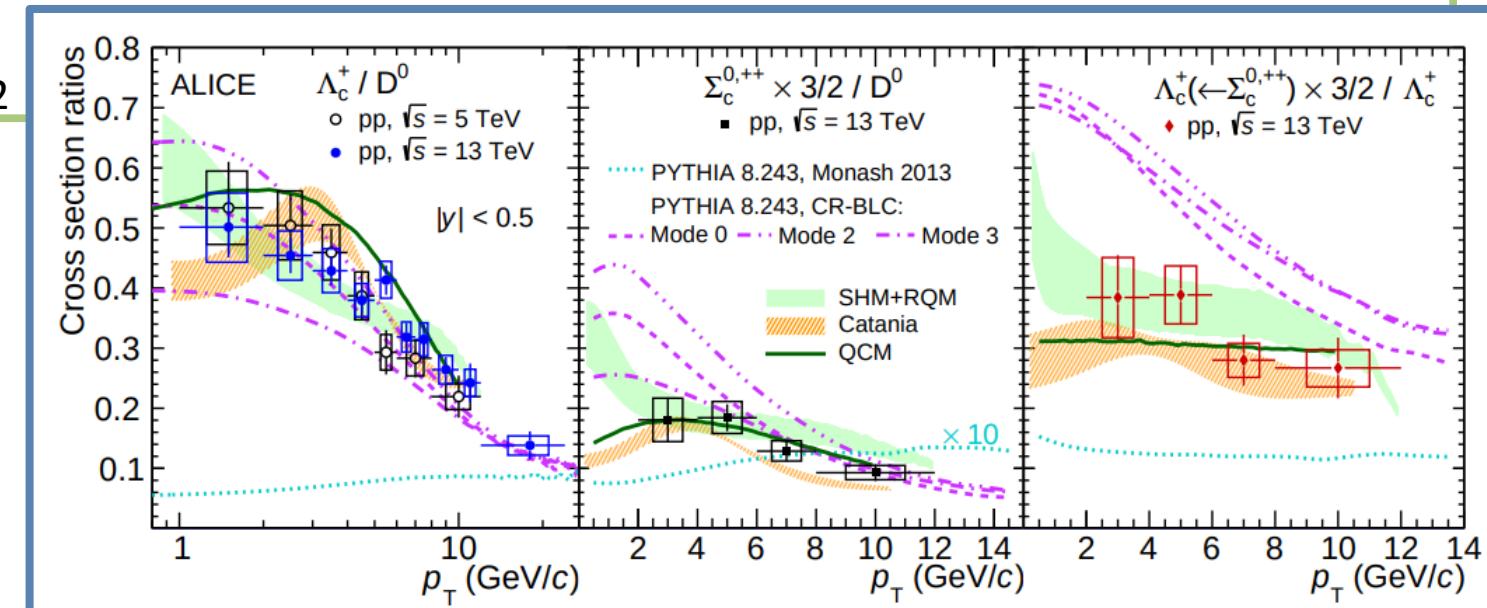
Reduction of rise-and-fall behaviour in  $\Lambda_c / D^0$  ratio:

-Confronting with AA: Coal. contribution smaller w.r.t. Fragm.

-FONLL distribution flatter w/o evolution through QGP

-Volume size effect

ALICE Coll., Physical Review Letters 128, 012001 (2022)



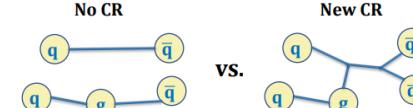
Other models:

He-Rapp, Phys.Lett.B 795 (2019) 117-121:

Increase  $\approx 2$  to  $\Lambda_c$  production: SHM with resonance not present in PDG

**PYTHIA8 + color reconnection**

CR with SU(3) weights and string length minimization



# Coalescence : Duke

Y. Xu, S. Cao, M. Nahrgang, W. Ke, G. Qin, J. Auvinen, and S. Bass, Nucl.Part.Phys.Proc. 276 (2016) 225.

S. Cao, G. Qin, and S. Bass, PRC92, 024907 (2015).

## Instantaneous coalescence model

### Mesons

$$\frac{dN_M}{d^3 p_M} = \int d^3 p_1 d^3 p_2 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} f_M^W(\vec{p}_1, \vec{p}_2) \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

$$f_M^W(q^2) = g_M \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-q^2\sigma^2} \quad \vec{q} \equiv \frac{E_2^{\text{cm}} \vec{p}_1^{\text{cm}} - E_1^{\text{cm}} \vec{p}_2^{\text{cm}}}{E_1^{\text{cm}} + E_2^{\text{cm}}}$$

### Baryons

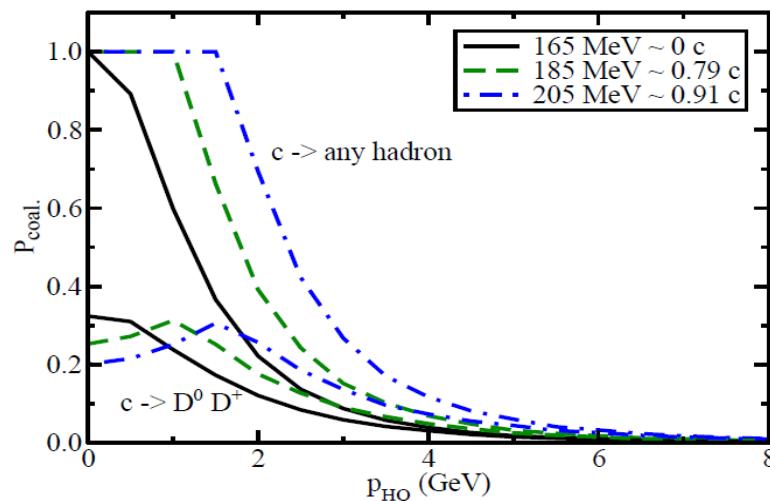
$$\frac{dN_B}{d^3 p_B} = \int d^3 p_1 d^3 p_2 d^3 p_3 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} \frac{dN_3}{d^3 p_3} f_B^W(\vec{p}_1, \vec{p}_2, \vec{p}_3)$$

$$\times \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2 - \vec{p}_3).$$

$$f_B^W(q_1^2, q_2^2) = g_B \frac{(2\sqrt{\pi})^6 (\sigma_1 \sigma_2)^3}{V^2} e^{-q_1^2 \sigma_1^2 - q_2^2 \sigma_2^2}$$

$$\vec{q}_1 \equiv \frac{E_2^{\text{cm}} \vec{p}_1^{\text{cm}} - E_1^{\text{cm}} \vec{p}_2^{\text{cm}}}{E_1^{\text{cm}} + E_2^{\text{cm}}},$$

$$\vec{q}_2 \equiv \frac{E_3^{\text{cm}} (\vec{p}_1^{\text{cm}} + \vec{p}_2^{\text{cm}}) - (E_1^{\text{cm}} + E_2^{\text{cm}}) \vec{p}_3^{\text{cm}}}{E_1^{\text{cm}} + E_2^{\text{cm}} + E_3^{\text{cm}}}$$



$$\sigma_{ri} = 1/\sqrt{\mu_i \omega} \quad \text{Harmonic oscillator relation}$$

$$\mu_1 = \frac{m_1 m_2}{m_1 + m_2}, \quad \mu_2 = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}.$$

These two parameters are obtained by requiring the coalescence probability through all possible hadronization channels to be unity for a zero momentum heavy quark.

# Coalescence : Duke

Y. Xu, S. Cao, M. Nahrgang, W. Ke, G. Qin, J. Auvinen, and S. Bass, Nucl. Part. Phys. Proc. 276 (2016) 225

S. Cao, G. Qin, and S. Bass, PRC92, 024907 (2015).

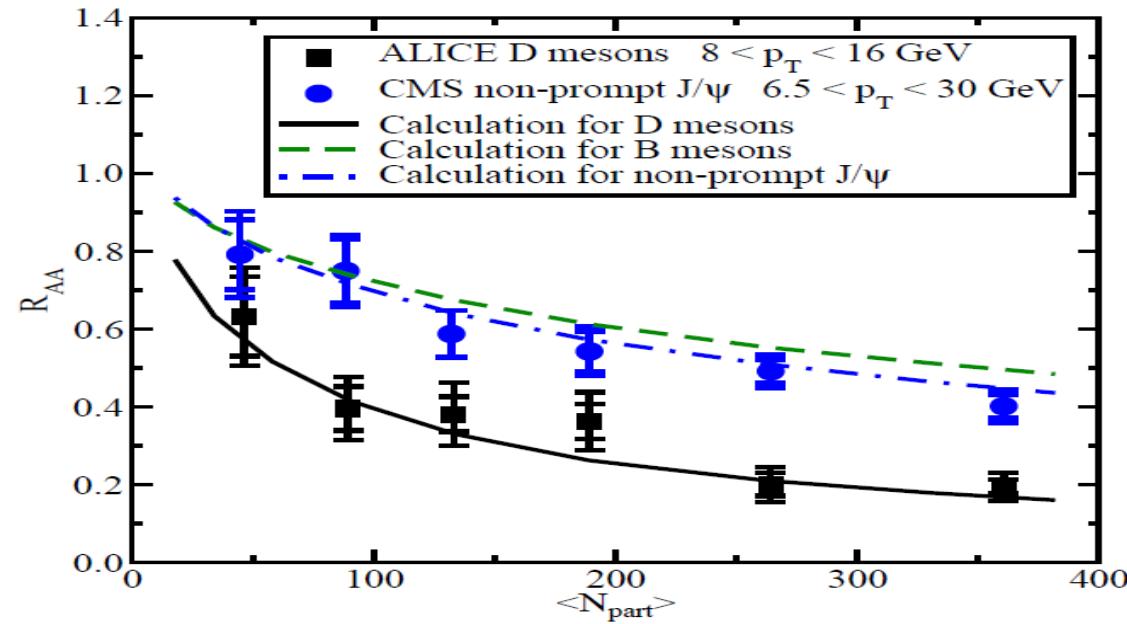
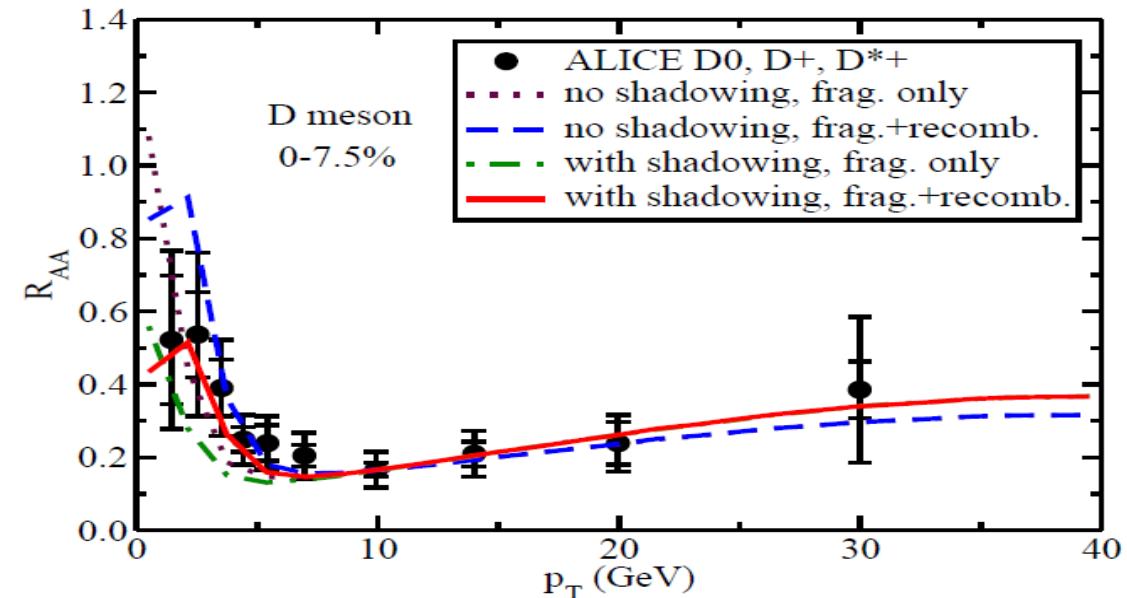
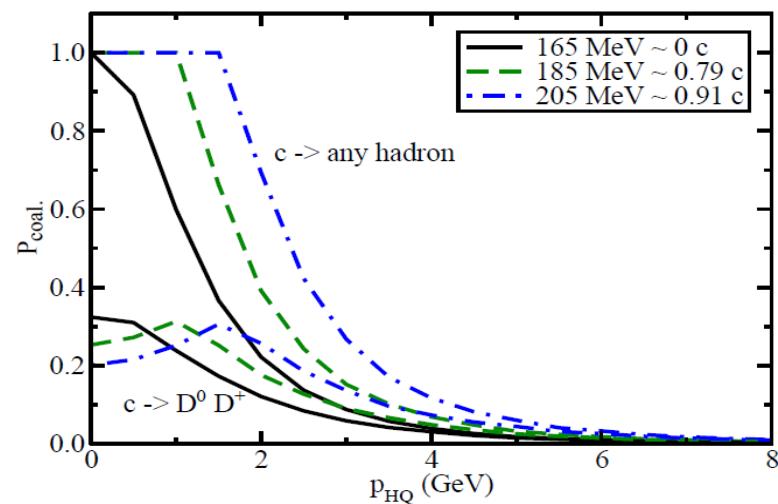
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### Baryons

$$\begin{aligned} \frac{dN_B}{d^3 p_B} = & \int d^3 p_1 d^3 p_2 d^3 p_3 \frac{dN_1}{d^3 p_1} \frac{dN_2}{d^3 p_2} \frac{dN_3}{d^3 p_3} f_B^W(\vec{p}_1, \vec{p}_2, \vec{p}_3) \\ & \times \delta(\vec{p}_M - \vec{p}_1 - \vec{p}_2 - \vec{p}_3). \end{aligned}$$



# Coalescence: LBT

S. Cao, K. Sun, S. Li, S. Liu, W. Xing, G. Qin, and C. Ko, PLB 807 (2020) 135561.

F. Liu, W. Xing, X. Wu, G. Qin, S. Cao, and X. Wang, EPJC 82 (2022) 4, 350.

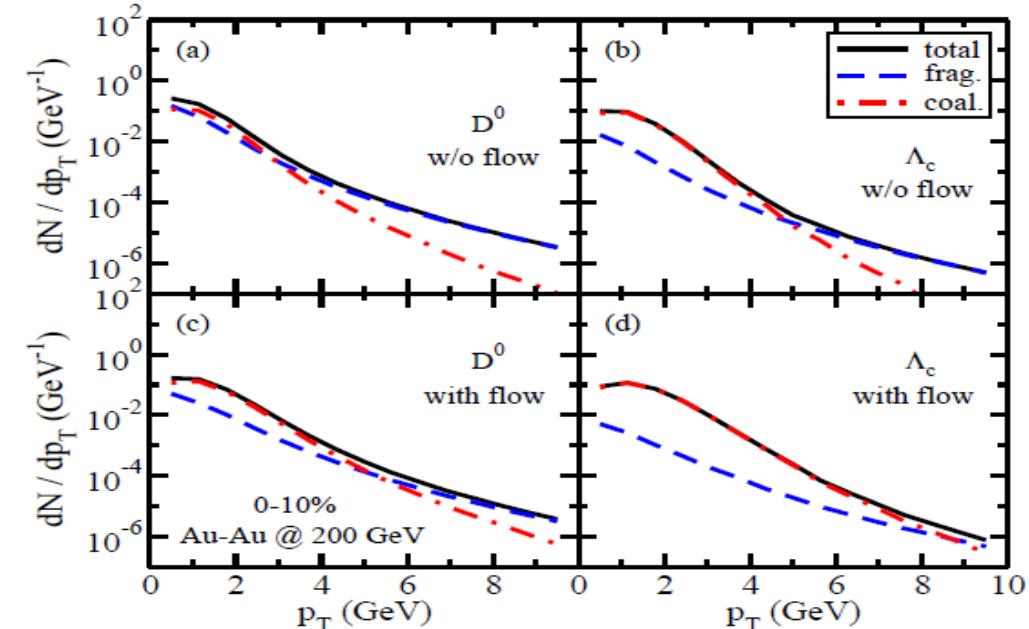
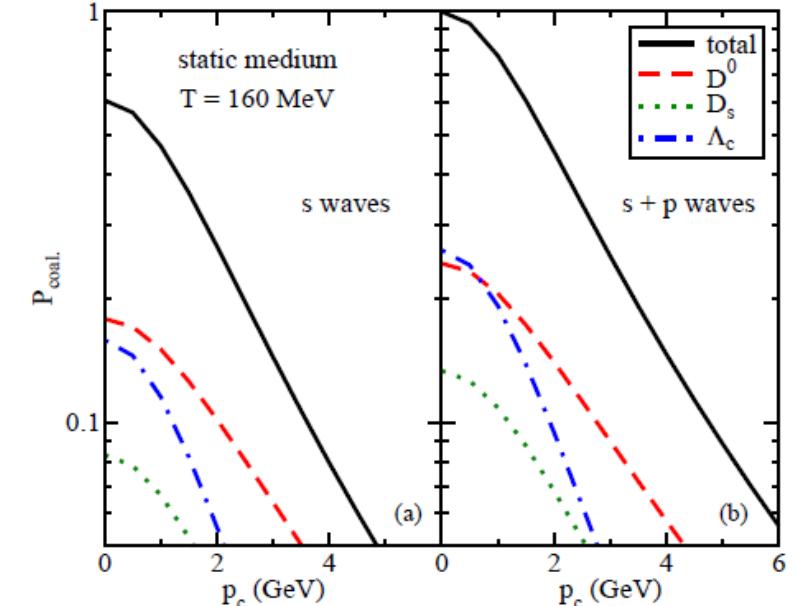
$$f_h(p'_h) = \int \left[ \prod_i dp_i f_i(p_i) \right] W(\{p_i\}) \delta(p'_h - \sum_i p_i)$$

- The quark wave functions in the meson is assumed to be those of a harmonic oscillator potential
- The Wigner functions for mesons are in the s and p-wave states

$$W_s = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 \mathbf{k}^2},$$

$$W_p = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3} \sigma^2 \mathbf{k}^2 e^{-\sigma^2 \mathbf{k}^2}$$

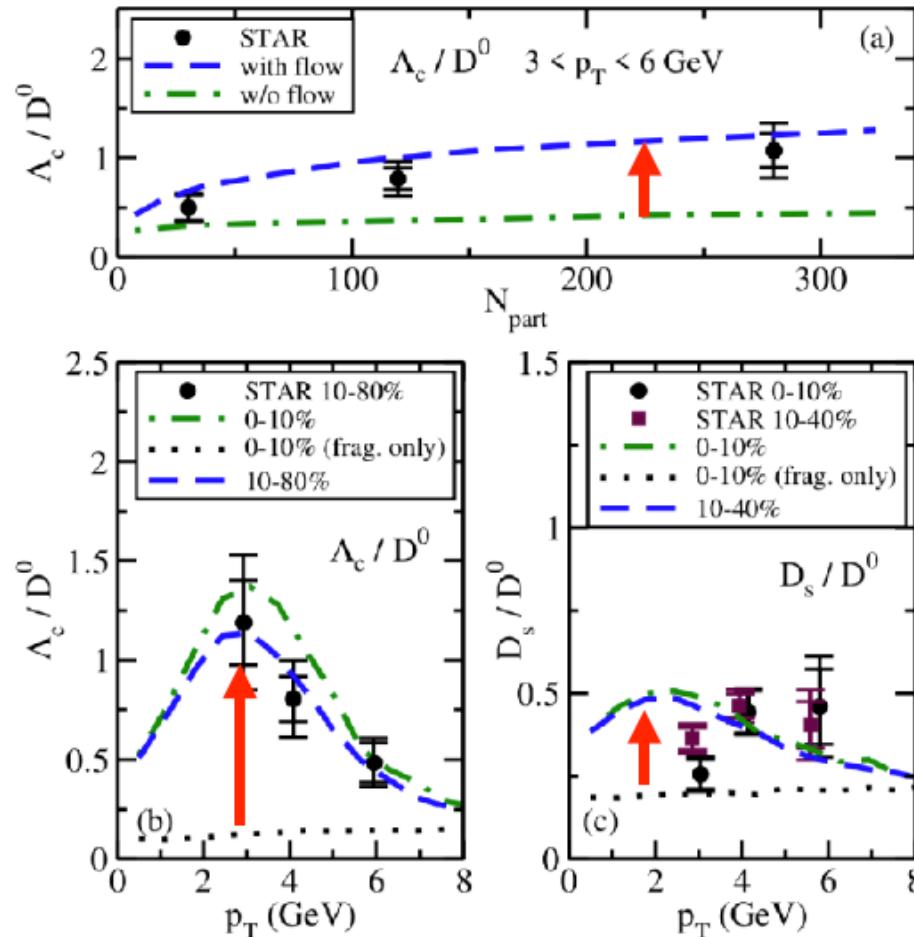
The oscillator frequency is fixed to impose that the total coalescence probability for zero-momentum charm quark is equal to 1 when s and p states are included.



# Coalescence: LBT

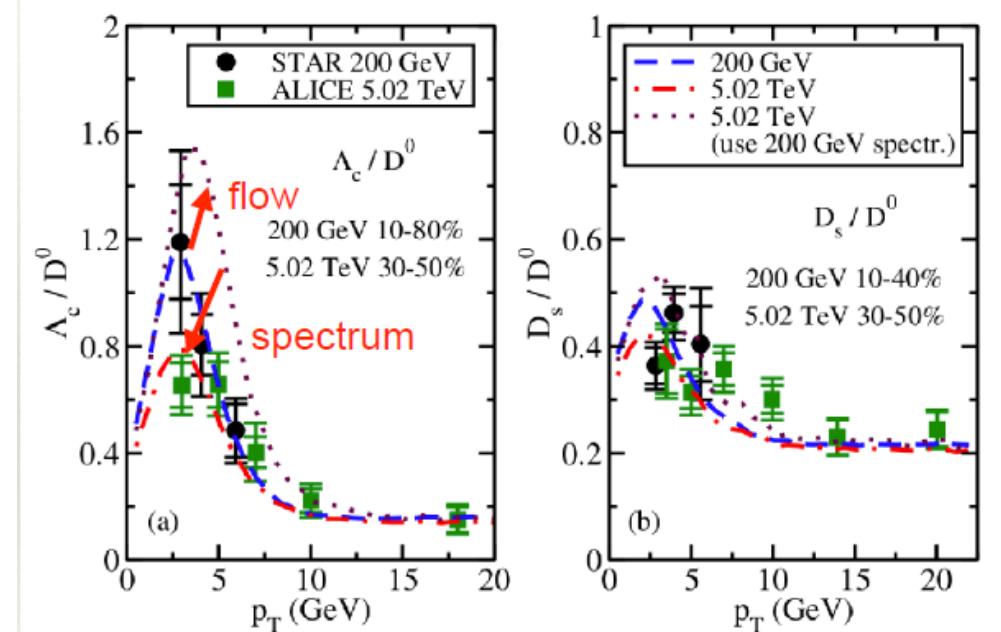
S. Cao, K. Sun, S. Li, S. Liu, W. Xing, G. Qin, and C. Ko, PLB 807 (2020) 135561.  
 F. Liu, W. Xing, X. Wu, G. Qin, S. Cao, and X. Wang, EPJC 82 (2022) 4, 350.

Recent improvements of the coalescence model Wigner function modified including both s and p wave states



**Stronger QGP flow boost on heavier hadrons  
 => increasing  $\Lambda_c / D^0$  ratio with  $N_{\text{part}}$**

harder initial charm spectra at LHC reduces the  $\Lambda_c / D^0$  ratio



# Coalescence : NANTES

M. Nahrgang, J. Aichelin, P.B. Gossiaux, and K. Werner, PRC 93 (2016) 4, 044909.  
 P.B. Gossiaux, R. Bierkandt and J. Aichelin, PRC 79 (2009) 044906.

**The heavy quarks form hadrons (D and B mesons) either by coalescence or by fragmentation.**

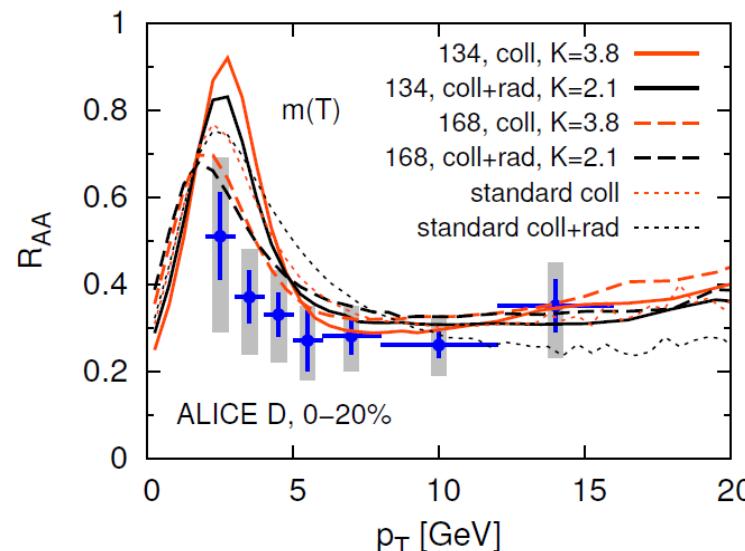
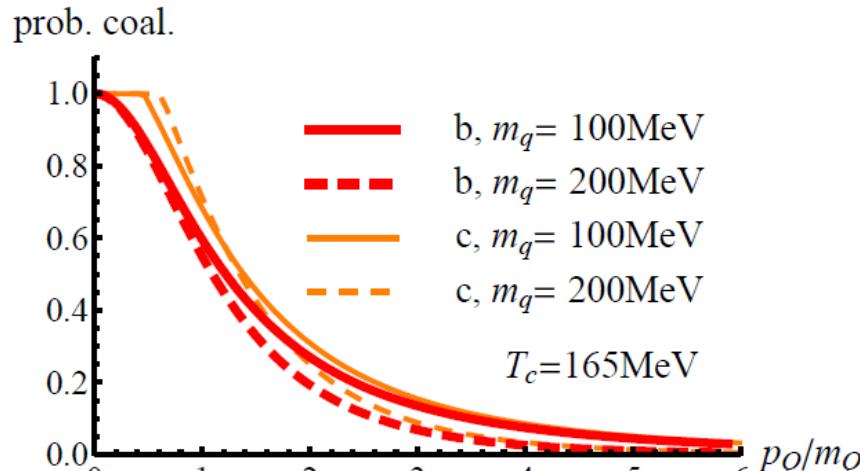
- The coalescence mechanism is based on the model of C. B.

Dover et al., PRC 44, 1636 (1991)

$$N_{\Phi=D,B} = \int p_Q \cdot d\sigma_1 p_q \cdot d\sigma_2 \frac{d^3 p_Q}{(2\pi\hbar)^3 E_Q} \frac{d^3 p_q}{(2\pi\hbar)^3 E_q} \\ \times f_Q(x_Q, p_Q) f_q(x_q, p_q) f_\Phi(x_Q, x_q; p_Q, p_q)$$

- The relative fraction depends on  $p_Q$ , on the fluid cell velocity and on the orientation of the hypersurface.

$$f_\Phi(x_Q, x_q; p_Q, p_q) = \exp\left(\frac{(x_q - x_Q)^2 - [(x_q - x_Q) \cdot u_Q]^2}{2R_c^2}\right) \exp(-\alpha_d^2(u_Q \cdot u_q - 1))$$



# Coalescence : PHSD

T. Song, H. Berrehrah, D. Cabrera, J. M. Torres-Rincon, L. Tolos, W. Cassing and E. Bratkovskaya, PRC 92, no.1, 014910 (2015).  
 T. Song, H. Berrehrah, D. Cabrera, W. Cassing and E. Bratkovskaya, PRC 93, no.3, 034906 (2016).

$$\frac{dN_{Hadron}}{d^2p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3p_i}{(2\pi)^3} f_q(x_i, p_i) W(x_1, \dots, x_n; p_1, \dots, p_n) \delta\left(p_T - \sum_i p_{iT}\right)$$

$$W_s(r, p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$$

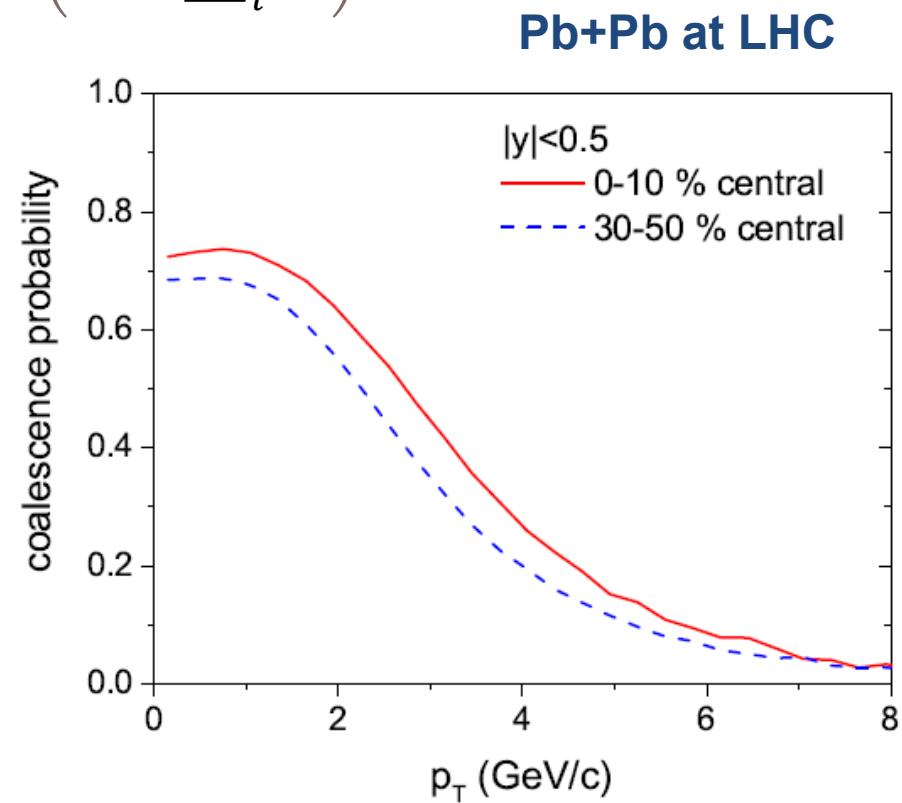
$$W_p(r, p) = \frac{2S+1}{36} \left( \frac{16}{3} \frac{r^2}{\sigma^2} + \frac{16}{3} \sigma^2 p^2 - 8 \right) e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2}$$

$$r = |\mathbf{r}_1 - \mathbf{r}_2|,$$

$$p = \frac{|m_2 \mathbf{p}_1 - m_1 \mathbf{p}_2|}{m_1 + m_2}$$

$$\langle r_M^2 \rangle = \frac{1}{2} \langle (\mathbf{R} - \mathbf{r}_1)^2 + (\mathbf{R} - \mathbf{r}_2)^2 \rangle$$

$$= \frac{1}{2} \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \langle r^2 \rangle = \frac{3}{4} \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \sigma^2$$



# Coalescence : PHSD

T. Song, H. Berrehrah, D. Cabrera, J. M. Torres-Rincon, L. Tolos, W. Cassing and E. Bratkovskaya, PRC 92, no.1, 014910 (2015).

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$$W_s(r, p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2} - \sigma^2 p^2},$$

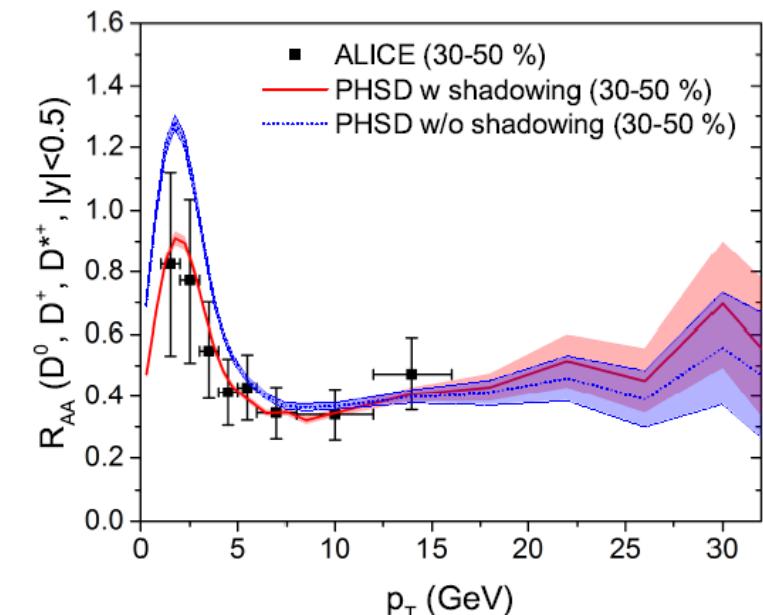
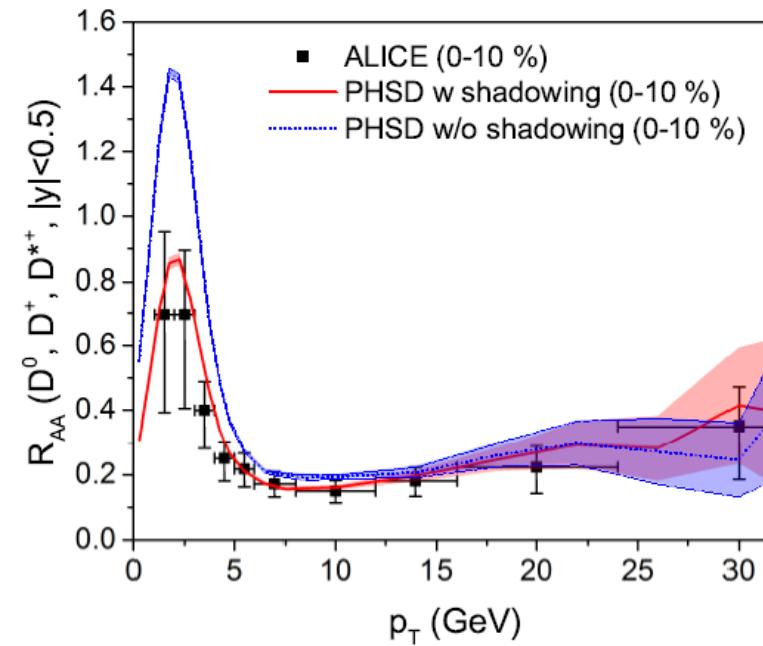
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$$\langle r_M^2 \rangle = \frac{1}{2} \langle (\mathbf{R} - \mathbf{r}_1)^2 + (\mathbf{R} - \mathbf{r}_2)^2 \rangle$$

$$= \frac{1}{2} \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \langle r^2 \rangle = \frac{3}{4} \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \sigma^2$$



# Resonance Recombination Model (RRM)

## Alternative dynamical realization of the coalescence approach

Hadronization proceeds via formation of resonant states when approaching the critical temperature

Starting point is the Boltzmann equation for the **meson**

$$\left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) F_M(t, \vec{x}, \vec{p}) = -\frac{\Gamma}{\gamma_p} F_M(t, \vec{x}, \vec{p}) + \beta(\vec{x}, \vec{p})$$

The gain term

$$g(\vec{p}) = \int d^3x \beta(\vec{x}, \vec{p}) = \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} \int d^3x f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \sigma(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2)$$

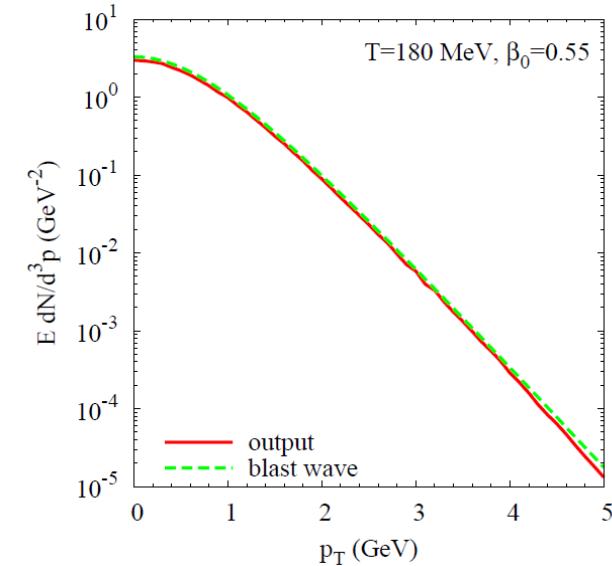
The cross section (**q+q → M**) is approximated by a relativistic Breit-Wigner

By imposing the stationarity condition at the equilibrium

$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^3} f_q(\vec{x}, \vec{p}_1) f_{\bar{q}}(\vec{x}, \vec{p}_2) \sigma_M(s) v_{\text{rel}}(\vec{p}_1, \vec{p}_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).

L. Ravagli, H. van Hees and R. Rapp, Phys. Rev. C 79, 064902 (2009).



Conserve energy in the hadron formation process  
Recover the equilibrium limit of hadron distributions

# Baryons in Resonance Recombination Model (RRM)

The 3-body hadronization process in RRM are conducted in 2 steps

## □ STEP 1

quark-1 and quark-2 recombine into a diquark,  
 $q_1(p_1) + q_2(p_2) \rightarrow dq(p_{12})$

The diquark spectrum in analogy to meson formation

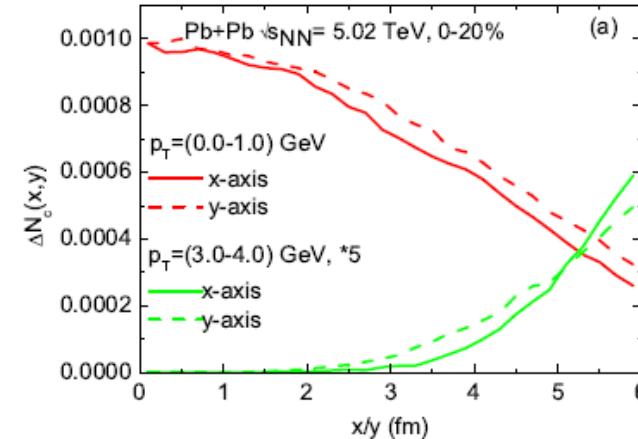
## □ STEP 2

the diquark recombines with quark-3 into a baryon  
 $dq_1(p_{12}) + q_3(p_3) \rightarrow B$

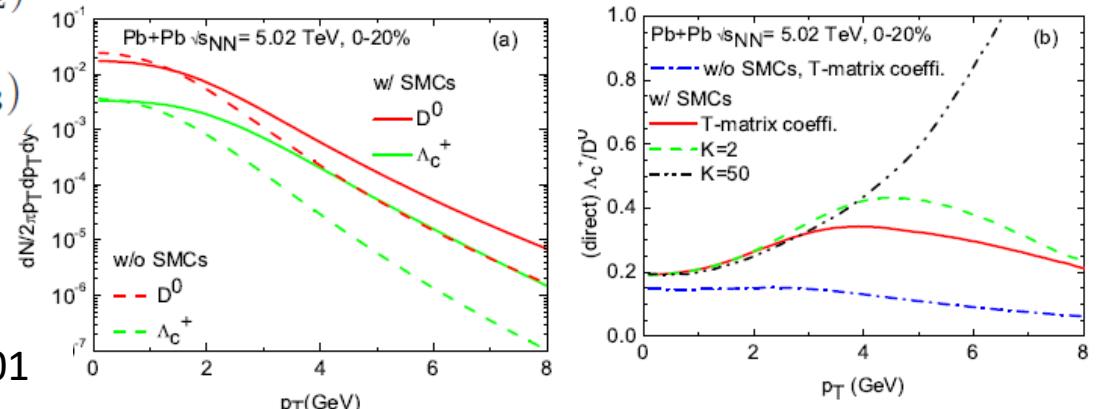
The baryon spectrum in analogy to meson formation

$$f_B(\vec{x}, \vec{p}) = \frac{\gamma_B}{\Gamma_B} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2 d^3 \vec{p}_3}{(2\pi)^6} \frac{\gamma_{dq}}{\Gamma_{dq}} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) \\ \times f_3(\vec{x}, \vec{p}_3) \sigma_{dq}(s_{12}) v_{\text{rel}}^{12} \sigma_B(s) v_{\text{rel}}^{dq3} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

Space-momentum correlations included



- low- $p_T$  (0-1 GeV) c quarks preferentially populate the inner regions of the fireball
- higher- $p_T$  (3-4 GeV) c quarks populate the outer regions of the fireball



# Baryons in Resonance Recombination Model (RRM)

The 3-body hadronization process in RRM are conducted in 2 steps

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 $q_1(p_1) + q_2(p_2) \rightarrow dq(p_{12})$

The diquark spectrum in analogy to meson formation

## □ STEP 2

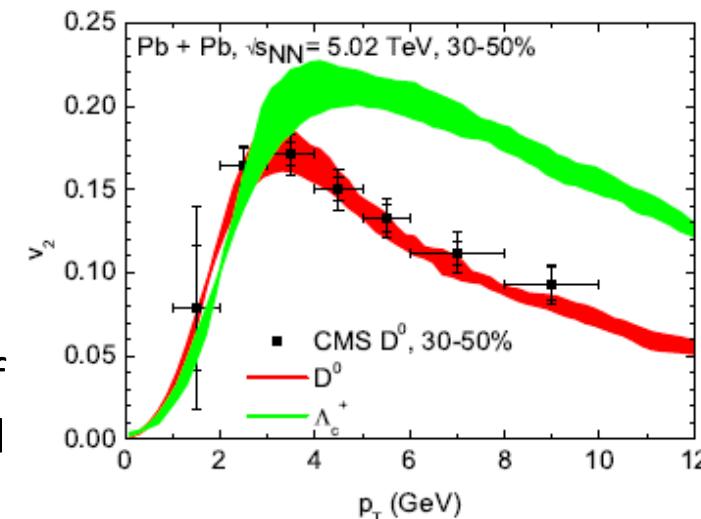
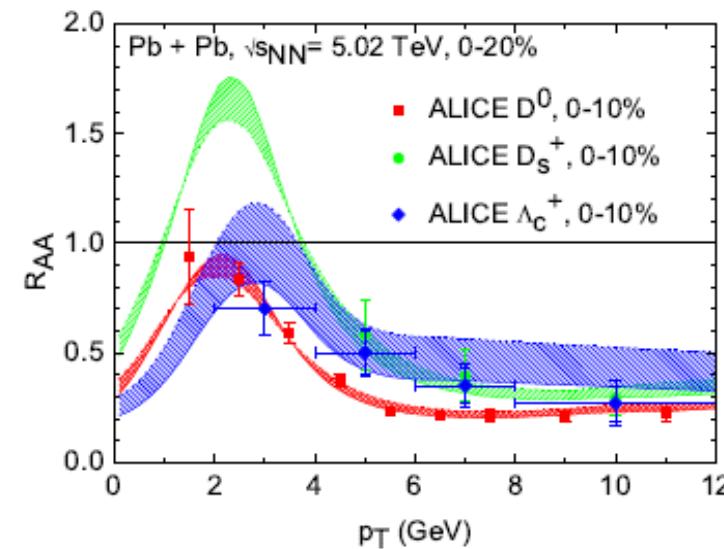
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The baryon spectrum in analogy to meson formation

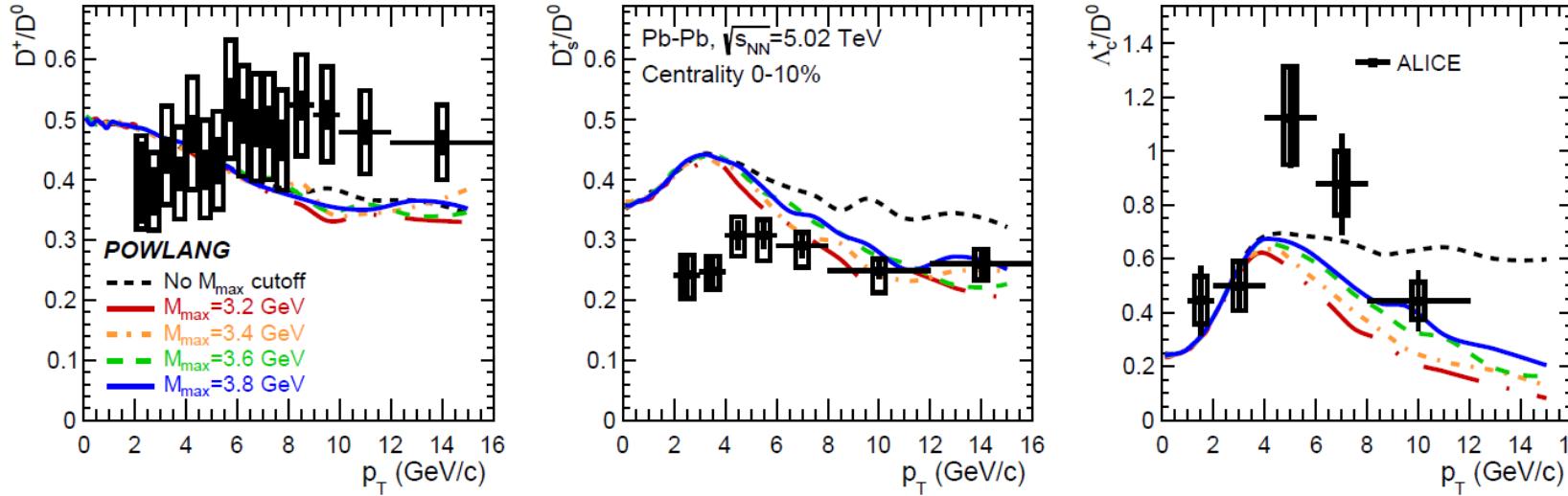
$$f_B(\vec{x}, \vec{p}) = \frac{\gamma_B}{\Gamma_B} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2 d^3 \vec{p}_3}{(2\pi)^6} \frac{\gamma_{dq}}{\Gamma_{dq}} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) \\ \times f_3(\vec{x}, \vec{p}_3) \sigma_{dq}(s_{12}) v_{\text{rel}}^{12} \sigma_B(s) v_{\text{rel}}^{dq3} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

HF hadro-chemistry improved by employing a large set of “missing” HF baryon states not listed by PDG, but predicted by the relativistic-quark model



# In-medium hadronization of heavy quarks

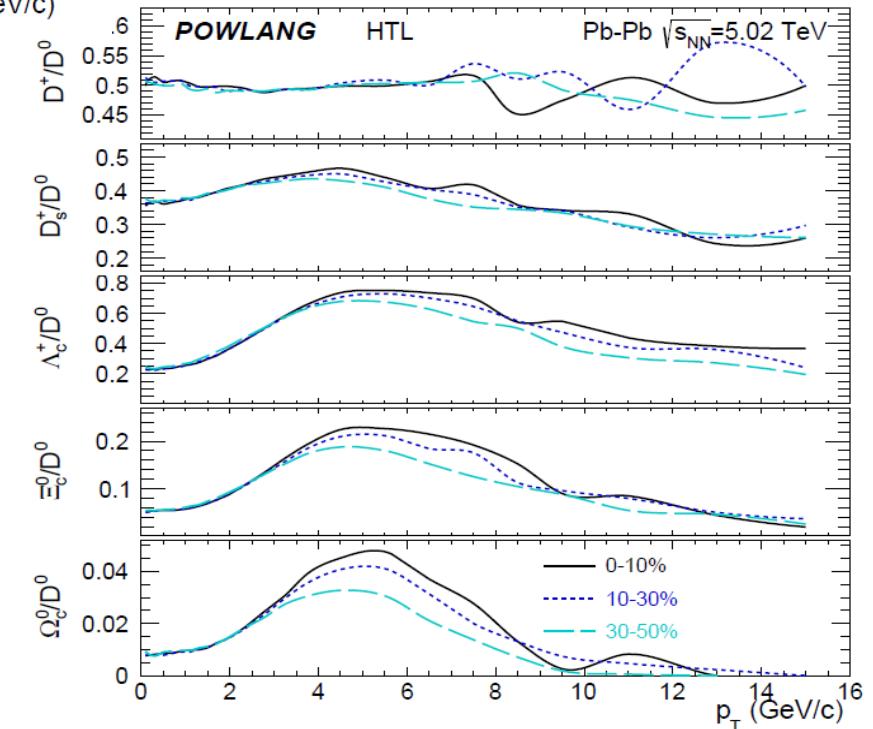
A. Beraudo et al., arXiv:2202.08732v1 [hep-ph]



HQ hadronization in the presence of a reservoir of lighter thermal particles:

Recombination of the HQ with light antiquark or diquarks:

- Color-singlet clusters with low invariant mass  $M$  ( $M < 4$  GeV) are assumed to undergo an isotropic 2-body decay in their local rest-frame.
- Heavier clusters are instead fragmented as Lund strings.
- Recombination with light diquarks  $\rightarrow$  enhances the yields of charmed baryons.
- The local color neutralization  $\rightarrow$  strong space-momentum correlation  $\rightarrow$  enhancement of the collective flow of the final charmed hadrons



## Conclusions

- **Charm hadronization in AA different than in  $e^+e^-$  and ep collisions**
  - Coalescence+fragmentation/Resonance Recombination Model enhancement of  $\Lambda_c$  production at intermediate  $p_T \rightarrow \Lambda_c/D^0 \sim 1$  for  $p_T \sim 3$  GeV
- ***In p+p assuming a medium:***
  - Coal.+fragm. good description of heavy baryon/meson ratio (closer to the data for  $\Lambda_c/D^0$ ,  $\Xi_c/D^0$ ,  $\Omega_c/D^0$ )

*Comparing different models is very useful for further understanding the hadronization mechanism*

