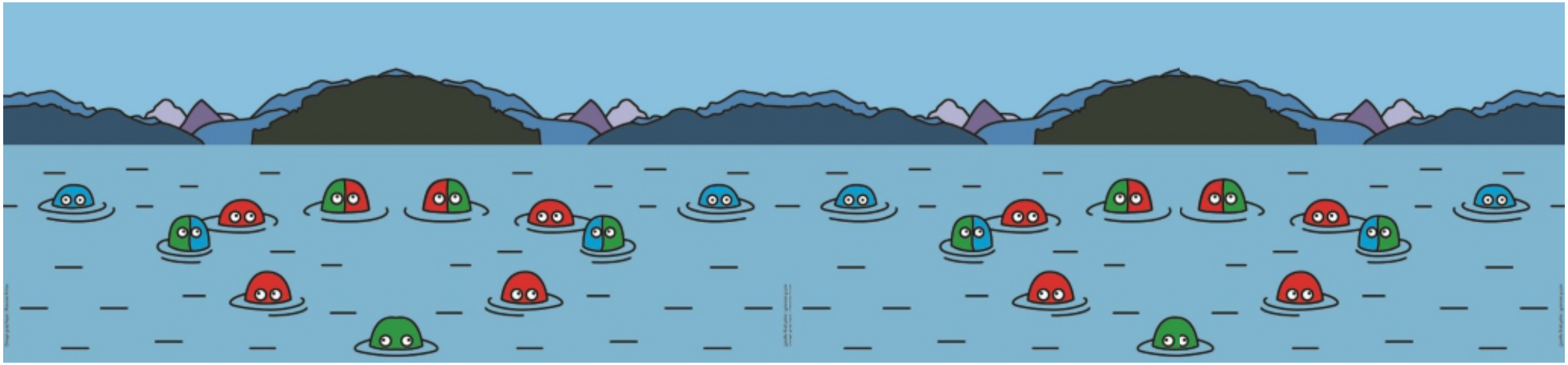
Theoretical overview of collective effects



Simple properties of the hydrodynamic description of large systems

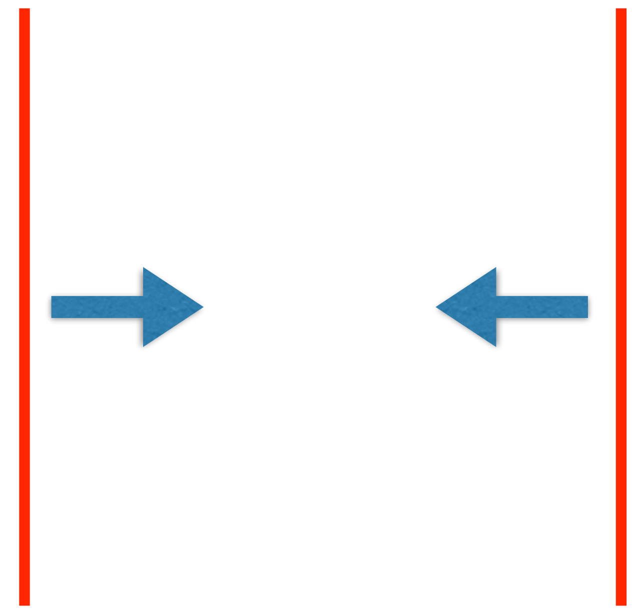
Jean-Yves Ollitrault

HF2022, Institut Pascal, October 19, 2022

Outline

1. Ideal hydrodynamics: scale invariance, equation of state from heavy-ion data
2. Viscous hydrodynamics: dimensional analysis, what we can and cannot learn about transport coefficients from data, problems with light-quark hadrons, the case of heavy quarks

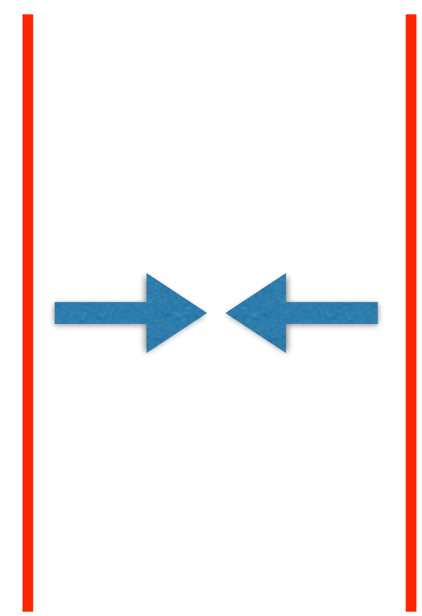
Head-on Pb+Pb collision at the LHC



Relativistic length contraction in the direction of motion, by a factor ~2700 at LHC

➔ Colliding spherical nuclei appears as disks

Head-on Pb+Pb collision at the LHC



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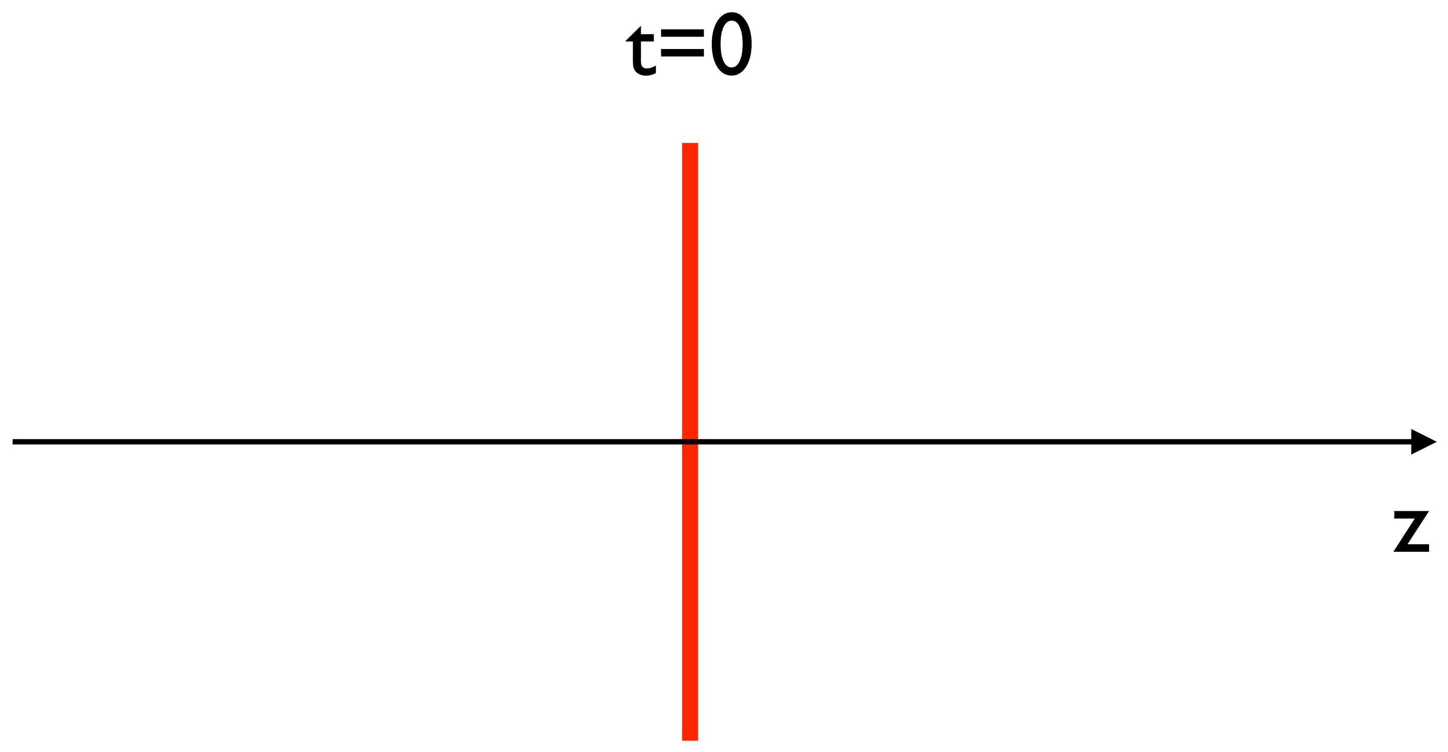
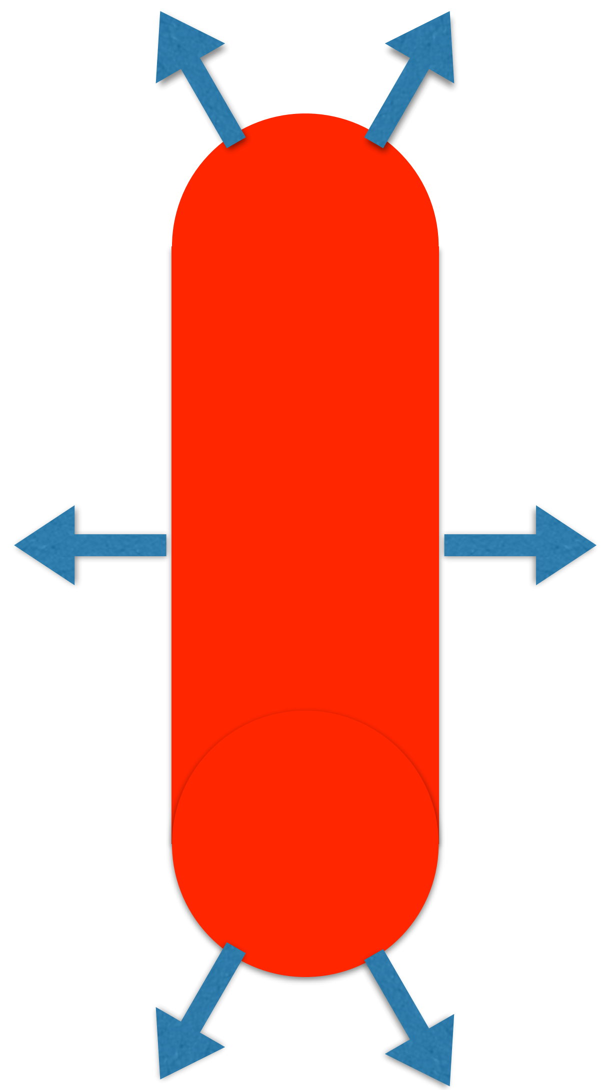
Head-on Pb+Pb collision at the LHC



Collision = instantaneous process at z=t=0

Head-on Pb+Pb collision at the LHC



- Strongly-coupled quark-gluon matter is created.
- Expands into the vacuum at ~ velocity of light

Why hydrodynamics

- Strongly-coupled system: cannot be described in terms of elementary particles or quasiparticles.
- Only valid first-principles description is macroscopic: a fluid expanding into the vacuum.
- Only description that explains experimental observations for light-quark hadrons to date. Now firmly established.

1. Ideal hydrodynamics = large-system limit

Equations of non-relativistic fluid dynamics:

1. Conservation of mass
2. Momentum equation (Euler equation)

   $\rho dv/dt = - \nabla P$

1. Ideal hydrodynamics = large-system limit

Equations of non-relativistic fluid dynamics:

1. Conservation of mass
2. Momentum equation (Euler equation)

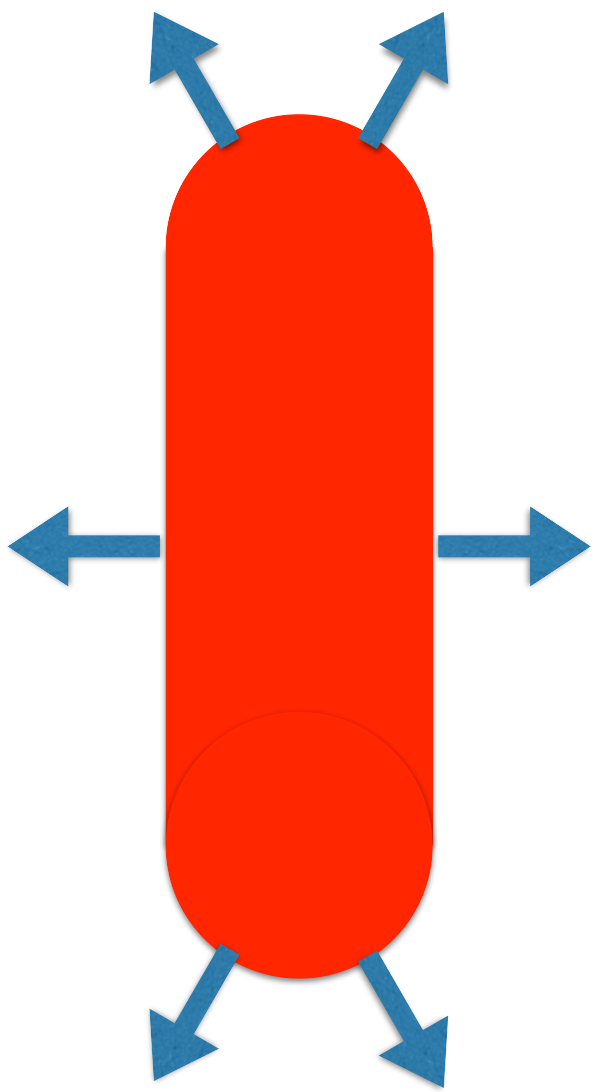
   $\rho dv/dt = - \nabla P$

Equations of relativistic fluid dynamics:

1. Conservation of energy
2. Relativistic Euler equation

   $(e+P) dv/dt = - \nabla P$

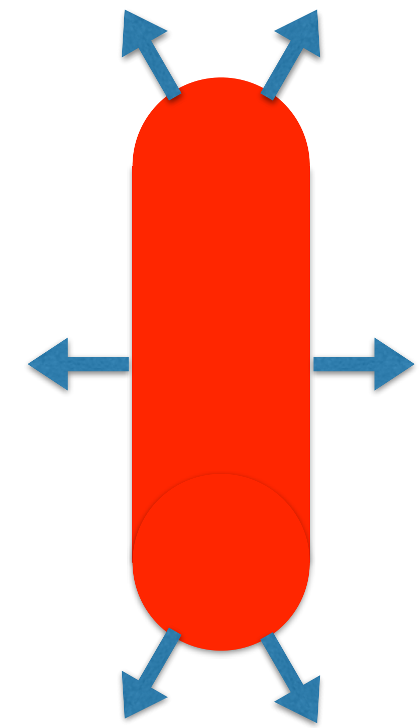
Scale invariance



Equations linear in space and time derivatives

if $v(\mathbf{x},t)$, $e(\mathbf{x},t)$ is a solution,

$v(\lambda\mathbf{x},\lambda t)$, $e(\lambda\mathbf{x},\lambda t)$ is also a solution for any λ.



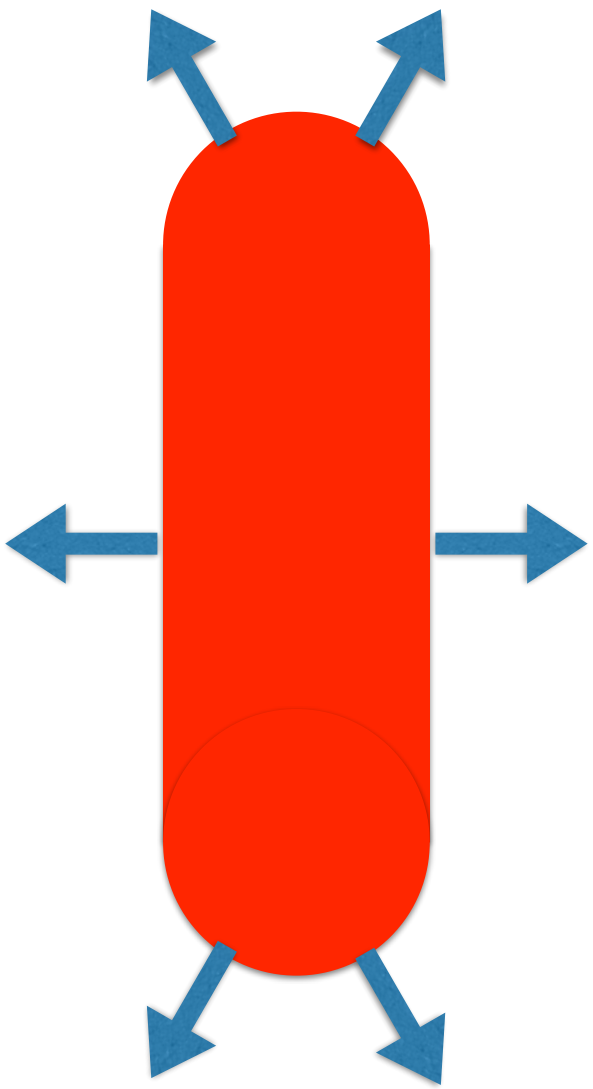
How an ideal hydro simulation works in practice

1. Initial condition: v and e at t≈0
2. Solve hydro equations with some equation of state (EOS: relation between e and P)
3. Convert the fluid locally, at some freeze-out temperature, into an ideal gas of hadrons at this temperature, boosted by the fluid velocity
   Implies: only dependence of spectra on hadron species is through the mass

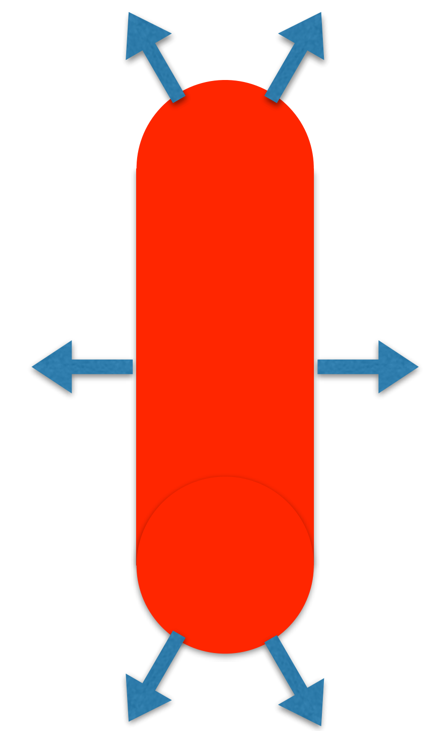
A robust description

1. Initial v: constrained by symmetry (v≈0 at midrapidity). Initial e: normalization constrained by final multiplicity, width of density profile constrained by nuclear radius.
2. Hydro evolution: only depends on equation of state, *only non-trivial input of an ideal hydro calculation, only place where QCD & its colour structure enter!*
3. Freeze-out temperature: constrained by relative hadron abundances

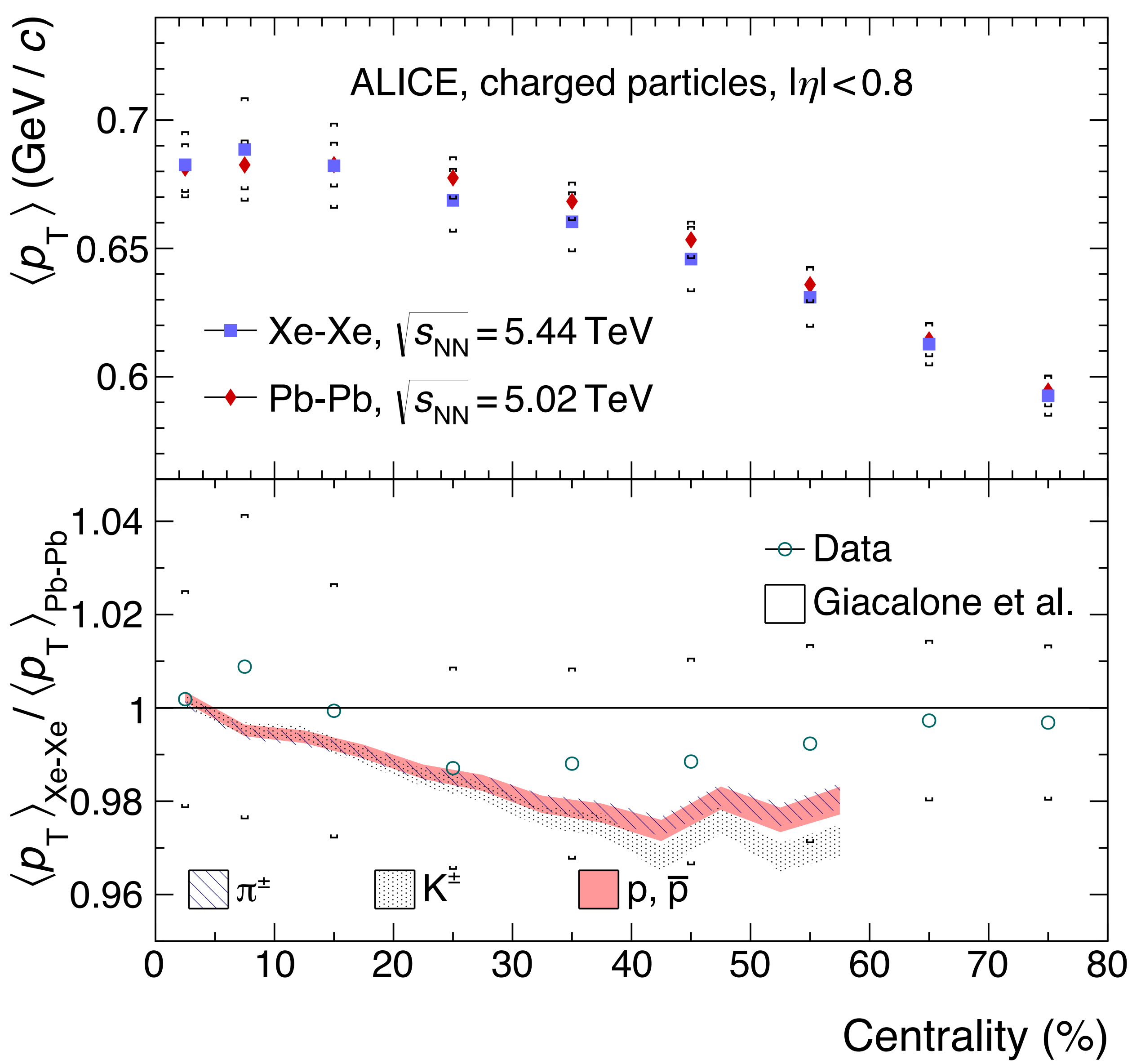
Scale invariance



Exact same hadron spectra, up to overall normalization λ^{-3}



Scale invariance seen in data



- Momentum spectra: Pb+Pb ≈ Xe+Xe collisions.
- ≈ independent of centrality, up to overall normalization.
- This in turn implies that a change in system size or centrality, at a given $\sqrt{s}$, amounts to a rescaling of space-time coordinates, at the same temperature.

Microscopic origin of scale invariance in heavy-ion collisions

- Nuclear density $\rho(r)$ is $\approx$ independent of mass A, implying nuclear volume proportional to A
- Hadron multiplicity: also proportional to A (Glauber N_{part} scaling)
- Implies hadron density roughly independent of A, where A=mass number of colliding nuclei (system size) or N_{part} (centrality).
- The independence of $\langle p_t \rangle$ on system size and centrality is a robust prediction of hydrodynamics.

Equation of state of QCD from heavy-ion data

- Since the EOS is the only non-trivial input of ideal hydrodynamic calculations, it is natural to expect that momentum spectra contain information about the EOS.
- For a thermal gas at rest, the picture is simple:
  $\langle p_t \rangle$ is proportional to the temperature T
  N_{ch} is proportional to the entropy S

Léon Van Hove, Phys.Lett.B 118 (1982) 138

Equation of state of QCD from heavy-ion data

- For a fluid in motion, correspondence between EOS and data is less trivial for two reasons:
- The hadron p_t gets a contribution from the transverse fluid velocity, in addition to the thermal contribution.
- The temperature spans the whole range from ∞ down to T_f as the system expands.

We have recently identified a simple and robust correspondence between observables and thermodynamic quantities in hydro calculations.

nature physics

LETTERS

https://doi.org/10.1038/s41567-020-0846-4

Check for updates

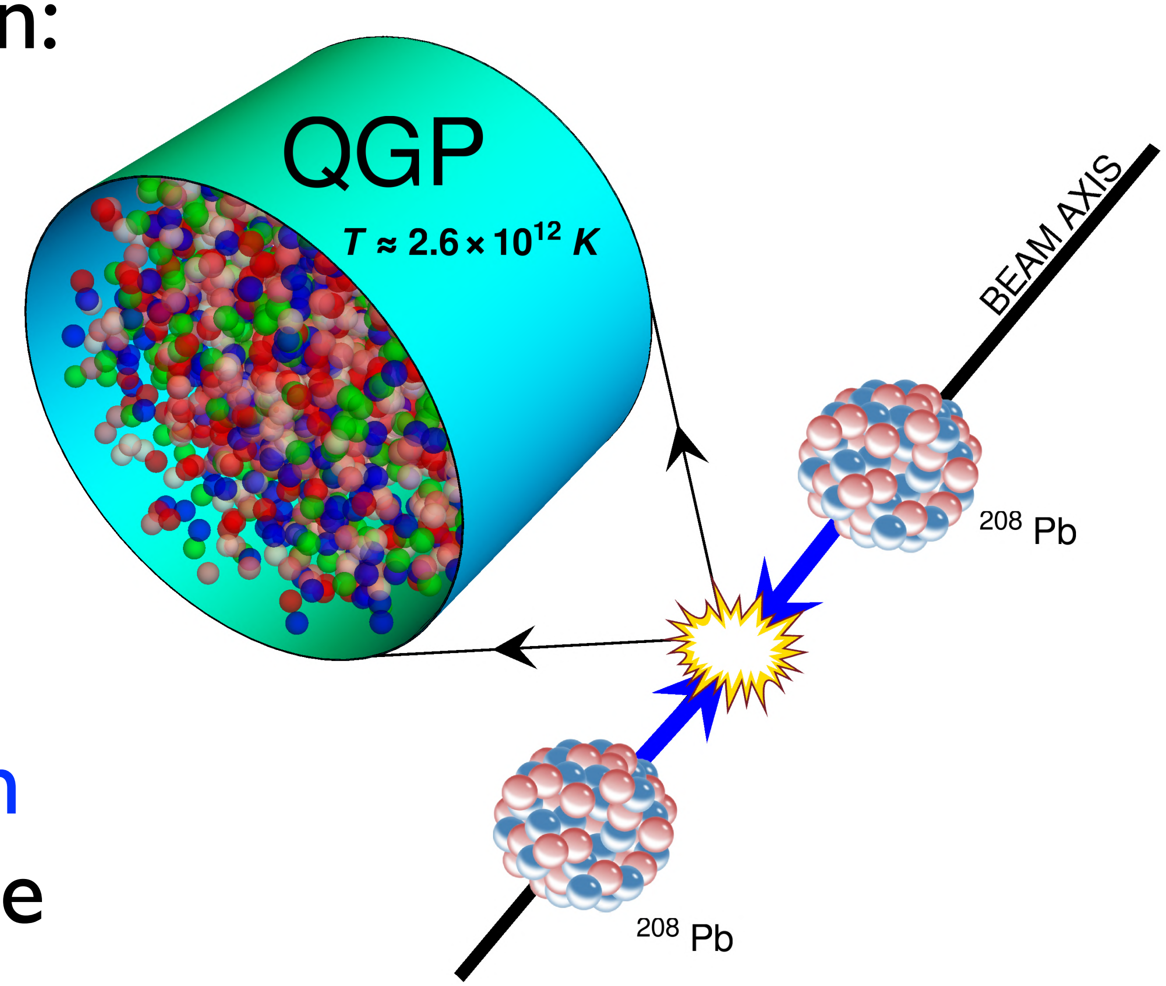
Thermodynamics of hot strong-interaction matter from ultrarelativistic nuclear collisions

Fernando G. Gardim[1,2], Giuliano Giacalone[2], Matthew Luzum[3] and Jean-Yves Ollitrault[2]

NATURE PHYSICS | VOL 16 | JUNE 2020 | 615–619 | www.nature.com/naturephysics

Idea: global analysis

- The output of the hydrodynamic calculation is a complex freeze-out hypersurface, where the fluid velocity depends on the space-time location.
- We only retain global information: the total energy E (thermal+collective) and total entropy S of the fluid (per unit rapidity) at freeze-out.
- We define the effective temperature, T_{eff} and effective volume, V_{eff}, as those of a uniform fluid at rest which would have the same energy E and entropy S.



Measuring T_{eff}

We calculate

- the effective temperature T_{eff}. Physically, it represents the average temperature in the fluid at at time $t \approx R$ (only relevant temperature for dimensional reasons)
- $\langle p_t \rangle$ of charged hadrons at freeze-out, after resonance decays

We find a simple and robust correspondence: $\langle p_t \rangle = 3.07\, T_{eff}$.
[cf. black-body thermodynamics: energy per particle=3T]

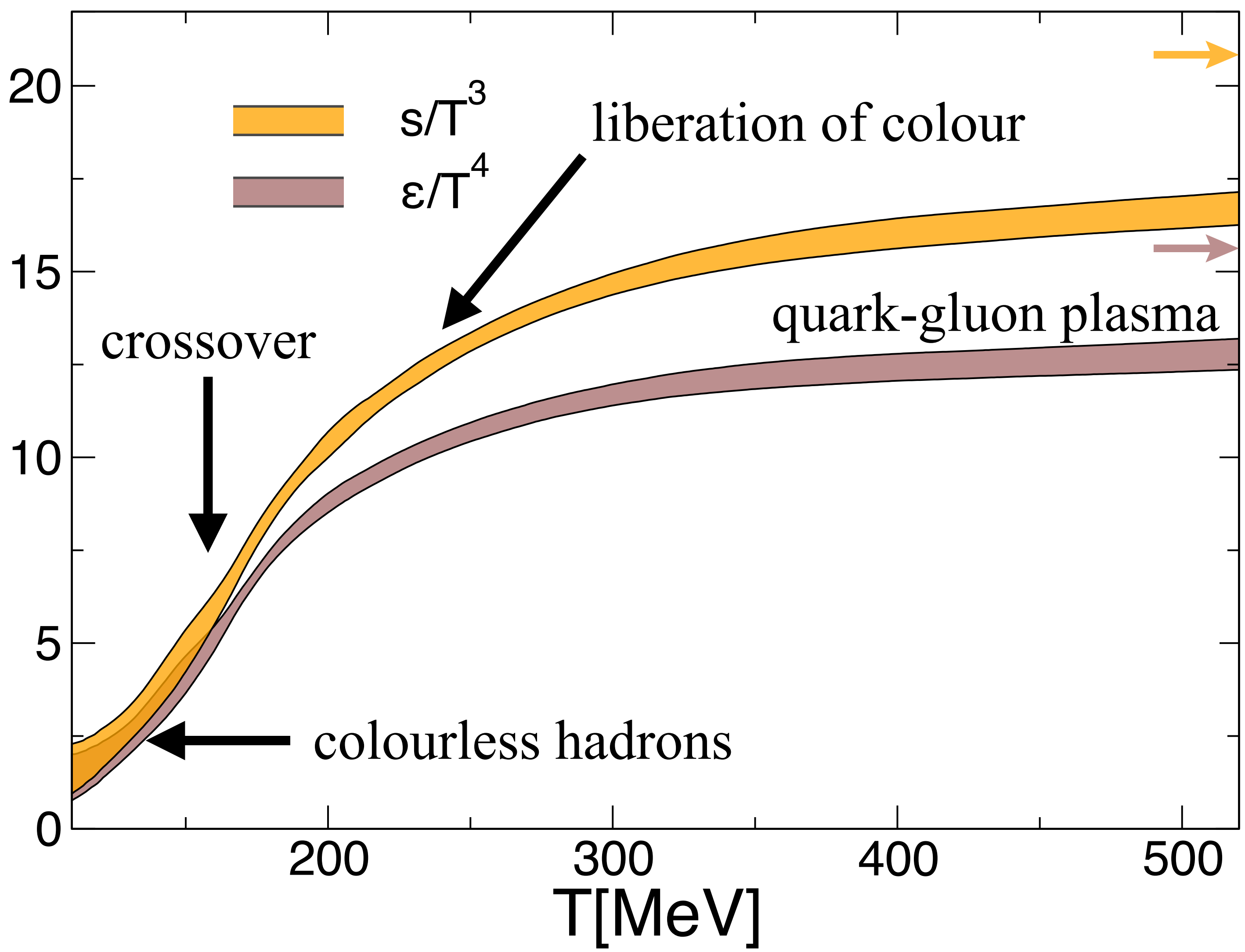
We have thus generalized the 1982 Van Hove argument to an expanding fluid

Measuring the entropy density at T_{eff}

The entropy S is related to the charged particle multiplicity N_{ch} using $S=6.7\ N_{ch}$. (Mazeliauskas et al. arXiv:1908.02792).

We take the volume V_{eff} from the hydrodynamic calculation (largest source of uncertainty) and we obtain the entropy density $s=S/V_{eff}$ at temperature T_{eff}

Equation of state from lattice QCD

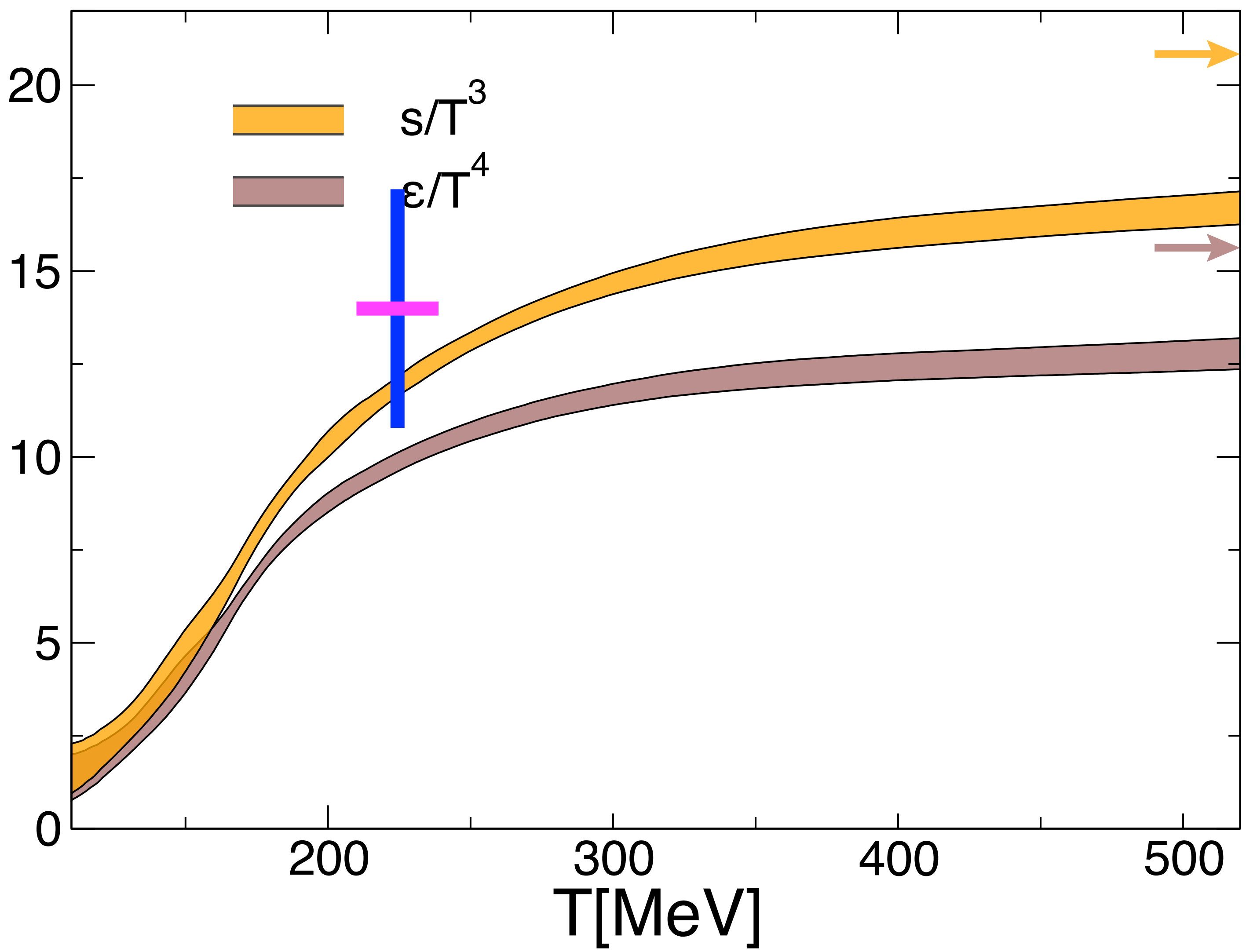


Borsanyi et al, 1309.5258

s/T^3 = dimensionless ratio, proportional to number of degrees of freedom:

- quark/antiquark/gluon
- spin
- flavor
- colour

Our result inferred from LHC data



T_{eff}=222 ± 9 MeV

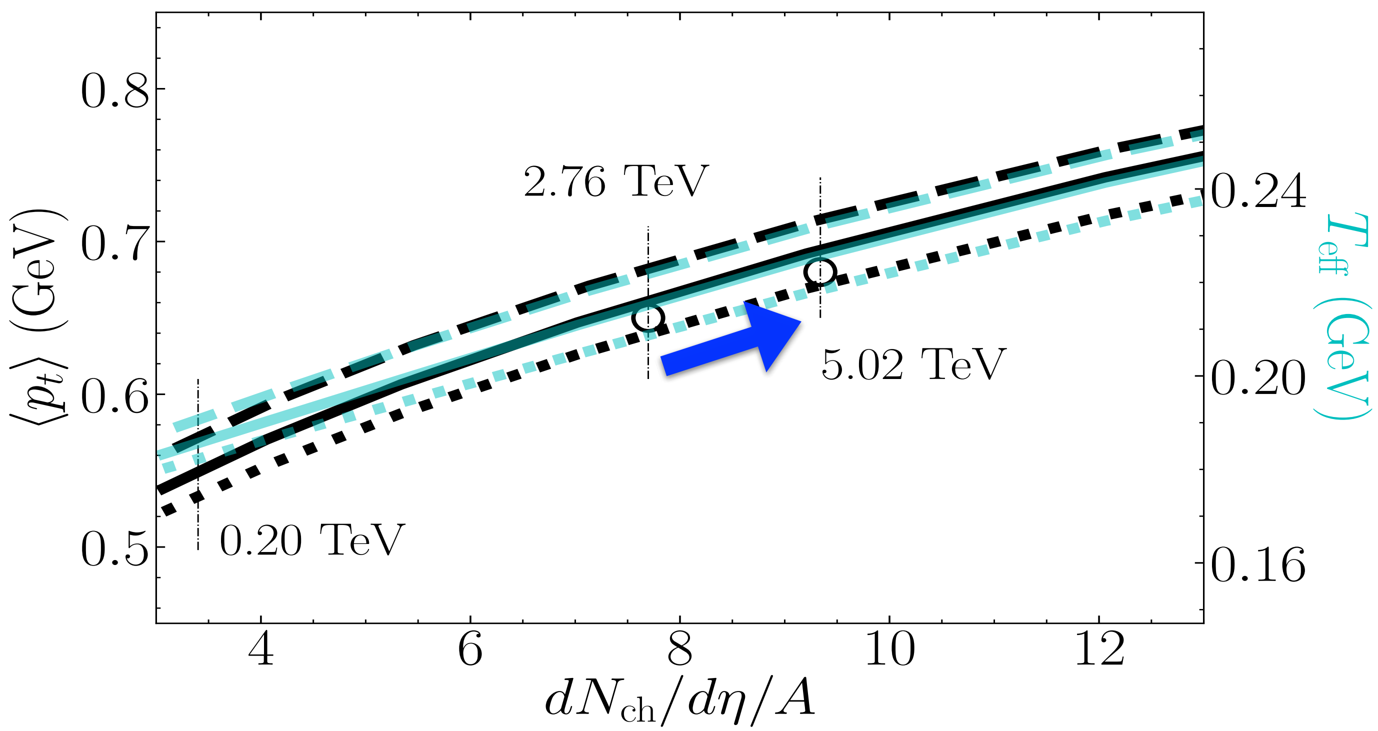
$s(T_{eff})/T_{eff}^3$ = 14 ± 3.5

compatible with lattice.

Confirms large number of degrees of freedom, implying that colour is liberated: deconfinement observed!

Speed of sound c_s in the QGP

We use results from two different collision energies at LHC.
Larger energy, same nuclei → more particles, fixed volume
→ compressibility = speed of sound.



$T_{\rm eff}$ proportional to $\langle p_t \rangle$

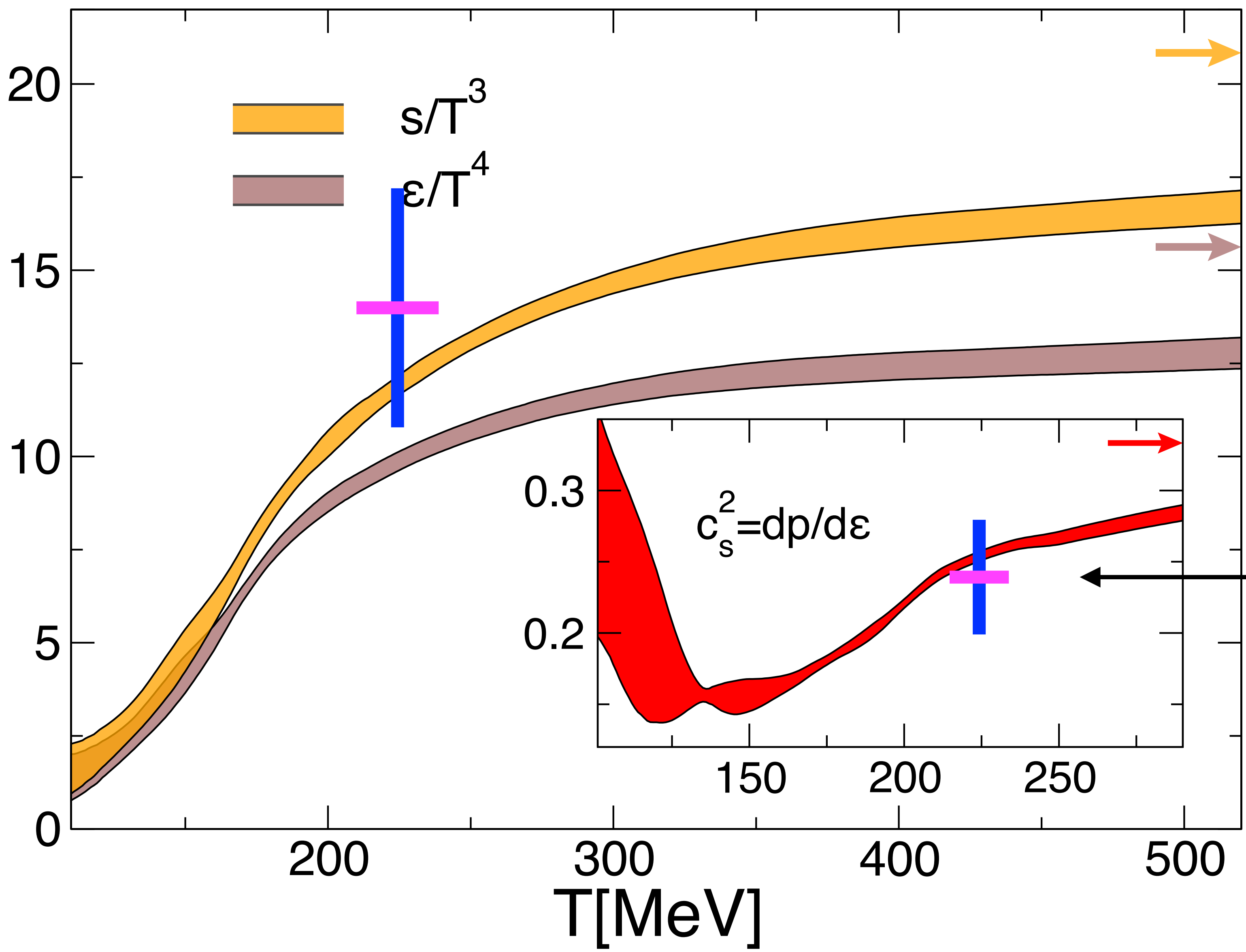
$s(T_{\rm eff})$ proportional to $dN_{ch}/d\eta$

$$c_s^2(T_{\rm eff}) \equiv \frac{dP}{d\varepsilon} = \left.\frac{s\,dT}{T\,ds}\right|_{T_{\rm eff}} = \frac{d\ln\langle p_t\rangle}{d\ln(dN_{\rm ch}/d\eta)}$$

Using LHC1 and LHC2 data: $c_s^2(T_{\rm eff}) = 0.24 \pm 0.04$

Sound velocity = 1/2 of light velocity.

Comparison with lattice QCD

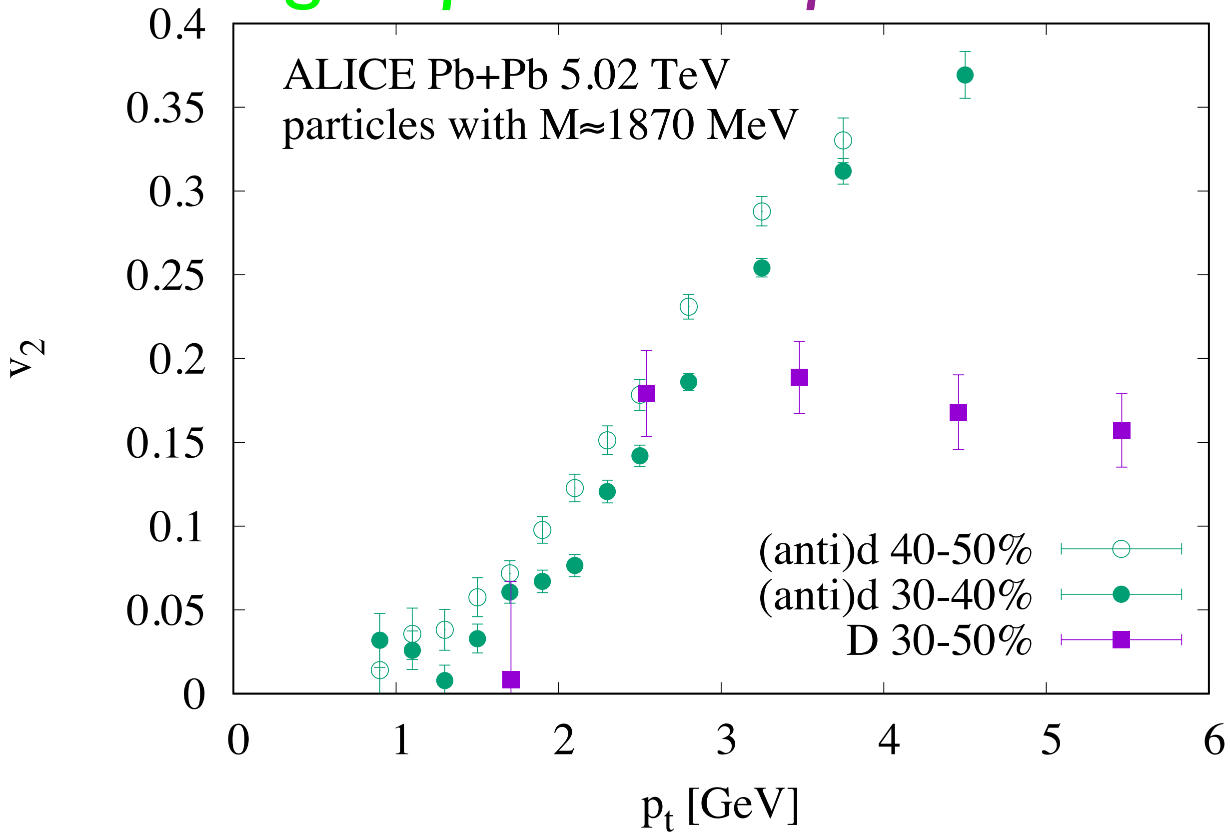


compatible with lattice

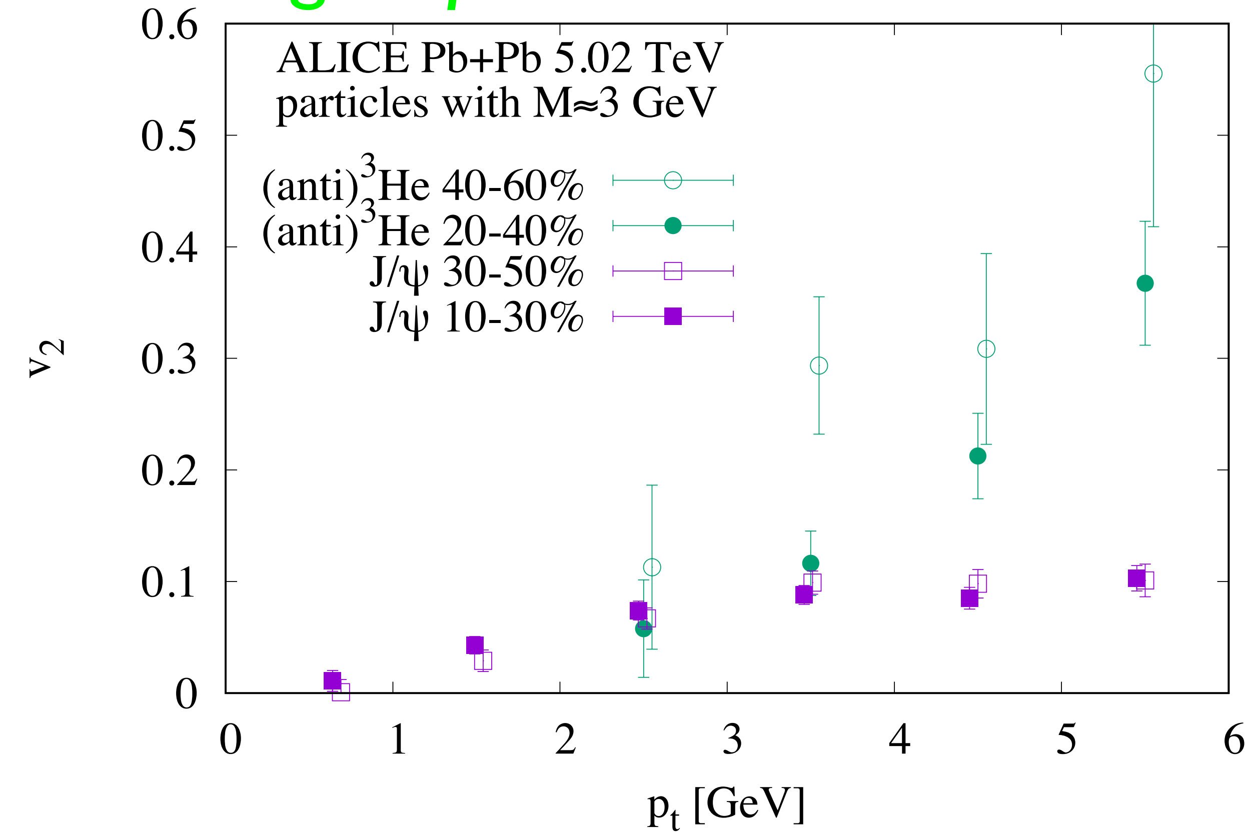
Do charm quarks flow with light quarks?

Robust prediction of ideal hydrodynamics:
same mass implies same momentum spectra and anisotropies.

light quarks vs *open charm*



light quarks vs *charmonium*



Difference expected in different initial densities
(≈participant nucleons for u,d,s, ≈binary collisions for c)
but way too small to explain the different v_2 observed at high p_t.

2. Viscous hydrodynamics
= finite-size corrections

Small departure from local thermodynamic equilibrium.

Enters in two places in our calculations:

1. hydro equations: Euler → Navier-Stokes

$$\rho\, dv/dt = -\nabla P + \eta\, \nabla^2 v$$

$$1/R \qquad 1/R^2$$

Note that viscous corrections break scale invariance:

relative correction proportional to $1/R$ (Reynolds number scaling)

In addition to EOS, viscous equations involve the temperature-dependent shear viscosity $\eta(T)$, and a similar bulk viscosity $\varsigma(T)$ term.

2. Viscous hydrodynamics
= finite-size corrections

2. Viscous corrections also enter the momentum distribution of hadrons at freeze-out. In the rest frame of the fluid,

$f(p)$ = Fermi-Dirac or Bose-Einstein + $\delta f(p)$

The transport coefficients η and ς do not constrain *at all* the p dependence of $\delta f(p)$, which depends on the details of hadronic interactions. All hydrodynamic calculations use arbitrary Ansätze!!

Unlike the equilibrium distribution, $\delta f(p)$ should have a non-trivial dependence on hadron species, but this has not yet been studied.

Dusling Moore Teaney 0909.0754

Light quarks versus heavy quarks

One typically models the evolution of charm quarks in the expanding quark-gluon plasma by simulating their Brownian motion in a background medium provided by an ideal hydrodynamic evolution.

Liu He Rapp 1806.05669

Compared with the arbitrary modeling of $\delta f(p)$ for light-quark hadrons, this description seems (to me) better motivated from a microscopic point of view.

Can data constrain $\eta(T)$ and $\zeta(T)$?

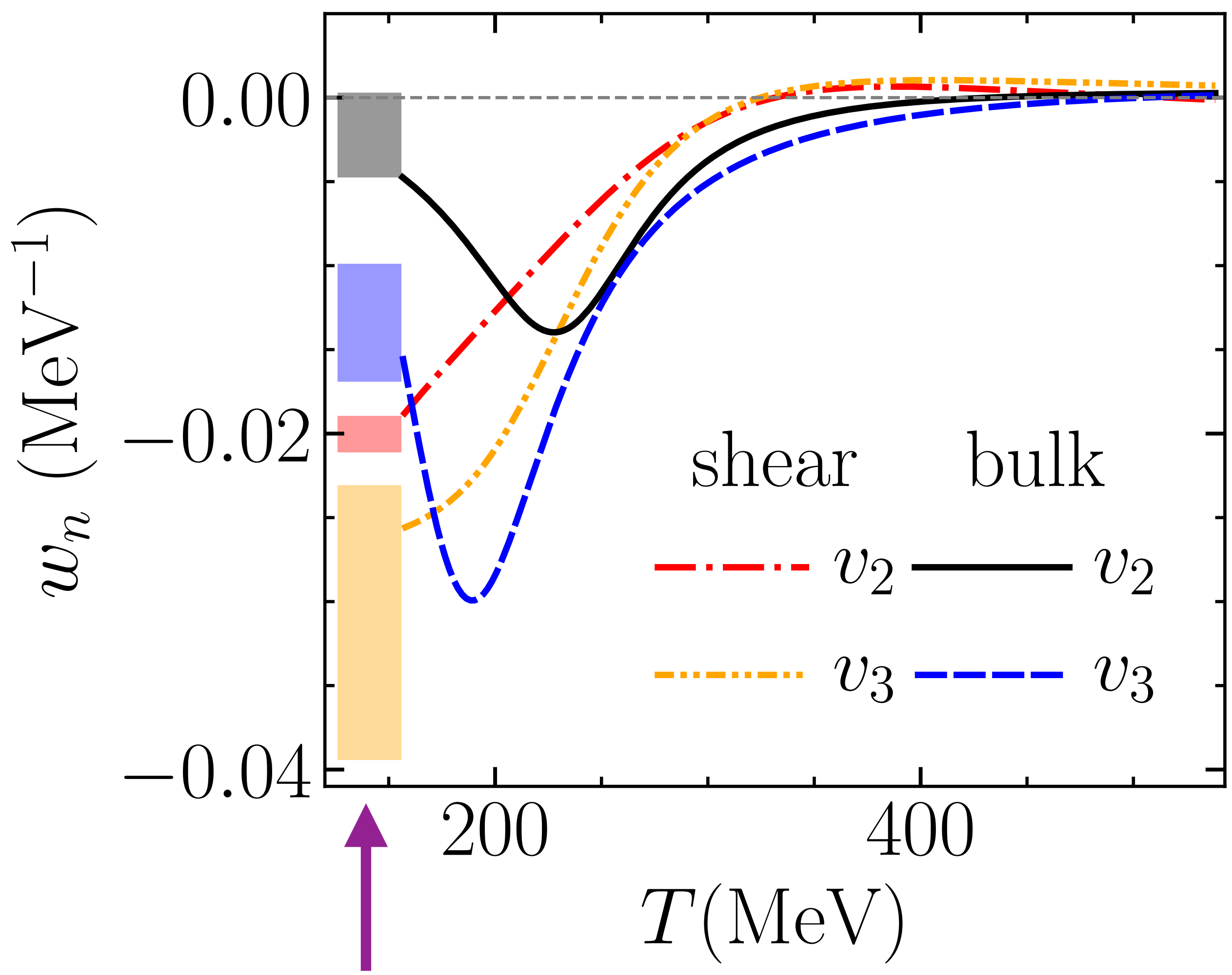
- First step: identify observables which depend weakly on the arbitrary $\delta f(p)$: Typically, p_t-integrated observables, v_2, v_3, $\langle p_t \rangle$.
- To leading order in viscosity, the relative change in an observable $\mathcal{O}$ due to viscosity must be linear:

$$\delta\mathcal{O}/\mathcal{O} = \int \left[w^{(\eta)}(T)(\eta/s)(T) + w^{(\zeta)}(T)(\zeta/s)(T) \right] dT,$$

where $w^{(\eta)}(T)$, $w^{(\zeta)}(T)$ are weight functions.

- scale invariance → T independent of system size and centrality
- → dependence of $\delta\mathcal{O}/\mathcal{O}$ on system size and centrality is just a global factor $1/R$ dictated by Reynolds number scaling.
- At fixed $\sqrt{s}$, we can at best access the integral, not the T dependence: an *effective viscosity* = weighted average over T.

Weights for $\mathcal{O}$ = v_2 or v_3

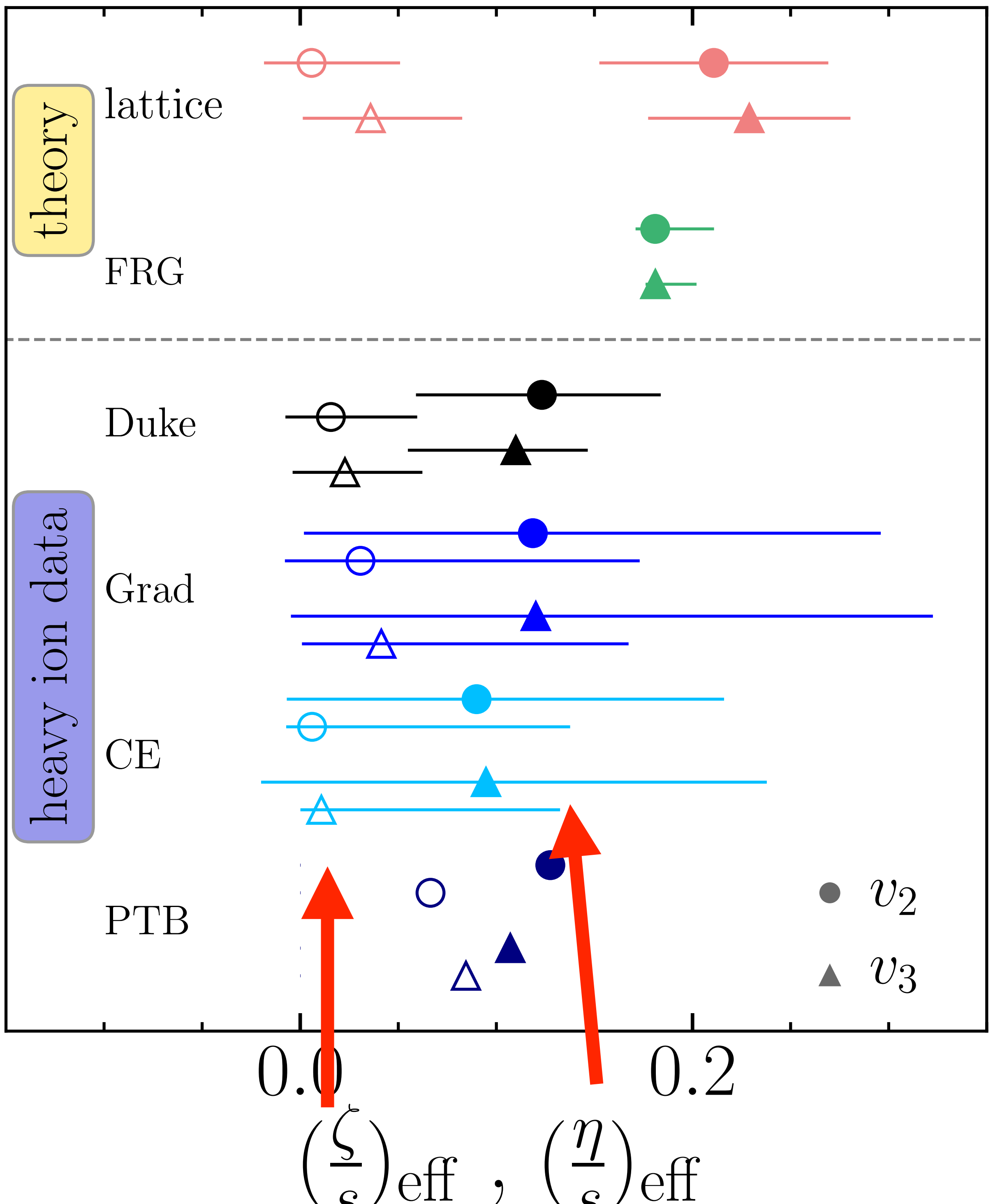


contribution of δf(p)

- We have computed the weight functions for v_2 and v_3 at LHC energy using viscous hydro simulations.
- Weights are largest arond 200 MeV, which means that the viscosity that one sees is the viscosity at this temperature.
- δf(p): contribution less than 20%: Good news since this part is not robust.

Gardim JYO 2010.1191

Effective viscosities: QCD versus LHC data



- Effective viscosities are very similar for v_2 and v_3
- Bulk viscosity less important than shear viscosity
- Functional renormalization group calculations imply effective QCD shear viscosity around 0.16-0.2, compatible with value extracted from Bayesian analyses of LHC data

Gardim JYO 2207.08692

Thank you

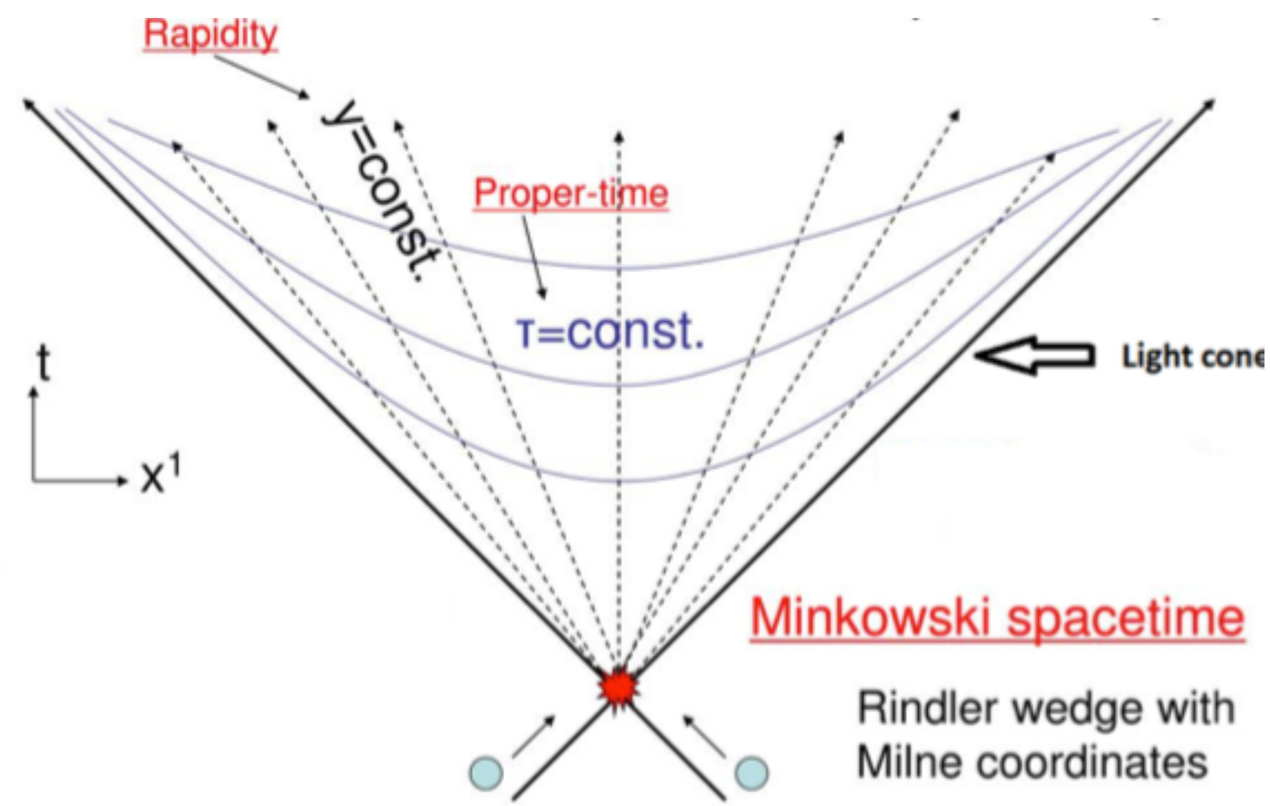
Supplemental material

Initial conditions I

Longitudinal symmetry

in appropriate coordinate system

Fluid initially *at rest:* v=0



(illustration: Marc Borrell, Bielefeld U.)

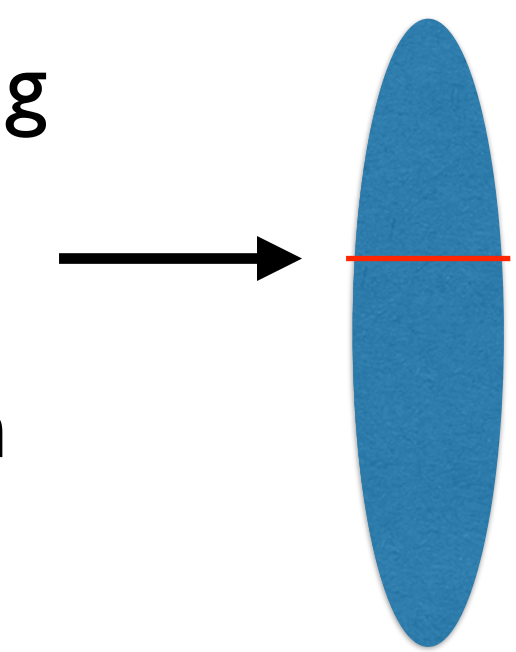
Initial conditions 2

Significant progress has been made in understanding the early stages of the collision from first principles.

We choose instead an *empirical* prescription for the initial entropy density: One *observes* that the number of particles produced in the collision is proportional to the mass of colliding nuclei.

We make this prescription local by assuming

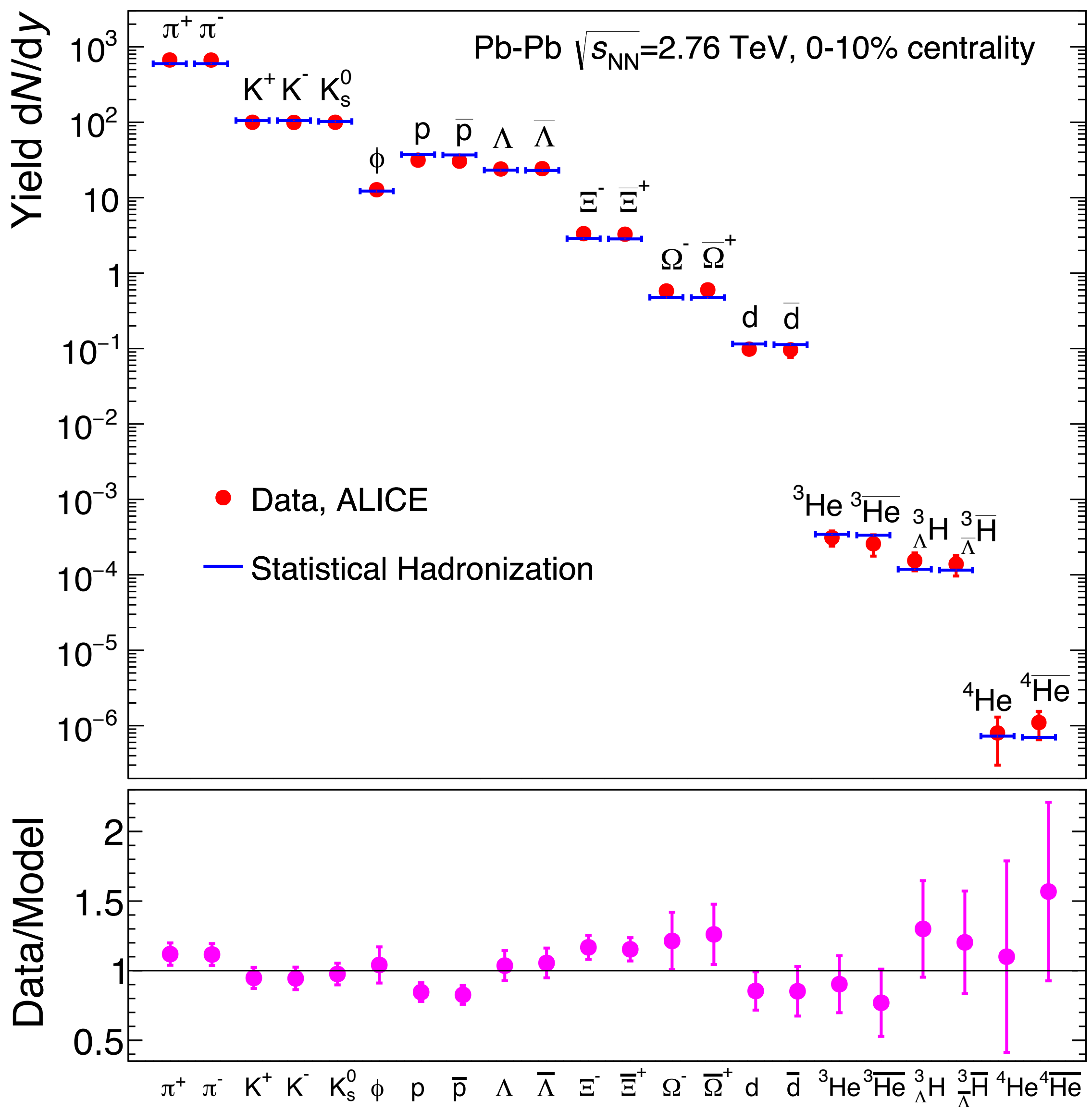
$s(x,y) \propto$ integral of nuclear density along z.



Proportionality constant: adjusted to match final particle multiplicity

Freeze-out temperature

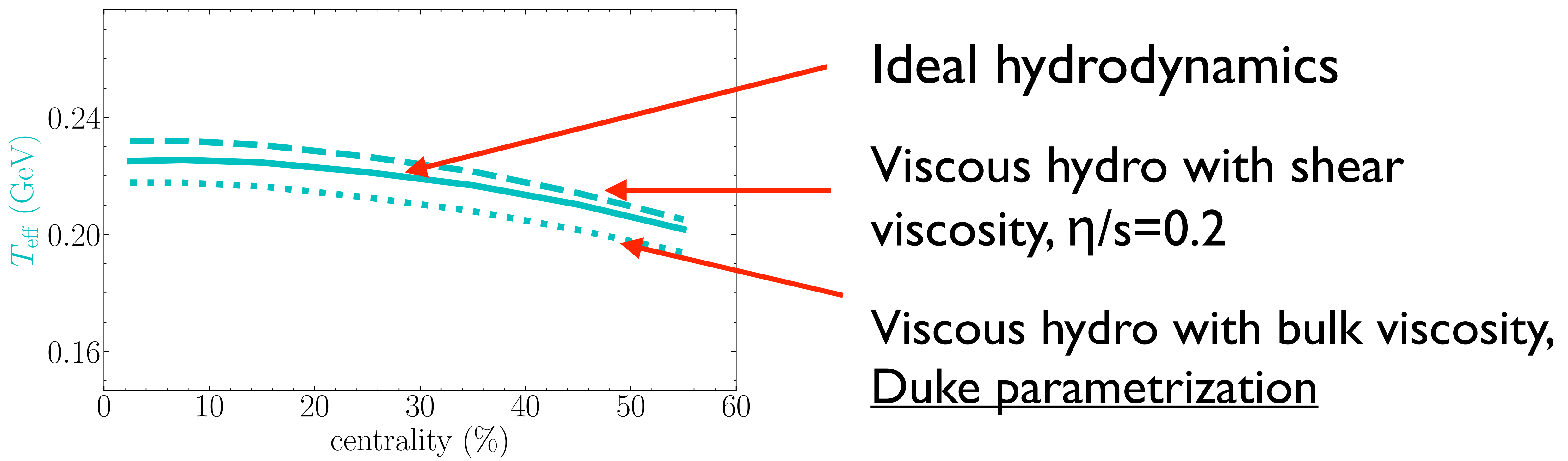
The value of T_f is adjusted empirically so as to match the observed relative abundances of hadrons: $T_f = 156 \pm 1.5$ MeV



Andronic et al, Nature 561 (2018) 7723, 321-330

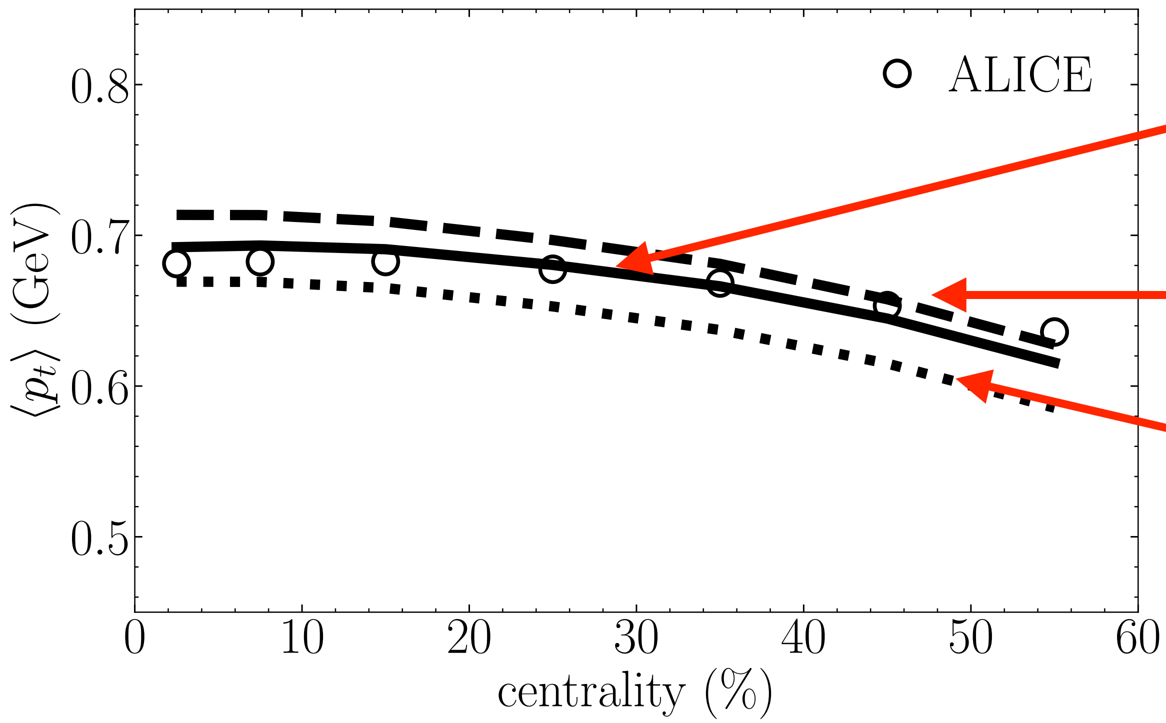
Value of T_{eff} in hydro simulations of Pb+Pb @ 5.02 TeV

We use the hydrodynamic code MUSIC, where the initial temperature is tuned to reproduce the charged multiplicity measured by ALICE for each centrality.



Value of <p_t> in hydro simulations of Pb+Pb @ 5.02 TeV

We compute the average transverse momentum of particles at the end of the fluid expansion (and after resonance decays)



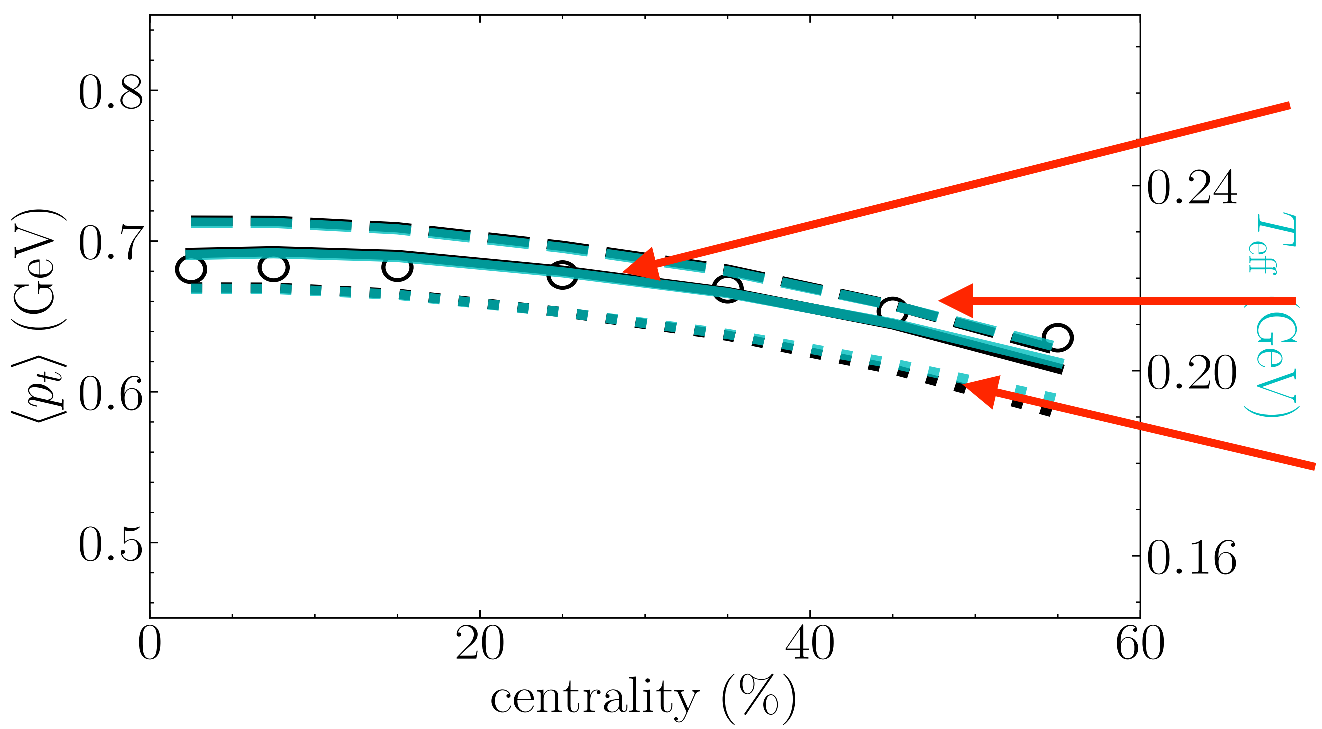
Ideal hydrodynamics

Viscous hydro with shear viscosity, $\eta/s=0.2$

Viscous hydro with bulk viscosity, Duke parametrization

Reviving Van Hove's 1982 idea

$\langle p_t \rangle = 3.07\, T_{eff}$ for all centralities, irrespective of bulk and shear viscosity!



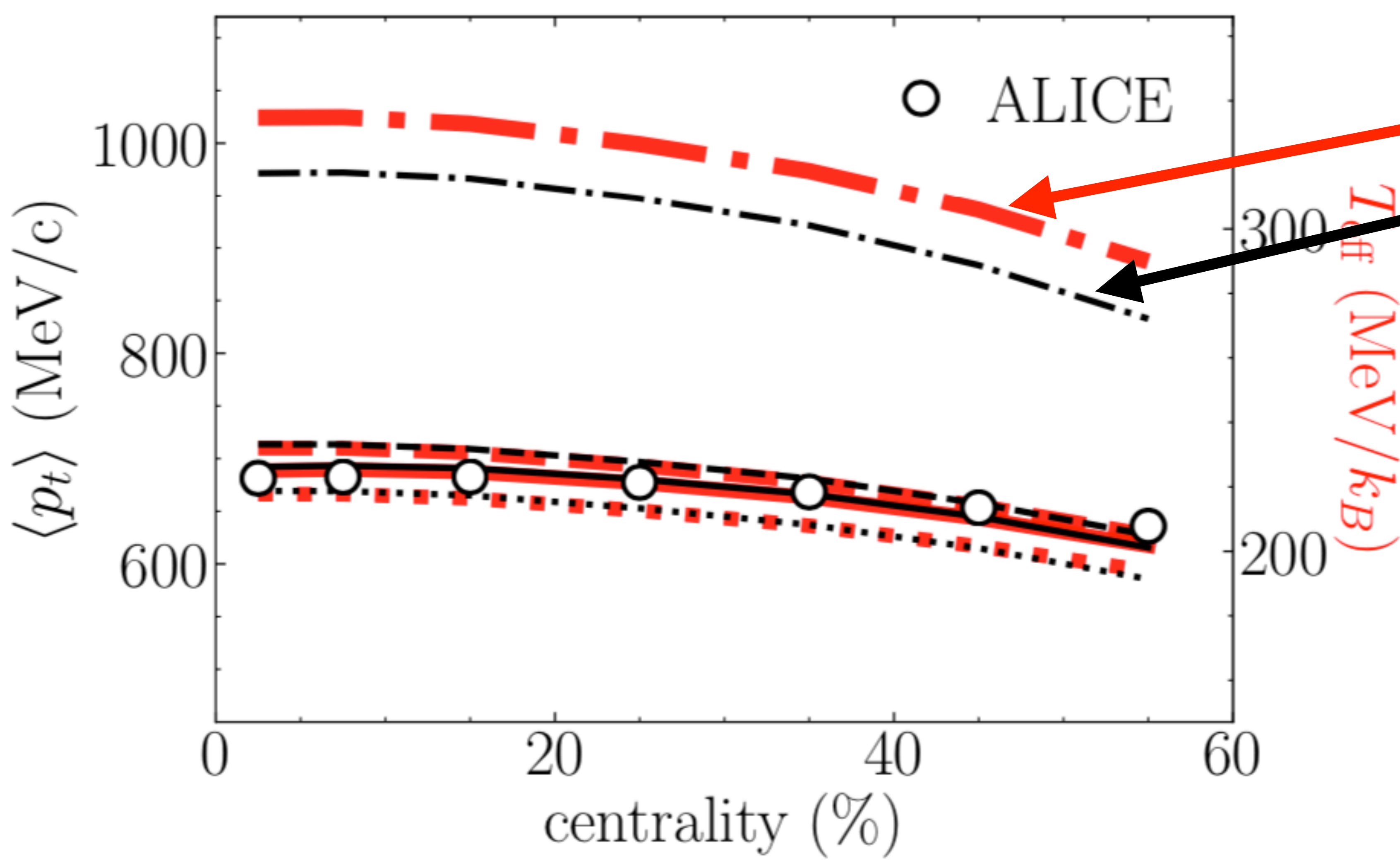
Ideal hydrodynamics

Viscous hydro with shear viscosity, η/s=0.2

Viscous hydro with bulk viscosity, <u>Duke parametrization</u>

Changing the equation of state

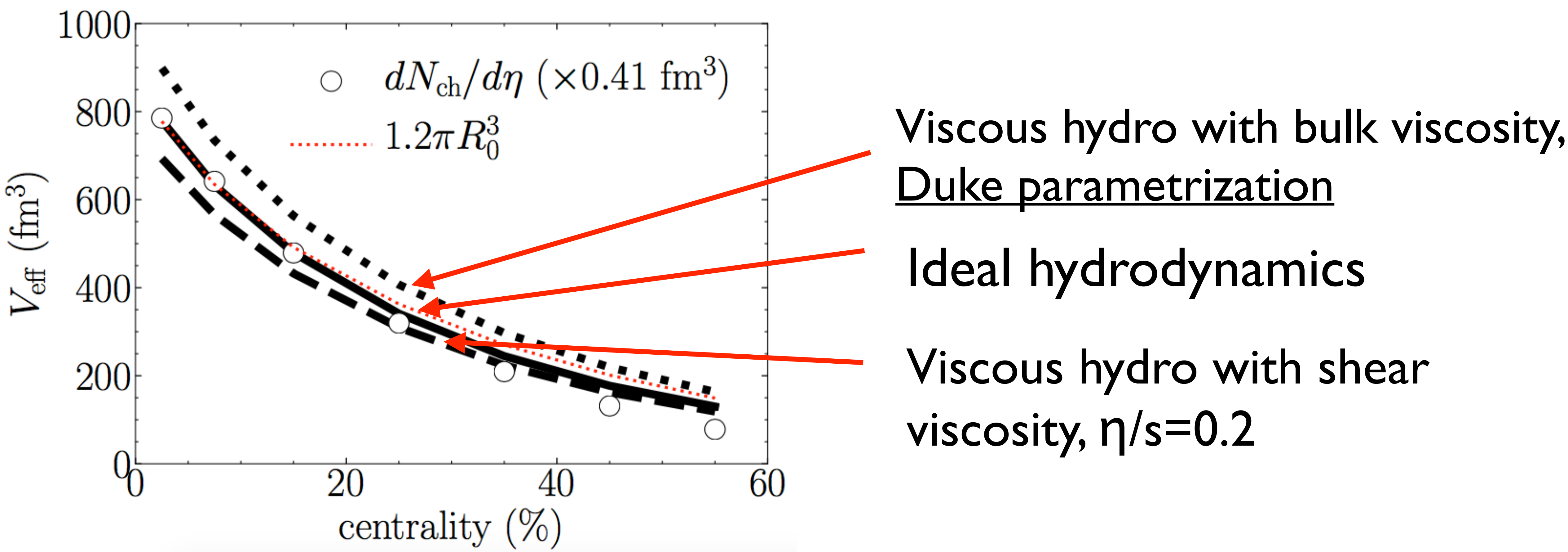
We test the robustness of the correspondence between <p_t> and T_{eff} by running ideal hydro with a stiff equation of state ε=3P+const.



Both T_{eff} and <p_t> increase.

The ratio is not strictly the same but almost: 2.90 instead of 3.07

Estimating the effective volume



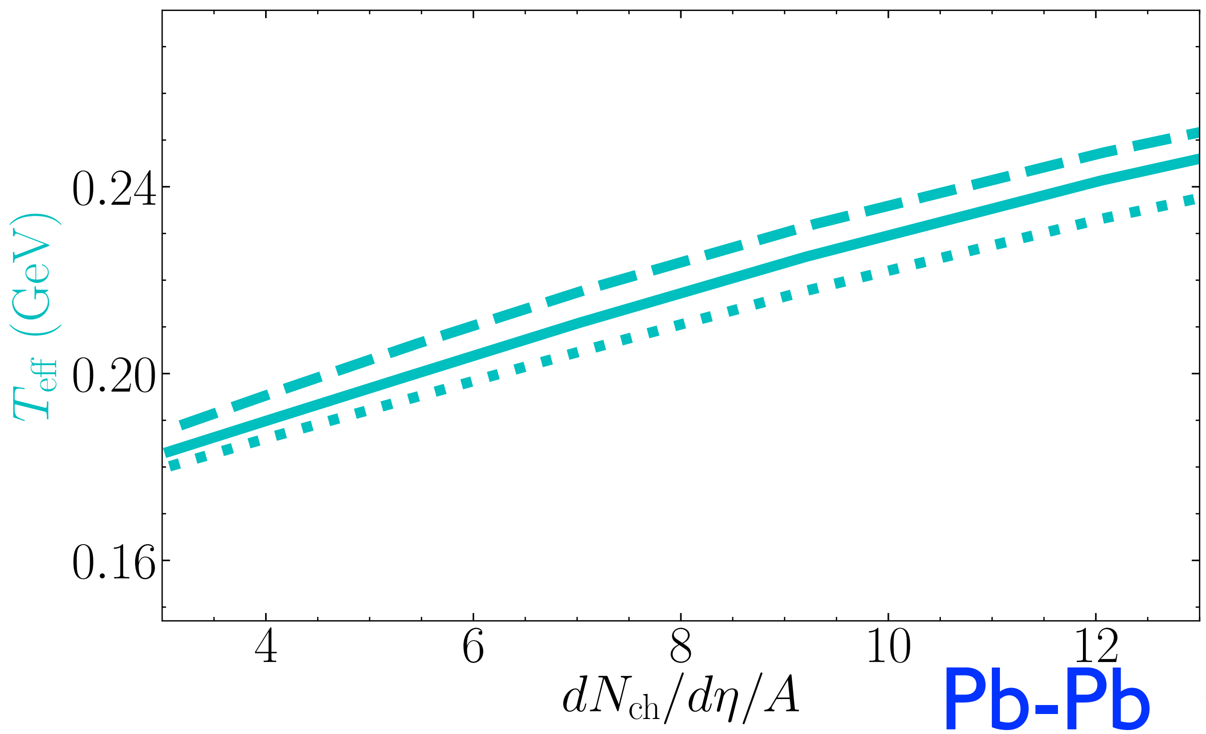
V_{eff} is proportional to R_0^3, where

R_0 = initial transverse size

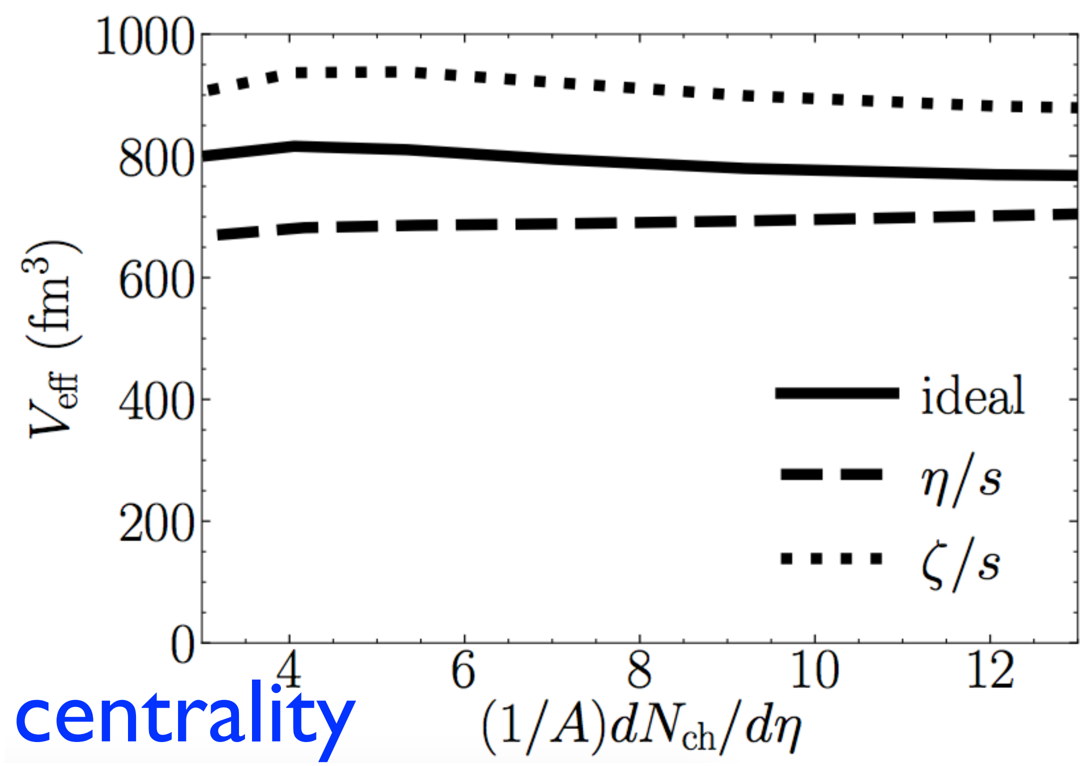
= nuclear size for central collisions

Varying the collision energy

(results are plotted as a function of number of produced particles, which itself depends on collision energy)



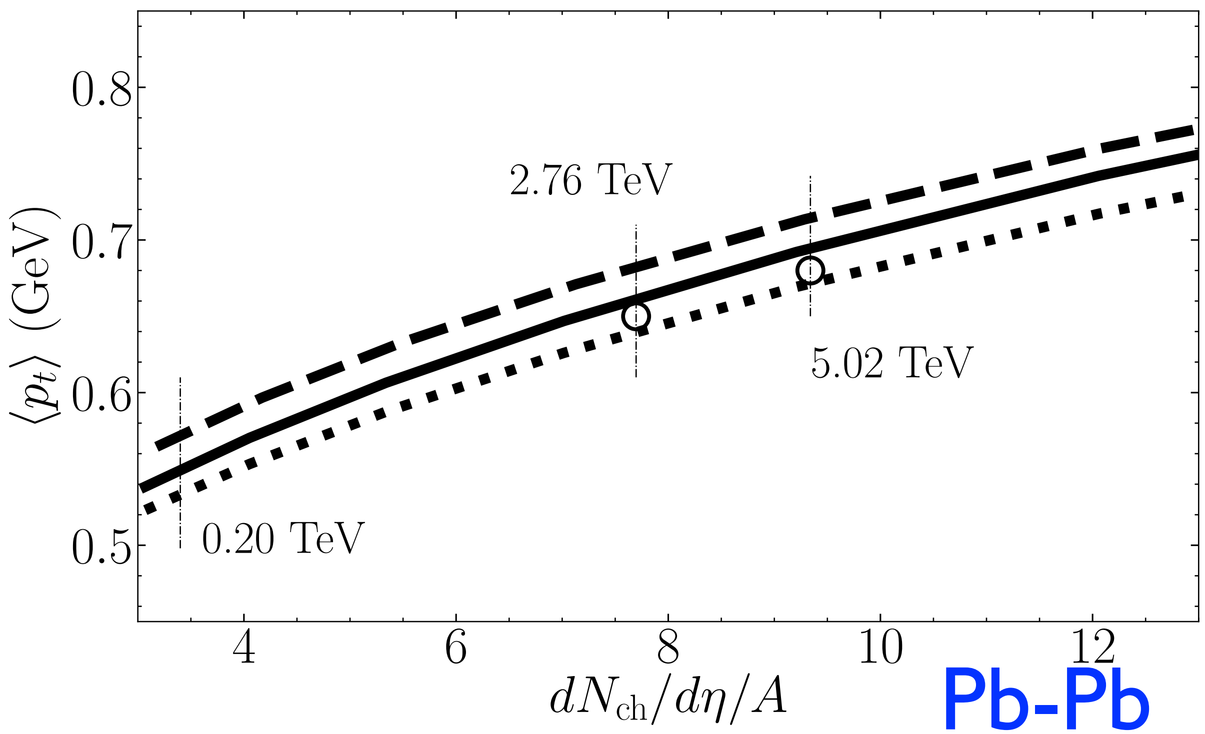
Pb-Pb 0-5% centrality

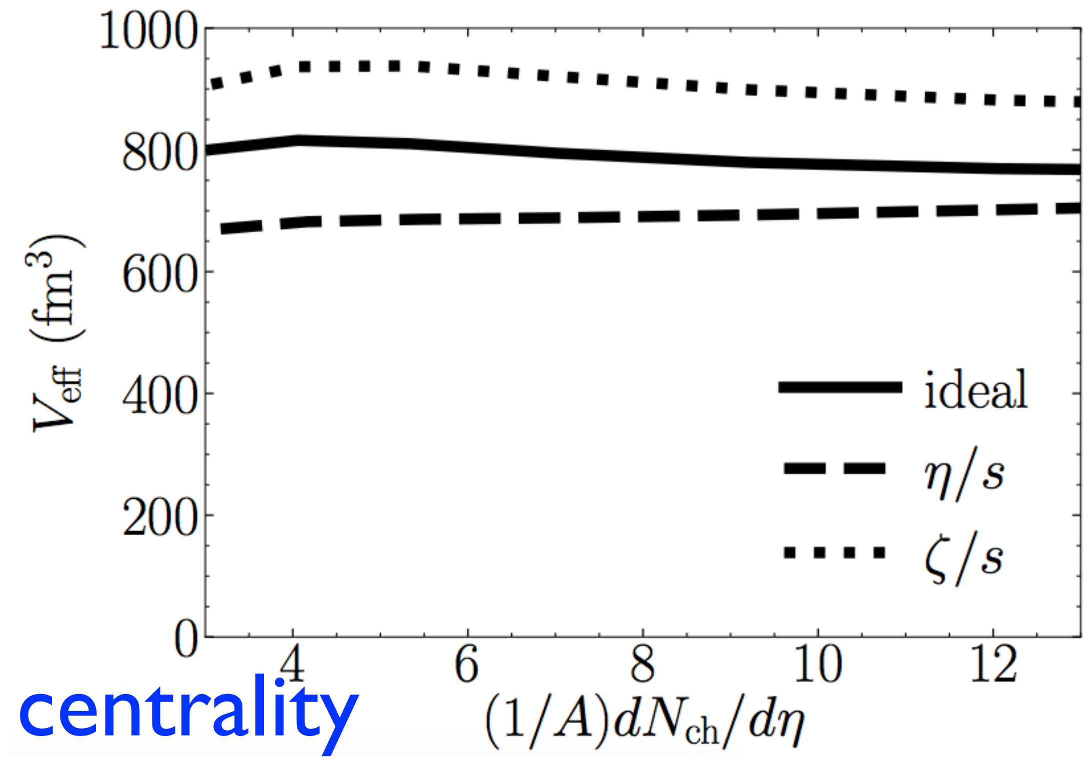


As energy increases, T_{eff} increases, V_{eff} remains constant. Increasing the collision energy amounts to heating the system at constant volume.

Varying the collision energy

(results are plotted as a function of number of produced particles, which itself depends on collision energy)



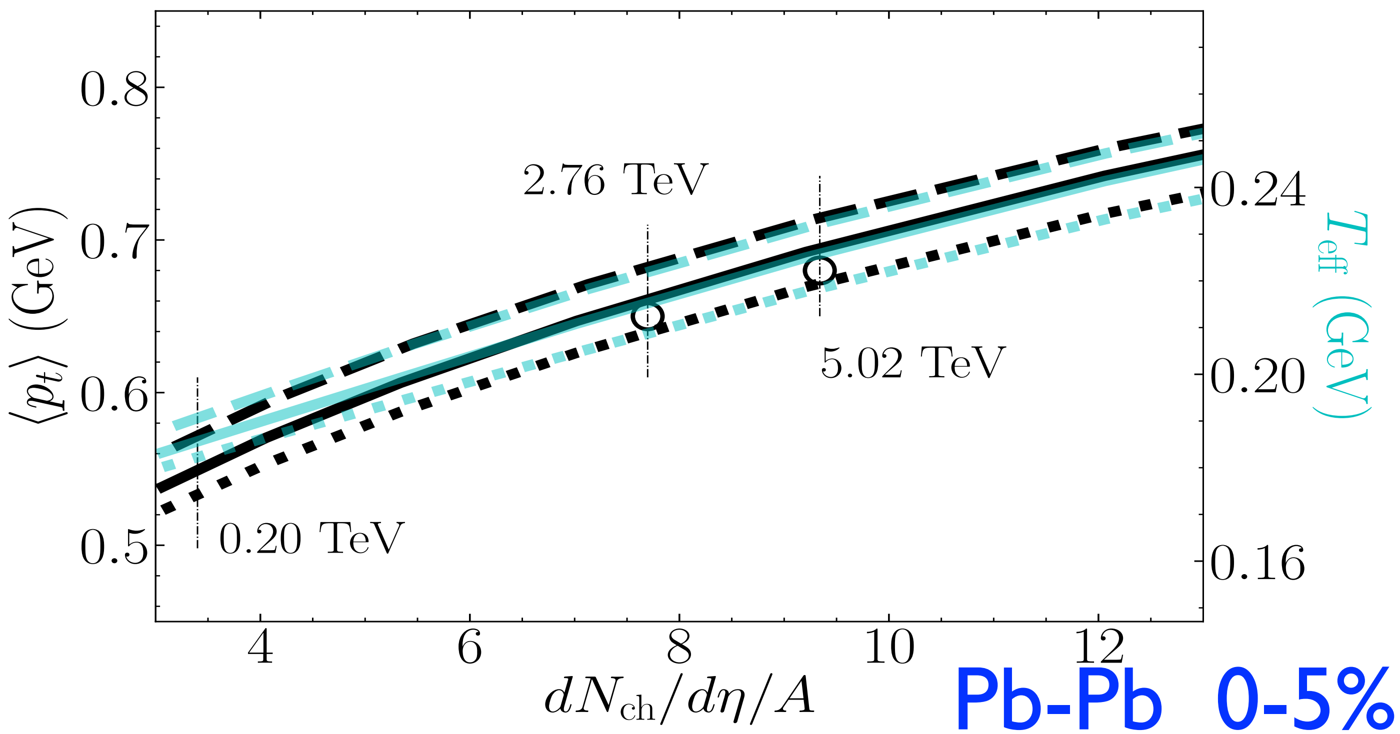


Pb-Pb 0-5% centrality

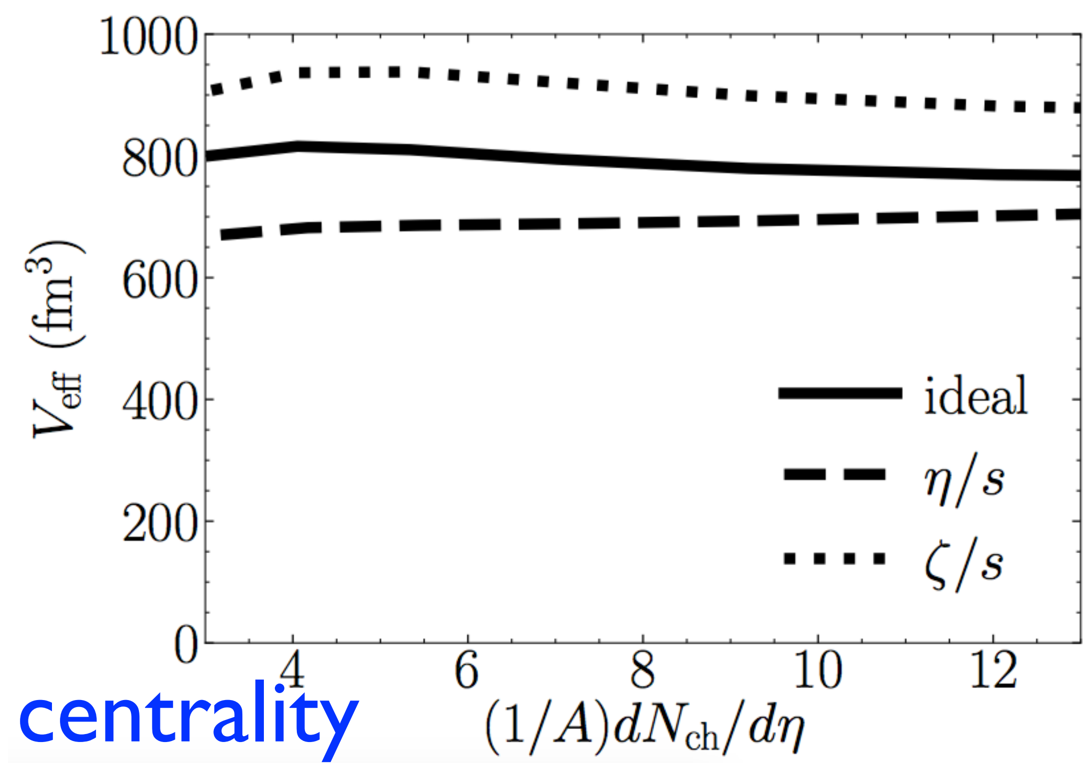
The variation of <p_t> still closely follows that of T_{eff}

Varying the collision energy

(results are plotted as a function of number of produced particles, which itself depends on collision energy)



Pb-Pb 0-5% centrality



Deviations from $<p_t>=3.07\ T_{eff}$ are negligible at LHC energy and beyond

Temperature dependence of transport coefficients