Modelling heavy-ion collisions with relativistic fluid dynamics

a very partial overview

Iurii KARPENKO

Faculty of Nuclear Sciences and Physical Engineering,
Czech Tecnhincal University in Prague (FJFI CVUT)

Supported by the Ministry of Education, Youth and Sports of the Czech Republic under grant „International Mobility of Researchers MSCA IF III at CTU in Prague“ CZ.02.2.69/0.0/0.0/16 027/0008465

Hydrodynamic equations

1) hydrodynamics of ideal fluid

- equations of motion come from local energy-momentum conservation:

$$\boxed{\partial_\mu T^{\mu\nu} = 0}$$

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p \cdot g^{\mu\nu}$$

2) viscous fluid dynamics:

- there are corrections to the energy-momentum tensor and new equations of motion:

$$T^{\mu\nu} = \underbrace{(\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi) \cdot g^{\mu\nu}}_{\text{ideal part}} + \underbrace{W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}}_{\text{viscous corrections}}$$

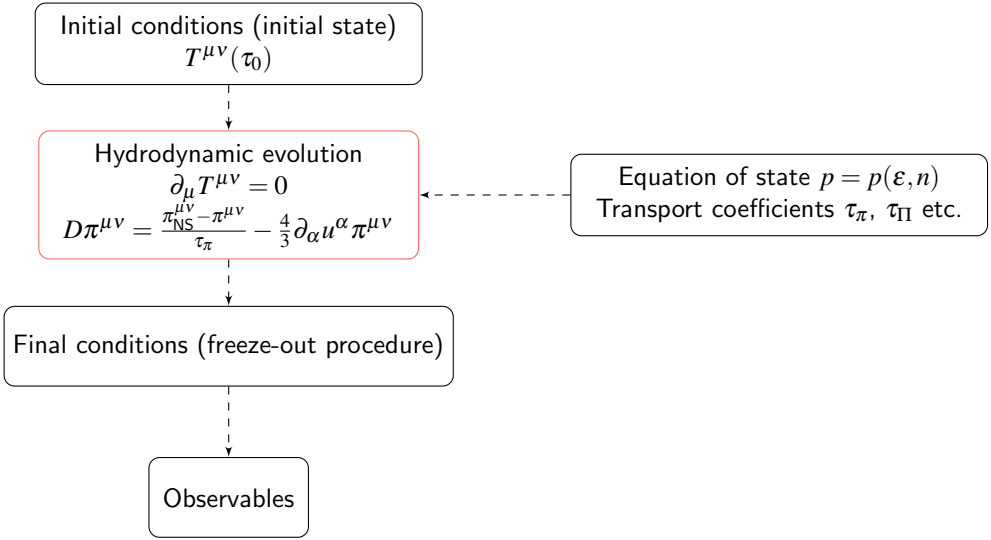
A way to model heavy-ion collisions with hydrodynamics

Hydrodynamic evolution

$$\partial_\mu T^{\mu\nu} = 0$$

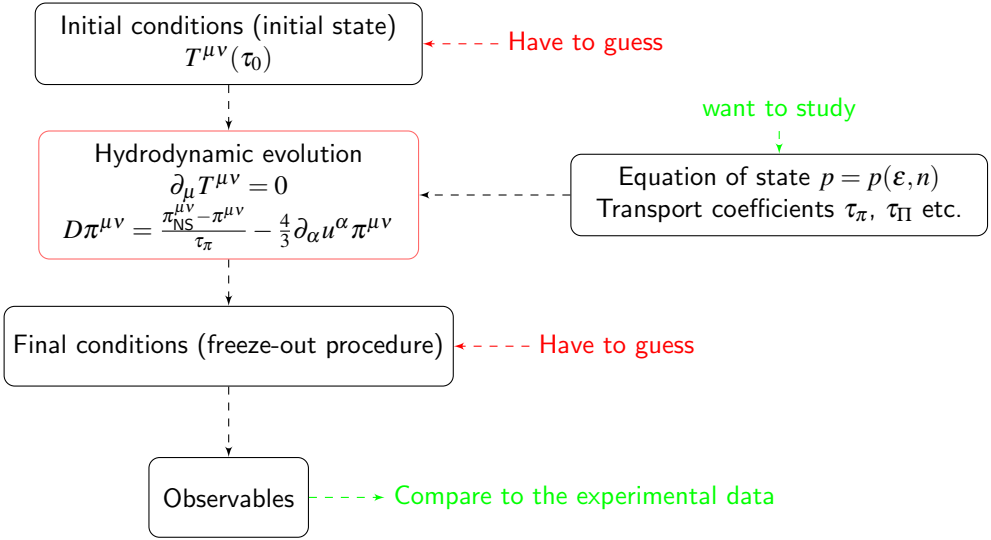
$$D\pi^{\mu\nu} = \frac{\pi_{\mathrm{NS}}^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3}\partial_\alpha u^\alpha \pi^{\mu\nu}$$

A way to model heavy-ion collisions with hydrodynamics



Boxes framed in black are all inputs to the hydrodynamic phase!

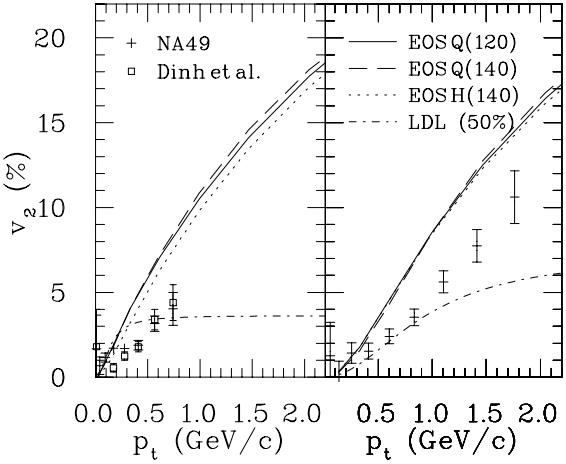
A way to model heavy-ion collisions with hydrodynamics

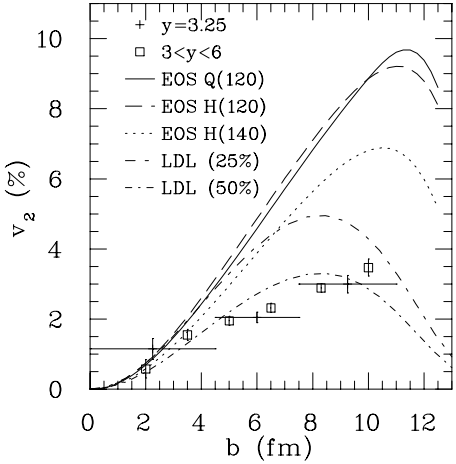


Boxes framed in black are all inputs to the hydrodynamic phase!

Hydrodynamics circa 2001 vs SPS data

Elliptic flow v_2 in $\sqrt{s_{NN}} = 17.3$ GeV Pb-Pb vs hydro calculation by Kolb, Huovinen, Heinz, Heiselberg, Phys.Lett.B500:232-240,2001



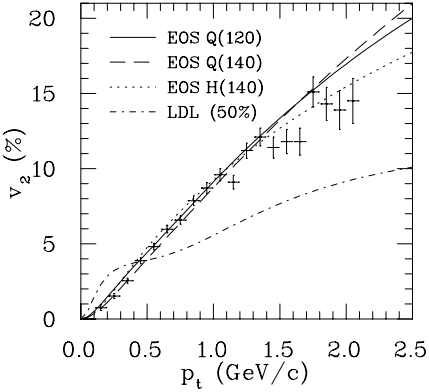


→ hydrodynamic model overshoots the data, LDL (low density limit) works much better.
(this all changed in 2015, see one of the next slides)

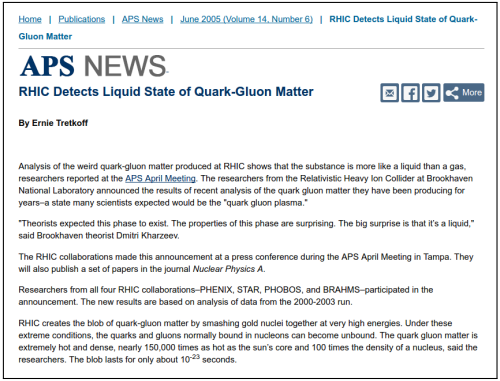
First $\sqrt{s_{\rm NN}} = 130$ GeV RHIC results

Before RHIC start-up: two scenarios possible:

- asymptotic freedom $\Rightarrow$ weakly interacting system
- stongly interacting system



Elliptic flow v_2 in $\sqrt{s_{\rm NN}} = 130$ GeV Au-Au minimum-bias vs hydro calculation by Kolb,Huovinen,Heinz,Heiselberg, Phys.Lett.B500:232-240,2001



Home | Publications | APS News | June 2005 (Volume 14, Number 6) | **RHIC Detects Liquid State of Quark-Gluon Matter**

APS NEWS

RHIC Detects Liquid State of Quark-Gluon Matter

More

By Ernie Tretkoff

Analysis of the weird quark-gluon matter produced at RHIC shows that the substance is more like a liquid than a gas, researchers reported at the APS April Meeting. The researchers from the Relativistic Heavy Ion Collider at Brookhaven National Laboratory announced the results of recent analysis of the quark gluon matter they have been producing for years--a state many scientists expected would be the "quark gluon plasma."

"Theorists expected this phase to exist. The properties of this phase are surprising. The big surprise is that it's a liquid," said Brookhaven theorist Dmitri Kharzeev.

The RHIC collaborations made this announcement at a press conference during the APS April Meeting in Tampa. They will also publish a set of papers in the journal *Nuclear Physics A*.

Researchers from all four RHIC collaborations--PHENIX, STAR, PHOBOS, and BRAHMS--participated in the announcement. The new results are based on analysis of data from the 2000-2003 run.

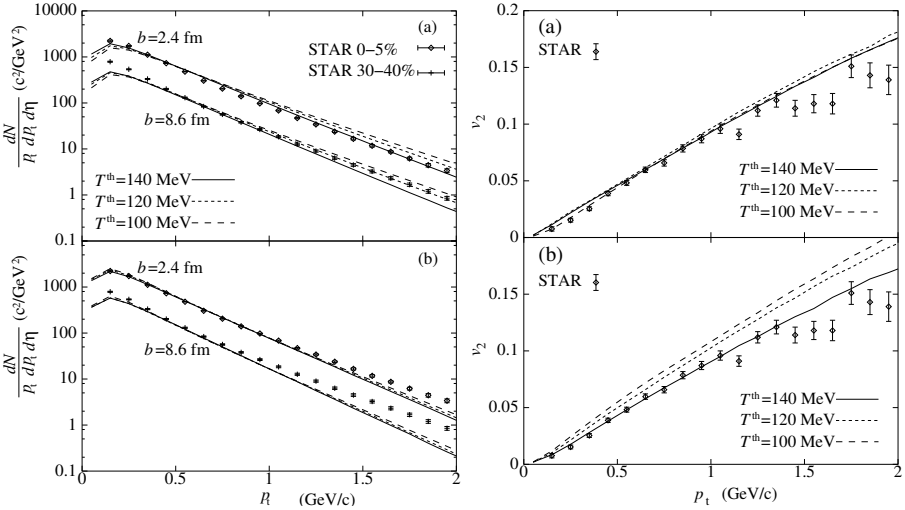
RHIC creates the blob of quark-gluon matter by smashing gold nuclei together at very high energies. Under these extreme conditions, the quarks and gluons normally bound in nucleons can become unbound. The quark gluon matter is extremely hot and dense, nearly 150,000 times as hot as the sun's core and 100 times the density of a nucleus, said the researchers. The blob lasts for only about 10^{-23} seconds.

Ingredients for the hydrodynamic calculation:

- initial ε profile from Glauber formula
- ideal 2+1D hydrodynamic expansion
- EoS Q: HG+QGP phases, 1st order PT
- freeze-out at $T = 120, 140$ MeV

Hydrodynamics circa 2002 vs early RHIC data ["Gen 1" calculations]

T. Hirano, K. Tsuda, Phys.Rev.C 66, 054905 (2002), nucl-th/0205043

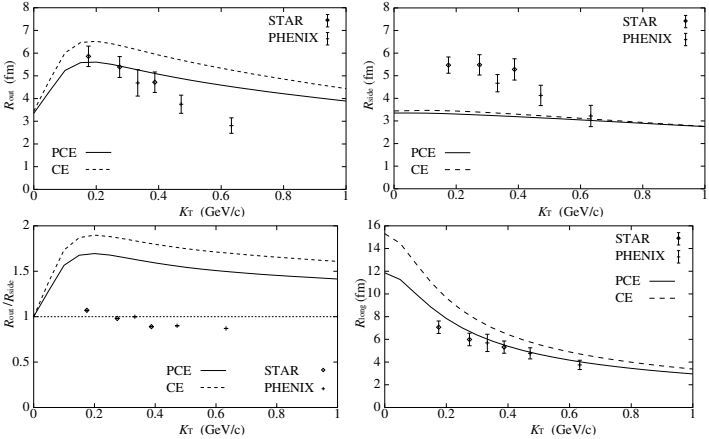


(a) CE EoS

(b) PCE EoS

Note: $\eta/s = 0$ (relativistic viscous hydro were non-existent in the field at the time).
Ideal hydrodynamics works!

However, HBT observables didn't work:

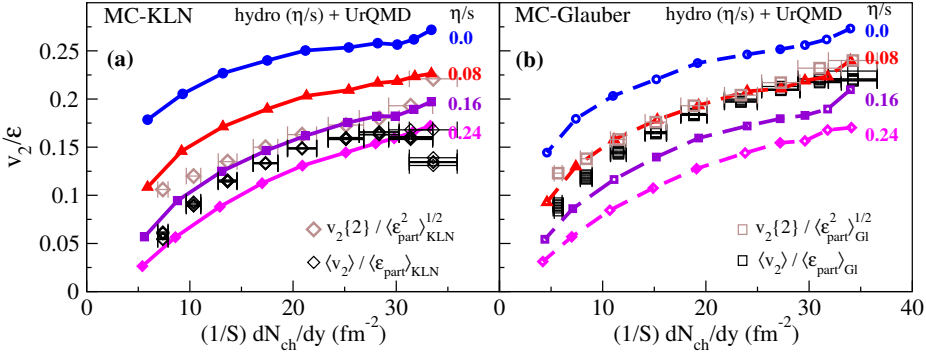


Large $R_{\text{out}}/R_{\text{side}}$ was predicted as a signature of system passing the QGP formation threshold. (e.g. Soff, Bass, Dumitru, nucl-th/0012085)

T. Hirano, K. Tsuda, Phys.Rev.C 66, 054905 (2002), nucl-th/0205043

Fast-forward to 2010s ["Gen 2" calculations]

H. Song, S. Bass, U. Heinz, T. Hirano, C. Shen, Phys.Rev.Lett. 106, 192301 (2011), arXiv:1011.2783



Ideal hydrodynamics strongly overshoots the data[1]??? What happened?

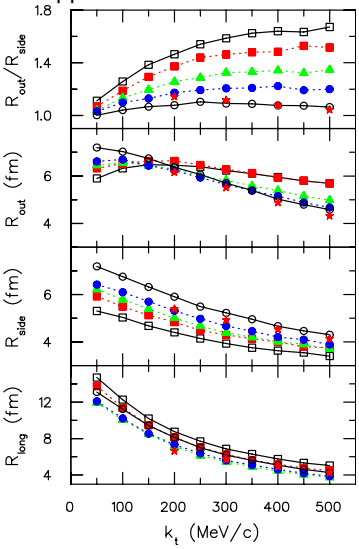
What happened is that the inputs to the hydro have changed:

- The initial state here is Monte Carlo Glauber / Monte Carlo KLN (a flavour of CGC), which results in steeper initial energy density profiles.
- The equation of state now has a crossover-type transition between the hadronic and the QGP phases.

[1] "The data" here is v_2/ε measured by experiment, whereas ε itself is completely impossible to measure.

Resolution of the "HBT puzzle" in the "Gen 2" calculations

With improved inputs to the fluid dynamical modelling, the discrepancy with the RHIC data simply disappeared.



The baseline ($\approx$ to the early hydro calculations): 1st-order PT EoS, no pre-thermal flow, no viscosity $\rightarrow$ open black squares

The updated ingredients:

- pre-thermal flow $\rightarrow$ red squares
- crossover EoS $\rightarrow$ green triangles
- viscosity $\rightarrow$ blue circles
- a more compact initial state $\rightarrow$ open black circles

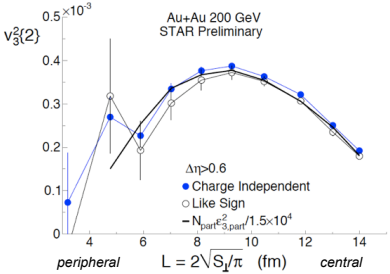
Scott Pratt, Nucl.Phys.A830:51c-57c,2009, arXiv:0907.1094

New observables'2010

Triangular flow

Paul Sorensen, Thu/26, 12:00

cos3(ϕ_1-ϕ_2) for $|\Delta\eta| > 0.6$



Related posters

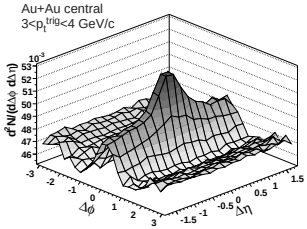
Tue/24
Li Yi, board 33
Jim Thomas, board 43

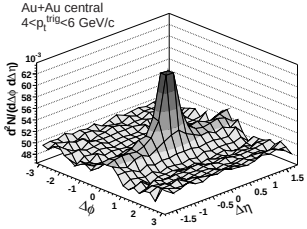
Thu/26
Chanaka De Silva, board 15

- v_3 observed. Peak at mid-central, similar to v_2
- Sensitive to the initial geometry, $v_3 \propto \varepsilon_{3part}$

STAR ☆ Hiroshi Masui *QM2011, Annecy, France, May 22-28, 2011* 21/29

Ridge (long-range rapidity corr)





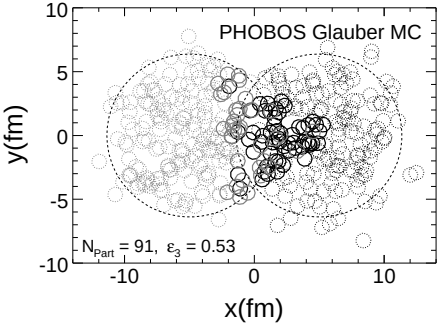
STAR collab, 0909.0191 (PRC 80, 064912)

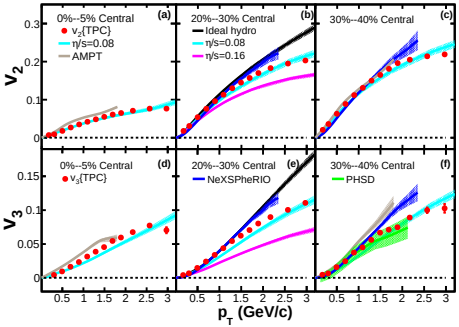
None of those could be reproduced in most of the hydrodynamic models circa 2008-2009.

Fluctuating initial state is required to describe the new observables

One of the inputs had to change: there have to be fluctuations in the initial state!

Initial nucleon distribution is irregular $\Rightarrow$ initial state triangularity ε_3 $\Rightarrow$ hydrodynamics $\Rightarrow$ final state v_3.



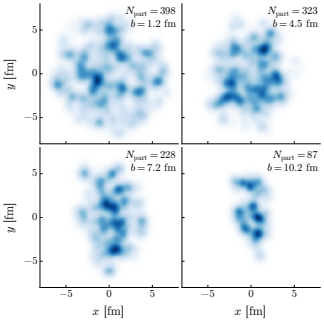


Note in the middle column of the right figure: the splitting between "ideal hydro" and "$\eta/s = 0.16$" cases is stronger for v_3 than for v_2 $\Rightarrow$ "triangular flow is more sensitive to shear viscosity".

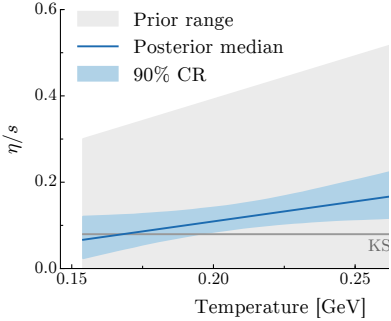
IP-Glasma and bulk viscosity [“Gen 3” calculations]

Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94, 024907 (2016) arXiv:1605.03954

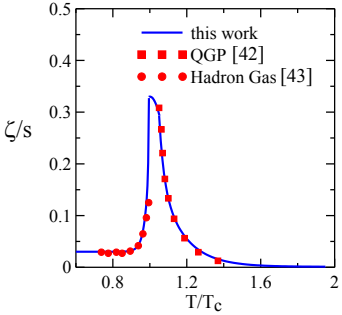
Initial state: T_RENTo



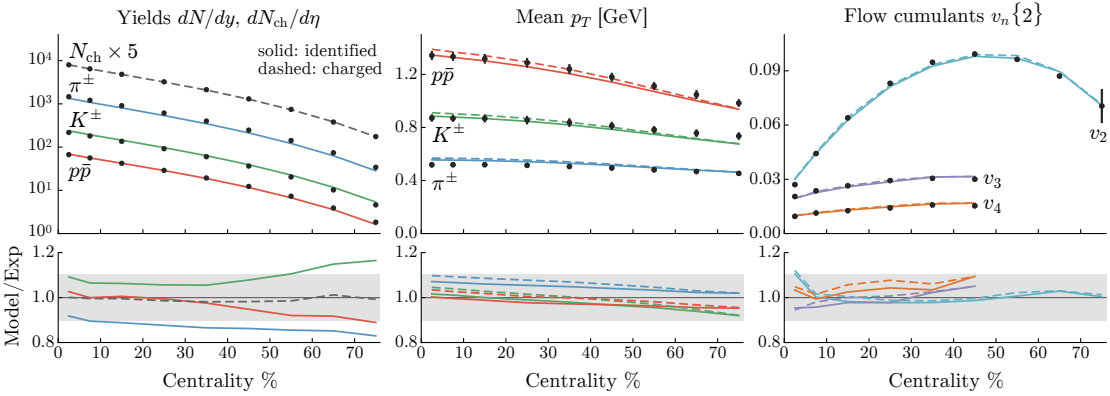
Temperature-dependent η/s



Finite ζ/s

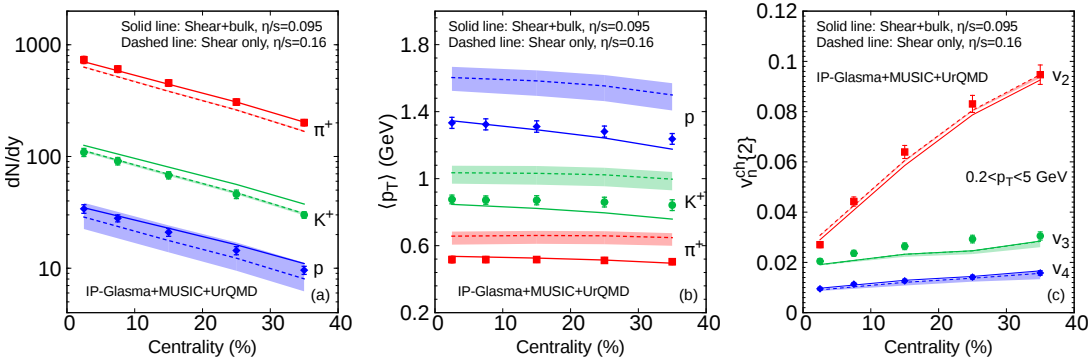


Gen 3: More observables involved, more model parameters to vary $\Longrightarrow$ Bayesian analysis



Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94, 024907 (2016) arXiv:1605.03954

p_T spectra of identified hadrons play an important role to constrain the bulk viscosity!
($\Rightarrow$ imporance of non-exciting non-smoking-gun observables)

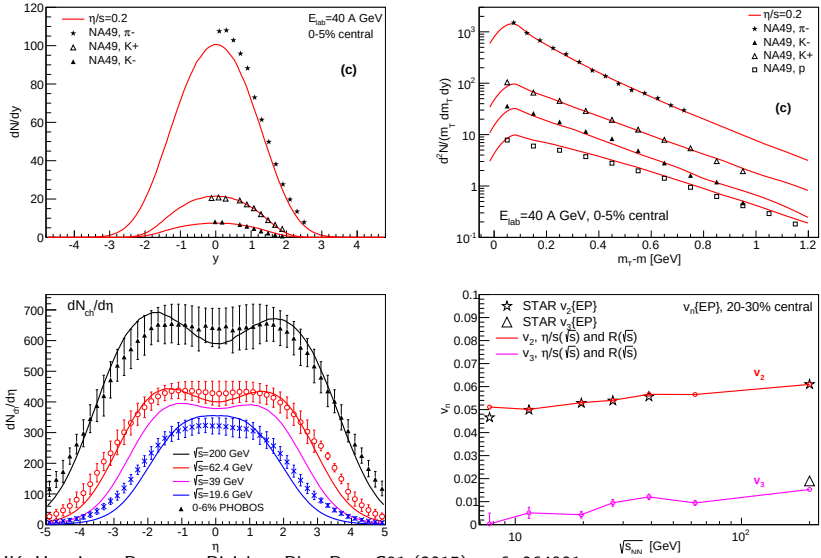


S. Ryu, J.-F. Paquet, C. Shen, G. S. Denicol, B. Schenke, S. Jeon, C. Gale, Phys. Rev. Lett. 115, 132301 (2015)

Towards lower energies: "Gen 2+" works for SPS!

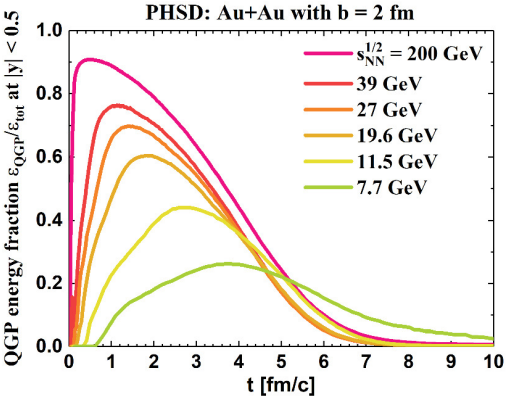
The inputs:

- 3D IS from UrQMD
- $\eta/s = 0.08 \ldots 0.2$
- crossover EoS
- event-by-event hydrodynamics
- +final-state hadronic cascade

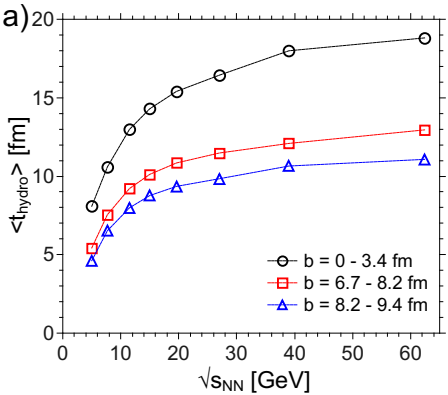


IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

It is understood now that a mélange of QGP and hadronic matter is created in heavy-ion collisions at energies lower than 200 GeV RHIC.



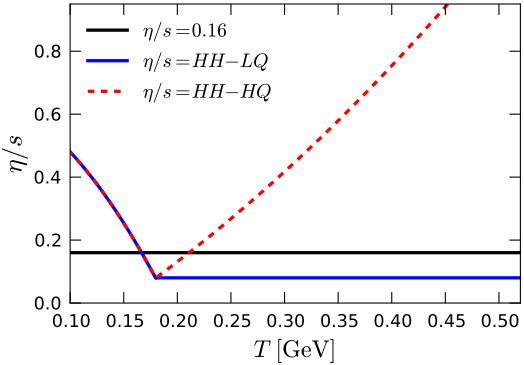
Moreau, Soloveva, Grishmanovskii, Voronyuk, Oliva, Song, Kireyeu, Coci, Bratkovskaya,
Astronomical Notes, 2021, 13988

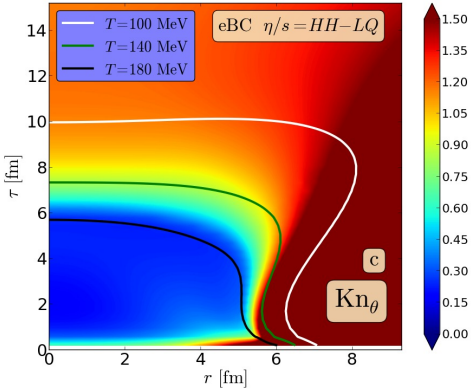


Hybrid UrQMD: Auvinen, Petersen,
Phys. Rev. C 88, 064908 (2013)

What worries (some of) us

When going from ideal to viscous fluid dynamics, a microscopic scale emerges (mean free path), so there is a Knudsen number $\mathrm{Kn} = l_{\mathrm{micro}}/l_{\mathrm{macro}}$ and the viscous fluid dynamics is generally applicable when $\mathrm{Kn} \ll 1$.

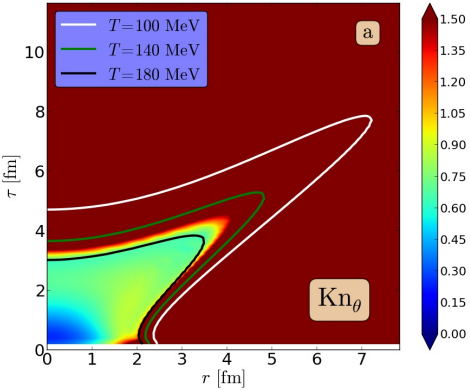


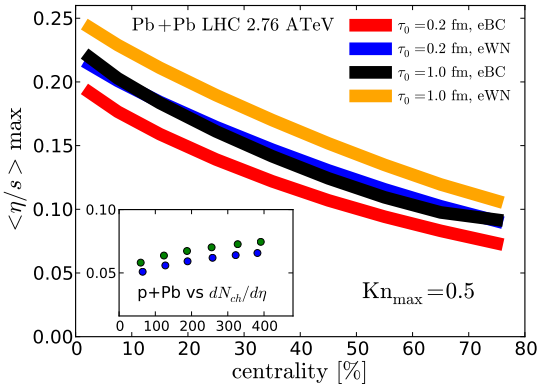


Niemi, Denicol, arXiv:1404.7327

Right plot: Knudsen number is mostly OK in a heavy-ion simulation.

What worries (some of) us

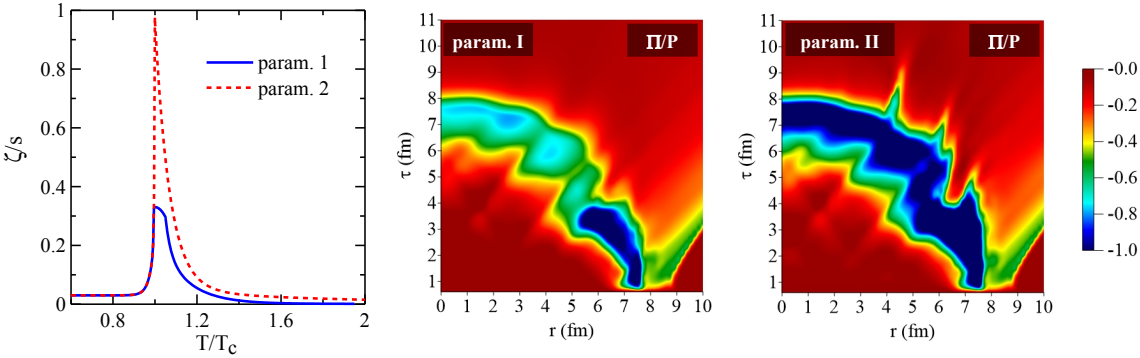




Niemi, Denicol, arXiv:1404.7327 Left plot: Knudsen number is on the edge in a p+Pb simulation.

Right plot: 'maximum allowed' η/s from the Knudsen number. Viscous hydrodynamics is running close to the limit of its validity already for a 'large' Pb+Pb system.

"Cavitation" due to bulk pressure



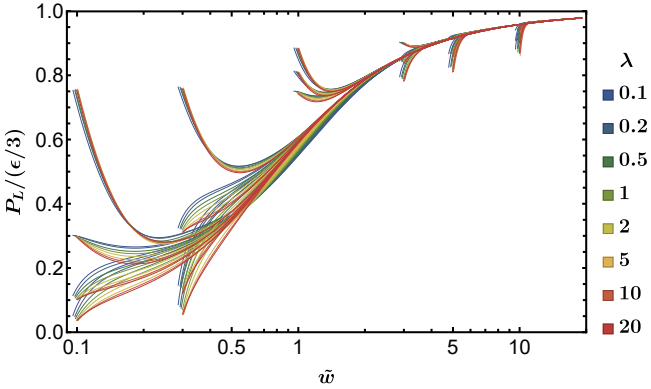
Gabriel S. Denicol, Charles Gale, Sangyong Jeon, arXiv:1503.00531

Initial state: approach to equilibrium

Hydrodynamic attractors

M. Heller and M. Spaliński, Phys. Rev. Lett. 115, 072501 (2015)
Müller-Israel-Stewart (MIS) theory for boost invariant conformal fluid
"system relaxes to an attractor regardless of when an initial condition is set"

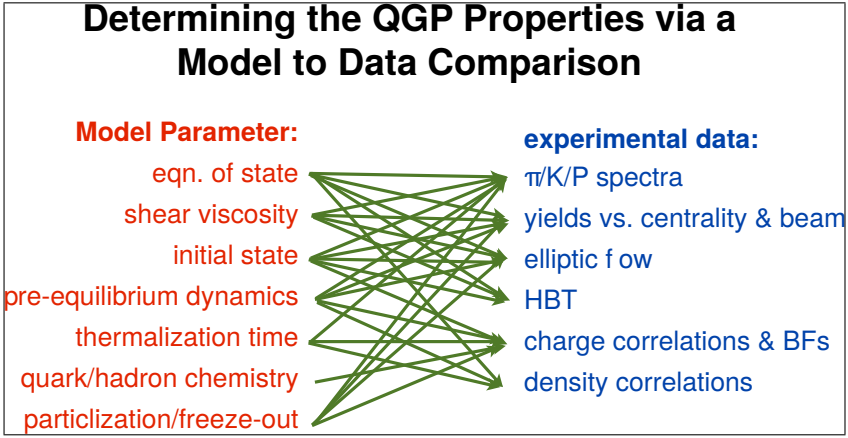
A particular example here:
X. Du, M. Heller, S. Schlichting, V. Svensson,
Phys. Rev. D 106, 014016 (2022)



Many more (toy) models and examples of attractors discovered by now.

Inputs and outputs: entangled dependencies

S. Bass, Quark Matter 2017



- evaluating model output for single point in the parameter space requires many CPU hours.

Summary of summary

There's no need for that.

Thank you for your attention!