Workshop HF2022: Heavy Flavours from small to large systems

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How clustering shapes the structure of tetraquarks

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Multiquark states in QCD

Growing experimental evidence, during the last two decades, about the existence of exotic hadrons or multiquark states, containing more valence quarks than ordinary mesons ($\overline{q}q$) and baryons (qqq).

Prototypes are tetraquarks with valence quark structure $\bar{q}\bar{q}qq$, pentaquarks, with structure $\overline{q}qqqq$, hexaquarks, with $qqqqqq$.

However, contrary to ordinary hadrons, multiquark states are not color-irreducible, in the sense that they can be decomposed along a finite number of combinations of ordinary mesonic or baryonic clusters.

$$\begin{aligned}
(\bar{q}\bar{q}qq) &= \sum (\bar{q}q)(\bar{q}q), \\
(\overline{q}qqqq) &= \sum (\overline{q}q)(qqq), \\
(qqqqqq) &= \sum (qqq)(qqq).
\end{aligned}$$

Hadronic clusters, being color-singlets, mutually interact by means of short-range forces, like meson-exchanges or contacts. They would form loosely bound states, in similarity with atomic molecules. These are called hadronic molecules or molecular states. (Törnqvist, 1994.)

In contrast, multiquark states formed directly from confining interactions acting on all quarks, would form compact bound states, called compact multiquark states. (Maiani *et al.*, 2005.)

Multiquark states can thus be formed by two different mechanisms, each leading to a different structure. The issue is to find, by theoretical justification, also guided by experimental data, the most faithful description.

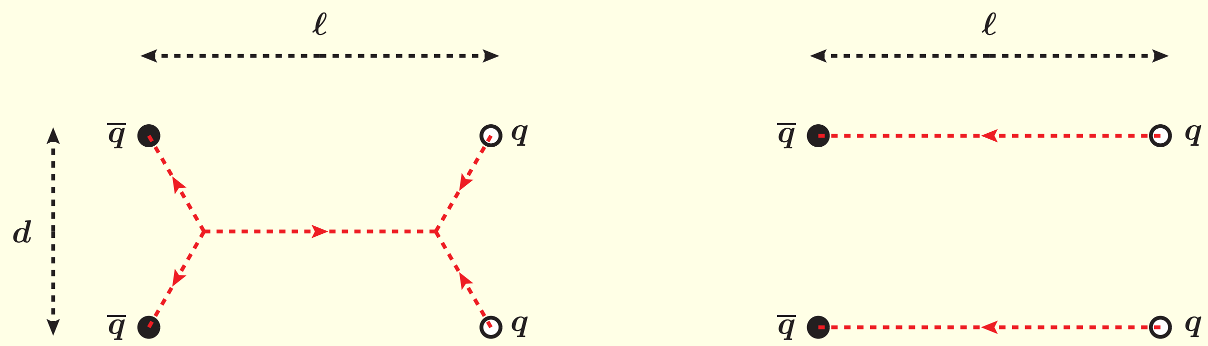
(In the following, we concentrate on tetraquark states, ignoring details coming from quark flavors and spins, which do not play a fundamental role.)

Energy balance

A first hint is provided by the study of the energy balance of the system. This is most easily done in the static limit of the theory, with very heavy quarks, fixed at spatial positions. The system would choose configurations with minimal energy.

The problem is analytically solved in the strong coupling limit of lattice theory (Dosch, 1983) and confirmed by direct lattice numerical calculations (Alexandrou *et al.*, 2005; Suganuma *et al.*, 2005; Bicudo *et al.*, 2011).

In the strong coupling limit, the potential energy is concentrated on the Wilson lines (path-ordered gluon field phase factors) with constant linear energy density.



(a) (b)

(a): Compact tetraquark, formed by confining interactions, through diquark–antidiquark global interaction.

(b): Two meson clusters.

The compact tetraquark formation is energetically favored if the diquark interdistance d is much smaller than the quark-antiquark interdistance ℓ:

$$d \ll \ell.$$

However, quarks have finite masses and move in space. Even if a compact tetraquark has been formed, there is a probability that the quarks, during their motion, reach the two-meson-cluster configuration, in which case the system dislocates or decays.

An interesting and nontrivial case is when the compact tetraquark mass lies below the two-meson threshold, which prevents dislocation. Nevertheless, there will be, through fluctuations, constant transitions to the two-meson virtual states. This will have the tendancy to expand the compact system into a more loosely bound system, close to a molecular-type state.

This is a dynamical mechanism, whose precise outcome necessitates the resolution of the four-body bound state problem, in the presence of the confining forces. For the time being, this problem has not yet been satisfactorily solved.

Effective field theory approach

The problem can be studied through an effective field theory approach. The quark and gluon degrees of freedom are integrated out in favor of the low-energy hadronic degrees of freedom. A perturbative expansion is then adopted with respect to the low-energy appropriate scale parameters. In the case of heavy quarks these are the nonrelativistic energies and momenta.

According to the energy balance analysis, there is always a probability that a compact tetraquark be formed from the action of the confining forces (actually an infinite tower of such states). We assume that in first approximation, because of the compactness of the state, the latter can be assimilated to a pointlike object and treated as an elementary particle.

We are mainly interested in the case where the mass of the latter particle lies below the lowest possible two-meson threshold. This might be the case when the compact tetraquark is the ground state of the corresponding spectrum.

The two mesons are M_1 and M_2, with masses m_1 and m_2.

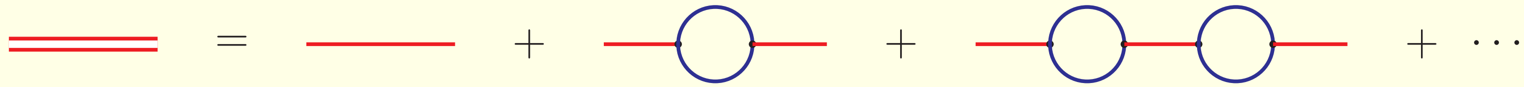
The bare mass of the tetraquark is m_{t0}.

Because of the initial general structure of the tetraquark, containing two-meson clusters, the pointlike tetraquark has necessarily a coupling to the mesons M_1 and M_2. The coupling is assumed scalar and is designated by g' (dimensionless, factored by $(m_1 + m_2)$).

The coupling g' generates, through meson loops, radiative corrections inside the tetraquark propagator, modifying the parameters of the bare propagator. In particular, the bare mass m_{t0} will be changed into a physical mass m_t.

Quark flavors and spins will be ignored, as not playing a fundamental role here.

Graphically:



Mutual interactions of mesons are neglected (in first approximation); they play a less important role than the direct tetraquark-two-meson coupling.

The full tetraquark propagator becomes

$$D_t(s) = \frac{i}{s - m_{t0}^2 + i(m_1 + m_2)^2 g'^2 J(s)},$$

where s stands for p^2 and J is the standard loop function. The divergence of J is absorbed by the bare mass term, yielding a renormalized mass m_{t1}:

$$m_{t1}^2 = m_{t0}^2 - i(m_1 + m_2)^2 g'^2 J^{\mathrm{div}}.$$

The renormalized tetraquark propagator is now

$$D_t(s) = \frac{i}{s - m_{t1}^2 + i(m_1 + m_2)^2 g'^2 J^{\mathrm{r}}(s)}.$$

Notice that g' does not undergo any renormalization. The mass term m_{t1} does not yet represent the physical mass of the tetraquark. The latter is determined from the pole position of the propagator.

We stick here to the case of heavy quarks and heavy mesons, treating the problem in its nonrelativistic limit, referred to the two-meson threshold. The nonrelativistic energy E is introduced through the standard definition

$$\sqrt{s} = (m_1 + m_2) + E.$$

The nonrelativistic energy corresponding to the renormalized mass m_{t1} of the tetraquark is defined similarly:

$$E_{t1} = m_{t1} - (m_1 + m_2), \qquad E_{t1} < 0.$$

Since we are considering the case of stable tetraquarks (under the strong interactions), the mass of the tetraquark is expected to lie below the two-meson threshold. The physical nonrelativistic energy of the tetraquark will be designated by E_t and is also expected to be negative.

To simplify notations, we shall use henceforth the reduced dimensionless energy variables e through the definitions

$$e \equiv \frac{E}{2m_r}, \quad e_{t1} \equiv \frac{E_{t1}}{2m_r}, \quad e_t \equiv \frac{E_t}{2m_r}, \quad m_r = \frac{m_1 m_2}{(m_1 + m_2)}.$$

The quantity $-e_t$ represents the nonrelativistic binding energy of the tetraquark by reference to the two-meson threshold.

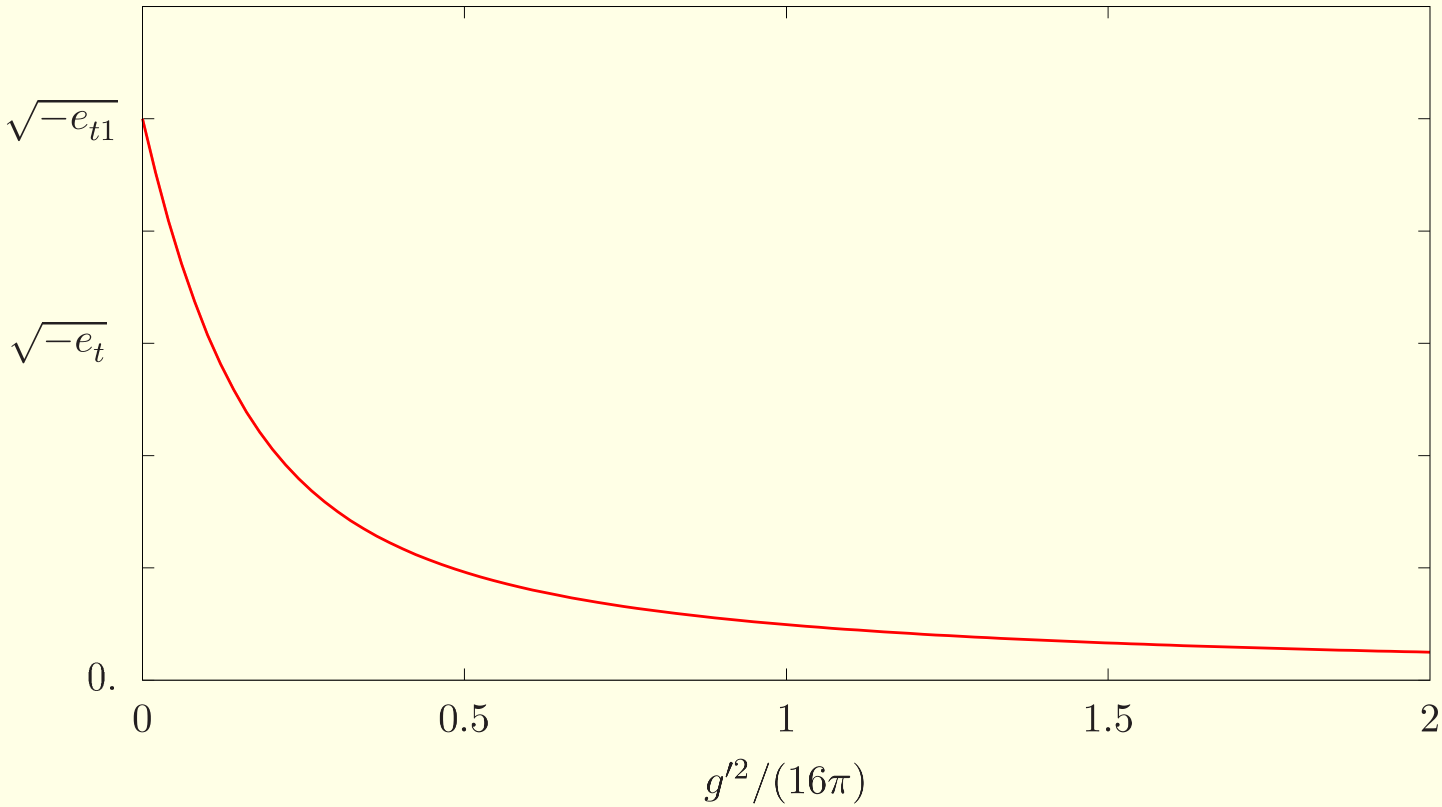
One finds for the nonrelativistic energy e_t of the tetraquark the equation

$$-e_t + e_{t1} + \frac{g'^2}{16\pi}\sqrt{-e_t} = 0,$$

whose solution is

$$\sqrt{-e_t} = \frac{1}{2}\Big[-\frac{g'^2}{16\pi} + \sqrt{\Big(\frac{g'^2}{16\pi}\Big)^2 - 4e_{t1}}\Big].$$

The binding energy $-e_t$ is a decreasing function of $g'^2/(16\pi)$, and comes out smaller than $-e_{t1}$, reaching the value 0 when $g' \to \infty$.



Very rapid decrease of the binding energy. With ordinary values of $g'^2/(16\pi)$ of the order of 1, the binding energy decreases by a factor of 1/100. The state takes the appearance of a shallow bound state.

Compositeness

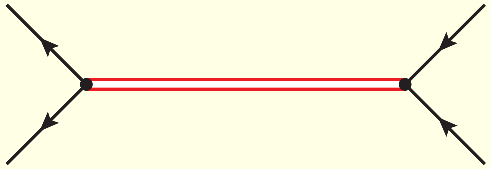
The comparison of the molecular and compact schemes is reminiscent of a general problem, already raised in the past in the case of the deuteron state, denoted under the term of compositeness (Weinberg, 1965).

Weinberg has shown that this question can receive, in the nonrelativistic limit, a precise and model-independent answer, by relating the probability of a state as being elementary (or compact) to observable quantities, represented by the scattering length and the effective range of the two constituents of the molecular scheme in the S-wave state of their scattering amplitude. Designating by Z this probability, one has the following relations for the scattering length a and the effective range r_e, adapted to the tetraquark problem:

$$a = \frac{2(1-Z)}{(2-Z)}R, \qquad r_e = -\frac{Z}{(1-Z)}R, \qquad R = (-2m_r E_t)^{-1/2},$$

where R is the radius of the bound state, m_r the reduced mass of the two-meson system, E_t the tetraquark nonrelativistic energy.

The contribution of the tetraquark state, in the s-channel, to the two-meson elastic scattering amplitude is obtained by inserting the tetraquark propagator between two tetraquark-two-meson couplings.

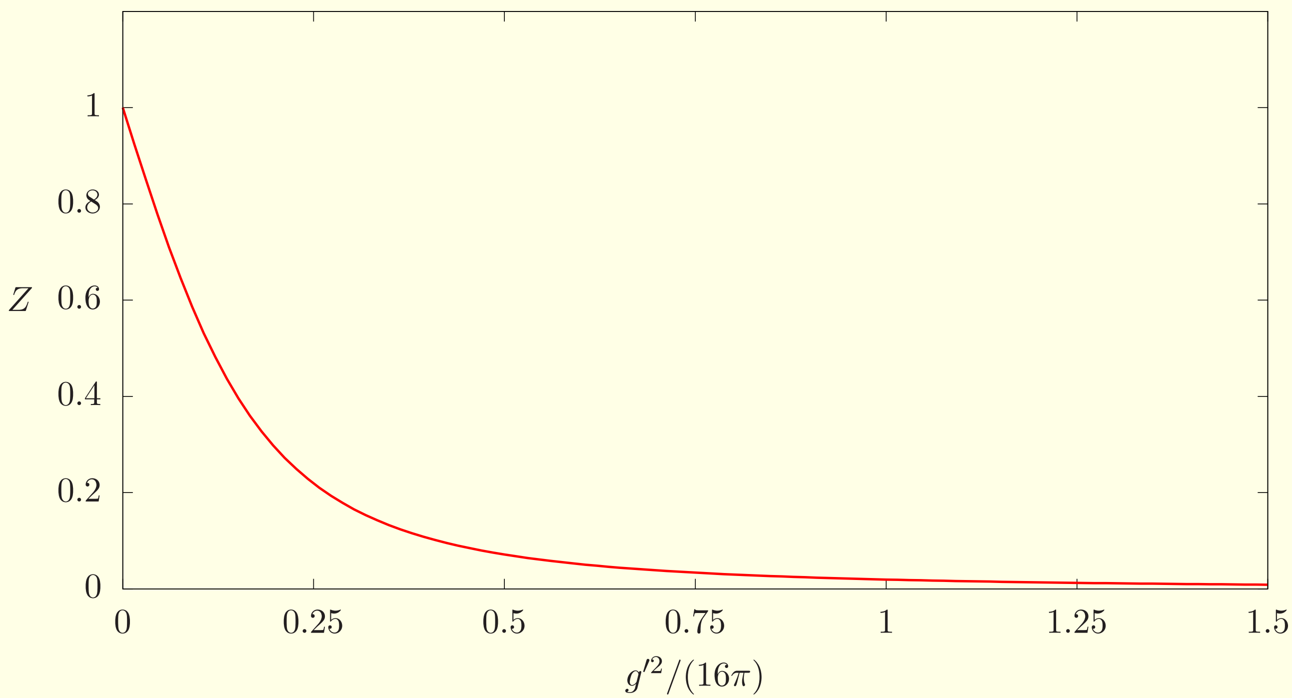


From the scattering length and the effective range one obtains Z:

$$Z = \frac{\sqrt{-e_t}}{\sqrt{-e_t} + \frac{1}{2}\frac{g'^2}{16\pi}}.$$

$Z = 1 \iff$ compact tetraquark.

$Z = 0 \iff$ molecular state.



Like the binding energy, Z is a rapidly decreasing function of $g'^2/(16\pi)$. For values of the latter of the order of 1, Z is very close to zero.

Summary

A compact tetraquark, formed from the confining forces acting between quarks and gluons, rapidly evolves, under the influence of the clustering phenomenon, towards a molecular-type state. The origin of the state is, however, the compact nature: nowhere, in the present model, did we consider direct interactions between mesons.

The experimental test for this phenomenon is provided by the measure of the elementariness coefficient Z. Pure molecular states are characterized by the value $Z = 0$. A value of $Z \neq 0$, reflects the existence of an original compact tetraquark. Many tetraquark candidates fall in this category (Oller *et al.*, 2018).

Shallowness of many bound states may receive a natural explanation from the above mechanism.

The present study can also be extended to the case of resonances and to the case when direct meson-meson interactions are incorporated.