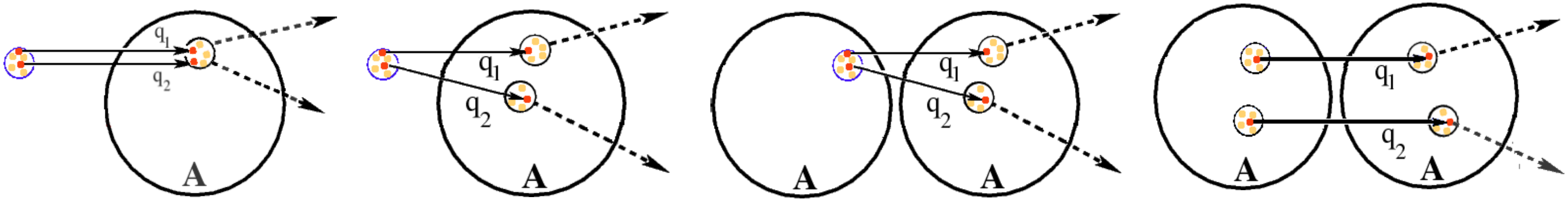


Double & triple heavy-flavour from N-parton scatterings in p-p, p-A, A-A

HF 2022

Institut Pascal, Orsay, 20th Oct. 2022

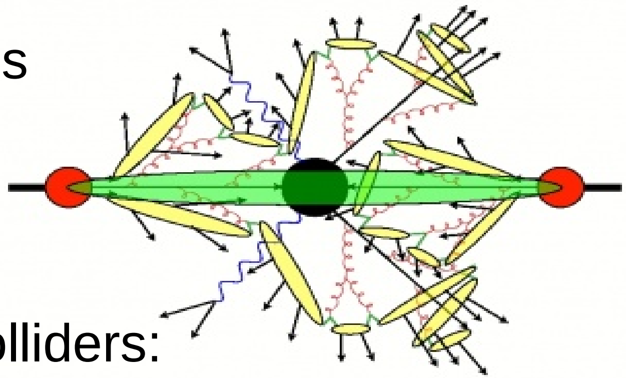
David d'Enterria

CERN

(*) Details in DPS/TPS/NPS in pp, pA, AA review:

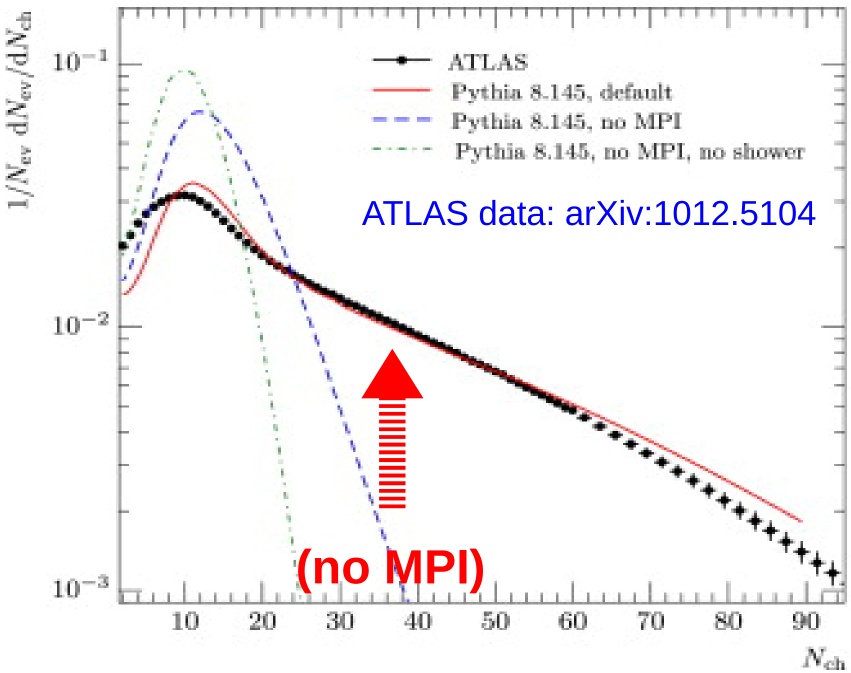
D.d'E & A.Snigirev: arXiv:1708.07519 [Adv.Ser.Direct.High.En.Phys. 29 (2018) 159]

Multi-parton interactions at the LHC

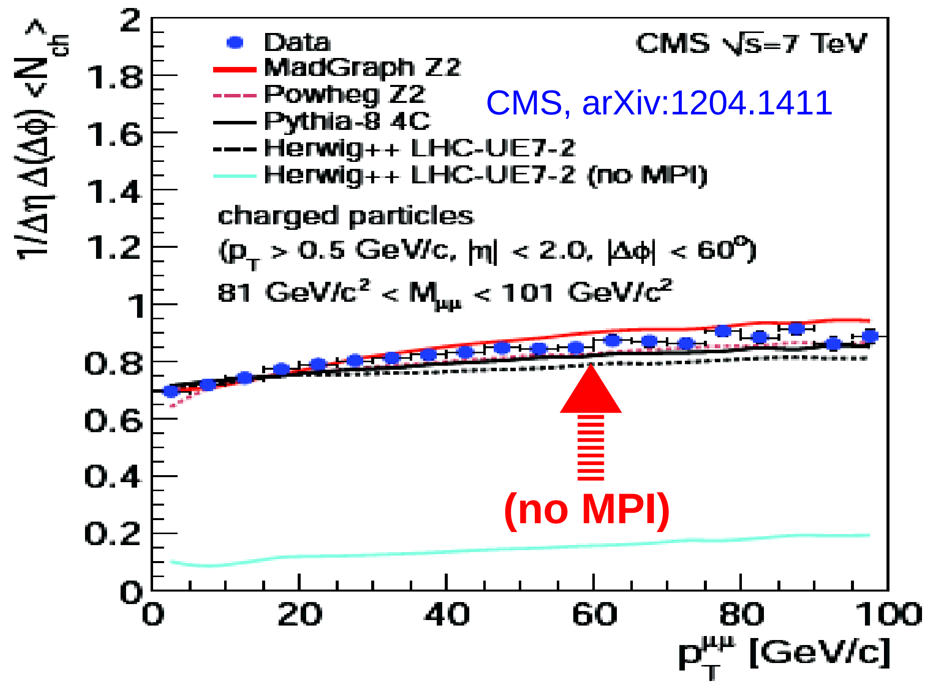


- MPI are intrinsic component of hadron collisions (p,Pb) = non-pointlike objects with finite transverse size and increasingly larger gluon density with $\sqrt{s}$.
- MPI O(1–3 GeV) clearly observed at hadron colliders:

~50% of total hadron production



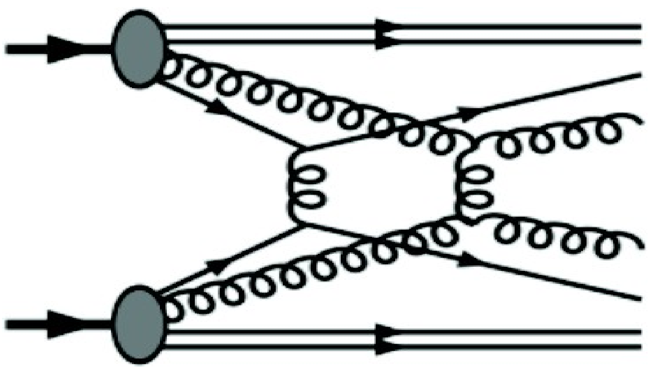
Underlying event in hard scatterings:



- Double hard parton scatts. (p_T,m_X>3 GeV) happen also & been observed

Double Parton Scattering x-sections (p-p)

- Assuming that the probability to produce two hard collisions is independent, one can simply write double parton scatterings (DPS) cross section as the product of two single-parton scatterings (SPS) ones:



$$\sigma^{\rm DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right)\frac{\sigma^{\rm SPS}_{(hh'\to a)}\cdot\sigma^{\rm SPS}_{(hh'\to b)}}{\sigma_{\rm eff}}$$

normalized by an effective x-section (σ_{eff}), with a trivial combinatorial factor (m) to avoid double-counting in case of same particles produced.

- How to interpret σ_{eff}? What values one would naively expect for it?

- Let's start with the most generic expression for DPS cross section:

$$\sigma^{\rm DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right)\sum_{i,j,k,l}\int \Gamma^{ij}_{h}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)\times\hat{\sigma}^{ik}_{a}(x_1,x_1',Q_1^2)\,\hat{\sigma}^{jl}_{b}(x_2,x_2',Q_2^2)$$
$$\times\Gamma^{kl}_{h'}(x_1',x_2';\mathbf{b_1}-\mathbf{b},\mathbf{b_2}-\mathbf{b};Q_1^2,Q_2^2)\,dx_1dx_2dx_1'dx_2'd^2b_1d^2b_2d^2b$$

Generalized PDFs = $f(x,Q^2,\mathbf{b})$

Double Parton Scattering x-sections (p-p)

- Assumption 1: Generalized PDFs factorize into longitudinal & transverse components:

transv. density = f(**b**)

$$\Gamma_h^{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathbf{b_1}) f(\mathbf{b_2})$$

p-p transv. overlap function (mb^{-1}): $t(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$

- Assumption 2: The longitudinal double-PDF is the product of 2 single PDF (i.e. no parton correlations in colour, momentum, flavour, spin,…)

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2)\, D_h^j(x_2; Q_2^2)$$

- σ_{eff} = <Interparton transv. separation>^{2}. Derivable from geometric p-p overlap with naive expected size of $\sigma_{eff} \approx 30$ mb

$$\sigma_{\rm eff} = \left[\int d^2 b\, t^2(\mathbf{b})\right]^{-1}$$

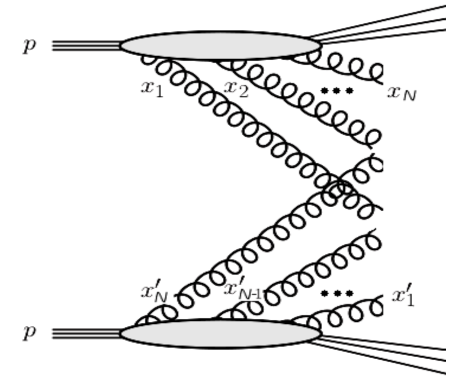
Model for density	Form of density, dN/d^3r	Predictions rms r	Predictions $\sigma_{\rm eff}$	Measurements Scale (fm)
Solid sphere	Constant, $r<r_p$	$\sqrt{3/5}r_p$	$4\pi r_p^2/4.6$	$r_p=0.73$
Gaussian	$e^{-r^2/2\Sigma^2}$	$\sqrt{3}\Sigma$	$4\pi\Sigma^2$	$\Sigma=0.34$
Exponential	$e^{-r/\lambda}$	$\sqrt{12}\lambda$	$35.5\lambda^2$	$\lambda=0.20$
Fermi, $\lambda/r_0=0.2$	$(e^{(r-r_0)/\lambda}+1)^{-1}$	$1.07r_0$	$4.6r_0^2$	$r_0=0.56$

- But experimentally: σ_{eff}(exp) ≈ 15 mb. proton "hard" radius: r = 0.3–0.7 fm appears smaller than e.m. one:

Understandable: Probability of 2nd scatt. is larger if 1st scatter already took place ("centrality bias").

N-parton scattering x-sections (p-p)

- Assuming that the probabilities for N hard collisions to be independent of each other, one can write a generic pocket-formula for NPS x-section:



$$\sigma^{\rm NPS}_{hh'\to a_1\ldots a_n} = \left(\frac{m}{n!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdots \sigma^{\rm SPS}_{hh'\to a_n}}{\sigma^{n-1}_{\rm eff,NPS}}$$

normalized by the N^th–1 power of an effective x-section ($\sigma_{\rm eff,NPS}$) plus a trivial combinatorial factor (m/n!) to avoid double,triple,N-counting in case of same particles produced:

- DPS: $m = 1$ if $a_1 = a_2$; and $m = 2$ if $a_1 \neq a_2$.
- TPS: $m = 1$ if $a_1 = a_2 = a_3$; $m = 3$ if $a_1 = a_2$, or $a_1 = a_3$, or $a_2 = a_3$; and $m = 6$ if $a_1 \neq a_2 \neq a_3$.

- Ignoring all parton correlations, $\sigma_{\rm eff,NPS}$ is the inverse N^{th}–1 power of the integral of the N^{th} power of the pp overlap function:

$$\sigma_{\rm eff,NPS} = \left\{\int d^2b\, T^n(\mathbf{b})\right\}^{-1/(n-1)}$$

- A generic framework for the most economical (geometrical) expressions for N-parton scattering cross sections is available.

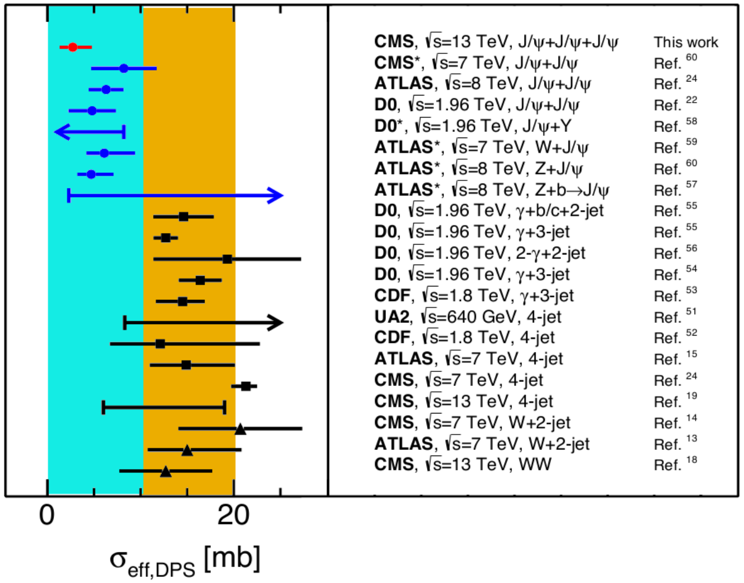
Double Parton Scatterings

DPS studies at the LHC

- Motivation for studies of multiple production of hard/heavy particles:
  (1) Generalized PDFs (x,Q^2 ,b) of the proton, in particular the unknown energy evolution of transverse proton profile.
  (2) Role of partonic correlations (in space, p, x, flavour, colour, spin,...) in hadronic wave functions.
  (3) Backgrounds for rare (B)SM resonance decays w/ multiple heavy particles
- "Pocket formula" results at the LHC:

$$\sigma_{\rm DPS}^{pp\to\psi_1\,\psi_2+X} = \left(\frac{m}{2}\right) \frac{\sigma_{\rm SPS}^{pp\to\psi_1+X}\,\sigma_{\rm SPS}^{pp\to\psi_2+X}}{\sigma_{\rm eff,DPS}}$$

$\sigma_{\rm eff}$ ~ <Interparton transv. separation>^{2}
derivable from p-p transverse overlap:
$\sigma_{\rm eff}$ ~ 20–30 mb (PYTHIA8/HERWIG p form-factor)
$\sigma_{\rm eff}$ ~ 15 mb (from DPS of jets, EWK bosons)
$\sigma_{\rm eff}$ ~ 5 mb (from di-quarkonia)

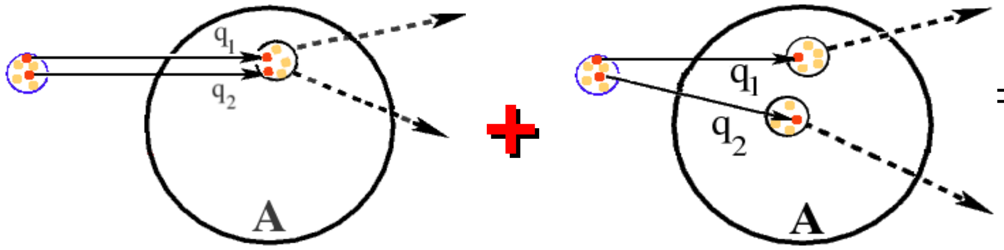


- Reasons: Parton correlations? x-,flavour-dependent transverse p profile?
- Novel observables: DPS with ions, Triple-parton scatterings (TPS) in particular with charm, bottom final states: largest pQCD cross sections

Double Parton Scattering x-sections in p-A

[DdE, Snigirev, PLB 718 (2013)1395]
[Also Treleani, Strikman, Blok...]

- Two contributions to DPS x-section in p-A:



$$\Rightarrow \sigma^{DPS}_{(pA)} = \sigma^{DPS,1}_{(pA)} + \sigma^{DPS,2}_{(pA)}$$

$$\sigma^{DPS,1}_{(pA\to ab)} = A \cdot \sigma^{DPS}_{(pN\to ab)} \quad + \quad \sigma^{DPS,2}_{(pA\to ab)} = \sigma^{DPS}_{(pN\to ab)} \cdot \sigma_{eff,pp} \cdot F_{pA}$$

p-A overlap function:

$$F_{pA} = \int d^2r\, T^2_{pA}(\mathbf{r}) = 30.4\ \mathrm{mb}^{-1}$$

Pb Woods-Saxon density (r=6.62 fm, a=0.546 fm)

Relative weight of DPS terms: $\sigma^{DPS,1}$:$\sigma^{DPS,2}$ = 0.7 : 0.3 (small A), 0.33 : 0.66 (large A)

- "Pocket" formula for DPS p-A x-section:

$$\sigma^{DPS}_{(pA\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{SPS}_{(pN\to a)} \cdot \sigma^{SPS}_{(pN\to b)}}{\sigma_{eff,pA}}$$

($\sigma_{eff,pp}$=13±2mb)

$$\sigma_{eff,pA} = \frac{\sigma_{eff,pp}}{A + \sigma_{eff,pp} F_{pA}} = 21.5 \pm 1.1\ \mu b$$

► Ratio of DPS p-Pb/p-p x-sections: $\sigma_{\mathrm{eff,DPS}}/\sigma_{\mathrm{eff,DPS,pA}} \approx [A + A^{4/3}/\pi]$

- DPS x-sections are large in p-A: a factor ×600 (not ×208) for p-Pb (!)
- Pb transverse density (F_{pA}) well known: Alternative extraction of $\sigma_{eff,pp}$

Examples: DPS x-sections in p-Pb (8.8 TeV)

[DdE, Snigirev, NPA 931 (2014) 303]

- Cross sections & rates for DPS processes with J/ψ,Υ & W, Z bosons
  [Also V. Goncalves (2018): double-J/ψ; Paukunen (2019): double-D,...]

pPb (8.8 TeV)	$J/\psi + J/\psi$	$J/\psi + \Upsilon$	J/ψ+W	J/ψ+Z
$\sigma^{\rm SPS}_{{\rm pN}\to a}, \sigma^{\rm SPS}_{{\rm pN}\to b}$	45 μb ($\times$2)	45 μb, 2.6 μb	45 μb, 60 nb	45 μb, 35 nb
$\sigma^{\rm DPS}_{\rm pPb}$	45 μb	5.2 μb	120 nb	70 nb
$\rm N^{\rm DPS}_{\rm pPb}$ (1 $\rm pb^{-1}$)	~65	~60	~15	~3
	$\Upsilon + \Upsilon$	Υ+W	Υ+Z	ss WW
$\sigma^{\rm SPS}_{{\rm pN}\to a}, \sigma^{\rm SPS}_{{\rm pN}\to b}$	2.6 μb ($\times$2)	2.6 μb, 60 nb	2.6 μb, 35 nb	60 nb ($\times$2)
$\sigma^{\rm DPS}_{\rm pPb}$	150 nb	7 nb	4 nb	150 pb
$\rm N^{\rm DPS}_{\rm pPb}$ (1 $\rm pb^{-1}$)	~15	~8	~1.5	~4

Leptonic final states: BR(J/ψ,Υ,W,Z) = 6%, 2.5%, 11%, 3.4%
Accept.*Effic.= 1% (J/ψ, |y|=0,2), 20% (Υ, |y|<2.5), 50% (W,Z |y|<2.4)

- Many double hard scatterings processes with visible p-Pb x-sections at the LHC. (Note: J/ψ values are per unit-|y|).
- Useful independent extraction of $\sigma_{\rm eff,pp}$!

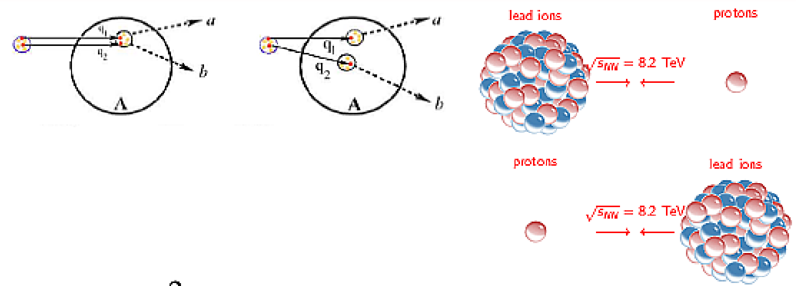
First study of DPS in p-Pb (LHCb, 8.2 TeV)

[LHCb, PRL 125 (2020) 212001]

double charm production in proton lead collisions

- select pairs of D^0, $\bar{D}^0$, D^+, D^-, D_s^+, D_s^- and J/ψ
- sort them into pair production and "DPS" categories

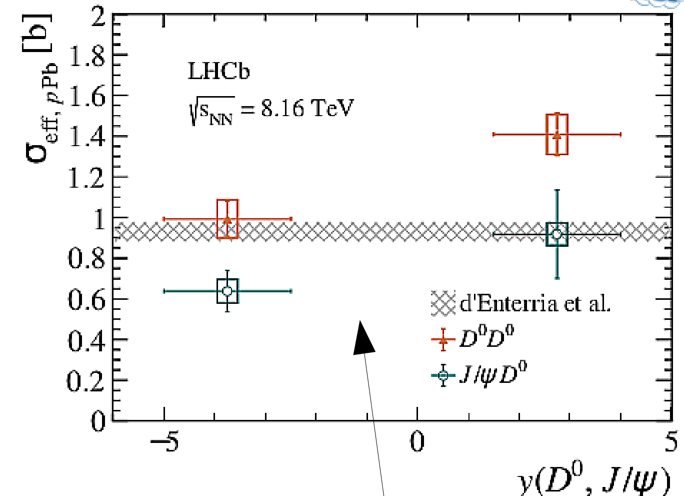
$$\sigma_{C_1,C_2} = \alpha \frac{\sigma_{C_1}\sigma_{C_2}}{\sigma_{\text{eff}}}$$



$$R^{D_1 D_2}_{forward} = \frac{\sigma_{D_1 D_2}}{\sigma_{D_1 \bar{D}_2}} = 0.308 \pm 0.015 \pm 0.010$$

$$R^{D_1 D_2}_{backward} = 0.391 \pm 0.019 \pm 0.025$$

$$R^{D^0 D^0}_{pp} = 0.109 \pm 0.008$$



Like sign charm fraction tripled!

$\sqrt{s_{NN}} = 8.2$ TeV *Phys. Rev. Lett. 125 (2020) 212001*

Albert Bursche | charming DPS | 10th October 2021 | 15 / 17

- Useful independent extraction of $\sigma_{\text{eff,pp}}$:

$$\sigma_{\text{eff,pA}} = \frac{\sigma_{\text{eff,pp}}}{A + \sigma_{\text{eff,pp}}\, F_{pA}}$$

$\sigma_{\text{eff,pp}}(D^0D^0)$ = 7–16 mb

$\sigma_{\text{eff,pp}}(J/\psi D^0)$ = 13–40 mb

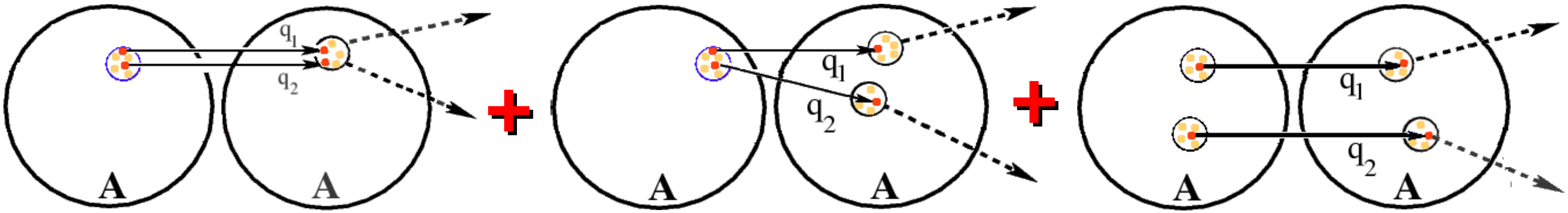
nPDF effects visible in -y/+y results.

(why LHCb does not quote the equivalent $\sigma_{\text{eff,pp}}$ values?)

Double Parton Scattering x-sections in A-A

[DdE, Snigirev, PLB727 (2013)157]

- Three contributions to DPS x-section in A-A:



$$\sigma^{DPS,1}_{(AA\to ab)} = A^2 \cdot \sigma^{DPS}_{(NN\to ab)}$$

$$\sigma^{DPS,2}_{(AA\to ab)} \approx 2\sigma^{DPS}_{(NN\to ab)} \cdot \sigma_{eff,pp} \cdot A \cdot T_{AA}(0)$$

$$\sigma^{DPS,3}_{(AA\to ab)} \approx \sigma^{DPS}_{(NN\to ab)} \cdot \sigma_{eff,pp} \cdot A^2 / 2 \cdot T_{AA}(0)$$

► Third "N_{coll} term" $\propto A^2 \cdot T_{AA}(0)$, clearly dominant (1:4:200 ratio for PbPb)

"Genuine" DPS (within same nucleon): ~2.5% (in Pb-Pb) or ~13% (Ar-Ar)

- "Pocket formula" for DPS A-A x-section:

$$\sigma^{DPS}_{(AA\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{SPS}_{(NN\to a)} \cdot \sigma^{SPS}_{(NN\to b)}}{\sigma_{eff,AA}}$$

(for Pb-Pb collisions)

$$\sigma_{eff,AA} = \frac{1}{A^2[\sigma^{-1}_{eff,pp} + \frac{2}{A} T_{AA}(0) + \frac{1}{2} T_{AA}(0)]} = 1.5\text{ nb}$$

► Ratio of DPS Pb-Pb/p-p x-sections: $\sigma_{eff,pp}/\sigma_{eff,AA} \propto A^{3.3}/5 \simeq 9 \cdot 10^6$!

- Strong centrality dependence:

$$\sigma^{DPS}_{(AA\to ab)}[b_1, b_2] \approx \left(\frac{m}{2}\right) \sigma^{SPS}_{(NN\to a)} \cdot \sigma^{SPS}_{(NN\to b)} \cdot f_{\%}\sigma_{AA} \cdot \langle T_{AA}[b_1, b_2]\rangle^2$$

Examples: DPS x-sections in Pb-Pb (5.5 TeV)

[DdE, Snigirev, NPA 931 (2014)303]

- Cross sections & rates for DPS processes with J/ψ,Υ & W, Z bosons:

PbPb (5.5 TeV)	$J/\psi + J/\psi$	$J/\psi + \Upsilon$	J/ψ+W	J/ψ+Z
$\sigma^{SPS}_{NN\to a}, \sigma^{SPS}_{NN\to b}$	25 μb ($\times$2)	25 μb, 1.7 μb	25 μb, 30 nb	25 μb, 20 nb
σ^{DPS}_{PbPb}	210 mb	28 mb	500 μb	330 μb
N^{DPS}_{PbPb} (1 nb^{-1})	~250	~340	~65	~14
	$\Upsilon + \Upsilon$	Υ+W	Υ+Z	ss WW
$\sigma^{SPS}_{NN\to a}, \sigma^{SPS}_{NN\to b}$	1.7 μb ($\times$2)	1.7 μb, 30 nb	1.7 μb, 20 nb	30 nb ($\times$2)
σ^{DPS}_{PbPb}	960 μb	34 μb	23 μb	630 nb
N^{DPS}_{PbPb} (1 nb^{-1})	~95	~35	~8	~15

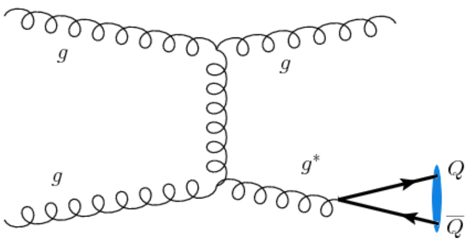
Leptonic final states: BR(J/ψ,Υ,W,Z) = 6%, 2.5%, 11%, 3.4%
Accept.*effic.= 1% (J/ψ, |y|=0,2), 20% (Υ, |y|<2.5), 50% (W,Z |y|<2.4)

- Visible rates for many double hard scatterings processes in Pb-Pb! (Note: J/ψ values are per unit-|y|).

Example: Pb-Pb → J/ψ J/ψ at 5.5 TeV

[DdE,Snigirev, PLB727 (2013)157]

- **FONLL+CEM (R.Vogt):** Single-parton J/ψ



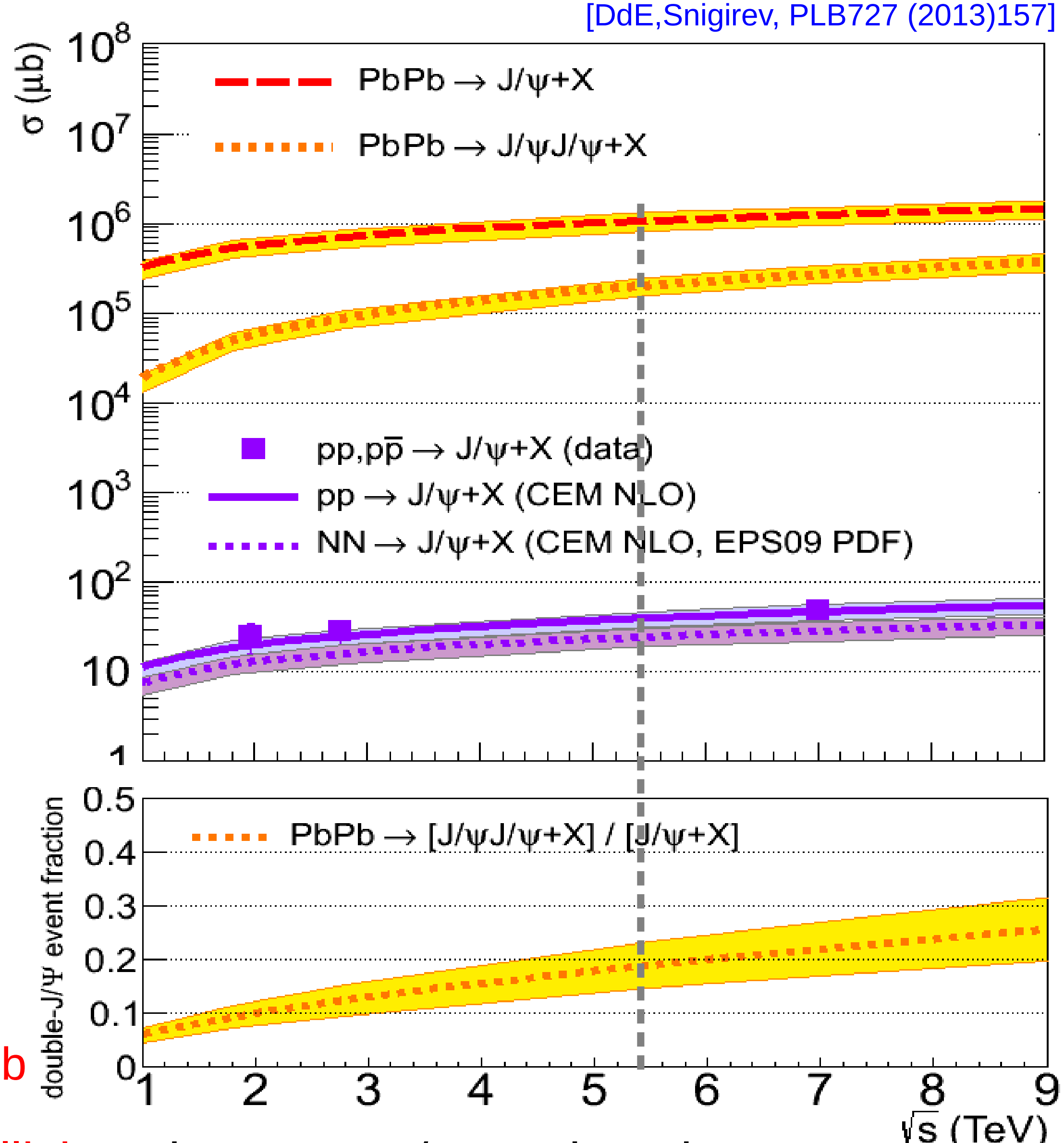
- NLO accuracy.
- Scales: $\mu_R = \mu_R = 1.5 \cdot m_c$
- Good agreement with Tevatron&LHC data

- EPS09 Pb nPDF

  20–35% shadowing x-section reduction

- At 5.5 TeV:

$\sigma^{DPS}(\text{Pb–Pb} \rightarrow J/\psi\, J/\psi\, X)$ =200±50 mb



20% of min.bias Pb-Pb collisions have two J/ψ produced !

Example: Pb-Pb → J/ψ J/ψ at 5.5 TeV

[DdE, Snigirev, PLB727 (2013)157]

- Visible rates:
  - ► Fiducial x-section per unit-y: $d\sigma_{J/\psi}/dy \approx \sigma_{J/\psi}/8$
  - ► BR(J/ψ→ l^+l^-) ≈ 6%
  - ► Typical ALICE/CMS acceptance & efficiencies: ε ≈ 1/12

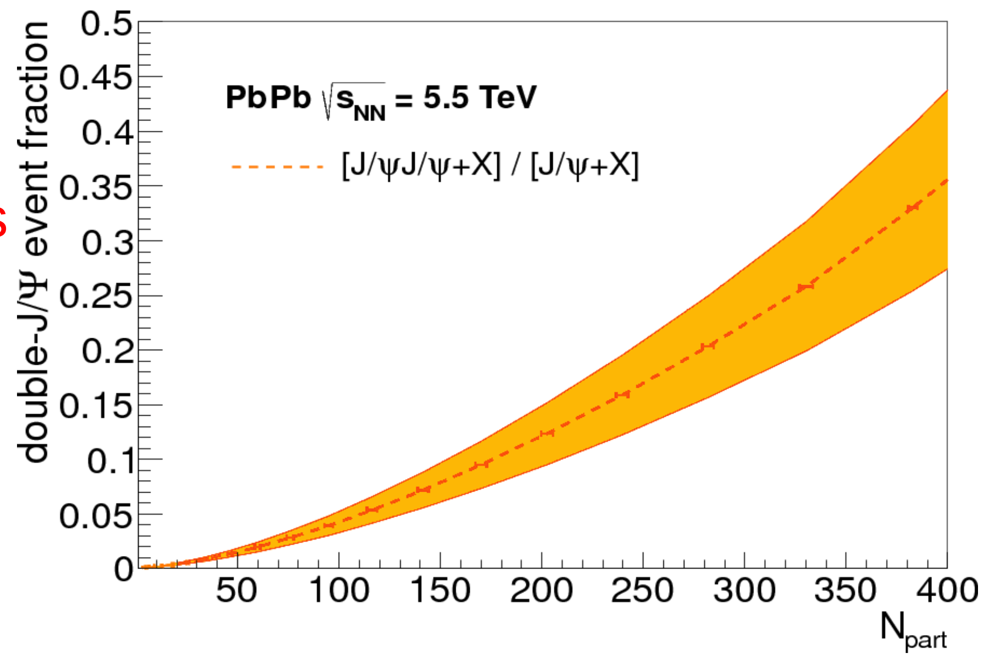
- Expected dimuon rates including yield all loses & 1 nb^{-1} integ. luminosity:

$$\mathcal{N} = \sigma^{\mathrm{DPS}}_{\text{Pb-Pb} \to J/\psi\, J/\psi'} / (\varepsilon \cdot \mathcal{L}_{\mathrm{int}}) \approx$$ 250 double-J/ψ per year (per unit-|y|)

(x2 less including final-state suppression)

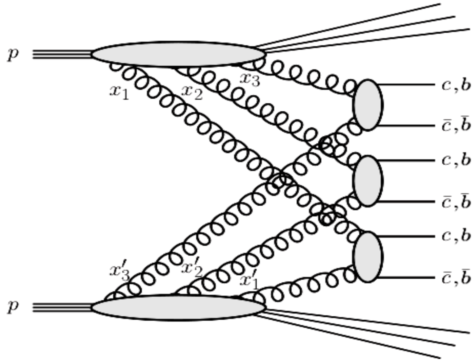
- Centrality dependence of double-J/ψ fraction:
  35% of central Pb-Pb collisions
  have two J/ψ produced !

Seeing 2 J/ψ on event-by-event basis not to be blindly taken as signal of c-cbar recombination.



Triple Parton Scatterings

Triple parton scattering x-sections (p-p)



- Assuming that the probabilities for 3 hard collisions to be independent of each other, one can again write a pocket-formula for TPS x-section:

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdot \sigma^{\rm SPS}_{hh'\to a_2} \cdot \sigma^{\rm SPS}_{hh'\to a_3}}{\sigma^2_{\rm eff,TPS}}$$

normalized by the square of an eff. x-section ($\sigma^2_{\rm eff,TPS}$) plus a trivial combinatorial factor (m/3!) to avoid triple-counting in case of same particles produced: $m = 1$ if $a_1 = a_2 = a_3$;
$m = 3$ if $a_1 = a_2$, or $a_1 = a_3$, or $a_2 = a_3$; and
$m = 6$ if $a_1 \neq a_2 \neq a_3$.

- How to interpret $\sigma_{\rm eff,TPS}$? Relationship with $\sigma_{\rm eff}$? What values to expect?

- Most generic expression for TPS cross section:

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \sum_{i,j,k,l,m,n} \int \Gamma^{ijk}_h(x_1, x_2, x_3; \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}; Q_1^2, Q_2^2, Q_3^2)$$
$$\times \hat\sigma^{il}_{a_1}(x_1, x_1', Q_1^2) \cdot \hat\sigma^{jm}_{a_2}(x_2, x_2', Q_2^2) \cdot \hat\sigma^{kn}_{a_3}(x_3, x_3', Q_3^2)$$
$$\times \Gamma^{lmn}_{h'}(x_1', x_2', x_3'; \mathbf{b_1} - \mathbf{b}, \mathbf{b_2} - \mathbf{b}, \mathbf{b_3} - \mathbf{b}; Q_1^2, Q_2^2, Q_3^2)$$
$$\times dx_1 dx_2 dx_3 dx_1' dx_2' dx_3' d^2b_1 d^2b_2 d^2b_3 d^2b.$$

Generalized PDFs = f(x,Q^2,**b**)

Triple parton scattering x-sections (p-p)

- Assumption 1: Factorize generalized Triple-PDF into longitudinal & transverse components:

$$\Gamma_h^{ijk}(x_1,x_2,x_3;\mathbf{b_1},\mathbf{b_2},\mathbf{b_3};Q_1^2,Q_2^2,Q_3^2)$$
$$= D_h^{ijk}(x_1,x_2,x_3;Q_1^2,Q_2^2,Q_3^2)f(\mathbf{b_1})f(\mathbf{b_2})f(\mathbf{b_3}),$$

p-p transv. overlap function (mb^{-1}): $T(\mathbf{b}) = \int f(\mathbf{b_1})f(\mathbf{b_1}-\mathbf{b})d^2b_1$, with $\int d^2b T(\mathbf{b}) = 1.$

- Assumption 2: Longitudinal triple-PDF is the product of 3 single PDFs (i.e. no parton correlations in colour, momentum, flavour, spin,…)

$$D_h^{ijk}(x_1,x_2,x_3;Q_1^2,Q_2^2,Q_3^2) \;=\; D_h^i(x_1;Q_1^2)D_h^j(x_2;Q_2^2)D_h^k(x_3;Q_3^2)$$

- Then, $\sigma^2_{\rm eff,TPS}$ is simply the inverse of the cube of the transv. pp overlap:

$$\sigma^2_{\rm eff,TPS} = \left[\int d^2b\, T^3(\mathbf{b})\right]^{-1}$$

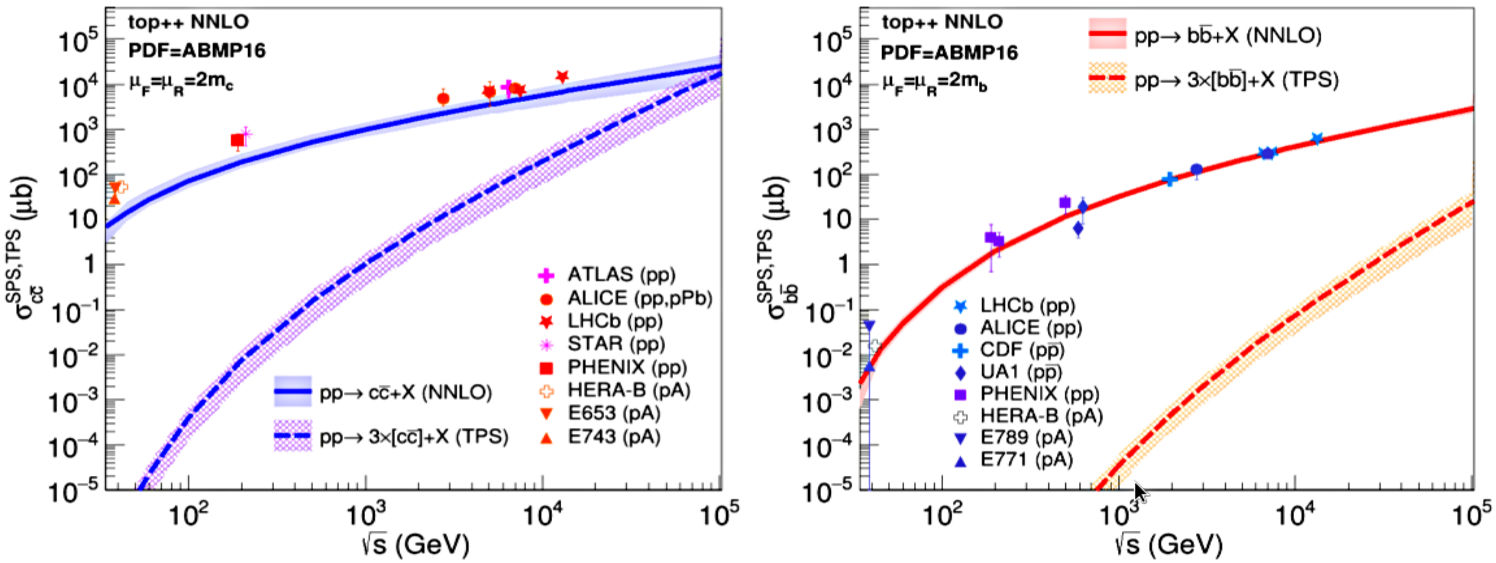
- By testing many proton overlaps/profiles (hard sphere, Gaussian, expo, dipole fit), we find a close relationship between $\sigma_{\rm eff,TPS}$ & $\sigma_{\rm eff}$:

$$\sigma_{\rm eff,TPS} = k \times \sigma_{\rm eff,DPS},\ \text{with}\ k = 0.82 \pm 0.11$$

- Measuring TPS provides independent info on $\sigma_{\rm eff}$ and p transv. profile.

Triple charm & beauty production (p-p)

- TPS x-sections are small: $\sigma(\mathrm{SPS})^3/\sigma(\mathrm{eff})^2 \approx 1$ fb for $\sigma(\mathrm{SPS}) \approx 1\ \mu$b, but rise fast (cube of SPS) with c.m. energy.
- Charm & beauty have large enough $\sigma(\mathrm{SPS})$ to attempt TPS observation:

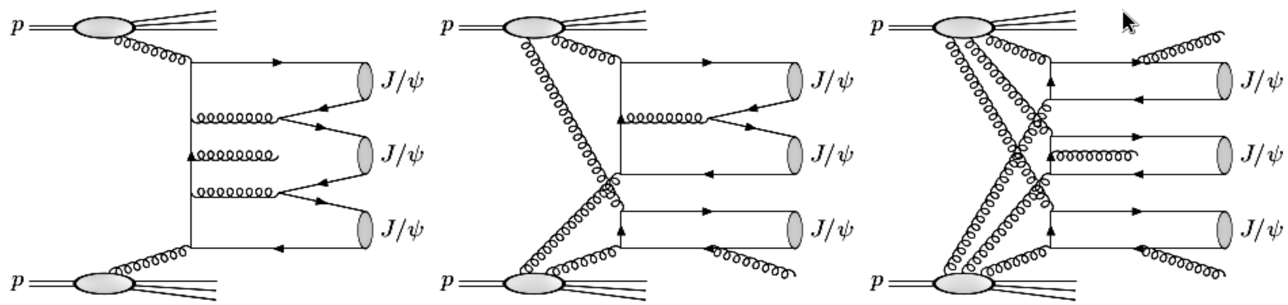


Final state	$\sqrt{s} = 14$ TeV	$\sqrt{s} = 100$ TeV
$\sigma^{\mathrm{SPS}}_{c\bar{c}+X}$	$7.1 \pm 3.5_{\mathrm{sc}} \pm 0.3_{\mathrm{PDF}}$ mb	$25.0 \pm 16.0_{\mathrm{sc}} \pm 1.3_{\mathrm{PDF}}$ mb
$\sigma^{\mathrm{TPS}}_{c\bar{c}\,c\bar{c}\,c\bar{c}+X}$	$0.39 \pm 0.28_{\mathrm{tot}}$ mb	$16.7 \pm 11.8_{\mathrm{tot}}$ mb
$\sigma^{\mathrm{SPS}}_{b\bar{b}+X}$	$0.56 \pm 0.09_{\mathrm{sc}} \pm 0.01_{\mathrm{PDF}}$ mb	$2.8 \pm 0.6_{\mathrm{sc}} \pm 0.1_{\mathrm{PDF}}$ mb
$\sigma^{\mathrm{TPS}}_{b\bar{b}\,b\bar{b}\,b\bar{b}+X}$	$0.19 \pm 0.12_{\mathrm{tot}}\ \mu$b	$24 \pm 17_{\mathrm{tot}}\ \mu$b

- Triple charm amounts to ~15% (50%) of inclusive charm x-sections at LHC (FCC). Contribution from triple-SPS, double-SPS processes?

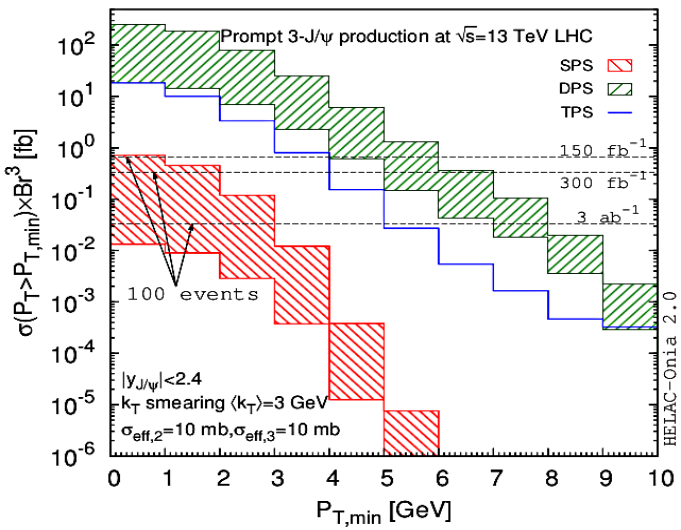
Triple-J/ψ from SPS production (p-p)

- H.-S. Shao et al. [arXiv:1902.04949, PRL 122(2019)192002] computed all triple-J/Ψ x-sections with SPS HELAC-ONIA plus TPS pocket formula:



(a) SPS (b) DPS (c) TPS

		inclusive	$2.0 < y_{J/\psi} < 4.5$	$\lvert y_{J/\psi}\rvert < 2.4$
13 TeV	SPS	$0.41^{+2.4}_{-0.34} \pm 0.0083$	$(1.8^{+11}_{-1.5} \pm 0.18) \times 10^{-2}$	$(8.7^{+56}_{-7.5} \pm 0.098) \times 10^{-2}$
	DPS	$(190^{+501}_{-140}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(7.0^{+18}_{-5.1}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(50^{+140}_{-37}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$
	TPS	$130 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$1.3 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$18 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$
27 TeV	SPS	$0.46^{+2.9}_{-0.39} \pm 0.022$	$(3.2^{+22}_{-2.8} \pm 0.21) \times 10^{-2}$	$(5.8^{+39}_{-5.1} \pm 0.29) \times 10^{-2}$
	DPS	$(560^{+2900}_{-480}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(19^{+97}_{-16}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(120^{+630}_{-100}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$
	TPS	$570 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$5.0 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$57 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$
75 TeV	SPS	$0.59^{+4.4}_{-0.52} \pm 0.016$	$(3.0^{+25}_{-2.7} \pm 0.23) \times 10^{-2}$	$(7.2^{+63}_{-6.5} \pm 0.38) \times 10^{-2}$
	DPS	$(1900^{+11000}_{-1600}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(57^{+340}_{-50}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(310^{+2000}_{-270}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$
	TPS	$3900 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$27 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$260 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$
100 TeV	SPS	$1.1^{+8.4}_{-1.0} \pm 0.044$	$(4.5^{+33}_{-4.0} \pm 0.72) \times 10^{-2}$	$(36^{+290}_{-32} \pm 1.8) \times 10^{-2}$
	DPS	$(3400^{+19000}_{-2900}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(100^{+550}_{-86}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$	$(490^{+3000}_{-430}) \times \frac{10\text{ mb}}{\sigma_{\rm eff,2}}$
	TPS	$6500 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$45 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$	$380 \times \left(\frac{10\text{ mb}}{\sigma_{\rm eff,3}}\right)^2$



- SPS negligible, DPS (TPS) dominates at low (high) p_T.

Clear sensitivity to $\sigma_{\rm eff}$!

TPS in p-p collisions (13 TeV, CMS)

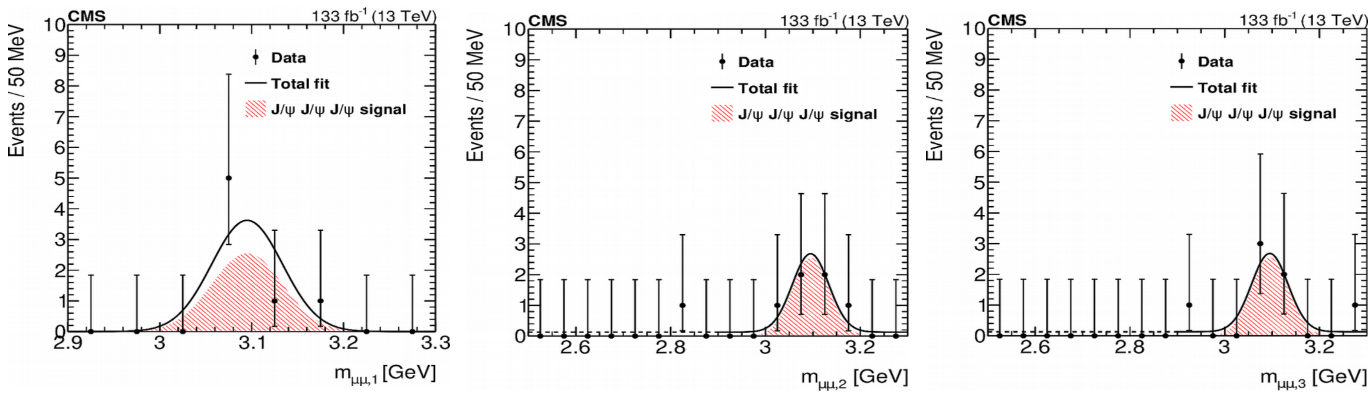
- Triple parton scatterings x-sections in p-p: alternative extraction of $\sigma_{\rm eff,DPS}$

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdot \sigma^{\rm SPS}_{hh'\to a_2} \cdot \sigma^{\rm SPS}_{hh'\to a_3}}{\sigma^2_{\rm eff,TPS}}$$

$$\sigma_{\rm eff,TPS} = (0.82 \pm 0.11)\, \sigma_{\rm eff,DPS}$$

[DdE, Snigirev, PRL 118(2017)122001]

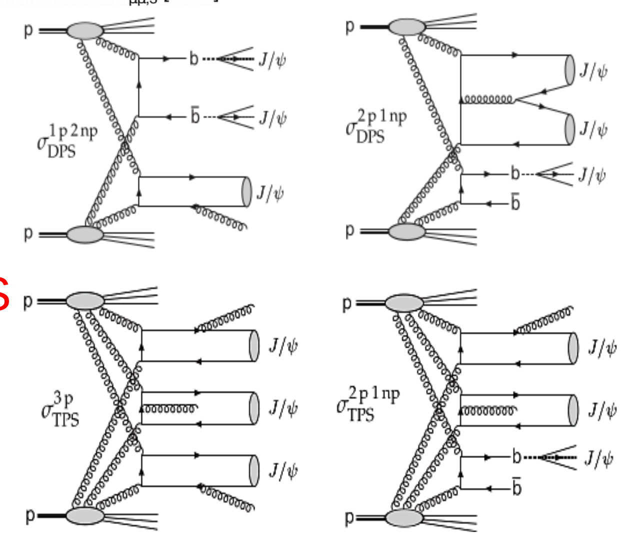
- First observation of triple-J/ψ production (CMS):



[arXiv:2111.05370
Nat. Phys. to appear]

- Measurement of fiducial cross section
$\sigma(pp \to 3J/\psi) = 272^{+141}_{-104}$ (stat) $\pm$ 17 (syst) fb

- Pocket formula with (N)NLO for single-,double-, triple-J/ψ SPS x-sections:

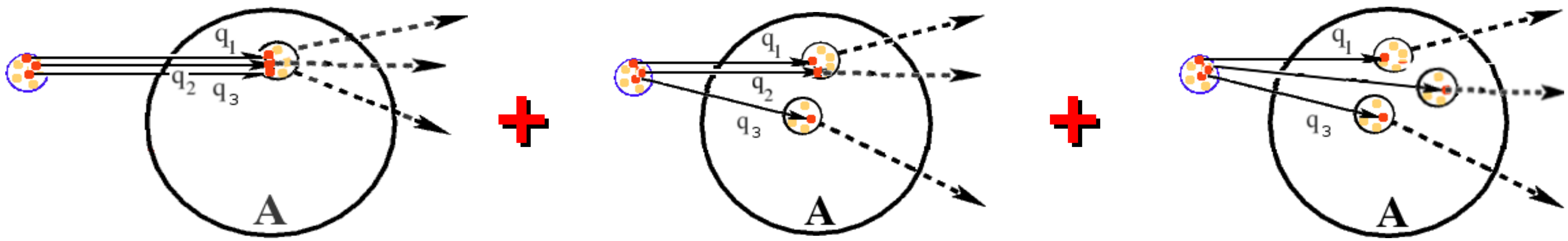


- Triple-J/ψ fractions: ~6% SPS, ~74% DPS, ~20% TPS
- $\sigma_{\rm eff,DPS} = 2.7\ ^{+1.4}_{-1.0}$ (exp) $^{+1.5}_{-1.0}$ (theo) mb consistent with for di-quarkonia (lower than jet/γ/W/Z DPS results):
  - ▶ q/g x-dependent transverse profile & correlations

Triple Parton Scattering x-sections in p-A

[DdE, Snigirev, EPJC 78 (2018)359]

- Three contributions to TPS x-section in p-A:



$$\sigma^{\rm TPS,1}_{\rm pA\to abc} = A\cdot\sigma^{\rm TPS}_{\rm pN\to abc} \qquad \sigma^{\rm TPS,2}_{\rm pA\to abc} = \sigma^{\rm TPS}_{\rm pN\to abc}\cdot 3\,\frac{\sigma^2_{\rm eff,TPS}}{\sigma_{\rm eff,DPS}}\,F_{\rm pA},$$

$$\sigma^{\rm TPS,3}_{\rm pA\to abc} = \sigma^{\rm TPS}_{\rm pN\to abc}\cdot\sigma^2_{\rm eff,TPS}\cdot C_{\rm pA},\quad \text{with}\quad C_{\rm pA} = \frac{(A-1)(A-2)}{A^2}\int d^2b\, T^3_{\rm pA}(\mathbf{b}),$$

Relative weight of TPS terms: $\sigma^{\rm TPS,1}_{\rm pA\to abc} : \sigma^{\rm TPS,2}_{\rm pA\to abc} : \sigma^{\rm TPS,3}_{\rm pA\to abc} = 1 : 4.54 : 3.56$

(TPS yields in pPb: 10% "genuine", 50% involve 2 nucleons, 40% involve 3 different Pb nucleons)

- “Pocket” formula for TPS p-A x-section:

$$\sigma^{\rm TPS}_{\rm pA\to abc} = \left(\frac{m}{6}\right)\frac{\sigma^{\rm SPS}_{\rm pN\to a}\cdot\sigma^{\rm SPS}_{\rm pN\to b}\cdot\sigma^{\rm SPS}_{\rm pN\to c}}{\sigma^2_{\rm eff,TPS,pA}}$$

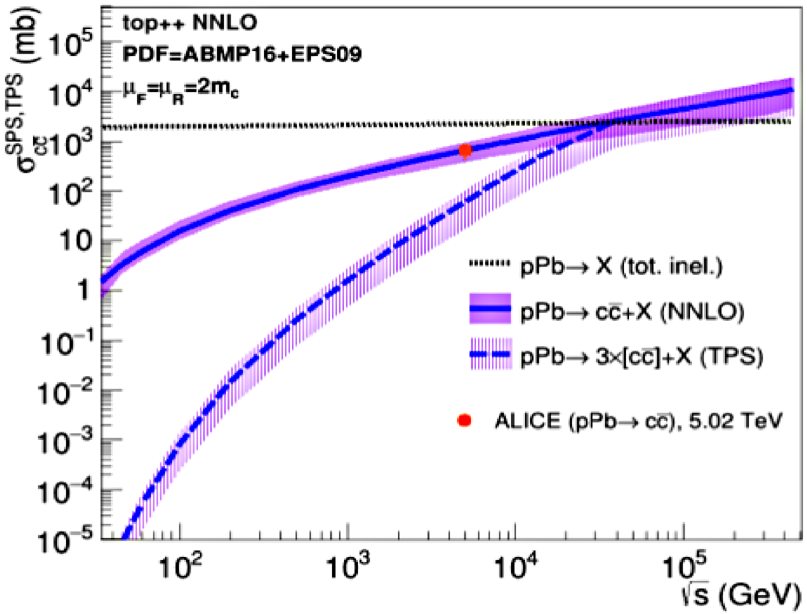
$$\sigma_{\rm eff,TPS,pA} = \left[\frac{A}{\sigma^2_{\rm eff,TPS}} + \frac{3\,F_{\rm pA}[{\rm mb}^{-1}]}{\sigma_{\rm eff,DPS}} + C_{\rm pA}[{\rm mb}^{-2}]\right]^{-1/2}$$

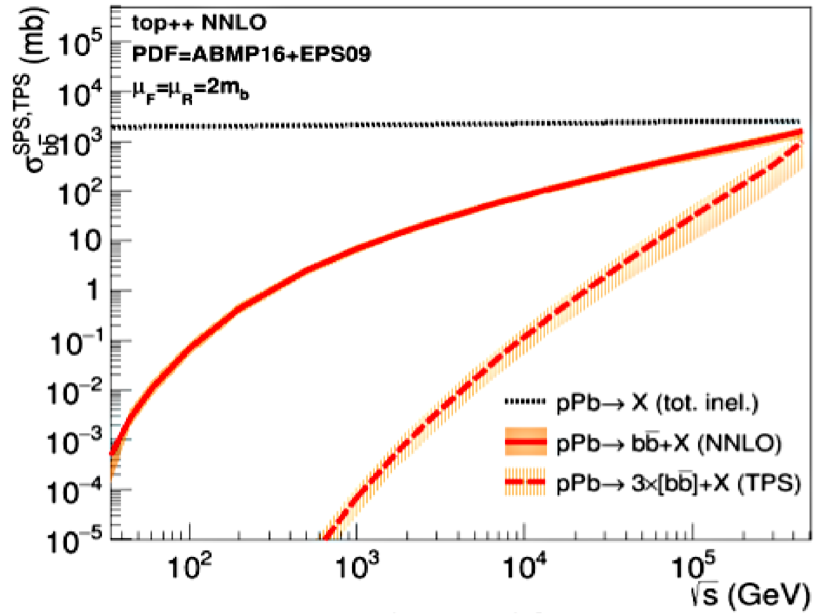
► $\sigma_{\rm eff,TPS,pPb} = 0.29 \pm 0.04$ mb ($\times$45 times the p-p case with $\sigma_{\rm eff,TPS}$= 12.5 mb)

- TPS x-sections are large in p-A: a factor $\times$45 for p-Pb compared to p-p
- Pb transv. density ($F_{\rm pA}$, $C_{\rm pA}$) well-known: Alternative extraction of $\sigma_{\rm eff,pp}$

Example: Triple charm & beauty in p-Pb colls.

- Charm & beauty have very large TPS x-sections at the LHC & above:





Process	pPb(8.8 TeV)	pPb(63 TeV)	p-Air(430 TeV)
$\sigma^{\rm inel}_{\rm pA}$	2.2 ± 0.4 b	2.4 ± 0.4 mb	0.61 ± 0.10 b
$\sigma^{\rm SPS}_{c\bar{c}+X}$	$0.96 \pm 0.45_{\rm sc} \pm 0.10_{\rm PDF}$ b	$3.4 \pm 1.9_{\rm sc} \pm 0.4_{\rm PDF}$ b	$0.75 \pm 0.5_{\rm sc} \pm 0.1_{\rm PDF}$ b
$\sigma^{\rm TPS}_{c\bar{c}\,c\bar{c}\,c\bar{c}+X}$	$200 \pm 140_{\rm tot}$ mb	$8.7^{*} \pm 6.2_{\rm tot}$ b	$5.0^{*} \pm 3.6_{\rm tot}$ b
$\sigma^{\rm SPS}_{b\bar{b}+X}$	$72 \pm 12_{\rm sc} \pm 5_{\rm PDF}$ mb	$370 \pm 75_{\rm sc} \pm 30_{\rm PDF}$ mb	$110 \pm 25_{\rm sc} \pm 5_{\rm PDF}$ mb
$\sigma^{\rm TPS}_{b\bar{b}\,b\bar{b}\,b\bar{b}+X}$	$0.084 \pm 0.045_{\rm tot}$ μb	$11 \pm 7_{\rm tot}$ μb	$17 \pm 11_{\rm tot}$ μb

- Triple charm amounts to ~20% (~100%!) of inclusive charm x-sections at LHC (FCC). Large triple J/Ψ production at FCC: $\sigma(J/\psi J/\psi J/\psi + X) \approx 1$ mb
- Triple beauty amounts to ~3% of inclusive beauty x-sections at FCC.

Summary: DPS studies

- What's the parton transverse density of a proton? Its energy evolution? How do partons correlate (kinemat., quantum numbers) transversely?
- Double hard parton scatterings in p-p collisions:

$$\sigma^{DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{SPS}_{(hh'\to a)} \cdot \sigma^{SPS}_{(hh'\to b)}}{\sigma_{\rm eff}}$$

In absence of parton correlations:

$$\sigma_{\rm eff} = \left[\int d^2b\, t^2(\mathbf{b}) \right]^{-1}$$

geom. overlap area of 2 proton transv. profiles

- σ_{eff}(exp) ≈ 2–20 mb at Tevatron/LHC. Can HI colls. help to clarify this?
- Available DPS x-sections "pocket formula" for p-A and A-A: ($\sigma_{eff,pp}$=13±2mb)

$$\sigma_{\rm eff,pA} = \frac{\sigma_{\rm eff,pp}}{A + \sigma_{\rm eff,pp}\, F_{\rm pA}} = 21.5 \pm 1.1\ \mu b$$

$$\sigma_{\rm eff,AA} = \frac{1}{A^2[\sigma^{-1}_{\rm eff,pp} + \frac{2}{A}T_{AA}(0) + \frac{1}{2}T_{AA}(0)]} = 1.5\ \rm nb$$

Huge enhancements! $\sigma_{\rm eff,DPS}/\sigma_{\rm eff,DPS,pA} \approx 600$, $\sigma_{\rm eff,pp}/\sigma_{\rm eff,AA} \propto A^{3.3}/5 \simeq 9 \cdot 10^6$

- p-Pb: Large DPS yields in p-A (in particular with quarkonia) provide many useful independent extractions of $\sigma_{eff,pp}$. 1st-ever measurement by LHCb.
- Pb-Pb: Large DPS but dominated by scatts. from different nucleons. (~16% sensitivity on $\sigma_{eff,pp}$ from DPS with lighter ions such as Ar-Ar).

Summary: TPS studies

- What's the parton transverse density of a proton? Its energy evolution? How do partons correlate (kinemat., quantum numbers) transversely?
- Derived a generic expression for NPS x-sections in p-p collisions:

$$\sigma^{\rm NPS}_{hh'\to a_1\ldots a_n} = \left(\frac{m}{n!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdots \sigma^{\rm SPS}_{hh'\to a_n}}{\sigma^{n-1}_{\rm eff,NPS}}$$

$$\sigma_{\rm eff,NPS} = \left\{ \int d^2b\, T^n(\mathbf{b}) \right\}^{-1/(n-1)}$$

- And used it to derive pocket formula for triple parton scatterings in p-p...

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdot \sigma^{\rm SPS}_{hh'\to a_2} \cdot \sigma^{\rm SPS}_{hh'\to a_3}}{\sigma^2_{\rm eff,TPS}}$$

$$\sigma^2_{\rm eff,TPS} = \left[\int d^2b\, T^3(\mathbf{b}) \right]^{-1}$$

Summary: TPS studies

- What's the parton transverse density of a proton? Its energy evolution? How do partons correlate (kinemat., quantum numbers) transversely?

- Triple hard parton scatterings in p-p collisions:

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdot \sigma^{\rm SPS}_{hh'\to a_2} \cdot \sigma^{\rm SPS}_{hh'\to a_3}}{\sigma^2_{\rm eff,TPS}}$$

(closely related to DPS in the absence of parton correlations):

$$\sigma_{\rm eff,TPS} = (0.82 \pm 0.11)\ \sigma_{\rm eff,DPS}$$

- Triple charm amounts to ~15% of inclusive charm x-sections in p-p collisions at the LHC. Triple-J/Ψ fully dominated by DPS/TPS: "golden channel" to extract $\sigma_{\rm eff,pp}$: 1st-ever observation by CMS.

- Derived TPS x-sections "pocket formula" for p-A:

$$\sigma^{\rm TPS}_{\rm pA\to abc} = \left(\frac{m}{6}\right) \frac{\sigma^{\rm SPS}_{\rm pN\to a} \cdot \sigma^{\rm SPS}_{\rm pN\to b} \cdot \sigma^{\rm SPS}_{\rm pN\to c}}{\sigma^2_{\rm eff,TPS,pA}}$$

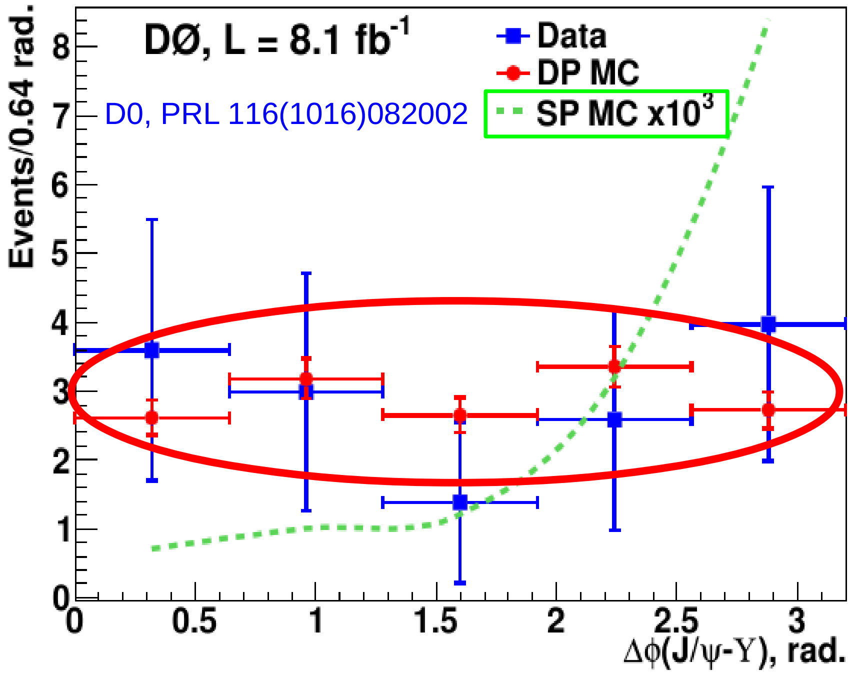
$$\sigma_{\rm eff,TPS,pA} = \left[\frac{A}{\sigma^2_{\rm eff,TPS}} + \frac{3\,F_{\rm pA}[\rm mb^{-1}]}{\sigma_{\rm eff,DPS}} + C_{\rm pA}[\rm mb^{-2}]\right]^{-1/2}$$

- Large TPS yields in p-Pb, e.g. $\sigma_{\rm TPS}$(triple-ccbar)=200 mb (~20% of incl. ccbar x-section): provide useful independent extractions of $\sigma_{\rm eff,pp}$. [Don't be shy to attempt a 1st-ever measurement in p-Pb...].

Backup slides

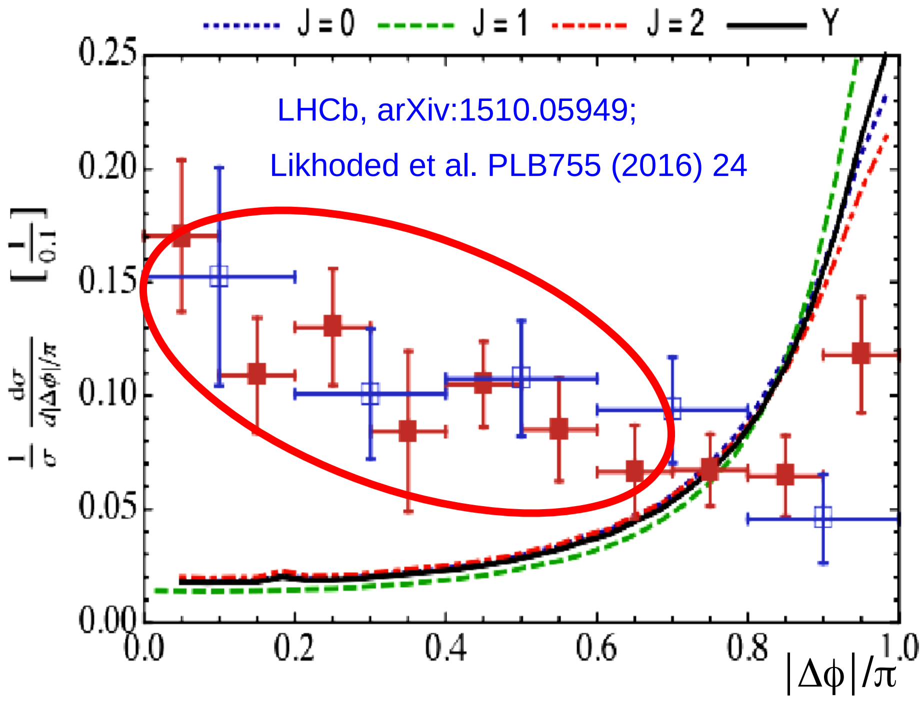
DPS studies with $Q\bar{Q}$: p-p → J/Ψ+Υ, Υ+D

- Uncorrelated J/Ψ+Υ azimuthal production in ppbar at 1.96 TeV:



$\sigma_{\rm eff} = 2.2 \pm 0.7\,({\rm stat}) \pm 0.9\,({\rm syst})$ mb.

- Uncorrelated Υ+D azimuthal production in pp at 7 TeV:

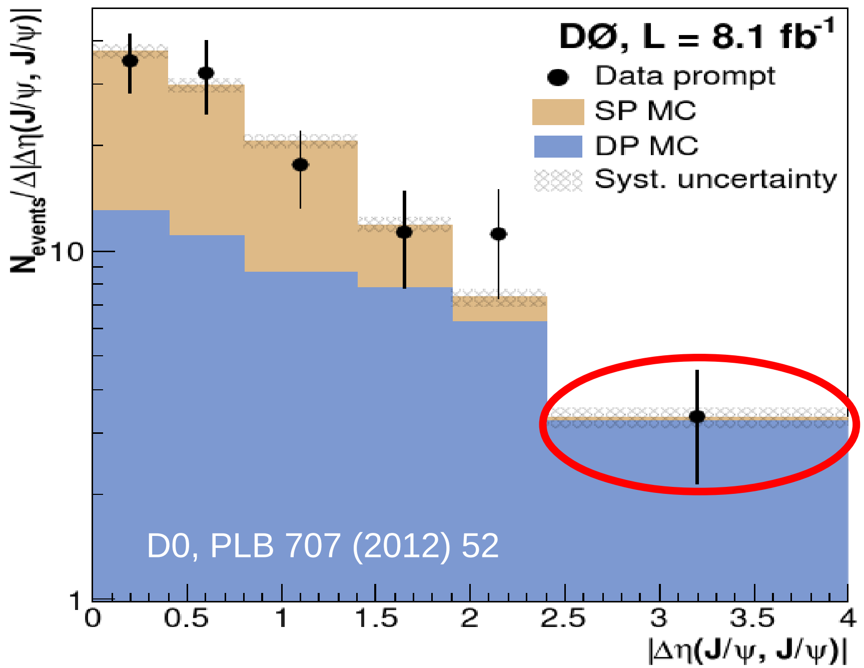


$\sigma_{\rm eff}|_{\Upsilon(1S)D} = 18.0 \pm 1.3\,({\rm stat}) \pm 1.2\,({\rm syst})$ mb

- Extracted σ_{eff} values differ by up to a factor of 8 for similar (g-induced) processes at 1.96 TeV & 7 TeV:
  – Energy-dependent parton transverse profile?
  – (Higher-order) SPS contributions under control?

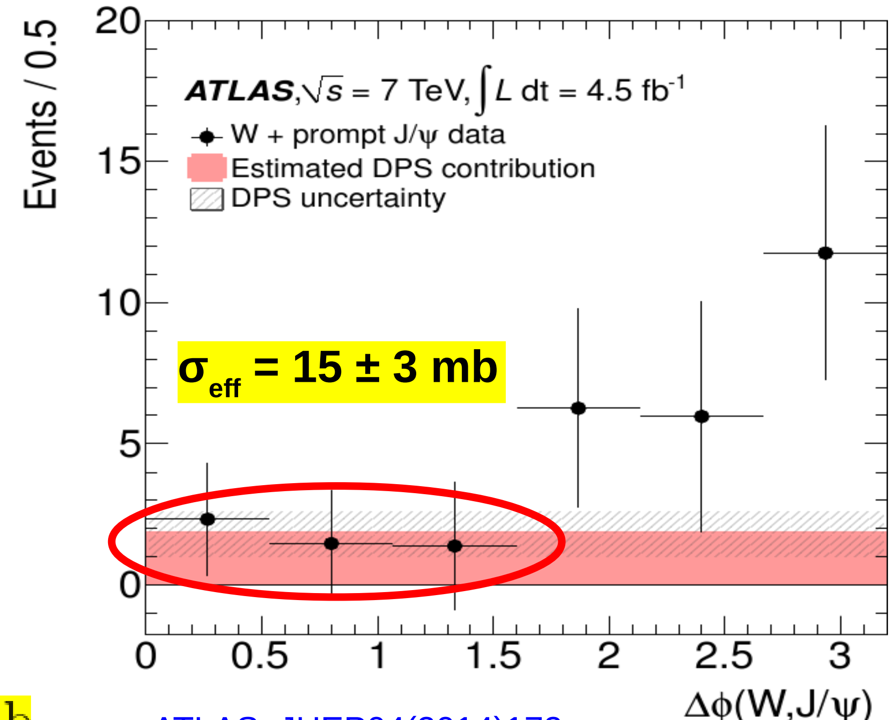
DPS studies with $Q\bar{Q}$: p-p → W^++J/Ψ, J/ΨJ/Ψ

- Uncorrelated J/Ψ+J/Ψ rapidity production in ppbar at 1.96 TeV:



$$\sigma_{\rm eff} = 4.8 \pm 0.5({\rm stat}) \pm 2.5({\rm syst})\ {\rm mb}$$

- Uncorrelated W+J/Ψ azimuthal production in pp at 7 TeV:

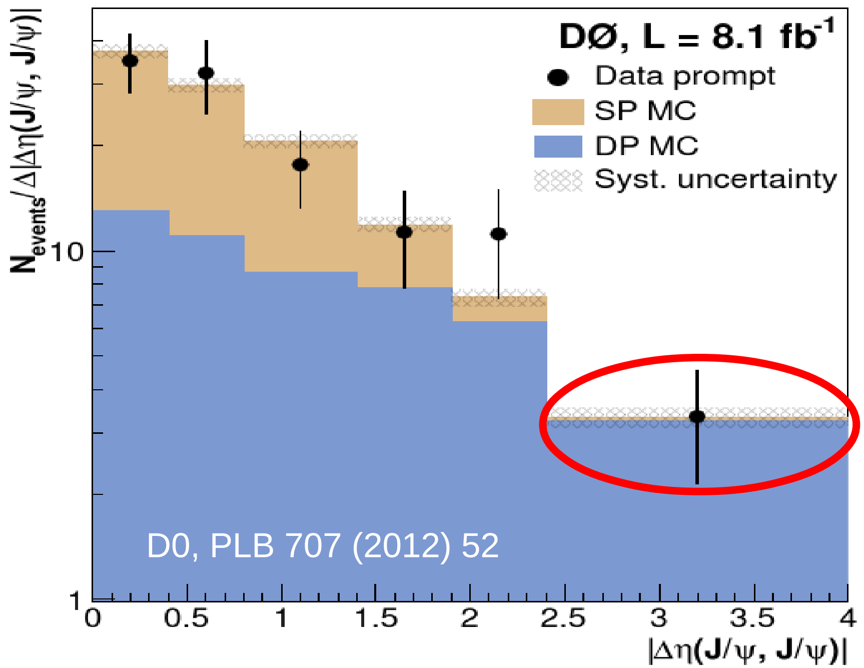


ATLAS, JHEP04(2014)172

- Extracted σ_{eff} values differ at 1.96 TeV & 7 TeV:
  – (Higher-order) SPS contributions under control?
  – Energy-dependent parton transverse profile? (Quark vs. gluon?)

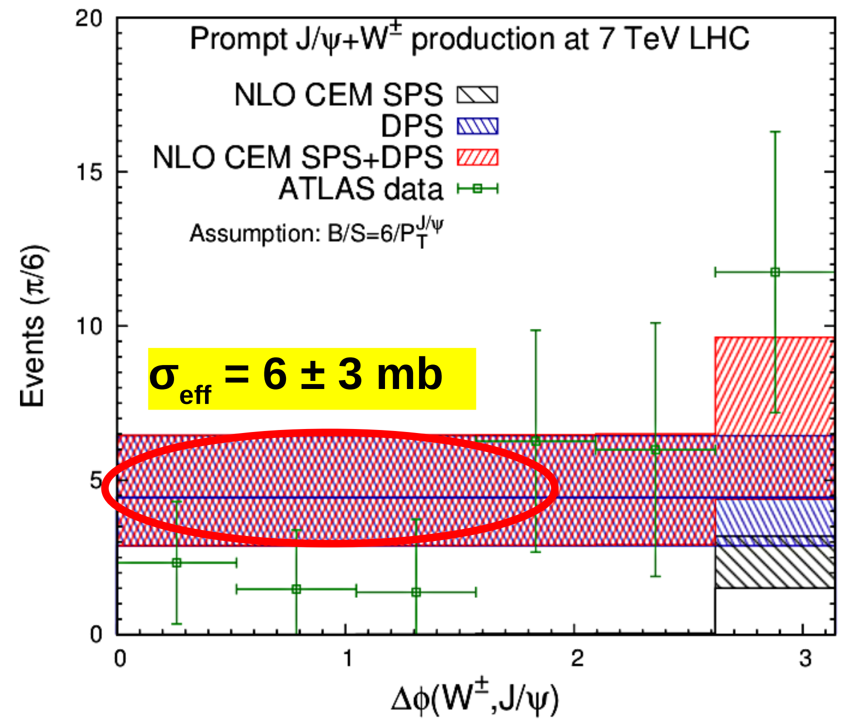
DPS studies with $Q\bar{Q}$: p-p → W^++J/Ψ, J/ΨJ/Ψ

- Uncorrelated J/Ψ+J/Ψ rapidity production in ppbar at 1.96 TeV:



$$\sigma_{\rm eff} = 4.8 \pm 0.5({\rm stat}) \pm 2.5({\rm syst})\ {\rm mb}$$

- Uncorrelated W+J/Ψ azimuthal production in pp at 7 TeV:



(a) NLO

Lansberg&Shao&Yamanaka, PLB781 (2018) 485

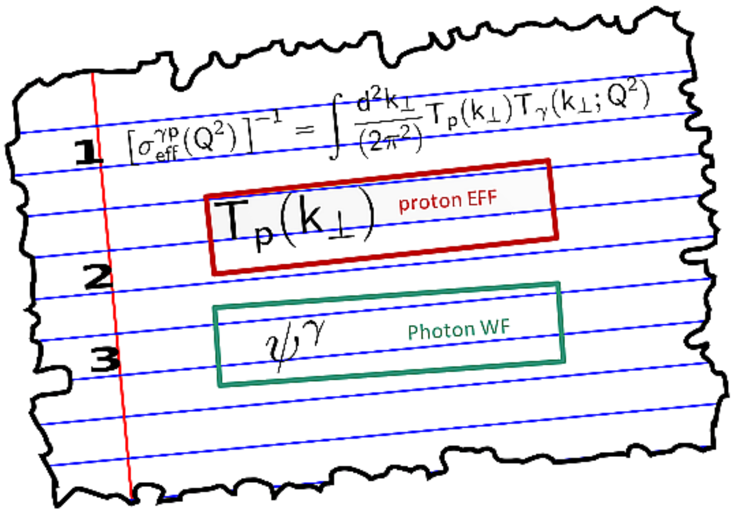
- Extracted σ_{eff} values differ at 1.96 TeV & 7 TeV:
  - (Higher-order) SPS contributions under control?
  - Energy-dependent parton transverse profile? (Quark vs. gluon?)

DPS in Ultraperipheral p-Pb collisions?

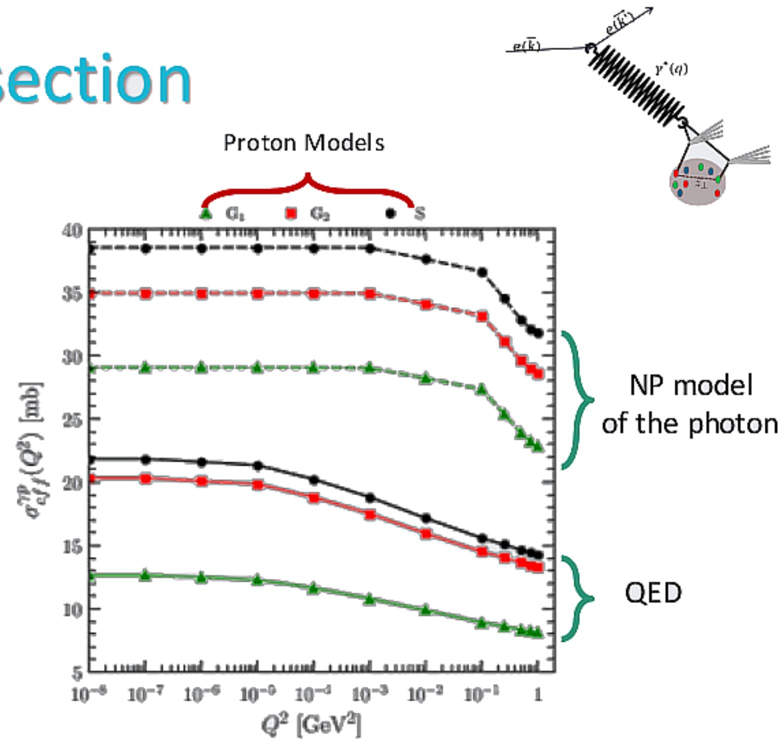
[M.Rinaldi, et al.]

- Rinaldi&Ceccopieri (also Blok & Strikman) have proposed to study DPS from photon-proton collisions (where photon = vector meson):

6 The γ-p effective cross section



M. R. and F. A. Ceccopieri, arXiv:2103.13480



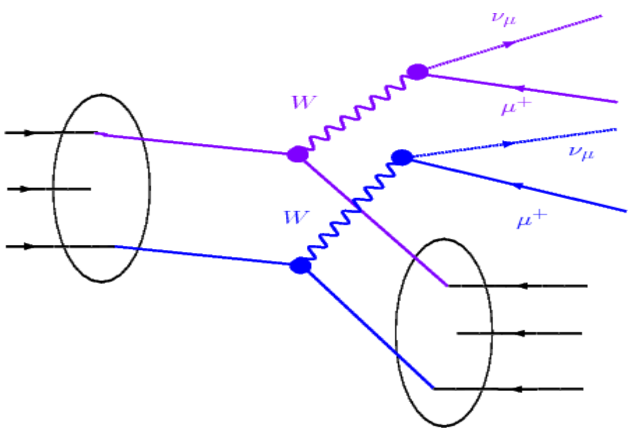
- Such studies (based on HERA data so far) could be tested with UPCs in p-Pb with the photon emitted from the Pb ion (we should go beyond searching for 'ridges' in UPCs, and extract some quantitative x-sections...)

DPS “golden channel”: Same-sign WW

[Kulesza, Stirling, Gaunt, Treleani, Del Fabbro, ...]

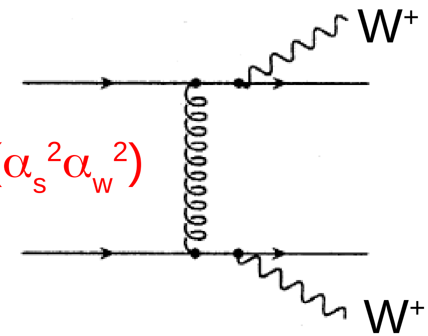
- Same-sign W-W production from 2 independent hard scatterings is a “golden” DPS signature:

  - Well controlled pQCD x-sections.
  - Clean experimental final-state: 2 like-sign leptons + missing-E_T

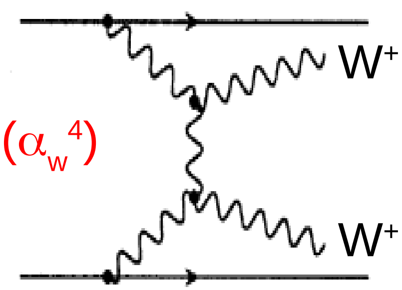


- Backgrounds: Same-sign W-W production in single parton scatterings (SPS) is higher-order and occurs only with 2 extra jets:

QCD ($\alpha_s^2\alpha_w^2$)



EWK (α_w^4)

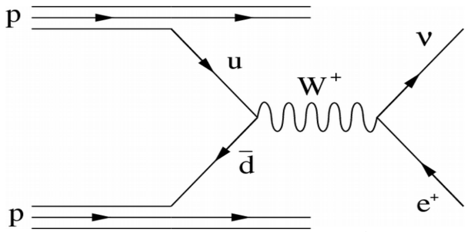


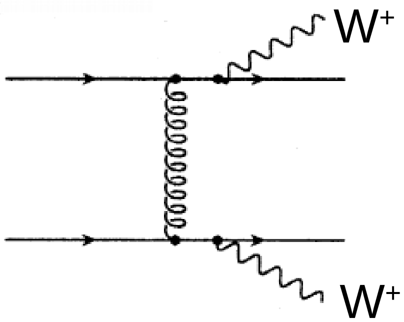
- σ(WW,DPS)~1/3·σ(WWjj,SPS), but SPS background reducible by more than x20 applying jet cuts.

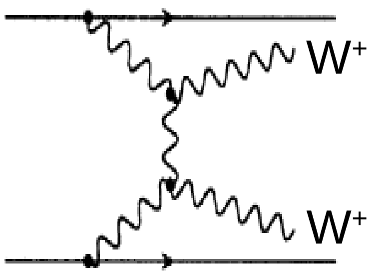
Case study: p-Pb → W^+W^+, W^-W^- at 8.8 TeV

[DdE,Snigirev, PLB718 (2013)1395]

- Theoretical setup:
  - MCFM 6.2: Single-parton W^+,W^-
    W^+W^+jj (QCD) background
    - NLO accuracy.
    - Scales: $\mu(W) = m_W$, $\mu(WW)$ = 150 GeV
    - CT10 proton PDF, EPS09 Pb nuclear PDF
    - Uncertainties: ~10%
  - VBFNLO 2.6.0: W^+W^+jj (EWK) background
    - NLO accuracy
    - Scales: $\mu^2 = t_{W,Z}$
    - CT10 PDF
    - Uncertainties: <10%







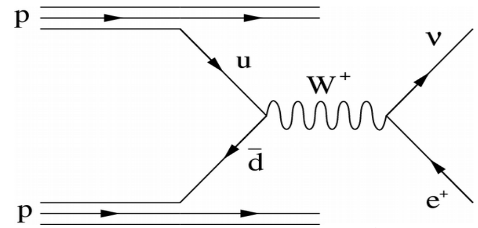
- Cross sections in pb (signal & background):

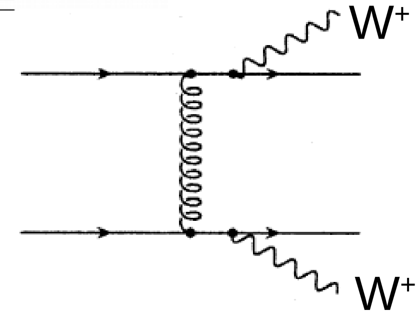
p-Pb final-state: Code (process #): Order (σ units):	W^+ MCFM (1) NLO (μb)	W^- MCFM (6) NLO (μb)	W^+W^- MCFM (61) NLO (nb)	W^+W^+jj (QCD) MCFM (251) 'NLO' (pb)	W^+W^+jj (VBF) VBFNLO (250) NLO (pb)	$W^\pm W^\pm$ (DPS) Eq. (15) (pb)
$\sqrt{s_{NN}}$ = 5.0 TeV	6.85 ± 0.68	5.88 ± 0.59	5.48 ± 0.56	12.1 ± 1.2	12.4 ± 0.6	44. ± 8.
$\sqrt{s_{NN}}$ = 8.8 TeV	12.6 ± 1.3	11.1 ± 1.1	13.0 ± 1.3	40.4 ± 4.0	51.8 ± 2.0	152. ± 27.

Case study: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

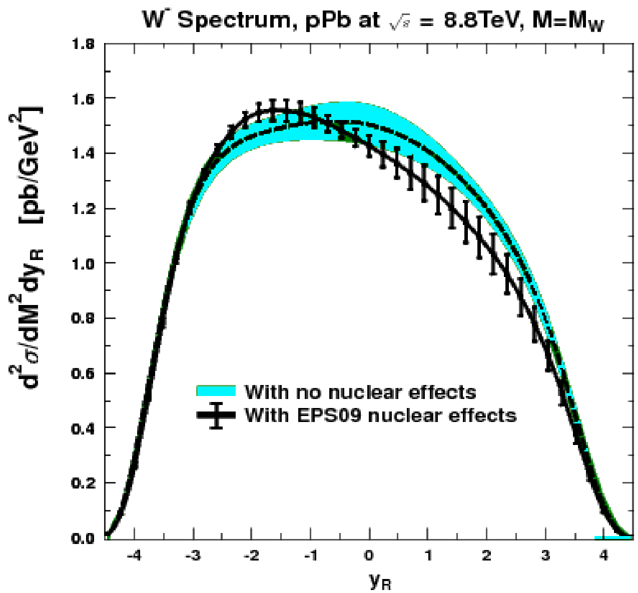
[DdE,Snigirev, PLB718 (2013)1395]

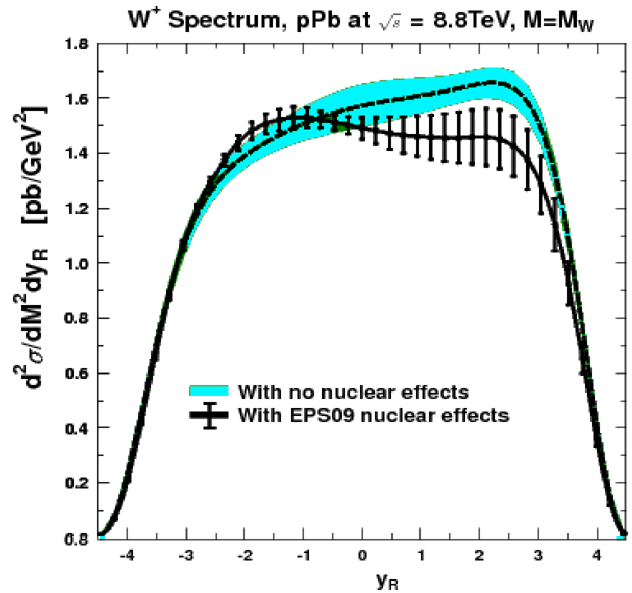
- Theoretical setup:
  - MCFM 6.2: Single-parton W^+,W^-
    W^+W^+jj (QCD) background
    - NLO accuracy.
    - Scales: $\mu(W) = m_W$, $\mu(WW)$ = 150 GeV
    - CT10 proton PDF, EPS09 Pb nuclear PDF:





~10% effects due nuclear (anti)shadowing alone:





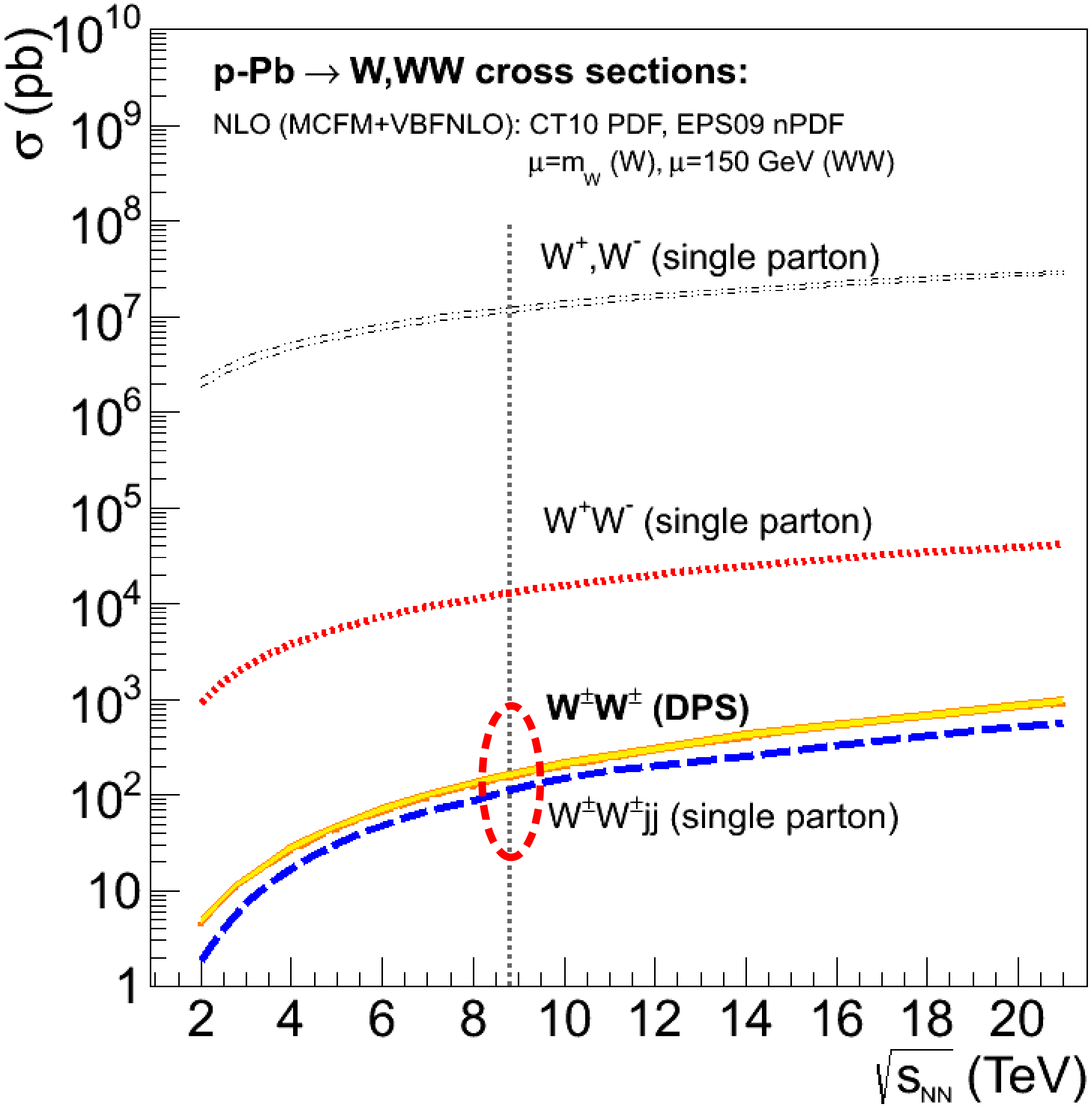
Isospin+shadow. effects on total inclusive x-sections:
W^- : +7%
W^+ : -15%
compared to p-p

[Paukkunen&Salgado JHEP 1103 (2011) 071]

Results: p-Pb → W^+W^+, W^-W^- at 8.8 TeV

[DdE,Snigirev, PLB718 (2013)1395]

- Cross sections for all relevant SPS & DPS processes vs sqrt(s):



p-Pb @ 8.8 TeV:

σ(WW,DPS) ≈ 150 pb

σ(WWjj) ≈ 100 pb

±18% uncertainties:

±15% for σ_{eff}

±10% for scales&PDFs

Results: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

[DdE,Snigirev, PLB718 (2013)1395]

- Measurable final-states:
  - W's branching ratios:
    - BR(W→ lν) ≈ 3 ×1/9, BR(W→qq') ≈ 2/3
    - Both leptonic: 4 final-states (μμ,ee,eμ,μe): $4\times(1/9)^2$ ≈ 1/20, 1/16 (+ τ)
    [1 leptonic + 1 hadronic (jet-charge): 2/9 ×4/3 ≈ 0.3]
  - Typical ATLAS/CMS acceptances & efficiencies:
    - Leptons: |y|<2.5, p_T >15 GeV ⇒ ε_{WW} ≈ 40%
- LHC p-Pb luminosities (note: very small pileup):
  - $\mathcal{L}_{\rm int}$ = 0.2–2 pb^{-1} (increase to nominal p intensity, reduce beam size)
- Expected (purely leptonic) rates including yield loses & luminosity:

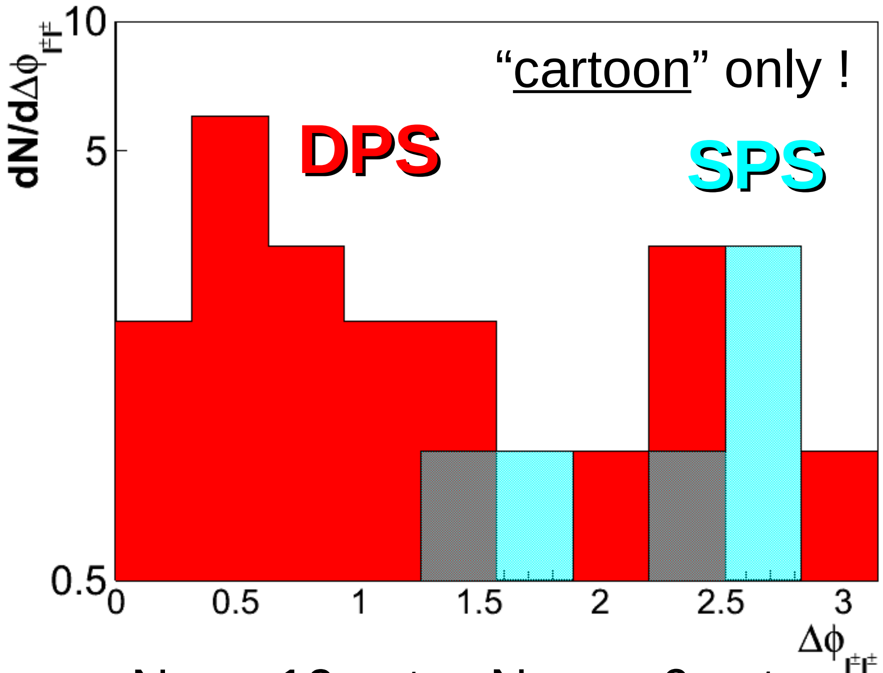
$$N_{\rm DPS} = \sigma^{\rm DPS}_{pPb\to WW}/(\varepsilon\cdot\mathcal{L}_{\rm int}) \approx$$ 1–10 same-sign WW pairs/year

(factor ×6 more in 1 lepton + 1-jet channel)

Results: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

- Typical DPS-sensitive kinematical distributions for signal & background:

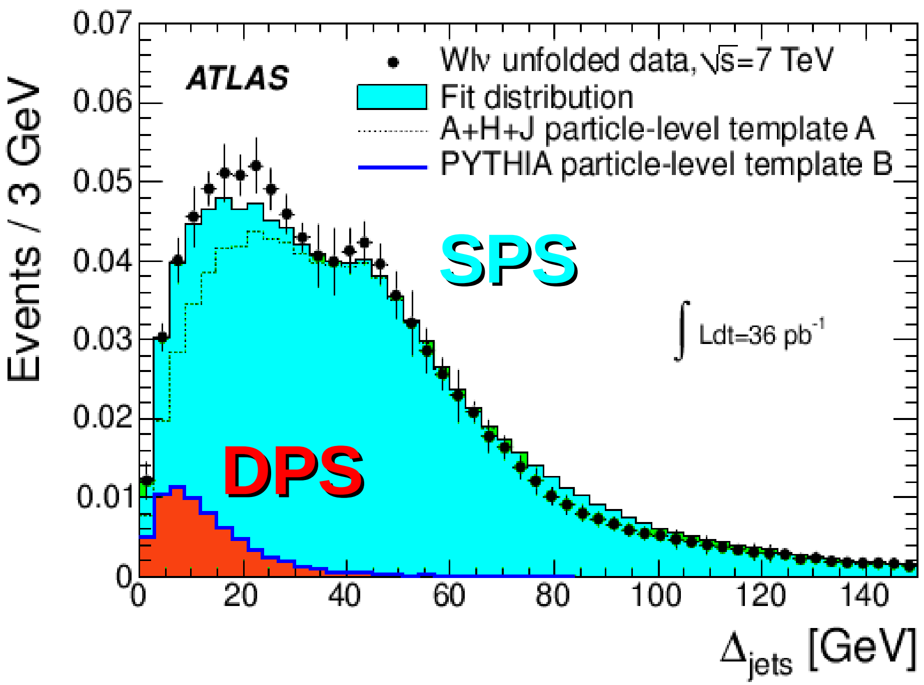
p-Pb @ 8.8 TeV (2 pb^{-1}):
Same-sign leptons
azimuthal separation:



N_{DPS}≈10 evts N_{SPS}≈ 6 evts

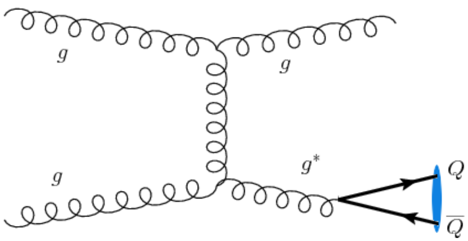
(Other reducible bckgds: WZ,$Z^{(*)}Z^{(*)}$,B^0B^0)

Compare to:
p-p→ W+2j @ 7 TeV (36 pb^{-1}):
dijet azimuthal separation



Example: Pb-Pb → J/ψ J/ψ at 5.5 TeV

- **FONLL+CEM (R.Vogt)**: Single-parton J/ψ

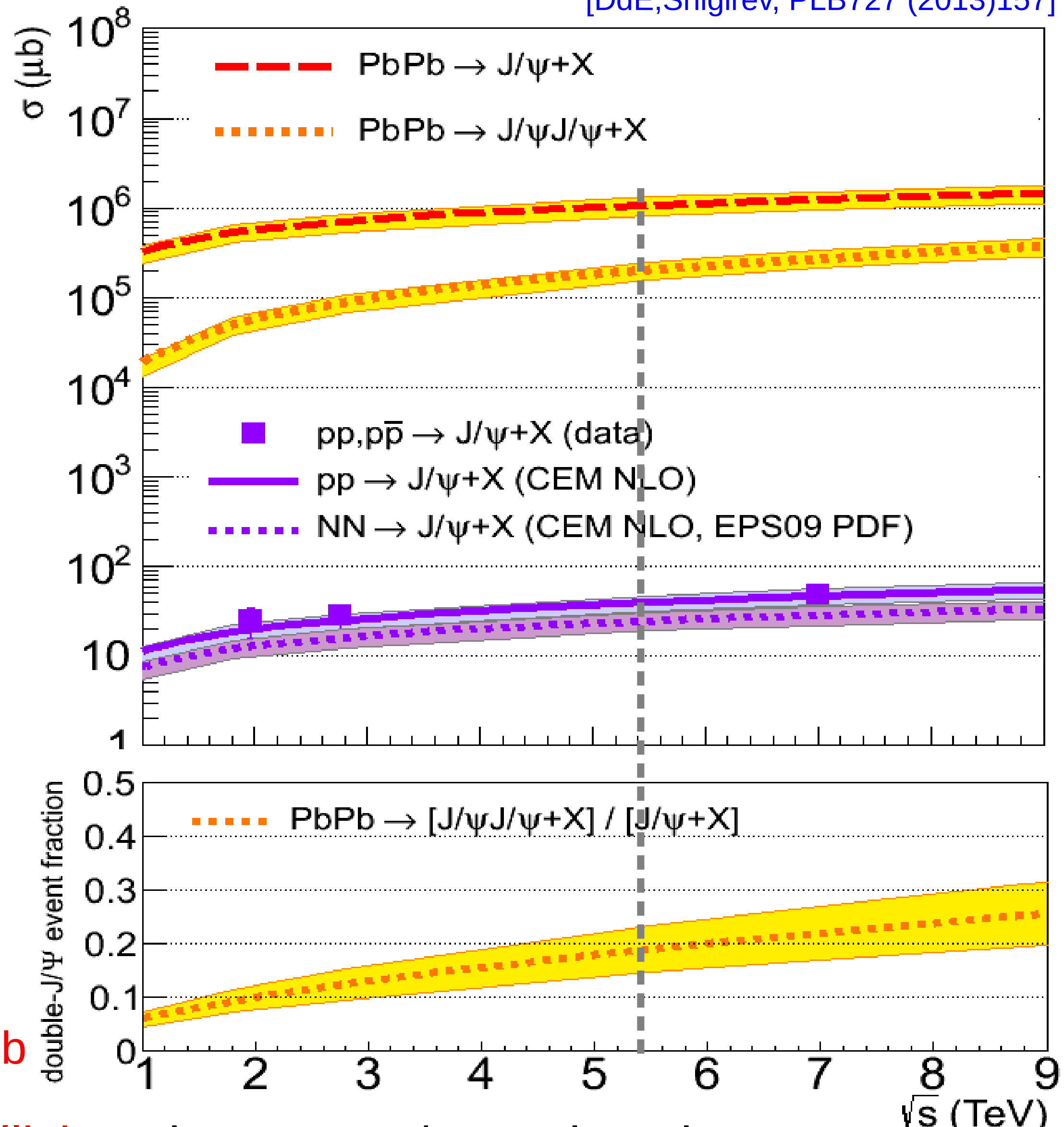


- NLO accuracy.
- Scales: $\mu_R = \mu_R = 1.5 \cdot m_c$
- Good agreement with Tevatron&LHC data
- EPS09 Pb nPDF
  20–35% shadowing
  x-section reduction

- At 5.5 TeV:

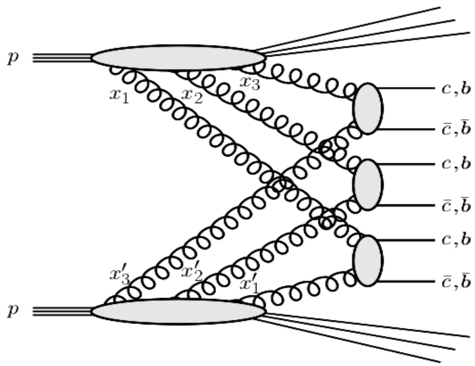
$\sigma^{DPS}(\text{Pb–Pb} \rightarrow J/\psi\, J/\psi\, X)$ =200±50 mb

[DdE,Snigirev, PLB727 (2013)157]



20% of min.bias Pb-Pb collisions have two J/ψ produced !

Triple parton scattering x-sections (p-p)



- Assuming that the probabilities for 3 hard collisions to be independent of each other, one can again write a pocket-formula for TPS x-section:

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \frac{\sigma^{\rm SPS}_{hh'\to a_1} \cdot \sigma^{\rm SPS}_{hh'\to a_2} \cdot \sigma^{\rm SPS}_{hh'\to a_3}}{\sigma^2_{\rm eff,TPS}}$$

normalized by the square of an eff. x-section ($\sigma^2_{\rm eff,TPS}$) plus a trivial combinatorial factor (m/3!) to avoid triple-counting in case of same particles produced:

$m = 1$ if $a_1 = a_2 = a_3$;
$m = 3$ if $a_1 = a_2$, or $a_1 = a_3$, or $a_2 = a_3$; and
$m = 6$ if $a_1 \neq a_2 \neq a_3$.

- How to interpret $\sigma_{\rm eff,TPS}$? What values one naively expects for it?

- Most generic expression for TPS cross section:

$$\sigma^{\rm TPS}_{hh'\to a_1 a_2 a_3} = \left(\frac{m}{3!}\right) \sum_{i,j,k,l,m,n} \int \Gamma^{ijk}_h(x_1, x_2, x_3; \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}; Q_1^2, Q_2^2, Q_3^2)$$
$$\times \hat{\sigma}^{il}_{a_1}(x_1, x'_1, Q_1^2) \cdot \hat{\sigma}^{jm}_{a_2}(x_2, x'_2, Q_2^2) \cdot \hat{\sigma}^{kn}_{a_3}(x_3, x'_3, Q_3^2)$$
$$\times \Gamma^{lmn}_{h'}(x'_1, x'_2, x'_3; \mathbf{b_1} - \mathbf{b}, \mathbf{b_2} - \mathbf{b}, \mathbf{b_3} - \mathbf{b}; Q_1^2, Q_2^2, Q_3^2)$$
$$\times dx_1 dx_2 dx_3 dx'_1 dx'_2 dx'_3 d^2b_1 d^2b_2 d^2b_3 d^2b.$$

Generalized PDFs = f(x,Q^2,**b**)

Triple parton scattering x-sections (p-p)

- Assumption 1: Factorize generalized Triple-PDF into longitudinal & transverse components:

$$\Gamma_h^{ijk}(x_1, x_2, x_3; \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}; Q_1^2, Q_2^2, Q_3^2) = D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) f(\mathbf{b_1}) f(\mathbf{b_2}) f(\mathbf{b_3}),$$

p-p transv. overlap function (mb^{-1}): $T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$, with $\int d^2 b T(\mathbf{b}) = 1.$

- Assumption 2: Longitudinal triple-PDF is the product of 3 single PDFs (i.e. no parton correlations in colour, momentum, flavour, spin,…)

$$D_h^{ijk}(x_1, x_2, x_3; Q_1^2, Q_2^2, Q_3^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2) D_h^k(x_3; Q_3^2)$$

- Then, $\sigma^2_{eff,TPS}$ is simply the inverse of the cube of the transv. pp overlap:

$$\sigma^2_{\rm eff,TPS} = \left[\int d^2 b\, T^3(\mathbf{b})\right]^{-1}$$

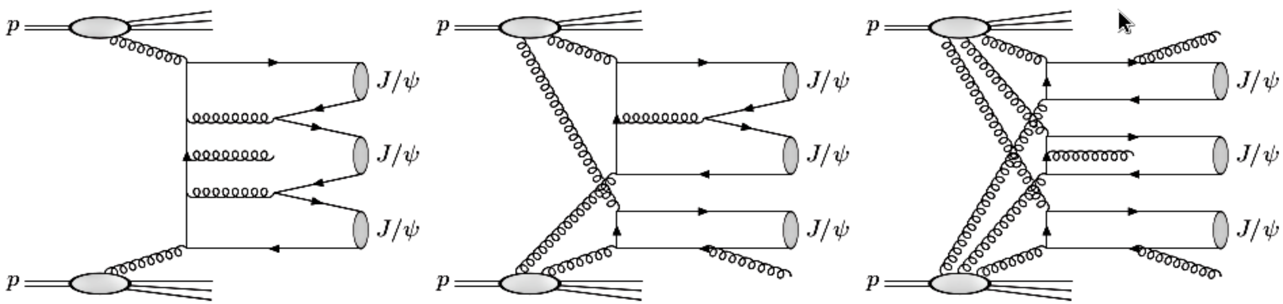
- By testing many proton overlaps/profiles (hard sphere, Gaussian, expo, dipole fit), we find a close relationship between $\sigma_{eff,TPS}$ & σ_{eff}:

$$\sigma_{\rm eff,TPS} = k \times \sigma_{\rm eff,DPS}, \text{ with } k = 0.82 \pm 0.11$$

- Measuring TPS provides independent info on σ_{eff} and p transv. profile.

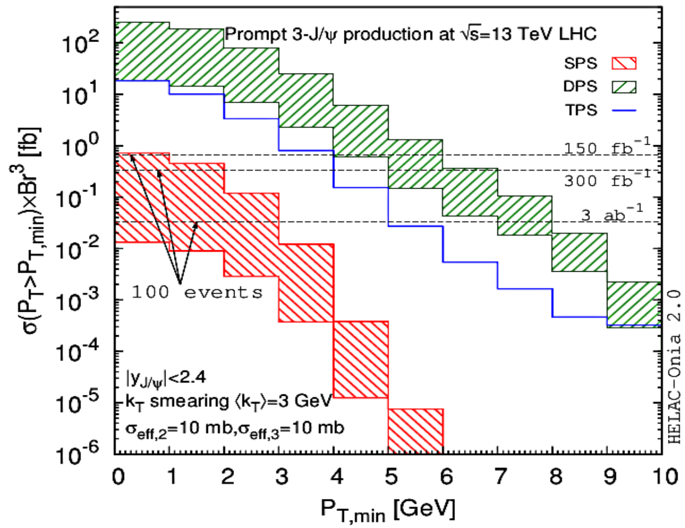
Triple-J/ψ from SPS production (p-p)

- H.-S. Shao et al. [arXiv:1902.04949, PRL 122(2019)192002] computed all triple-J/Ψ x-sections with SPS HELAC-ONIA plus our pocket formulas:



(a) SPS (b) DPS (c) TPS

		inclusive	$2.0 < y_{J/\psi} < 4.5$	$\lvert y_{J/\psi}\rvert < 2.4$
13 TeV	SPS	$0.41^{+2.4}_{-0.34} \pm 0.0083$	$(1.8^{+11}_{-1.5} \pm 0.18) \times 10^{-2}$	$(8.7^{+56}_{-7.5} \pm 0.098) \times 10^{-2}$
	DPS	$(190^{+501}_{-140}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(7.0^{+18}_{-5.1}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(50^{+140}_{-37}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$130 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$1.3 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$18 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
27 TeV	SPS	$0.46^{+2.9}_{-0.39} \pm 0.022$	$(3.2^{+22}_{-2.8} \pm 0.21) \times 10^{-2}$	$(5.8^{+39}_{-5.1} \pm 0.29) \times 10^{-2}$
	DPS	$(560^{+2900}_{-480}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(19^{+97}_{-16}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(120^{+630}_{-100}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$570 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$5.0 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$57 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
75 TeV	SPS	$0.59^{+4.4}_{-0.52} \pm 0.016$	$(3.0^{+25}_{-2.7} \pm 0.23) \times 10^{-2}$	$(7.2^{+63}_{-6.5} \pm 0.38) \times 10^{-2}$
	DPS	$(1900^{+11000}_{-1600}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(57^{+340}_{-50}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(310^{+2000}_{-270}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$3900 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$27 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$260 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
100 TeV	SPS	$1.1^{+8.4}_{-1.0} \pm 0.044$	$(4.5^{+33}_{-4.0} \pm 0.72) \times 10^{-2}$	$(36^{+290}_{-32} \pm 1.8) \times 10^{-2}$
	DPS	$(3400^{+19000}_{-2900}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(100^{+550}_{-86}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(490^{+3000}_{-430}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$6500 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$45 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$380 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$



- SPS negligible, DPS (TPS) dominates at low (high) p_T.

Clear sensitivity to σ_{eff}!