School on *Disorder in Complex Systems* at the Institut Pascal — Université Paris-Saclay
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Statics and Dynamics of Disordered Elastic Systems

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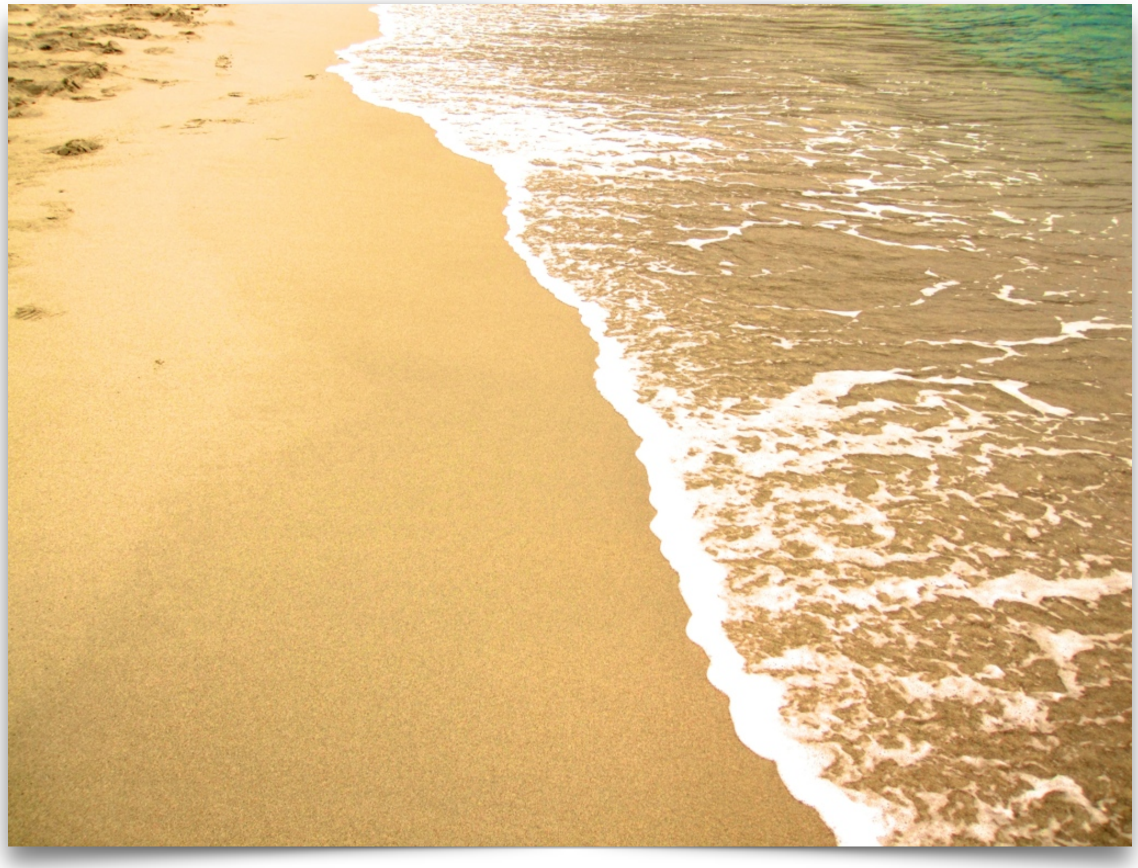
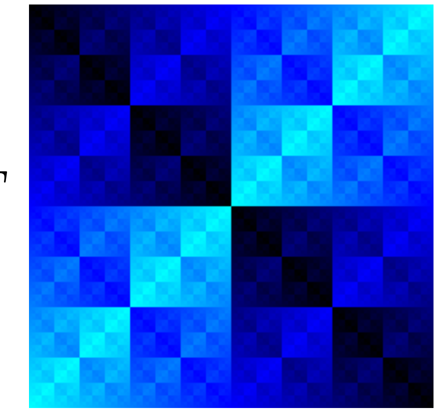
Outline — Part II: Dynamics

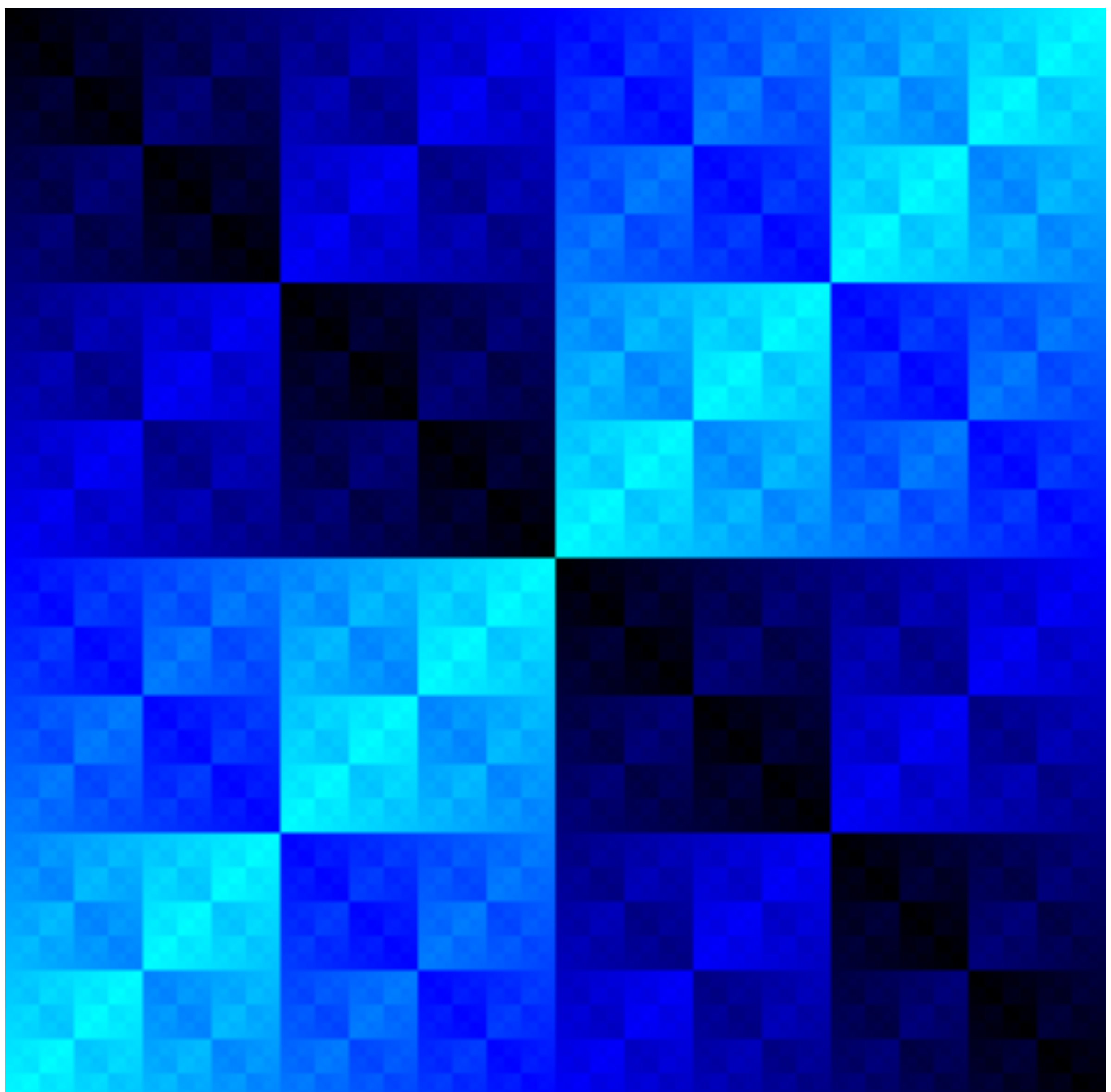
- Quenched Edwards-Wilkinson
- Equilibrium dynamics: 2D→1D
- Fast-flow regime
- Creep regime
- Depinning / Thermal rounding
- Moving Bragg glass



C) Gaussian Variational Method (GVM) roughness computations

- GVM in a nutshell

$$\mathcal{H}[u,V] \ \& \ \text{Random } V \ \xrightarrow{\text{Replicæ}} \ \widetilde{\mathcal{H}}[u_1,\dots,u_n] \ \& \ \lim_{n\to 0} \ \overset{\text{GVM}}{\approx} \sum_{\substack{\text{Fourier}\\ \text{modes}}} \vec{u}^T$$

$$\vec{u}$$



- Hierarchical matrices:

  invariance upon permutation of replica indices
  ⇒ every line/column with reshuffled coefficients

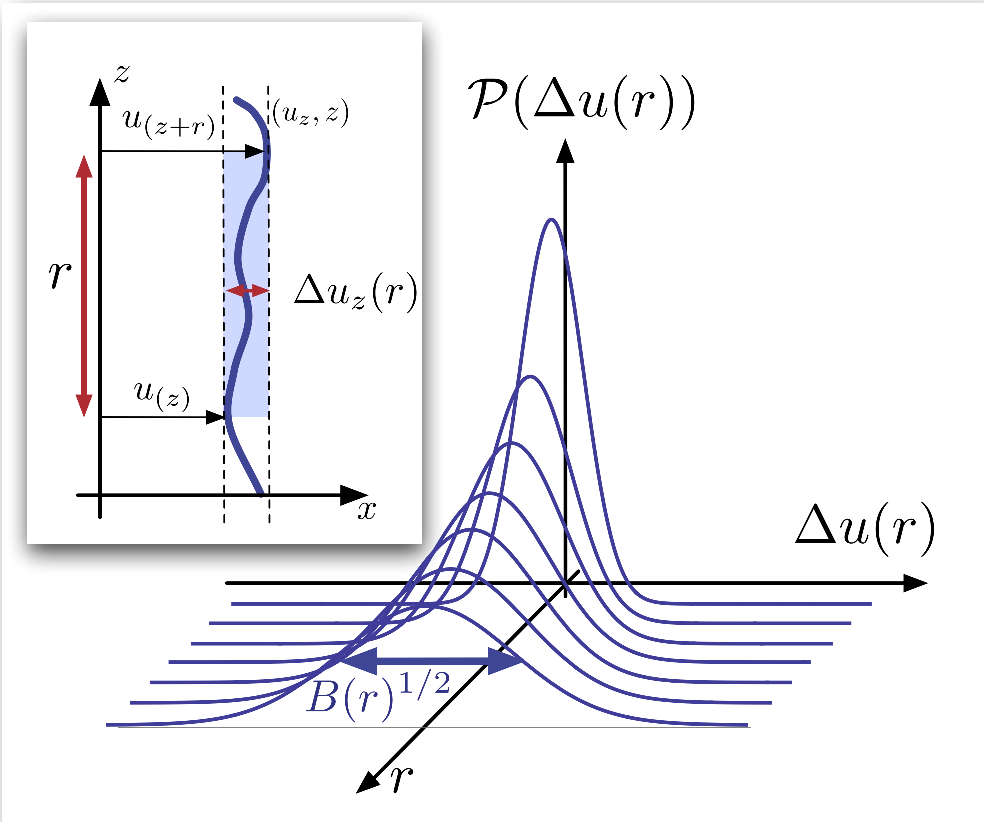
  Example here: color code = coefficient values
  (continuous blend on the first line/column)

- Algebra of inverting such $n \times n$ matrices in limit $n \to 0$:

  M. Mézard & G. Parisi, *J. Phys. I* 1, 809 (1991) [Append. II]

  E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 82, 184207 (2010) [Appendix B]

Roughness function & Structure factor (and related quantities)



- Our roughness = 'height-height correlation function"

$$B(r) = \overline{\left\langle [u(z+r) - u(z)]^2 \right\rangle} = \left\langle \frac{1}{N_r} \sum_{\text{pairs}} \Delta u_z(r)^2 \right\rangle_{\text{samples}}$$

- Working in Fourier: structure factor

$$S(q) = \overline{\langle \tilde{u}_{-q} \tilde{u}_q \rangle} = \left\langle |\tilde{u}_q|^2 \right\rangle$$

$$\Rightarrow B(r) = \int \frac{d^d q}{(2\pi)^d} 2\left[1 - \cos(qr)\right] S(q)$$

- Issues in experimental (and also numerical) studies of roughness:
  - **Finite statistics:** # of samples / N_r= #pairs for a given lengthscale: $N_r \searrow$ when $r \nearrow$
  - **Finite system size:** e.g. in ferroelectric domain walls, L=512 pixels ⇒ relevance of power-law exponents?
  - **Finite-time saturation:** glassy behaviour, *a priori* not fully relaxed systems (experimentally/numerically)
  - **Beware of strong impurities:** might break locally the elastic description ⇒ non-Gaussian artefacts

A.-L. Barabàsi & H. E. Stanley, *Fractal Concepts in Surface Growth,* Cambridge University Press, 1995.
Cf. Preprint: J. Guyonnet, E. Agoritsas, P. Paruch, S. Bustingorry, arXiv:1904.11726 [cond-mat.dis-nn].
S. Bustingorry, J. Guyonnet, P. Paruch, E. Agoritsas, *J. Phys. Condens. Matter* 33, 345001 (2021).

Usual approaches to study these DES problems

- **Scaling analysis: 'Flory/Imry-Ma'-like arguments**
  - E. Agoritsas, V. Lecomte, *J. Phys. A: Math. Theor.* 50, 104001 (2017)
    "Power countings versus physical scalings in disordered elastic systems - Case study of the one-dimensional interface"

- **Gaussian Variational Method (GVM) approaches for the statics**

  Cf. Refs. in previous slides. Exact in the limit of infinite dimension in the transverse direction.

- **Special mappings/limits when available**

  E.g. Mapping of the 1D static interface (short-range elasticity & random-bond disorder) on the 1+1 Directed Polymer and the 1D Kardar-Parisi-Zhang equation

- **Functional Renormalisation Group (FRG) in $\epsilon = 4 - d$ or non-perturbative**
  - K. J. Wiese arXiv:2102.01215 [cond-mat.dis-nn], "Theory and Experiments for Disordered Elastic Manifolds, Depinning, Avalanches, and Sandpiles"
  - P. Le Doussal, *Annals of Physics* 335, 49 (2010), "Exact results and open questions in first principle functional RG"
  - I. Balog, G. Tarjus, M. Tissier, *J. Stat. Mech.: Th. Exp.* 2019, 103301 (2019), "Benchmarking the nonperturbative functional renormalization group approach on the random elastic manifold model in and out of equilibrium"

- **Numerical studies e.g. on quenched Edwards-Wilkinson dynamics**
  - E. E. Ferrero, S. Bustingorry, A. B. Kolton, A. Rosso, *Comptes Rendus de Physique* 14, 641 (2013)
    "Numerical approaches on driven elastic interfaces in random media"

Model: 1D interface with finite width / short-range correlated disorder

- Short-range elasticity & Elastic limit / Quenched random-bond weak disorder

$$\boxed{\mathcal{H}_{\mathrm{DES}} = \mathcal{H}_{\mathrm{el}} + \mathcal{H}_{\mathrm{dis}}}$$

Hamiltonian:

$$\mathcal{H}[u,\widetilde{V}] = \int_{\mathbb{R}} dz \cdot \left[\underbrace{\frac{c}{2}(\nabla_z u(z))^2}_{\text{Elasticity}} + \underbrace{\int_{\mathbb{R}} dx \cdot \rho_\xi(x-u(z))\,\widetilde{V}(z,x)}_{\text{Effective random potential } V(z,u(z))} \right]$$

- Density $\rho_\xi(x-u(z))$ & random potential $\widetilde{V}(z,x)$

$$\overline{\widetilde{V}(z,x)} = 0$$

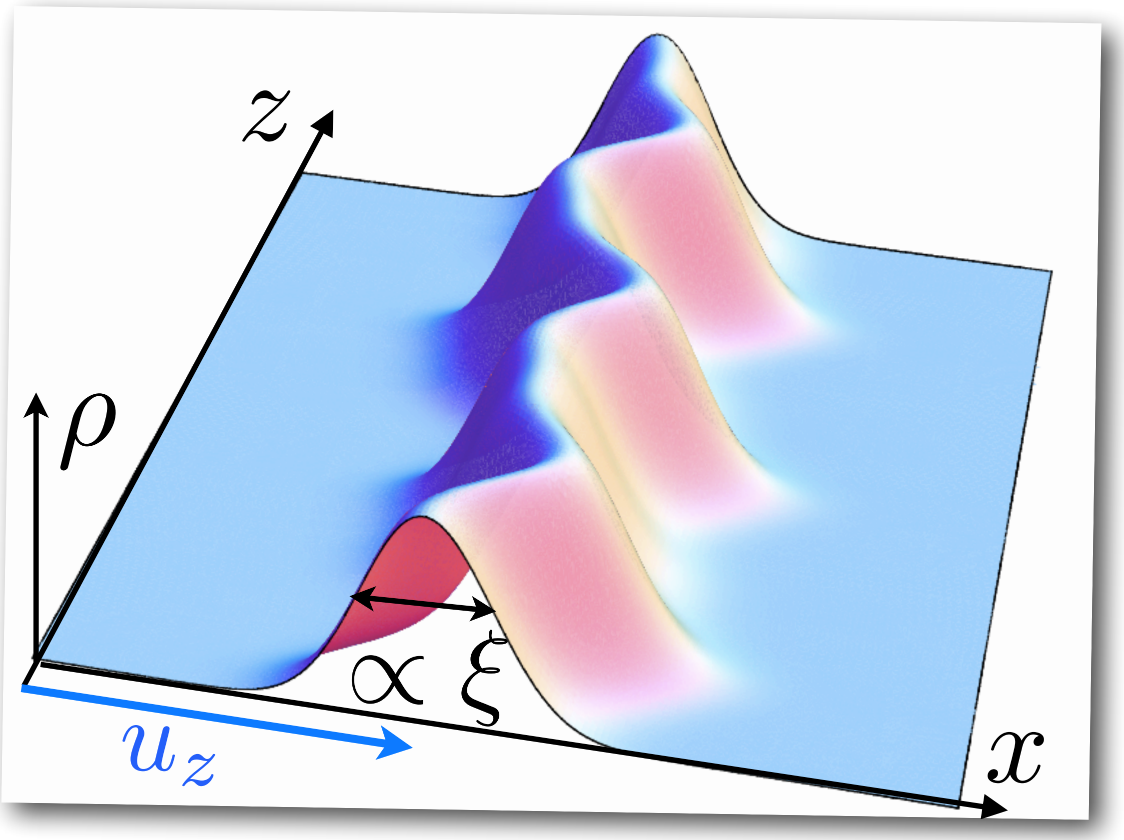
$$\overline{\widetilde{V}(z,x)\widetilde{V}(z',x')} = D\,\delta(z-z')\delta(x-x')$$

- Alternative: ***correlated*** effective potential $V(z,u(z))$

$$\boxed{\overline{V(z,x)V(z',x')} = D\,\delta(z-z')R_\xi(x-x')}$$

- Exponentially decaying with following scaling:

$$\boxed{R_\xi(x) = \xi^{-1}R_1(x/\xi)} \quad \text{e.g.} \quad R_\xi^{\mathrm{G}}(x) = \frac{e^{-x^2/(2\xi^2)}}{\sqrt{2\pi}\xi}$$



Elastic constant c / Width ξ / Disorder strength D / Temperature T

- Overdamped dynamics: 'quenched Edwards-Wilkinson'

$$\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\mathrm{dis}}(z,u(z)) + f_{\mathrm{ext}} + \eta_{\mathrm{thermal}}(z,t)$$

$$\langle \eta_{\mathrm{th}}(z,t)\eta_{\mathrm{th}}(z',t')\rangle = 2\gamma T\delta(z-z')\delta(t-t')$$

$$F_{\mathrm{dis}}(z,x) = -\partial_x V(z,x)$$

Overdamped dynamics / RB vs RF disorder

$$\boxed{\mathcal{H}_{\rm DES} = \mathcal{H}_{\rm el} + \mathcal{H}_{\rm dis}}$$

- **Overdamped dynamics for numerics / field-theory:**

$$\cancel{\underbrace{m\partial_t^2 u(z,t)}_{\text{Acceleration}}} + \gamma\partial_t u(z,t) = \underbrace{-\frac{\delta\mathcal{H}_{\rm el}[u]}{\delta u(z,t)}}_{F_{\rm el}(z,t)} \underbrace{-\frac{\delta\mathcal{H}_{\rm dis}[u,V]}{\delta u(z,t)}}_{F_{\rm dis}(z,t)} + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

- Gaussian noise of zero mean and 2-pt correlator: $\langle \eta_{\rm th}(z,t)\eta_{\rm th}(z',t')\rangle = 2\gamma T\delta(z-z')\delta(t-t')$

- Here focus on 'quenched Edwards-Wilkinson' (qEW):

$$\boxed{\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\rm dis}(z,u(z)) + f_{\rm ext} + \eta_{\rm thermal}(z,t)}$$

- Random force: $F_{\rm dis}(z,x) = -\partial_x V(z,x)$

$$\boxed{\Delta(x) = -D\,R''(x)}$$

$$\overline{V(z,x)V(z',x')} = D\delta(z-z')R(x-x')$$

$$\overline{F_{\rm dis}(z,x)F_{\rm dis}(z',x')} = \delta(z-z')\,\Delta(x-x')$$

$\int dx\,\Delta(x) = 0$ random-bond (RB)

$\int dx\,\Delta(x) > 0$ random-field (RF)

- Typically with periodic boundary condition: $u(z+L_z) = u(z)$, and also $L_x \sim L_z^\zeta$

Numerics: E. E. Ferrero, S. Bustingorry, A. B. Kolton, A. Rosso, *Comptes Rendus de Physique* 14, 641 (2013)

Overdamped dynamics & MSR dynamical action

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- Martin-Siggia-Rose (MSR) dynamical action: $\overline{\langle \mathcal{O}(t)\rangle} = \int \mathcal{D}u_{zt} \int \mathcal{D}\hat{u}_{zt}\, \mathcal{O}[u,\hat{u}]\, e^{-S[u,\hat{u}]}$

$$S[u,\hat{u}] = \int_{zt} i\hat{u}_{zt}(\gamma\partial_t - c\partial_z^2)u_{zt} - \frac{1}{2}\int_{ztt'} i\hat{u}_{zt}\, i\hat{u}_{zt'}\,\Delta(u_{zt}-u_{zt'}) - f_{\rm ext}\int_{zt} i\hat{u}_{zt} + \gamma T\int_{zt} i\hat{u}_{zt}\, i\hat{u}_{zt}$$

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Force-force correlator = central object for FRG

Experimental example: moving ferromagnetic domain wall

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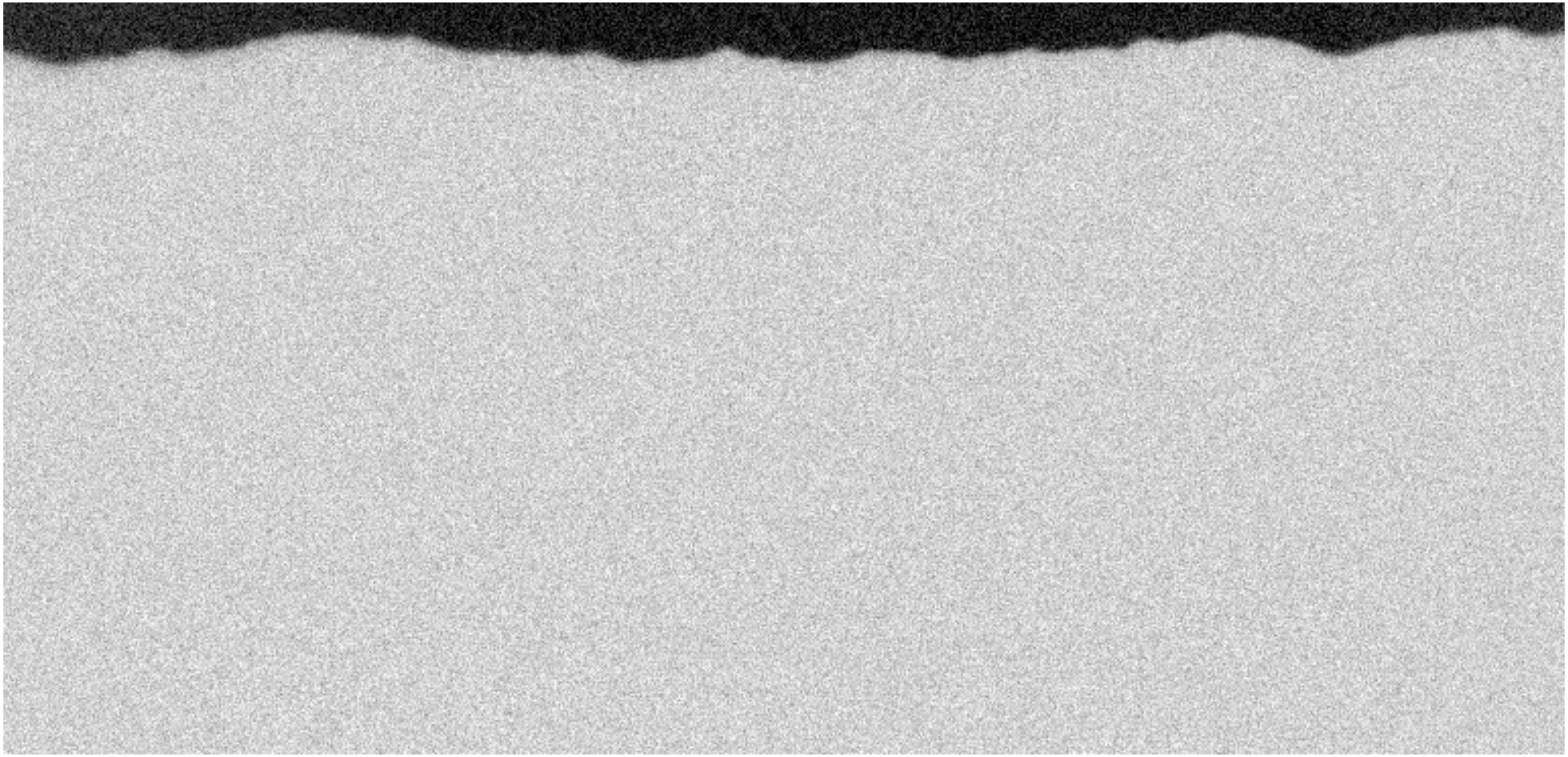
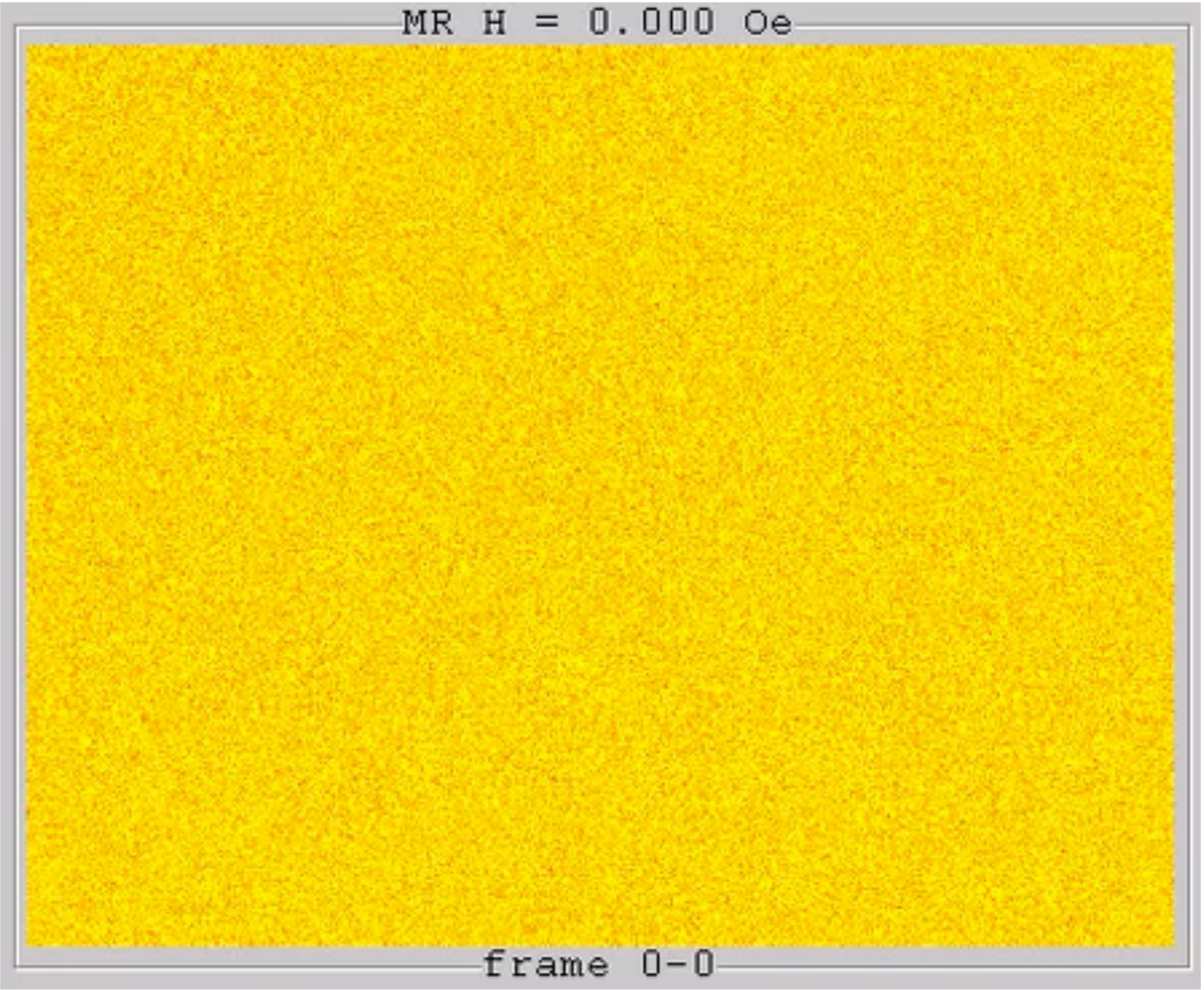
- V. Repain et al. @ Orsay:



Experimental example: moving ferromagnetic domain wall

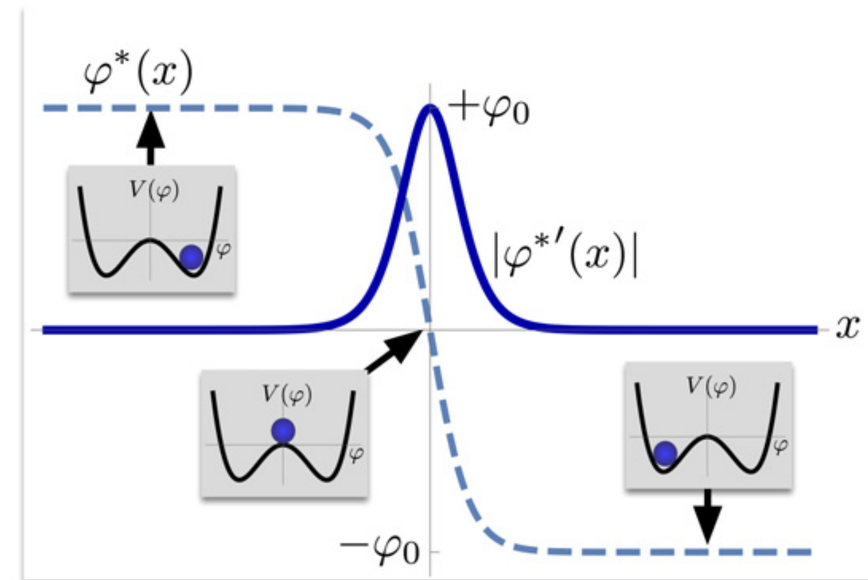
- V. Repain et al. @ Orsay: **Pt/Co(0,5 nm)/Pt/SiO_2**

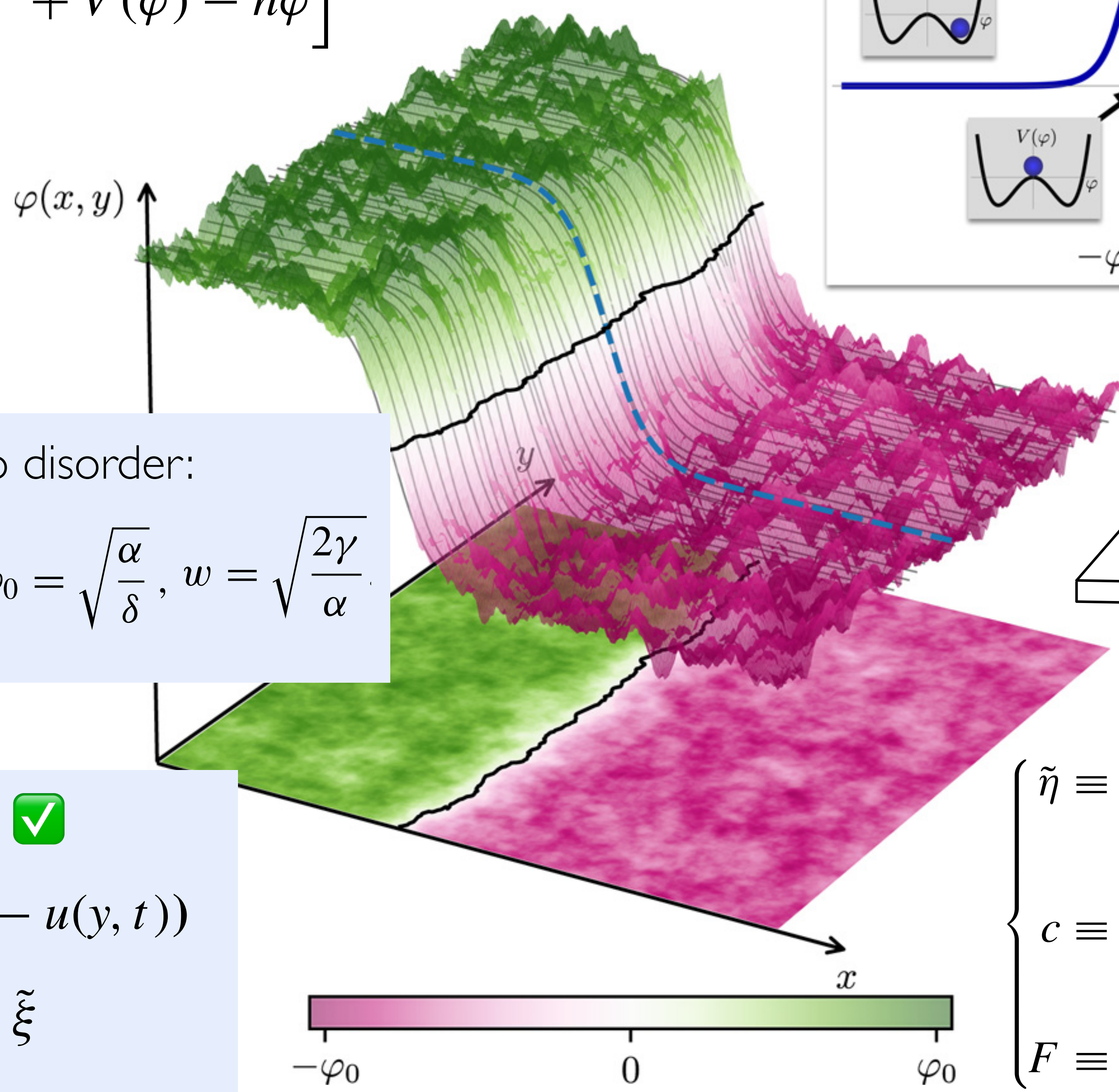


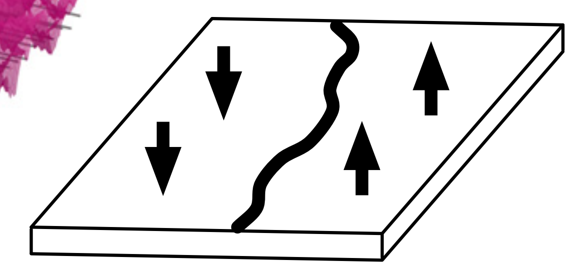
Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

- Let's start from a 2D Ginzburg-Landau description of the magnetisation in thin films:

$$\begin{cases} \eta\partial_t\varphi = -\dfrac{\delta\mathcal{H}_{\mathrm{GL}}[\varphi]}{\delta\varphi} + \xi \qquad \text{“}\langle\xi\xi\rangle = 2\eta T\,\delta\,\delta\text{”} \\ \mathcal{H}_{\mathrm{GL}}[\varphi] = \displaystyle\int d\mathbf{r}\left[\frac{\gamma}{2}|\nabla_{\mathbf{r}}\varphi|^2 + V(\varphi) - h\varphi\right] \\ V(\varphi) = -\dfrac{\alpha}{2}\varphi^2 + \dfrac{\delta}{4}\varphi^4 \end{cases}$$







- Ideal profile at T=0/h=0/no disorder:

$$\boxed{\varphi^*(x) = -\varphi_0 \tanh\left(\frac{x}{w}\right)} \quad \varphi_0 = \sqrt{\frac{\alpha}{\delta}},\ w = \sqrt{\frac{2\gamma}{\alpha}}.$$

- Model reduction onto EW ✅

<u>Ansatz</u>: $\varphi(x,y,t) = \varphi^*(x - u(y,t))$

$$\tilde{\eta}\partial_t u = c\partial_y^2 u + F + \tilde{\xi}$$

$$\begin{cases} \tilde{\eta} \equiv \eta\mathcal{N}_1 = \eta\dfrac{2\sqrt{2}}{3}\dfrac{\alpha}{\delta}\sqrt{\dfrac{\alpha}{\gamma}} \\ c \equiv \gamma\mathcal{N}_1 = \dfrac{2\sqrt{2}}{3}\dfrac{\alpha}{\delta}\sqrt{\alpha\gamma} \\ F \equiv h\mathcal{N}_3 = -2\sqrt{\dfrac{\alpha}{\delta}}h. \end{cases}$$

N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* <u>102</u>, 104204 (2020).

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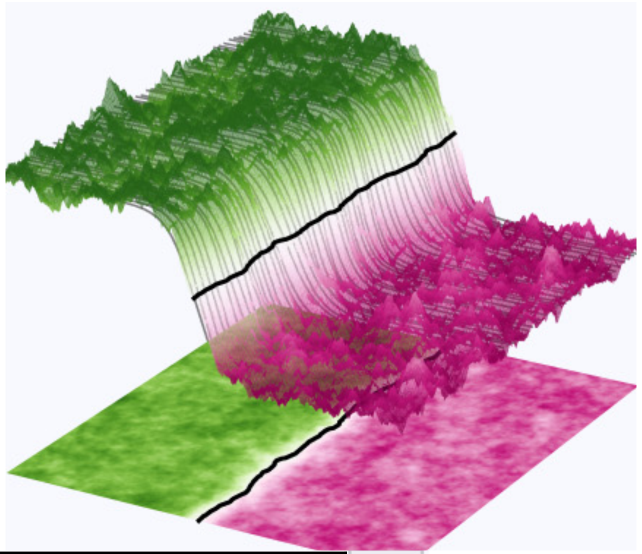
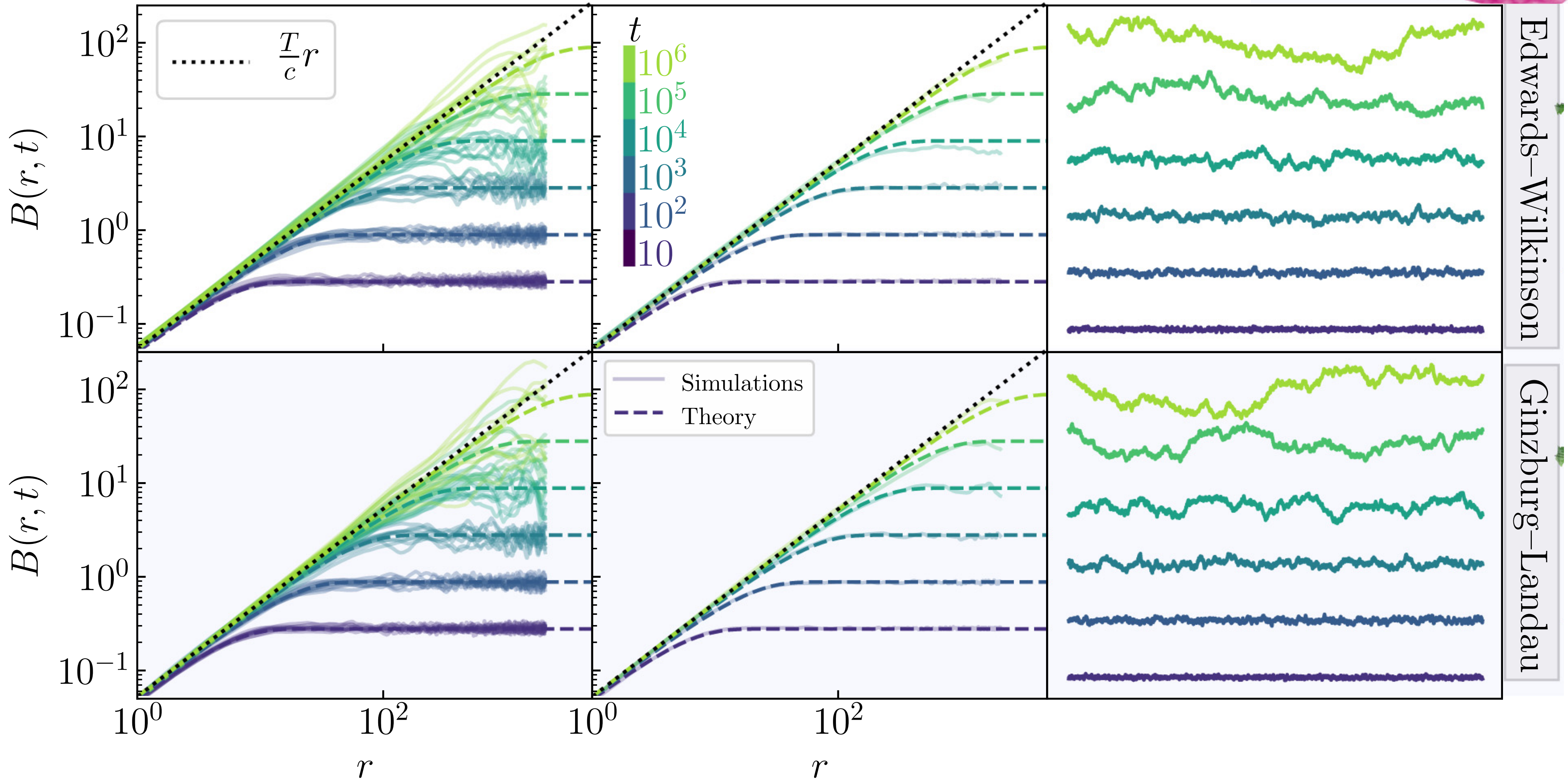


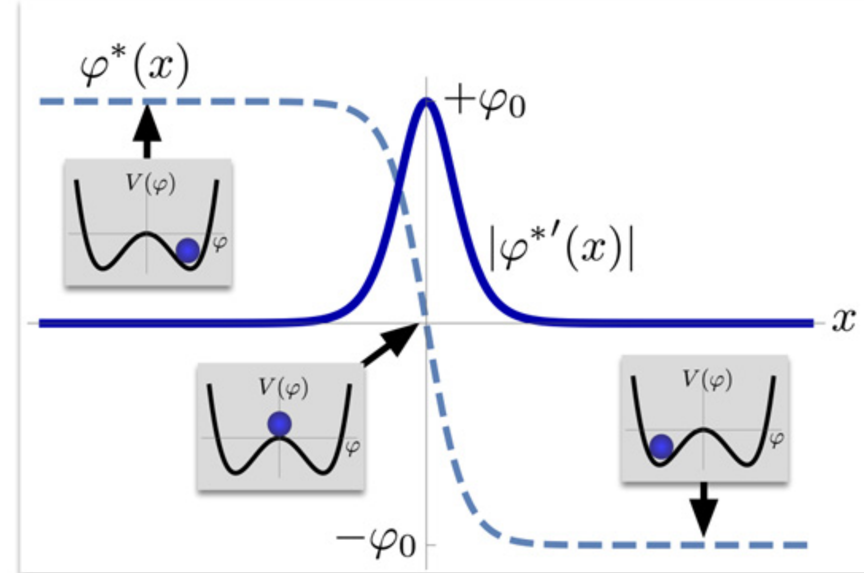


N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 102, 104204 (2020).

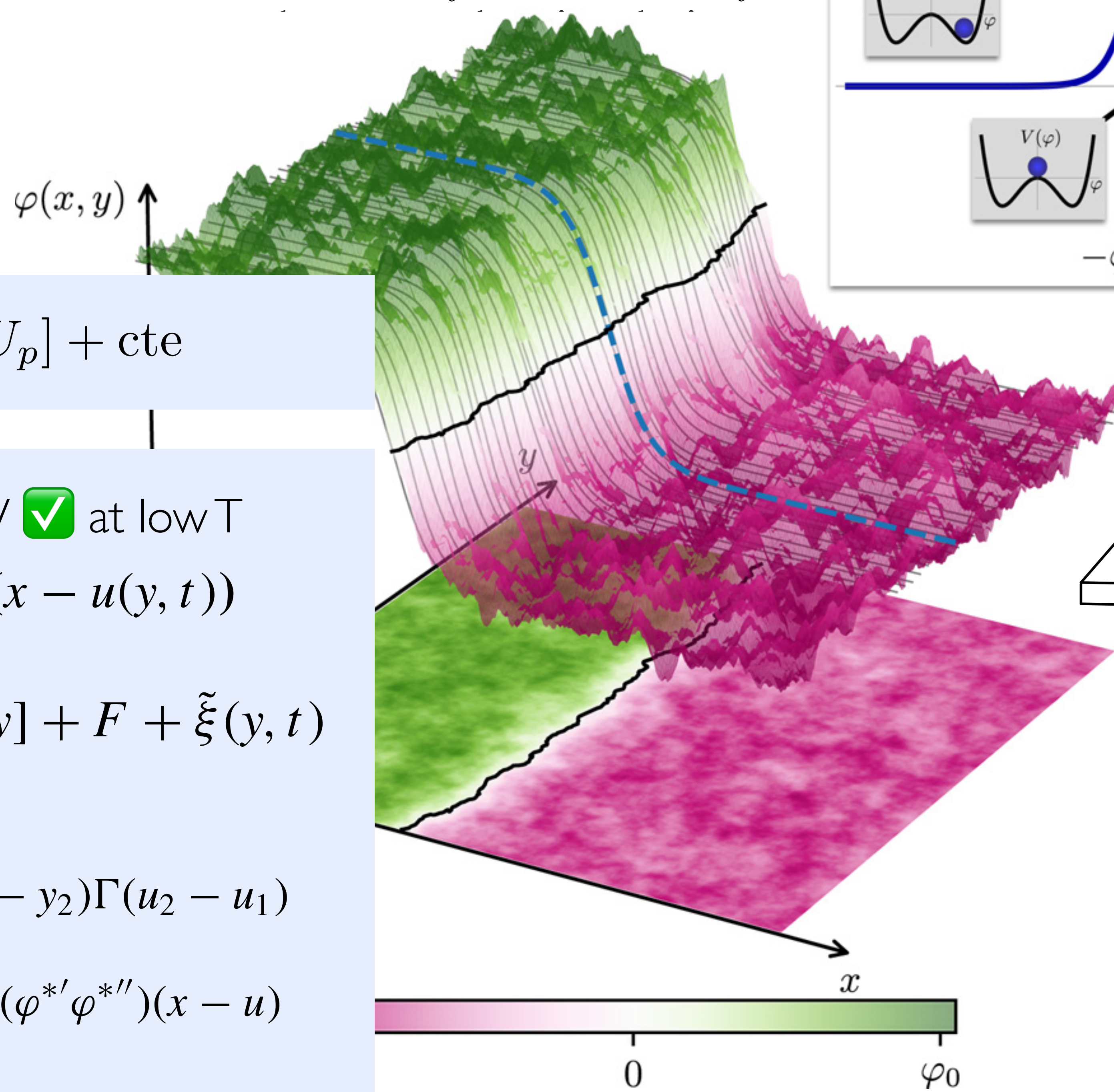
Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

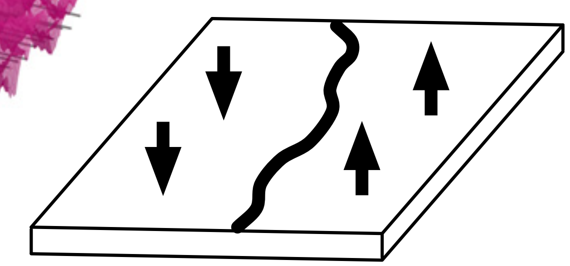
- Let's allow for fluctuations of the barrier height in the local double-well potentials:

$$\begin{cases} V(\varphi) = -\dfrac{\alpha}{2}\varphi^2 + \dfrac{\delta}{4}\varphi^4 \\ V_\zeta(\varphi(\mathbf{r})) = V(\varphi(\mathbf{r}))[1+\epsilon\zeta(\mathbf{r})] \end{cases} \qquad \overline{\zeta(\mathbf{r}_i)\zeta(\mathbf{r}_j)} = \delta^2(\mathbf{r}_i - \mathbf{r}_j)$$



$$\mathcal{H}_{\mathrm{GL}}[\varphi_u, \zeta] = \mathcal{H}_{\mathrm{DES}}[u, U_p] + \mathrm{cte}$$





- Model reduction onto qEW ✅ at low T

  Ansatz: $\varphi(x,y,t) = \varphi^*(x - u(y,t))$

$$\tilde{\eta}\partial_t u = c\partial_y^2 u + F_p[u(y,t),y] + F + \tilde{\xi}(y,t)$$

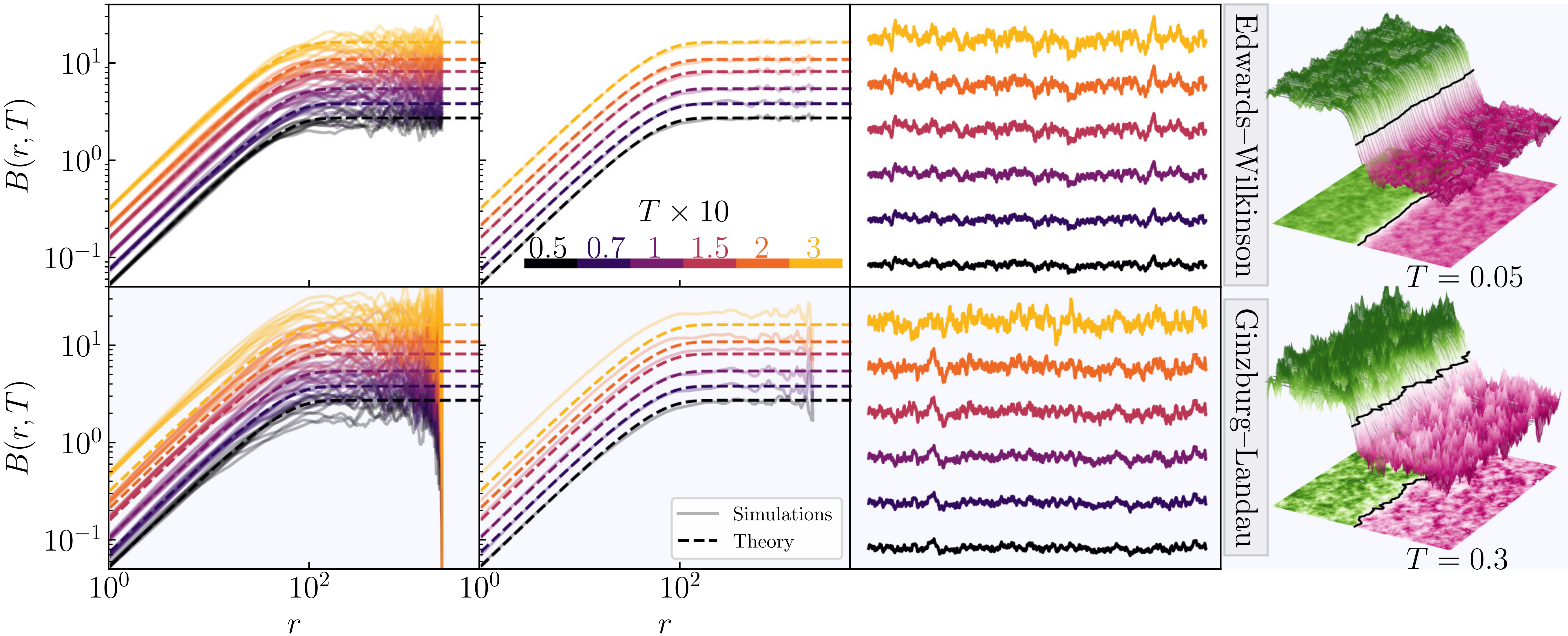
$$\begin{cases} \overline{F_p(u_1,y_1)F_p(u_2,y_2)} = \epsilon^2\delta(y_1-y_2)\Gamma(u_2-u_1) \\ \Gamma(u) = \gamma^2 \displaystyle\int_{-\infty}^{\infty} dx\,(\varphi^{*\prime}\varphi^{*\prime\prime})(x)(\varphi^{*\prime}\varphi^{*\prime\prime})(x-u) \end{cases}$$

N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 102, 104204 (2020).

Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

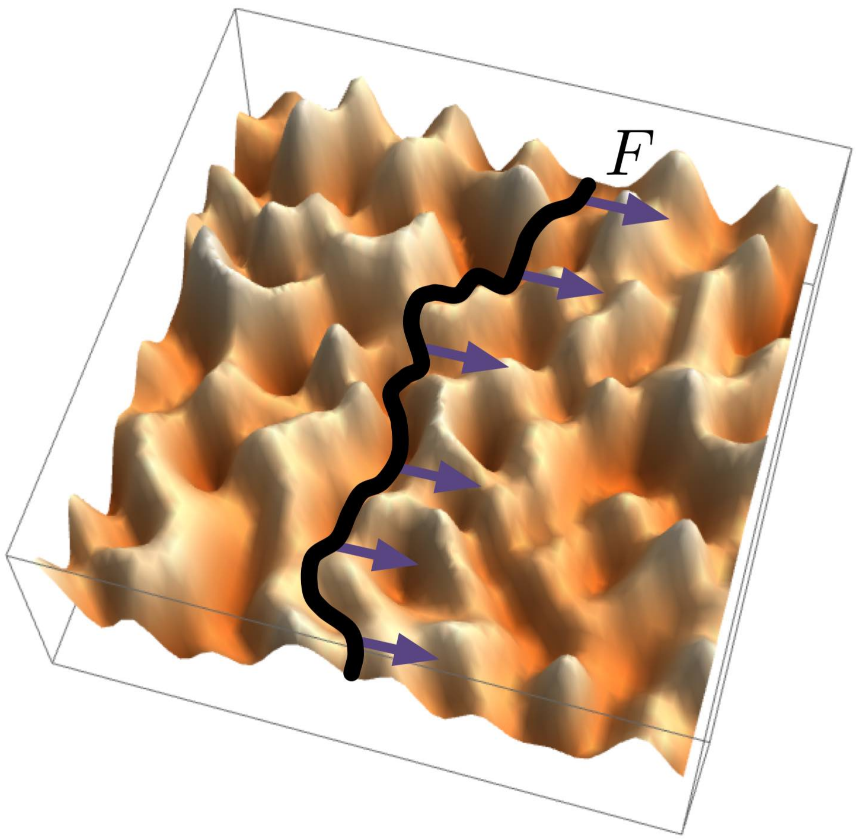
- Let's allow for fluctuations of the barrier height in the local double-well potentials:

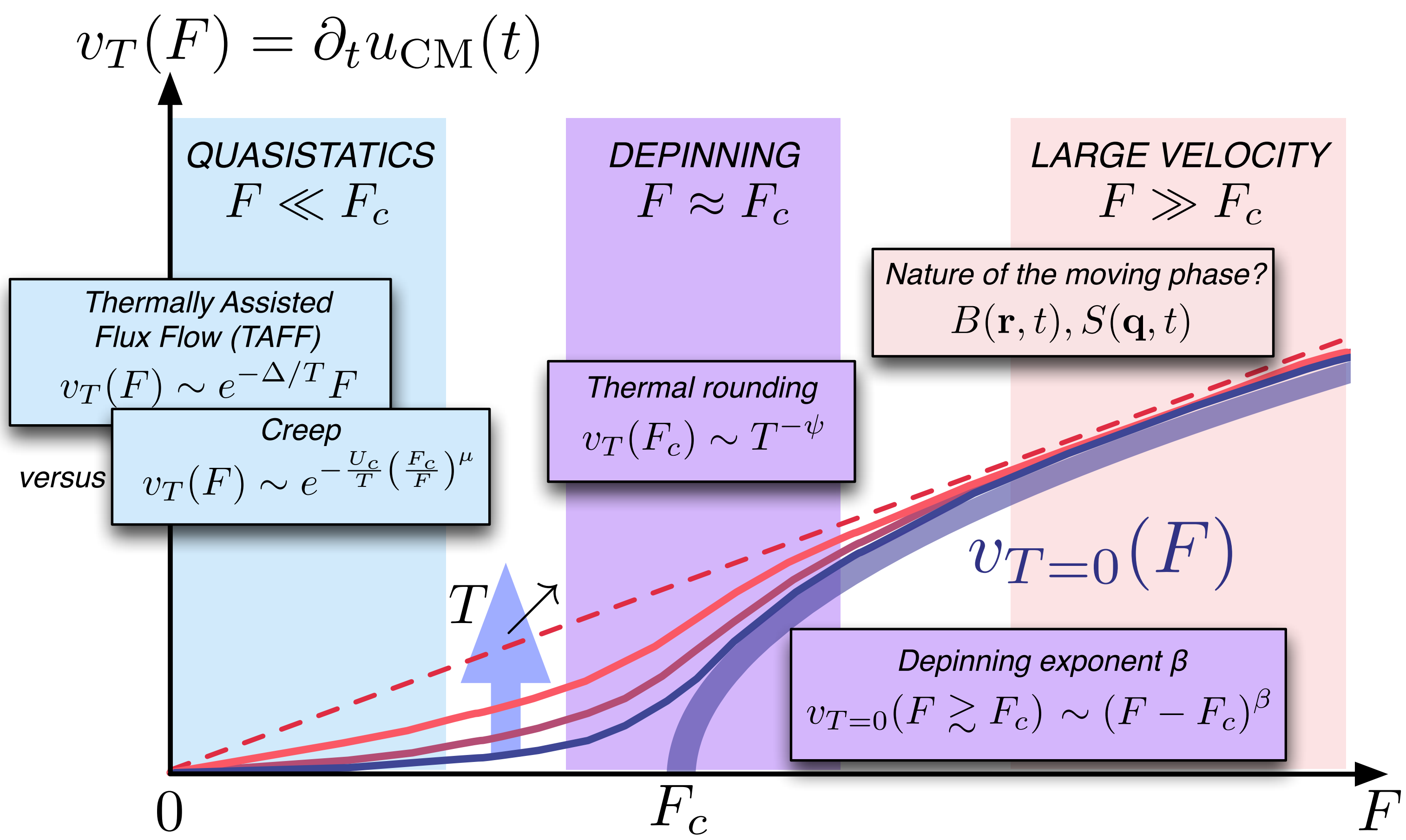
$$\begin{cases} V(\varphi) = -\frac{\alpha}{2}\varphi^2 + \frac{\delta}{4}\varphi^4 \\ V_\zeta(\varphi(\mathbf{r})) = V(\varphi(\mathbf{r}))[1 + \epsilon\zeta(\mathbf{r})] \qquad \overline{\zeta(\mathbf{r}_i)\zeta(\mathbf{r}_j)} = \delta^2(\mathbf{r}_i - \mathbf{r}_j) \end{cases}$$



N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 102, 104204 (2020).

@Large velocity: 'fast flow' regime





$$\gamma\, \partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\rm dis}(z,u(z)) + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

- Large force ⇒ large steady-state velocity for the center of mass

$$\boxed{u(z,t) = v(t-t_0) + \delta u(z,t)}$$

$$F_{\rm dis}(z,u(z,t)) = F_{\rm dis}(v(t-t_0)+\delta u(z,t)) \approx F_{\rm dis}(v(t-t_0))$$

$$\overline{F_{\rm dis}(v(t-t_0))F_{\rm dis}(v(t'-t_0))} = \Delta(v(t-t')) \begin{cases} = \dfrac{1}{v^3}\Delta_{\xi/v}(t-t') & (\text{‘RB’}) \\ = \dfrac{1}{v}\Delta_{\xi/v}(t-t') & (\text{‘RF’}) \end{cases}$$

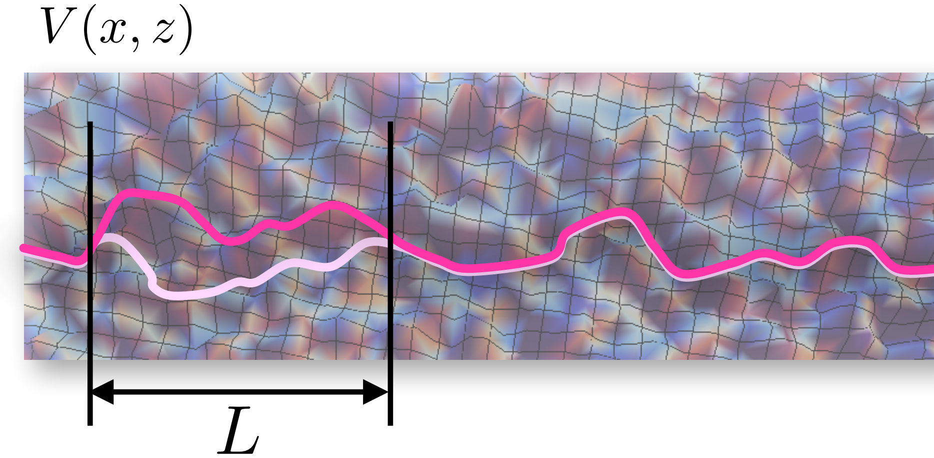
$$\boxed{F_c \sim \frac{c\xi}{L_c^2}}$$

P. Chauve, T. Giamarchi, P. Le Doussal, *Phys. Rev. B* 62, 6241 (2000) **[cf. Appendices B-C]**

Quasistatic ‘creep’ regime - Standard scaling argument

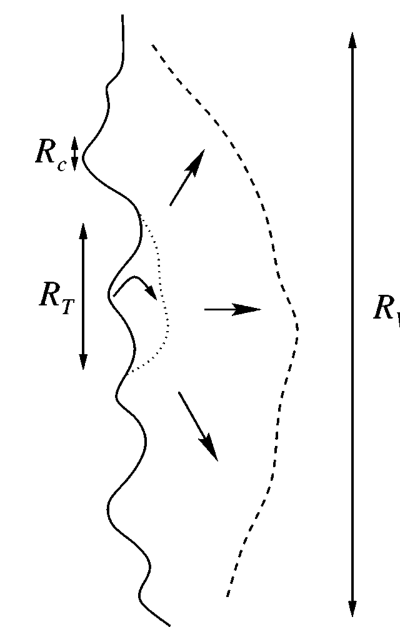
- Focus on low temperature T / small force f / large system size L: creep prediction?

- Scaling argument initially presented in L. B. Ioffe & V. M. Vinokur, *J. Phys. C* 20, 6149 (1987)
  T. Nattermann, *Phys. Rev. Lett.* 64, 2454 (1990)



- *Quasistatic assumption*: in order to move a segment of length *L* of the interface, we can estimate the energy barrier to cross from the *equilibrium* free energy.
  - Typical transverse displacement deduced from the roughness at equilibrium: $\boxed{u(L) \sim L^{\zeta}}$
  - Elastic (free-)energy associated to this displacement: $E_{\mathrm{el}}(L) \sim L^d \cdot c\dfrac{u(L)^2}{L^2} \sim L^{d-2+2\zeta}$
  - Under an external force, corresponding (free-)energy: $E_f(L) \sim f L^d u(L) \sim f L^{d+\zeta}$
  - Minimum size *L* for which it is worth to overcome a barrier, thus depends on the force:

$$E_{\mathrm{el}}(L_{\mathrm{opt}}) = E_f(L_{\mathrm{opt}}) \Leftrightarrow \boxed{L_{\mathrm{opt}}(f) \sim f^{-(2-\zeta)}} \Leftrightarrow \boxed{E_{\mathrm{el}}(L_{\mathrm{opt}}(f)) \sim f^{-\mu}}$$



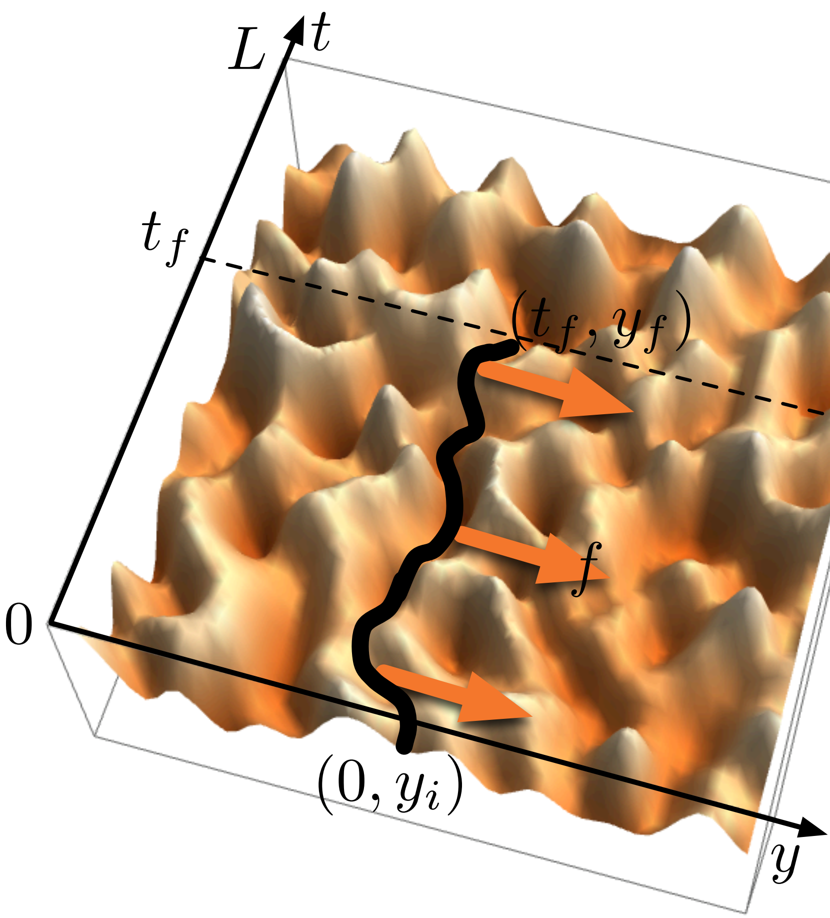
- Mean steady-state velocity controlled by the typical ‘largest barrier’, which controls the mean first passage time (MFPT). Under an Arrhenius assumption:

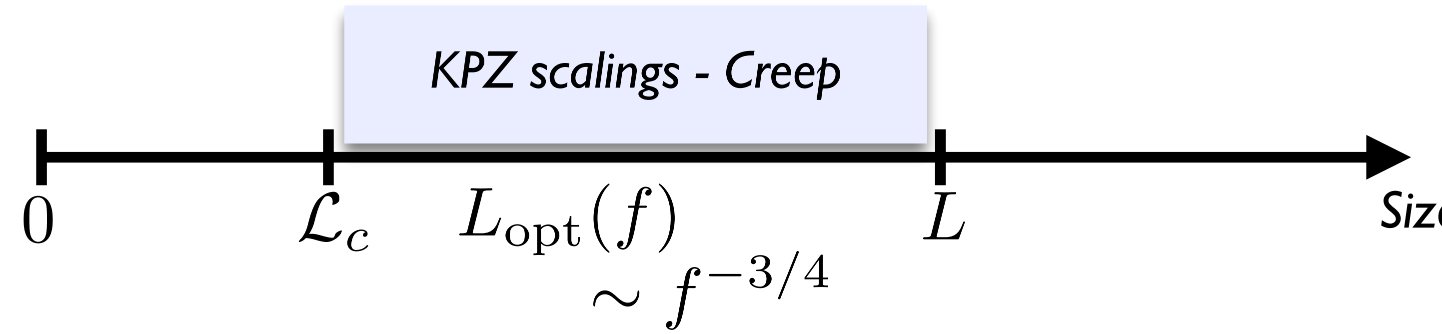
$$\boxed{v_T(F) \sim e^{-\frac{1}{T}E_{\mathrm{el}}(L_{\mathrm{opt}}(F))} \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}}} \qquad \mu = \frac{d-2+2\zeta}{2-\zeta} \overset{(\zeta=2/3)}{=} \frac{1}{4}$$

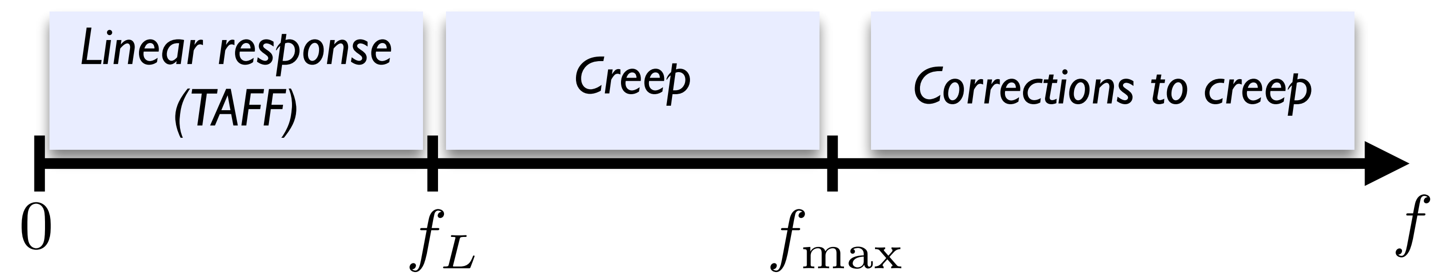
P. Chauve, T. Giamarchi, P. Le Doussal, *Phys. Rev. B* 62, 6241 (2000) **[very nice intro as well!]**

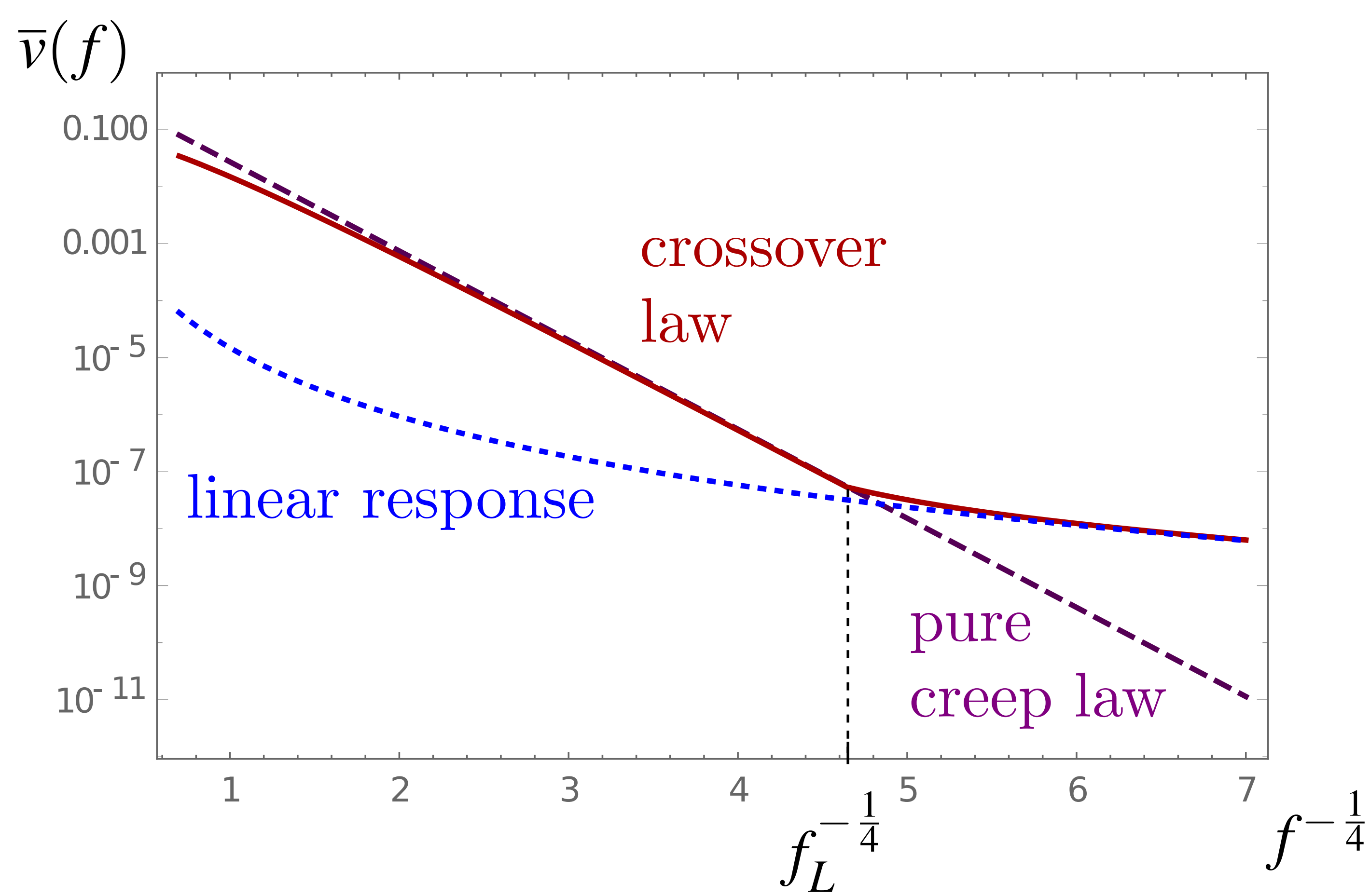
E. Ferrero, L. Foini, T. Giamarchi, A. B. Kolton, A. Rosso, *Annu. Rev. Cond. Math. Phys.* 12, 111 (2021) **[recent review]**

Quasistatic 'creep' regime - Regime of validity of the creep prediction









E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

Quasistatic ‘creep’ regime - Model reduction?

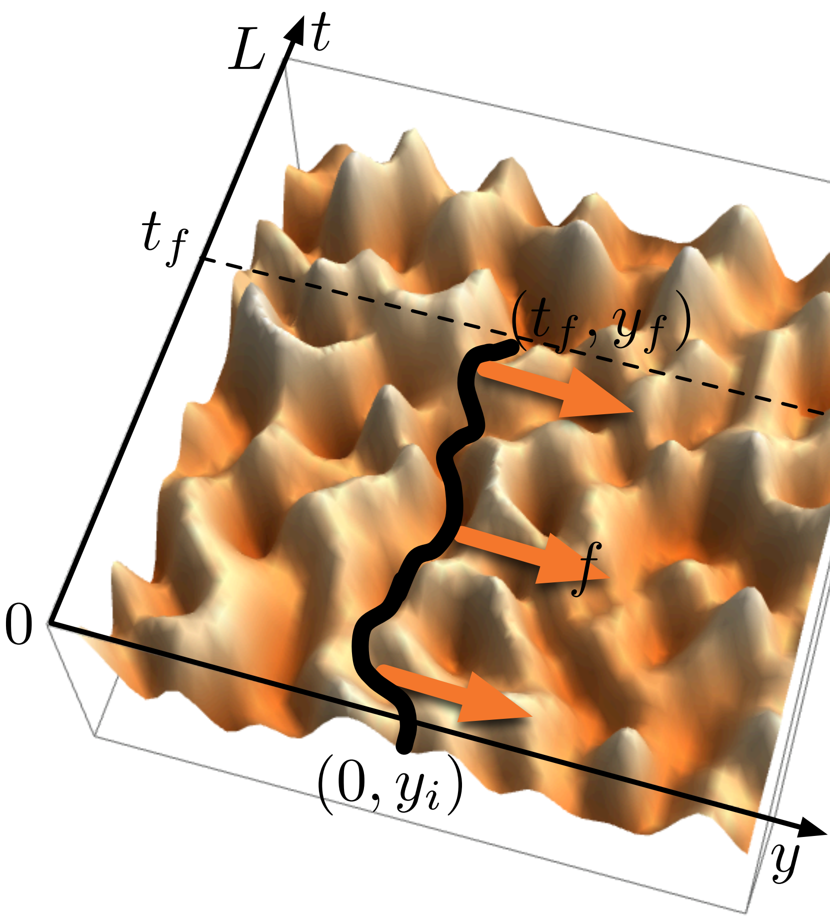
- Focus on low temperature T / small force f / large system size L: creep prediction?

$$v_T(F) \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}}$$

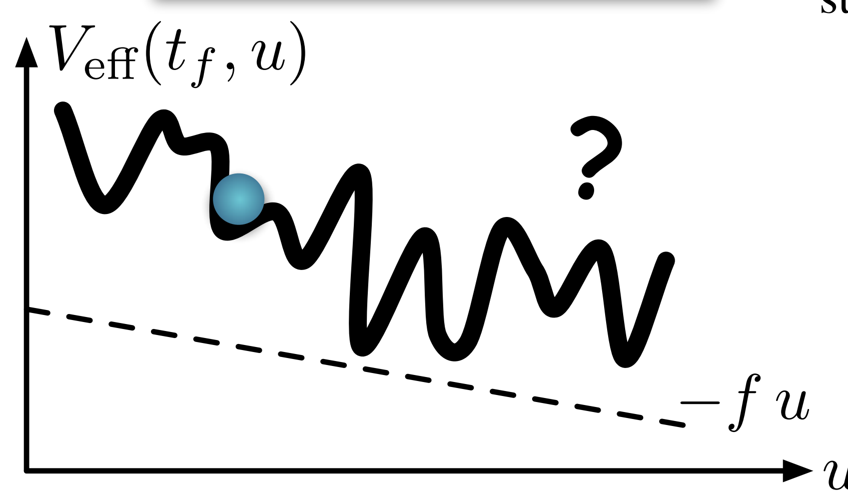
1D interface, short-range elasticity (elastic limit), random-bond disorder

$$\mu = \frac{d-2+2\zeta}{2-\zeta} \overset{(\zeta=2/3)}{=} \frac{1}{4}$$

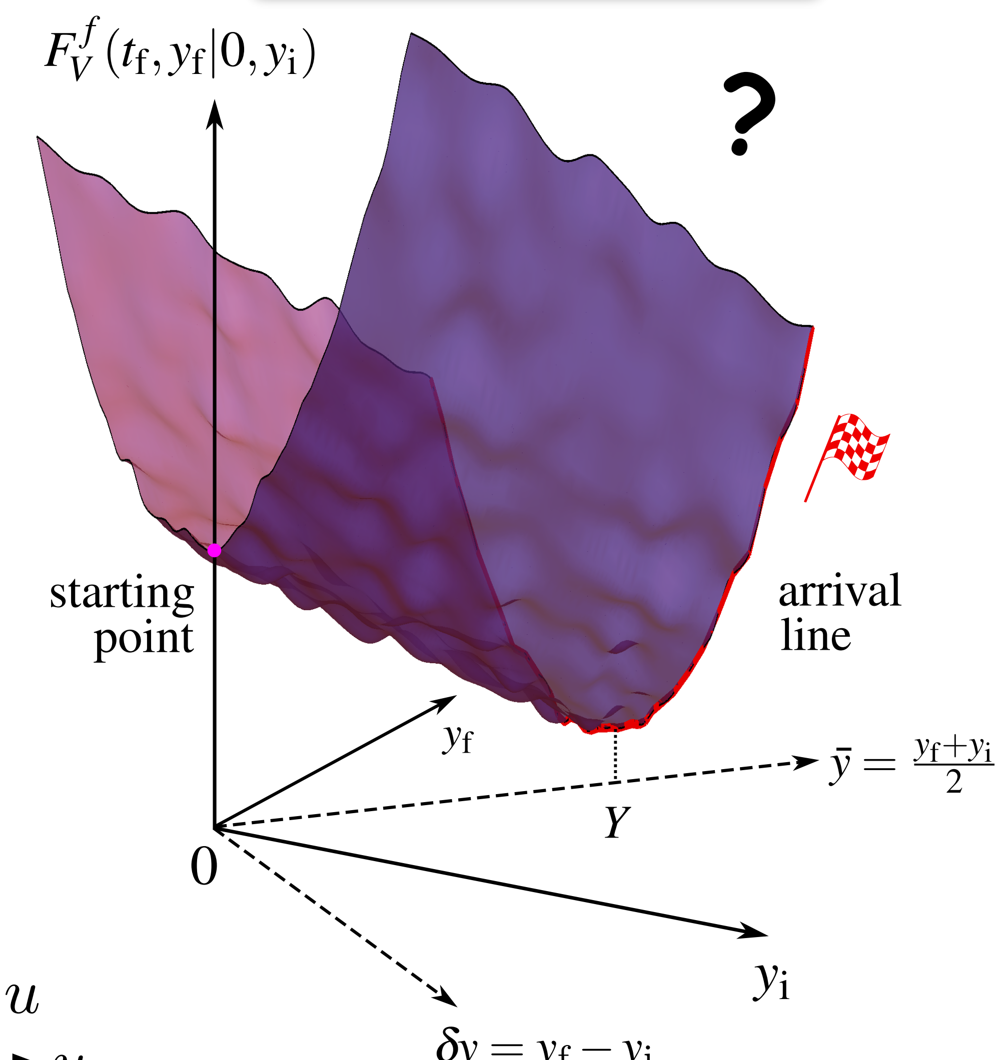
- 1D elastic interface in a 2D disordered energy landscape



1 DOF: center of mass?



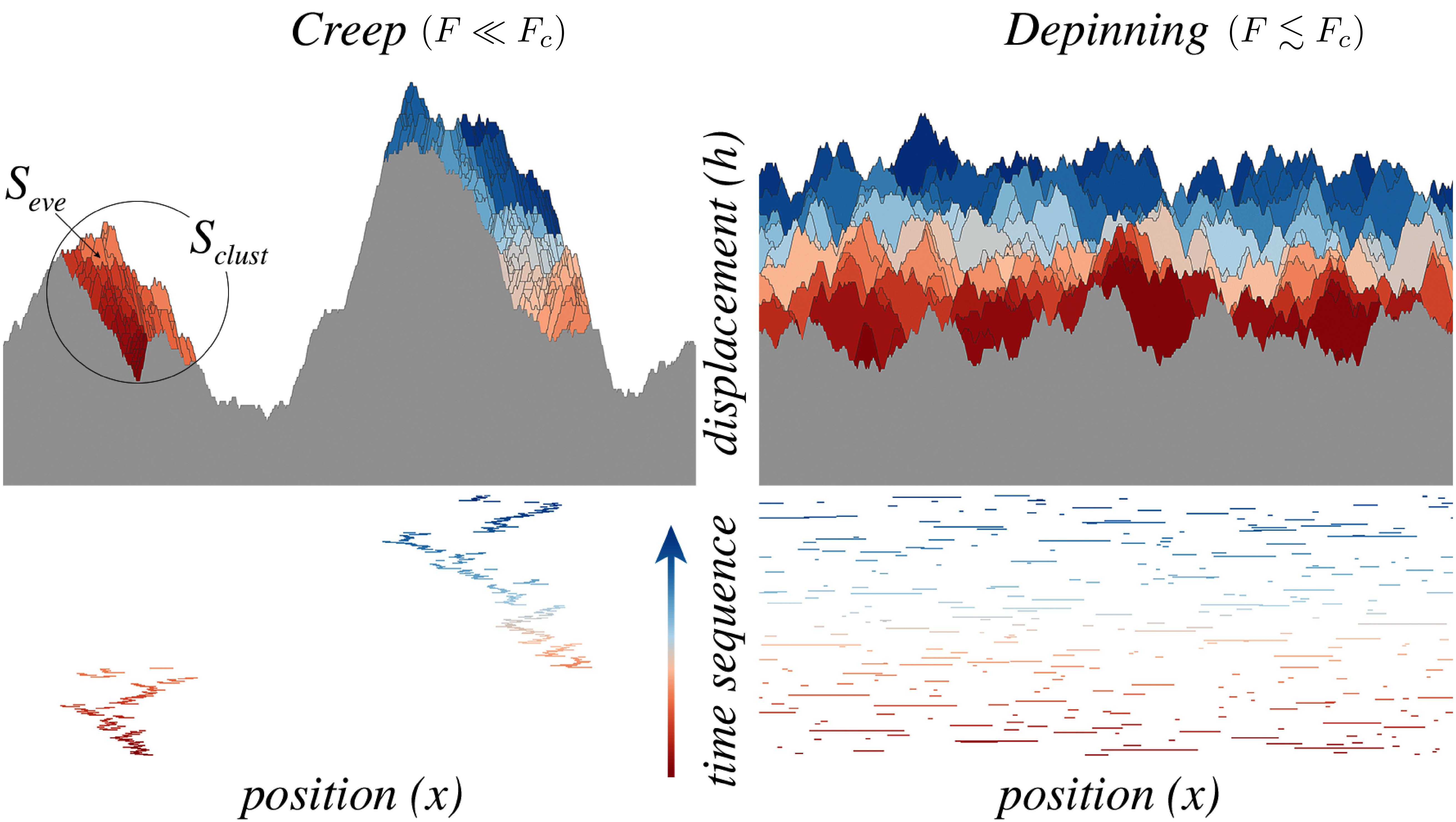
2 DOFs: center of mass & elasticity?



E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

Quasistatic 'creep' regime - Numerical study on avalanches

E. Ferrero, L. Foini, T. Giamarchi, A. B. Kolton, & A. Rosso, *Phys. Rev. Lett.* 118, 147208 (2017),
"Spatiotemporal Patterns in Ultraslow Domain Wall Creep Dynamics".



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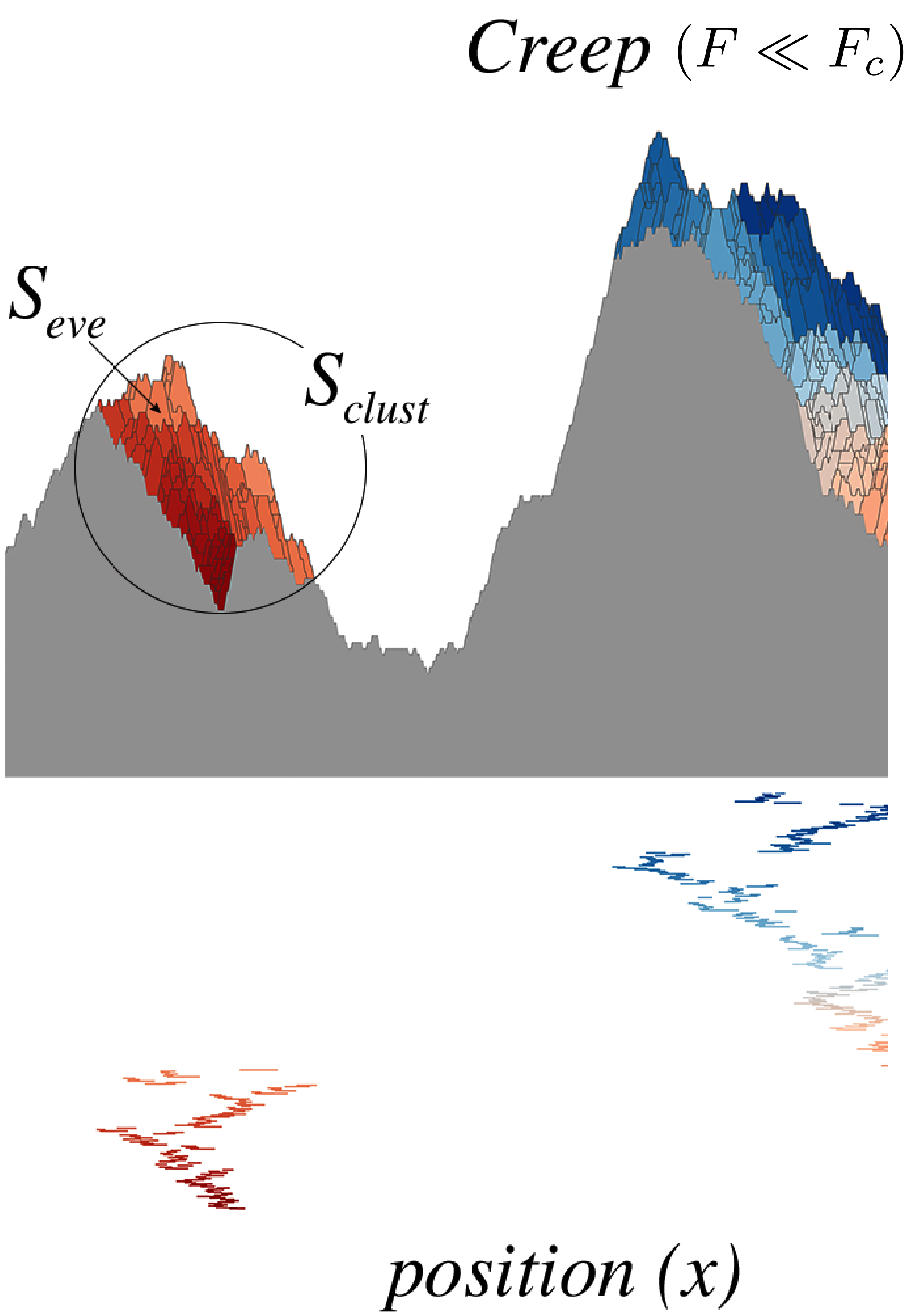

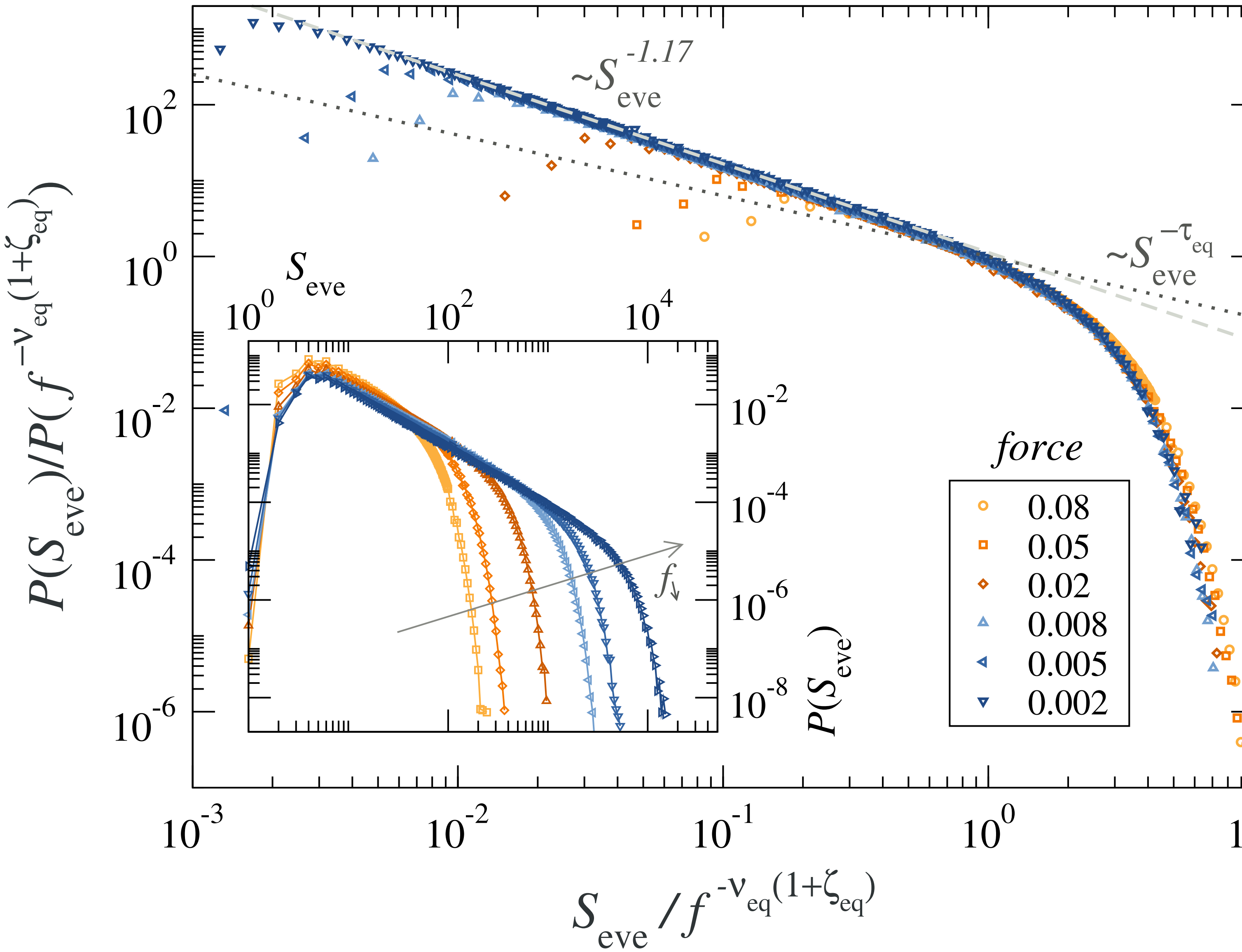

FIG. 3. Events' size distributions $P(S_{\rm eve})$ at different forces (inset), collapsed by plotting $P(S_{\rm eve})/P(S_c)$ vs $S_{\rm eve}/S_c$, with $S_c(f) = f^{-\nu_{\rm eq}(1+\zeta_{\rm eq})}$ (main panel), therefore validating the expected creep scaling $L_{\rm opt} \sim f^{-\nu_{\rm eq}}$, given $S_c \sim L_{\rm opt}^{(1+\zeta_{\rm eq})}$.

Quasistatic 'creep' regime - Numerical study on avalanches

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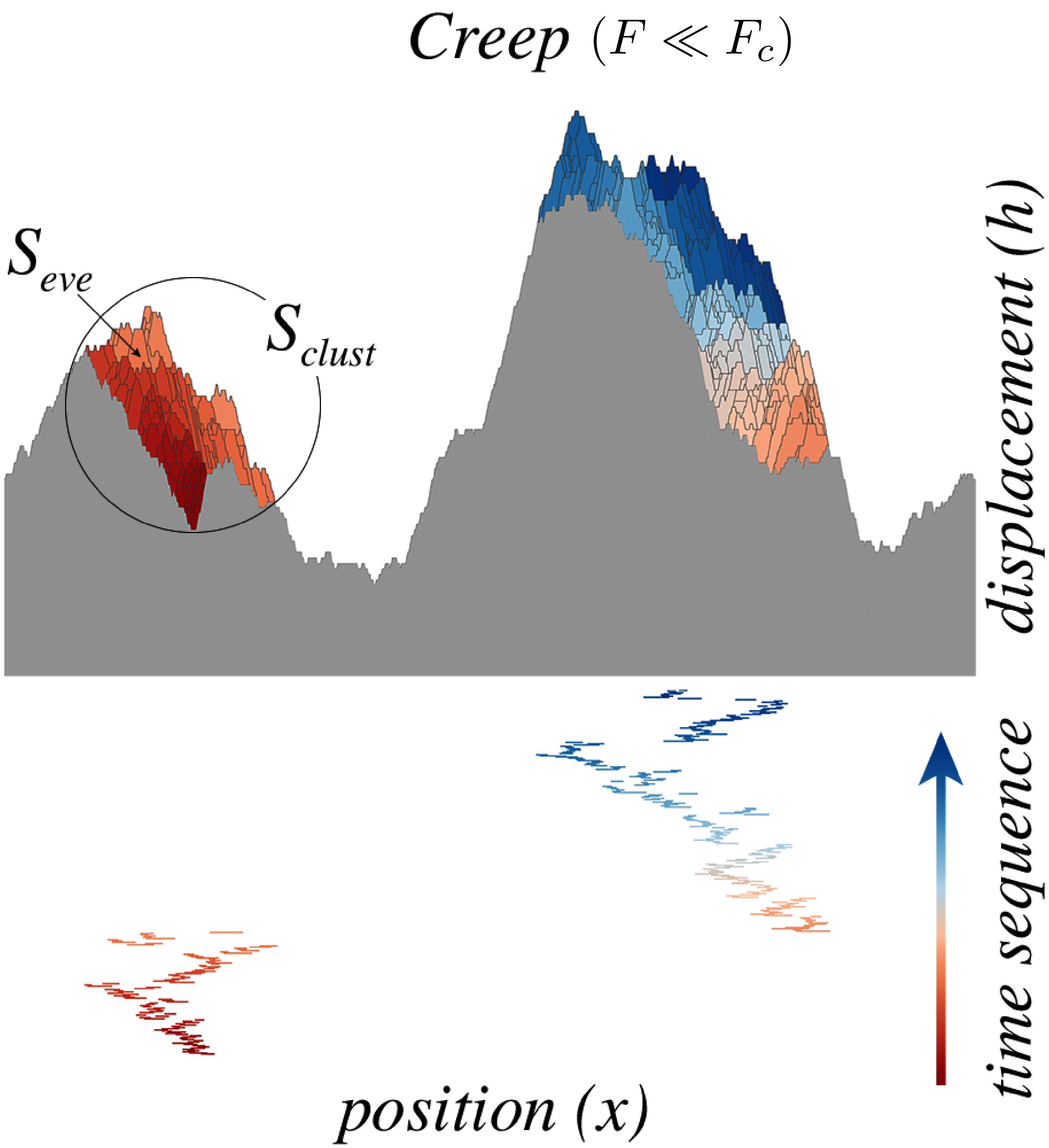

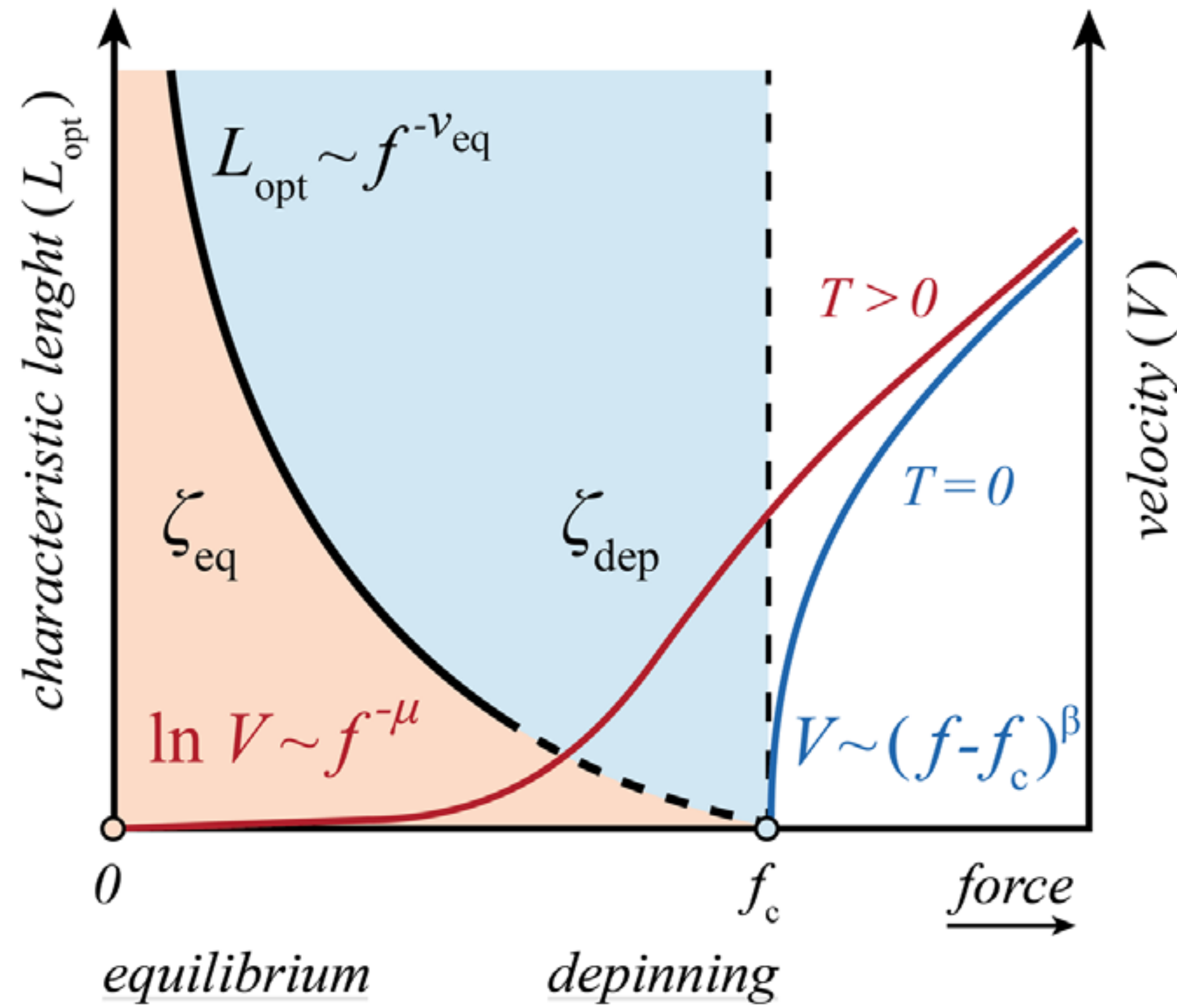

School on *Disorder in Complex Systems* at the Institut Pascal — Université Paris-Saclay
June 7-17, 2022

Statics and Dynamics of Disordered Elastic Systems

Elisabeth Agoritsas
(Physics of Complex Systems Laboratory, EPFL)

Thierry Giamarchi
(DQMP , University of Geneva, Switzerland)

Outline — Part II: Dynamics

- Quenched Edwards-Wilkinson
- Equilibrium dynamics: 2D→1D
- Fast-flow regime
- Creep regime
- Depinning / Thermal rounding
- Moving Bragg glass