Infinite dimensional solution for AQRD dynamics

(a sketch)

from Agoritsas, J. Stat. Phys 2021

$$x_i(t) = x_i(0) + \underbrace{\gamma(t) c_i}_{\text{"affine" motion along the applied field.}} + u_i(t)$$

nonaffine motion (from particle interactions in solid, c.f. Dynamical Matrix)

$$\therefore r_{ij}(t) = r_{ij}(0) + \gamma(t)(c_i - c_j) + w_{ij}(t)$$

global shear is a special case, where

$$c_i = x_{i,z}(0)\,\hat{x}, \quad c_{ij} = r_{ij,z}(0)\,\hat{x},$$

for AQRD c_i is a random function on $x_i(0)$;

$$c_i = c(x_i(0)) \text{ with } \overline{c(\vec{x})} = 0 \quad (\text{no net displacement})$$

and assume gaussian for 2pt:

$$\overline{c(x)\cdot c(x')} = \frac{\ell^2\, \Xi\, e^{-|x-x'|/2\xi}}{\sqrt{2\pi}\,\xi}$$

Now individual local displacements:

$$\Rightarrow \overline{c_i} = 0$$

$$\overline{c_i \cdot c_j} = \frac{\ell^2\, \Xi\, e^{-\frac{r_{ij}(0)}{2\xi}}}{\sqrt{2\pi}\,\xi}$$

tunable amplitude w/ units of length

distance b/w two particles

Now, take infinite-dimensional limit, so that there are no correlations b/w neighboring strains:

$$d\,\overline{c_{ij}\cdot c_{i'j'}} \xrightarrow{d\to\infty} 0 \quad \text{for } (ij)\neq(i',j')$$

$$d\,\overline{c_{ij}^2} \xrightarrow{d\to\infty} 2\ell\,\Xi\left[\frac{1}{\sqrt{2\pi}\,\varepsilon} - \frac{e^{-\ell^2/2\xi^2}}{\sqrt{2\pi}\,\varepsilon}\right]$$

$$= \mathcal{F}(\Xi,\ell,\xi)$$

This gives many body dynamics:

$$\underbrace{\zeta\left[\dot{x}_i(t) - \dot{\gamma}(t)\,c_i\right]}_{\text{local drag force}} = F_i(t) + \overbrace{\xi_i(t)}^{\text{Noise with memory}} \qquad (A)$$

$$F_i(t) = -\sum_{j(\neq i)} \nabla \underbrace{V}_{\text{potential}}\left(x_i(t) - x_j(t)\right)$$

$$\langle \xi_{i\mu}(t)\rangle = 0$$

$$\langle \xi_{i\mu}(t)\,\xi_{j\nu}(s)\rangle = \delta_{ij}\,\delta_{\mu\nu}\left[2T\zeta\,\delta(t-s) + \Gamma_c(t,s)\right]$$

$$\Gamma_c(t,s) \sim e^{-\frac{|t-s|}{\tau_p}} \; (!)$$

Assume $u_i(t) \sim w_{ij}(t) \sim O(1/d)$

Goal is to take (A) (a many body equation) and write it as a scalar process:

can write in terms of the gap between pairs h_{ij}:

$$r_{ij}(t) = \ell\left(1 + \frac{h_{ij}(t)}{d}\right)$$

$\Downarrow$ lots of math

$$h(t) = h_0 + y(t) + \Delta r(t)$$

this depends on 3 self-consistent kernels:

$$K(t) \sim \langle \nabla^2 V \rangle$$

$$M_C(t,s) \sim \langle \nabla V \cdot \nabla V \rangle$$

$$M_R(t,s) \sim \delta \langle \nabla V \rangle / \delta P$$

$\Downarrow$

all dynamics is rescaled by $\mathcal{F}(\Xi, \ell, \xi) / \ell^2$