Combination of Higgs boson pair production and single Higgs production in the ATLAS experiment

# <u>Alkaid Cheng</u> (University of Wisconsin-Madison) on behalf of the ATLAS collboration

September 13, 2022





Alkaid Cheng

1/19



- The Higgs self-couplings \u03c6<sub>i</sub> are one of the properties of the Higgs boson that are still largely unconstrained experimentally.
- ► These couplings provide key information on the **shape of the Higgs potential** V(H) which has important physics implications (e.g. stability of the universe [JHEP08(2012)098]).

$$V(H) = \frac{1}{2}m_{H}^{2}H^{2} + \lambda_{3}\nu H^{3} + \frac{1}{4}\lambda_{4}H^{4} \quad \lambda_{3} = \lambda_{4} \text{ in SM}$$

- ► The quartic coupling  $\lambda_4$  is out of reach for the LHC [PRD 72, 053008], but a handle on the **trilinear coupling**  $\lambda_3$  is possible.
- In particular,  $\lambda_3$  can be directly accessed through the production of Higgs boson pairs (HH).
- Additional contributions also come from single Higgs production (H) via NLO EW corrections.
- ► Deviation of the coupling strength from SM (measured by the coupling modifier  $\kappa_{\lambda} = \lambda_3 / \lambda_3^{\text{SM}}$ ) may point to **new physics**.

イロト イポト イヨト イヨト

## Higgs boson pair (HH) production

- The two dominant HH production modes are gluon-glupn fusion (ggF) and vector boson fusion (VBF) which contribute to over 95% of the HH production cross section in the SM.
- In addition to the Higgs self coupling (κ<sub>λ</sub>), the ggF production mode is also sensitive to the top Yukawa coupling (κ<sub>t</sub>), while the VBF production mode allows the probe of couplings with one or two vector bosons (κ<sub>V</sub> and κ<sub>2V</sub>).



SM  $\sigma_{\rm HH}^{\rm VBF} = 1.72 \text{ fb}^{-1}$  at N3LO ( $m_{\rm H}$ =125.09 GeV)

The SM HH production is an extremely rare process, with only ~4000 events expected in Run 2 (∫ L=139 fb<sup>-1</sup> @ 13 TeV).

< ロ > < 同 > < 回 > < 回 >

## Single Higgs (H) production

▶ The Higgs self-coupling also contributes to single Higgs processes via NLO EW corrections.



 These corrections affect the inclusive cross-sections, Higgs-boson branching fractions and differential distributions.

Alkaid Cheng	Higgs Hunting 2022	Sep	tember	13, 2022	4/19

4 D N 4 B N 4 B N 4 B N

= nan

### Contribution of $\kappa_{\lambda}$ in single Higgs: cross section and branching ratio

• The single Higgs production cross section and decay branching ratios can be written in terms of  $\kappa_{\lambda}$ :

$$\begin{split} \mu_i(\kappa_\lambda,\kappa_i) &= \frac{\sigma^{\text{BSM}}}{\sigma^{\text{SM}}} = Z_H^{\text{BSM}}(\kappa_\lambda) \left[ \kappa_i^2 + \frac{(\kappa_\lambda - 1)C_1^i}{K_{EW}^i} \right] \\ \mu_f(\kappa_\lambda,\kappa_i) &= \frac{\text{BR}_f^{\text{BSM}}}{\text{BR}_f^{\text{SM}}} = \frac{\kappa_f^2 + (\kappa_\lambda - 1)C_i^f}{\sum_j \text{BR}_j^{\text{SM}} \left[ \kappa_j^2 + (\kappa_\lambda - 1)C_1^i \right]} \\ Z_H^{BSM}(\kappa_\lambda) &= \frac{1}{1 - (\kappa_\lambda^2 - 1)\delta Z_H}, \text{with } \delta Z_H = -1.536 \times 10^{-3} \end{split}$$

 $\kappa_i, \kappa_f$ : Additional coupling modifiers such as  $\kappa_F$  or  $\kappa_V$ 

ż

 $Z_{H}^{\text{BSM}}$ : **Universal**  $O(\lambda_{3}^{2})$  correction from wavefunction renormalization

- $C_1^i$ : Linear  $O(\lambda_3)$  correction from interference between LO amplitude and the virtual NLO EW  $\lambda_3$  correction (process and kinematic dependent)
- $K_{EW}^i$ : k-factor for the full set of NLO EW correction



Alkaid Cheng

September 13, 2022 5/19

## Contribution of $\kappa_{\lambda}$ in single Higgs: differential distribution

The production modes with the largest kinematic dependence on λ<sub>3</sub> are VH, tt
H [Eur. Phys. J. C (2017) 77]. They provide additional sensitivity on λ<sub>3</sub> at differential level.



► The differential information is encoded through the simplified template cross-section (STXS) framework available for tt

, VH and VBF processes. Only inclusive information is available for ggH and tHj due to missing differential calculations.

• Combination of HH and H productions using the **full Run 2 data**  $(126-139 \text{ fb}^{-1})$  of pp collisions at  $\sqrt{s} = 13$  TeV from the ATLAS detector [ATLAS-CONF-2022-050].

Channel	Integrated luminosity [fb <sup>-1</sup> ]	Reference	
$HH \rightarrow b\bar{b}\gamma\gamma$	139	[PRD 106, 052001]	
$HH \rightarrow b\bar{b}\tau\bar{\tau}$	139	[ATLAS-CONF-2021-030]	
$HH  ightarrow b ar{b} b ar{b}$	126	[ATLAS-CONF-2022-035]	
$H \rightarrow \gamma \gamma$	139	[CERN-EP-2022-094]*	
$H \to ZZ^* \to 4\ell$	139	[Eur. Phys. J. C 80 (2020) 957]	
$H \rightarrow \tau^+ \tau^-$	139	[JHEP 08 (2022) 175]	
$H \rightarrow WW^* \rightarrow e\nu\mu\nu$ (ggF, VBF)	139	[CERN-EP-2022-078]**	
$H \rightarrow b\bar{b}$ (VH)	139	[Eur. Phys. J. C 81 (2021) 178]	
$H \rightarrow b\bar{b}$ (VBF)	126	[Eur. Phys. J. C 81 (2020) 537]	
$H \rightarrow b\bar{b}$ $(t\bar{t}H)$	139	[JHEP 06 (2022) 097]	

\*to appear in JHEP \*\*to appear in PRD

- In addition to κ<sub>λ</sub>, the coupling modifiers κ<sub>t</sub>, κ<sub>V</sub>, κ<sub>2V</sub> (κ<sub>t</sub>, κ<sub>b</sub>, κ<sub>τ</sub>, κ<sub>V</sub>) are considered in the double (single) Higgs processes.
- ► The **overlap** between/within HH and H analyses is **negligible** or has minor impact on the statistical results.
- Uncertainties across channels are correlated when relevant.

イロト 不得 トイヨト イヨト

э.

## HH Combination Results: Limit on HH Production

- Upper limit on  $\mu_{HH} = \sigma_{HH} / \sigma_{HH}^{SM}$  at 95% CL assuming no *HH* production.
- ▶ The HH cross section includes both ggF and VBF production modes.



 Improvement of around 50% on top of the luminosity increase compared to the 2019 combination [PRB 800 (2020) 135103].

8/19

< ロ > < 同 > < 回 > < 回 >

## HH + H Combination Results: $\kappa_{\lambda}$ -only and $\kappa_{\lambda}$ generic constraints

- Assuming **BSM only affect**  $\kappa_{\lambda}$ .
- Expected result derived from Asimov dataset generated under the SM assumption with all coupling modifiers at unity.
- Two scenarios considered:

 $\kappa_{\lambda}$  only: Fit with  $\kappa_{\lambda}$  floating and all other coupling modifiers fixed to unity.

 $\kappa_{\lambda}$  generic: Fit with all coupling modifiers ( $\kappa_{\lambda}$ ,  $\kappa_t$ ,  $\kappa_b$ ,  $\kappa_\tau$ ,  $\kappa_V$ ) floating with the exception of  $\kappa_{2V}$  that is fixed to unity (no available parameterisation of single-Higgs NLO EW correction as a function of  $\kappa_{2V}$ )



Dominant contribution from the combined HH channels.

► The constraints on  $\kappa_{\lambda}$  is still substantial even in the generic model (where  $\kappa_t, \kappa_b, \kappa_\tau, \kappa_V$  are floating).

Alkaid Cheng

Higgs Hunting 2022

### HH + H Combination Results: Summary of $\kappa_{\lambda}$ constraints



- ► The single Higgs processes allow the constrain of  $\kappa_{\lambda}$  with fewer model-dependent assumptions (by allowing other coupling modifiers to be free parameters)
- Improvement of around 50% over the 2019 combination [PRB 800 (2020) 135103].

Image: A matrix

- Combination of single and double Higgs boson production using the full Run 2 dataset in the ATLAS experiment to constrain the Higgs boson self-coupling λ<sub>3</sub>.
- Input analyses considered

single Higgs:  $\gamma\gamma$ ,  $ZZ^*$ ,  $WW^*$ ,  $\tau^+\tau^-$  and  $b\bar{b}$  final states

double Higgs:  $b\bar{b}\gamma\gamma$ ,  $b\bar{b}\tau\bar{\tau}$  and  $b\bar{b}b\bar{b}$  decay channels

• Constraints in terms of the coupling modifier  $\kappa_{\lambda} = \lambda_3 / \lambda_3^{\text{SM}}$ :

 $\kappa_{\lambda}$ -only:  $-0.4 < \kappa_{\lambda} < 6.3$  (obs. 95% C.L.), best fit  $\kappa_{\lambda} = 3.0^{+1.8}_{-1.9}$ 

 $\kappa_{\lambda} + \kappa_t, \kappa_V, \kappa_b, \kappa_{\tau}$ :  $-1.4 < \kappa_{\lambda} < 6.1$  (obs. 95% C.L.), best fit  $\kappa_{\lambda} = 2.3^{+2.1}_{-2.0}$ Big improvement (~ 50%) over the previous combination (2019).

- The results provide the most stringent constraint on the Higgs self-coupling to-date.
- Stayed tuned for Run3 and the High Luminosity LHC (HL-LHC)!

## Appendix: The Simplified Template Cross Section (STXS) framework



- ▶ Physical cross sections defined in mutually exclusive regions of phase space ("bins").
- Provide more fine-grained measurements for individual Higgs production modes in various kinematic regions, and reduce the theoretical uncertainties that are directly folded into the measurements.
- Allow for the use of MVA techniques and provide a common framework for the combination of measurements in different decay channels and experiments.

Alkaid Cheng

Higgs Hunting 2022

- ► The C<sup>1</sup><sub>1</sub> coefficients on the single Higgs production has been extended to the latest STXS 1.2 framework with higher phase-space granularity. It allows the use of more differential information for additional sensitivity [LHCHWG-2022-002].
- ▶ The  $C_1^i$  coefficients are computed for the  $t\bar{t}H$ , V(lep)H and  $H_{jj}$  processes. The coefficients for the **ggH and tHj STXS bins are not computed** because the theoretical calculations at the differential level are not available (the inclusive coefficients are used instead).
- ▶ No available  $C_1^i$  calculation on tHW, bbH and ggZH processes.
- ▶ Events are generated at LO using MadGraph5 for each single-Higgs process. They are classified into STXS bins via Rivet. In each STXS bin, the coefficient C<sup>1</sup><sub>1</sub> is computed as

$$C_1^i = \frac{\sum_j w_{\mathsf{NLO}}^j}{\sum_j w_{\mathsf{LO}}^j}$$

where  $w_{LO}^j$  is the weight from LO cross section, and  $w_{NLO}^j$  is the weight from LO cross section corrected for the  $\kappa_{\lambda}$  effect through NLO EW correction. The sums run over all the events in *i*-th STXS bin.

#### Appendix: Previous Combination Results (2019)

 In 2019, a combination of single-Higgs and double-Higgs production by ATLAS was performed using the partial Run2 dataset [ATLAS-CONF-2019-049].

Channel	Integrated luminosity [fb $^{-1}$ ]	Reference	
$HH \rightarrow b\bar{b}\gamma\gamma$	36.1	[JHEP 11 (2018) 040]	
$HH \rightarrow b\bar{b}\tau\bar{\tau}$	36.1	[Phys. Rev. Lett. 121 (2018)]	
$HH \rightarrow b\bar{b}b\bar{b}$	27.5	[JHEP 01 (2019) 030]	
$H \rightarrow \gamma \gamma$ (excluding $t\bar{t}H$ )	79.8	[Phys. Rev. D 98 (2018)]	
$H \to ZZ^* \to 4\ell$	79.8	[JHEP 03 (2018) 095]	
$H \rightarrow \tau^+ \tau^-$	36.1	[Phys. Rev. D 99 (2019)]	
$H \rightarrow WW^* \rightarrow e\nu\mu\nu$	36.1	[Phys. Lett. B 789 (2019) 508]	
$H \rightarrow b\bar{b}$ (VH)	79.8	[Phys. Lett. B 786 (2018) 59]	
$H \rightarrow b\bar{b}$ $(t\bar{t}H)$	36.1	[Phys. Rev. D 97 (2018)]	
$H \rightarrow$ multilepton $(t\bar{t}H)$	36.1	[Phys. Rev. D 97 (2018)]	



Alkaid Cheng

September 13, 2022

14/19



exp.

-6.2, 11.6

イロト イ理ト イヨト イヨト

## Appendix : HH Combination - Constraints on $\kappa_{\lambda}$ and $\kappa_{2V}$

- Upper limit on HH production cross section at 95% CL as a function of  $\kappa_{\lambda}$  and  $\kappa_{2V}$
- In the  $\kappa_{2V}$  case, the ggF production mode is considered as a background.



- The sensitivity on  $\kappa_{\lambda}$  mainly comes from the  $b\bar{b}\gamma\gamma$  and  $b\bar{b}\tau^{+}\tau^{-}$  channels.
- The  $b\bar{b}b\bar{b}$  channel has a dominant contribution to the constraint on  $\kappa_{2V}$ .

< ロ > < 同 > < 回 > < 回 >

A reweighting procedure is used to model any value of  $\kappa_{\lambda}$  using a basis of 3 samples. This is possible due to the structure of ggF HH production. The two contributing diagrams are the box and triangle diagrams. These generate a differential cross-section of

$$\frac{d\sigma(\kappa_{\lambda})}{dm_{HH}} = |A(\kappa_{\lambda})|^2 = |\kappa_{\lambda}M_{\triangle}(m_{HH}) + M_{\square}(m_{HH})|^2$$

which can be re-written as

$$\frac{d\sigma(\kappa_{\lambda})}{dm_{HH}} = \kappa_{\lambda}^2 a_1(m_{HH}) + \kappa_{\lambda} a_2(m_{HH}) + a_3(m_{HH})$$

where  $a_1, a_2$ , and  $a_3$  depend only on  $m_{HH}$ .

While the above equation should hold for any variable,  $m_{HH}$  is selected due to the assumption that acceptance times efficiency only depends on  $m_{HH}$ . The rest of the kinematics are assumed to vary coherently with the truth  $m_{HH}$  distributions. The values of the  $a_i$  are solved for using  $\kappa_{\lambda} = 0, 1$  and 20 samples as a basis, with each sample consisting of over 10 million truth-level events simulated using Powheg at NLO+FT in  $m_{HH}$  bins of 10 GeV with a bin range of 200 to 1000 GeV. These samples have no decays or cuts and are common to all di-Higgs analyses.

The  $m_{HH}$  distributions of the basis set are linearly combined using the obtained values of the  $a_i$  to create distributions for each  $\kappa_{\lambda} \in [-30, 30]$  in increments of 0.2. Weights for each  $\kappa_{\lambda}$  are then defined as the ratio of the given distribution to that of  $\kappa_{\lambda} = 1$ . These weights are applied to the generated SM samples based on their truth  $m_{HH}$  values to obtain reweighted samples corresponding to the full range of  $\kappa_{\lambda}$  under investigation. The reweighting procedure is only used to determine the signal shapes, with the results normalized to the theoretical cross-section.

A B A B A B A
 A B A
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 A
 A

#### Appendix: HH $\kappa_{\lambda}$ Parameterisation - VBF

The full cross section for the VBF to HH process involves three diagrams,

$$\sigma(\kappa_{VV},\kappa_{\lambda},\kappa_{V}) = |A|^{2} = |\kappa_{V}\kappa_{\lambda}M_{s} + \kappa_{V}^{2}M_{t} + \kappa_{VV}M_{x}|^{2}$$

Expanding the absolute square of the amplitude A then yields six terms:

$$\sigma = \kappa_V^2 \kappa_\lambda^2 a_1 + \kappa_V^4 a_2 + \kappa_{VV}^2 a_3 + \kappa_V^3 \kappa_\lambda a_4 + \kappa_V \kappa_\lambda \kappa_{VV} a_5 + \kappa_V^2 \kappa_{VV} a_6$$

This requires the combination of multiple different MC samples (in this case, six) in order to model the signal hypothesis at any arbitrary point in  $\kappa_{VV}$ ,  $\kappa_{\lambda}$ ,  $\kappa_{V}$  space. Unlike with ggF, in the case of VBF these MC samples cannot be truth-level samples, but rather must be samples that have been run through reconstruction and selection.

Using the basis samples  $(\kappa_{\lambda}, \kappa_{2V}, \kappa_{V}) = (1, 1, 1), (2, 1, 1), (10, 1, 1), (1, 1, 0.5), (-5, 1, 0.5)$ , the differential cross section can be expressed as

$$\begin{split} \frac{d\sigma_{\text{VBF}}}{d\Phi}(\kappa_{\lambda},\kappa_{2V},\kappa_{V}) &= \left(\frac{68\kappa_{2V}^{2}}{135} - 4\kappa_{2V}\kappa_{V}^{2} + \frac{20\kappa_{2V}\kappa_{V}\kappa_{\lambda}}{27} + \frac{772\kappa_{V}^{4}}{135} - \frac{56\kappa_{V}^{3}\kappa_{\lambda}}{27} + \frac{\kappa_{V}^{2}\kappa_{\lambda}^{2}}{9}\right) \times \frac{d\sigma_{\text{VBF}}}{d\Phi}(1,1,1) \\ &+ \left(-\frac{4\kappa_{2V}^{2}}{5} + 4\kappa_{2V}\kappa_{V}^{2} - \frac{16\kappa_{V}^{4}}{5}\right) \times \frac{d\sigma_{\text{VBF}}}{d\Phi}(1,1,5,1) \\ &+ \left(\frac{11\kappa_{2V}^{2}}{60} + \frac{\kappa_{2V}\kappa_{V}^{2}}{3} - \frac{19\kappa_{2V}\kappa_{V}\kappa_{\lambda}}{4} - \frac{53\kappa_{V}^{4}}{30} - \frac{13\kappa_{V}^{3}\kappa_{\lambda}}{6} - \frac{\kappa_{V}^{2}\kappa_{\lambda}^{2}}{8}\right) \times \frac{d\sigma_{\text{VBF}}}{d\Phi}(2,1,1) \\ &+ \left(-\frac{11\kappa_{2V}^{2}}{140} + \frac{11\kappa_{2V}\kappa_{V}\kappa_{\lambda}}{216} + \frac{13\kappa_{V}^{4}}{270} - \frac{5\kappa_{V}^{3}\kappa_{\lambda}}{54} + \frac{\kappa_{V}^{2}\kappa_{\lambda}^{2}}{72}\right) \times \frac{d\sigma_{\text{VBF}}}{d\Phi}(10,1,1) \\ &+ \left(\frac{88\kappa_{2V}^{2}}{45} - \frac{16\kappa_{2V}\kappa_{V}^{2}}{3} + \frac{4\kappa_{2V}\kappa_{V}\kappa_{\lambda}}{9} + \frac{152\kappa_{V}^{4}}{45} - \frac{4\kappa_{V}^{3}\kappa_{\lambda}}{9}\right) \times \frac{d\sigma_{\text{VBF}}}{d\Phi}(1,1,0.5) \\ &+ \left(\frac{82V^{2}}{45} - \frac{4\kappa_{2V}\kappa_{V}\kappa_{\lambda}}{9} - \frac{8\kappa_{V}^{4}}{45} + \frac{4\kappa_{V}^{3}\kappa_{\lambda}}{9}\right) \times \frac{d\sigma_{\text{VBF}}}{d\Phi}(-5,1,0.5) \end{split}$$

Alkaid Cheng

A combined likelihood  $\mathcal{L}(\mathcal{D}|\mu, \theta)$  is constructed as a product of likelihoods from each channel.

D: Data  $\mu$ : Parameter of Interest (POI)  $\theta$  Nuisance Parameters

Profile likelihood ratio 
$$\tilde{\lambda}(\mathcal{D}|\mu) = \begin{cases} \frac{\mathcal{L}(\mathcal{D}|\mu,\hat{\theta}(\mu))}{\mathcal{L}(\mathcal{D}|\hat{\mu},\hat{\theta})} & \hat{\mu} \ge 0, \\ \frac{\mathcal{L}(\mathcal{D}|\mu,\hat{\theta}(\mu))}{\mathcal{L}(\mathcal{D}|0,\hat{\theta}(0))} & \hat{\mu} < 0, \end{cases}$$
(1)

$$\text{Test statistics} \qquad \tilde{q}_{\mu} = \begin{cases} -2\ln\tilde{\lambda}(\mathcal{D}|\mu) & \hat{\mu} \leq \mu, \\ 0 & \hat{\mu} > \mu, \end{cases} = \begin{cases} -2\ln\frac{\mathcal{L}(\mathcal{D}|\mu,\hat{\theta}(\mu))}{\mathcal{L}(0,\hat{\theta}(0))} & \hat{\mu} \leq 0, \\ -2\ln\frac{\mathcal{L}(\mathcal{D}|\mu,\hat{\theta}(\mu))}{\mathcal{L}(\mathcal{D}|\hat{\mu},\hat{\theta})} & 0 \leq \hat{\mu} \leq \mu, \end{cases}$$
(2)

- $\hat{\theta}$  and  $\hat{\mu}$  are the values of  $\theta$  and  $\mu$  that maximizes the likelihood (i.e. unconditional ML).
- *θ*(μ) is the value of θ that maximizes the likelihood for the specified μ (i.e. conditional maximum-likelihood (ML) estimator of θ).
- Exclusion upper limits on the POI are set following the CLs prescription.

э

A B A B A B A
 A B A
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 A
 A