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Top pair associated Higgs boson production and the top-to-Higgs fragmentation function

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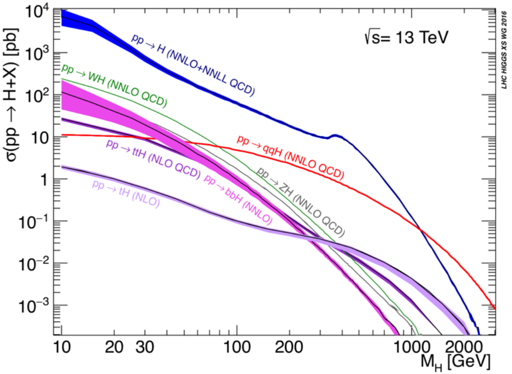
Based on: arXiv:2106.06516

Higgs Hunting, Orsay-Paris

September 12th, 2022

Introduction

$t\bar{t}H$ PRODUCTION

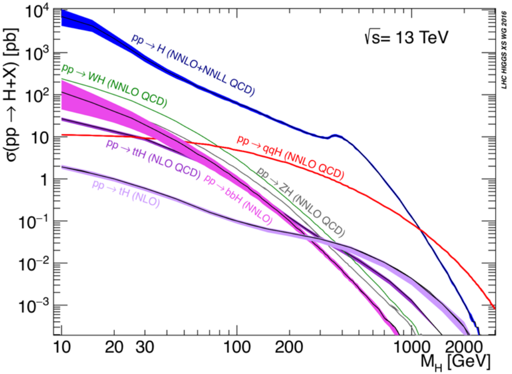


Investigating $t\bar{t}H$[1,2] for:

- ◇ Studying the SM,
- ◇ top-Higgs coupling,
- ◇ BSM physics.

[1] ATLAS, 2018, [2] CMS, 2018.

$t\bar{t}H$ PRODUCTION



$t\bar{t}H$ in literature:

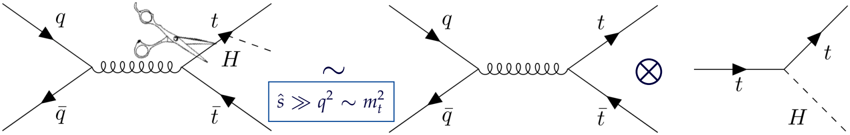
- ✓ NLO QCD corrections[3,4],
- ✓ NLO QCD corrections of $t\bar{t}H + jets$[5],
- ✓ NLO EW corrections[6].

[3] W. Beenakker, S. Dittmaier, M. Krämer, B. Plümper, M. Spira, P. M. Zerwas, 2001,
[4] L. Reina, S. Dawson and D. Wackeroth, 2002,
[5] H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro, 2014,
[6] S. Frixione, V. Hirschi, D. Pagani, H. S. Shao and M. Zaro, 2014.

Method

FINAL STATE FACTORISATION

Hard scattering and collinear emission factorise in the collinear limit:



$$\boxed{\mathrm{d}\hat{\sigma}_{q\bar{q}\to t\bar{t}H}(p_q, p_{\bar{q}}, p_H) = \int_0^1 \mathrm{d}z \; \mathrm{d}\tilde{\sigma}_{q\bar{q}\to t\bar{t}}(p_q, p_{\bar{q}}, p_t; \mu)\mathrm{D}_{t\to H}(z;\mu)},$$

with $z = \dfrac{n\cdot p_H}{n\cdot p_t}$, $n^\mu = \dfrac{1}{\sqrt{2}}(1,0,0,1)$ light-cone vector in the Higgs direction.

- Analogous to the initial state factorisation (PDFs).
- $\mathrm{D}_{t\to H}(z;\mu)$ can be perturbatively computed.

LEADING POWER FACTORISATION FORMULA

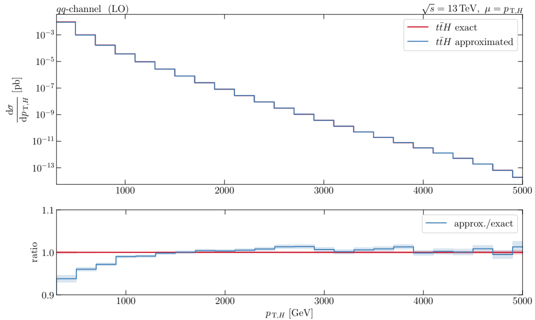
The LO $q\bar{q} \to t\bar{t}H$ cross section is decomposed as:

$$\mathrm{d}\hat{\sigma}_{q\bar{q}\to t\bar{t}H}(P) \approx \underbrace{\mathrm{d}\tilde{\sigma}_{q\bar{q}\to t\bar{t}H}(P,\mu)}_{\substack{\text{Direct Contribution} \\ q^2 \sim \hat{s}}} + \underbrace{2\int_0^1 \mathrm{d}z\, \mathrm{d}\tilde{\sigma}_{q\bar{q}\to t\bar{t}}(p = P/z) \mathrm{D}_{t\to H}(z,\mu)}_{\substack{\text{Fragmentation Contribution} \\ q^2 \ll \hat{s}}} \,.$$

The direct (infrared-safe) contribution is computed as:

$$\mathrm{d}\tilde{\sigma}_{q\bar{q}\to t\bar{t}H}(P,\mu) \approx \lim_{m_t\to 0}\Bigg(\lim_{m_H\to 0} \mathrm{d}\hat{\sigma}_{q\bar{q}\to t\bar{t}H}(P) - 2\int_0^1 \mathrm{d}z\, \mathrm{d}\tilde{\sigma}_{q\bar{q}\to t\bar{t}}(p = P/z) \lim_{m_H\to 0} \mathrm{D}_{t\to H}(z,\mu)\Bigg).$$

FRAGMENTATION APPROACH

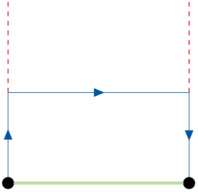


- ◇ **Good approximation at large $p_{\mathrm{T},H}$** $\rightarrow$ errors decrease to below 5% for $p_{\mathrm{T},H} > 600\,\mathrm{GeV}$.
- ◇ Enables to **resum logarithms** at high $p_{\mathrm{T},H}$ $\rightarrow$ necessary for future colliders.

[7] H. Zhang, E. Braaten, 2015.

Fragmentation function

Definition of the fragmentation function



[LO example of the following general formula]

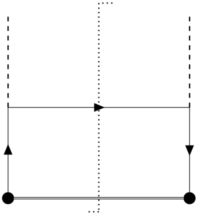
$$D_{q\to H}(z) = \frac{z^{d-3}}{4\pi}\int dx^- e^{-ip_H^+ x^-/z}\frac{1}{2N_c} Tr_{color} Tr_{Dirac}\Big[\not{n}\,\langle 0|\,\psi_q(0)$$
$$\bar{\mathrm{P}}\exp\left(ig\int_0^\infty dy^- n\cdot A_a(y^- n)T_a^T\right) a_H^\dagger(p_H)a_H(p_H)$$
$$\mathrm{P}\exp\left(-ig\int_{x^-}^\infty dy^- n\cdot A_b(y^- n)T_b^T\right)\bar{\psi}_q(x^- n)\,|0\rangle\Big]$$

Wilson Lines

This definition is gauge invariant!

[8] J.C. Collins, D.E. Soper, 1982.

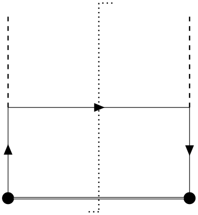
EXAMPLE: LO FRAGMENTATION FUNCTION



Applying the formula of the previous slide at LO, the fragmentation $\mathrm{D}_{t\to H}(z)$ reads:

$$\mathrm{D}_{q\to H}(z) = \frac{z^{d-3}}{4\pi}\int \frac{d^d p_t}{(2\pi)^d}(2\pi)\delta^+(p_t^2 - m_t^2)(2\pi)\delta^+(p_H^+/z - (p_t + p_H)^+)\ \frac{y_t^2\tilde{\mu}^{2\epsilon}}{2N_c}$$
$$\times \sum_{spins,colors} \mathrm{Tr}\left[\not{n}\frac{\not{p}_t + \not{p}_H + m_t}{(p_t+p_H)^2 - m_t^2}(\not{p}_t + m_t)\frac{\not{p}_t + \not{p}_H + m_t}{(p_t+p_H)^2 - m_t^2}\right].$$

EXAMPLE: LO FRAGMENTATION FUNCTION



Using reverse unitarity, the phase-space becomes a loop integral[9]:

$$\delta(x) \to \frac{1}{2\pi i}\left(\frac{1}{x - i\varepsilon} - \frac{1}{x + i\varepsilon}\right)$$

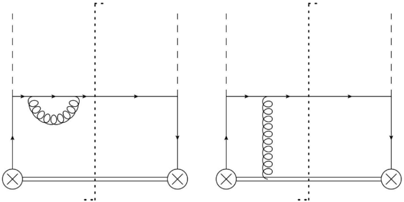
$$\mathrm{D}_{t\to H}(z) = \frac{z^{d-3}}{4\pi} \int \frac{d^d p_t}{(2\pi)^d} (2\pi) \boxed{\delta^+(p_t^2 - m_t^2)} (2\pi) \boxed{\delta^+(p_H^+/z - (p_t + p_H)^+)} \frac{y_t^2 \tilde{\mu}^{2\epsilon}}{2N_c}$$

$$\times \sum_{spins,colors} \mathrm{Tr}\left[\not{n} \frac{\not{p}_t + \not{p}_H + m_t}{(p_t + p_H)^2 - m_t^2} (\not{p}_t + m_t) \frac{\not{p}_t + \not{p}_H + m_t}{(p_t + p_H)^2 - m_t^2} \right].$$

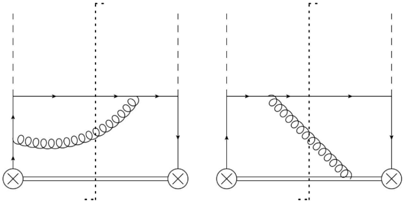
[9]C. Anastasiou, K. Melnikov, 2002.

Loop Computation

Real corrections



Virtual corrections



- **Reduction to MIs** performed with standard techniques[10],
- computation of differential equations in **Canonical form**[11]

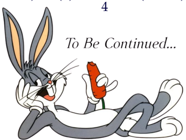
$$\mathrm{d}\vec{f}(\vec{x},\epsilon) = \epsilon \, \mathrm{d}A(\vec{x}) \, \vec{f}(\vec{x},\epsilon),$$

- analytic solution in **GPLs**,
- integration constants matched at $m_t^2 \to \infty$,
- simplyfication of polylog. expressions with **symbols**[12].

[10]*FIRE*. A. V. Smirnov, 2015, [11] J. Henn, 2013,
[12]A. Goncharov, 2009, C. Duhr, H. Gangl, J.R. Rhodes, 2012.

Master Integrals Results

$$
\begin{aligned}
I_{0,0,1,1,1,1,1} = &-\frac{1}{4\epsilon^2}\ln(z) + \frac{1}{\epsilon}\Big\{ -\mathrm{Re}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] - \frac{1}{8}\arg^2\Big(\frac{x^+}{x^-}\Big) - \frac{1}{8}\ln^2(1-r) - \frac{1}{8}\ln^2(r) - \frac{1}{8}\ln^2(1-z) - \frac{1}{8}\ln^2(z \\
&-\frac{1}{4}\ln(r)\ln(1-z) + \frac{1}{4}\ln(1-r)\ln(r) + \frac{\pi^2}{12}\Big\} + \Big\{ -\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z-1}{z-x^+}\Big)\Big] - 4\,\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z}{z-x^+}\Big)\Big] - \mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{z(1-x^+)}{z-x^+}\Big) \\
&-\mathrm{Re}\Big[\mathrm{Li}_3\Big(1-\frac{z}{1-w^+}\Big)\Big] + \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{r+z(w^+-1)}{zw^+}\Big)\Big] + \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{zw^+}{r+z(w^+-1)}\Big)\Big] - \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{rw^+}{w^+z-z+r}\Big)\Big] \\
&-\frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{w^+z-z+r}{rw^+}\Big)\Big] - \frac{1}{2}\mathrm{Re}\Big[\mathrm{Li}_3\Big(\frac{w^+}{w^-}\Big)\Big] + 2\arg\Big(\frac{x^+}{x^-}\Big)\,\mathrm{Im}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] - 2\,\mathrm{Im}\Big[\ln\Big(1-\frac{z-x^+}{z-x^-}\Big)\Big]\,\mathrm{Im}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+} \\
&-\mathrm{Im}\Big[\ln\Big(1-\frac{z-x^+}{z-x^-}\Big)\Big]\,\mathrm{Im}\Big[\mathrm{Li}_2\Big(1-\frac{x^+}{x^-}\Big)\Big] + \frac{1}{2}\ln(z)\mathrm{Li}_2\Big(1-\frac{1-r}{1-z}\Big) - \frac{1}{2}\ln(r)\mathrm{Li}_2\Big(1-\frac{1-r}{1-z}\Big) - \frac{1}{2}\ln(z)\mathrm{Li}_2(r) + \frac{1}{2}\ln(\\
&+\frac{1}{2}\ln(r)\mathrm{Li}_2\Big(1-\frac{r}{z}\Big) + 2\ln\Big(\frac{m_t^2}{\mu^2}\Big)\,\mathrm{Re}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] - 2\ln(r)\,\mathrm{Re}\Big[\mathrm{Li}_2\Big(\frac{z}{x^+}\Big)\Big] - \frac{1}{48}\ln^3(1-z) + \frac{7}{24}\ln^3(z) + \frac{1}{48}\ln^3(1-r) - \\
&+\frac{1}{4}\ln^2(1-z)\ln\Big(\frac{m_t^2}{\mu^2}\Big) + \frac{1}{4}\ln^2(z)\ln\Big(\frac{m_t^2}{\mu^2}\Big) + \frac{1}{4}\ln^2(1-r)\ln\Big(\frac{m_t^2}{\mu^2}\Big) + \frac{1}{4}\ln^2(r)\ln\Big(\frac{m_t^2}{\mu^2}\Big) + \frac{1}{4}\arg^2\Big(\frac{x^+}{x^-}\Big)\ln\Big(\frac{m_t^2}{\mu^2}\Big) - \\
&-\frac{1}{16}\ln(1-z)\ln^2(1-r) - \frac{1}{16}\ln(z)\ln^2(1-r) - \frac{3}{4}\ln(1-z)\ln^2(r) + \frac{5}{8}\ln(z)\ln^2(r) + \frac{5}{8}\ln(1-r)\ln^2(r) - \frac{1}{16}\arg^2\Big(\frac{x^+}{x^-} \\
&+\frac{1}{16}\ln^2(1-z)\ln(1-r) + \frac{9}{16}\arg^2\Big(\frac{x^+}{x^-}\Big)\ln(1-r) - \frac{1}{4}\arg^2\Big(\frac{x^+}{x^-}\Big)\ln(r) - \frac{3}{4}\ln^2(1-z)\ln(r) - \frac{1}{4}\ln^2(1-r)\ln(r) + \frac{1}{4}
\end{aligned}
$$



[13] CB, M. Czakon, T. Generet, M. Krämer, 2021.

Conclusions

CONCLUSIONS

SUMMARY

- Analytic computation of $\mathrm{D}_{t\to H}(z)$ fragmentation at $\mathcal{O}(y_t^2\alpha_s)$,
- analytic computation of $\mathrm{D}_{g\to H}(z)$ fragmentation at $\mathcal{O}(y_t^2\alpha_s)$,
- computation of $\mathrm{P}_{tH}(z)$ and $\mathrm{P}_{gH}(z)$ splitting function at $\mathcal{O}(y_t^2\alpha_s)$.

OUTLOOK

- Testing the fragmentation approximation for $t\bar{t}H$ production at NLO (work in progress),
- use the formalism to resum large logs appearing in $t\bar{t}H$ production.

Back up

CANONICAL BASIS APPROACH

- It is possible to choose a basis of MIs, the **Canonical basis**[1], such that:

$$\mathrm{d}\vec{f}(\vec{x},\epsilon) = \epsilon\, \mathrm{d}A(\vec{x})\, \vec{f}(\vec{x},\epsilon),$$

  with

$$\mathrm{d}A(\vec{x})_{ij} = \sum_k c_{ijk}\, \mathrm{dlog}(\alpha_k(\vec{x})).$$

- The **solution of the differential equations system** is:

$$\vec{f}(\vec{x},\epsilon) = \mathrm{P}\exp\left[\epsilon \int_\gamma \mathrm{d}\tilde{A}(\vec{x}')\right] \vec{f}(\vec{x}_0,\epsilon).$$

  Canonical MIs can be expanded in Taylor series around $\epsilon = 0$:

$$\vec{f}(\vec{x},\epsilon) = \sum_{k=0}^{\infty} \epsilon^k\, \vec{f}^{(k)}(\vec{x}) \rightarrow \vec{f}^{(k)}(\vec{x}) = \int_\gamma \mathrm{d}\tilde{A}(\vec{x}')\, \vec{f}^{(k-1)}(\vec{x}') + \vec{f}(\vec{x}_0,\epsilon)$$

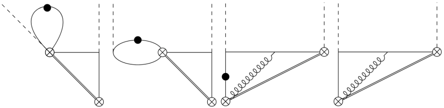
[1] Henn, 2013.

CANONICAL MATRIX FOR VIRTUAL CORRECTIONS

- Generic canonical master: $\boxed{f_i^{\text{virt}} = \epsilon^{n_i}\, B_i(m_t, m_h, z)\; T_i^{\text{virt}}}$.
- Semi-algorithmic approach:
  - ✓ T_i^{virt} found by **maximizing symmetries**,
  - ✓ $B_i(m_t, m_h, z)$ found by applying **Magnus transformations**[2].
- **Rationalizing** roots: $m_t^2 \to m_h^2/4\,(-\tau^2+1)$.
- Canonical matrix in **dlog-form**:

$$\begin{aligned} \mathrm{d}A_\tau = {} & M_1 \,\mathrm{dlog}\,(\tau) + M_2 \,\mathrm{dlog}\,(1-\tau) + M_3 \,\mathrm{dlog}\,(1+\tau) + M_4 \,\mathrm{dlog}\,(2 - z\,(1-\tau)) \\ & + M_5 \,\mathrm{dlog}\,(-2 + z\,(1+\tau)) + M_6 \,\mathrm{dlog}\left(-4 + z\left(3+\tau^2\right)\right). \end{aligned}$$

Examples of pre-canonical T_i^{virt}:



$I_{0,2,1,0,0,1,1}$ $\quad I_{0,2,1,1,0,1,1}$ $\quad I_{0,1,0,2,1,1,1}$ $\quad I_{0,1,0,1,1,1,1}$

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[2] Argeri *et al.*, 2014

SIMPLIFYING POLYLOG. EXPRESSIONS WITH SYMBOLS

Symbols[3] ($\mathcal{S}$) were used as a systematic way of simplifying the polylogarithms appearing in the MIs analytic expressions.

Definition

- $\mathcal{S}(\mathrm{Li}_n(x)) = -\ln(1-x)\otimes \underbrace{\ln(x)\otimes \ldots \otimes \ln(x)}_{\text{n-1 times}}$,
- $\mathcal{S}(\ln(x)\ln(y)) = \ln(x)\otimes\ln(y) + \ln(y)\otimes\ln(x)$,
- $\ldots\otimes\ln(x\cdot y)\otimes\ldots = (\ldots\otimes \ln(x)\otimes\ldots) + (\ldots\otimes\ln(y)\otimes\ldots)$,
- $\mathcal{S}(\pi^n) = 0$ with $n \geq 2$,
- Symbols are unique up to $\sim \pi^n$.

[3] Goncharov, 2009, Duhr, Gangl, Rhodes, 2012.

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- $\ldots \otimes \ln(x \cdot y) \otimes \ldots = (\ldots \otimes \ln(x) \otimes \ldots) + (\ldots \otimes \ln(y) \otimes \ldots),$
- $\mathcal{S}(\pi^n) = 0$ with $n \geq 2$,
- Symbols are unique up to $\sim \pi^n$.

Example

$$\mathcal{S}\left(-\mathrm{Li}_2(x) - \ln(1-x)\ln(x) + \frac{\pi^2}{6} \right)$$

$$= \ln(1-x) \otimes \ln(x) - (\ln(1-x) \otimes \ln(x) + \ln(x) \otimes \ln(1-x)) + 0$$

$$= -\ln(x) \otimes \ln(1-x)$$

$$= \mathcal{S}(\mathrm{Li}_2(1-x))$$

$$\downarrow$$

$$\boxed{\mathrm{Li}_2(1-x) = -\mathrm{Li}_2(x) - \ln(1-x)\ln(x) + A\pi^2}$$

$A = \frac{1}{6} \rightarrow$ Euler's reflection formula

[3] Goncharov, '09, Duhr, Gangl, Rhodes, '12.

COLLINEAR RENORMALIZATION

The bare $t \to h$ fragmentation function:

$$D_{h\to h} = \delta(1-z) + \mathcal{O}(y_t^2)$$

$$D^B_{t\to h}(z) = (Z_{th} \otimes D_{h\to h})(z) + (Z_{tt} \otimes D_{t\to h})(z) + \mathcal{O}(y_t^2\alpha_s^2, y_t^4).$$

The renormalization constants in terms of splitting functions are:

$$\begin{aligned}
Z_{th}(z) =& \frac{y_t^2}{16\pi^2}\frac{1}{\epsilon}P^{(0)}_{th}(z) \\
&+ \frac{y_t^2}{16\pi^2}\frac{\alpha_s}{2\pi}\left(\overbrace{\boxed{\frac{1}{2\epsilon}P^{(1)}_{th}(z)}}^{?} + \boxed{\frac{1}{2\epsilon^2}(P^{(0)}_{qq} \otimes P^{(0)}_{th})(z)} - \boxed{\frac{\beta^{(0)}_{th}}{4\epsilon^2}P^{(0)}_{th}(z)}\right) \\
&+ \mathcal{O}(y_t^2\alpha_s^2, y_t^4), \\
Z_{tt}(z) =& \delta(1-z) + \boxed{\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P^{(0)}_{qq}} + \mathcal{O}(\alpha_s^2, y_t^2).
\end{aligned}$$

$P^{(0)}_{qq}$, $P^{(0)}_{th}$ known $\to$ $P^{(1)}_{th}$ derived as a by-product of our computation.

Collinear renormalization

The bare $t \to h$ fragmentation function:

$$D_{h \to h} = \delta(1-z) + \mathcal{O}(y_t^2)$$

$$D^B_{t\to h}(z) = (Z_{th} \otimes D_{h\to h})(z) + (Z_{tt} \otimes D_{t\to h})(z) + \mathcal{O}(y_t^2\alpha_s^2, y_t^4).$$

The renormalization constants in terms of splitting functions are:

$$Z_{th}(z) = \frac{y_t^2}{16\pi^2}\frac{1}{\epsilon}P^{(0)}_{th}(z)$$

$$+ \frac{y_t^2}{16\pi^2}\frac{\alpha_s}{2\pi}\left(\boxed{\frac{1}{2\epsilon}P^{(1)}_{th}(z)} + \boxed{\frac{1}{2\epsilon^2}(P^{(0)}_{qq} \otimes P^{(0)}_{th})(z)} - \boxed{\frac{\beta^{(0)}_{th}}{4\epsilon^2}P^{(0)}_{th}(z)}\right)$$

$$+ \mathcal{O}(y_t^2\alpha_s^2, y_t^4),$$

$$Z_{tt}(z) = \delta(1-z) + \boxed{\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P^{(0)}_{qq}} + \mathcal{O}(\alpha_s^2, y_t^2).$$

$P^{(0)}_{qq}$, $P^{(0)}_{th}$ known $\to$ $P^{(1)}_{th}$ derived as a by-product of our computation.

$$P^{(1)}_{th}(z) = C_F\Big[-8z\,\mathrm{Li}_2(z) + z\,\ln^2(1-z) - \frac{1}{2}z\,\ln^2(z) + 3z\,\ln(1-z) - 4z\,\ln(z)\,\ln(1-z) + (-1 + \frac{1}{2}z)\,\ln(z) + (-\frac{13}{2} + 15z)\Big]$$